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MATHEMATICAL THINKING STYLES AND THEIR INFLUENCE ON TEACHING AND LEARNING MATHEMATICS

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A mathematical thinking style is the way in which an individual prefers to present, to understand and to think through, mathematical facts and connections by certain internal imaginations and/or externalized representations. In which way mathematical thinking styles (analytic, visual and integrated) are influence factors on the learning and teaching of mathematics is described on the basis of selected qualitative empirical studies from primary up to secondary school.
Within the current MaTHSCu-project the styles are measured quantitatively by comparing mathematical thinking styles in eastern and western. This study is introduced and finally conclusions and implications for school are drawn.

INTRODUCTION

„*I experienced myself, how mathematics can be opened by one teacher and closed by another.*" (Wagenschein 1983)

This citation comes from Martin Wagenschein (1896-1988), who was a pedagogue, math and physics teacher and a lecturer at several universities in Germany. Parts of his work also influenced the discussion in mathematics education in Germany. This citation is an appropriate starting point for thinking about *preferred* ways to learn and understand mathematics for one's own and as from researcher's perspective as well. How do you *like* to learn and understand mathematics? This is quite a simple question – but offers a lot of interesting answers, if pupils and students from primary up to secondary and vocational schools and university were asked:

"I understand mathematics the best way, when Mrs. D. is drawing pictures on the table, because I need these pictures also in my mind." (David, 10 years, Grade 4)

„I like to learn mathematics with numbers and symbols. Sketches do not help me really in my process of understanding." (Gloria, 16 years, Grade 10)

"My previous teacher explained fast and much and did not make any drawings… My new teacher always makes a drawing and now I understand how to come to the result, not like only by formulae and calculation." (Sarah, 15 years, Grade 9)

There are kinds of explanations which cause young people to understand mathematical methods well and others through which they only understand a little. Some individuals like pictures and visualizations or some prefer formulae and variables and again others like something in between pictures and formulae. Having a good or bad understanding of mathematics is not an unusual matter of fact happening during school time and is influenced by a lot of factors. Not only teachers also many psychologists and pedagogues still share the opinion that success and failure of learning are exclusively caused by individually different learning abilities. Similarly, it remains still unanswered why the same pupil produces bad results in a multiple-choice task in mathematics while within a math-project he or she produces extraordinary results. Mathematical abilities are probably the first explanation, which comes in mind. But mathematical abilities are not the whole answer to these phenomena. Another, meantime well-funded explanation, is based on *mathematical thinking styles (visual, analytic and integrated thinking style)*, which are preferences for using our mathematical abilities.

So in the following, the theory of *M*athematical *T*hinking *S*tyles (*MTS*) is described theoretically and on the basis of qualitative and quantitative empirical studies. These demonstrate how mathematical thinking styles influence the teaching and learning of mathematics. The MaTHSCu-Project (Mathematical Thinking Styles in School and Across Cultures (since 2012); Project leader: Rita Borromeo Ferri) focuses on the question, if there are differences in mathematical thinking styles of 15 year old pupils and their math teachers in eastern and western cultures (South Korea[1], Japan[2] and Germany).

THEORETICAL BACKGROUND OF MATHEMATICAL THINKING STYLES

The term mathematical thinking style is characterised as follows:

“A mathematical thinking style is the way in which an individual prefers to present, to understand and to think through, mathematical facts and connections by certain internal imaginations and/or externalized representations. Hence, a mathematical style is based on two components: 1) internal imaginations and externalized representations, 2) on the wholist respectively the dissecting way of proceeding.” (Borromeo Ferri 2004, 2010)

A central characteristic of the construct mathematical thinking style is the distinction between abilities and preferences as mentioned before. Mathematical thinking styles are about how a person *likes* to understand and learn mathematics and not about how good this person understands mathematics. This approach is based on the theory of thinking styles of Sternberg (1997). So in the sense of Sternberg (1997), “A style is a way of thinking. It is not an ability, but rather, a preferred way of using the abilities one has.” That means that thinking styles are not viewed as being unchangeable, but they may change depending on time, environment and life demands. Sternberg and Wagner (1991) created a Thinking Styles Inventory for testing 13 different thinking styles based in Sternbergs (1997) theory of mental-self-government.

[1] Many thanks to Prof. Oh Nam Kwon and Prof. Mi-Kyung from the Seoul National University, South Korea.
[2] Many thanks to Prof. Toshikazu Ikeda from the Yokohama National University, Japan.

These styles were not mathematical thinking styles, but he used this test in schools, universities and other professional area. Besides Sternberg's theory of thinking styles also Riding & Rayner (1998) and Riding (2001) have to be mentioned, who dealt with cognitive style dimensions (verbalizer-imager, wholist-anayltics) and questions how information can be differently understood on an internal and external level. For Riding & Rayner (1998, 8) a "cognitive style is seen as a preferred and habitual approach to both organizing and representing information." So one part of the characterization of mathematical thinking styles make clear, that a preferred way of learning and understanding mathematics can also be distinguished in the way of proceeding. This means a task can be solved in a dissecting or in a wholistic way in combination to several modes of representation for example analytically or visually. The named representations were from great interest for gaining the theory of mathematical thinking styles. In the literature one can find a lot of classifications of thinking.

In 1892, in German, Klein constructed a typology of three different thinking styles. This classification was based on observations in cooperation with other mathematicians and not based on empirical studies:

"1) The philosopher who constructs on the basis of concepts

2) The analyst who essentially "operates" with a formula

3) The geometer whose starting point is a visual one ("Anschauung")"

(quoted from Tobies 1987, 44) (Original in German, translated by the author)

A similar typology, restricted to visual and analytic thinking styles, is found in Hadamard (1945). Unlike Hadamard, but similar to Klein, Burton (1995, 95) identified three, and not two, styles of thinking: visual, analytic and conceptual thinking style. Furthermore Skemp (1987) distinguished between verbal and algebraic symbols for understanding mathematics and many other researchers are dealing with the question, which kind of representation could be effectively on learning mathematics. The analysis of the typologies or classifications of thinking and of how they were evolved illustrated, that they were not reconstructed with pupils at school. So in the first qualitative study (Borromeo Ferri 2003, 2004) the goal was to reconstruct visual, analytic and conceptual thinking styles with pupils from Grade 9 and 10 during their pair-problem solving process. The design of the study was very complex for grasping the construct of the construct "style" (preference) itself and the representation (visual, analytic, conceptual) and the way of proceeding (wholist, dissecting) out of the data, stimulated recall and interview. The aim was not only to reconstruct these preferences, but to find explanations, what it means to be a visual or an analytic thinker. So Grounded Theory (Straus & Corbin 1990) was the appropriate method for analysing the data. The results of the study showed, that the conceptual thinking style could not be reconstructed, but the following table, makes the two components of the characterisation of mathematical thinking styles again clear:

	Internaly oriented types			Externaly oriented Types				
				congruent			incongruent	
1) / 2)	picture	symbolic	mixed	picture-picture	mixed	Symbolic-symbolic	Picture/ symbolic	Symbolic/ picture
Wholists								
Combiners								
Dissecters								

Figure 1: Model to describe the construct mathematical thinking styles and their different kinds of mathematical thinking styles.

Some explanation of this model: Component 1) includes internal imaginations and externalized representations. Through this, I define internal types as individuals who mainly assimilate facts internally and who do not see the necessity for representations (except if they serve as means of communication). External types, however, make external representations. If their internal imaginations match with the externalized representations (e.g. picture-picture) they are called congruent, if these do not match (e.g. picture-symbolic), they are called incongruent. Component 2) examines the process of solving the task which can be understood in a wholist way (task is exploited from the whole to parts of it), in a dissecting way (task is exploited from parts to the whole) and in ways combining these two “pure” ways.

Based on this an empirical grounded description of the characteristics of the visual, analytic and integrated thinking style could be developed:

- Visual thinking style: Visual thinkers show preferences for distinctive internal pictorial imaginations and externalized pictorial representations as well as preferences for the understanding of mathematical facts and connections through holistic representations. The internal imaginations are mainly effected by strong associations with experienced situations.
- Analytical thinking style: Analytic thinkers show preferences for internal formal imaginations and for externalized formal representations. They are able to comprehend mathematical facts preferably through existing symbolic or verbal representations and prefer to proceed rather in a sequence of steps.
- Integrated thinking style: These persons combine visual and analytic ways of thinking and are able to switch flexibly between different representations or ways of proceeding.

Mathematical Thinking Styles have principles, they

- are not mathematical abilities, but preferences how these abilities like to be used;
- are attributes of the personality, because preferences are connected with positive affects;

- are not mathematical problem solving strategies, because strategies are on a higher level of consciousness;
- are partly influences by (mathematical) socialisation[3], which means, that parents or in most cases teachers give guidelines how mathematics has to be learned and represented during lessons or tests, for example with or without visualisation, pictorial sketches etc.;

THE INFLUENCE ON THE TEACHING AND LEARNING OF MATHEMATICS – RESULTS OF EMIRICAL STUDIES

In this chapter results of two empirical studies will demonstrate at first, in which way mathematical thinking styles influence the teaching and learning of mathematics. Then the mentioned comparative study within the MaTHSCu-Project is described and so is highlighting the cultural perspective.

Mathematical Thinking Styles in Primary School

After reconstructing mathematical thinking styles in grade 9 and 10 for creating the theory, the question aroused, if these styles could also be reconstructed with primary kids[4]. On the one hand the relevance for learning elementary mathematical ideas, ways of thinking and algorithms in primary school was evident, but on the other hand the conceptual development could be not enough to grasp different styles. So an explorative qualitative study was done with two classes in at the beginning of grade 4 (sample: 40 pupils and their teachers). For this study a questionnaire (open items) was developed and used, which firstly should give some ideas for preferences for certain styles. Both classes were then observed for two weeks during mathematics and science lessons. Four kids from each class solved mathematical problems on their own and were videotaped and also interviewed afterwards. The combination of these instruments and the analysis of the several kinds of data made it possible to reconstruct different mathematical thinking styles. Having in mind, that in primary school mathematical hands-on materials and several kinds of representation-modes of numbers are used for constructing amongst others the number sense, it was interesting to see personal preferences at all. For getting a short insight, mathematical thinking styles of two children, Hanna (8 years) and Jens (9 years) are briefly described. Both solved the following problem:

> A cochlea is at the bottom of a 20m deep standpipe. She wants to crawl to the top. During the day she crawls 5m up, but in the night she glides 2m down. At which day does the cochlea arrive at the top?

This problem can be solved in a wholistic and dissecting way in combination with using only numbers or also with a sketch for drawing the crawling of the cochlea up and down and of

[3] Due to the restriction of pages, empirical finding concerning mathematical socialisation in combination with mathematical thinking styles could not be included in this paper. This would be another contribution.
[4] In Germany primary school is from grade 1 to grade 4, ages from 6-10 years.

course without writing something down. Seven of the eight videotaped children got the right solution (on the 6th days in the evening).

Hanna (8 years) could be reconstructed as an analytic thinker on the basis of the questionnaire, the observations, problem solving processes and the interview. She likes numbers and operating with number and also when her teacher is writing calculations on the board. Hanna solved the problem without writing something down, so very internal oriented, although her formal-dissecting way became apparent: "…three metres, early in the morning, second day. ... seven metres ...no, eight metres, late in the evening at the second day.." Hanna did not speak about the cochlea; she had numbers and calculation processes in her mind.

Jens (9 years) could be reconstructed as a visual thinker on the basis of the questionnaire, the observations, problem solving process and the interview. In the questionnaire he marked "I like to understand mathematics, if my teacher makes drawings and pictures on the board". In the interview Jens confirmed this way of understanding and learning mathematics again. For solving the problem he makes a sketch (at the left one can see a part of the drawing). During the solving process he said: "Yeah I see the cochlea crawling up and down and I need to draw it down."

Summarizing this explorative study in primary school, it was in particular of high interest to see *different* preferences for understand and learning mathematics at that age and so supports the principle of a mathematical thinking style as an attribute of personality.

Influence of mathematical thinking styles on the mathematical modelling behaviour

The aim of the project COM² ("Cognitive-psychological analysis of modelling processes in mathematics lessons"; 2005-2010), directed by the author, was to analyse teachers' and students' actions, ways of thinking, in sum, their behaviour while working on modelling problems in mathematics lessons from a cognitive perspective (see Borromeo Ferri 2010, 2011). For that aim, mathematical thinking styles were used as "theoretical glasses" for analysing this behaviour and to interpret this. Without going deeply in the discussion of mathematical modelling, the central research questions of the COM²-project were:

- How do grade 10 pupils solve modelling tasks, and what influences do the mathematical thinking styles of the learners have on their modelling processes in reality-oriented mathematics lessons?
- How do mathematical thinking styles of teachers influence their way of dealing with mathematical modelling problems in the classroom? Are there differences with respect to the various phases of the modelling cycle (real situation, situation model, real model, mathematical model, mathematical results, real results)?

The design of this qualitative study was highly complex, because both teachers and pupils were in the focus. Three grade 10 classes from different Gymnasien (German Grammar Schools) were chosen. The sample was comprised of 65 pupils and 3 teachers (one male, two female). Each individual in a class had to complete a questionnaire on mathematical thinking styles, which has been developed on the basis of the Ph.D. thesis Borromeo Ferri (2004). The pupils were given three different modelling tasks, one per lesson (three lessons altogether). In each of the classes one group was videotaped during the whole lesson. Focused interviews were conducted with the teachers to reconstruct in each case his or her mathematical thinking style (the male teacher was analytic thinker, one female teacher was a visual thinker and the other female teacher was an integrated thinker). As one appropriate method within the field of qualitative research I used Grounded Theory (Strauss & Corbin 1996) because one central aim is the possibility to generate a theory on a code-based procedure.

As a quantitative result, when looking at the statement given from visual and analytic thinkers during their modelling processes altogether 87 verbal statements of analytic thinkers in the realm of mathematics and only 48 statements in reality where their preference. The spread of the visual thinkers is not so high (mathematics: 65; reality: 73), their preference on reality becomes nevertheless apparent. On the basis of deep qualitative analysis different kinds of modelling behaviour became visible, which are summarized in their central characteristics:

- Analytic thinkers usually change to the mathematical model immediately and return to the real model only afterwards when the need arises to understand the task better. They work mainly in a formalistic manner and are better at "perceiving" the mathematical aspects of a given real situation.
- Visual thinkers mostly imagine the situation in pictures and use pictographic drawings. Their argumentation during the modelling process is mostly very vividly even they work within the mathematical model. They often follow the normative modelling cycle.

But also the mathematical thinking style of the three investigated teachers had great influence on their teaching behavior. To make this more concrete on the basis of the analysis: A teacher's mathematical thinking style can be reconstructed and manifests itself during individual pupil-teacher conversations, as well as during discussions of solutions, and while imparting knowledge of mathematical facts. Very interesting was that teachers who differ in their mathematical thinking styles have preferences for focusing on different parts of the modelling cycle, while discussing the solutions of the problems and while helping students during their modelling processes. For pupils who share the mathematical thinking style of the teacher will have a better way of understanding, because both are talking in the same "mathematical language". If there is a mismatch between teacher' and pupils' style this also can have consequences for the learning processes and at least in learners' performance. It is obvious for teachers to reflect about their own mathematical thinking style to be flexible in their way of teaching. In the COM^2-project we showed all teachers some clips of their lessons and did a stimulated-recall asking them, if they recognized personal "teaching-patterns". But mostly teachers were not aware of their behaviour during modelling activities in the classroom, and were astonished about their preferences for certain parts of the modelling process, connected to their mathematical thinking style.

Mr. Peters for example was reconstructed as an analytic thinker on the basis of the interview and observations. In the stimulated-recall he was asked:

Interviewer: Do you think that you have a preference for formalising?

Mr. Peters: "I didn't think about that yet, for me that is mathematics, yes, Iam doing mathematics. But yes, I like formalising mathematics."

Mrs. Heidkamp especially recognized her strong preference for visual thinking (she was reconstructed as a visual thinker) after looking the video-clips.

Interviewer: You recognized that you are a visual thinker. Do you have experiences concerning situations "speaking not the same language" with some pupils?

Mrs. Heidkamp: "Yes, I had a girl who came from another school in my math class. After a while she came to me and told me that she is not able to understand me. She did not understand me! Now I think, that she means my explanations, my mathematical explanations, perhaps they were too visual for her or not concrete enough."

The situation Mrs. Heidkamp is reporting about is a wonderful example for this mismatch between teachers' and pupils' mathematical thinking styles mentioned earlier. Consequences concerning mathematical performance are obvious, because this mismatch induces even though unconsciously impressions of weak mathematical abilities of pupils. Already Zhang & Sternberg (2001, 204) pointed out:

"Findings from a third study indicated that teachers inadvertently favored those students whose thinking styles that were similar to their own."

Mathematical Thinking Styles in School and Across Cultures (MaTHSCu)

The described studies using the theory of mathematical thinking styles for looking at thinking, teaching and learning processes were all qualitative. So a lack of research was the construction of appropriate scales for mathematical thinking styles, which was directly connected with the open question, if the construct of mathematical thinking style can be determined quantitatively. When Sternberg started working on his Thinking Styles Inventory, he made clear that "styles do indeed appear to be largely distinct from intelligence or aptitudes." (Sternberg & Grigorenko 1997, 708) Anyhow Sternberg emphasized these distinctions for his thirteen different styles this is also applied for mathematical thinking styles. But the areas of conflict of style with abilities, intelligence and aptitudes were considered when developing a psychometric test for mathematical thinking styles for pupils and teachers.

In doing so the aim was to find answers for a lot of research questions who could not attended so far on a general level such as:

- Are there differences in the stylistic patterns of boys and girls?
- Are there correlations between mathematical thinking styles and beliefs?
- Are there correlations between preferences for certain mathematical thinking styles and mathematical performance?
- Are there cultural differences in mathematical thinking styles, in particular between eastern and western cultures?

Before the focus will be on the last question, some aspects of the construction of the mathematical thinking style scales are described.

On the basis of the theory of mathematical thinking styles (have a look at figure 1 again) 27 items could be developed for pupils (for grade 9 and 10) and specified for teachers. The "thinking-style-scale" comprised four different sub-scales:

1) visual (5 Items) 2) analytic/formal (5 items) (kinds representation)

3) wholistic (4 Items) 4) dissecting (5 items) (ways of proceeding)

Additionally the following two sub-scales will be correlated for generating the stylistic patters:

5) internal (4 items) 6) external (4 items) (types of assimilating information)

Examples for items for the subscale "analytic thinking" are: Variables and formulae are helpful for me to understand mathematics; I like to use a formula, when I have to solve a mathematical problem. For estimating a four-step interval-scale is used. All items were piloted several times with pupils, teachers and students. After the final pilot study the scales had a good till satisfied reliability[5] (cronbachs alpha): visual (.77), analytic (.90), wholist (.80), dissecting (.60), internal (.65), external (.77). Besides these scales, also four problem solving tasks (open format) were integrated in the test and therefore a coding manual was developed concerning the way of representation and the way of proceeding accompanied with items who asked after the kind of associations or ideas pupils had directly after reading the problem and after they had solved the problem. Furthermore scales from PISA (PISA-Consortium 2003) were integrated in the test, in particular scales of beliefs, self-efficacy, motivation, emotion and concerning exercising mathematics. The questionnaire of the teachers includes besides the scales of mathematical thinking styles also scales of beliefs and how they exercise mathematics. Furthermore a semi-structured interview will give more informations about several teachers.

One central goal of MaTHSCu-project is the comparison of mathematical thinking styles of 15 year old pupils and their math teachers in western cultures (Germany) and eastern respectively Confucian cultures (South Korea, Japan, Singapore, Taiwan and China; see Biggs 1996, 46). In particular results in the area of the culture comparative research area of psychology, which often compared thinking processes of individuals in China vs. USA, pointed out over and over again the preference of Asian people for seeing situations very holistically. On the contrary western individuals have preferences for analytic perspectives (Nisbett 2001, Nisbett & Masuda 2001). Besides Schwank (1996) in particular Cai (1995, 1998, 2002) conducted studies concerning mathematical thinking in eastern and western cultures using routine and problem solving tasks in an open format. As central results Cai emphasised, that individuals from the USA often used pictorial and Chinese individuals rather numeric or symbolic solving processes. As an open question Cai (2002, 281) is asking: "Is it possible that these Chinese students might have used visualization mentally, but hey expressed their solutions in non-visual forms (e.g. algebraic equations)? On the other hand, is

[5] results of the students' questionnaire

it possible that U.S. students have just used drawing strategies because teachers told them so and they did not necessarily think visually?" For answering these questions Cai did not enforced further studies for investigating these phenomena. Even though in the actual discussion are no findings, in which way individual preferences of mathematical thinking styles could be measured in in eastern and western cultures. Although there are differences between both cultures concerning education as well as learning and teaching which are deeply fixed, but mathematical thinking styles are individual preferences and so can be independent of the cultural background. Looking at the societal development of both cultures (as a western culture the Greece stressed individual freedom, "so Chinese see themselves as a part of manifold networks" Kühnen 2003, 14) the conception of education become apparent. Central terms like integration and harmony characterises the teaching and learning situation of the East Asian philosophy (Leung 2001, 44). Vollstedt (2011a, 2011b) reconstructed in her qualitative comparative study of pupils from Hongkong and Germany different types of sense-construction of mathematics, which reflected the described cultural background.

So, all these studies highlighted important, interesting and different aspects of both cultures concerning learning and teaching mathematics. Investigating mathematical thinking styles of teachers and students will give again further findings. Due to the fact that the data collection and data analysis is not finished at the moment, I cannot implement these in this proposal, but in the final form of the ICME-12-proceedings and for the presentation. Elements of the design of this study were mentioned earlier. The data is analyzed statistically with the software SPSS and the method of latent-class analysis will give insight when comparing the stylistic patterns of both cultures and should test one of the obvious hypotheses, that pupils of eastern cultures have preferences for the analytic thinking style and for the wholistic way of proceeding and pupils of western cultures have preferences for the visual thinking style and for the analytic way of proceeding.

SUMMARY AND CONCLUSION

This article is aimed to show the theory of mathematical thinking styles and how different mathematical thinking styles (visual, analytic and integrated) could be reconstructed in several qualitative studies. Also in primary school preferences for mathematical thinking styles of pupils became visible and in the COM²-project the influence of these styles on the modelling behaviour of pupils and teachers were explicit. Finally the current quantitative oriented and comparative study shall answer more questions concerning mathematical thinking styles as one influence factor on the teaching and learning of mathematics also in a cultural context.

Furthermore, the presented results obtained up to present indicate a highly didactical relevance of these kinds of studies: Its significance for mathematics lessons is obvious. Pupils who are not sharing the mathematical thinking style with their teacher may have problems of understanding, but if the teacher is conscious of his own style and arranges mathematical facts in different ways, problems of understanding could be prevented. These results correlate with results from other empirical studies (Zhang & Sternberg, 2001)

Therefore, it is necessary that teachers become conscious about their own mathematical thinking style, on the one hand in order to guarantee equality of chances among pupils, and on the other hand to develop their own mathematical potentials. Doing this and so coming back to the citation of Wagenschein at the beginning, mathematics would not be closed for him from teacher to teacher.

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