Topic Study Group 3

Mathematics Education for Gifted Students

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1. Aims of the TSG

The goal of TSG-3 was to promote research and practice in the field of mathematical ability, mathematical potential, and giftedness in different cultures and contexts. The topic study group involved educational researchers, research mathematicians, mathematics teachers, teacher educators, curriculum designers, doctoral students, and others in a forum for exchanging insights related to the research and practice in mathematics education. The main purpose was to contribute to the development of our understanding of the nature and nurture of high mathematical ability in individuals.

1.1. Submissions

We received 28 submissions from 16 countries: Austria, Canada, China, Germany, Israel, Japan, Peru, Romania, Russia, Singapore, Slovenia, South Africa, USA, Sweden, Thailand, and The Netherlands, thus reaching our goal of diverse cultural representation. Of the submissions, five were accepted as long-paper presentations, seventeen were accepted as short-oral presentations, and six as posters. Among these, 21 papers have been presented.

1.2. Sessions

Throughout the three days of the TSG sessions, participants' dialogue and networking were focused on identifying emerging research themes, potential interdisciplinary approaches, and future research opportunities. To create a diversity of approaches and interactions, chairing the sessions was distributed among the TSG-3 organizers. Thus, first day: Florence Singer and Joseph Li, second day: Viktor Freiman, Florence Singer, Joseph Li, and third day: Joseph Li, Viktor Freiman, Florence Singer alternatively chaired the respective sessions. During the third day, we succeeded to have an interesting ad-hoc interaction between the online community and the participants in situ, in Shanghai, thanks to the excellent translation provided by Joseph Li. This online — offline interaction has shown the large interest of our TSG topic and the desire to extend the discussions concerning important issues related to high achievement and giftedness in mathematics.

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1.3. Paper topics

As mentioned above, of the 28 accepted papers, 21 papers have been presented during the online conference. A list of these papers and authors, organized in Tab. 1 in the order of presentation, is next included.

Tab. 1. List of the presented papers

Papers and author(s)
[1] Student perceptions of support provided by a summer math camp. *Michael Hicks, **Hiroko Kawaguchi Warshauer**, and Max Warshauer* (USA).
[2] Derivation of regression equations predicting Japan mathematical olympiad preliminary qualifiers from within arbitrary groups. ***Atsushi Tamura*** (Japan).
[3] How do math students use informal representations? A comparison between gifted and not gifted. ***Florence Mihaela Singer**, Cristian Voica* (Romania).
[4] School stages of educating the mathematician-investigator. ***Aleksandr Vasilevich Yastrebov*** (Russia).
[5] Problem solving and creativity among talented students from a multi-age perspective. ***Odelya Uziel**, Miriam Amit* (Israel).
[6] Educating prospective teachers in the field of mathematical giftedness - comparing experiences. ***Matthias Simon Brandl**, Attila Szabo, Elisabeth Mellroth, and Ralf Benölken* (Germany).
[7] Questions about the identification of mathematically gifted students. ***Marianne Nolte*** (Germany).
[8] What do prospective teachers express as to mathematical giftedness? An exploratory study. *Daniela Assmus and **Ralf Benoelken*** (Germany)
[9] Mathematical thematic content and didactic skills for the teaching of mathematics of students of the primary education career of the Catholic University Sedes Sapientiae, Peru. *Norma Fuentes Supanta De Fukunaga and **Patricia Edith Guillen Aparicio*** (Peru).
[10] Role of peer and teacher recognition for students' talents in STEM projects. ***Viktor Freiman** and Jacques Kamba* (Canada).
[11] Egalitarianism in inclusivity: thwarting the intellectual growth of mathematically gifted students in South African schools. ***Michael Kainose Mhlolo*** (South Africa).
[12] Activities for the mathematically gifted and their evaluation in Slovenia. ***Bostjan Kuzman**, Mojca Juriševič, and Urška Žerak* (Slovenia).
[13] Using interdisciplinary problem posing to promote gifted students in the regular classroom. ***Sara Hinterplattner**, Zsolt Lavicza, and Marca Wolfensberger* (Austria).
[14] Mathematically gifted students: challenges and opportunities in the primary years. ***Ban Har Yeap*** (Singapore).
[15] Discovering and educating the gifted students with excellent problems. ***Xiangrui Chan*** (China).
[16] Mathematical culture and teaching of equation. ***Yanchun Liu**, Lili Gao, and Peng Zhao* (China).
[17] Study of construction by quadratic curve addition method. ***Hideyo Makishita*** (Japan).
[18] Intuitive sense constructions of children with mathematical giftedness. ***Alena Witte**, Franziska Strübbe* (Germany).
[19] LEMAS — a joint initiative of Germany's Federal Government and Germany's Federal States to foster high-achieving and potentially gifted pupils. *Friedhelm Käpnick, Philipp Guillaume Girard, Julia Kaiser, Yannick Ohmann, **Lea Martina Schreiber**, and Wiebke Auhagen* (Germany).
[20] University students self-evaluation: digital solutions for identifying highly motivated students. ***Mirela Vinerean Bernhoff**, Yvonne Liljekvist, and Elisabet Mellroth* (Sweden).
[21] Experimental study on intellectual development in elementary school students. ***Yuwen Li*** (China).

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2. Conference Themes

The congress papers can be summarized across five main themes:

- analysis of high-achieving students' skills and perceptions,
- identification and prediction of mathematical giftedness,
- informal training programs for the gifted and talented students in mathematics,
- adequate training for teachers and prospective teachers dealing with mathematically gifted students, and
- educational policies related to mathematical giftedness in different countries.

Concerning the *analysis of high-achieving students' skills and perceptions*, Uziel and Amit[5] explored the problem-solving capacities of 118 talented students in grades 5–12 who took part in the enrichment program known as "Kidumatica", examining the students' solutions from a multi-age perspective. Data was gathered from students' products and teacher observations during a series of workshops devoted to 10 non-routine problems with multiple solution paths. Their findings revealed a troubling phenomenon: as the age of students rises, they are less prone to looking for creative and holistic solutions when solving problems, and more likely to be "held hostage" by their habitual use of algebra. Somehow strengthening this conclusion, Witte and Strübbe[18] noticed that very young mathematically gifted children show a strong fascination for mathematical questions, and they develop intuitive conceptual constructions regarding various mathematical relationships. For older students, in a qualitative case study, Hicks, Warshauer and Warshauer[1] examined the perceived support as described by three female African American students enrolled in a summer camp for high achievers with an interest in STEM. The students' perceptions on the given support for developing their mathematical competence and sense of belonging to a community of mathematics learners highlighted that the key solution for progress in learning is adequately addressing individual needs at the right time. From another perspective, Singer and Voica[3] investigated students' representations of abstract mathematical concepts beyond reproducing definitions and theorems. They exposed 51 undergraduate university students to tasks that required the association of mathematical concepts as limits or convergence to as many as possible images that potentially generate suggestive mental representations in school students. The researchers found that high-performing university students have focused on mathematical properties of concepts, for which they found meaningful images that revealed deep mathematical meanings, compared to low performers who only pointed some surface characteristics of the mathematical concepts involved.

Identification and prediction of mathematical giftedness was a topic of high interest in the TSG-3 community, as the discussions have shown. Thus, in her review article, Nolte[7] gave an overview of the questions on diagnostics and procedures of high mathematical talent. Various methods such as intelligence tests, school achievement tests and checklists were presented and discussed. The conclusions

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favored multidimensional approaches with a focus on specially designed mathematical tests. Identification of the gifted students is important when we talk about Mathematical Olympiads. Thus, Tamura[2] developed a model based on derivation of regression equations that could predict Japan Mathematical Olympiad preliminary qualifiers from within arbitrary groups. Within the presentation, he explained how his mathematical talent checklist was used to identify preliminary qualifiers among a group of average high school students and Mathematical Olympiad preliminary qualifiers at a true discriminant ratio of over 93% and how this tool was refined through a logistic based on regression analysis to improve the result. As he pointed out, an analysis of the characteristics found in the sample of students with advanced mathematical abilities are indispensable in the development of “math for excellence” educational materials. Beyond complex mathematical tools for the identification of talented students, Bernhoff, Liljekvist, and Mellroth[20] found that it is possible to identify highly motivated individuals among engineering students by exposing them to the use of Learning Management System (LMS) to self-evaluate their work on recommended tasks, which then provided the lecturer with some statistical data. The research is just at the beginning, we wait for the next steps of it.

Some of the TSG-3 papers were focused on the presentation of *informal training programs for the gifted and talented students* in mathematics, giving the audience some concrete hints and procedures. Thus, Li[21] described a training organized for students who participated on a voluntary basis from first to fifth grade at Changhe Elementary School in Dezhou, China, and compared their mathematics achievement with students who did not participate in the training. Through comparison studies on all participated students and prospective longitudinal studies on randomly selected students from both the experimental group and control group, he found that students who participated in the training had significantly better mathematical achievement and abilities in geometry, logic, and innovation than their comparable peers. Some relevant tasks for talented students could come also from history and traditions. Makishita[17] refered to a traditional Japanese mathematics from the Edo period described in Wasan books, showing that people learned mathematics for fun to solve quizzes, puzzles, and other entertainment problems, as well as for monetary exchange, and other everyday work activities. The author has used Wasan contents and applications in modern mathematics education, recording success with various types of mathematics classes. Diverse and substantially rich examples of working with mathematically gifted students exist in many countries. Kuzman, Juriševič, and Žerak[12] from Slovenia presented activities such as mathcamps, research projects, competitions provided by different parties (schoolteachers, educational institutions, expert groups, learning societies), concluding that often the success of these activities largely depended on the enthusiasm and competencies of the involved individuals. These activities were evaluated within the PROGA project (2017–2020), and a support system for high school students with special talents in mathematics was established. While in many countries, alternative programs are searched to improve students’ mathematical abilities at higher levels, Yastrebov[4] made a three-folded plea: 1) Shaping of skills

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and habits of the future mathematician can be started at the early stages of his/her education and upbringing within school; 2) Educating the mathematician-investigator at the early stages of education contributes to the implementation of general goals of school education, regardless of the student's future profession; and 3) Experience of the pedagogical society is sufficient for shaping the uniform, integral system of educating the mathematician-investigator at the school level.

The issue of *adequate training for teachers and prospective teachers dealing with mathematically gifted students* was also one of high interest for the TSG-3 community. Brandl, Szabo, Mellroth and Benölken[6] focused on how prospective teachers can be trained in the field of mathematical giftedness. By comparing three independently developed concepts to deduce cornerstones of appropriate seminar concepts, they concluded that a combination of theoretical and practical parts is particularly important for a sustainable education in the context of giftedness. As Freiman and Kamba[10] have stressed, over the past decades, novel integrated STEM learning spaces have emerged in K-12 schools providing students with new interdisciplinary enrichment opportunities to express their gifts and talents. Based on the study of provincial makerspaces and maker projects they have conducted since 2016, they identified a clear trend related to an increasing role of an expertise of some students who became recognized by their peers and was essential for the collective success of the projects they developed. Therefore, enhancing the monitoring role of high achieving students among their peers can promote better learning in the whole class. Within a study investigating prospective teachers' perceptions regarding mathematical giftedness, Assmus and Benölken[8] found that future teachers' ideas and knowledge are rather shadowy compared to the state of research on the modeling of mathematical giftedness. Yeap[14] has proposed a model of professional development of teachers comprising three phases. In the first phase, teachers learn by experiencing the doing of challenging mathematics. In the second phase, they observe students engaging in mathematically challenging tasks. In the third phase, they teach lessons that include such tasks. Following this strategy, three categories of opportunities emerged: for the mathematically students the teachers teach, for the other students who are also taught by these teachers, and the final category includes opportunities for the teachers, their colleagues, and the school culture. De Fukunaga and Aparicio[9] analyzed the mathematical thematic content and the didactic skills of prospective primary education teachers from Universidad Católica Sedes Sapientia, Lima-Peru. Their results show that student-teachers require training in mathematical content and personalized pedagogical help, as well as the necessary didactic material for their children to achieve meaningful learning and develop their mathematical potential applied to daily life and society. Still, mathematical culture is the treasure of human culture, and its contents, ideas, methods, and language are important components of modern civilization; following this assumption, Liu et al.[16] insisted on the fact that teachers in the new era should think deeply about how to integrate mathematics culture into the teaching practice, so that students can be influenced by the mathematics culture in the process of learning. They provided an example of introducing mathematical

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culture into equation teaching for students in different grades. They explored ways to maximize the charm of cultural acquisitions to stimulate students' interest in learning, and their passion for mathematics and mathematical culture.

The topic of *educational policies related to mathematical giftedness* in different countries inevitably goes into the eternal debate between egalitarianism and the promotion of giftedness — how this two can be approached and complemented. As Mhlolo[11] has stressed, since 1994 the focus moved from separate and specialized education for gifted learners to inclusive education with all learners being educated in regular classrooms. Although inclusive education policy initiatives in theory aimed at ensuring quality education for all, current empirical evidence shows that in many African countries including South Africa, excellence and egalitarianism have become out of balance as gifted students from previously disadvantaged communities do not reach their full potential in regular classrooms. Similarly, Hinterplattner et al.[13] noticed that meeting the needs of all students in an inclusive classroom is a rather challenging task. Gifted students often need more or different tasks and activities than teachers in regular classes can or are willing to offer. In such situations, it may happen that gifted students face boredom, which may lead to various behaviors driven by unsatisfaction. To prevent boredom and misbehavior, an experiment with 10 gifted secondary school students was carried out based on findings from neuro-didactics, to ensure a good learning experience. The students were taken out from one of their three regular mathematics classes per week for 9 weeks and challenged with an interdisciplinary problem that was based on STEAM ideas. To solve this problem, a combination of problem-posing abilities, the capacity of gathering knowledge from various fields, and their applied interdisciplinary ideas were necessary. After this experiment, students were asked about their experiences concerning the project itself, and its impact on their regular classes. Results show that the project was described as quite challenging and motivating. Also, its impacts on their regular classes were described as highly positive. Students reported that they used the time they were usually waiting in the regular classrooms for solving the problems they found in their project. Educational policies addressing strategies to foster high-achieving and potentially gifted students might not be only developed from a research perspective, but also from a systemic one. Käpnick et al.[19] presented a joint initiative of Germany's federal government and Germany's federal states for an interdisciplinary network of scientists from 16 universities, together with 300 schools, under the name of "LEMAS" to develop guiding principles and adaptive concepts to support gifted and talented students. This long-term program has just started, and the entire community is expecting relevant results.

3. Future Directions for Research and Practice

The participants agreed that they will continue the international exchange of ideas related to research on the identification of mathematical talent, didactics of teaching highly able students, as well as the promotion of mathematical challenge and

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enrichment for all. The focal topics will continue to include empirical, theoretical, and methodological issues related to students' excellency in mathematics. Discussions and research within community will aim at better understanding of: useful tools for identifying and assessing mathematically gifted students; the educational approaches and organizational settings more effective for training gifted individual students or groups of students of various ages; the nature of mathematical tasks and activities that are challenging, free of routine, inquiry-based, and rich in authentic mathematical problem solving and posing; the relationship between exceptional mathematical abilities, motivation and mathematical creativity; the relationship between mathematics education for the gifted and equity of education for all students; teacher education aimed at mathematics teaching that encourages and promotes mathematical talents, and the development of interdisciplinary programs (STEAM included), for gifted students. New areas of research will be opened towards the relationship between mathematics education for gifted students and the talent development in the areas that are important in the future, such as artificial intelligence, genetic technology, computational thinking, big data, cryptocurrencies, etc. We hope that the network of professionals in the field will continue to increase, for the benefit of the students around the world.

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