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FROM A VISUAL TO A SYMBOLIC OBJECT IN ALGEBRA AND GEOMETRY: ERP STUDY WITH MATHEMATICALLY EXCELLING MALE ADOLESCENTS

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In this paper we performed a comparative analysis of brain activity associated with transition from visual objects to symbolic objects in algebra and geometry. Algebraic tasks required translation from graphical to symbolic representation of a function whereas tasks in geometry required transition from a geometrical object to symbolic representation of its property. Geometry and Algebra Tests were designed of batteries of short choice-reaction tasks. 32 right-handed male-students who excel in mathematics were chosen for comparative data analysis. We found that non gifted (NG) participants had higher brain activity than their gifted (G) counterparts. This difference was significant in the first stage of the task, 300-400 ms post stimulus at parieto-middle areas of the cortex. Moreover, higher brain activity was found during the geometry test as compared to the algebra test.

Key words: Mathematical giftedness, Brain activity, Algebra, Geometry, Visual and symbolic representations

INTRODUCTION

A considerable body of research has been conducted towards understanding the neural foundation of mathematical cognition (e.g. Dehaene et al., 2003; Santens et al., 2010, Dankner & Anderson, 2007). In addition, there has been extensive neuroscientific research on human intelligence including individual differences in general intelligence (e.g. Deary et al., 2010) as well as on mathematical giftedness (O'Boyle, 2005). However, these studies have not gone beyond arithmetic, logic and mental rotations. That is why we focus our attention on brain activation associated with solving advanced mathematical tasks by adolescents excelling in mathematics who differ in their general giftedness level.

BACKGROUND

Algebra and geometry in mathematics education

Mathematics educators generally differentiate between five (main) types of objects in mathematics (NEA, 2003): Number and Quantity, Shape and Space, Pattern and Function, Chance and Data, and Arrangement. Among the skills considered to be essential for successful mathematical performance they name the following mathematical actions: Modeling and Formulating, Manipulating and Transforming, Inferring and Drawing

Conclusions, and Communicating. These actions may be presented differently in different mathematical tasks; however, generally speaking, students are required to be competent in all of these skills (NEA, 2003). Mathematics education psychologists have collected a wealth of research data in the fields of learning and teaching algebra, numerical thinking, measurement and complexity of learning geometry (e.g., Gutierrez & Boero, 2006). Our study focuses on translations between visual and symbolic representations in two fields in school mathematics: algebra and geometry. Among topics which are central in studying school algebra are functions and multiple representations (Kieran, 2006) while in studying geometry visual reasoning and mathematical deduction are among the main elements focused on in mathematics education research (Owens & Outhred, 2006).

Mathematical abilities, cognitive skills and brain research

Literature review demonstrates quite consistent findings that associate different mental operations with location of brain activation: memory retrieval (Dobbins & Wagner, 2005), attention control processes and general task difficulty (Delazer et al., 2003) are associated with the prefrontal cortex and representation (Danker & Anderson, 2007), whereas verbal encoding (Clark & Wagner, 2003), mental rotation (Heil, 2002), and visuo-spatial strategies in mathematics (Sohn, et al., 2004) are associated with the parietal cortex. However, when complexity of the problems rises, more brain areas simultaneously support the solving process. For example, Danker and Anderson (2007) found that both regions (prefrontal cortex and parietal cortex) are involved in both the transformation and retrieval stages of any given algebra problem solving task. There is a region of the lateral inferior prefrontal cortex that is particularly involved in more advanced tasks involving topics like algebra or geometry (e.g., Qin et al., 2004; Kao et al., 2008). In a variety of studies involving tasks like algebra equation solving and geometry proof generation (for review, see Anderson, 2007), activity in the posterior parietal cortex mostly correlates with problem complexity, while activity in the lateral inferior prefrontal cortex proves to be the best correlate of student proficiency.

The brains of mathematically gifted students show enhanced development and activation of the right hemisphere (Prescott et al., 2010). Another characteristic of the mathematically gifted is enhanced brain connectivity (O'Boyle, 2005) and an ability to activate task-appropriate regions in both brain hemispheres in a well-orchestrated and coordinated manner (O'Boyle, 2005). There is strong evidence for special development of the prefrontal and posterior parietal regions of the brain (e.g. Desco, 2011) and enhanced intra-hemispheric fronto-parietal connectivity (Prescott et al., 2010). Moreover, there is strong empirical evidence that individuals with higher intelligence exhibit lower total brain activation compared with individuals who have lower intelligence (e.g. Neubauer & Fink, 2009).

MATERIALS AND METHODS

Participants

Thirty-two male high school students from the northern part of Israel (16-17 years old) participated in this study. The students excelled in mathematics but differed in general intelligence, as presented in Table 1.

Table 1: Research population

	Gifted (G) IQ>135 and Raven >28 of 30	**Non Gifted (NG)** 100<IQ<130 Raven < 26	Total
Excelling in mathematics (E) SAT-M >26 of 35 or Math score > 92 in high level mathematics	19	13	32

All participants were native speakers of Hebrew, right-handed, without a history of learning disabilities and neurological disorders and had normal to corrected vision.

Stimuli and Procedure

A computerized Geometry test and Algebra test were designed using E-Prime software (Schneider, Eschman, & Zuccolotto, 2002). Each test included 60 tasks (trials). All tasks were presented visually at the center of the computer screen and were displayed in black characters on white background. Each task on each test was presented in two windows with different stimuli (S1 – Task condition; and S2 – Suggested answer) that appeared consecutively. The sequence of events and examples of the tasks is presented in Figure 1.

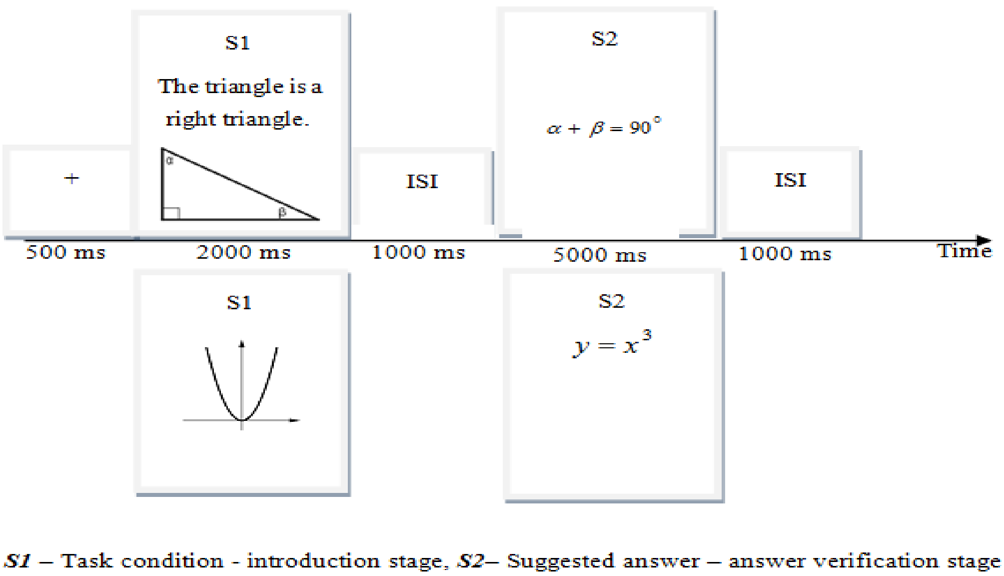

Figure 1: The sequence of events and examples of the tasks

ERP Recording and Analysis

Scalp EEG data was continuously recorded using a 64-channel BioSemi ActiveTwo system.

The ERP waveforms were time-locked to the onset of S1 and to the onset of S2. The averaged epoch for ERP, including a 200 ms pre-trigger baseline, was 1200 ms for S1 and 2200 ms for S2 (for which only the correct answers were averaged). The resulting data were baseline-corrected, and global field power (RMS) was calculated. Each condition resulted in around 40 trials.

First, we examined the differences between tests for G and NG participants as they were manifested in initial ERP components. Based on the preliminary examination of grand average waves on each electrode and on the observation of ERPs topographical maps, we detected early components: P1 component at parieto-occipital electrodes at S1and S2 stages and P2 at frontal and central electrodes at S2 stage. MANOVAs were used for latencies and mean amplitudes on the chosen electrodes with Giftedness as a between-subjects factor and the Test as a within-subjects factor. This was done for each trial's stage.

Second, we examined the mean amplitude differences among G and NG participants. Based on an inspection of the ERPs grand mean waveforms and topographic maps, the mean amplitudes of 5 time intervals of 100 ms were measured (300-400 ms, 400-500 ms, 500-600 ms, 600-700 ms, 700-800 ms). This was done because there were no clear peaks after 300 ms. The representative six electrode regions of interest (ROI) over anterior and posterior regions of both hemispheres (including midline) were chosen. The mean amplitudes were averaged over these regions (right posterior (P4, PO4, O2), middle posterior (Pz, POz), left posterior (P3, PO3, O1), right anterior (AF4, F4, FC4), middle anterior (AFz, Fz, FCz) and left anterior (AF3, F3, FC3)). Repeated measures ANOVA was performed on the ERP mean amplitude considering the Test (algebra, geometry), Time (five time intervals) and ROI (six sites) as within-subject factors and the G factor (Gifted, Non Gifted) as between-subject factors. The examination of the time course was done for each of the two stages of a task (S1 and S2).

For all consequent ANOVAs, results were corrected for deviations according to Greenhouse-Geisser.

RESULTS

Behavioral data

Table 2 demonstrates reaction times and accuracy (mean and SD) of the performance on algebraic and geometric tasks of the excelling in mathematics participants.

Repeated measures MANOVA was performed on Acc, RT and RTc taking test (algebra, geometry) as within-subject factors and the G factor (Gifted, Non Gifted) as between-subject factors. MANOVA showed significant effect of the test (F (3, 28) = 28.825, p = .000). Subsequent ANOVA showed a significant effect of the test on Acc (F (1, 30) = 5.384, p = .017) and RT (F (1, 30) = 8.495, p = .007) and not on RTc. Excelling in mathematics participants were more accurate and faster on geometric tasks compared to algebraic tasks. There was no significant effect of the G factor and no significant interaction involving test and G factor.

Table 2: RT and Accuracy is different for gifted and non gifted excelling in mathematics participants

	Accuracy (Acc)		Reaction time (RT)		Reaction time for correct responses (RTc)	
	Algebra	Geometry	Algebra	Geometry	Algebra	Geometry
G	82.7(8.1)	86.9(6.7)	1638(412.9)	1527.3(283.4)	1554.7(408.3)	1527.3(283.4)
NG	84(8.6)	82.3(7.4)	1704.3(302.8)	1527.9(326.3)	1617(313)	1527.9(326.3)
Total	83.2(8.2)	86.7(6.9)	1664.9(368.2)	1527.6(296.4)	1580(368.4)	1527.6(296.4)

Electrophysiological scalp data

Posterior P1 and Anterior P2 ERP components:

Figure 2 demonstrates the ERP waveforms of the global field power for two stages (S1 and S2) for Gifted vs. Non Gifted participants and for Algebra vs. Geometry.

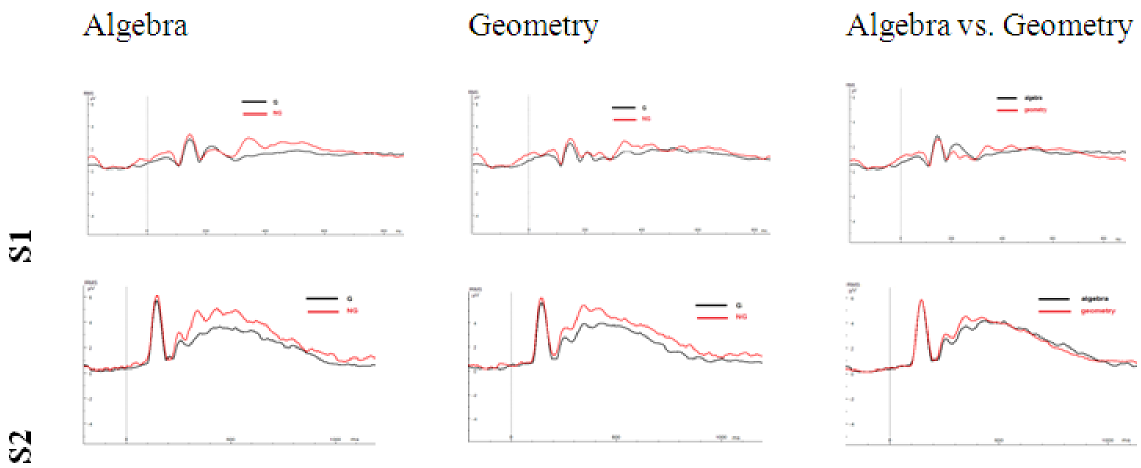

Figure 2: ERP waveforms of the global field power for two stages (S1 and S2) for Gifted vs. Non Gifted participants and for Algebra vs. Geometry.

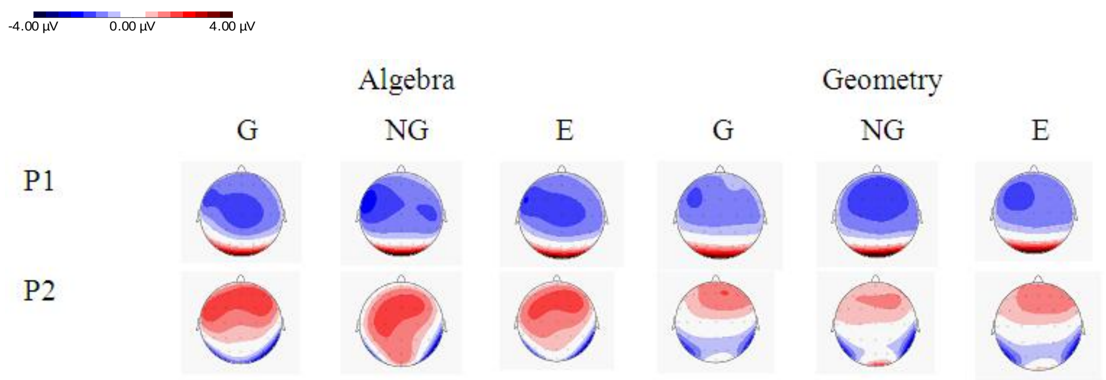

Figure 3: ERP grand-average topographical maps, showing amplitude variation of P1 and P2 at S1 stage in algebra and geometry test for GE, NGE and E participants.

No significant differences were found for latency and amplitude of P1 in stage 1 or stage 2.

MANOVA showed a significant effect of Test on the latency of anterior peak P2 (F (6, 25) = 3.514, p=.012) at Fz, FCz electrodes (Table 3) and on the amplitude of P2 (F (6, 25) = 5.085, p=.002) at all chosen electrodes (Table 3). The latency of P2 in the geometry test was shorter and the amplitude higher than in algebra test. In addition, there was significant interaction of G factor and Test on amplitude of P2 (F (6, 25) = 6.824, p=.000). The amplitude of P2 of NG participants was the same for both tests; however the amplitude of P2 of G participants was significantly higher on the algebra test than on the geometry test.

Table 3: Test effect on latency and amplitude of P2 peak

	F3	Fz	F4	FC3	FCz	FC4
Latency F(1,30)	N.S.	12.902^{***}	N.S.	N.S.	19.701^{***}	N.S.
Amplitude F(1,30)	11.567^{**}	16.582^{***}	6.804^{*}	29.163^{***}	15.703^{***}	11.656^{**}

[1] *p <.05, ** p <.01, *** p <.001

Slow potential components:

Following the P1 and P2 components, a slow late potential was observed.

To examine the time course of these effects, we divided the slow potential component into smaller time windows, each lasting 100 ms, that is, 300–400, 400–500, 500–600, 600–700, and 700–800 ms.

Main findings for task introduction stage (S1):

We observed a significant effect of the G factor (F (1, 30) = 4.433, p = .044). The mean amplitude of NG participants was significantly higher than G participants. There was a significant interaction of Time × G factor (F (1.733, 51.979) = 3.473, p = .045). Pair-wise comparisons corrected to Bonferoni adjustment revealed that the mean activity of NG was significantly larger than that of G at time interval 300-400 ms (F (1, 30) = 7.651, p = .010). See Figure 4.

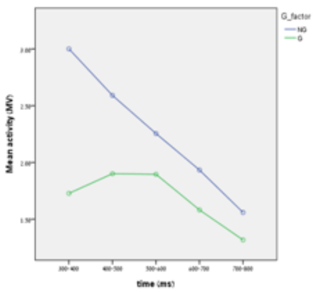

Figure 4: Mean activity of G vs. NG participants in different time intervals in stage S1.

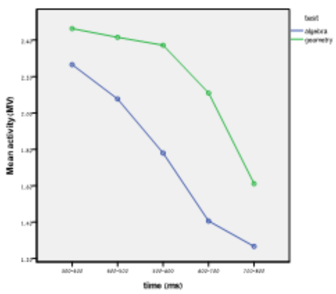

Figure 5: Mean activity of Algebra test vs. Geometry test in different time intervals in stage S1.

A significant effect of the Test (*F* (1, 30) = 5.501, *p* = .026) was also found. The mean amplitude of the geometry test was significantly higher than of the algebra test. Pair-wise comparisons corrected to Bonferroni adjustment revealed that the mean activity of geometry test was significantly larger than that of the algebra test at time interval 600-700 ms (*F* (1, 30) = 9.592, *p* = .005). See Figure 5.

In addition, there was a significant interaction of ROI×G factor (*F* (2.562, 76.868) = 5.564, *p* = .003). Pair-wise comparisons corrected to Bonferroni adjustment revealed that the mean activity of NG participants was significantly larger than their G counterparts at posterior-middle ROI (*F* (1, 30) = 10.075, *p* = .003).

Furthermore, there was significant interaction of Test ×Time ×ROI (*F* (7.244, 217.332) = 2.206, *p* = .033*)*. Pair-wise comparisons corrected to Bonferroni adjustment revealed that the mean activity of the geometry test was significantly larger than that of the algebra test at posterior-middle ROI in time interval 600-700 ms(*F* (1, 30) = 11.921, *p* = .002).

Main findings for answer verification stage (S2):

We found a significant effect of Test (*F* (1, 30) = 5.719, *p* = .023). The mean amplitude of the geometry test was significantly larger than that of the algebra test. Moreover there was significant interaction of Test × Time (*F* (2.763, 111.351) = 7.040, *p* = .000). Pair-wise comparisons corrected to Bonferroni adjustment revealed that the mean activity of the geometry test was significantly larger than that of the algebra test at time interval 300-400 ms (*F* (1, 30) = 14.158, *p* = .001).

Second, there was significant interaction of ROI × Test (*F* (3.989, 119.679) = 6.063, *p* = .000). Pair-wise comparisons corrected to Bonferroni adjustment revealed that the mean activity of the geometry test was significantly larger than the algebra test at posterior-left ROI (*F* (1, 30) = 14.644, *p* = .001). See Figure 8.

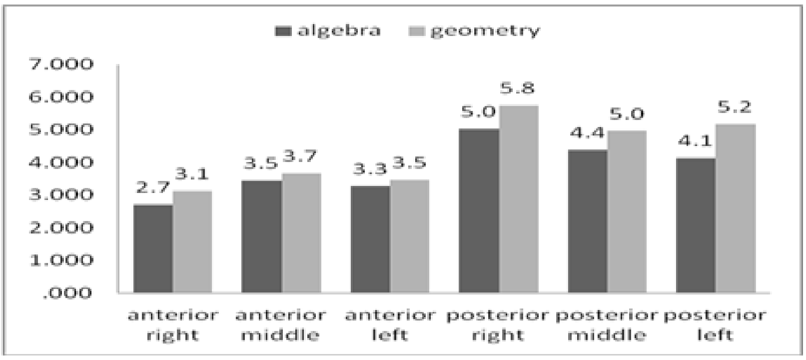

Figure 8: Mean activity on the algebra and geometry tests in different ROI in stage S1.

DISCUSSION

The goal of present study was to examine differences in ERPs between Gifted and Non Gifted excelling in mathematics adolescents while solving relatively advanced mathematical tasks in algebra and geometry.

The first major finding regarding the Giftedness effect was that, relative to G participants, NG participants produced greater brain activity in both stages of the two tests. This activity was significantly greater in NG participants at task introduction stage (S1). This finding is consistent with the neural efficiency hypothesis of intelligence, stating that brighter individuals display lower (more efficient) brain activation while performing cognitive tasks (for review see Neubauer & Fink, 2009). The differences in brain electrical activation between G and NG participants were most prominent in the posterior parieto-middle site. The posterior parietal cortex is known to reflect representational activities (Danker & Anderson, 2007). Therefore the aforementioned differences in parieto-middle sites may be connected to the differences between G and NG in their visual-spatial strategies of processing in the introduction stage.

There were no significant differences in latency and amplitude of P1 peak associated to the Test and G factors. This was the case in both stages of the tasks'. The posterior P1 is related to visual processing and is sensitive to physical stimulus characteristics (e.g. Di Russo et al., 2002). Therefore, our finding suggests that the basic visual processing of the stimuli (visual and symbolic) in both tests and for both groups of participants was the same.

For the frontal P2 in visual task introduction stage, there was significant difference between geometric tasks and algebraic tasks for both amplitude and latency. Algebraic tasks demonstrated higher P2 amplitude and longer P2 latency compared to geometric tasks. This suggests that the perceptual load for algebraic task was larger than that of the geometric task. Literature on mental arithmetic processing suggests that early ERP components (P1, P2) are generally considered to be a reflection of attention to digit-pattern and the identification of numbers and their meaning (Iguchi & Hashimoto, 2000).

The second major findings regarding Test effect reveal that there was significantly higher brain electrical activity connected to geometry test compared to algebra test. It has been found that late positive components in the ERP are correlated with retention operations in working memory (King & Kutas, 1995) and the larger the processing demands to retain the object information in working memory, the greater the slow wave activity (Berti, 2000). Therefore, we can argue that geometric tasks increase the participants' working memory load by keeping the visual geometric object in working memory until the problem is solved.

Our study shows the differences in activity among different types of population on different types of tests, which is a first step in understanding the underlying brain processes in different aspects of mathematical processing.

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