12th International Congress on Mathematical Education
Program Name XX-YY-zz (pp. abcde-fghij)
8 July – 15 July, 2012, COEX, Seoul, Korea (This part is for LOC use only. Please do not change this part.)

USE OF STUDENT MATHEMATICS QUESTIONING TO PROMOTE ACTIVE LEARNING AND METACOGNITION

Khoon Yoong WONG

National Institute of Education, Nanyang Technological University

Singapore

khoonyoong.wong@nie.edu.sg

Asking questions is a critical step to advance one's learning. This lecture will cover two specific functions of training students to ask their own questions in order to promote active learning and metacognition. The first function is for students to ask themselves mathematical questions so that they learn to think like mathematicians who often advance knowledge by asking new questions and trying to solve them. This is also called problem posing, an important component of the "look back" step in Polya's problem solving framework. The second function is for students to ask their teachers learning questions during lessons when they do not understand certain parts of the lessons. Students who are hesitant to ask learning questions need to be inducted into the habit of doing so, and a simple tool called Student Question Cards (SQC) will be described to achieve this. Four types of mathematics-related questions are designed to cover meaning, method, reasoning, and applications and these questions are printed on laminated cards given to the students. The teacher will pause at specific parts of a lesson and require the students to select questions to ask to clarify their doubts. This reverses the typical roles of the teacher and the students during classroom interactions. Lessons learnt from a small study that trialed this approach with Grades 4 and 7 students in Singapore will be discussed. These two functions have the potentials to promote active learning of mathematics among school students through strengthening their metacognitive control. Teachers need to pay attention to the science, technology, and art of student questioning.
Student questions, problem posing, metacognition, Confucius, Socratic dialogue

MAIN THEME

Children are naturally curious about themselves and their environment. A natural way with which they try to satisfy their curiosity is to ask questions. The main theme of this regular lecture is to argue that student questioning should be made a stronger part of classroom teaching and learning of mathematics than is currently practiced in many countries.

There are many reasons why questioning is important in knowledge construction and its learning. Student questioning can serve two different but related functions. The first function is to help students think like mathematicians by posing their own mathematical questions and trying to solve them. This could lead to "new" knowledge constructed by the students. The second function is to train students to develop the good learning habit of asking their teachers mathematics-related questions about things that they do not understand. This learning function reverses the typical role of the teacher and students: instead of "teacher asks questions and students answer them", the new role becomes "students ask questions and the teacher answers them". Examples will be given below to illustrate these two functions. The lecture will conclude that student questioning, just like other teaching techniques, requires that teachers pay attention to the *science*, *technology,* and *art* of using it in their classrooms.

I shall begin by tracing the historical traditions in the East and West with regards to the use of questioning in education.

QUESTIONING IN EAST AND WEST EDUCATION

It is commonly held nowadays that students in the Confucian Heritage Culture are passive learners and the teachers in this culture are authorities not to be questioned by their students. This was not the case in ancient China. The Chinese expression for learning or knowledge is made up of two characters, meaning to learn and to ask questions (学问). Etymologically this term can be traced to the *Book of Change* (I Ching, 易经), which Confucius (551 – 479 BC) had written some commentaries on: the superior person accumulates knowledge through learning and validates it through asking questions (君子学以聚之, 问以辩之). In the *Book of History* (*Shang Shu*, 尚书 or *Shu Jing* 书经) edited by Confucius and used by him as a text, one finds the well-known Chinese proverb, 好问则裕; whoever asks questions will become abundant (in knowledge). This point is clearly illustrated by the story of one of his eminent disciples, Zheng Zi (曾子 or 曾参) (505－ 435 BC). Zheng was a "low ability" student but he was diligent in his studies and believed in the efficacy of asking questions as a helpful learning technique: 弗知而不問焉，固也 (If you do not know and yet do not ask, your mind will become stuck.). Zheng was well known for his filial piety to his mother, and he edited several classics of Confucianism, including the classic on filial piety, *Xiao Jing* (孝经). The way to ask a question is explained in the *Book of Rites* (*Li Ji*, 礼记): Those skillful in asking begin as if attacking hard wood: begin with the easy parts, then move on to the hard ones. Given time, the various parts will be separated (善问者，如攻坚木，先其易 者，后其节目，及其久也，相说以解). Unfortunately this ideal that students should be active inquirers has been distorted over thousands of years in China, Korea, and Japan leading to passive learning in the so-called Confucian Heritage Culture.

The Buddha (ca 563 – 483 BC) is revered as a great teacher, and the Buddhist canon (*Sutras*) shows that many of his teachings were in response to the questions asked by his disciples and other seekers. The huge collection of Chinese Zen Buddhist writings describes numerous events when the masters answered questions asked by their followers. Thus, asking questioning had been widely used in learning in ancient Eastern eras. This effective activity seems to have been neglected in many contemporary Asian classrooms. For example, in December 2011, a survey of 300 Korean school children conducted by *Chosun Ilbo* found that 42% of them never asked questions in class, and 45% were scolded or ignored by their teachers when they asked questions[1]. This might be so in other countries as well.

In the West, the ancient Greek philosopher, Socrates (ca 470 – 399 BC), was famous for developing the teaching technique named after him (Socratic dialogue). In this one-to-one dialogue, he would ask the student a series of questions to help him think critically and arrive at the answers himself. The underlying epistemological assumption is that the answers are innate in the mind of the students, and thus, the teacher's role is that of a midwife (*maieutics*) to help the students to "give birth" to that knowledge. If the student has not been taught the knowledge prior to the dialogue, then his knowledge must have come from his past lives, even in non-human forms! His attempt to lead Meno's slave to "discover" that the area of a large square is double that of a smaller one[2] has become one of the most influential dialogues in Western philosophy. It has prompted the famous mathematician Hadamard (1905/2004), who called this the "heuristic method", to extend it from one-to-one dialogue to larger groups. Proponents of student-centered teaching in recent years often include Socratic questioning as a key method to have students think deeply and justify their answers. However, the eminent philosopher of the past century, Bertrand Russell (1872 – 1970), noted that Socrates had used in the Meno's dialogue "leading questions which any judge would disallow" (1961, p. 110). He maintained that the discovery of scientific and mathematical knowledge can hardly "be elicited from a previously ignorant person by the method of question and answer" (ibid). Thus, it is unlikely that Socratic dialogue can help school students to come up with mathematically sophisticated ideas like the definition of prime numbers, although it might be used to solicit students' views about how mathematics is used in their daily lives. Unlike the original spirit of Socratic dialogue, the "teacher asks and students answer" mode has evolved in the West to become a key component of direct instruction to find out what students know about the lessons after they have been taught rather than to induce students to "recover" knowledge from an innate source.

A lesson from the brief historical sketch above about questioning in ancient East and West is that it has been thought to be an active factor in constructing knowledge and in learning it, irrespective whether the questioning is done by the teacher or the student. I will begin with the function of student questioning used to construct knowledge.

STUDENT QUESTIONING I: CONSTRUCT NEW KNOWLEDGE

Importance of Asking Mathematical Questions

Albert Einstein commented that "The important thing is not to stop questioning. Curiosity has its own reason for existing"[3]. Curiosity is the innate motivator of a person to search for

[1] http://english.chosun.com/site/data/html_dir/2011/12/07/2011120700522.html
[2] http://www.cut-the-knot.org/proofs/half_sq.shtml
[3] http://rescomp.stanford.edu/~cheshire/EinsteinQuotes.html

knowledge, and asking questions is an important initial step to satisfy this curiosity and it should be encouraged.

First of all, students must be stimulated to see how mathematics can help them know about the world, and this involves helping them to habitually adopt a mathematical lens in their observations of everyday situations. With well-developed mathematical literacy, they can form considered opinions about statements that cite quantities and make informed decisions about real-life situations that involve numbers and spatial relationships. For example, when they hear that people use only 10% of their brain capacity, they will recognize that percents are defined with respect to a reference base and ask what might be 100% of brain capacity[4]. Since it is not clear what is the brain capacity for learning and how to measure it, the given claim does not add new knowledge about ourselves. Pondering over such questions will help students (and adults as well) to *de*construct *mis*leading information to avoid being *mis*led by *mis*use of statistics. However, students may not have developed such a critical mindset. This seems to be the case in relation to understanding messages conveyed by graphs. In her doctorate study, Wu Yingkang (2005) administered a questionnaire to about 900 Grades 7 to 9 students in Singapore about their critical views toward statistical graphs. About one third of them hardly asked themselves questions about the reliability of the data, whether the graph was suitable for the given data, or whether the graph supports claims made in the accompanying report. This lack of critical analysis through asking relevant questions is a weakness that can prevent students from learning about the world through quantitative data.

Historically mathematics evolves when mathematicians ask questions and try to solve them. Classic mathematics texts often consist of list of questions. For example, the Chinese classic, *Nine Chapters on the Mathematical Art* (九章算术, ca 200 BC) is a collection of problems about taxation, pricing, land measurements and other situations, their solutions, and explanations of the procedures used. This mode is now recognized as the problem posing approach in the construction of knowledge. Indeed, the recent KOM Project of Denmark (Niss & Højgaard, 2011) has emphasized “being able to ask and answer questions in and with mathematics” as one of the eight mathematics competencies in that curriculum reform (p. 53). This problem posing approach has several advantages. First, students are more likely to become engaged working on their own questions. This will help to deepen their mastery of the skills and thinking. They may have posed problems that cannot be solved, and this encounter, supported by discussion with teachers, demonstrates to the students a common situation called unsolved problems (conjectures) faced by working mathematicians. Indeed, to tackle interesting conjectures has spurred the development of mathematics. This experience may change students’ beliefs about the nature of mathematical thinking. Finally, these questions may reveal “a good deal about what learners are attending to and what they

[4] See Chabris and Simons (2010) for further questions one can ponder over about this and other misleading claims.

are aware of" (Mason & Johnston-Wilder, 2004, p. 310). What is revealed may go beyond mathematical skills. Let me illustrate this with the following example.

A class of Grade 8 students in Brunei Darussalam had just completed some work on quadratic equations using a multi-modal thinkboard (Wong, 1999). They were required to write "stories" that could illustrate the use of quadratic equations. In Figure 1, the student wrote a story that revealed certain feelings about neighbors. These may or may not be her true feelings, but such stories can help teachers further understand the values and interests held by their students.

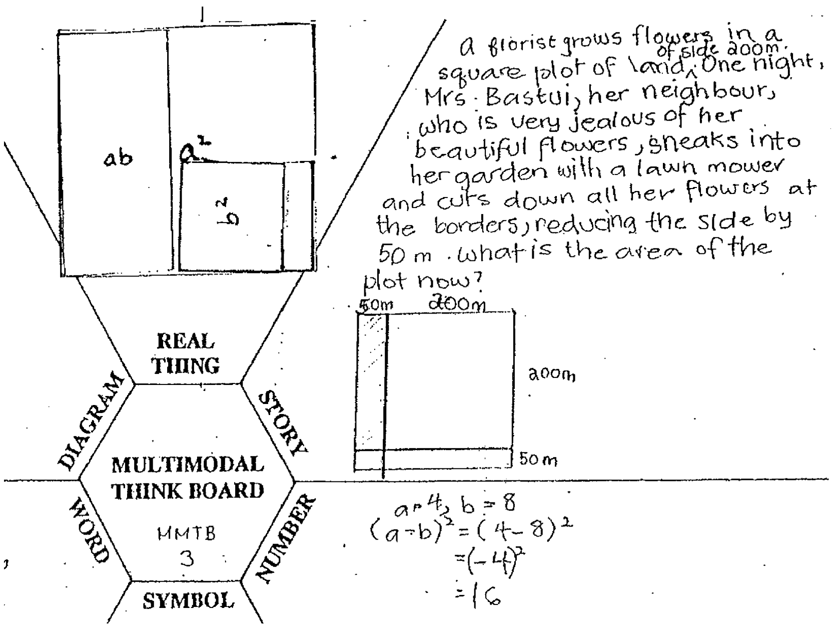

Figure 1. A multi-modal "story" with embedded values

Ways to Incorporate Problem Posing into Lessons

School students can learn to pose their own questions under two conditions in mathematics lessons. First, the Polya's (1957) method of problem solving has "looking back" as the last of a 4-step process. Having solved a problem, students are advised to think of using the same result or method to solve other problems, either given or posed by the students. Wilson, Fernandez, and Hadaway (1993) had proposed a helpful framework that includes problem posing after the "looking back" step so that it loops back to the first step of Polya's method to begin another cycle of problem solving. Secondly, the "problem posing" approach has gained wide acceptance among mathematics educators (e.g., Brown & Walter, 2005; Silver, 1994), and students learn to ask "what if" and "what if not" questions to help them generate new problems to solve. Students can learn to change the numbers (becoming larger or using different types of numbers), conditions of the problem (generalize or specialize), the contexts or storylines, representations (verbal or pictorial but rarely concrete), and operations (reverse them). These different ways can give rise to problems of different levels of sophistication and difficulty.

In a recent doctorate study, Chua (2011) developed a framework to classify solvable problems posed by secondary students by problem types (relational or direct recall), problem information (edit information, add object, over-conditioning, implicit assumption), solution types (multi-step, algebra), and domain knowledge. The unsolvable problems were due to ambiguity in language and contradictions in the information provided by the students. Given that the students in the study had not been trained to pose problems, it is not surprising that the most common type of problems was direct recall that requires only a direct link between the initial state and the final state of the problem.

Trainee teachers may be expected to be better problem posers than school students, but this is not necessarily so. In my mathematics methodology course for the BSc (Ed) program at the National Institute of Education, trainee teachers for secondary level had to complete an assignment about problem solving. Each of them was given a problem suitable for secondary school students and had to solve it using two different ways. Then they planned a problem solving lesson for that problem, including posing an extension problem, as in Polya’s final step of looking back. Most of the problems posed by these trainee teachers involved straightforward changes in numbers and the storyline. In a typical case, a trainee teacher was given the following problem:

> To play a treasure hunt game, 90 boys and 105 girls formed mixed groups with the same proportion of boys and girls in each group. Find the greatest number of groups that could be formed. In this case, how many boys and girls are there in each group?

She posed her problem by adding another option and set it within the context of racial fairness, which is related to what is called National Education in Singapore (Wong, 2003).

> In a nationwide treasure hunt in Singapore, to ensure racial fairness, 108 Chinese, 72 Malays, 96 Indians formed mixed groups with the same proportion of each race in each group. Find the greatest number of groups that can be formed. In this case, how many people of each race are there in each group?

Another technique which can be used to stimulate problem posing is to give students an answer or a set of operations and ask them to formulate possible questions (Herrington, Wong, & Kershaw, 1994). Yet another activity to build on student questions is to ask them play the examiner’s role. Tell them to submit their own questions (with answers) about the recent lessons, circulate these questions among the students, and select some of these questions for the upcoming test. This will help them revise important mathematics knowledge and skills to be mastered, leading to better learning outcomes. Finally, it is possible to achieve higher performance by improving learning behaviors, which is the second function of student questioning.

STUDENT QUESTIONING II: LEARNING IN CLASSROOMS

Current Practice of Teachers as Questioners

Learning and construction of knowledge are intricately intertwined. For this section, I will consider learning to encompass behaviors that students undertake in mathematics lessons, and the focus here is to explore the role of student questioning as a desirable learning behavior in mathematics lessons.

Teachers ask questions in lessons to satisfy many purposes, one of which is to find out what students have understood from the lessons, although such questioning often ends up with finding out what the students do *not* know. In the latter case, Einstein felt that it would be a waste of teacher's time and that the teacher should discover through questioning what the students are capable of knowing[5]. There is already a large literature in general education (e.g., Kerry, 2002) and mathematics education (e.g., Schuster & Anderson, 2005; Small, 2009; Watson & Mason, 1998) about how to improve teacher questioning techniques. These include asking a combination of closed and higher order questions, using wait times to stimulate deeper thinking, providing immediate and meaningful feedback, and moving beyond the standard IRF (Teacher Initiates, Student Responds, Teacher Feedback) sequence. Numerous studies have described the types of questions teachers often ask, usually closed ones, and their effects on student achievement (e.g., Hattie, 2009). By participating in these teacher-initiated questioning, students become more active and engaged in their learning compared to just listening and taking notes, even though at times they may just guess at what the teacher wants to hear rather than reveal their true answers in front of their classmates for fear of being wrong. Nevertheless, the whole interaction is under the control of the teacher, which is the pedagogical norm in many classrooms all over the world.

Scant Discussion about Students as Questioners

Compared to the comprehensive coverage of practice and research about teacher questioning, there is relatively less attention given to investigate student questioning as a standard teaching technique or to provide practical guidance on its use in lessons. For example, in Kerry's book (2002), there are 58 pages on questioning and only seven pages deal with student questions. Furthermore, "student questions" does not appear in the index of a recent international survey of expertise in mathematics instruction (Li & Kaiser, 2011), books about exemplary practices for mathematics teachers (Posamentier, Jaye, & Krulik, 2007), and pedagogy books in general (Feldman & McPhee, 2008; Marzano, 2007). Godinho and Wilson (2006) wrote a book with a title about helping pupils to ask questions, but most of the contents are about teacher questions. In a comprehensive review of student questioning in science education, Chin and Osborne (2008) concluded that "the explicit teaching of questioning skills to students can lead to improved performance on a range of science-related tasks" (p. 34) and

[5] http://einstein.biz/quotes

noted that “there is still a lot of scope for pedagogies that exploit the potential value of students’ questions” (p. 35). Perhaps, student questioning may have been embedded as an “invisible” part of classroom discussion; if so, it should be made more “visible” to the teachers and students alike. The following sections cover a few examples.

About 20 years ago, Dillon (1988) noted that “few student questions are asked [in class] and even fewer are answered” (p. 8). There is a chapter entitled “Pedagogy of Student Questions”, in which the author suggested that teachers should welcome questions from the class, provide time for them to ask questions, and find ways to sustain the asking. This requires changing the culture of the classrooms so that the students feel that it is alright, even expected, to ask questions when they do not understand and need to seek help from the teachers in front of the class.

The PEEL (Project for Enhancing Effective Learning) project was a comprehensive teacher-led action research that begun in Melbourne in 1985 and has spread to many schools in Australia and elsewhere[6]. It aims to inculcate in secondary students good learning behaviors. For example, students are encouraged to seek assistance by telling their teachers when and what they do not understand and asking their teachers why they go wrong (Mitchell, 1992, p. 63). Similar good learning behaviors are also discussed by Postman and Weingartner (1969). They proposed a Question Curriculum and wrote: “Once you have learned how to ask questions – relevant and appropriate and substantial questions – you have learned how to learn and no one can keep you from learning whatever you want or need to know” (p. 34). This highlights that questioning is a desirable attribute for life-long learning, which nowadays is considered critical for workers of the 21st century.

Focusing on mathematics study skills in particular, Ooten (2010) discussed seven types of questions to ask in mathematics classes. For examples, she suggested that the students prepare a prioritized list of questions, ask often, and show their work in public.

Foster (2011) explained how he began a mathematics lesson by writing on the board questions generated by the students and letting them decide which questions to tackle. He provided several interesting examples related to linear equations. Since this is a new learning experience for the students, he recommended patience on the part of the teacher.

Piccolo, Harbaugh, Carter, Capraro, and Capraro (2008) described study that covers both teacher-generated and student-initiated questioning as part of mathematics discourse at middle school level. They also found that “teacher talk was dominant and student talk was mainly a response to teacher questioning” (p. 404) and they called for “further research on how better to provide students with the skills and mathematical competence to ask and engage in rich mathematical discourse with teachers” (ibid.). This agrees with what Commeyras (1995) believed: “Children are naturally rich with questions, but when teachers take primary

[6] http://www.peelweb.org/index.cfm?resource=about

responsibility for questioning, student questioning becomes something to be taught" (p. 105). Chin and Osborne (2008) also made a similar suggestion. The next section describes a small study to help students learn to ask questions in mathematics lessons. I will call this the SQC (Student Question Cards) technique.

The SQC (Student Question Cards) Technique: Issues and Procedure

This technique was developed for a project conducted with my colleague, Quek Khiok Seng; see Wong and Quek (2006; 2009; 2010). It was an exploratory study to investigate to what extent student questioning could be incorporated into normal mathematics lessons to help students become more metacognitive about their learning. Two Grade 4 and two Grade 7 teachers volunteered to participate in this study in 2005. Four issues related to student questioning as a learning behavior in the classrooms are addressed.

Issue 1. Students are able to ask mundane questions such as "Do we have to underline our answers?" or "When do we have to hand in our homework?" But, as noted above, they are poor in asking questions targeted at specific contents. Some of them would ask over-encompassing questions such as, "Teacher, can you explain everything from the beginning again?" Thus, their ability to ask specific mathematics questions when they do not understand is not inborn (despite the claims of Socratic dialogue?) and it has to be nurtured. In our investigation, we created a set of standard questions that focus on four key aspects of mathematical learning: Meaning, Method, Reasoning, and Application. See Figure 2.

Meaning		**Method**	
M1:	What do you mean by ….	Md1:	Can you show us how to do this problem in another way?
M2:	What is the difference between …. and ….	Md2:	Can you explain/show us this step (….) again?
M3:	Can you use a diagram to show ….	Md3:	What will you do next?
M4:	(Your own question)	Md4:	(Your own question)
Reasoning		**Application**	
R1:	Why do you do that ….?	A1:	Why do we study this topic (….)?
R2:	What happens if you change …. to ….?	A2:	How do we use this (….) in everyday life?
R3:	(Your own question)	A3:	(Your own question)

Figure 2. The Student Question Cards (SQC)

For each aspect, the research team devised two or three questions in simple expressions that can be asked of most topics, where the students fill in the ellipses (…) with words related to the specific lesson. The (Your own question) option allows students to frame questions in their own words. While these specific questions serve as a scaffold at the initial stage to promote student questioning, it is hoped that with sufficient practice the scaffolding using these question prompts can be steadily faded so that the students will become better at asking their own questions in a format that is specific and relevant to the mathematics being taught. These questions are printed on laminated cards with one aspect per card; the label of each aspect appears on one side and the sample questions on the reverse side of the same card. These laminated cards were given to the students during the trial.

Issue 2. Some students are apprehensive about asking questions in front of their classmates for fear of looking "stupid" because of their "dumb" questions. This might be a universal or a particularly Asian face-saving concern. This issue can be partially alleviated by letting students choose questions from an "approved" list.

Issue 3. This issue refers to the situation where many students have been "conditioned" to answer questions but not to ask them. As pointed out by Dillon (1988), time must be specifically created in lessons to allow students to ask questions. During the trial, the participating teachers planned their lessons to include pauses called Question Times (QT) after about 15 minutes of teaching or class activity. During these QTs, the students were asked to refer to the SQC to find a question to ask about that part of the lesson prior to the QT. This was the scaffolding part, and doing it two or three times per lesson was an intensive way to help bring about this new habit. The ways in which the teachers applied this technique might influence how this habit develops among their students.

The four teachers were found to handle the QT in different ways. They tended to select students who were normally quiet or hardly asked questions in class. At times they focused on one particular aspect; for example, Reasoning questions because they were working on justification of certain results. If over several lessons, the teacher could go through all the four aspects, this should help students become familiar with these four significant ways of thinking about mathematics. One teacher found that her students were able to insert the appropriate terms in the prompts (…), while the students of another teacher had difficulty with this, i.e., they just asked "what do you mean by *that*" without being able to state precisely what is "that". These various ways of using the SQC increased the opportunities for students to develop their skills in asking questions.

Issue 4. Well-prepared teachers do not have difficulty answering questions related to how to solve the problems they assign to their students. However, the SQC technique includes questions other than Methods, and this can be challenging to teachers who are not strong in mathematics content knowledge or how mathematics can be applied in the real-life contexts. Indeed, Application questions were very popular, suggesting that the students in this study were eager to know why they studied the topics. The teachers seemed to have difficulty answering these questions. Some of them were also caught off guard by questions about different representations (Meaning), for example, "can you draw a diagram to show …?". When this happens, teachers ought to work out more appropriate answers outside class time, and this is a meaningful professional development activity for them. Hopefully, they will incorporate these new insights into subsequent lessons. Indeed, as Hattie (2009) wrote, "the biggest effects on student learning occur when teachers become learners of their own teaching, and when students become their own teachers" (p. 22). The SQC technique provides a concrete and systematic tool for both teachers and students to learn about mathematics by playing these new reverse roles with regards to questioning.

The SQC (Student Question Cards) Technique: Responses from Teachers and Students

Teachers' feedback. Table 1 provides some feedback from the teachers with respect to the use of SQC. They agreed that it was useful and they may try it occasionally in the future.

Table 1: Teachers' feedback to Student Question Cards (SQC)

	T1 (Primary)	T2 (Primary)	T3 (Secondary)	T4 (Secondary)
Usefulness in helping students understand lesson	Through the questions they asked, realised most students did not understand topic. QT breaks monotony of teaching so students more engaged/ likely to listen.	Generally quiet and unresponsive class but now more active; thought they were having fun, game-like.	Yes, if students asked correct questions.	Good to use cards at beginning of topic and in remedial class; Students tended not to listen when their friends were asking questions.
Integrated into practice?	Not yet a repertoire in her teaching but will continue to use question cards (but not the checklist).	Cannot use for every lesson; might use QT now and then.	Yes, but students asking questions could only be sustained when it is a school culture.	Will have QT even if cards not used. Some teachers already doing it but not as structured and detailed.

Students' feedback. The students responded to a questionnaire about their experience on the 6-point scale: 1 = Strongly Disagree; 6 = Strongly Agree. The mean scores are reported in Table 2. Their responses were slightly positive, with the two secondary classes less positive than the primary classes. Although the students found the cards easy to use, they did not really enjoy using it. In their open-ended responses, the students expressed positive opinions about being able to choose questions to ask and having the chances to ask questions. On the other hand, some of them were worried that the teacher would pick on them to ask the questions or conversely were disappointed when they were not chosen. Some found this time-consuming (they were also asked to write down in a short checklist what question they had chosen at each QT) and preferred to simply raise their hands if they wished to ask questions.

Table 2: Students' feedback to Student Question Cards (SQC)

	Questionnaire Items	T1 (P)	T2 (P)	T3 (S)	T4 (S)
Q18:	The question cards were easy to use.	4.24	5.04	4.03	3.74
Q20:	I could understand the questions on the cards.	4.12	4.58	4.32	4.00
Q22:	This "question cards" method helped me to understand mathematics better.	4.10	4.91	3.45	3.16
Q21:	I would like my teacher to use this "question cards" method to teach mathematics.	3.96	4.04	3.18	3.11
Q19:	I enjoyed using these question cards to learn.	3.76	4.64	3.61	3.03

CONCLUSION

I wish to conclude this lecture by considering the *science, technology,* and *art* of this under-utilized pedagogic technique.

The *science* of student questioning should build on sound theories. Relevant theories include metacognition, self-regulated learning, and inquiry learning through asking mathematical questions and learning questions. Gibboney (1998) also provided a theoretical point: "unless the learner herself raises questions, no meaningful learning can occur" (p. 32). To what extent these theories are able to support learning via student questioning has to be tested against empirical findings. Evidence is available about the efficacy of problem posing (e.g., Brown & Walter, 1993), but research about students playing the role of questioners in the classrooms and the impacts of student-generated examples and questions on various measures of learning outcomes has hardly started. Teachers can spearhead this by conducting collaborative action research with colleagues or engage in lesson study about applying this technique.

Scientific experiments require the use of robust tools. This is the *technology* of student questioning.

The SQC is just one tool of this technology. It is easy to use and covers the key aspects of mathematical learning. It is also flexible enough to allow teachers to decide how it fits into their teaching styles as reported above. Postman and Weingartner (1969) suggested that teachers initially focus on the quantity of questions generated by the students "to get them to begin formulating questions" (p. 185) and only later to help them develop criteria "by which the quality of a question can be evaluated" (p. 186). This is similar to the approach taken by Foster (2011) in his lessons. More tools and processes of using these new tools need to be developed and their impacts investigated.

Technology is a critical bridge between the *science* and *art* of teaching. The *art* of student questioning refers to how individual teachers decide to use or not to use it for the target students. The *art* depends on personal styles, and this takes many hours of consistent practice to develop. Given that the science and technology of student questioning are still underdeveloped at this stage, teachers have to develop their own art of using this technique through trial and error, support from peers, and personal reflection.

Teacher questioning will always be a key feature of mathematics lessons. It is necessary to find out to what extent the current practice of teacher as questioner and students as answerers may jeopardize the nurturing of curiosity in students. The loss of curiosity is aptly described as "children enter school as question marks and leave as periods" (Postman & Weingartner, 1969, p. 67). Student questioning is offered here as a complement to teacher questioning with the aim to generate stronger students' curiosity about mathematics and to place this curiosity at the center of their learning. Training them to apply the habit of asking questions about the Meaning, Method, Reasoning, and Application of mathematics is one route to satisfy that curiosity. This effort could lead them to become more effective life-long learners. In *The Analects* (论语 8.17), Confucius said: Learn as if you could not catch up and afraid of losing what you have learned (学如不及，犹恐失之). This mindset will prepare students to meet future challenges that require strong mathematics competence.

References

Brown, S. I. & Walter, M. I. (Eds.). (1993). *Problem posing: Reflections and applications*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Brown, S. I. & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Chabris, C., & Simons, D. (2010). *The invisible gorilla and other ways our intuition deceives us*. London: HarperCollins.

Chin, C., & Osborne, J. (2008). Students' questions: A potential resource for teaching and learning science. *Studies in Science Education, 44*(1), 1 – 39.

Chua, P. H. (2011). *Characteristics of problem posing of grade 9 students on geometric tasks*. Unpublished PhD thesis, Nanyang Technological University, Singapore.

Commeyras, M. (1995). What can we learn from students' questions? *Theory into Practice, 34*(2), 101 – 106.

Dillon, J. T. (1988). *Questioning and teaching: A manual of practice*. London: Croom Helm.

Feldman, J., & McPhee, D. (2008). *The science of learning & the art of teaching.* Clifton Park, New York: Thomson.

Foster, C. (2011). Student-generated questions in mathematics teaching. *Mathematics Teaching, 105*(1), 26 – 31.

Gibboney, R. A. (with Webb, C. W.). (1998). *What every great teacher knows: Practical principles for effective teaching.* Brandon, VT: Holistic Education Press.

Godinho, S., & Wilson, J. (2006). *Helping your pupils to ask questions.* London: Routledge.

Hadamard, J. S. (2004). Thoughts on the heuristic method. In R. G. Ayoub (Ed.), *Musings of the masters: An anthology of mathematical reflections* (pp. 31 – 43). Washington, DC: Mathematical Association of America. (Original work published 1905)

Hattie, J. A. C. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* London: Routledge.
Herrington, T., Wong, K. Y., & Kershaw, L. (1994). *Maths Works: Fostering mathematical thinking and learning*. Adelaide: Australian Association of Mathematics Teachers.
Kerry, T. (2002). *Explaining and questioning.* Cheltenham: Nelson Thornes Ltd.
Li, Y., & Kaiser, G. (Eds.). (2011). *Expertise in mathematics instruction: An international perspective*. New York: Springer.
Marzano, R. J. (2007). *The art and science of teaching: A comprehensive framework for effective instruction*. Alexandria, VA: ASCD.
Mason, J., & Johnston-Wilder, S. (Eds.). (2004). *Fundamental constructs in mathematics education.* London: Routledge-Falmer.
Mitchell, I. (1992). The class level. In J. R. Baird & J. R. Northfield (Eds.), *Learning from the PEEL experience* (pp. 61-104). Melbourne: Monash University.
Niss, M., & Højgaard, T. (Eds.). (2011). *Competencies and mathematical learning ideas and inspiration for the development of mathematics teaching and learning in Denmark* (English ed.). (Original work in Danish published 2002). Roskilde, Denmark: Roskilde University, Department of Science, Systems and Models, IMFUFA.
Ooten, C. (with Moore, K.). (2010). *Managing the mean math blues: Math study skills for student success.* Upper Saddle River, NJ: Pearson Education.
Piccolo, D. L., Harbaugh, A. P., Carter, T. A., Capraro, M. M., & Capraro, R. M. (2008). Quality of instruction: Examining discourse in middle school mathematics instruction. *Journal of Advanced Academics, 19*(3), 376 – 410.
Polya, G. (1957). *How to solve it* (2nd ed.). New York: Doubleday & Company.
Posamentier, A. S., Jaye, D., & Krulik, S. (2007). *Exemplary practices for secondary math teachers.* Alexandria, VA: ASCD.
Postman, N., & Weingartner, C. (1969). *Teaching as a subversive activity*. New York: Delacorte Press.
Russell, B. (1961). *History of western philosophy* (2nd ed.). London: George Allen & Unwin.
Schuster, L., & Anderson, N. C. (2005). *Good questions for math teaching: Why ask them and what to ask, Grades 5 – 8.* Sausalito, CA: Math Solutions Publications.
Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19 – 28.
Small, M. (2009). *Good questions: Great ways to differentiate mathematics instruction*. New York, NY: Teachers College Press.
Watson, A., & Mason, J. (1998). *Questions and prompts for mathematical thinking*. Derby: Association of Teachers of Mathematics.
Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical problem solving. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 57 – 78). New York: Macmillan.
Wong, K. Y. (1999). Multi-modal approach of teaching mathematics in a technological age. In E. B. Ogena & E. F. Golia (Eds.), *8th Southeast Asian Conference on Mathematics Education, technical papers: Mathematics for the 21st century* (pp. 353 – 365). Manila: Ateneo de Manila University.

Wong, K. Y. (2003). Mathematics-based National Education: A framework for instruction. In S. Tan & C. B. Goh (Eds.), *Securing our future: Sourcebook for infusing National Education into the primary school curriculum* (pp. 117 – 130). Singapore: Pearson Education Asia.

Wong, K. Y., & Quek, K. S. (2006, May). *Encouraging student questioning among mathematically weak students*. Paper presented at ERAS Conference 2006: Diversity for Excellence: Engaged Pedagogies. Singapore.

Wong, K. Y., & Quek, K. S. (2009). *Enhancing Mathematics Performance (EMP) of mathematically weak pupils: An exploratory study.* (Unpublished Technical Report). Singapore: National Institute of Education, Centre for Research in Pedagogy and Practice. Available from http://repository.nie.edu.sg/jspui/handle/10497/2900

Wong, K. Y., & Quek, K. S. (2010). Promote student questioning in mathematics lessons. *Maths Buzz, 11*(1), 2 – 3. Available from http://math.nie.edu.sg/ame/

Wu, Y. K. (2005). *Statistical graphs: Understanding and attitude of Singapore secondary school students and the impact of a spreadsheet exploration*. Unpublished PhD thesis, Nanyang Technological University, Singapore.