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SEMIOTIC ANALYSIS OF COLLECTIVE CHAT-BASED PROBLEM-SOLVING PROCESSES

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The development of 'Semiotic Process Cards' based upon Charles Sanders Peirce's triadic sign relation is illustrated here. The instrument for analyzing mathematical chat sessions is developed in a project called 'Math Chat', which is based on the use of mathematical inscriptions in an experimental setting. What is characteristic of this chat setting is that pupils are required to document their attempts at solving mathematical problems as mutual inscriptions in written and graphical form. To develop a suitable instrument an interactionist approach was combined with a semiotic perspective. The development and also the structure of Semiotic Process Cards are explained.

Semiotics, C.S. Peirce, Inscription, Framing, Digital Media

INTRODUCTION

Particularly in mathematics, the learning process depends considerably on written-graphical communication. Morgan (1998) describes the widespread significance of writing in mathematical learning processes and Pimm (1987) also mentions, that mathematics depends on written forms of communication. This is due to the fact that the depiction and description of many mathematical operations can be seen as the mathematical idea or procedure itself and do not necessarily have to be understood as its sole representation in the form of a symbol or sign. Writing and written presentations are integral elements of mathematical communication. Krummheuer (2000) refers to the fleetingness of spoken utterances in learning situations in mathematical education and suggests that:

> […] the quick evaporation and the situational uniqueness of verbal accomplishments impedes reflection on such interactive procedures […]. Complementing such reflections with a written presentation of the result (especially of the work process) seems helpful. (p. 31)

In order to access the written products of a problem solving process, I had pupils solve mathematical problems in a specific setting, namely using two tablet PCs that were connected via internet chat. Oral communication between participants is not possible, which makes the writing of questions, tips, suggestions and different approaches necessary in order to communicate with the other participant. We can hereby study theoretical and methodological questions on the solving of mathematical problems by analyzing written elements. It must be noted that this particular setting was chosen only due to its usefulness for the analysis of written portrayal of mathematical problem-solving strategies and is not to be viewed as a suggestion for a particularly innovative setting for teaching and learning. The aim is solely to encourage pupils to engage in written communication, in order to analyze the meaning and importance of this communication in collective problem solving strategies. But there were a lack of instruments to analyses the written products of the pupils. So, as part of the 'Math Chat'[1] project, a semiotic instrument of analysis was developed which enables an accurate examination of the mutually produced processes of written problem-solving and communication. This instrument also makes the analysis of any oral utterances of the participants during this process possible. Thus, this instrument was even more versatile than was initially intended.

In the following sections, I firstly describe the technical and organizational requirements and specifications of the project, thereafter I explain what is described as mathematical inscriptions by Latour & Woolgar and I present the aspects of Charles S. Peirce's semiotics necessary for the development of the instrument of analysis. Then I describe the 'Semiotic Process Cards' that were developed based upon these theoretical approaches.

THE 'MATH CHAT' PROJECT

The main focus of the 'Math Chat' project lay in the examination of the fundamental problem of the written depiction of collective strategies for the solving of mathematical problems in an experimental setting. The research centered on the type of inscription (see Latour & Woolgar 1986) pupils compiled during the mutual problem-solving process, how these inscriptions were used and developed and what part they played in structuring the problem-solving process.

The fourth-grade pupils that took part in the project used two tablet PCs to communicate during the chat sessions. There were one or two pupils on each side of the setting. The two computers were situated in different locations, connected by a wireless internet connection. No oral communication was possible between the chat participants, which made it necessary to enter questions, tips, methods and proposals of resolution in written form in order to communicate with the other participant in the joint problem-solving process. The program NetMeeting (Microsoft) is used to facilitate the chat setting. This program enables participants to enter data in two different forms: Alphanumeric data entered using the keyboard appears in the 'chatbox', a special marker is used to enter data on the 'whiteboard' (see Figure 1 for use of terms). The chatbox and the whiteboard appear on screen side by side as two separate windows.

The aim was to induce pupils to communicate the problem-solving process non-orally, in order to investigate the importance of the written product of the collective problem-solving process, as described above. In this way, the chat setting offers a new perspective on a number of fundamental questions concerning the teaching and learning of mathematics. Chatting is a form of interaction that is based on the written word and graphics, but its interactive nature means it also has similarities to spoken interaction. Thus, through the

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medial written form of communication it is possible to gain insight into a conceptually oral situation. Both theoretical and methodological questions on mutually created, written aspects of a mathematical problem-solving process can be examined, since no oral communication between the participants was possible.

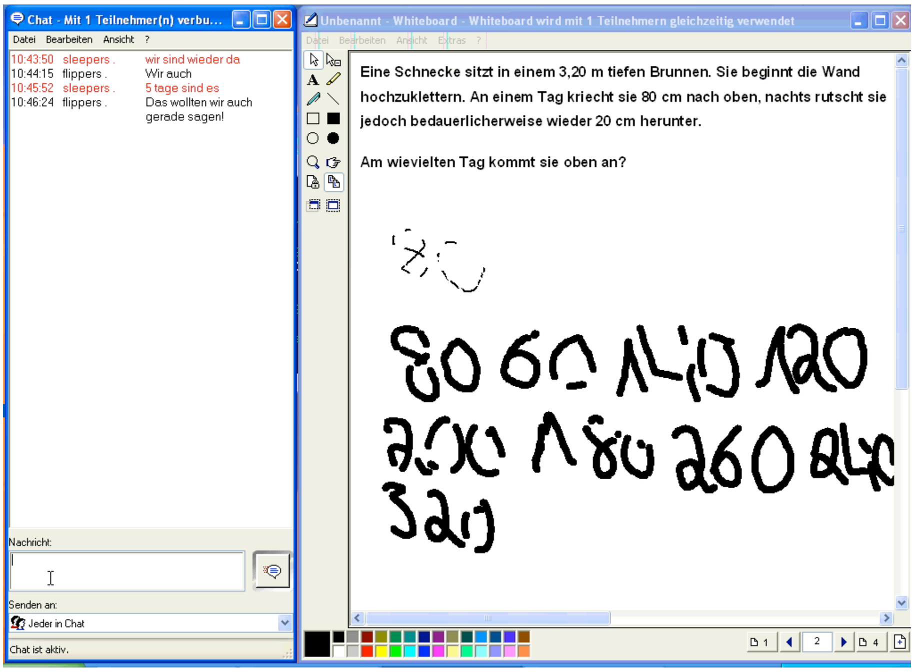

Figure 1. Screenshot of a 'net meeting'

During the research project 'Math Chat', all operations that were carried out on the computer screens and all oral utterances of the participants were recorded. Scenes deemed relevant to the research problem were then transcribed, in order to enable a detailed analysis.

MATHEMATICAL INSCRIPTIONS

In terms of solving problems based on the research of mathematical information and correlations, the participating pupils worked mainly on the written products (of interaction) in the setting described above. I refer to the written-graphical products generated by the pupils in the chat setting as "inscriptions" (Latour & Woolgar, 1986). Latour and Woolgar study the development and evolution of knowledge in laboratories. The different kinds of models, pictures, icons, and notations used in the laboratories are classified by Latour and Woolgar as 'inscriptions'. They describe several characteristics of inscriptions (Latour & Woolgar, 1986; Latour, 1987):

- Inscriptions are mobile because they are recorded in materials and can be sent by mail, courier, facsimile, or computer networks.
- They are immutable during the process of moving to different places. Inscriptions remain intact and do not change their properties.
- The fact that they can be integrated in publications just after a little cleaning up is described as one of the most important advantages of inscriptions.

- The scale of inscriptions can be modified without changing internal relations.
- It is possible to superimpose several inscriptions of different origins.
- They can be reproduced and spread at low cost in an economical, cognitive and temporal sense.
- Inscriptions can be merged with geometry because of their two-dimensional character. Latour (1990) mentions this advantage as the greatest one.

Inscriptions are seen by Latour and Woolgar as a very ductile means of representation that is continuously changing and improving. In this way they represent aspects of the conceptual development during the research process (see also Schreiber, 2004). Latour (1990) talks about "cascades of ever more simplified" (p. 20) inscriptions. Latour's and Woolgar's definition of the term 'inscription' applies exactly to the subject matter being researched in the 'Math-Chat' project. The interest is centered on a detailed analysis of the inscriptional aspect in mathematical interactions, both on the interactive origination of the inscriptions as well as the meaning and importance of the developing inscriptions for the interaction process.

Roth & McGinn (1998) point out that the use of inscriptions is closely interconnected with the social practice in which they originated:

> Inscriptions are pieces of craftwork, constructed in the interest of making things visible for material, rhetorical, institutional, and political purpose. The things made visible in this manner can be registered, talked about and manipulated. Because the relationship between inscriptions and their referents is the matter of social practice … students need to appropriate the use of inscriptions by participating in related social practices. (p. 54)

Herein lies the basis of the interactionist approach in relation to the learning of mathematics, which forms the foundation of this project. What is unique about the approach described here, is that the focus lies on the process of genesis of individual inscriptions: pupils externalize their ideas in a chat-based dialogue using alphanumeric and/or graphic notations. The reactions of their chat partners enable the gradual development of a single inscription into a joint or mutual inscription. The internet chat method is conducive to this process of text compilation, as this process becomes both collective and interactive. This process can be viewed as an important component of the chat-based interaction and it generates the "taken-as-shared-meaning" of the chat partners (Cobb & Bauersfeld, 1995). There are many other publications that deal with interactively created inscriptions (see Roth & McGinn, 1998; Lehrer, Schauble & Carpenter, 2000; Sherin, 2000; Meira, 1995; 2002; Gravemeijer, Cobb, Bowers & Whitenack, 2000; Gravenmeijer, 2002 and Fetzer, 2007), although all of these focus on face-to-face situations. The focus during this project, however, lay solely on an inscription-based communication between the two sides of the chat setting, which was facilitated by the experimental design.

ASPECTS OF CHARLES SANDERS PEIRCE'S SEMIOTICS

For the analysis of the commonly accomplished inscriptions in the chat-based solving processes it is refered to Peirce's sign model. The Peircean sign model is a very differentiated classification and it is applied by some researchers of didactics of mathematics (e.g. Hoffmann 1996, 2003; Dörfler 2004, 2006) as well as by pedagogical researchers (i.e. Zellmer 1979).

Peirce's triadic sign relation

Peirce's triadic sign relation consists of a „triple connection of sign, thing signified and cognition produced in the mind" (CP, 1.372). That which Peirce refers to as „sign" or "representamen" can be understood as the external, visually, aurally or otherwise perceptible depiction of a sign, while the "interpretant" is a sort of inner sign, which is associated with the external perception of the observer. The „object" is to be understood as that which the observer of an external sign believes what was its creator's purpose. For Peirce, these three correlates are integral parts of a sign and none of the three is superfluous. The sign itself only becomes a sign when it is perceived by an observer to be such (CP 2.228).

The foundation of the triadic sign relation

The interpretant is determined by the concepts, theories, habits and skills of the observer, this is what is meant by the 'idea' or 'ground' in the Peircean sign model. The way in which I use this concept in my semiotic analyses (Schreiber 2006), suggests the substitution of this expressions by the term "frame", which is used by Goffman (1974, p. 7). This "ground of the representamen" is to be integrated into an interactionist perspective: each individual creates interpretants against the background of his or her own subjective interpretation experiences and under a specific perspective. Goffman points out the importance of standardization and the formation of a routine during the "definition of the situation" (1974, p. 1f.) and introduced the term "frame" to describe interpretation processes (Goffman 1974, p. 7). These framing procedures can be taken as the "ground of representamen", as defined by Peirce (see figure 2).

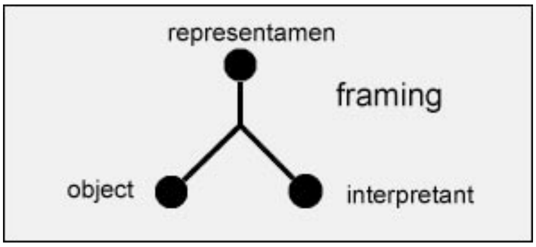

Figure 2: framing as the basis of Peirce's triadic sign relation

The variance of the interpretant is limited by the frame, which is triggered within each individual by the representamen. The representamen does not stand for this object in every regard, but rather only in regard to an activated frame. The connection of Peirce's semiotics with Goffman's frame analysis makes the empirical analysis of frames on the detailed level of Peircean triads possible.

The chainig process

Peirce describes meaning as a constantly developing process, in which the interpretant of a given triadic sign relation becomes the representamen of a further triad.

> Anything which determines something else (its *interpretant*) to refer to an object to which itself refers (its *object*) in the same way, the interpretant becoming in turn a sign, and so on *ad infinitum*. (CP, 2.303; italics Peirce's own)

Peirce believed that every interpretant within a triad could also be interpreted within another (figure 3). This continuous process of semiosis is potentially endless. It cannot be brought to an end but can be interrupted (CP, 5.284). Peirce noted that the identification of a first or final sign is not possible in this context.

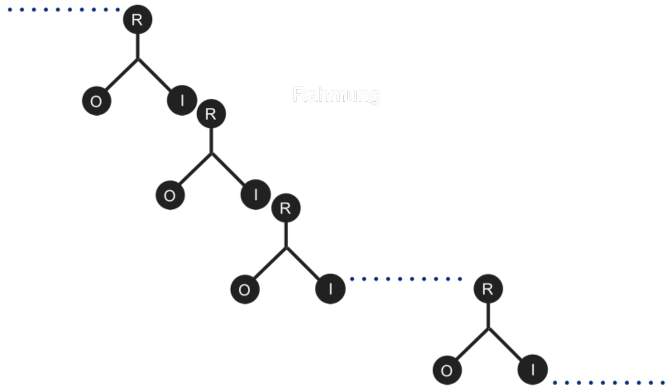

Figure 3: The unlimited process of semiosis. (Schreiber 2010, p. 37)

However, the processes I have reconstructed are not in all cases linear. According to my analyses, there are interpretants that serve as representamen in the following triad, and groups of sign triads that serve as representamen in a new sign triad. Furthermore, there are sign triads that are connected with one another, because they correspond with the same representamen. This is depicted in Figure 4 witch is a detail of the semiotic process card (see also Figure 5).

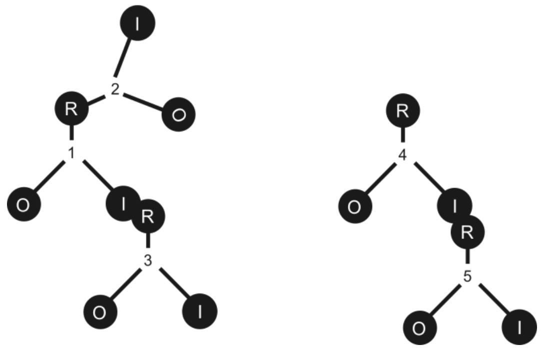

Figure 4: The complex semiotic process.

Due to the non-linear alignment of the process in my example, I rejected the term 'chaining' and chose instead to use the term 'complex semiotic process'. In my opinion, this term reflects the development of the interpretation process accurately, although these processes are partially linear.

SEMIOTIC PROCESS CARDS

Interaction analyses were carried out based on written transcriptions of the 'Math Chat' sessions. These analyses allow a detailed description of the sessions, which in turn facilitate a summarized interpretation (see Krummheuer & Naujok 1999). These interpretations were then used to describe the complex semiotic process. The results of these descriptions are demonstrated below as 'Semiotic Process Card' (henceforth SPC). In the SPC the elements described in the sections above are all accounted for: Peirce's triadic sign relation, embedded in an underlying 'frame', and its development as part of a complex semiotic process. The format of the SPC is illustrated in Figure 5. The SPC used here for demonstration will be analyzed in detail on the conference.

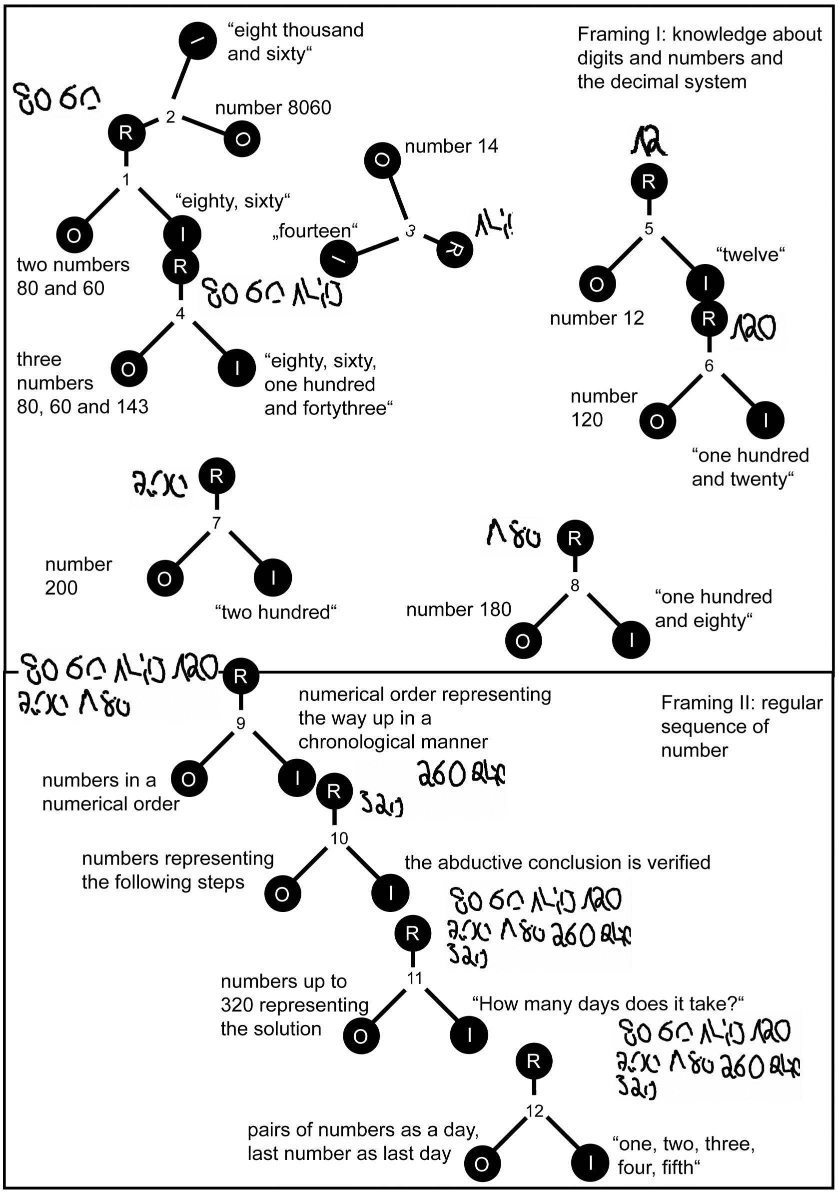

Figure 5: A Semiotic Process Card

The SPC should be read from above to below and in general from left to right. In order to make orientation easier, the triads are chronologically numbered. The letters 'R' for representamen, 'I' for interpretant and 'O' for Object indicate which part of the triad is being referred to. Words or text and images will be used to display and demonstrate the three correlates. In some cases the interpretant of a triad will be supplemented by further aspects and thus become a new representamen (e.g. figure 5, triad 3 & 5).

The framing, which is recreated through the interaction analysis is referred to in the semiotic analysis and is indicated in the SPC. After the compilation of several SPC, the various reconstructed frames were able to be classified as ”mathematical”, “argumentative”, “formal” and “social” frames (see Schreiber 2010).

The complex semiotic process is represented by the configuration of the triads. As figure 5 shows, the process can progress very differently. Where the progress is linear, the subsequent triad with the correlate 'representamen' is positioned at the correlate 'interpretant' of the preceding triad (see Figure 5, triads 5 and 6). Where two parts of the process relate to the same representamen, I assign the representamen in question to two triads (see Figure 5, triads 1 and 2). If the representamen of a triad corresponds with the entire previous process, from an accumulation of sub-processes, then the correlate representamen is placed on a line of the box which underlies the hitherto existing process (e.g. Figure 5, triad 9).

The empirical example related to the Semiotic Process Card in Figure 5 and also some results and findings will be presented on the conference.

References

Cobb, P. & Bauersfeld, H. (ed.), (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum.

Dörfler, W. (2006). Diagramme und Mathematikunterricht. [Diagrams and mathematics instruction]. *Journal für Mathematik-Didaktik 27*(3/4), 200-219.

Dörfler, W. (2004). Mathematical reasoning: Mental activity or practice with diagrams. In M. Niss (ed.), *Proceedings of the Tenth International Congress on Mathematical Education*. Roskilde: IMFUFA, Roskilde University.

Fetzer, M. (2007). *Interaktion am Werk. Eine Interaktionstheorie fachlichen Lernens, entwickelt am Beispiel von Schreibanlässen im Mathematikunterricht der Grundschule. [Work-based interaction. A theory of interaction of subject learning, developed using writing impulses in primary mathematical education].* Bad Heilbrunn: Klinkhardt.

Goffman, E. (1974). *Frame analysis. An essay on the organisation of experience.* Cambridge, MA: Harvard University Press.

Gravemeijer, K. (2002). Preamble: from models to modeling. In K. Gravemeijer et al (ed.), *Symbolizing, modelling and tool use in mathematics education* (pp. 7-22). Dordrecht: Kluwer Academic Publishers.

Gravemeijer, K., Cobb, P., Bowers, J. S., & Whitenack, J. (2000). Symbolizing, modelling and instructional design. In P. Cobb, E. Yackel, & K. Mc Clain (ed.), *Symbolizing and communicating in mathematics classrooms* (pp. 225-273). London: Lawrence Erlbaum.

Hoffmann, M. (2006). What is a „semiotic perspective“, and what could it be? Some Comments on the contributions to this spezial issue. *Educational Studies in Mathematics 61* (1-2), 279-291.

Hoffmann, M. (ed.). (2003). *Mathematik verstehen. Semiotische Perspektiven. [Understanding mathematics. Semiotic perspectives]*. Hildesheim: Franzbecker.

Hoffmann, M. (1996). *Eine semiotische Modellierung von Lernprozessen. [A semiotic modeling of learning processes]*. Occasional Paper 160, retrieved from http://www.unibi.de/idm/serv/dokubib/occ160.pdf

Krummheuer, G. & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung. [Basic principles and examples of interpretativ classroom teaching research]*. Opladen: Leske und Budrich.

Krummheuer, G. (2000). Mathematics learning in narrative classroom cultures: studies of argumentation in primary mathematics education. *For the Learning of Mathematics, 20*, 1. Kingston, Ontario, Canada: FLM Publishing Association, 22-32.

Latour, B. (1987). *Science in action: How to follow scientists and engineers through society.* Cambridge, MA: Harvard Press.

Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (ed.), *Representation in scientific practice* (pp. 19-68). Cambridge. MIT Press.

Latour, B. & Woolgar, S. (1986). *Laboratory life. The construction of scientific facts.* 2nd edition. Princeton: Princeton University Press.

Lehrer, R., Schauble, L. & Carpenter, S. (2000). The interrelated development of inscriptions and conceptual understanding. In Cobb, P.; Yackel, E. & Clain, K. (ed.), *Symbolizing and communicating in mathematics classrooms* (pp. 325-360). London: Lawrence Erlbaum.

Meira, L. (1995). The microevolution of mathematical representations in children´s activities. *Cognition and Instruction, 13* (2), 269-313.

Morgan, C. (1998). *Writing Mathematically – The Discourse of Investigation.* London: Falmer Press.

Nöth, W. (2000). *Handbuch der Semiotik. 2 – vollständig neu bearbeitete und erweiterte Auflage. [Handbook of semiotics. Second edition]*. Stuttgart, Weimar: Verlag J.B. Metzler.

Peirce, C. S.: *Collected Papers of Charles Sanders Peirce Volumes I-VI*, edited by C. Hartshorne & P. Weiss (1931-1935). Cambridge, MA: Harvard University Press.

Peirce, C. S.: *Collected Papers of Charles Sanders Peirce Volumes VII-VIII,* edited by A. W. Burks (1958). Cambridge, MA: Harvard University Press.

Pimm, D. (1987). *Speaking mathematically. Communication in mathematics classrooms.* London, New York: Routledge.

Roth, W.-M. & McGinn, M. (1998). Inscriptions: Toward a theory of representing as social practice. *Review of Educational Research, 68* (1). 35-59.

Schreiber, Chr. (2010). *Semiotische Prozess-Karten – chatbasierte Inskriptionen in mathematischen Problemlöseprozessen. [Semiotic Process Cards – chatbased inscriptions in mathematical problem-solving processes]*. Dissertation. Münster: Waxmann

Schreiber, Chr. (2006). Die Peirce'sche Zeichentriade zur Analyse mathematischer Chat-Kommunikation. [Using the Peircean sign-triad to analyse mathematical chat communication]. *Journal für Mathematikdidaktik 27* (3/4). Wiesbaden: Vieweg + Teuber Verlag, 240-267.

Sherin, B. (2000). How students invent representations of motion – a genetic account. *Journal of Mathematical Behavior, 19* (4), 399-441.

Zellmer, S. (1979). *Pädagogische Semiotik in der Mathematik-Didaktik. [Pedagogical semiotics in the didactics of mathematics]*. Baden-Baden: Agis.