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GENERAL ABILITY VS. EXPERTISE IN MATHEMATICS: AN ERP STUDY WITH MALE ADOLESCENTS WHO ANSWER GEOMETRY QUESTIONS

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ABSTRACT

This paper represents one part of a large-scale Multidimensional Examination of Mathematical Giftedness. It presents research on brain activity (using ERP – Event-Related Potentials – methodology) associated with solving mathematical problems that require transition from a geometrical object to a symbolic representation of its property. Out of a total of 170 adolescents with varying levels of general giftedness (Gifted-G, Non-gifted-NG) and mathematical expertise (Excelling-E, Non-excelling-NE) who took part in the study, 43 right-handed male students were chosen for comparative data analysis presented in this paper. We aimed to investigate the differences in time investment and form of activation among four groups of participants (G-E, G-NE, NG-E, and NG-NE). In the study presented herein ERP methodology was used to compare brain activation when students performed the tasks. We found different patterns of brain activity in six regions of interest (ROI) in two subsequent stages of problem solving among the four experimental groups. The more distinct difference between two stages of tasks associated with chosen ROI (introduction and verification stages) was found in the NG-NE group.

1. INTRODUCTION

A considerable body of research has been conducted which contributes to the understanding of the neural foundation of mathematical cognition (Dehaene et al., 2003; Zago, 2001; Grabner et al., 2009; Santens et al., 2010, Dankner & Anderson, 2007). In addition, there has been extensive neuroscientific research on human intelligence including individual differences in general intelligence (e.g., Jung & Haier, 2007; Neubauer & Fink, 2009; Deary et al., 2010) as well as on mathematical giftedness (O'Boyle, 2008; Grabner et al., 2009). However, these studies have not gone beyond arithmetic, logic and mental rotations. That is why we have chosen to focus our attention on brain activation associated with solving advanced mathematical tasks. Additionally, to the best of our knowledge, differences between giftedness and expertise have not been addressed in brain research. This observation resulted in the integration of 4 groups of research population, divided according to general giftedness and mathematical expertise.

2. BACKGROUND

Studying geometry in school

Studying geometry in high school involves analyzing geometric structures, characteristics and relationships (NCTM, 2000). School geometry as a mathematical subject is concerned with geometrical figures and their properties. Mental images of geometrical figures represent mental constructs possessing simultaneously conceptual and figural properties (Fischbein, 1993). A diagram with a geometric figure, or part of it, can serve as a representation of a theorem. Geometrical reasoning is usually associated with visual and logical components which are mutually related.

Mathematical abilities, cognitive skills and brain research

Recent cognitive studies have demonstrated the complex character of mathematical abilities. Particularly, they have shown that the precise acquisition of mathematical abilities involves a broad range of general skills, including spatial perception, visuo-spatial ability, attention, memory and so on (Butterworth et al., 2003). Together, these skills enable the acquisition, understanding, and performance of various mathematical activities (Ardila & Rosselli, 2002; Dehaene, 1997).

Literature review demonstrates quite consistent findings that associate different mental operations with the location of brain activation: memory retrieval (Badre & Wagner, 2005; Dobbins & Wagner, 2005; Thompson-Schill, 2003), attention control processes and general task difficulty (Delazer et al., 2003) are associated with the prefrontal cortex and representation (Danker & Anderson, 2007), whereas verbal encoding (Clark, & Wagner, 2003), mental rotation (Heil, M., 2002), and visuo-spatial strategies in mathematics (Dehaene, Spelke, Pinel, Stanescu, Tsivkin, 1999; Sohn, Goode, Koedinger, Stenger, et al., 2004) are associated with the parietal cortex. Kao and Anderson (2008) found that the left parietal and right prefrontal cortices were the areas of the brain which were the most responsive in complex geometric tasks. Kao and Anderson (2008) demonstrated that there is strong involvement of right hemispheric regions, particularly in the retrieval of geometric knowledge. It has also been found that the strength of brain activity is related to the individual's mathematical skills (Ravizza et al., 2008).

The brains of the mathematically gifted show enhanced development and activation of the right hemisphere (Singh & O'Boyle, 2004; Prescott et al., 2010). Another characteristic of mathematically gifted people is enhanced brain connectivity (Jung & Haier, 2007; Geake, 2009; O’Boyle, 2008) and an ability to activate task-appropriate regions in both brain hemispheres in a well-orchestrated and coordinated manner (Dehaene et al, 1999; O'Boyle, 2005). There is strong evidence for special development of prefrontal and posterior parietal regions of the brain (Jung & Haier, 2007; Geake, 2009, Desco, 2011) and enhanced intra-hemispheric fronto-parietal connectivity (Jung & Haier, 2007; Prescott et al., 2010).

Our study addresses individual differences in general intelligence as well as in mathematical competence. In what follows we describe findings related to performance of geometric tasks involving transition of the mathematical object to its property. Moreover, the study uses electrophysiological measures which can shed light on the temporal characteristics of geometric reasoning.

3. MATERIALS AND METHODS

Participants

Table 1: Research population

	Gifted (G) IQ>135, Raven >28 of 30	Non-Gifted (NG) 100<IQ<130, Raven < 26 of 30	Total
Excelling in mathematics (E) SAT-M >26 of 35 or Math score > 92 in high level mathematics	12	12	24
Non-Excelling in mathematics (NE) SAT-M <21 of 30 or Normal level of mathematics instruction	12	7	19
Total	24	19	43

Forty-three male high school students from the northern part of Israel (16-17 years old) participated in this study. The students belonged to 4 groups as presented in Table 1.

All participants were native speakers of Hebrew, right-handed, without a history of learning disabilities and neurological disorders and had normal to corrected vision.

Stimuli and Procedure

A computerized geometry test was designed using E-Prime software (Schneider, Eschman, & Zuccolotto, 2002). Each test included 60 tasks (trials). All tasks were presented visually at the center of the computer screen and were displayed in black characters on white background. Each task on each test was presented in two windows with different stimuli (S1 – Task condition – introduction stage; and S2 – Suggested answer – verification stage) which appeared consecutively.

Each task started with the fixation cross. The cross was replaced by problem introduction (S1) after 500 ms. The problem introduction was visible for 2000 ms and separated from the answer by a blank time period (ISI) of 1000 ms. The answer remained visible until the participant responded or for a maximum of 5000 ms. Time periods were determined by a pilot study with 30 participants.

S1 included drawings of geometric figures with the angles marked by Greek letters α and β. S2 presented a statement about the relationship between α and β. At the S2 stage the participants had to determine the correctness of the statement. Figure 1 presents the sequence of events and examples of the tasks.

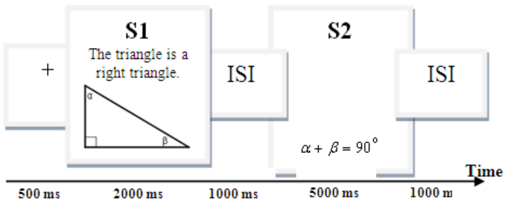


Figure 1: The sequence of events and examples of the tasks

ERP Recording and Analysis

Scalp EEG data were continuously recorded using a 64 channel BioSemi ActiveTwo system (BioSemi, Amsterdam, The Netherlands) and ActiveView recording software. Pin-type electrodes were mounted on a customized Biosemi head-cap, arranged according to the 10–20 system. Two flat electrodes were placed at the side of the eyes in order to monitor horizontal eye movement. A third flat electrode was placed underneath the left eye to monitor vertical eye movement and blinks. During the session electrode offset was kept below 50 µV. The EEG signals were amplified and digitized with a 24 bit AD converter. A sampling rate of 2048 Hz (0.5 ms time resolution) was employed.

ERPs were analyzed offline using the Brain Vision Analyzer software (Brain- products). ERPs were Zero Phase Shift filtered offline (bandpass: 0.53 Hz–30 Hz) and referenced to the common average of all electrodes. Epochs with amplitude changes exceeding ±80 µV on any channel were rejected. Ocular artifacts were corrected using the Gratton, Coles & Donchin (1983) method. The ERP waveforms were time-locked to the onset of S1 and to the onset of S2. The averaged epoch for ERP, including a 200 ms pre-trigger baseline, was 1200 ms for S1 and 2200 ms for S2 (for which only the correct answers were averaged). The resulting data were baseline-corrected, and global field power (RMS) was calculated for each segment. Each condition resulted in aapproximately 40 trials.

Based on inspection of the ERPs grand mean waveforms and topographic maps, the mean amplitudes of seven intervals of 100 ms were measured. A repeated measure ANOVA was performed on the ERP mean amplitude at seventeen electrodes (AF3, AFz, AF4, F3, Fz, F4, FC3, FCz, FC4, P3, Pz, P4, PO3, POz, PO4, O1, O2) taking Excellency (Excelling, Non-Excelling) and Giftedness (Gifted, Non-Gifted) as between-subject factors and Time (seven time intervals), Caudality (anterior, posterior) and Laterality (left, middle, right) as within-subject factors. Examination of the time course was done for two stages of a task (introduction stage and answer verification stage). On the basis of grand mean waveforms (Global Field Power) for each of the problem-solving stages (introduction stage and answer verification stage) the peaks P1, N2 and N4 were identified (see Findings section). The electrodes for statistical analysis were chosen based on the preliminary examination of global field power (RMS) on each electrode and on the observation of ERP topographical map (Table 2).

Table 2: Electrodes chosen for statistical analysis

Stage / Peak	Introduction Stage		Answer Verification Stage	
	time epoch	chosen electrodes	time epoch	chosen electrodes
P1	100-200	P5, P3, P4, P6, PO3, POz, PO4	90-200	PO3, POz, PO4, O1, O2
N2	150-260	P5, P3, P4, P6, PO3, POz, PO4	200-300	F3, Fz, F4, AF3, AFz, AF4
N4	300-500	AF3,AFz, AF4, F3, Fz, F4	300-500	AF3,AFz, AF4, F3, Fz, F4

First, MANOVA were used for latencies and mean amplitudes on the chosen electrodes with Giftedness (gifted vs. non-gifted) and Excellency (excelling vs. non-excelling) as between-subject factors. This was done for each identified peak and for each trial's stage.

Second, repeated-measures MANOVA was conducted for N4 latency and mean amplitude with Giftedness (G vs. NG) and Excellency (E vs. NE) as between-subjects factors, and Problem stage (Introduction vs. Answer verification) and Laterality (left electrodes vs. middle electrodes vs. right electrodes) as within-subject factors. This was done for each identified peak. Because using data from multiple electrode sites may lead to a violation of the sphericity assumption, all subsequent ANOVA results were corrected using the Greenhouse–Geisser correction system.

4. RESULTS

Behavioral data

Table 3 demonstrates reaction times and accuracy (mean and SD) of the performance on geometric tasks found for the four groups of participants, for group G (gifted) as compared to group NG (non-gifted) and for group E (excelling in math) as compared to group NE (non-excelling in math).

Table 3: RT and Accuracy in different groups of participants

	Reaction Time (RT) Mean (SD) in ms			Reaction Time (RT) for Correct Responses Mean (SD) in ms			Accuracy Mean (SD) in %		
Groups	G	NG	Total	G	NG	Total	G	NG	Total
E	1583.7 (255.9)	1385.6 (247.4)	1484.6 (266.1)	1524.2 (262.3)	1345.6 (244.8)	1434.9 (264.4)	84.9 (7.2)	86.1 (7.2)	85.5 (7.1)
NE	1694.8 (371.1)	1677.2 (386.0)	1688.3 (365.9)	1625.1 (369.7)	1596.8 (387.5)	1614.7 (365.8)	82.5 (4.41)	80.2 (8.6)	81.7 (6.2)
Total	1639.2 (316.8)	1493 (328.6)		1574.7 (317.7)	1438.1 (319.6)		83.7 (5.9)	84.0 (8.1)	

ANOVA for accuracy (percentage of correct answers), Reaction Time (RT) and Reaction Time for correct responses (RTC) were performed with Excellency and Giftedness as the between-participant factors. The ANOVA for accuracy did not show a significant effect, neither for E factor ($F(1, 39) = 3.743$, $p = .060$) nor for G factor ($F < 1$). However, as shown in Table 3, the accuracy of E-male adolescents is higher than the accuracy of their NE counterparts. The E participants were significantly faster than their NE counterparts ($F(1, 39) = 4.237$, $p = .046$). However, the RT for correct responses did not reach the significance level for the E-factor ($F(1, 39) = 3.221$, $p = .080$). The effect of Giftedness was found to be not significant in all the aforementioned measures (accuracy, RT and RTC). It is notable that the gifted participants were slower than their non-gifted counterparts. However, the accuracy was the same for gifted and non-gifted students. There were no main effects or interactions involving E and G factors.

Electrophysiological scalp data

Figure 2 and Figure 3 demonstrate the ERP waveforms of the global field power for the introduction stage (S1) for gifted vs. non-gifted and excelling vs. non-excelling subjects, respectively. Figure 4 demonstrates the ERP waveforms of the global field power for verification stage (S2) for G, NG, E and NE male adolescents.

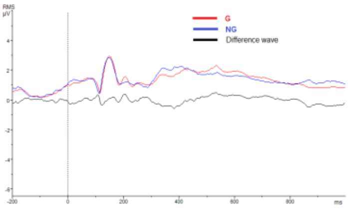


Figure 2:
The ERP waveforms of the global field power for S1 (G vs. NG)

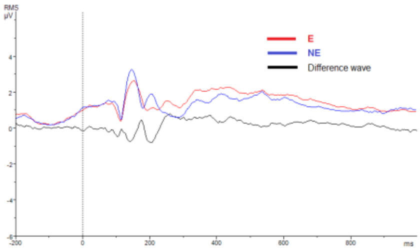


Figure 3:
The ERP waveforms of the global field power for S1 (E vs. NE)

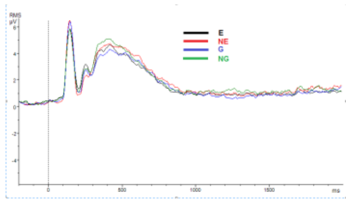
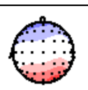
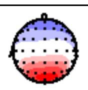
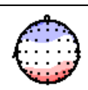
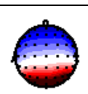
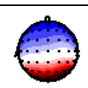
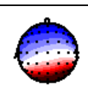
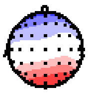
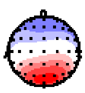
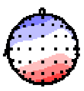
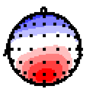
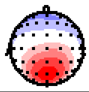
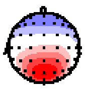
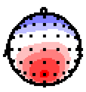
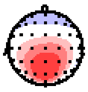


Figure 4:
The ERP waveforms of the global field power for S2 stage (G-NG, E-NE)

MANOVA demonstrated the main effect of *Stage* (F (7, 33) = 7.438, p < .001). Subsequent ANOVAs revealed that this effect was significant in all time epochs except 0-100 ms. The mean activity in these time frames was significantly less in the introduction stage (S1) than in the verification stage (S2). This finding suggests that the introduction stage demanded fewer cognitive resources than the verification stage. MANOVA demonstrate neither main effects of *E* and *F* factors nor significant interaction of *E* × *G* (p < .05). MANOVA demonstrated neither significant interactions of *Stage* × *E* and *Stage* × *G* nor significant interaction of *Stage* × *E* × *G* (p < .05). However, examination of mean activity for interaction of *Stage* × *E* revealed that the mean activity of E students was greater than NE students in the introduction stage (S1) while, in contrast, the mean activity for NE students was greater in the verification stage (S2). ANOVA revealed this phenomenon (*Stage* × *E*) to be significant in time frame 600-700 ms (F (1, 39) = 4.795, p < .05).

Table 4: Scalp topology in the Stage × Laterality × Caudality × G × E interaction.

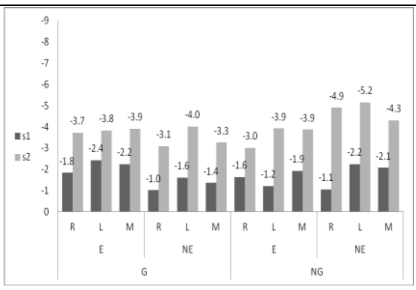
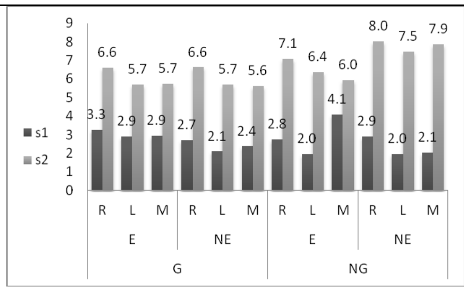
Time epoch	**Introduction stage**				**Verification stage**				***Stage x Laterality x Caudality x G x E*** *(F (14, 26) = 1.267, Wilks Λ = .576, p = .238)*
	G		**E**		**G**		**E**		
	G	**NG**	**E**	**NE**	**G**	**NG**	**E**	**NE**	
300-400 ms									F (1.720, 67.083) = 3.383*
400-500 ms									F (1.845, 71.950) = 3.599*
500-600 ms									F (1.907, 74.389) = 6.361**
600-700 ms									F (1.720, 67.087) = 4.776*

[1] E – E, G – G, NE – NE, NG – NG,

[2] *p <.05, ** p <.01, *** p <.001

Moreover MANOVA revealed significant interaction of *Stage*×*Caudality*×*Laterality* (*F* (14, 26) = 4.167, p = .001). Subsequent ANOVA demonstrated this interaction to be significant in all chosen time epochs. This suggests different patterns of activation in the six regions of interest as manifested in the *Caudality*×*Laterality* interaction between two different stages. Notably, in all time frames in the introduction stage (except 0-100 ms) activation of the right posterior region of the scalp was the greatest among the left posterior and middle posterior. This supports the findings that the right posterior cortex region is involved in geometric processing (Kao & Anderson, 2008). However, there is a different pattern of brain activation in the anterior part of the scalp associated with laterality.

MANOVA revealed a non significant multivariate interaction of *Stage*×*Laterality*×*Caudality* ×*G*× *E* (*F* (14, 26) = 1.267, *p* = .238). However, subsequent ANOVA demonstrated significant interaction of *Stage*×*Laterality*×*Caudality*×*G*×*E* in four time frames, beginning at 300 ms and ending at 700ms (See Table 4).

Time Frame	Anterior Part	Posterior Part
400- 500 ms		

[1] L – Left, R – Right, M – Middle; E – E, NE – NE, G – G, NG – NG; S1- Introduction stage, S2 – Verification stage

Figure 5: Mean Activity in time frame 400-500 ms for *Stage*×*Laterality*×*Caudality*×*G*×*E* interaction.

Notably, the mean brain activity is lower for the introduction stage (S1) than for the verification stage (S2). Moreover, the difference between the activities in the two stages for both anterior and posterior parts is the greatest for NG students who do not excel in mathematics (NG – NE). (Figure 5)

MANOVA did not reveal a significant difference in P1, N2 peaks on *E*, *G* and *E x G*. This suggests that the phase of the mathematical object perception is similar for the experimental groups. The difference was found in N4 peak (semantic processing). In order to investigate the nature of this difference more deeply we performed additional MANOVA for analysing the difference in N4 peak for two stages and for different study groups (Table 5).

The interaction *Laterality* × *E* is significant for amplitudes and latencies of N4 peak. Notably, the latency of N4 peak is shorter for E students than for non-E students for each laterality category: Left, Middle, and Right. MANOVA revealed a significant interaction *Stage*×*Laterality*×*G*×*E* on amplitude of *N4* peak (*F* (4, 36) = 3.888, *p* = .010). Subsequent ANOVA revealed this interaction for AF electrodes (*F* (1.739, 67.825) = 6.147, *p* <.01) and for F electrodes (*F* (1.724, 67.221) = 3.882, *p* <.05).

Table 5: Significant Main Effects in Latency and Amplitude (found for N4 peak only)

<table>
<tr><th colspan="3" rowspan="2"></th><th colspan="2">Introduction Stage</th><th colspan="2">Verification Stage</th></tr>
<tr><th>MANOVA</th><th>ANOVA</th><th>MANOVA</th><th>ANOVA</th></tr>
<tr><td rowspan="6">N4</td><td rowspan="3">Latency
F (6, 34)</td><td>E</td><td>.123</td><td></td><td>1.118</td><td>F(1, 39)
F3: 5.327*
E:388 (48.7) vs. NE: 422.4 (51.3)</td></tr>
<tr><td>G</td><td>.121</td><td>F(1, 39)
AF3: 4.339*;</i>
<i>G: 423.6 (58.2) vs. NG: 388.7 (44.3)</i>
<i>AF4</i><i>: 6.139*; 425.1</i> (53.7) vs. 381.4 (50.5)
Fz: 5.280*; 411.6 (67.2) vs. 363.5 (47.5)
F4: 6.174* ; 417.7 (60.2) vs. 371.1 (47.2)</td><td>2.086</td><td></td></tr>
<tr><td>E x G</td><td>.152</td><td>F(1, 39)
Fz: 4.742*
G-E: 425.6(6.6)
G-NE:397.5 (68.7)
N-GE: 345 (36.3)
NG-NE 396.7 (61)
F4: 4.755*</td><td>2.227</td><td>F(1, 39)
AF3: 6.889*
AF4: 4.893*
AFz: 4.863*
F4: 7.637**
Fz: 9.151**
G-E: 429.6 (53.4) G-NE: 365.7 (52.41
NG-E 379.4 (47.9) NG-NE 415.7 (60.9)</td></tr>
<tr><td rowspan="3">Amplitude
F (6, 34)</td><td>E</td><td>1.057</td><td></td><td>1.098</td><td></td></tr>
<tr><td>G</td><td>1.266</td><td></td><td>.396</td><td></td></tr>
<tr><td>E x G</td><td>2.578*</td><td></td><td>2.299
p=.057</td><td>AF4: 4.021 p = .052</td></tr>
</table>

5. CONCLUSIONS

The present study investigates the differences in brain activity in G versus NG and E versus NE male adolescents while performing a geometric task involving transition from the mathematical object to its property.

Behavioral data of the study demonstrated that there were no main effects of G and E on the accuracy measure. However, a significant main effect of E was found in the RT measure. The E participants were significantly faster than their NE counterparts. This implies that E and NE students should be provided with different amounts of time for performance of geometry problems and probably with different collections of problems in geometry lessons.

Electrophysiological data revealed that both G and E factors caused different patterns of brain activity in later time epochs (starting from 300 ms after stimuli presentation) of the process of geometric problem solving. These differences were shown both in the strength of electrical activity and in its topographic distribution. In this case, in two subsequent stages of problem-solving, different patterns of brain activity (in the examined regions) among the four study groups were obtained. We assume that these findings should be devolved to mathematics teachers, who should be made aware of the existence of between-group differences, which are usually imperceptible in the mathematics classroom.

Additionally, the findings demonstrated that both in anterior and posterior parts of the brain, differences between two subsequent stages of problem-solving were significantly less prominent in the G-E group compared to the three other groups, while the more significant difference was found between the G-E and NG-NE groups.

Finally, because there were no significant differences in early ERP components' (P1 and N2) amplitudes for G and E factors, it may be suggested that the early processing in these stages is quite similar in all four experimental groups.

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