Topic Study Group 33

Knowledge in/for Teaching Mathematics at Secondary Level

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ABSTRACT The chapter summarizes the results of TSG-33: "Knowledge in/for teaching mathematics at secondary level". We provide all titles of all the scientific contributions in the program overview and address suggestions for future directions of research in this field in the report about the discussions.

Keywords: Mathematics teacher knowledge; Knowledge for teaching mathematics; Secondary education.

1. Themes and Description

The TSG-33 assembled international mathematics educational researchers on the topic of "Knowledge in/for teaching mathematics at secondary level". Since ICME-13 in Hamburg 2016 (Even et al., 2017), research, theory, and practice in this research topic have evolved and, in particular, questions about the relationship between teachers' knowledge and the practice of mathematics teaching at secondary level have been taken up by many researchers around the world. The goal of TSG-33 was to focus on a number of critical issues in research on knowledge in/for teaching mathematics at secondary level and to foster international discussion about the findings and challenges researchers, mathematicians, teacher educators, teachers, and policy makers face in addressing issues in this area of research. In particular, the role of teachers' knowledge in practice and practical implications for teacher training and professional development also played a stronger role. The discussions in TSG-33 focused on the following issues and respective key questions driving current research in this research field:

(1) Conceptualization of knowledge in/for teaching mathematics at secondary level

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- What kind of knowledge in/for teaching mathematics should be considered to become a proficient/effective mathematics teacher at secondary level?
- What core characteristics, basic abilities, attitudes, and beliefs are in play? (Are there some normative orientations?)
- What aspects are considered in the various existing theoretical frameworks?

(2) Measurement of knowledge in/for teaching mathematics at secondary level
- What aspects are measured in the study of knowledge in/for teaching mathematics at secondary level?
- How is knowledge in/for teaching mathematics at secondary level measured?
- Are these measurements/instruments appropriate for different contexts?

(3) Relationships between knowledge in/for teaching mathematics at secondary level and teaching practice in mathematics, including instructional quality in mathematics teaching, and student achievement
- What distinguishes (theoretical) knowledge in/for teaching mathematics at the secondary level from teaching practice (enacted knowledge)?
- What are the relationships between teachers' affect and their knowledge in/for teaching mathematics at the secondary level?
- What kind of situational knowledge and skills are needed or observable in practice and how does teachers' knowledge influence the quality of mathematical instruction and relate to students' mathematical achievement?

(4) Practical implications for teacher education and professional development and validation of research findings on knowledge in/for teaching mathematics at secondary level
- What are appropriate measures in teacher education and professional development to develop knowledge in/for teaching mathematics at secondary level?
- What kind of knowledge is relevant for teaching practice and where and how do teachers learn this knowledge?
- How can studies on teachers' knowledge be used to improve the quality of teacher education and professional development?

2. Program Overview

As one of the larger Topic Study Groups at ICME-14, TSG-33 was able to benefit from the possibility of a fourth program session. Due to the ongoing COVID-19 pandemic, the program was offered in a hybrid format (i.e., presentations were given both remotely and on-site). The program provided an opportunity for participants to discuss the questions stated in Section 1 in depth. However, as many researchers participated remotely, the time difference presented challenges for the scientific discussion with researchers working in different parts of the world. The names of the presenters in TSG-33 are listed in Tab. 1, along with the country of their affiliated institution and the title of their presentation.

Tab. 1. List of papers presented

Paper and author(s)
[1] Critical remarks on the notion of unpacking mathematics in discourses of teacher knowledge. ***Thorsten Scheiner*** (Australia).
[2] What subject matter knowledge do Chinese in-service junior middle school teachers lack? ***Dandan Sun*** (China).
[3] Assessing the relationship between teachers' knowledge and classroom practices in the use of ICT in the secondary mathematics classroom. ***Mailizar Mailizar*** (Indonesia).
[4] Number sense of teachers in different school levels. ***Rahmah Johar****, Munirah Ghazali, Mailizar, and Suci Maulina* (Malaysia).
[5] Arts integrated pedagogy for meaningful mathematics teaching and learning. ***Binod Prasad Pant****, Bal ChandraLuitel, and Indra Mani Shrestha* (Nepal).
[6] Uncovering mathematics teaching knowledge of out-of-field mathematics teachers. ***Achmad Nizar***[6]*, Merrilyn Goos, Miamh O'Meara, and Ciara Lane* (Ireland).
[7] A study of Sri Lanka's pre-service mathematics teachers' pedagogical content knowledge. ***G.M. Wadanambi*** (Sri Lanka) *and Frederick K. S. Leung* (Hong Kong SAR, China).
[8] Interweaving mathematics-news-snapshots in class: implications for teachers' horizon content knowledge. ***Ruti Segal****, Atara Shriki, Boaz Silverman, and Nitsa Movshovitz-Hadar Oranim* (Israel).
[9] Comparing German and Slovak teachers' knowledge of content and students related to functions. ***Veronika Hubeňáková****, Ute Sproesser, and Ingrid Semanišinová* (Slovakia).
[10] A focus on the specificities of teachers' knowledge for improving teacher education: the case of the MTSK conceptualization. ***Miguel Ribeiro****, Marlova Caldatto, and Milena Policastro* (Brazil).
[11] The influence of teaching experience on mathematical teacher content knowledge at middle school level. ***María D. Cruz Quiñones****, Mourat Tchoshanov, Héctor Jesús Portillo Lara, Carlos Paez, and Rocio Gallardo* (Mexico).
[12] Implementation of eight teaching practices for teaching problem solving. ***Sarah Sparks****, Alees Lee, Katie Morrison, and Gulden Karakok* (USA).
[13] A preservice secondary mathematics teacher's specialized knowledge: the case of limit. ***Rüya Savuran*** *and Mine Işıksal-Bostan* (Turkey).
[14] What do teachers learn about what mathematics is in academic mathematics courses? ***Anna Hoffman*** *and Ruhama Even* (Israel).
[15] Mathematical quality of geometry instruction of a novice high school teacher in terms of richness of mathematics. ***Fetma Aslan-Tutak*** *and Buket Semercioglu Kapcak* (Turkey)
[16] Investigation of preservice mathematics teachers' translations among multiple representations. ***Zeynep Pehlivan*** *and Fetma Aslan-Tutak Achmad* (Turkey).
[17] Preservice secondary school teacher's errors when translating between representations. ***Florence Thomo Mamba*** (Malawi).
[18] Connecting knowledge for teaching geometry at the secondary level with instructional quality in mathematics teaching. ***Agida Manizade*** (USA) *and Dragana Martinovic* (Canada).
[19] Upgrading learning for teachers in real analysis (ULTRA): an instructional model for secondary teacher education. ***Nicholas H. Wasserman****, Keith Weber, Juan Publo, Mejia-Romos, Timothy Fukawa-Connelly* (USA).
[20] Applications of teaching secondary mathematics in undergraduate mathematics courses. ***Elizabeth G. Arnold****, Elizabeth A. Burroughs, Elizabeth W. Folton, James A. Mendoza* (USA).
[21] Mathematics teachers' perceptions of teaching competencies: a study of grades 5 through 8. ***Heather Bleecker*** *and Polly Dupuis* (USA)
[22] Identifying mathematical learning opportunities in a task as a missing, essential skill of teaching. ***Michelle King****, Jodie D. Novak, Robert A. Powers, Alees T. Lee, Adam Ruff, and Shweta Naik* (USA).
[23] The validation of an assessment instrument for measuring mathematical knowledge for teaching (MKT). ***Mihyun Jeon*** (USA). (Poster)
[24] The specialized knowledge of a new generation of mathematics teachers under STEM training. ***Jenny Patricia Acevedo-Rincon*** (Colombia). (Poster)
[25] A case study on MPCK of junior middle school mathematics teachers with different characteristics. ***Ruifang Zhao*** (China). (Poster)

[6]Achmad Nizar unfortunately passed away a few days before the conference, but we would like to highlight his contribution to TSG-33 nevertheless.

The presentations were accompanied by introductory remarks and discussion prompts from the team members.

3. Results of the Discussion, Future Directions and Suggestions

The discussions of the presentations led to various observations that will be documented here as a result of the conference. Of course, such summaries can only reflect a subjective impression of the organizing team. Compared to the presentations of the corresponding Topic Study Groups at ICME-12 in Seoul and ICME-13 in Hamburg, an increase of studies on the differences between mathematical knowledge at the university level and secondary school level could be observed. Considerations of the nature and kind of subject matter knowledge for teaching have evolved in various directions, including directions that diverge from Shulman's original notion of pedagogical content knowledge. Moreover, with the increasing recognition of the situated nature of teacher knowledge (Even et al., 2017), it is not surprising that the role of teachers' school mathematics knowledge is again becoming the focus of several research contributions.

The greater importance of professional practice for the development of teacher knowledge in teacher education and professional development is also accompanied by a stronger focus in research. The need to conduct studies of the development of teachers' knowledge over the course of a teacher's career (especially the need for longitudinal studies of high scientific standard) was again evident.

There have also been developments in research at the level of conceptualizing theoretical frameworks for teacher knowledge, partly due to the influence of the significance of the unfolding of teacher knowledge in situated (and culturally shaped) teaching practice[7]. On the basis of the still prevalent work of Shulman, various dimensions of teacher knowledge were distinguished and delineated, depending on certain content-related aspects or aspects of teaching practice. New conceptualizations are partly more fine-grained or take into account specific cultural conditions, such as the teaching knowledge of indigenous populations or national traditions. Overall, a more critical use and enrichment of common conceptual frameworks was observed. The frameworks discussed were not mainly analytical, but reflected both normative and descriptive approaches. However, for research on situated teacher knowledge, the more context-oriented knowledge is analyzed in teacher practice, the more difficult it becomes to empirically distinguish the knowledge from other factors like teachers' personality or affect. As the field seeks to better account for the ways in which teacher knowledge and its frameworks are culturally constituted, the question of how findings of studying teacher knowledge can be generalized to other cultural contexts becomes even more significant. In this context, there is a great need for further development of

[7] For further research in this field, it is worth looking at the developments in research on teacher knowledge in other disciplines (i.e., Science education), where consensual models of practice-relevant teacher knowledge have emerged in recent years (Hume et al., 2020).

research methods (both quantitative and qualitative) to study situated and contextual teacher knowledge that meet high scientific quality criteria.

References

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