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RIDING THE THIRD WAVE: NEGOTIATING TEACHER AND STUDENTS' VALUE PREFERENCES RELATING TO EFFECTIVE MATHEMATICS LESSON

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The "Third Wave" is an ongoing international collaborative mathematics education research project, involving 10 countries conducted over the years 2009-2011. Adopting the theoretical framework of social cultural perspective, the project aimed to explore the contextually-bound understanding and meaning of what counts as effective mathematics lesson from both the teachers and pupils' perspectives. This paper will begin with a brief description of the Third Wave Study Project, the research framework and the general methodology used. Thereafter, it will concentrate on the main focus of the paper featuring a detailed discussion of the related findings from the Malaysian data. The data involved six mathematics teachers and 36 pupils from three types of primary schools. Multiple data sources were collected through classroom observations, photo-elicited focus group interviews with pupils and in-depth interviews with teachers. During each class lesson observation, the six selected pupils (as predetermined by their teacher) were given a digital camera to capture the moments or situations in the observed lesson that they perceived as effective. Pupils were then asked to elaborate what they meant by effective mathematics lesson based on the photographs that they have taken. Teachers were also interviewed individually immediately after each lesson observation and pupil's focus group interview. Findings of the study show that both teachers and pupils shared two co-values and two negotiated values in what they valued as an effective mathematics lesson. The two co-values are "board work" and "drill and practices" while the two negotiated values are "learning through mistakes" and "active student involvement". However, there are minor differences in teachers' and pupils' value preferences, for instance, pupils valued more of "clear explanation" from their teachers and active participation in classroom activities whereas teachers put emphasis on using different approaches to accommodate different types of pupils. More importantly, it was observed that an effective mathematics lesson is very much shaped by the continuous negotiation between teachers' and pupils' values and valuing. This paper will end with reflections on some possible implications and significant contributions of the study in mathematics education.

Keywords: Effective lesson; Expert/Excellent teacher; Mathematics Education; Primary School; Photo-elicited interview; Third Wave, Values.

INTRODUCTION

As reflected in the title, this paper aims to discuss the value negotiation between teachers and pupils about what constitutes an effective mathematics lesson. More specifically, what are the characteristics of a mathematics lesson that are valued as effective by the teacher and his/her pupils in a mathematics classroom? Do they co-value the same characteristics? How do the teacher and his/her pupils negotiate to shape an effective mathematics lesson?

To set the context, this discussion will begin with a brief introduction of the Third Wave Project, in which the data of discussion were based upon, followed by the research framework and methodology used.

THE THIRD WAVE PROJECT AND ITS RESEARCH FRAMEWORK

The Third Wave Project was a multinational research project coordinated by Seah Wee Tiong from Monash University, Australia (see Seah & Wong, 2012 for more details). The project was conducted by 12 research teams in 11 economies. These 11 economies included Australia, the Chinese mainland (2 teams from 2 different provinces), Hong Kong, Japan, Macau, Malaysia, Singapore, Sweden, Taiwan, Thailand, and USA. Conducted over the years 2009–2011, the project aimed "to investigate the harnessing of relevant values to optimize school mathematics teaching and learning" (Seah & Wong, 2012, online first version, no page number). Considering the cognitive dimension as the first wave and the affective dimension as the second wave, the project adopted the value perspective as the third wave dimension for examining effective mathematics teaching and learning.

The study was framed by three main knowledge domains, namely effectiveness in mathematics learning, values (in mathematics education) and the role of interaction in education. First, the project argued that the notion of effectiveness in mathematics teaching could be cultural related and thus value laden. This cultural context is not restricted to only the national cultures, but also the ethnic, occupational, religious and gender culture. For example, it was found by Cai and Wang (2010) that mathematics teachers from the East (Chinese mainland, in particular) and their Western counterparts (such as the USA) differed in their view of effective mathematics teaching, and this difference was closely related to their values about the nature of mathematics. Moreover, results of a few international comparative studies such as PISA 2003 (OECD, 2004). have also shown that effective teaching is more about responding to the socio-cultural aspects of the learning environment rather than adopting a particular pedagogy. Therefore, unpacking what each culture values and considered as important will help to identify the characteristics of effective mathematics teaching that are really suited to that particular cultural context.

Second, recognizing that values play a central role in any culture, values (in mathematics education) is thus regarded not as an affective factor, but more a socio-cultural product, drawing their form and meaning from discourses, practices and norms of participants and the interaction among themselves (Seah & Wong, 2012). Consequently, mathematics classroom is considered an important venue where teachers and students negotiate their values, and where complex interaction and discourse exchanges are being carried out. It is, therefore

suggested that classroom observation becomes the major means of data collection in this project.

Next, the interaction between the teacher and pupils, particularly in mathematics classroom activities is proposed as the place where social mediation and negotiation of what these teacher and pupils considered as important is taking place. What the teacher values as important for his/her pupils will be enacted and emphasised in teaching. Likewise, pupils also bring in their individual values about what an effective mathematics lesson should be based on their own prior knowledge and experiences. However, the teacher's values might sometimes be in agreement but at other times in conflict with his/her pupils' values. It is through active interaction and negotiation between the teacher and pupils that what needs to be valued by the class as a whole will be perceived as being co-valued by both parties.

PURPOSES OF THE MALAYSIAN STUDY

In concomitant with the Third Wave Project, there were three main purposes in the Malaysian study: (i) to identify the elements of mathematics lessons considered as effective primary mathematics lessons by the teachers and their pupils; (ii) to identify any common value(s) underlying effective primary mathematics lessons across different cultural groups; and (iii) to explore the negotiation and co-emphasis of values in effective mathematics lessons between mathematics teachers and students. However, the discussion of this paper will focus mainly on the third purpose mentioned.

METHODOLOGY

The research framework of the study as discussed in the earlier section suggested a socio-cultural approach to explore teacher and pupils' preferred values in an effective mathematics lesson. The following is a brief discussion of the participants and methods of data collection.

Participants

This study involved six primary school mathematics teachers selected from the latest published list of "Excellent Teacher" released by the Ministry of Education in 2009. In Malaysia, 'Excellent Teacher' (or "Guru Cemerlang" in the Malay language) is a promotion scheme introduced by the Malaysian Ministry of Education since 1993 (Malaysian Ministry of Education, no date). Under this scheme, the award is conferred on a teacher who exhibits these four main characteristics: a) possesses high expertise, knowledge and skills in his/her subject area; b) dedicated and responsible; c) highly motivated especially in teaching and learning in the classroom; and d) had at least five years of teaching experience in school. This study deliberately chose the excellent teachers as the participants based on the assumption that these teachers were more likely to produce quality or effective teaching than the normal teachers in schools. Even though the study acknowledged that not every lesson taught by these "Excellent Teacher" ought to be an effective lesson.

Table 1 provides brief background information of the six teachers. Both teachers G and K are Indian male teachers who are teaching in two different Tamil Primary schools whereas teachers C and L are Chinese female teachers teaching in two different Chinese primary schools. Teacher Z is a Malay man teacher while teacher R is a female Malay teacher, both are teaching in two different national primary schools. Among these teachers, Z, C and L are very experienced teachers with more than 30 years of teaching experience while R, G and K are younger teachers and newly accredited as "Excellent Teacher".

Table 1: Respondents' Profile

Teacher	Z	R	C	L	G	K
Gender	Male	Female	Female	Female	Male	Male
Race	Malay	Malay	Chinese	Chinese	Indian	Indian
No. of years of teaching mathematics	30 years	10 years	30 years	31 years	11 years	9 years
Year awarded with ET	1999	2009	2001	2008	2008	2008
Types of primary school	Malay	Malay	Chinese	Chinese	Tamil	Tamil

ET=Excellent Teacher

Besides the teacher participants, there were also 36 pupils participated in this study. Each teacher was asked to select 6 pupils (two high performing; two averages and two low performing) from the class that was observed for teaching. These pupils were given a camera each to capture the critical moments of the lesson and to elaborate in the focus group interview after the observed lessons. More detailed discussion about this method of data collection or better known as photo-voice will be elaborated in the next section.

METHODS OF DATA COLLECTION

This study employed three methods of data collection as briefly described below:

(1) Observations of classroom teaching

Two video cameras were used to record the classroom teaching so as to provide a holistic view and the teaching process of the lesson. One video camera focused on the teacher while the other focused on the pupils. Each teacher was observed for three mathematics lessons. These lessons were prepared by the Excellent Teachers on a particular mathematics topic that they choose and would like the researchers to observe. The duration of each lesson ranged from 50 minutes to one hour. Table 2 displays the grade level and topic of each lesson taught by each teacher.

Table 2: Grade level and topics taught on each lesson observation by each teacher

Teacher	Grade of Class taught	Lesson 1	Lesson 2	Lesson 3
G	Grade 6	Mixed operation	Area	Pie chart
K	Grade 5	Percentage	Mass	Perimeter
C	Grade 5	Multiplication	Percentage	Mass
L	Grade 3	Volume	Volume of liquid	3-D shapes
Z	Grade 4	Fraction	Division	Money
R	Grade 4	Length	Perimeter	Volume of liquid

(2) Photo voice or photo-elicited interview with pupils

In view of the limitation of vocabulary, language competency and expressive ability of young children, this study adopted the concept and method of photo voice which was first introduced by Wang and Burris (1997). They defined photo voice as a participatory research methodology that uses photographic technique that allows the participants to "identify, represent and enhance their community" (p. 369). Photo voice might be new as a data collection tool in mathematics education research, nevertheless it has been widely used in community research studies (e.g. Wang, 1999; Young & Barrett, 2001; Wang, Cash & Powers, 2000). In fact, Darbyshire, MacDougall and Schiller (2005) had adapted photo voice as one of their data collection methods to explore the Australian children's perceptions and experiences of place, space and physical activities. The children were provided with a disposable camera and asked to take photographs on any physical activities. They would then write a brief comment and reasons that they took each photograph. The children were found to express themselves better during the interview based on the photographs taken.

In this study, during each lesson observation, the six selected pupils were given a camera each to capture any moment that they deemed the teacher was teaching effectively. At this junction, my research team faced a methodological challenge. On one hand, we expect the pupils to give as verbatim as possible their meaning of "effective mathematics lesson ". Thus, we did

not provide them a definition or an example. However, on the other hand, most pupils would ask for an example of an effective lesson or the meaning of "effective". To resolve this problem, we opted to give the pupil an analogy of medicine. We told the pupils that when they have stomach pain, the doctor prescribe them a kind of medication. They take the medicine and they are get cured. So they would say that medicine is effective. Similarly, what kind of teaching you would like to have or you think it is important to be an effective teaching. We acknowledged that this ambiguity could be a limitation for this study. Therefore, after each lesson observation, we asked each pupil to explain based on the photographs that they have taken. The aim of this focus group pupil interview was to explore the characteristics that the pupils valued as an effective mathematics lesson.

(3)Teacher Interview after each lesson observation

Immediately after each lesson observation, the pupils were interviewed then followed by the teacher. During the teacher interview, the teacher was first asked to reflect whether the objectives of his/her lesson were achieved. The teacher concerned was also asked to indicate the moments of teaching that he/she perceived as effective. Later, the teacher was shown the photographs taken by their pupils to check if they agreed with the pupils' view about the elements of an effective mathematics lesson.

FINDINGS AND DISCUSSION

In total, 18 lessons were observed, 18 individual teacher interviews as well as 18 focus group pupil interviews were video recorded and transcribed for analysis. Each of the 36 selected pupils took photographs of effective moments during the three observed lessons for each school.

Table 3: Number of photographs taken by each pupil from the six schools

Pupil	SK A	SK B	SJKT L	SJKT P	SJKC T	SJKC M	Total
A1	15	22	11	85	26	18	177
A2	15	19	26	48	8	17	133
B1	44	40	26	64	9	18	201
B2	27	28	34	41	42	13	185
C1	19	32	56	38	4	16	165
C2	14	51	7	81	14	29	196
Total	134	192	160	357	103	111	1057

As shown in Table 3, each pupil took photographs ranging from as few as four photographs to 85 photographs. Altogether 1057 photographs were taken. There was no significant observed difference in the number of photograph taken by the pupils in terms of their ability levels or type of school. Comparatively, pupils from SJKC T appears to take the least number of photographs, perhaps these pupils came from the younger grade (they were Grade 3 pupils). Meanwhile, pupils from the SJKT P seem to capture the most number of photographs, but many of the pictures taken were repetitive.

The data were then analysed separately based on the video recorded classroom teaching and individual interview for teachers while photo-elicited focus group interviews and photographs taken for pupils. A list of themes emerged from the analysis of the data on the classroom observation (a total of 59 themes); teacher interviews (a total of 40 themes) and student interviews (a total of 83 themes). For the purpose of this paper, only the five most common themes from each data source were compared. Table 4 and Table 5 below display the distribution of themes for teachers and pupils respectively.

Table 4: Themes representing espoused and observed classroom practice across different schools

Theme	Type of school					
	SK		SJKC		SJKT	
School	A	B	M	T	L	P
Teacher	Z	R	C	L	G	K
1. Board work	✓〇	✓〇	✓〇	✓〇	✓〇	✓〇
2. Drill and practice	✓〇	✓〇	✓〇	✓〇	✓〇	✓〇
3. Different approaches for different types of pupils		✓	✓		✓	✓
4. Use of real or concrete objects	✓〇	✓〇	✓〇	✓〇	✓〇	✓〇
5. Engaging ICT or courseware	✓〇	✓〇		✓〇	✓〇	✓〇

✓ = espoused in interview 〇= observed in classroom teaching

As displayed in Table 4, all the six participating teachers espoused (in the interviews) as well as enacted (as observed in classroom teaching) five common themes: namely board work, drill and practice, different approaches for different pupils, use of concrete objects and integration of courseware. Similarly, the 36 pupils (see Table 5) co-valued board work, exercise or practice, learning through mistakes, explanation and students' involvement as the

five common elements of an effective mathematics lesson. Hence, two out of the five common themes were co-valued by both teachers and pupils, that are, board work and exercises or practices.

Table 5: Themes valued by pupils from all schools

Theme	School					
	SK A	SK B	SJKT L	SJKT P	SJKC T	SJKC M
Board work	√	√	√	√	√	√
Exercise or practice	√	√	√	√	√	√
Learning through mistakes	√	√	√	√	√	√
Explanation	√	√	√	√	√	√
Students Involvement	√	√	√	√	√	√

Co-value 1: Board work

In this study, board work refers to asking pupils to come in front of the class to demonstrate their working or writing out their solutions on the board. There were 85 photographs taken by the pupils exhibiting board work. During the focus group pupil interviews, the following reasons were given:

> "when he (the pupil) works in front, everyone in the class can see what he is doing]" (translated from Malay language, SK B_B1).
>
> "The mistakes made can be discussed together" (translated from Malay language, SK B_B2).
>
> "Sir will call the students who know how to do it to do so that whoever don't know, they also will learn from it" (translated from Tamil, SJKT P_A1).
>
> "Maybe she don't understand it well…to make her clearer, sir called her out to do it" (translated from Tamil, SJKT P_C1).
>
> "In this photo, teacher asked a pupil to come out and do,…Teacher called the boy who can do to come out and do, or else we may not understand later" (translated from Mandarin, SJKC M_B1).
>
> "Teacher asked a pupil to come out and do, if he did wrongly, teacher can then correct him" (translated from Mandarin, SJKC M_A1).

Analyzing these pupils' reasons indicates that pupils valued board work as a platform for them to learn from their peers. This can happen in two ways. If the better ability pupils were asked to demonstrate their solutions in front, the weaker peers can learn from them the correct methods of solving the problem. Conversely, if the weaker ability pupils were called out, the other peers can learn from the mistakes made.

Meanwhile, engaging pupils in board work was also commonly mentioned as one of the elements of an effective mathematics lesson during the teacher interviews. For instance, Teacher R from SK B espoused in the interview that she liked to call pupils to show their work in front of the class, particularly the academically weaker pupils. Teacher R mentioned in her interview that, "*the children come to show in front, their friends can spot where they have made mistakes*" (translated from Malay; *SK, Teacher R interview after 1st lesson).* Teacher K from SJKT P also echoed that asking pupils to present their solutions in front is a good strategy as "*when one group had learned, if got some mistakes, they can share…* "(SJKT, Teacher K interview after 3rd lesson). Hence, it is observed that what the teachers have espoused was coherent with what were mentioned earlier by their pupils. In brief, engaging pupils in board work allows pupils to learn from each others, particularly from the peers' mistakes. Therefore teachers would encourage pupils to show their workings through board work while their pupils also support this kind of activity. Thus, both teacher and pupils co-valued the importance of board work as an important element of an effective mathematics lesson.

Co-value 2: Exercises and Practices

Giving exercises for drill and practice was observed to be a common norm for most mathematics classes in all the three types of primary schools in Malaysia. Thus, it was not surprising to notice that many of the pupils took photographs on worksheet or textbook exercises that were assigned by their teachers (see some sampled photos below):

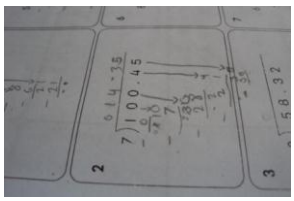

Figure 1: photograph taken by SK_B1

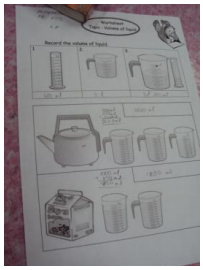

Figure 2: photograph taken by SK B_C1

Figure 3: photograph taken by SJKC T_A2

During interview, pupils gave practical reasons such as:

"Sir Vanthu,(Sir) give us exercises to [do and to see if] students understand or not" (SJKT L_B2).

> "He [the teacher] will give them [the pupils] more exercises and allocate time for them and explain to them" (translated from Tamil, SJKT P_B1).

> "Teacher gave us worksheets to practice. If we do, during examination, there are a lot of questions. If those that the teacher photocopied are similar to the one in examination, then we will know how to do" (translated from Mandarin, SJKC T_C2).

The reasons given indicate that pupils valued exercises and practice for enhancing their understanding, and more importantly so that they can score well in examinations. The examination-oriented culture, particularly in Chinese primary schools, is clearly rooted as early as in Grade 3 (illustrated from the interview with the Grade 3 pupils of the selected school –SJKC T as displayed above).

The value of drill and practice was also clearly reflected in teachers' interview data. For instance, Teacher R highlighted that in one of the interview that, "In my mindset, learning mathematics must do a lot of exercises... if lack of exercise, we will be weak." (Translated from Malay, SK, Teacher R interview after 2nd lesson). Her opinion was supported by Teacher K that, "that is my secret for me, I give more activities, more exercises…I apply [this] for the Year 1 [pupils] also, I think that is a better way, they can do well." (SJKT, Teacher K interview after 3rd lesson). Teacher K further added that, "Before the exam, I have given them 3 or 4 books, the exercise books, the small squares, so many activities, so many exam papers, I prepared them to do well." In brief, these teachers appeared to hold a strong belief of "practice make perfect" and drill and practice is one of the effective ways of ensuring their pupils do well in examination.

Clearly, this co-sharing of the value of drill and practice has allowed both the teacher and their pupils to negotiate and to support each other's effort. The teachers gave a lot of drill and practice to their pupils in the forms of individual exercises, group exercises, memorization of multiplication tables and home work. The pupils were willing to carry out these practices in school as well as at home as they hold the same belief of practice makes perfect, and practice as the way to ensure better performance in examination.

Besides the above two clear-cut co-values: board work and exercises or practices, further analysis of the data show that there were a few negotiated values as discussed below:

Negotiated value 1: Learning through mistakes

"Learning through mistakes" is classified as a negotiated value because this value was explicitly mentioned by the pupils but only implicitly reflected in teachers' interviews. For example, a pupil (A1) from one of the national primary school (SK A) took a photograph that depicts a comparison of four pupils' solutions written on a small board (see Figure 4). During the interview, the pupil (A1) explained that, "Can correct what is wrong or not?" (Translated from Malay language). His explanation was supported by his peer (B1) that, "Teacher want us to see and to show which is right which is wrong" (Translated from Malay language, SKA_B1).

Figure 4: A photo taken by SK A_A1 showing a comparison of different solutions on the blackboard.

Similar photograph content and similar argument was also given by a group of the Chinese primary school (SJKC M) pupils as illustrated in their interview conversation below: (The following conversations were translated from Mandarin):

R: Why does the teacher want to compare one right and one wrong?

C2: So that we can know…

A1: Which is wrong, which is right.

B1: Because (the teacher) wants us to know where the mistake is.

A2: Can get us remember, because after teacher discussed about it, we can remember it better.

The above pupils' reflections show that pupils valued their teachers correcting their mistakes immediately in class or in their exercise books, so that they can learn through their own or their peers' mistakes.

However, no teacher mentioned explicitly about the value of learning through mistakes. Instead, they negotiated this value through the effective use of board work. Teachers liked to call pupils, particularly the academic weak pupils to display their working or solution steps in front of the class. Their purpose is to assess to what extent these pupils have achieved the teaching and learning objectives. Three of the teachers explicitly mentioned that,

"I choose mostly weak pupils" (translated from Malay; SK, Teacher R interview after 2nd lesson); "Just now I chose all are weak or slow learners." *(*SJKT, Teacher K interview after 2nd lesson); *and "*those who are lagged behind, I will call them especially" (translated from Chinese, SJKC, Teacher L interview after 3rd lesson).

Nevertheless, their intentions were well received by their pupils who took them as opportunities for learning.

Negotiated value 2: Active student involvement

Student involvement refers to the moments or situations whereby the pupils mentioned explicitly in the interviews or in the photographs taken that they were asked by their teachers to go out to the front of the class to be involved in teaching and learning activities such as demonstration, role play, testing memorising of multiplication table or playing games. In total,

there were 85 photographs taken depicted pupils' active involvement in some activities. Figure 5-7 illustrate some of the scenes taken.

Figure 5: A pupil was asked to fill in the table on the board, regarding the number of sides and angle of different polygons. (Photograph taken by SJKC T_A2)

Figure 6: Two pupils were asked to role play as seller and buyer in the lesson on 'mixed operation'. (Photograph taken by SK A_C2)

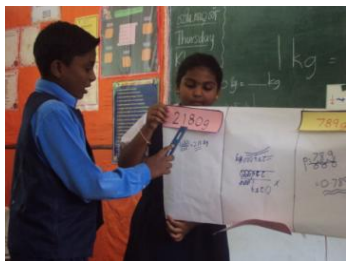

Figure 7: Pupils were asked to explain their solution in front of the class. (Photograph taken by SJKT P_A2)

The finding that pupils like to be involved in activity did not surprise the teachers. Teacher C mentioned that, "They (her pupils) are very active. They like to come in front (of the class) and show" (translated from Mandarin, SJKC_M, interview after 1st lesson). Her response was echoed by another male teacher from SJKT_P that, " Because this class, they every...every day they ask me to call (them) in front. Students they want to do in front" (SJKT, Teacher K interview after 2nd lesson). Likewise, Teacher L also found some of her pupils liked to show their works in front of the class. She recalled, "They like to involve

themselves in learning, so they like to come out. I gave them chances to answer, come out to the front, they like to write it out at the front. *(Translated from Chinese, SJKC, Teacher L interview after 3rd lesson).*

Thus, the above teachers' reflection indicated that the teachers have tried to fulfil their pupils' value of student involvement through the negotiation of giving board work and group activities. However, most teachers were constrained by the shortage of time and the overloaded syllabus that they could only provide selective activities for their pupils.

IMPLICATIONS AND CONCLUSION

The above analysis implies that both the teachers and their pupils shared two co-values and two negotiated values. The two co-values are "board work" and "drill and practices" while the two negotiated values are "learning through mistakes" and "active student involvement".

In fact, board work provides a platform for the pupils to display their solution or working steps in front of the class. In this way, the teacher can assess the progress or achievement of the pupils as well as diagnose the mistakes of his/her pupils. At the same time, board work allows pupils to learn through their peers' mistakes, as well as provide the pupils with opportunities to engage themselves actively in the teaching and learning process. Indeed, board work was commonly observed in many Asian mathematics classrooms. For examples in Japanese classroom (Shimizu, 2009) and Australian classroom (Seah, 2007), students are commonly asked to come in front to present their works too. Likewise, Lim (2007) observed that Shanghai mathematics teachers also like to "call individual student to demonstrate in front of class, answer or explain orally" (p. 81 in Table 1) as a way to engage the pupils.

More importantly, to make board work as a platform for effective learning, the teacher needs to carefully pose appropriate questions and choose pupils' solution steps to display on the board for discussion. As highlighted by Takahashi (2011), in a typical Japanese mathematics lesson, the teacher usually presents a problem to his/her pupils without demonstrating a procedure or method. The aim is to encourage pupils to bring in different approaches and different possible solutions. The experienced Japanese teacher will then go round the class to select appropriate exemplar solution methods. These anticipated solution methods should include both the most efficient methods and also those caused by misconceptions. In this way, the teacher can then facilitate the pupils to compare and justify which is the best method to use, as well as correcting some misunderstandings of pupils.

The next implication is that board work could be particularly effective for mathematics teaching and learning as compared to other subjects. Solving mathematical problems requires showing the working or solution steps precisely. Pupils are expected to derive step by step logically and to clearly demonstrate how a solution is arrived. Unlike other subjects such as language (e.g. English) or even art or science subjects (e.g. history, biology) might be too content laden or need too much space and time to demonstrate their solutions on the board. Therefore, this finding implies that using board work could be an effective teaching approach that provides a platform to engage pupils actively as well as providing a two way

communication between teacher and pupils, particularly suitable for mathematics teaching and learning.

The second characteristic that was co-valued by both teachers and pupils as effective mathematics lesson was the beliefs about the role of practice and exercises in mathematics learning. Mathematics composes of both concepts and skills. It is generally believe that drill and practice is an effective way to sharpen mathematical skills. Particularly, drilling of basic facts can help pupils to memorise the multiplication tables and formulae which consequently resulted in automatic response, which could be useful for recalling of facts and formulae during problem solving. Indeed, this belief of "practice makes perfect" is also a common norm in many of the examination oriented Asian countries (such as China and Singapore). Holding strongly to this belief, therefore the teacher tends to give a lot of exercises and problems for his/her pupils to practice and solve. As the pupils also hold strongly to the same belief of practice makes perfect, they do not complain about the amount of exercises or home work that they were given. Thus the teacher strives hard to provide more exercises for the pupils, while the pupils work hard to complete them, so as to achieve excellence in the mathematics examinations.

However, one major implication to be cautious is that drilling without conceptual understanding or some called it as procedural learning might not lead to effective mathematics learning in the long run. The amount of time spent and the quantity of exercises given do not necessarily equate to the quality of learning. Therefore careful selection of exercises and variation of exercises given are crucial in applying the belief of practice makes perfect.

Finally, the findings of this study imply that there were minor differences in teachers' and pupils' value preference in relating to an effective mathematics lesson. Pupils valued more of "clear explanation" from their teachers and active participation in classroom activities whereas teachers place emphasis on using different approaches to accommodate individual differences of pupils. Nevertheless, to ensure an effective mathematics lesson, both teachers and pupils will have to negotiate continuously in order to suit the pupils' and the teachers' value preferences. As in this study, the teacher negotiated pupils' value preference of active participation through board work and group activities occasionally. Pupils valued clear explanation from their teacher which is negotiated with their teacher's value of "using concrete objects". Through the use of concrete objects, the teachers believed that they would give a more concrete and clearer explanation to their pupils that would further enhance their understanding of abstract mathematical concepts.

In brief, for any mathematics lesson to be effective, ideally, both teachers and pupils should share and co-value collectively. Or else, they will have to negotiate continuously between teachers' and pupils' value preferences.

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