Topic Study Group 44

Mathematics and Interdisciplinary Education

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1. Aims of the TSG

The preceding TSG on "Interdisciplinary Mathematics Education" at ICME-13 in Hamburg produced, among other things, a conceptual classification of the general notion of "discipline" as well as the terms used to indicate various degrees of interaction between mathematics and other disciplines (Williams et al., 2016).

The present TSG, as the slight change in title suggests, took a more specific point of departure in mathematics in its current educational and societal shapes, viewed as social realities: namely, mathematics as taught from preschool to higher education, and mathematics as a more widely established set of social practices (such as in academic research) viewed broadly to include also statistics and what is sometimes referred to as "applied mathematics". The main goal of the group was to share studies and in-depth cases of the ways in which mathematics currently interacts — or is supposed to interact with other educational practices, in part reflecting the role of mathematical practices in society at large. We focused in particular on current or potential contributions of mathematical theory and techniques to elucidate the "big" questions which are increasingly emphasized in general schooling in many countries, such as sustainable development, and the roles and nature of digital technologies in modern society. This concerns how school mathematics functions as preparation for general citizenship, and also for more specific professional specializations. In other words, we focused on ways in which mathematics is or could be taught in a "paradigm of questioning the world", rather than in a "paradigm of visiting monuments" (Chevallard, 2015).

2. Submissions and Sessions

We received 26 submissions from 19 countries (Europe: 9; North America: 7; Asia: 6; Australia 3; Africa: 1). Out of these 26 submissions, 7 were accepted as long oral presentations, 16 as short oral presentations, 2 as posters, and 1 was rejected. These numbers reflect the total number of papers and posters, submitted mainly at the ordinary deadline in 2020, but with two being added at the extraordinary deadline in 2021.

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The sessions were initially (for 2020) planned with parallel session for short orals. Unfortunately, at the online conference in 2021, not all accepted contributions could be presented due to the authors' choice of not attending the congress. In the actual congress, we ended up with having 6 long oral presentations (LO) and 6 short oral presentations (SO), see Tab. 1, all followed by an opportunity to raise questions and comments in a discussion with the presenter(s). The three sessions were attended by about 25 online participants and 2–5 onsite participants. Posters were presented at the general poster presentation of the congress.

Tab. 1. List of papers presented

Paper and author(s)
Session 1
[1] Interdisciplinary mathematics education: some reflections from the anthropological theory of the didactic. ***Francisco Javier García*** (Spain). (LO)
[2] Interdisciplinary Inquiry-Based learning with queueing situations: investigating the questions triggering mathematical activities. ***Yuici Nezu*** *and Takeshi Miyakawa* (Japan). (LO)
[3] A classroom experience: vector concept. ***Viana Nallely García-Salmerón*** *and Flor Monserrat Rodríguez Vásquez* (Mexico). (SO)
[4] Students' use of geometric cues in an art studio: scaling of artworks. ***Mehtap Kus*** *and Erdinc Cakiroglu* (Turkey). (SO)
Session 2
[5] Posing a generating question within the pedagogy of questioning the world: the case of GPS coordinates. *Lenin Augusto Cepeda,* ***Avenilde Romo Vázquez****, and Luis Ramón González* (Mexico). (LO)
[6] Mathematics and financial education: how do they intersect together? ***Annie Savard*** *and Alexandre Cavalcante* (Canada). (LO)
[7] Task design features for integrating covariational reasoning with science. ***Debasmita Basu*** *and Nicole Panorku* (U.S.A.). (SO)
[8] STEM projects as didactical situations in mathematics: theoretical frame to construct algebraic institutional meanings. ***Aitzol Lasa****, Miguel Wilhelmi, Olga Belletich, Jaione Abaurrea, and Haritz Iribas* (Spain). (SO)
[9] Physcial measurements as an environment supporting primary pupils' reasoning about central tendency. ***Lúbomíra Valovičová*** *and Janka Medová* (Slovakia). (SO)
Session 3
[10] Questioning interdisciplinarity within teacher education: a module on the evolution of the COVID-19 pandemic. *Laura Branchetti and* ***Eleonora Barelli*** (Italy), *Berta Barquero, and Oscar Romero* (Spain). (LO)
[11] A situation of interdisciplinary mathematics education in context of protection of water resources. ***Thi Nga Nguyen****, Thien Thanh Lam, and Minh Dung Tang* (Vietnam). (LO)
[12] Transdisciplinary and interdisciplinary mathematics in the international baccalaureate. ***Sarah Christina Phillips*** (Canada) *and Jan Mills* (New Zealand). (SO)

3. Paper Topics

Following the call for papers, the papers were classified as far as possible according to the three subthemes of the TSG:

- Subtheme 1: *Mathematics and the study of nature*: here we consider the ways in which mathematics interacts with teaching and learning of subjects such as physics, biology, chemistry etc.
- Subtheme 2: *Mathematics and technology*: interactions with the study and use

of technology in a broad sense, comprising digital technologies, technological innovation and engineering
- Subtheme 3: *Mathematics and the study of human activity and society*, including business and enterprises, economy, creative fields such as art and music, philosophy, history etc.

The specific contents of the presented papers allow to identify several new tendencies and approaches, as is to some extent reflected from the paper titles listed in Tab. 1. The papers certainly included examples and results corresponding to all three subthemes, reflecting a variety of experimental and emergent practices and concerns in mathematics teaching around the world. At the same time, the challenge of mathematics and other disciplines is considered in wider perspectives than has been traditionally the case in this area of research. In particular, 4 of 12 papers refer to the "paradigm of questioning the world" (Chevallard, 2015) and base their experiments and reflections on associated constructs from the Anthropological Theory of the Didactic, such as "study and research paths" and "ecology and economy" (of teaching designs). The range and originality of interdisciplinary phenomena examined by the papers is impressive, and relates to all grade levels including higher education. It is also clear that despite the variation in institutional constraints across the world, the same phenomena (from water resources to satellite based navigation and art design) are increasingly of importance to all of humanity. Several of the studies involve real life data and activities involving data collection, so that the numerical or geometrical aspect of a phenomenon is not merely shown through a table or drawing in a textbook. As always, researchers and special resources are deployed in such experiments, and the question of sustainability arises. In this respect, some of the papers reflect on the fact that teachers face new requirements and challenges if they wish or need to organize interdisciplinary teaching.

4. Discussion and Areas for Future Research

The discussions at the congress allowed participants to get more insight into research methods and teaching practices from around the world. In all countries, interactions between mathematics and other school disciplines seems to be officially encouraged or even prescribed, at least to some degree. However, in some countries, this impetus has come more recently. Both in such countries, but also in other ones, there are still a number of constraints that can inhibit or weaken students' experiences of such interactions. Teachers' educational background is frequently not geared towards independent inquiry across disciplines, or even towards recognizing mathematical components in questions that involve other disciplines as well. Teachers may also be hesitant to orient their teaching towards such inquiry, for many other reasons — including perceived lack of time. Teaching resources for mathematics are, when it comes to relations with other disciplines, often limited to rather sterile examples made to "illustrate" some mathematical technique. On the other hand, experiments as carried out by the authors of several of the papers, do confirm that these constraints can be modified locally. With appropriate support, teachers can be successful with much more ambitious designs for students' experience of the many ways in which mathematical techniques and models appear in other disciplines and, more generally, in descriptions and solutions of important problems for humans and their societies.

There was a strong agreement among participants in the group that teacher education needs to focus more on interdisciplinary learning and teaching of mathematics in school. At present, few if any teacher education programs are geared for interdisciplinary teaching. One reason for this is that teachers need to study more than one discipline to gain some depth in view of the interdisciplinary setting. The study of several disciplines for being a teacher at secondary school is not included in the teacher education programs of most countries, with some exceptions like Germany and Denmark, where two subjects must be studied. But even teachers who studied two disciplines may not teach more interdisciplinary subjects, it does make it easier to combine the two subjects studied, such as mathematics and geography, in the lessons. Doing a project with colleagues from other disciplines makes sense for the learners, but is difficult to implement as a common element in everyday school life. Whether two disciplines are studied or a project is implemented with colleagues, teachers should be made aware of the importance of this interdisciplinarity. Learners benefit especially if they also experience the interrelationships of, for example, the STEAM disciplines on a metacognitive level. This could mean, on the one hand, that teachers have designed their learning unit in terms of the cross-link approach (Borromeo Ferri et al., 2019). One can speak of cross-linking, if at least two (scientific) disciplines are combined during one lesson/project or within the whole lesson-unit/project and are reflected with learners on a metacognitive level (Borromeo Ferri et al., 2019).

It becomes obvious that besides the challenge of interdisciplinarity, there is always the question of which methods can be used in the classroom. Inquiry-based learning, for example, touched upon in several papers presented in this group, is often a promising approach. Reflectively, however, the question should also be asked whether all disciplines are suitable for interdisciplinary work? Which ones fit together particularly well? These questions are useful with regard to the development of interdisciplinary learning environments for specific issues. However, it is important to note that it is through interdisciplinary work that different perspectives should be required of learners, to develop their critical engagement with a topic. Critical engagement or thinking is again one of the central four skills described in the 21st Century Skills, the 4 Cs: Creativity, Critical Thinking, Communication, and Collaboration (e.g., Rotherham et al., 2010). However, creativity, communication, and collaboration can also be promoted and fostered when it comes to issues that learners can solve in an interdisciplinary learning environment.

Furthermore, there is still a research desideratum for more studies that also explore the effects of interdisciplinary learning, for example regarding motivation, or performance in mathematics. If different effects of interdisciplinary learning were investigated more systematically, then one would have a good basis for impact on educational policy including the necessary changes in teacher training.

The topic of interdisciplinarity is becoming more and more prominent in mathematics education, especially for the reasons mentioned above, such as the promotion of 21st Century Skills for the present and future generations. During ICME 14, in the TSG on mathematical modeling, the topic of interdisciplinarity was alo discussed in a separate session in a panel. Real-world problems, which do not necessarily have to be very complex, require solutions from different disciplines. The focus on interdisciplinarity in mathematics learning is interesting for many research areas in mathematics education. The clarification of basic concepts, such as mono-, multi-, inter-, trans- and meta-disciplinarity, has already been well elaborated by Williams et al. (2016). Based on these terms, each research area that focuses on the topic of interdisciplinarity could now work out specifics regarding the topic area in terms of research questions and finally also regarding the practical implementation.

The dialogue between the papers and their authors should continue. The shared character of both constraints and potential resources makes it evident that more, and more international, research is needed in this area — research that goes beyond the local and national contexts considered in the vast majority of studies, here and elsewhere. We also note that only a few of the papers presented here involve, for instance, physics education researchers among the authors. As already mentioned, collaboration between teachers from different disciplines is most evidently needed at schools in order to realize truly interdisciplinary experiences for students. So perhaps more collaboration of researchers from different didactical horizons — not only from mathematics education — is equally needed to engage more efficiently with the issues at the international level?

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