

Topic Study Group 60

Semiotics in Mathematics Education

TSG-60 Working team¹

1. Aims of the TSG

The TSG-60 aimed at exploring the significance of semiotics and the diverse uses of signs in the teaching and learning of mathematics at all levels. The importance of semiotics is reflected in a large body of literature within mathematics education, an overview of which is to be found in the ICME-13 monograph “Signs and Signification: Semiotics in Mathematics Education Research.” The goal of TSG-60 was to expand on prior work, addressing the following themes and sub-themes:

- *Themes*
 1. Semiotic perspectives within mathematics education;
 2. Sign use and mathematics meaning-making processes;
 3. Modes of mathematical narrative through different sign systems;
 4. Relationships between sign systems (e.g., natural language, diagrams, pictorial and alphanumeric systems) and transformations between sign systems in mathematics thinking and learning;
 5. Inventing and generalizing with visual, alphanumeric, and other sign systems;
- *Sub-themes*
 1. Semiotics and Technology (e.g., Design of activities and tasks based on visual-kinesthetic interactions; interplay between physical manipulatives and virtual entities, and roles of animation and video as instructional tools)
 2. Semiotics in Specific Areas of Mathematics (e.g., Episodes of sign-use in calculus, geometry, algebra, arithmetic, etc.)
 3. Semiotics Inside and Outside Mathematics Education (e.g., differences and similarities between semiotic usages in art, linguistics, or cinema, and mathematics)
 4. Semiotics in Relation to Feeling and Expression (e.g., gestures, embodiment, more-than-human agencies, affects, aesthetics, and rituals)

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2. Submissions, participation, and sessions

2.1. Submissions and participation

We received, in 2019, 13 submissions from 8 countries (South America: 1; North America: 4; Asia: 3; Europe: 4; Africa: 1). Of these 13 submissions 10 were accepted as paper presentations and 3 as posters. Of the 13 accepted submissions, only 7 papers were able to be presented during the online conference. We list the papers in Tab. 1.

Tab. 1. The list of papers presented

Paper and author(s)
<i>Session 1: Embodied aspects, gestures, movement</i>
[1] Collaborative gestures among secondary students conjointly proving geometric conjectures. Candace Walkington, Min Wang, and Mitchell Nathan (USA).
[2] Conceptualization of co-emergent curriculum in a mathematics lesson. Kazuma Kageyama and Masataka Koyama (Japan)
[3] Can a movement notation be a mathematical notation? Giulia Ferrari and Francesca Ferrara (Italy)
<i>Session 2: Language, meaning making, social factors</i>
[4] Semiotic character and issues in the learning and teaching of linear functions in Japan: The influence of terminology. Hiroaki Hamanaka, Masayoshi Yoshikawa, Hisae Kato, and Mitsunobu Kawauchi (Japan).
[5] A semiotic lens on learning math in sign languages. Christina M. Krause (USA/Germany) and Annika M. Wille (Austria).
[6] Semiotic chaining in Linear Algebra. Hamide Dogan (USA).
[7] Interference between artifacts in semiotic chains. Andrea Maffia and Mirko Maracci (Italy).
<i>Session 3: Workshop</i>

2.2. Themes prominent during the sessions

The themes that became prominent during the three sessions can be outlined along five categories:

2.2.1. Gestures, body, and their annotations

Walkington et al.^[1] elaborated on the notion of “collective gestures”, in reference to gestural actions bodily coordinated among several students, arguing that they can express important mathematical insights emerging from distributed cognition. *Ferrari and Ferrara*^[3] shared a notation for body motion stimulated, in part, by the Laban notation, that they propose to enrich research practices.

2.2.2. Co-Emergent Curriculum

Kageyama and Koyama^[2] distinguish between a hypothetical learning, as it can be traced in a mathematical textbook, and the real learning that incorporates spontaneous contributions from interactions among students and teachers. Their case study focused on the word-usage in a mathematics lesson. They characterize the resultant process as a co-emergent curriculum.

2.2.3. *Language and mathematical concepts*

Hamanaka et al.^[4] discussed the influence of the Japanese phrase for “linear algebra” which, as opposed to the English one, does not connote straight lines. *Krause and Wille*^[5] elaborated on different semiotic approaches to analyze the use of sign language among Deaf students in the context of a mathematics lesson.

2.2.4. *Semiotic chaining*

Dogan^[6] traced the emergence of signified-signifier pairs and how they facilitate the emergence of new concepts in linear algebra. *Maffia and Maracci*^[7] incorporated a Peircean semiotic perspective to analyze the enchaining process. They introduced the notion of “interference” to characterize how different artifacts interact in the formation of a semiotic chaining.

2.2.5. *Abstraction and Mathematics*

This was the theme of the workshop that took place in Session 3. The inquiry centered on how semiotics may cast light on the concept of abstraction in the context of two selected video episodes. The complexity inherent in this investigation emerged from the use of physical materials and tools and the expression of ideas in Sign Language, all of which seem to reflect a bodily and “concrete” ground for thinking, allowing for the articulation of abstract ideas.

3. **Areas for future research and outlook**

The topic of Abstraction and Mathematics led to numerous research questions for future work. These included how the word “abstraction”, as uttered in different languages, might lead to distinct ways of conceptualizing it, how to complicate the almost automatic association between “material” and “bodily” with “concrete”, and how to question the presumption of abstraction as preventing inclusiveness. Other areas for future research are those that were touched only tangentially, or not at all, by the presented papers, such as semiotics in relation to feelings and aesthetic expressions, the relationship between the uses of sketches and diagrams in mathematics and in other disciplines, or the significance of students’ inventing mathematical notations.

The group — TSG leaders and participants alike — agreed that it might be worthwhile to work on a joint publication, for example a special issue of a journal in which will be focused on the topics as they emerged during the discussions. Furthermore, a future seminar and or topic conference/symposium has been considered. Potential contributors met shortly after the conference to elaborate on possible outlets and topics for an open call. The endeavor is still in the planning stage as of March 2022.