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# ACTIVITIES FOR LEANING TRANSFORMATION BASED ON VISUALIZATION

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*The idea of transformation is very important for electronic engineering, computer science, computer graphics, etc. But at the secondary school in Japan, idea of transformation is not emphasized and students learn mainly the conditions of congruence and similarity between two triangles. Therefore, most students compare the shape of two figures and then only think about the positional relation between each side, angle, etc. It is very hard for students to think that one figure is transformed to the other. We will propose the activities to emphasize the idea of transformation through visualized circle inversion. In this activities even junior high school students drew Steiner's rings by thinking the transformed figure how inverted an original one.*

*Visualization, DGS, Transformation, circle Inversion, Steiner's rings.*

## 1. INTRODUCTION

For more than 20 years, many kinds of educational software have been developed to visualize mathematics and show mathematics dynamically. And geometrical figures have been dynamically visualized and graphs, algebraic expressions and data for leaning functions have been visually integrated. Then new approaches with using these kinds of software have been developed in school mathematics. Then students' activities have been changed from manipulating difficult mathematical expressions or writing deductive proof to exploring a relation between figures or behaviors of functions and thinking inductive proof. Then by using technology new interesting activities are developed and students' activities are expanded. Students jump out to a new mathematical world. And many interesting activities to visualize mathematics will be introduced more and more. In this paper, we would like to show activities for a circle inversion to be visualized and to be worked by from secondary students to college ones.

## 2. LEARNING TRANSFORMATION.

The idea of transformation is very important for electronic engineering, computer science, computer graphics, etc. In the new Japanese Education Ministry guidelines announced in 2008, sixth grade students are expected to understand "stretching", "shrinking" and "symmetry". In junior high school, the first grade students learn "translation" and "rotation" and it is emphasized to understand how to move one figure to another. And second grade students learn "congruence" and third grade students learn "similarity". Each topic has been taught separately. The Japanese word "ido", which means "move from one position to another", is used for transformation of figures. Students mainly learn the conditions of congruence and similarity for two triangles. Therefore, most students compare the shape of two figures, and then only think about the positional relation between each side, angle, etc. They never think one figure is transformed to the other. Students major in engineering course in a high school and a university have a hard time to learn various transformations. In these days, it has become easy to visualize the relation of transformations using dynamic geometry software (DGS). Many materials using technology for learning translation, rotation, reflection, line symmetry, point symmetry, dilation and similar transformations which are taught in elementary and secondary schools have been developed. Through these materials students can see how each point is transformed, understand the relation between points on the figure and find what parts are variant or invariant. The experience of this kind of activities will help them to learn advanced math. There are very few materials for learning an inversion. An inversion causes unexpected results and unpredictable situations. Inversion is not taught in elementary and secondary schools. But there are many interesting topics related to inversion and very interesting activities for investigation are expected and it is available for even secondary students to learn it, when they use technology. And these activities continue to advanced mathematics.

In the "Erlangen Program", Klein, Felix said geometric properties are not changed by the transformations of the principal group (Felix Klein, 1892-1893). And, conversely, geometric properties are characterized by their remaining invariant under the transformations of the principal group. It means transformation is the way to find the invariant properties in a geometric figure. Therefore, we expect students to focus on the variant or invariant properties of a geometric figure by transformation.

## 3. WHAT IS CIRCLE INVERSION?

Draw a point P and a reference circle R (T, t). In this paper, "circle R (T, t)" means a circle R which has the central point T and radius t. When point P' exists on the line TP with (TP) · (TP') = $t^2$, point P ' is the inversion point of point P with respect to the circle R. This circle R used for the definition is called "the reference circle". Fig. 1(a) shows point P is inside the reference circle and Fig. 1(b) shows point P is outside the reference circle.

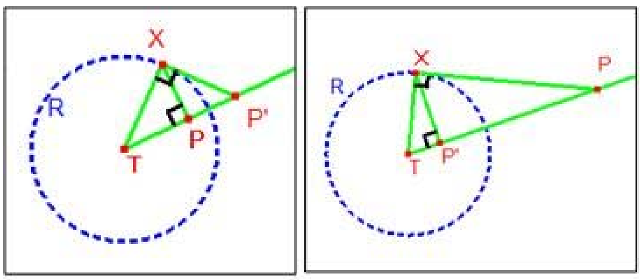


Fig. 1(a) and 1(b) Point P is inside or outside the circle

In the next section we would like to show activities which assist students to learn inversion. Students will learn how to draw these beautiful figures from easy simple figures. Students will find rules for the relationship between an original figure and the transformed figure by inversion through activities. And then Steiner's rings will be able to draw by students even though they are junior high school students. We tried to do these activities with junior high school students and university students

## 4. ACTIVITIES FOR LEARNING INVERSION

### (1) Draw inverted figures and Observe the basic movement of inversion

When a point P moves on a line L, the locus of point P’ which is inverted using the reference circle R is a circle inside the circle R (Fig.2 (left)). When a point P moves on a circle, the locus of the point P’ is also the circle inside the circle R (Fig.2 (middle)). And when the point move on a triangle ABC, the locus of P’ is the funny shape inside the circle R (Fig.2 (right)). This figure must be made by the part of arc to be transformed by the each side of the triangle. When the shape of triangle is changed a different shape can be shown. And students draw and observe these figures and find this kind of inversion keeps any big figure in the given the reference circle. Students examine these drawing by Cabri’s function “inversion”, “locus”, ”trace” and “animation” besides drawing tool.

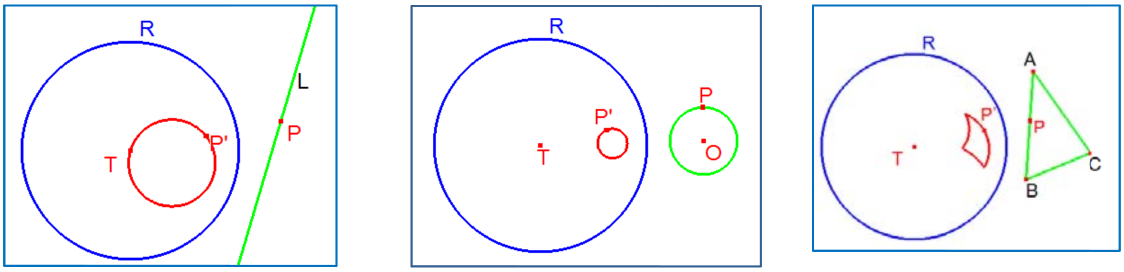


Fig. 2 inversion of a point on a line, on a circle and a point on a triangle

### (2) Categolize relations between an original figer and thn transformed one

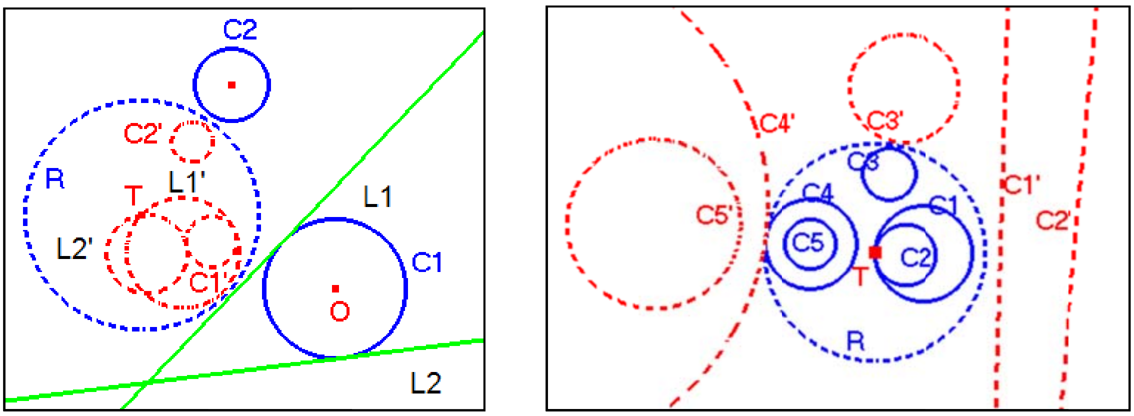


Fig. 3 original figures outside of the reference circle R or inside it

We ask students to categorize the relation between an original figure and the transformed one. At the first step, students draw two lines and two circles outside the reference circle R and draw transformed figures (Fig. 3(left)). Then, students are asked if there are any differences between the circle C1’ and C2’ or L1’ and L2’? At the second step, students draw many circles inside the reference one and then draw the inverted figure for each circle (Fig. 3(right)). Students categorize the position and shape of transformed figures with the original circle. Students can find there is the straight line transformed by the original circle at the special position.

**(3) Explore the position of the original circle for transformed one.**

In this step, students move one original circle on the line through the center of the reference circle R and draw the trace of the transformed figure. They can see the area of transformed figures (red part of the Fig.4) . When they use the “animation” in Cabri, they can see the locus of transformed figures for all through the line. Students find that there are parts where transformed figures do not exist. And then they move the origial circle on the line not through the center of the reference circle . They find that an inversion confines any circle which exixts far away in infinity to the closed area(red part of the Fig.5) and see this movoment interestingly.

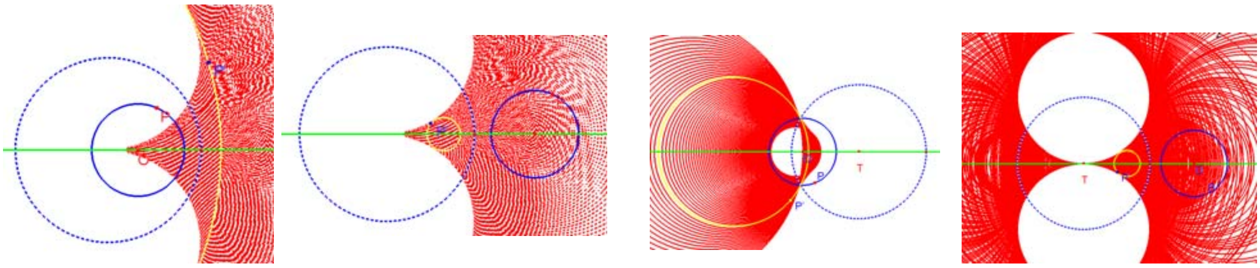

Fig.4 trace of transformed figures by moving the original circle O on the line through the center of the reference circle

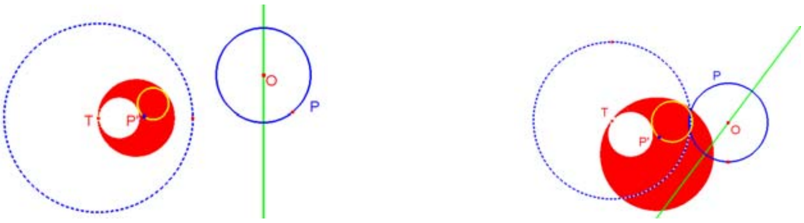


Fig. 5 trace of a transformed figure by moving the original circle O on the line not through the center of the reference circle

They can also find there is the position where the original circle O overlaps with the inverted one. Or students will find that there is the position where an original figure is transformed to a line. If they are high school or upper level studnts, they can use the Cartesian coordinate and then make sure it by the eqauation showed by the Cabri’s “equation and coordinate”(Fig. 6) . And then they find the special case, i.e. when the original cirle is through the center point of the reference circle, transformed figure is line(Fig.7). They think the center of this transformed cirle must be the point at infinity. This fact must be connected with spherical geometry. And they can also generalize this relation and get the equation by themselves. When one student found this fact he said “man who found this geometry must have drawn the figure on the earth not on the desk!”

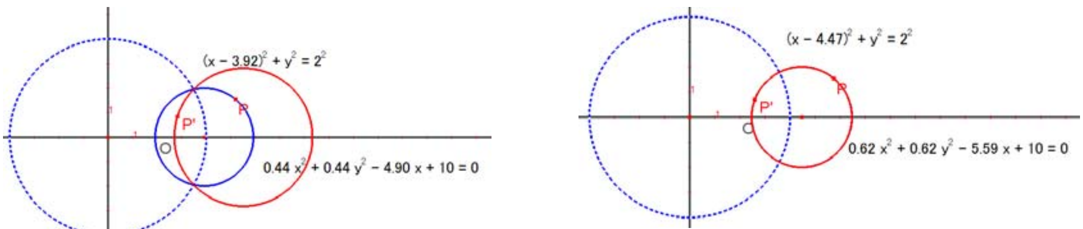


Fig. 6 a transformed figure overlapped with the original circle.

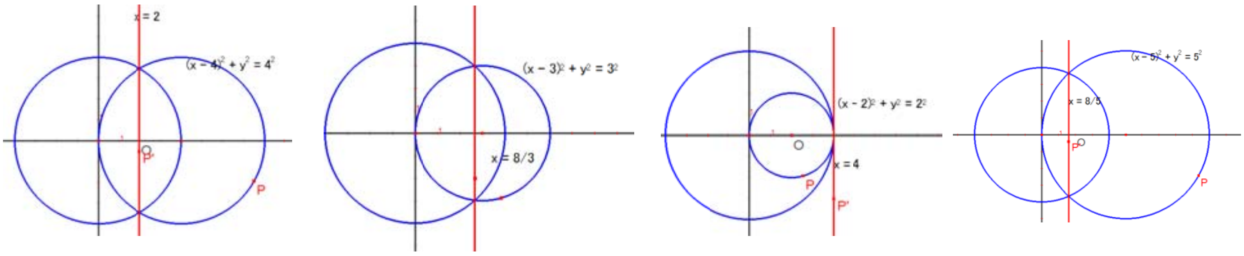


Fig. 7 the examples of the transformed figure which becomes a line

**(4) Draw Steiner's rings**

Through these activities, students learn transformed figures have important rules of size and location between the original figure and the reference circle. And also they find that circles with the same size between parallel lines are transformed to circles between two circles. Moreover they show that curves transformed by inversion keep touching each if original figures are tangent each other. Then even junior high school students could draw Steiner's rings like the fig. 8.

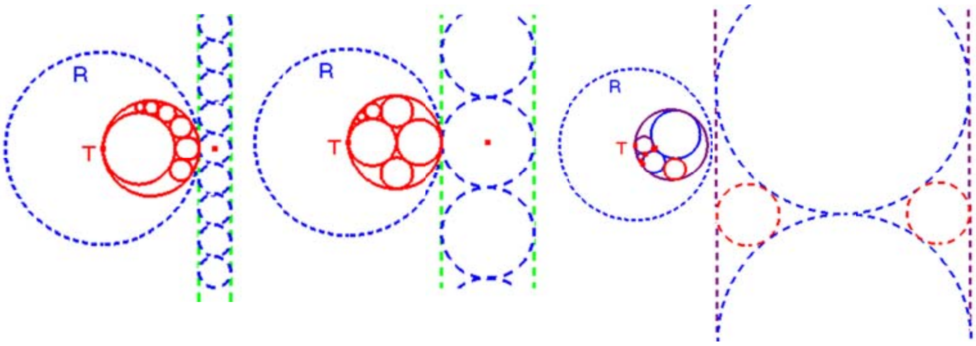


Fig. 8 examples of the Steiner's rings drawn by inversion.

## 5. RESULT AND DISCUSSION

In Japanese secondary schools, students learn translation, rotation and similarity in Euclidean geometry and learn these topics separately. When they use Dynamic Geometry Software like Cabri Geometry or GeoGebra, they can understand the relation between these transformations. And even secondary students can see the behaviours of inversion through these activities we showed. As these activities help for students to understand transformation visually and dynamically, we can expand these actives to solve Wasan Problem and to see spherical geometry. When we use GeoGebra, we can draw geometrical figures and get an algebraic expression in a complex plane. So we can also expand this activity to express the transformation of a complex plane (Fukuda and Kakihana, 2009). If we handle inversion using complex functions, inversion can be treated as a linear fractional transformation and it's described very simply. The software facilitates drawing the transformed figures by the expression of complex functions. Then we can use both geometrical and analytical strategies to solve a problem

Circle inversion is an important and powerful geometrical method. In addition, when we express this transformation by complex number, it's a good introduction to "complex plane" which Johann Carl Friedrich Gauss connected "complex number" with. Although, complex numbers had been thought of as only a symbol to express a number formally, they were later connected to geometry and this idea has expanded to "complex analysis", electromagnetism and other applications. We will continue to develop materials of learning transformation for this advanced mathematics

## 6. CONCLUSION

In this paper we showed the examples which difficult problems become much more tractable and one activity is expanded to outside of regular course of study by using technology. The use of technology in education started from CAI for helping basic learning and has changed to a tool to explore mathematics and construct their knowledge. In these days Internet can be used any place and any time and a mobile terminal and game equipment are used in education. Our activities are able to do on the Moodle course through Internet. The game equipment is used for practice of basic calculations in a class (Suehiro, 2011). And also technology is used to present students' idea or communicate each other even in mathematics education. Materials and activities with technology will change and expand students' mathematical world more and more

### Software for Education

Cabri Geometry II Plus. Dynamic Geometry Software, Product of Cabrilog. http://www.cabri.com/

GeoGebra. Developed by Markus Hohenwarter, It's a multi-platform dynamic mathematics software for all levels of education that combines arithmetic, geometry, algebra and calculus. Recently includes spread sheets. Free download from following website. http://www.geogebra.org/

### References

Iwata Shikou, ed. (1971), *Comprehensive dictionary in geometry*, Tokyo, Japan. Maki-shoten

Felix Klein (1892-1893). *A comparative review of recent researches in geometry*, trans., M. W. Haskell, *Bull. New York Math, Soc. 2*, 215-249.

Electronic version: http://math.ucr.edu/home/baez/erlangen/erlangen_tex.pdf,

http://math.ucr.edu/home/baez/erlangen/

Tanigutchi, H. and Okumura,Y. (1996), *Invitation to hyperbolic geometry, with complex number*, Tokyo, Japan. Baifukan

Fukazawa Hidetosi & Dan Sokolowsky (1991), *Japanese Geometry –How many can you prove?* Tokyo, Japan. Morikita Syuppan.

Fukazawa Hidetosi and Dan Sokolowsky (1994), *Japanese Geometry –How many can you prove? Vol. 2: Geometric Problems (Triangles, Circles and ellipses).* Tokyo, Japan, Morikita Syuppan.

Fukuda Chieko & Kakihana Kyoko (2009), Solving Wasan Problems with an Inversion Method- Development of teaching materials for approaching complex geometry -, Proceeding of ATCM15, pp. 153-162

Steen, L.A. (1990), Pattern, in “On the Shoulder of Giants- New Approaches to Numeracy -”, National Academy Press, pp.1-10

Suehiro, Satoshi(2011), Role with new ICT equipment in mathematics class- Basic scholastic attainments by use of “Nintendo DS classroom”, the proceeding of 35th Japan Society for Science Education

Yerushalmy, M. (1999), Making Exploration Visible: On software Design and school Algebra Curriculum, International Journal of Computers for Mathematical Learning, Vol. 4, Nos.2-3, 1999, pp.169-189