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APPLICATIONS AND MODELLING RESEARCH IN SECONDARY CLASSROOMS: WHAT HAVE WE LEARNT?

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This paper focuses on my 20 year program of research into the teaching and learning of applications and modelling in secondary classrooms. The focus areas include the impact of task context and prior knowledge of the task context during the solution of applications and modelling tasks, mathematical modelling in secondary school, metacognition and modelling and applications, curriculum change with respect to applications and modelling and affinity of pre-service secondary teachers with modelling in lower secondary school. Some of the analysis tools used in this research will be presented.

Modelling, secondary school, applications

INTRODUCTION

Research into teaching and learning through mathematical modelling and applications has been quite strong for several decades now (Blum, Galbraith, Henn, & Niss, 2007; Kaiser, Blum, Borromeo Ferri, & Stillman, 2011) and a feature in regular lectures at recent congresses (e.g., Blum, 2008; Galbraith, 2008). However, in many countries it would be true to say, "there is still a substantial gap between the forefront of research and development in mathematics education, on the one hand, and the mainstream of mathematics instruction, on the other" (Blum, 1993, p. 7) when it comes to this area of teaching and learning. My colleagues and I have been researching in this area for many years. In my work I have focused on a range of significant issues that impact on the field of applications and modelling in mathematics education, and I take this opportunity to reflect on some of these from the perspective of my own work and that of others. The focus areas include the impact of task context and prior knowledge during applications and modelling tasks, mathematical modelling in school particularly in secondary school, metacognition and modelling and applications, curriculum change with respect to applications and modelling and affinity of pre-service teachers with modelling in lower secondary school.

Given the various idiosyncrasies associated with some localised curricular initiatives within and between countries the meanings and interpretations ascribed to terms such as *applications* and *mathematical modelling* in my work will first be clarified. These meanings are consistent with those adopted by the International Community for the Teaching of Mathematical Modelling and Applications (ICTMA), which is an Affiliated Study Group of ICMI.

> Various intermediate stages exist between completely structured word problems and open modelling problems where the structuring must be supplied entirely by the modeller. One such stage involves contexts where the aim of the problem is well defined, where the problem is couched in everyday language, but where some additional mathematical information must be inferred on account of the real world setting in which the problem is presented. This is a level between textbook word problems and modelling problems contextualised fully within real-life settings. (Stillman & Galbraith, 1998, p. 158)

These I call applications tasks. With *applications* the direction (mathematics → reality) is the focus. “Where can I use this particular piece of mathematical knowledge?” The model is already learnt and built. With *mathematical modelling* the reverse direction (reality → mathematics) becomes the focus. “Where can I find some mathematics to help me with this problem?” The model has to be built through idealising, specifying and mathematising the real world situation. Both types of task have their place in school classrooms.

The term mathematical modelling when used in curricular discussions and implementations is often interpreted differently. One interpretation sees mathematical modelling as motivating, developing, and illustrating the relevance of particular mathematical content (e.g., Chinnappan, 2010). A second perspective views the teaching of modelling as a goal for educational purposes not a means for achieving some other mathematical learning end. The models and modelling perspectives of Lesh and Doerr (2003), for example, while clearly associated with the first interpretation, extend beyond to include elements of the second. My own approach sees the second interpretation as encompassing the first. Both approaches agree that modelling involves some overall process that involves formulating, mathematisating, solving, interpreting, and evaluating as essential components.

IMPACT OF TASK CONTEXT AND PRIOR KNOWLEDGE DURING APPLICATIONS TASKS

There is a plethora of meanings that the word *context* conveys in mathematics education. Two of these are *situation context*, the “context for learning, using and knowing mathematics” (Wedege, 1999, p. 207) and *task context*, “representing reality in tasks, word problems, examples, textbooks, teaching materials” (p. 206). Although the influence of the situation context on student solutions cannot be denied, the focus here is on the effects of task context using the meaning above.

The location of mathematical tasks in meaningful contexts for both teaching and assessment purposes can be enriching according to Van den Heuvel-Panhuizen (1999). She claims that contexts can achieve this by enhancing accessibility, contributing to "the latitude and transparency" (1999, p. 136) of the tasks, and by providing students with solution strategies inspired "by their imagining themselves in the situation[s]" (1999, p.136) portrayed in the tasks. On the other hand, she acknowledges that the use of tasks embedded in familiar contexts is not always supportive of students' solution attempts and may also create difficulties, particularly in assessment. Students sometimes refuse to engage with the intended mathematical interpretation of the problem by appealing to plausible alternative realistic scenarios that resolve the task non-mathematically (see Gravemeijer, 1994). At other times they ignore the task context entirely and therefore exclude their "real-world knowledge and realistic considerations" (Van den Heuvel-Panhuizen, 1999, p. 137).

Results from a study by Busse and Kaiser (2003) indicate that task context effects can be very individual and unpredictable. They found that at times emotional involvement with issues, such as environmental destruction, which students associated with the situation portrayed in a task context had a distracting effect. A rich store of knowledge about the task context also sometimes became a hindrance as it expanded, rather than limited, the real-world associations a student was able to activate, resulting in confusion. These extraneous associations were occasionally used by students to accept incorrect results. For example, a decrease in global oil consumption was seen as reasonable when there should have been an increase based on extrapolation from given data as the imposition of an "eco-tax" could explain such a spurious result. On the positive side, other students in their study reported the use of contexts of interest to them as being highly motivational, whilst others used knowledge of the task context to correctly verify results. Busse (2011) concludes that it might be useful "to use the notion of *contextual idea* … to indicate the mental representation of the real-world context *offered* in the task" (p. 42). He points out that such ideas are dynamic changing as the student works on the task.

Abstraction Within Versus Abstraction Away From the Task Context

Students often have great difficulty formulating adequate mathematical representations of applications tasks for a variety of reasons. The traditional view of the mathematisation of a task context necessitates the extraction of the inherent mathematics from the situation through a process of abstraction. An alternative viewpoint involves "abstracting *within*, not *away from*," the task context (Noss & Hoyles, 1996, p. 125). In approaching the task, students are viewed as having the potential to activate a web of connections between the situation described in the task statement, their mental models of that situation (both contextual and mathematical) (see Figure 1) and the written mathematical model which they construct (Noss & Hoyles, 1996).

As applications tasks are presented to students as written text, text comprehension strategies come into play. Nathan, Kintsch and Young (1992) theorise that for the comprehension and solving of worded problems to be successful three mental representations of the problem need to be constructed (a) a *textbase* from the textual input in the problem statement, (b) a *situational model* of the events described in the *task statement* and inferred or elaborated from it using the task solver's general knowledge base, and (c) a model of its mathematical structure (the *problem model*). Thus, to understand an applications task in order to construct a *mental situational model* (Nathan et al., 1992), a student must possess sufficient resources to comprehend the situation described together with the appropriate strategies to generate the necessary *inferences* and *elaborations* to fully specify the situation being modelled or mathematised. The situational model draws on the student's prior knowledge to fill in the gaps in the description presented (see Figure 1). Prior knowledge of task contexts can be from (a) vicarious experiences in other academic subjects (*academic* knowledge), (b) general encyclopaedic knowledge of the world (*encyclopaedic* knowledge), or (c) truly experiential knowledge developed from personal experiences outside school or in practical school subjects (*episodic* knowledge). Episodic prior knowledge is personal but it may be derived directly from actions or from observation of actions. This derivation can have implications for the role of episodic prior knowledge in model formulation (see Stillman, 2000, for further details).

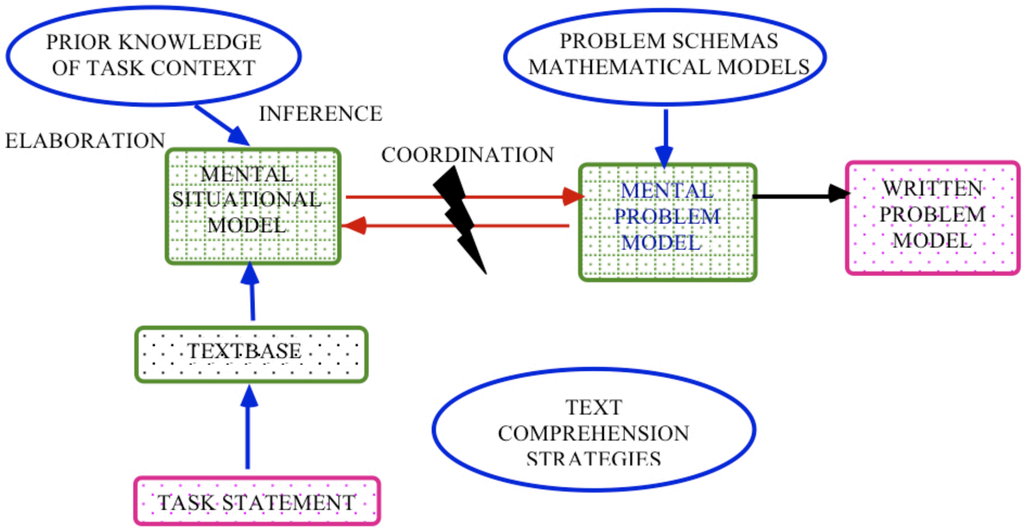

Figure 1. Abstraction within or away from task context (Stillman, 2002b)

According to Nathan et al. (1992), a set of *problem schemas* guide construction of a *mental problem model* from the mental situation model. This is where abstraction away from the context can occur with the mental situational model becoming divorced from the mental problem model instead of the two being integrated if coordination fails (i.e., jagged arrow in Figure 1). If this occurs, the task solver could use mathematical procedures that are invalid within the task context. "There is not necessarily any smooth transition from the situation model" to the problem model and this might not even be possible if the student does not possess the necessary mathematical schemas, selects inappropriate ones, or a mathematical representation of the relationship involved does not exist (MacGregor & Stacey, 1998, p. 58). In these instances the student must recognise the need to discard the situational model (MacGregor & Stacey, 1998), make new inferences, and construct another model.

In a study of a group of Australian senior secondary school students (Years 11 and 12) solving applications tasks (Stillman, 2002a, 2002b), many of the students appeared to use a general problem solving schema rather than specific schemas when attempting the tasks, except when the problem was recognised as being parallel to one they had attempted previously. Different students in the study reasoned within the task context for the purposes of (a) initial comprehension and problem representation, (b) progressing throughout the solution, (c) verifying the final solution and/or (d) recovering from errors. Other students ignored or backgrounded extra-mathematical knowledge. This is a sensible tactic with injudicious problems where the context is not authentic to reality or in problems where the context acts as a border around the mathematics and the two are completely separable. However, even though a task may be appropriately classified as a border problem, in practice the separation process may not be obvious for a student inexperienced in applying mathematics to real situations. Mathematically equivalent contextualised and decontextualized forms of such tasks give rise to quite different success rates in solution. The contextualised task involves sorting out the order in which contextual cues need to be applied, determining their relevancy, coordinating and integrating. The integration of information is the critical aspect of the difference in cognitive demand of the two versions of the task which may account for differences in success rates in mathematically equivalent contextualised and decontextualized forms of tasks when mathematical and language competency are adequate (see Stillman, 2001, for examples). A student who was able to abstract the mathematics away from the contextual detail would not have to address the high cognitive load that is imposed by the need of students with little previous experience in doing so, having to integrate data cues.

Engagement with the Task Context

The impact of engagement with the task context of these applications tasks on the students' performance on the tasks was also investigated (see Stillman, 1998b). Moderate to high engagement with a task context was not often associated with poor performance which was more likely to be associated with no to low engagement. High engagement with task context was not a necessary condition for success as the degree of engagement necessary for success appeared to be task specific. Students identified a sense of realism and having an objective to work towards as facilitators of their engaging with task context. Many students in the study were unable to engage with the context of an applications task to any significant degree and only a few of these students were successful at solving them. Engagement with task context alone was not of sufficient explanatory power to account for all the patterns in the data. Other factors clearly came into play. To develop the meta-knowledge associated with the successful modelling of situations in their environment, students need tasks that require them to engage with the context in order to solve them successfully. They need experience in abstracting within, not away from the situation described in the task statement so engagement with a task context continues throughout the solution process. This requires the setting of tasks that allow this and the modelling of this process by teachers.

Task Accessibility

A cognitive/metacognitive framework developed in Stillman (2002a) proved useful in identifying and examining the conditions that facilitated or impeded task access for the students in the study through an analysis of students' responses to the tasks (see Stillman, 2004a). When the conditions facilitating task access were examined some conditions reduced the difficulty level of a task for particular students whilst others were related to reduced complexity of a task. Personal conditions that reduced difficulty were personal attributes of the task solver such as being able to visualise, possessing metacognitive knowledge that encouraged task access, or prior knowledge of the task context. Other conditions reducing task difficulty were attributes of the task that were susceptible to individual variation when a particular student interacted with the task (e.g., how recent a particular piece of mathematics required in the task had been studied or how well rehearsed the required mathematics was). On the other hand, facilitating conditions associated with tasks being of lower complexity occurred when particular task attributes were present (e.g., the presence of salient cues, Kaplan & Simon, 1990, in the form of trigger words or visual features). Similarly, impeding conditions that increased the difficulty of a task for particular individuals often resulted from the interaction of a student's personal attributes with the attributes of the task (e.g., reluctance to make assumptions, cuing words not being salient, recall difficulties, interference from prior knowledge, metacognitive task knowledge which discouraged access) but sometimes were purely personal (e.g., possessing metacognitive personal knowledge that discouraged access). Impeding conditions that were associated with increased complexity of the task were task attributes such as the mathematics or the goal of the task not being obvious, the need to integrate given and derived contextual information in order to construct a mental representation of the situation described in the task, or the need to make assumptions in order to formulate a mathematical model.

Task difficulty varied from student to student whilst task complexity was fixed as it was determined by the attributes of the task. This is in agreement with Williams' (2002) distinction between these two terms. It is foreshadowed, however, that these attributes may be related to particular solution methods rather than the task per se (e.g., one solution approach may require a deeper level of integration of information than another). Personal attributes of the student also appear to act as intervening conditions between task complexity and task difficulty. These would explain the different consequences that occur (e.g., different difficulty levels or whether or not impeding conditions were overcome) when different students attempt tasks of the same complexity.

For students to benefit from facilitating conditions in applications tasks, they need: a well-developed repertoire of cognitive and metacognitive strategies as well as a rich store of mathematical concepts, facts, procedures, and experiences; vicarious general encyclopaedic knowledge of the world and word meanings; and truly experiential knowledge from personal experiences outside school or in more practical school subjects. In particular, a variety of retrieval, recognition, mental imaging, perceptual, and integration strategies together with metacognitive strategies for monitoring, regulating, and coordinating the use of these cognitive strategies is necessary (see Stillman, 2004a, for further details). The tasks used in this study were highly reading-oriented and thus, as suggested by Nathan et al. (1992), relied on students (a) accessing a good store of relevant prior knowledge for generating the inferences and elaborations necessary for understanding the situation fully and (b) having good comprehension skills to enable the student to specify a valid problem model for the task through the application of mathematical procedures. In some instances use of both cognitive and metacognitive strategies was enhanced by students (e.g., in a task about ice hockey) imagining or pretending to be in the situation described confirming Van den Heuvel-Panhuizen's (1999) assertion cited previously. This facilitation of access was also enhanced by the development of metacognitive knowledge which encouraged students to engage with the task (e.g., knowledge of task structure that has facilitated access to applications tasks for this task solver previously). However, once a modest degree of skill has been achieved in accessing complex tasks such as applications, the work of Schneider and Detweiler (1988) and Carlson et al. (1990) point to coordination and integration of multiple representations, further cues, and mathematical processes and procedures as becoming critical as the solution attempt progresses.

Failure of a student to possess a well-developed strategic repertoire or rich store of mathematical, encyclopaedic, semantic, or experiential knowledge can lead to the situation where conditions that facilitate one student's access to the task become impeding conditions for another. For example, a task about road construction, with apparent obvious mathematical and contextual cues for one student, may prove inaccessible for another who does not have the appropriate knowledge base or fails to activate an appropriate knowledge base because of a poorly developed strategic repertoire. If the student does have the appropriate knowledge and strategic bases but fails to activate either initially, an initial period of difficulty may be experienced but then overcome. At other times, particular attributes of an applications task such as unusual wording, a lengthy problem statement, or the required mathematical model or method not being obvious, can impede access. These difficulties can be overcome, however, by students possessing and activating a well-developed strategic store together with an appropriate knowledge base. A wide variety of cognitive strategies that include retrieval, comprehension, information organising, attention focussing, information representing, and visualising are necessary for overcoming the potential array of impeding conditions that a student may encounter in attempting to access an applications task. The effective use of these strategies is enhanced by an equally rich and varied store of metacognitive strategies. See Stillman (2004a) for further details.

MODELLING IN SECONDARY SCHOOLING

Figure 2, developed from a corresponding diagram in Galbraith and Stillman (2006), has multiple purposes as described below. It will provide a useful launching point for issues discussed later in this paper. Modelling in schooling has two concurrent purposes – (a) to solve a particular problem at hand, and (b) over time to develop modelling skills, that empower students to describe and solve problems in their personal and social worlds. These purposes have characterised my work in the field. The task begins with the messy real world situation [A]. The respective entries B-G in Figure 1 represent stages when particular products (e.g., the mathematical model [C] or decisions about acceptance or rejection of the model [F]) have been produced in the modelling process. The thicker arrows signify transitions between the stages. The overall solution process is described by following these arrows clockwise around the diagram from the top left. It culminates either in the report of a successful modelling outcome, or a further cycle of modelling if evaluation indicates that the solution is unsatisfactory in some way. The kinds of mental activity that individuals engage in as modellers attempt to make the transition from one modelling stage to the next are given by the broad descriptors of cognitive activity 1 to 7 in Figure 2.

The light double-headed arrows are included to emphasise that thinking within the modelling process is far from linear, or unidirectional as has been confirmed empirically by Oke and Bajpai (1986) and Borromeo Ferri (2006). The light arrows indicate the presence of reflective metacognitive activity as widely recognised and articulated by many researchers (e.g., Maaß, 2007; Stillman, 2011; Tanner & Jones, 1993). Such reflective activity can look both forwards and backwards with respect to stages in the modelling process. The double-headed arrows in Figure 2 are indicative rather than exhaustive. In theory they connect every pair of stages, for example C to E, but diagrammatic clarity precludes inclusion of them all as was done previously in Stillman (1998a, p. 145) where a regulatory mechanism was included in a process rather than a product diagram as here.

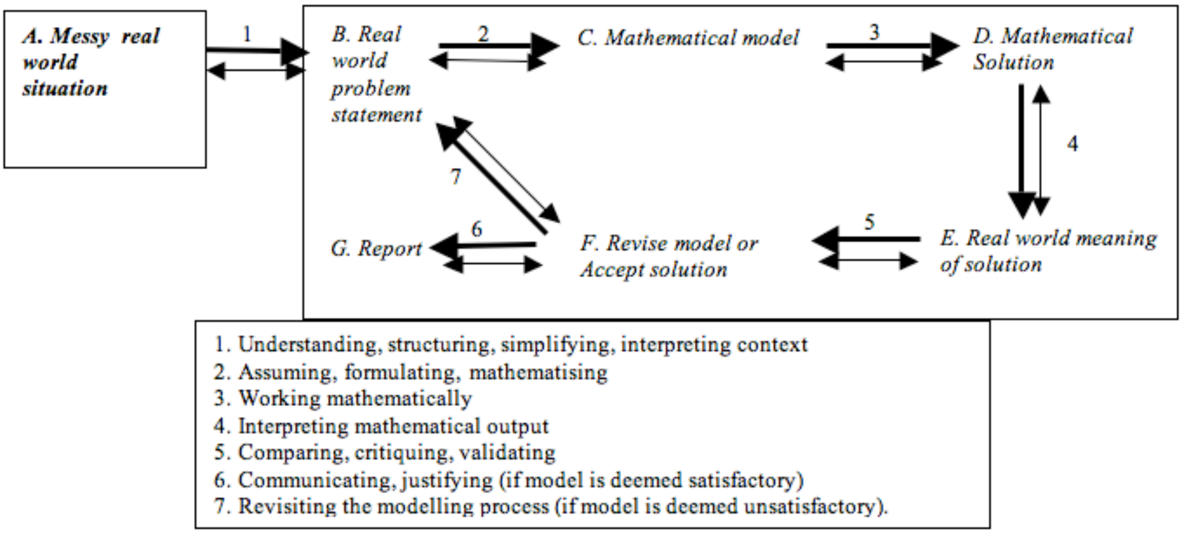

Figure 2. Modelling cycle from Stillman, Galbraith, Brown and Edwards (2007)

Foci for teaching and research

Such a modelling diagram, as Figure 2, serves a variety of purposes. At its most fundamental level it captures and depicts the modelling cycle familiar to those who work within the paradigm of modelling as real world problem solving. It thus shares commonalities with many of these. Secondly, it can be employed as a scaffolding device to articulate and support the practice of modelling for those beginning modelling (e.g. as described in Galbraith & Clatworthy, 1990). Used in this way it both defines and helps to bridge zones of proximal development that exist for beginning modellers. Thirdly, as Blum (2008) points out, such a representation is indispensable as "an instrument for teachers for diagnosis and well-aimed intervention" (p. 8). Fourthly, it serves to define and identify key foci for research with respect to individuals learning mathematical modelling. For example, with colleagues I have developed a research tool from the diagram and researched factors seen as blockages to progress, when students have difficulty in making transitions between stages of a modelling task as will be overviewed below.

Issues in formulating models

I now briefly indicate how my own work and interests relate to aspects of the structure displayed in Figure 1. A key part of the modelling process relates to the transition between stages B and C. This transition has been known as a "bottleneck" in modelling for quite some time (see, e.g., Hickman, 1986). Arguably making this transition is the most demanding part of the majority of modelling projects and its presence as a prime focus separates modelling as real world problem solving, from other educational approaches that also use the term 'modelling'. Helping students to achieve this transition is a continuing and major teaching and research priority. Differences of emphasis, and indeed of structure, have been introduced (e.g., Blum & Leiß, 2005, as cited in Leiß & Wiegard, 2005), through the addition of a 'real model' between B and C. Clearly, there is a connection to the worded applications tasks abstraction models that were based on text comprehension models. As "in applied mathematics one does not distinguish a real model from a mathematical model, but regards the transition from real life situation into a mathematical problem as a core of modelling" (Kaiser, 2005, p. 100), my colleagues and I have maintained the transition as shown. This has proved a useful basis for researching this transition nevertheless.

Development of Framework for Identifying Blockages in Transitions

A new research tool was developed with colleagues from Figure 2 and refined in attempting to address our goals of (a) identifying and classifying critical aspects of modelling activity within transitions between stages in the modelling cycle, and (b) identifying pedagogical insights for implementation through task design and organisation of learning. Typically, a genuine modelling task begins with a messy real world problem which is then transformed and solved mathematically as the modeller carries out various processes in the modelling cycle. Initially, the tool consisted of an empty frame of the transitions between stages as shown in Figure 2. Using preliminary data analysis an emergent framework was developed (Galbraith & Stillman, 2006; Galbraith, Stillman, Brown, & Edwards, 2007) for identifying potential places where student blockages could occur in these transitions. Details for the first two transitions are shown in Figure 3. These have been illustrated with data from tasks used at year 9 level. Generic elements are shown in ordinary type, instantiations for a particular task in small capitals. Continuing analysis of task implementations led to further refinements such as a further construct, level of intensity of a blockage, to explain the robustness of particular blockages to change and to identify student or teacher interventions to overcome these (Stillman, Brown, & Galbraith, 2010; Stillman, 2011).

1. MESSY REAL WORLD SITUATION → REAL WORLD PROBLEM STATEMENT
1.1 Clarifying context of problem [ACTING OUT, SIMULATING, DISCUSSING PROBLEM SITUATION]
1.2 Making simplifying assumptions [NO GOALKEEPER, RUNNERS WILL MOVE IN STRAIGHT LINES]
1.3 Identifying strategic entit(ies) [RECOGNISING LENGTH OF LINE SEGMENT AS THE KEY ENTITY]
1.4 Specifying correct elements of strategic entit(ies) [IDENTIFYING SUM OF TWO CORRECT LINE SEGMENTS]
2. REAL WORLD PROBLEM STATEMENT → MATHEMATICAL MODEL
2.1 Identifying dependent and independent variables [TOTAL RUN LENGTH AND DISTANCE FROM CORNER]
2.2 Representing formulae in terms of 'knowns' [LENGTH EXPRESSED IN TERMS OF FIELD EDGE DISTANCES]
2.3 Realising independent variable must be uniquely defined [X-CANNOT BE DISTANCE FROM BOTH A AND B]
2.4 Making relevant assumptions [LINEAR MODEL APPROPRIATE EVEN WHEN DATA POINTS APPEAR TO FOLLOW CURVE]
2.5 Choosing technology to enable calculation [RECOGNISING HAND METHODS ALONE ARE IMPRACTICAL]
2.6 Choosing technology to automate formulae for multiple cases [LISTS HANDLE MULTIPLE X-VALUES]
2.7 Choosing technology to produce graphical representation of model [SPREADSHEET OR GRAPHING CALCULATOR]
2.8 Choosing to use technology to verify algebraic equation [RECOGNISING GRAPHING CALCULATOR FACILITY TO GRAPH L VERSUS X]
2.9 Perceiving a graph can be used on function graphers but not data plotters to verify an algebraic equation [GRAPHING CALCULATOR CAN PRODUCE GRAPH OF FUNCTION TO FIT POINTS – SPREADSHEET CANNOT]

Figure 3. Initial transitions in Galbraith, Stillman, Brown, and Edwards (2007) Framework.

The Framework systematically documents activities and content with which modellers need competence in order to successfully apply mathematics at their level. Given that the elements in the framework were identified by observing students working, (and in particular wrestling with blockages to progress), there are two immediate potential applications. First are the insights obtained into student learning, and how these can inform our understanding of the ways that students act when approaching modelling problems. Second, closely allied to this are associated pedagogical insights. By identifying difficulties with generic properties, the possibility arises to anticipate where, in given problems, blockages of different types might be expected. This understanding can then contribute to the planning of teaching and task design, in particular the identification of prerequisite knowledge and skills, preparation for intervention at key points if required, and scaffolding of significant learning episodes. For example, blockages of low intensity appear to be able to be resolved by students themselves engaging in genuine reflection so tasks can be scaffolded in such a way to allow this to occur (e.g., by asking students to pause and write about their interim results in terms of what they would expect from real world experiences) (see Galbraith, Brown, & Stillman, 2010). Blockages of high intensity, on the other hand, might need direct intervention by the teacher to facilitate revision of mental models by students when they have resisted doing so for some time by failing to accommodate new contradictory information (see Stillman, 2011).

Evolving perceptions: 'assumptions' and 'technology'

Reflection on the modelling diagram calls to mind two other areas that have evolved over time. The first is the role of assumptions. Originally assumptions were perceived as confined to the process of setting up a mathematical model in the first place. While they play a major role in formulation, it is now realised that they permeate the whole of the modelling process (see detailed examples in Galbraith, 1996; Galbraith & Stillman, 2001; Galbraith, Stillman, & Brown, 2010). Galbraith and Stillman (2001), for example, identified three different classes of assumption. Firstly, *assumptions associated with model formulation* are those which have traditionally been accorded the descriptor 'assumption'. Secondly, *assumptions associated with mathematical processes* are also identified as important in the solution process. Examples here would include that domain requirements for mathematical functions invoked in the solution process will be satisfied by the real world values existing within the problem context, or that the sign of terms in inequalities can be confidently assigned from real data such that they can be manipulated unambiguously. Thirdly, *assumptions associated with strategic choices in the solution process,* influence the progress of a solution. These assumptions are central in providing global choices to the modeller, and determine how the direction of a solution path may change. Typically they are required when an interim result has been obtained which creates a temporary impasse that was not foreseen at the outset. This can occur either from a mathematical impasse, or within a process of evaluating a model against the real context. Examples include a physically impossible outcome, which might suggest either an incorrect use of a particular model (e.g., the particle model in mechanics), a need to reformulate the modelling question, or indeed to revisit the original assumptions used to set up the model.

Increased sensitivity to the different and pervasive types of assumptions at work in modelling problems have come to influence the ways in which such problems are addressed, notably in respect to different approaches that are sometimes possible.

The second area to benefit from reflection on Figure 2 is the role of technology in modelling. Within an approach to modelling as mathematical content, appropriate use of technology is essential. With colleagues I have been particularly interested in exploring the intersections in Figure 4. Galbraith, Stillman, Brown and Edwards (2007) argue that for beginning modellers the mathematics required for solution needs to be within the range of known and practised knowledge and techniques. However, it might not be known exactly which mathematics is appropriate for a situation under investigation but such decisions are part of the modelling process. Additionally, the presence of a technology-rich teaching and learning environment impacts on the modelling process, changing the mathematics that is accessible to students.

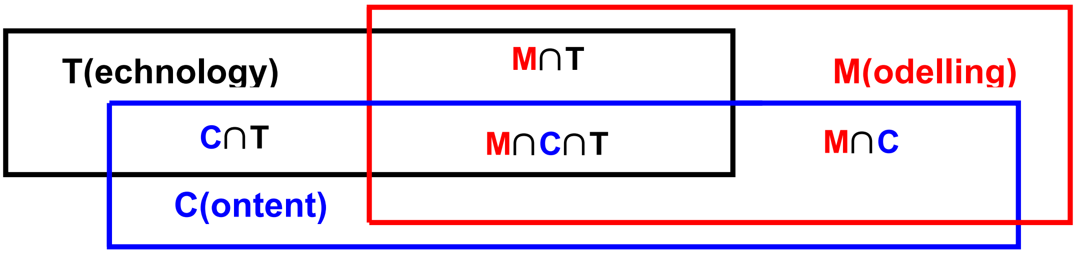

Figure 4. Interactions between modelling, mathematics content, and technology (Galbraith, Stillman, Brown & Edwards, 2007)

The use of electronic technologies in real-world settings for example as analysis tools such as graphing and CAS calculators and real-world interfaces (e.g., image digitisers) can reduce the cognitive demand of applications and modelling tasks. This can be achieved through supplementation and reorganisation of human thought (Tikhomirov, 1981) by carrying out routine arithmetic calculations, algebraic manipulations, graph sketching, acting as an external store of interim results, or overlaying visual images (e.g., digital photographs of real phenomena) within an interactive coordinate system to facilitate analysis. However, the use of these technologies has the potential to bring in a degree of complexity as they transform classroom activity and allow new forms of activity to occur. Regulation of this complexity allows teachers a further opportunity to mediate the cognitive demand of lessons involving real world contexts through the careful crafting of tasks for teaching, learning and assessment (Stillman, Edwards, & Brown, 2004). Figure 5 provides a list of questions that teachers could consider in relation to complexity of technology use to inform decisions related to the regulation of cognitive demand over time. These are particularly important in an introductory modelling environment where there is the intention of a progression from "supplantive modelling where the modelling structure is supplied by the task setter to generative modelling where the students generate the modelling themselves" (p. 490).

COMPLEXITY OF TECHNOLOGY USE	
General Attributes	***Dimensional Ranges***
LEVEL OF COMPLEXITY	simple … complex
Specific Attributes	
How many electronic technologies are involved?	1…many
How are these technologies used?	analysis tool, real-world interface
How much technological knowledge is required?	little…a lot
How easy is the technology to use?	easy…very difficult
How obscure is the choice of techniques?	fairly obvious…fairly obscure
How complex is each technique?	quite simple…quite complex
How complex is the combination of techniques?	all quite simple…most quite complex
How visible are the links between techniques?	fairly apparent…quite obscure
How many steps are involved?	1…many
How many features of the technology are involved?	1…many
What amount of guidance is given?	none…high
How much decision making is necessary?	none…a lot
How many representations can the technology provide?	1…many

Figure 5. Complexity of technology use in real world settings (Stillman, Edwards, & Brown, 2004, p. 494).

METACOGNITIVE ACTIVITY AND APPLICATIONS AND MODELLING

One of the most significant aspects of the additional detail of Figure 2 is the focus it supports on metacognitive activity, which permeates every aspect of the modelling process as proposed by Stillman (1998). Earlier work (Andrews & McLone, 1976) foreshadowed a regulatory mechanism validating processes and products throughout the modelling cycle. On-going research has confirmed the presence of this metacognitive activity.

Stillman and Galbraith (1998) inferred from a previous study of applications tasks (Stillman, 1993), that applications teaching should focus on reducing the time students spend on orientation activities. This was suggested as being able to be achieved by "developing cognitive skills that facilitate more effective problem representation and analysis, and by promoting the development of metacognitive strategy knowledge" (p. 185) to facilitate appropriate decision making during orientation. Since this time there has been a limited number of studies in the area.

Goos (2002) identified three generic types of metacognitive failure during problem solving that she called *red flag* situations. These were (a) *lack of progress*, (b) *error detection*, and (c) *anomalous results*. Red flag situations can emerge at any stage of a problem solving process, where their occurrence should elicit metacognitive monitoring, and regulatory actions. In mathematical modelling such red flags may be triggered by incorrect mathematics, or outcomes that, while mathematically accurate, are inconsistent with real world aspects of the problem. Goos typified three prevalent forms of metacognitive failure. *Metacognitive blindness* occurs when a red flag situation is not recognised, and so no appropriate action is taken. *Metacognitive vandalism* occurs when a perceived red flag results in drastic and often destructive actions being taken that not only fail to address the issue, but also alter or invalidate the problem itself. *Metacognitive mirage* occurs when solvers take unnecessary actions derailing a solution, because they perceive a non-existent difficulty. Two more classifications have been added from our work. *Metacognitive misdirection* describes a potentially relevant but inappropriate response to a perceived red flag that represents inadequacy, not vandalism. *Metacognitive impasse* occurs when progress stalls, and no amount of reflective thinking or strategic effort by the problem solver(s) alone is able to release the blockage. All five forms of metacognitive failure have been identified in my modelling work with students (see Stillman, 2011).

Meta - metacognition

Given that metacognitive activity is located heavily at the transitions in Figure 2, how pedagogy addresses the fostering of associated metacognitive competencies is crucial to producing consistently able modellers. This leads to the concept of *meta-metacognition* as a significant factor in teaching (Stillman, 2011). For a teacher in a classroom where mathematical modelling is being undertaken, a key task is monitoring progress of individuals or groups, and intervening strategically where necessary. One needs to appraise the enactment of metacognitive activities on the part of students – whether, for example, a student is undertaking sufficiently perceptive and rigorous reflection in considering the approach to, or quality of, a solution. In considering whether metacognitive activity on the part of students is appropriate, or if appropriate is being properly conducted, teachers are reflecting on metacognitive activity itself, both situation specific and with respect to its role in the overall modelling process. That is, they may be thought of as undertaking mental activity that is *meta – metacognitive* in nature.

At the macro level, how teachers generally undertake such *meta-metacognition* in relation to student activities, and subsequently act, is crucial to the way mathematical modelling is nurtured or stifled in their classrooms generally. At the micro level, students' capacity to develop skills in making transitions between modelling stages and to release blockages in the process, depends critically upon how they are facilitated and supported in learning and applying the modelling process, and the metacognitive strategies central to it. This in turn depends upon the perceptiveness and skill with which teachers assess, mediate, and provide for student metacognitive activity. This extends beyond intervening to help with the solution of a specific problem, to ensuring that the intervention also contributes to the long term goal of developing modelling competency. "What should this student be asking her/himself at this point in the modelling process?" is a meta-metacognitive reflective prompt that more is required than a suggestion about how to progress past a problem specific obstacle.

CURRICULUM CHANGE

Australian states and territories have their own educational jurisdictions and curricula. Several of these have attempted to include applications and modelling topics more centrally within their curricula in the past or currently do so. An on-going study by Stillman (2004b, 2007) has examined the implementation of applications and mathematical modelling curricula in Victoria and Queensland where the path of curriculum change has been different with markedly different outcomes.

Findings resulting from the Victorian case study were reported in Stillman (2007). Subsequent to the examination of extant curriculum documents and materials from the 1980s onwards, purposeful samples of 6 key curriculum figures (e.g., expert advisory committees members or curriculum managers), 6 teachers in key implementation roles (e.g., as seconded project officers and state or regional chairs of verification panels), and 6 classroom teachers were selected. These 18 participants were interviewed about their experiences during the change and subsequently and about their beliefs regarding conditions that promoted or hindered introduction and ongoing use of mathematical applications and modelling in upper secondary classrooms. Practising teachers were also asked about the impact of the changing role of applications and modelling in the various mathematics study designs on their teaching and assessment practices. Classroom artefacts typifying their current practice in applications and modelling at the upper secondary level were collected. Two major conditions identified that threatened the ability of the Victorian system to sustain change were (a) the extent of the change and (b) the rapid pace of the change (Stillman, 2007). These contrast with the Queensland implementation which is on-going.

Findings from the Queensland case study were reported in Stillman and Galbraith (2009, 2011) and Stillman and Brown (2011a). Curriculum documents from the late 1980's to the latest syllabus implementation in 2009 were examined. In addition purposeful samples of 5 key curriculum figures (e.g., non-teacher members of expert advisory committees, curriculum officers, statutory board or authority officers overseeing syllabus implementation), 6 secondary mathematics teachers in key implementation roles (e.g., state or district review panel chairs or state panel members), and 12 secondary mathematics classroom teachers were selected. Interview questions covering the introduction, state-wide implementation and modification were asked of these 23 interviewees. In addition, practising teachers provided artefacts that typified their use of real world applications and modelling in teaching and assessment, and their use of technology in these contexts.

Tasks collected varied from those that were open, with the making of assumptions, choice of mathematics, and interpretation in context essential aspects to ones that merely asked students to carry out specified mathematical calculations with no assessment of whether the proposed models made sense in the real problem context. The latter are examples of applications in which activities central to modelling are absent. From participant responses, it was noted that opportunities provided by modelling were grasped by some who welcomed the legitimacy of a syllabus requirement in supporting a desire for change. Others wanted to remain within the comfort provided by interpretation of 'applications' as little different from previous activity as was found in the early implementation of the 1992 syllabus (Stillman, 1998b). Several teachers welcomed a symbiotic relationship between modelling and technology use, with these two curricular elements seen as mutually supportive.

With respect to the extent of technology use in teaching and alternative assessment involving real world contexts, some teachers clearly had welcomed the opportunity to expand their teaching and assessing repertoires with respect to applications and modelling that technology brought. Others saw technology providing little more than a computational device removing tedium and potential inaccuracies of repetitive calculations or graphing associated with the solution of a mathematical model. The latter was usually associated with a view of modelling as no different from using mathematical applications and opportunities for technology use being more prominent in assessment than teaching. In classrooms where technology was said to play a significant role in teaching applications and modelling, the classroom culture was described as very different as the “internet generation” was more engaged by immediate feedback and dynamical displays available with technology. The constructive function of technology in concept formation was acknowledged by these teachers. Exploration, sustaining interest and engagement, and playing with the mathematical ideas and the situation being explored were mentioned as elements of classroom culture where technology was readily available and its use expected. Again, the teacher’s view of modelling limited the perceived potential and promoted use to solving or expanded it to pervade the modelling cycle. Some saw technology use as essential to successfully fulfilling syllabus intentions with respect to modelling. Even though several saw this as enriching the whole teaching/learning experience as intended by the syllabus, the unfulfilled potential of a borderless learning community networking amongst modelling groups across geographical boundaries further enriching that experience was mentioned.

The elaboration of modelling assessment criteria in a later syllabus revision was generally viewed as helpful in providing enhanced guidelines for task design and assessing performance. On the other hand, the reduction of rich criteria to box ticking procedures, flagged a desire by some to continue with minimalist approaches, attempting to assimilate challenging new requirements into traditional conservative practices. In this respect the impact of review panels emerged as ambivalent facilitators of change. If panels are viewed as agents for change and guardians of comparability, the appearance of implementations along the continuum from minimalist to very rich, suggests these functions require further work. The assessment by interviewees of current practice reflected this variety. In this sense the acceptance of more conservative implementations over time than some would wish, was seen as assuring that at least some progress occurred, perhaps more than in an environment requiring full immediate compliance where ‘failure’ is a real possibility. On the other hand some respondents enthused that schools were thereby enabled to do “fantastic things”.

PRE-SERVICE TEACHERS’ AFFINITY WITH USING MODELLING TASKS

Results with respect to teaching modelling from Australian data collected in an international study of pre-service mathematics teachers, *Competencies of Future Mathematics Teachers*, are reported in Stillman and Brown (2011b). Data were collected from 73 volunteer pre-service secondary mathematics teachers from 6 cohorts at 5 university sites in eastern Australia. Questionnaire responses targeting affinity of pre-service teachers with using modelling tasks in Years 8-10 were analysed from the perspective of possible differences associated with length of preparation program undertaken.

The general belief is that longer pre-service preparation programs such as a 4-year double degree are of more benefit and that those exiting from short programs such as 1-year postgraduate diplomas are underprepared for teaching. In some areas such as discerning whether the modelling task was appropriate for the target schooling level and whether they were prepared to use such a task in classrooms at this level, there was little difference in the responses of these groups with a high level of agreement reflecting the emphasis on these types of tasks in Australian curricula for this level of schooling. Pre-service teachers who were educated in Australia would perhaps have experienced such tasks when they were in schooling as well as in their practicum experience.

With respect to diagnostic competencies and the ability to analyse student responses for appropriateness of modelling approach, surprisingly a much higher proportion of 4-year program students could not do this. As they had more experience in schools over an extended period, they could be expected to acquire more pedagogical content knowledge in this regard than their 1-year counterparts. However, all students in 4-year programs prepare for two teaching areas one of which is mathematics and this can be a minor focus in mathematics. They thus may place less emphasis on their preparation for mathematics teaching than another area of more personal interest. The 1-year programs in contrast included some students preparing only as mathematics teachers. Usually, these students had a strong mathematics background and could be expected to place a strong emphasis on being as prepared as possible to teach mathematics.

Differences with respect to beliefs about the nature of mathematics were also surprising especially as a higher proportion of pre-service teachers in 1-year programs acknowledged the dual nature of mathematics. Perhaps this is related to the higher proportion of pre-service teachers in the 1-year groups basing this view on mathematically oriented arguments being commensurate with a deeper orientation towards mathematics teaching. The number of students in the 4-year programs who gave no reasons for their position is disappointing as pre-service programs emphasise the importance of reflection on practice.

CONCLUSION

Throughout this paper I have focused on issues that are both productive and challenging that I have researched with respect to mathematical applications and modelling in secondary schooling. By conducting research in classrooms about issues perceived by teachers as of concern I have endeavoured in my research program to closing the gap allude to by Blum (1993). In addition, I have begun researching the new generation of teachers in whose hands is future mainstream classroom instruction. By understanding how these future teachers are formed mathematically and pedagogically might give use more purchase on scaling up changed practices throughout educational systems.

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