12th International Congress on Mathematical Education
Program Name XX-YY-zz (pp. abcde-fghij)
8 July – 15 July, 2012, COEX, Seoul, Korea (This part is for LOC use only. Please do not change this part.)

EFFECTIVE USE OF SAGE-MATH IN VISUALIZATION OF LINEAR ALGEBRA CONCEPTS

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In this paper, the prime aim is to introduce visualization of linear algebra concepts which have been developed with Sage-Math (a free web site to do mathematical computations). Sage-Math has been developed at the University of Washington in 2005. Sage-math includes many features and functions which are based on Web-Mathematica and enough to surpass. Students don't need any program installed on their computer, since they can access any avaliable Sage server in the world from anywhere at any time to perform their mathematical computations and simulations. These can be used for students' better understanding in college mathematics education as well as a secondary school mathematics education. You can find these contents for your test or teaching in the following web address http://matrix.skku.ac.kr/2012-sage/sage-la

Key words: Mathematics Education, Sage-Math, Linear Algebra, Visualization

Introduction

Linear Algebra is the first abstract mathematics subject for most of new college students. Hence, most of students face some difficulties to deal with various novel mathematical

concepts. We know that there are many effective ways to deliver more mathematical contents and ideas to students by using the state of art 21st century multimedia educational environment. In these days our students prefer to engage in learning of mathematical concepts by visual understanding. Visualization makes it easy to understand the abstract concepts easy. It is well known that audiovisual learning content and teaching methods not just enhance in-class chemistry but also ensures an intuitive understanding of concepts (Chae, 2001, Lee & Ham, 2005, Neil Pitcher, 1991, Shin & Lew, 1999).

Sage-Math is a server program published on February 24, 2005, being a software for solving mathematical problems in various areas including Calculus, Linear Algebra, Differential Equations, Discrete Math, Number theory, Geometry and Numerical Analysis etc. Now this Sage software is widely used in Mathematics, Engineering, and other areas of science. And it is freely available in 24 hours at anywhere. Users have shared their knowledge on using it (http://math1.skku.ac.kr) (Gray, 2008, Ko et al., 2009). Students don't need any program installed on their computer, since they can access any avaliable Sage server in the world from anywhere at anytime to perform their mathematical computations and simulations. While we were using a new Sage-Math, we found some possibility that we could utilize it for student's better understanding of Linear Algebra concepts (Kim, Bak & Jung, 2007) .

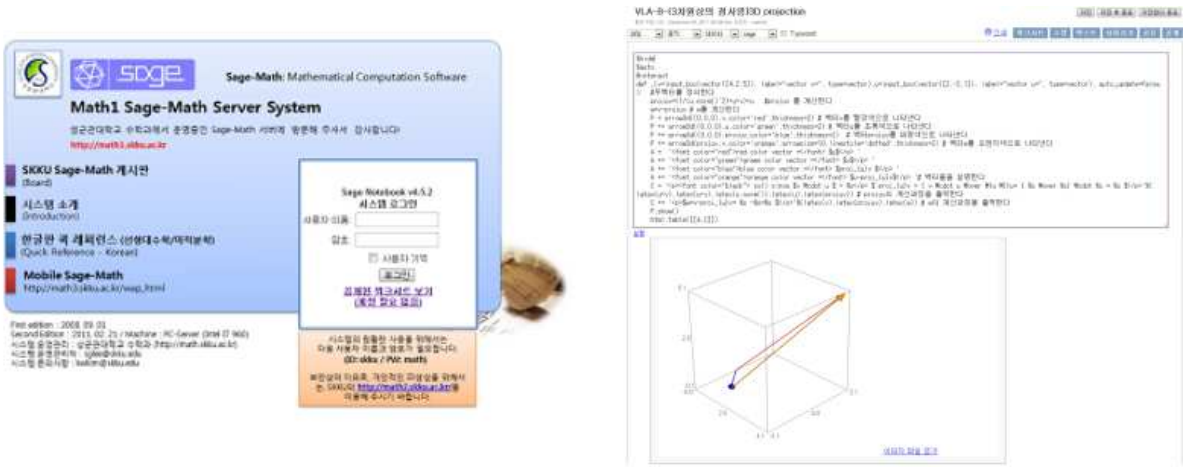


Figure 1. Sage-Math Korean version (http://math1.skku.ac.kr)

Visualization of mathematical concepts is a process that uses mathematical simulations for discovery and understanding of mathematical concepts. Mathematical visualization isn't just a supplementary method for illustrating what students are interested in but it also contributes in a major way to a complete understanding and utilization of mathematical concepts. Furthermore, it provides a way to find a way for discovery of new problems. Understanding of mathematical concepts can be achieved in both algebraic and geometric approach. If students can utilize visual simulation in

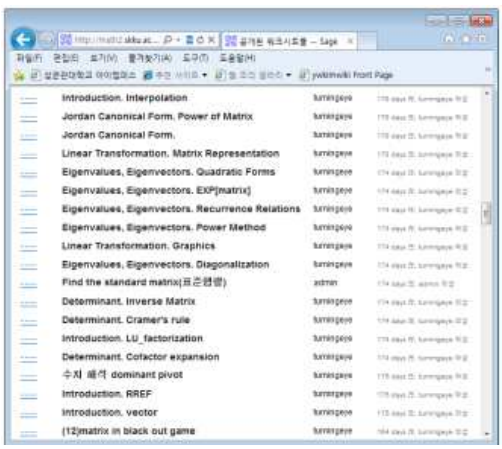


Figure 2. http://math2.skku.ac.kr/pub

their understanding of linear algebra concepts, then we may expect a lot more creative thinking among students (Lew & Lee, 1993). We can help students to extend those ideas to general concepts. Now we have many choices on visualization software of mathematics. We considered Sage-Math which is free open source software because we found it is competitive to all other commercial software. And we can share published programs on the web (http://math2.skku.ac.kr/pub). Furthermore, we were able to use our many preprogramed commands in it, so students do not have to have language barrier in using it.

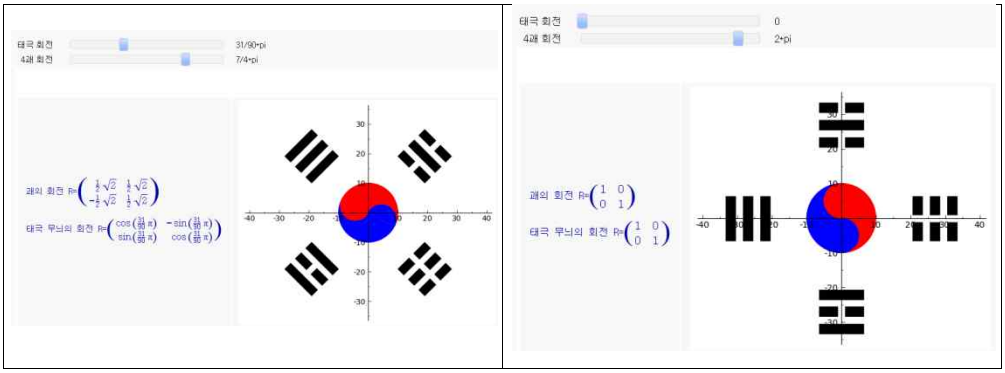


Figure 3. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/207.htm

Visualization of Linear Algebra with Sage

Here, we show some examples of Sage-Math visualization on several important concepts in linear algebra text book such as (Lee, 2009) and (Anton, 2002). In addition, we can easily extend our experience to other discipline in mathematics.

① Vector equation of a plane

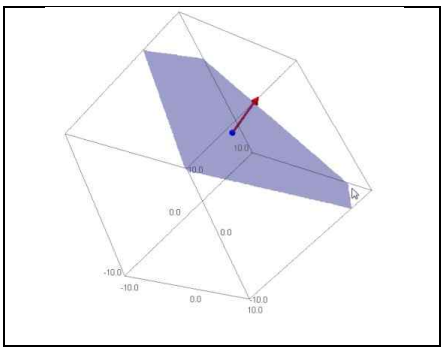


Figure 4. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/260.htm

② Linear Equation

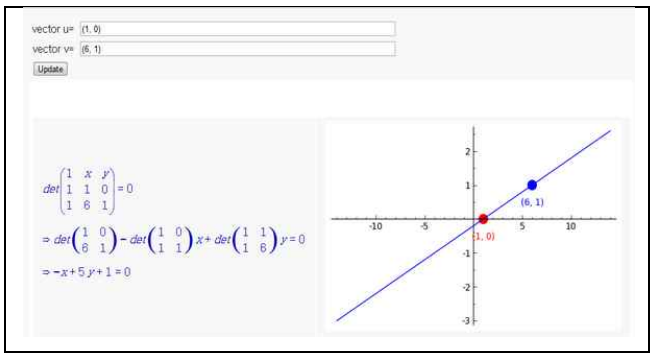
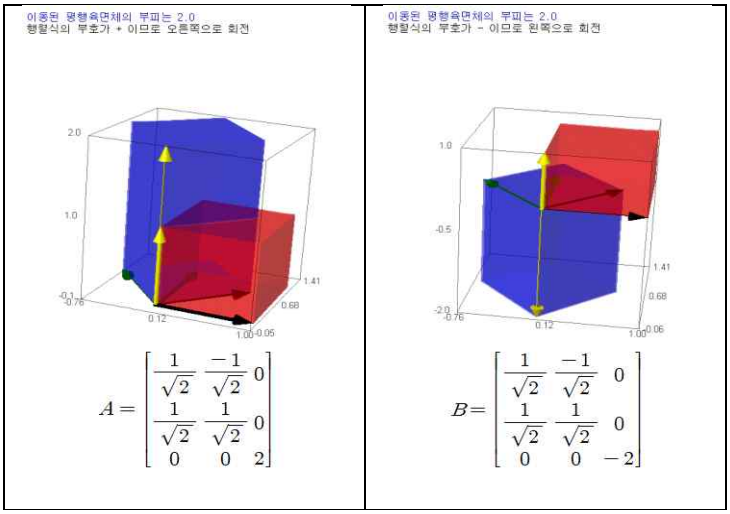


Figure 5. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/195.htm

③ Volume of parallelepiped



$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Figure 6. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/198.htm

④ Linear Combination in R^3

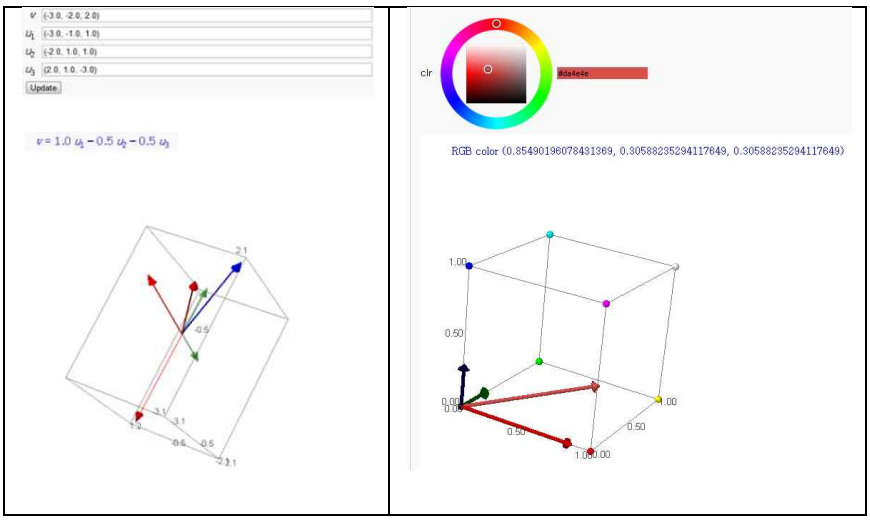


Figure 7. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/163.htm
Figure 8. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/179.htm

⑤ Dilation

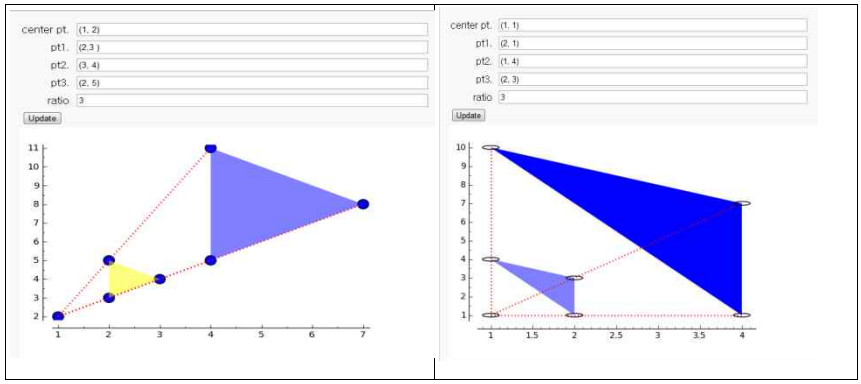


Figure 9. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/202.htm

⑥ Geometric transformation

right shear	x-axis symmetry	right shear	down shear

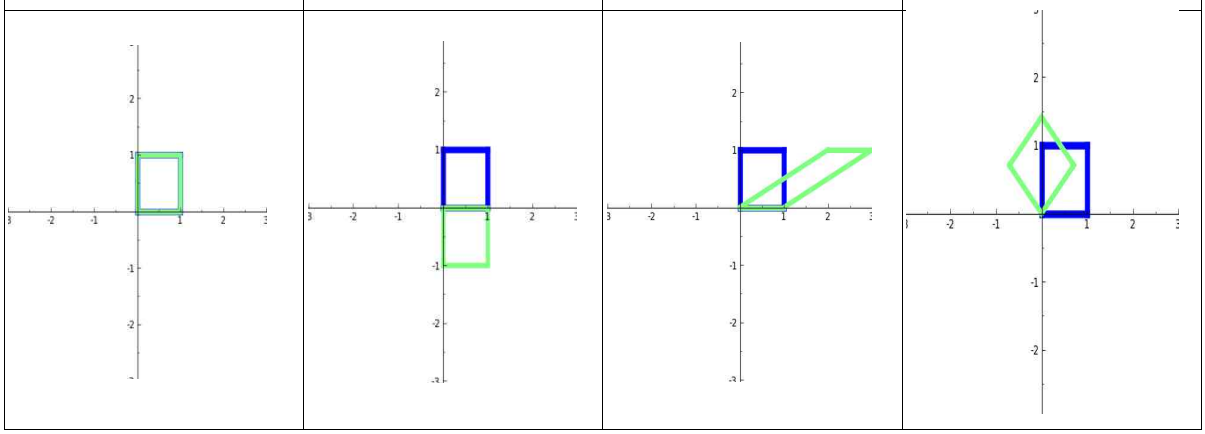


Figure 10. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/353.htm

⑦ Eigensystem

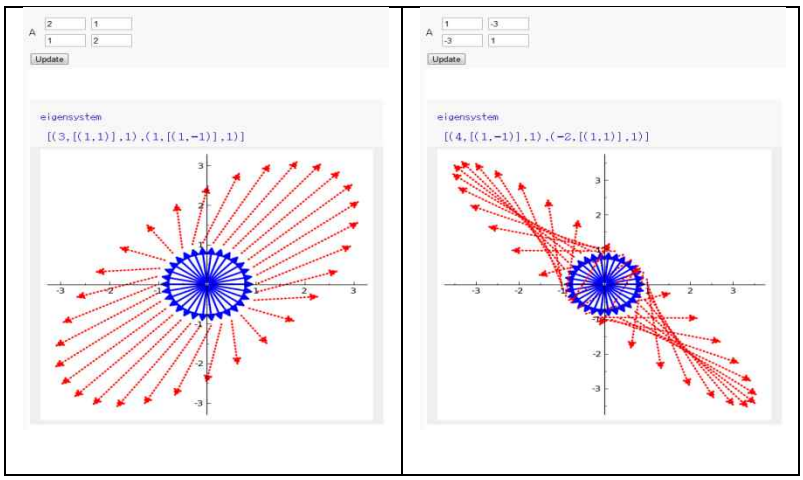


Figure 11. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/352.htm

⑧ Markov chain

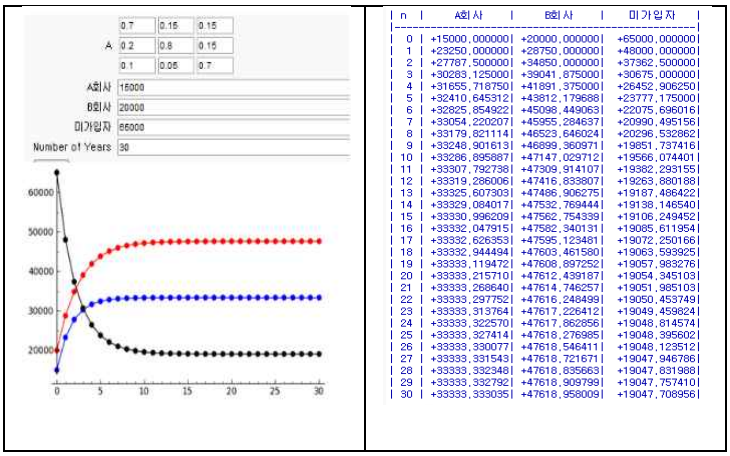


Figure 12. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/289.htm

⑨ Least squares line

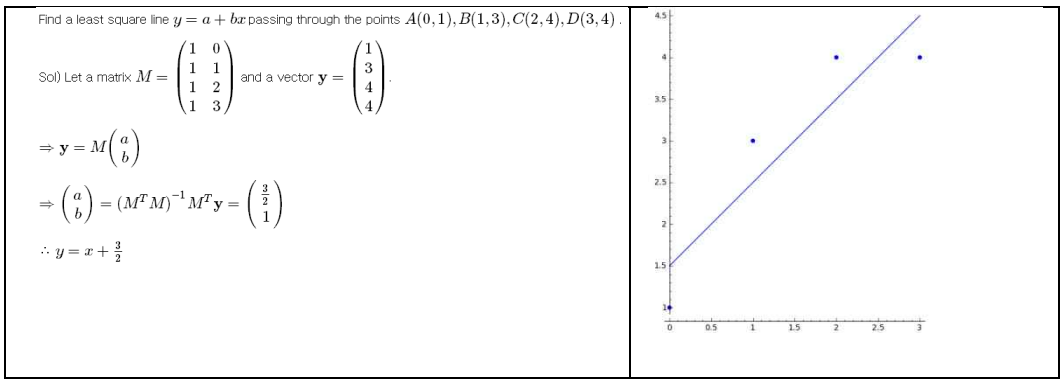


Figure 13. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/288.htm

⑩ Application of Linear Transformation

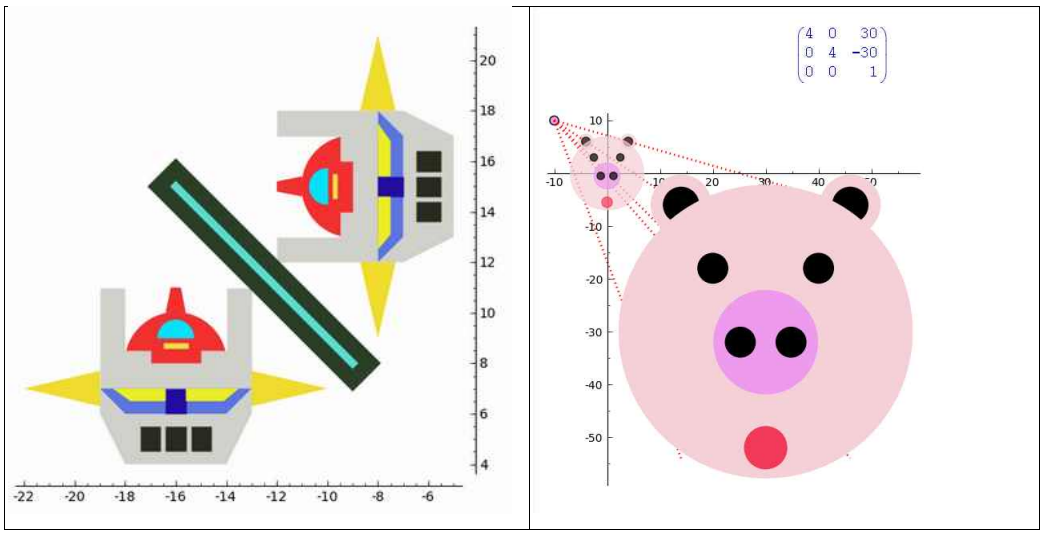


FIG 14. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/354.htm
FIG 15. http://matrix.skku.ac.kr/2012-sage/sage-la/visual/349.htm

Conclusion

Students can save times to learn or memorize most of Sage commands and programming languages from http://matrix.skku.ac.kr/2012-Sage/visualization.htm. They only have to find in the organized web page or published pages, copy and paste their commands, they may revise when it is necessary. <Table 1> shows the address of visualizations in linear algebra concepts

Table 1: Address of visualizations in linear algebra concepts (http://matrix.skku.ac.kr/2012-sage/sage-la/visualization.htm)

Title	Address
Vector Sum	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/194.htm
Scalar Multiplication	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/363.htm
Vector equations of plane	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/260.htm
Linear Equation	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/195.htm
Plane Equation	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/204.htm
Area of triangle	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/185.htm
Volume of parallelepiped	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/198.htm
Eigensystem	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/352.htm
Linear Combination in R^2	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/193.htm
Linear Combination in R^3	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/163.htm
Dilation	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/202.htm
Linear Transformation	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/353.htm
Matrix Multiplication	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/238.htm
Projection in R^2	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/183.htm
Projection in R^3	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/184.htm
Markov chain	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/289.htm
Least squares line	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/288.htm
Quadratic form	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/351.htm
RGB Color Space	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/179.htm
Application- Reflection	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/354.htm
Application- Rotation	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/207.htm
Application-Pororo	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/206.htm
Application- Dilation 1	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/355.htm
Application- Dilation 2	http://matrix.skku.ac.kr/2012-sage/sage-la/visual/349.htm

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