THE ROLE OF TURTLE SCHEMES FOR THE PROCESS OF THE SPATIAL VISUALIZATION

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The theory of embodied cognition assumes that behaviors, senses and cognitions are closely connected, and there is a growing interest in investigating the significance of embodied cognition in the field of mathematics education. This study aims to investigate what role embodied turtle metaphor and symbols play when students visualize three-dimensional objects. We used MRT(Verdenberg & Kuse, 1978) & SVT for this research and both tests turned out that turtle schemes are useful to the students in a low level group. In addition, students found turtle schemes more useful in SVT which requires constructing three-dimensional objects, than in MRT which requires just rotating the image of three-dimensional objects in their mind. These results suggest that providing students who are less capable of spatial visualizing with the embodied schemes like turtle metaphor and symbols can be an alternative to improve their spatial visualization ability.
Keywords: embodied cognition, spatial visualization, turtle metaphor, turtle symbols.

INTRODUCTION

Spatial ability, which is considered as an important factor in human intelligence, has a prominent role in mathematics, engineering, architecture and medical domains. It refers to the essential skills including practical life skills such as driving and swimming skills, also cognitive skills which can be used in scientific and engineering activities (Wang et al, 2007). In recent years, the development of technologies in engineering and computers enables the exploration of the spatial abilities. Over the last decade, great advances have been made in the understanding of 3D virtual worlds and numerous cyberspaces have emerged, including Second Life and Active Worlds so on(Hew & Cheung, 2010).

Just like the same number can be represented by the two different schemes -Arabic numerals and Roman numerals, cognitive operations are not independent of the symbols that instigate them(Gonzalez & Kolers, 1982). In the same manner, the cognitive process of the visualization varies according to the views and symbol systems. For example, O'Boyle et al (2005) found the different brain activation of the normal students and math gifted students by analyzing fMRI when they were asked to do mental rotation activities. The result revealed that most students could only activate their right brains which control the visual and spatial aspects, whereas gifted students could activate both sides of brains. The finding indicates that students may use specific "rules" for discovering, manipulating and transforming the spatial relationships without employing visuo-spatial strategies. In this study, both embodied turtle metaphor and symbols were provided to the students and teachers to help them understand three-dimensional objects. We aimed to examine what kind of complimentary roles this new scheme and visuo-spatial strategies play in understanding three-dimensional objects.

EMBODIED COGNITION & TURTLE METAPHOR

As there has been a change in cognitive science, it is now granted that cognition is not a passive reflection of the universe(Lakoff & Núñez, 1997). Instead, it is resulted from the interactive relationship between a physical body and environment. This stance holds that mind should be understood as an interactive function of human brain, body and environment and it led to the view of 'embodied cognition'. The theory of embodied cognition assumes that behaviors, senses and cognitions are closely connected, and there is a growing interest in investigating the significance of embodied cognition in the field of mathematics education. As Núñez(2007) stated, Mathematics is perhaps the most abstract conceptual system that we can think of, but even this it is ultimately embodied in the nature of our bodies, language and cognition. In similar context, Radford et al(2005) stressed that thinking is not only mediated by, but also located in signs, artifacts, and body.

In Amorim et al(2006) study of embodied spatial transformation, spatial embodiment was exemplified to understand their contributions. Body analogy can be implemented through spatial embodiment, or what Lakoff & Johnson(1999) have called the "bodily projection" of reference frames, whereby body axes(head-feet, front-back, and left-right) are mapped onto the embodied object. In case of LOGO is a programming language that projects learners onto turtles on Microworld. Learning with turtle metaphor was called syntonic learning which involves body-syntonicity and ego-syntonic(Papert, 1980). Turtles can not only be said to body syntonic but also ego syntonic; in this understanding a turtle's motion that makes a circle is related to learners' knowledge and understanding of their own bodies, it may also be ego syntonic in that it is coherent with children's sense of themselves as people with intentions, goals, desires, likes, and dislikes. In this manner, the representation of body syntonicity can be considered as embodied cognition.

SEMIOTIC MEDIATION & SPATIAL VISUALIZATION

Mathematical signs are seen mainly as 'instruments' for coding and describing mathematical knowledge, communicating mathematical knowledge, and generalizing it(Steinbring, 2006). In this sense, we could think about a tool that can represent and visualize three-dimensional objects. As graphs and formulas can be used in calculus, we can also think out signs that function as medium language to visualize three-dimensional objects.

According to Vygotsky(1978), externally oriented tools maybe transformed into internally oriented ones through a process of internalization that occurs through semiotic processes, in particular by the use of a semiotic system in social interaction. This complex semiotic process explains the sense in which a specific artifact may be considered a means of semiotic mediation. The term 'means of semiotic mediation' does not refer to the concrete act of using a tool to accomplish a task, but rather to the fact that new meanings related to the use of a tool may be generated an devolve(Botzer & Yerushalmy, 2008).

For visualization, it is mainly focused on the image of object in common psychology, though the 'process of forming images' is more highlighted in mathematical visualization.

Dreyfus(1993), for instance, claimed that it is necessary to have mental representations by distinguishing external representations and mental representations, and external representations could also influence to the process of forming mental representations. More recently, Presmeg(2006) stressed that both symbolic information and visual information, as external representations, are important to understand spatial visualization. According to her argument, visualization plays an important role in mathematical education especially when visual components combined with symbolic components. In this view, we would like to consider semi-visual and semi-symbolic turtle schemes which learners can perceive intuitively and manipulate at the same time.

TURTLE SCHEMES AS EMBODIED SEMIOTICS

LOGO expresses two-dimensional geometry by decomposing it into two basic geometric modules-segment and angle. Although LOGO has been successfully made a great achievement in two-dimensional learning, a number of researchers pointed out that it has difficulty understanding turtle movement in the three-dimensional context relating to their everyday gestures(Kynigos & Latsi, 2007). Also, the use of programming languages such as 'repeat', 'forward' and 'rotate' triggered the learners' cognitive burden(Hoyles & Noss, 2003). Thus, Cho et al(2011) proposed turtle symbols for the construction of three-dimensional blocks which consists of the symbols, **s**(moving forward), **l**/**r**(moving to the left/right),**u**/**d**(moving upward/downward) using turtle metaphor. An example of constructing three-dimensional objects with these turtle symbols is shown in Figure1.

The purpose of this study is to investigate what role turtle schemes play when students visualize three-dimensional objects. Turtle schemes involve turtle metaphor and turtle symbols. Learners are projected onto Microworld and construct turtle blocks by moving to the front, right, left and upside. The turtle symbols- **s**,**r**, **l**, **u**- can reflect the turtle movement and they are considered as 'embodied symbols'.

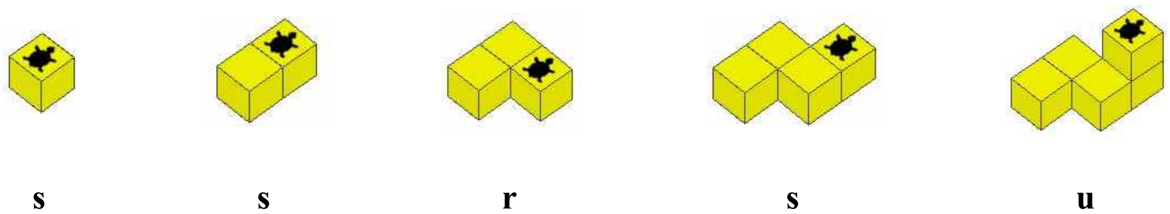


Figure 1. The system of the embodied symbols and visualization

METHOD

In a result reminiscent of Piaget's classification of imagery, at least two distinct cognitive processes were found to underlie performance on tests of spatial ability; one corresponding to the constructionof an image and the other to its transformation in memory. Both of the processes were investigated in this study when we provided turtle metaphor and embodied symbols. We conducted a test in order to explore whether it can be used as a cognitive tool of visualization.

Spatial visualization test can be employed without any prior knowledge on specific areas so it can be applied to the learners of all ages. A number of 20 first graders of middle school (13-year old) and 18 secondary school teachers recruited in this study. The students were selected from the gifted children centre of Seoul National University and most of them were high-performing students; the teachers are consisted of math or science teachers who have 5-20 years teaching experiences. Two kinds of pre-tests regarding spatial ability were conducted in a given time, a lesson about turtle metaphor and symbols was provided, and the post-test which shared the same items with the pre-tests was also conducted. Moreover, a simple survey which verifies the usefulness of turtle metaphor and symbols was carried out after the post-test.

According to Thurstone(1944), spatial ability can be classified as spatial visualization, spatial orientation and spatial relation. MRT can be used as a tool to examine the spatial visualization ability since this ability refers to the metal rotation which convert 2D objects into 3D objects and manipulate them. In this sense, we used Verdenberg & Kuse(1978)'s Mental Rotation Test (MRT) as our first tool, which has been developed based on Shepard & Metzler's(1971). There are a total number of 20 items in both part 1 and 2 in Verdenberg & Kuse(1978)'s MRT, but we only chose the most distinctive ten items and conducted the pre-test and post-test using the same items. In MRT, a 3D object was rotated in a three-dimensional environment which was consisted of 10 blocks, and students were asked to choose two of the corresponding 3D objects among four options. If both options they chose were wrong, they got 0 point; if get one option correct, they earned 1 point; if they chose both of the correct options, they earned 2 points. Maximum score of this test is 20.

Moreover, we developed a second test regarding to the spatial visualization, which was based on the MRT. This test divides the 3D objects, which are derived from MRT, into several floors and then visualize 3D objects using 2D diagrams. If MRT is designed to test the metal rotation and manipulation of 3D objects in learners' mind, the second test we proposed is more focused on the process of construction of 3D objects using 2D diagrams. This test is called Soma Visualization Test (SVT), since $3 \times 3 \times 3$ cubes are made of 7 pieces of soma cube and 2D diagrams are used to depict the soma cube solutions which is shown in Figure 2.

SVT includes four sets and each set comprises three items, each item is equivalent to 1 point and there are 12 items in total. While the 3D objects of MRT contain three articulations with 10 blocks, SVT has different number of blocks and articulations according to difficulty level of the task in Set 1, 2, 3, 4 (See Table 1).

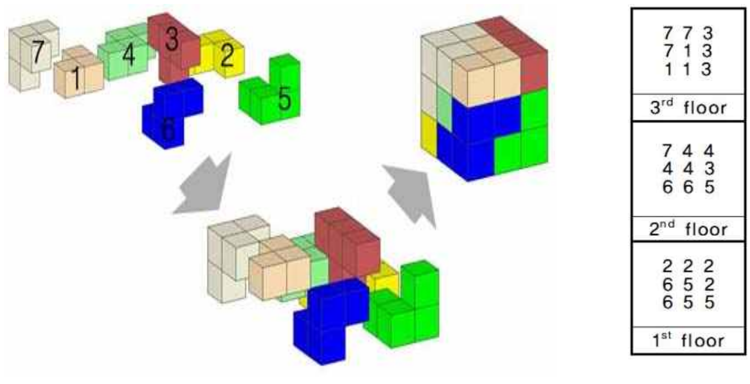
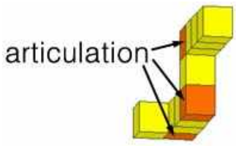
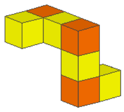
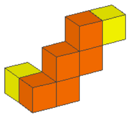
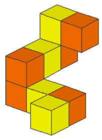
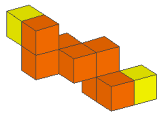


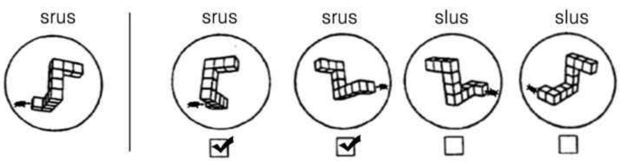
Figure 2. An example of the soma cube solution

Table 1.The structure of test items

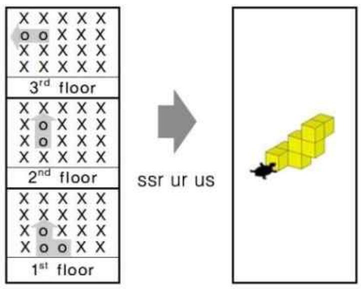
type of test	MRT	SVT			
		set 1	set 2	set 3	set 4
example					
the number of blocks	10	7	7	9	9
the number of articulations	3	3	5	5	7

A high credibility (Cronbach α=.84) of SVT was reported. Likewise it showed a very significant and positive correlation between the pre-tests of MRT and SVT with a high correlation coefficient(r = .74). Thus, we believe SVT can be considered as a highly reliable tool to test spatial visualization ability.

Between the pre-test and post test, a lesson were employed. Students took the test after having sufficient explanation about the items using examples. The lesson which took place between the pre-test and post-test was developed to enable students to understand 3D objects with turtle metaphor and symbols. The participants used the 'turtle agent' to move toward, right, left and upside to build blocks in JavaMAL microworld[1], which can help them get used to the turtle schemes and experience an embodied situation. Although the pre-test and post-test share the same items, students could still visualize the 3D objects with sequential



a. MRT



b. SVT

Figure 3. Examples of post-test: the use of turtle schemes

1 www.javamath.com

turtle movement since the turtle image was provided in each item which was intended to elicit turtle schemes from learners (see Figure 3).

In Figure 3a, turtle symbols of the MRT of the picture can be depicted as 'ssrrrruuuss'. Four options presented on the right side seemed to have the same 3D structures, however they could be different if the students considered the directions. For example, the turtle moved forward and left in 1st& 2nd options, while it moved forward and right in the 3rd& 4th options so the right answers should be the 1st& 2nd options.

Likewise, in case of SVT, the 2D diagram on the left can be depicted as the turtle symbols- 'ssrurus'. We expected this kind of use of metaphor and symbols would provide learners views and strategies which differ from the visuo-spatial strategies used in the existing spatial visualization tests.

DATA ANALYSIS

In order to verify the usefulness of the turtle metaphor and symbols as mediation tools of spatial visualization ability, we analyzed the pre-test and post-test results. The actual data was collected from 32-36 people although we were supposed to analyze the data of 20 teachers and 18 students, because some students were absent for and some outliers were excluded as well.

The percentage of correct answers of pre-test and post-test in MRT did not show a significant difference. However, there was a significant difference in pre-test and post-test in SVT, which showed 61.58% and 87.42% respectively (t=-6.11, p=0.00)(see Table 2).

Table 2. Result of MRT & SVT

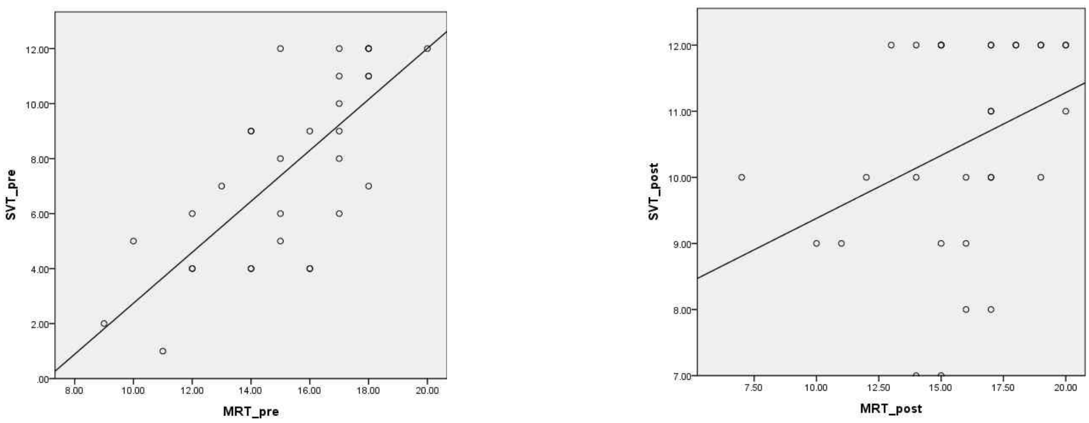
type of test	pre-test			post-test		
	Mean	SD	%	Mean	SD	%
MRT(Max. 20)	15.31	2.66	76.55	15.67	2.99	78.35
SVT(Max. 12)	7.39	3.41	61.58	10.49	1.67	87.42

Also, we divided the whole students into high, middle and low level groups based on the pre-test results. This enabled us to further analyze which group students showed the most significant differences in both pre-, post-tests. As revealed in Table 3, in the case of MRT, a statistically significant difference(t=-3.27, p=.01) was found in low level group. The percentages of correct answers of their pre-test and post-test were 60.5% , 75% , respectively. On the other hand, both low level and middle level students reported a significant increase in their test scores in SVT(t=-12.18, p=.00;t=-5.59, p=.00) . This increase can also be observed in Figure 4 which showed the relationships between MRT and SVT. As shown in Figure 4, ten students got 2-6 points relatively low scores in SVT pre-test, whereas all students got above 7 points in SVT post-test.

Table 3. Result of MRT & SVT by groups of students' levels

type of test	level	pre-test			post-test		
		Mean	SD	%	Mean	SD	%
MRT (Max. 20)	high	17.64	.84	88.20	16.71	2.73	83.55
	middle	15.43	.53	77.15	15.86	2.61	79.30
	low	12.1**[2]	1.73	60.50	15.0**	2.45	75.00
SVT (Max. 12)	high	11.64	.50	97.00	11.37	1.03	94.75
	middle	7.38**	1.33	61.58	10.54**	1.71	87.83
	low	3.59**	1.24	29.92	9.67**	1.72	80.58

An important pedagogical implication can be drawn from the analysis in terms of students' levels regarding spatial visualization ability. Students who got relatively higher scores in pre-tests were assumed to have better spatial visualization ability and they tended not to apply new cognitive strategies and tools. In addition, some of them even got lower scores in post-tests because they were not familiar with newly-introduced tools. On the other hand, students whose scores were lower than average scores in pre-test were more likely to employ turtle metaphor and symbols to help them solve spatial visualization problems. In other words, low spatial visulizers showed poor performance in pre-tests, but they showed a remarkable improvement in post-tests after acquiring the schemes about turtle metaphor and symbols.



a. pre-tests

b. post-tests

Figure 4. Scatter diagram and regression line of pre-tests and post-tests

[2] double asterisk means p-value is less than .01, which is a statistical significant level

This change implied turtle schemes may boost the students' visuo-spatial abilities who are lack of this skill.

In addition to the pre-,post-test, we also conducted a survey with the purpose of examining students' understandings toward turtle schemes. The survey composed of two categories to investigate how students perceive the use of turtle schemes, including the items regarding to turtle metaphor and turtle symbols. As a result, students reported turtle metaphor was more helpful than turtle symbols. It can be concluded that the problems what students were asked to solve using turtle symbols were not that complicated. If they were given more complicated problems which may cause cognitive burden to them, they would value turtle symbols rather than turtle metaphor. Moreover, turtle scheme was proved to be helpful in SVT and a significant difference between pre-test and post-test had been reported.

Both qualitative and quantitative results of this research demonstrated that turtle schemes are useful to the students in a low level group. In addition, students found turtle schemes more useful in SVT which requires constructing three-dimensional objects, than in MRT which requires rotating the image of three-dimensional objects in their mind. These results suggest that providing students who are less capable of spatial visualizing with the embodied schemes like turtle metaphor and symbols can be an alternative to improve their spatial visualization ability.

Table 4. The survey result regarding the use turtle scheme

type of test	perceived usefulness	NU	SCU	UN	U	STU	average
		1	2	3	4	5	
MRT	turtle metaphor	3	3	7	12	8	3.58
	turtle symbols	4	3	13	9	3	3.13
SVT	turtle metaphor	3	0	5	14	14	4.00
	turtle symbols	3	3	10	8	12	3.64

NU: never useful, SCU: scarcely useful, UN: unsure, U: useful, STU: strongly useful

CONCLUSION & DISCUSSION

As Ramads(2009) claimed, mental image is unlike a picture in the mind, it is a scheme for depicting and processing visual, spatial, temporal, causal, or other types of information. The understanding of 3D objects varies according to the different schemes; it could be understood as vague configuration or it can be sequentially analyzed. The new schemes- turtle metaphor and symbols- help the low spatial visualizers solve the spatial visualization problems. The result from this study supports Stieff's(2007) claims that visuo-spatial strategies give insights to the spatial relations while analytic strategies help students solve problems effectively and correctly.

The work presented here advances previous work, yet some major limitations are still remained. Firstly, a detailed analysis could not be achieved due to the small size of participants. Secondly, turtle metaphor and symbols proposed in this study triggered cognitive burdens to some students due to the insufficient practice about these new schemes. We assume the effectiveness of these schemes could be better proved if the participants were given more practice before taking post-tests.

The outcomes created by a given tool is closely related to the interaction between the tool and the user's mind(Norman,1998). The finding of this study provides the evidence how turtle metaphor and symbols used as cognitive tools by the students who have low spatial visualization abilities. Moreover, providing the new schemes such as turtle metaphor and symbols to the low-performing students may suggest an effective way to solve the spatial visualization problems in accordance with visuo-spatial strategies.

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