Topic Study Group 57

Diversity of Theories in Mathematics Education

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ABSTRACT This report presents an overview about the themes of the Topic Study Group 57 on the diversity of theories in mathematics education. Main topics, which were addressed, are the networking of theories in theories related to the use of technology, to design research and beyond. The program, format, contributions, discussions and the main results as well as some future implications are presented.

Keywords: Theory; Networking of theories; Axiology; Epistemology; Ontology; Methodology; Ethics.

1. Themes

1.1. General overview of the topic

Mathematics education is a scientific field with many theory cultures. This diversity can be regarded as richness but it also challenges research as well as communication and cooperation in the field. This is specifically the case when different theories are to be included into research. How the scientific community can cope with this diversity with scientific integrity remains an open question, specifically when research results from different theory cultures are used. Researchers working with the networking of theories (Bikner-Ahsbahs and Prediger, 2014) have started to investigate this question by conducting concrete research. The TSG-57 builds on previous lines of thought on the diversity of theories addressed in various conferences (e.g., ICME-12, ICME-13, CERME 6, 7, 8, 9, 10, 11) and wants to continue this discussion (e.g., Kidron et al., 2018). It aims at exploring how the diversity of theories can be used in mathematics education, how this may influence research theoretically, methodologically and

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epistemologically and how the diversity of theories may impact on the use of theories and research results in school practice. The TSG-57 wanted to collect concrete research examples addressing diversity of theories in order to obtain typical 'argumentative grammars' (structure of argumentation to substantiate evidence) for qualitative and theoretical research, research on technology-based teaching and learning, design research, research addressing different educational levels and networking of theories. The TSG welcomed also further ideas going beyond the subthemes briefly described below.

1.2. Subthemes

The TSG-57 called for contributing to four subthemes where the notion of "theories" not only means grand theories but also theory elements or theoretical models of a restricted scope addressing specific perspectives or phenomena.

Subtheme 1 addresses the diversity of theories in the digital era — using technology and other resources in teaching and learning: Technology use often requires theorizing tools, instruments, hence semiotic resources in connection with theorizing teaching and learning mathematics. That is, diversity of theories is an issue with respect to the use of technology and other resources. Some case studies of networking theories are also discussed.

Subtheme 2 addresses the diversity of theories for design research: Steps in design research often require different kinds of theories, for example normative theories for justifying aims, descriptive or explanatory theories for conducting design experiments and prescriptive theories for deciding about means for the design of instruction, hence, diversity of theories is relevant to consider.

Subtheme 3 addresses the diversity of theories at different educational levels including teacher education. Different educational levels (e.g., pre-school vs. university and teacher education) may require the use of various theoretical perspectives to capture the complexity and nature of its teaching and learning, for instance in the classrooms or for professional development.

Subtheme 4 addresses the networking of theories, which may investigate the relationship and function of theory elements in concrete research cases focusing on specificities of theories and their usages to gain insight on theory cultures in the field and meta-theoretical knowledge about how various theories can be related in research.

2. Program Overview

The TSG-57 had 14 submissions consisting of nine papers, two posters and three invited presentations (Tab. 1 on the next page). All submissions were accepted. Finally, the sessions had six long presentations and three invited ones of 20 min each, two short presentations of 10 min each and two poster presentations of 10 min each. We arranged the program according to the topics of the contributions rather than according to the subthemes.

Each of the three sessions started with an introduction bridging previous work within and beyond the TSG-57 with the presentations in the sessions and the aims and scope for the discussions. In each session, an invited talk addressed the subtheme of the day informing the final discussion.

Tab. 1. The list of papers presented

Paper and author(s)
Session 1: Why do we need a diversity of theories?
[1] Facing the challenge of theoretical diversity: the digital case. ***Michèle Artigue*** (France).
[2] Role of feedback when learning with an artefact. ***Angelika Bikner-Ahsbahs****, Estela Vallejo-Vargas, and Steffen Rohde* (Germany)
[3] Constructing mathematical knowledge by means of analogy: connecting Fischbein's theory on the role of intuition in mathematics and the theory of abstraction in context. ***Ivy Kidron*** (Israel)
[4] Seeking a "Theory" of networking praxeologies in mathematics education: a meta-theoretical discussion. ***Yusuke Shinno*** *and Tatsuya Mizoguchi* (Japan).
Session 2: Methodological approaches to the diversity of theories in design research
[5] Vertical analysis as a strategy of theoretical work: from philosophical roots to instrumental and embodied branches. ***Anna Shvarts*** *and Arthur Bakker* (The Netherlands).
[6] Configuration of the theoretical-methodological construct «the teaching model» by affinity between theories. ***Ulises Salinas-Hernández*** (France and Mexico)*, and Luis Moreno-Armella and Isaias Miranda* (Mexico).
[7] The holistic instructional design model of the unit knowledge structure of elementary school mathematics based on core competencies. ***Shiqi Lu*** *and Wenbin Xu* (China).
[8] Networking theories and methodology: identifying argumentative grammars in design research. ***Arthur Bakker*** (The Netherlands) *and William R. Penuel* (USA).
Session 3: Reconsidering basic commitments in the diversity of theories, specifically ethics
[9] Mathematics teaching and learning as an ethical event. ***Luis Radford*** (Canada).
[10] How can we classify teachers' paradidactic praxeologies in different institutional settings? ***Tatsuya Mizoguchi****, Yusuke Shinno, and Toru Hayata* (Japan).
[11] Theoretical networking in a large-scale Danish and a large-scale Norwegian intervention study: TMTM and PBG. ***Lena Lindenskov*** (Denmark).

2.1. Session 1: Why do we need a diversity of theories?

The chair, Angelika Bikner-Ahsbahs, and the cochair, Ivy Kidron, introduced the TSG-57. They distinguished the notions of foreground and background theory (Mason and Waywood, 1996), clarified the term "theory" based on the work of Radford (2008, 2012) as well as the notion of the networking of theories (Bikner-Ahsbahs and Prediger, 2014).

In her invited talk[1], Artigue focused on her research about teaching and learning in a digital environment and used two conceptual tools for her reflection on issues of theoretical diversity: the landscape of networking strategies and the concept of research praxeology. She presented two research cases where theory diversity was relevant, studies on the instrumental approach and the documentational approach to didactics.

Bikner-Ahsbahs et al.[2] combined Activity Theory and Instrumental Genesis to investigate the feedback of a digital tool designed for the teaching/learning of integers. Reflection on the way the two approaches are related revealed a layering model, which

describes how the students in the study developed their knowing in the activity of teaching/learning negative numbers mediated by feedback.

Kidron[3] presented a case of Networking of Theories, which showed how she linked the two originally separated foci into one comprehensive focus, the focus of constructing knowledge by means of analogy.

Based on the concept of research praxeology (Artigue & Bosch, 2014), Shinno and Mizoguchi[4] showed via three case studies of theory networking that theoretical concepts and the language used differ with respect to the kind of study they undertook, empirical study, design study, and theory development.

The main outcome in the discussion was that networking of theories can be a source of tensions between theoretical discourses, but it can also offer the flexibility to establish new theoretical discourses.

2.2. Session 2: Methodological approaches to the diversity of theories in design research

Yusuke Shinno (a member of the TSG-57 team) bridged Session 1 and Session 2 by recalling tension and flexibility in the networking of theories suggesting a dialectical way of working with different notions of the same term, such as scheme. Session 2 was then dedicated to the networking of theories in design research (Bakker, 2019), its gains and pitfalls, relating it to ontological, methodological and epistemological issues.

Shvarts and Bakker[5] pointed to the need of undertaking historical analyses to inform conceptualizing when comparing and contrasting theories. They called this analysis 'vertical analysis'. Their aim was to identify the grand theories behind local approaches and their ontological and epistemological philosophical presumptions. Anna illustrated this approach by unpacking the roots of action scheme used in the instrumental approach and compared it with embodied approaches distinguishing finally between enactive and mental schemes.

Salinas-Hernández et al.[6] took the idea of "affinity" as an alternative to networking strategies in order to configure the methodological construct "teaching model" by three theories based on a semiotic-mediating view. After identifying related elements (affinities) between the theories and configuring the teaching model, the authors used a qualitative investigation in physics education to analyse the teaching practice of two teachers of grade 11 with different experience.

Lu and Xu[7] presented a holistic instructional design model for elementary school mathematics based on core competencies. The core competence concept relates to the development of mathematical thinking. The basic theory underlying the model is a thick epistemology including five characteristics: unit knowledge structure, learning psychological process, teaching objectives, learning evaluation, learning activities.

Finally, Bakker and Penuel[8] talked about networking theories and methodology. Acknowledging the networking of theories approach they stressed also the danger of this approach of being tied to epistemological justifications of choices in design research and simultaneously ignoring the relevant role of ontology. Because of the

transformative nature of design research there is the necessity to include ontological and axiological aspects into justifications of design choices.

Main result from the discussion was the need for an increasing sensitivity to more comprehensively consider methodological as well as epistemological, and ontological issues in justifications of theoretical choices in transformative research in mathematics education.

2.3. Session 3: Reconsidering basic commitments in the diversity of theories, specifically ethics

In her bridging introduction, Erika Bullock (a member of the TSG-57 team) proposed to take up work from the philosophy of science. She referred to Patterson and Williams (1998) when she argued that there is a normative structure to scientific research holding certain commitments. These philosophical commitments involve theories about:

a) The nature of reality and what really exists (ontology)
b) The relationship between the knower and what is known (epistemology)
c) What we value and how we determine that value (axiology)
d) The strategy and justifications in constructing a specific type of knowledge (methodology), as linked to individual techniques (methods) (Daene, 2018, adapted from Patterson & Williams, 1998)

In his invited talk[9], Radford addressed the issue of ethics for teaching and learning in general. For the theory of objectification, he proposed and elaborated a communitarian conception of ethics that joins responsibility, commitment and care in the teaching and learning of mathematics.

Mizoguchi et al.[10] reported about two case studies of networking theories exploring the role of categorisation in a methodological approach, asking: "What is a theory for" and "How does it function". They used the two case studies to differentiate the kind of knowledge being achieved, in one case they revealed "descriptive" and the other case "explanatory" theory elements (Prediger, 2019).

Lindenskov[11] talked about theoretical networking in large-scale Danish and Norwegian Intervention studies. As teachers were included in these studies, culture-specific views of teachers on theories, goals and organizations played a significant role making visible the contradictions between researchers and teachers' view of how theories are meaningful.

What we learned from the discussion was the conviction that key commitments in the use of theories should be reflected and made explicit. Ways to do this are vertical analyses, grounding research in philosophical work and merging commitment and responsibility.

3. Future Directions

Networking theory research has led us in the past years to revisit our epistemological assumptions. The recent interest in ontology can complement the advances made on

the epistemological side. There are a number of future directions to think about in discussions on theory use in the different kinds of research genres in mathematics education.

There are certain commitments related to theories (i.e., axiological, ontological, and epistemological) and to teaching and learning of mathematics (e.g., ethical). These commitments seem to be relevant for methodological choices in research that influence what kind of knowledge can be achieved about a certain research object. We ask how these commitments become relevant when various theories are taken up e.g., when theories are networked.

Ethical issues are particularly relevant for classroom design. As design research is a transformative way of doing research and development, where do the decisions for or against a specific learning goal come from and what are the criteria in what directions changes of the design are to be made? Here axiological issues come into play affecting the ontology as well as the epistemology of the research conducted. Beyond that, how far is it necessary or even mandatory to take ethical issues of the teaching/learning into consideration for theorizing and what are the relevant concepts of ethics for that? More generally, what are the pitfalls when such commitments go unnoticed?

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