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MATHEMATICAL LITERACY FOR LIVING IN THE HIGHLY INFORMATION-AND-TECHNOLOGY-ORIENTED IN THE 21ST CENTURY: MATHEMATICS EDUCATION FROM THE PERSPECTIVE OF HUMAN LIFE IN SOCIETY

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This paper discusses mathematical literacy for living in our highly information-and-technology-oriented society in the 21st century. First, it inquires into the significance of thinking about mathematical literacy in terms of how it benefits modern individuals, as well as modern society. A summary of the past trends of mathematical literacy in Japan is given. This is followed by a consideration of a framework for thinking about mathematical literacy in the future. Here, the focus is on mathematical methods and the need to re-visit the meaning of studying mathematics. This is followed by a discussion of the design of school mathematics curricula that aim to nurture mathematical literacy. The discussion includes an examination of the general structure of school mathematics as it pertains to mathematical literacy, and the framework of school mathematics that addresses diversity. Concrete examples of the designs of school mathematics curricula based on research on mathematics education in Japan to date are given. Lastly, the maintenance and development of mathematical literacy outside school is touched upon.

1. INTRODUCTION

It is thought that Japanese mathematics education has produced definite results. In fact, Japan has been ranked high in the international mathematics studies in which Japan has been a participant since 1960. On the other hand, IEA and PISA found that Japanese students in secondary school tend to see mathematics as fixed knowledge (in the First International Mathematics Study in 1964), and their motivation to study mathematics is low (in the Second International Mathematics Study in 1980). More recently it was found that Japanese students' recognition of the significance of the study of mathematics is low (PISA2003). Furthermore, it has been found that Japanese adults are low in terms of awareness of the cultural value of mathematics. These factors in Japan now contribute to a phenomenon referred to as "the drift away from science and technology". Of course, behind these issues, there is a long tradition of attaching the utmost importance to testing centered around entrance examinations (Amano, 2007).

These tendencies bring to mind a discussion on mathematization and dematematization (Jablonka et al., 2007). Japanese students get high marks for performance in school, at least in mathematics examinations. However, seeing the previous tendencies, I cannot help thinking that dematematization in Japan has its roots in school education.

As we entered into the 21st century, the crisis of the earth's sustainability became the subject of public outcry. We are called on as a society to grapple with many such issues in a democratic way. Especially in modern society, where the role of science and technology is of vital importance, scientific and technological literacy is expected of all adults (Project "Science for All", 2008a).

In this paper, the concept of mathematical literacy for lifelong enrichment of all adults as an objective of mathematics education will be investigated and clarified. By reviewing many papers authored by researchers in Japan and overseas (Abe, 2010, etc.), we found that this concept of mathematical literacy and its importance for all adults is regarded as a common idea (AAAS, 1989; Project "Science for All", 2008a).

2. THE SIGNIFICANCE OF MATHEMATICAL LITERACY

Mathematical literacy is aimed at all adults who are already functioning in a society. Mathematical literacy seems to benefit people who are trying to better themselves. Also, mathematical literacy may benefit people who are trying to better society in a certain age. In the former, mathematical literacy depends more on humans, even though society in each era has an effect upon them. In this sense, mathematical literacy in a way transcends generations. In the latter, since mathematical literacy depends more on society in the present and near future, mathematical literacy may be different according to various images drawn of a society. In this section, the significance to think mathematical literacy will be considered from these two viewpoints keeping all adults in mind.

2.1 Addressing the individual

If we examine human history, it is clear that people have always had the will and potential to improve themselves. In this section, we will investigate that drive by looking into two kinds of people who strive to improve themselves; those who broaden their horizons to see themselves, and those who broaden their horizons to see the world.

2.1.1 People who broaden their horizons to see themselves

Learning mathematics makes us aware of human potential (Hirabayashi, 1987). In other words, learning mathematics broadens our view of ourselves. For example, when we learn mathematics, we engage ourselves in finding mathematical rules, solving and drawing up mathematical problems, and thinking of problems in the real world mathematically. By using mathematical expressions to present problems and by brainstorming together to come up with solutions, we become aware of our individual potential. We also come to realize that we are subjects working in our own ways to construct mathematics (Earnest, 1991).

Furthermore, there may be situations where people think mathematically about various problems they encounter in daily life. People are capable of broadening their ideas of themselves through recognizing themselves as individuals who can think mathematically.

2.1.2 People who broaden their horizons to see the world

Learning mathematics also broadens people's perspectives on the world (Steen, 1990). Looking back to elementary school, as children's lives and worlds broaden, mathematical content and methods also expand. At the end of compulsory education, students proceed from the world of concrete numbers to the world of abstract mathematical expressions. They enter into the world of two dimensions as they are introduced to problems that are solved with quadratic equations, Pythagorean theorem and quadratic functions. Furthermore, students proceed from the realm of informal demonstration to the realm of formal demonstration mainly through learning geometry.

People encounter more complex problems when they are out there in the real world. However, basic contents and methods to deal with the problems are included in what they already learned in school. Therefore, students' view of the world is broadened by learning mathematics in school. Also, from time to time, they learn more mathematics out in the world. We can expect this kind of cycle to develop and continue, with people broadening their views of themselves and their views of the world.

2.2 Addressing present society

It is said that our present society is highly information and technology-oriented and will continue to be so in the near future. Development of science and technology, especially development of information technology such as the internet, has gone beyond our imagination. Also, the decline of the birth-rate and increase of the aging population continue in Japan. In this section, such issues as sustainable society, democratic society, and a society of lifelong learning, both important for Japan and profoundly related to learning mathematics, will be considered.

2.2.1 Sustainable society

Realization of a sustainable society is a global challenge in the 21st century. To protect the global environment and construct a sustainable society, each and every adult is called upon to use their judgement when faced with various challenging situations. In order to think and judge, scientific and technological literacy is needed. Think of what happened on March 11, 2011 in Japan. North-eastern Japan was struck by a catastrophic earthquake and tsunami. These disasters caused nuclear power plants in Fukushima to break down. It shocked not only Japanese people, but the whole world.

The disaster at the nuclear power plants made us realize how important it is to make information available to the public, especially in a crisis. It made us reflect upon ourselves - how stunned we were, and how important it is to keep thinking for ourselves (Fromm, 1976).

2.2.2 Democratic society

Further development of a democratic society has been a continuing challenge since the beginning of the 20th century for Japan, as well as the world. Though Japan has nearly 70 years of experience as a democratic state, it seems to lack a sense of democracy. Moreover, in times of economic uncertainty, there seems to be a welcoming atmosphere for dictatorial leaders.

To further construct this democratic society, it is imperative that people consider diverse perspectives and engage in logical dialogue about current problems. Critical thinking must be emphasized in these arguments (Freire, 1970).

2.2.3 Lifelong learning in society

Responding to a rapid change in society, an demand for lifelong education began world-wide in the 1960's. Changes that took place after that period, particularly the development of information technology, are remarkable. It is causing massive information overload, as well as big changes in how we process information.

The need for lifelong learning and the lifelong education that supports lifelong learning is more and more increasing. Even today, learning *how to learn*, which was advocated in lifelong education remains important (Lengrand, 1970).

3. TRENDS IN MATHEMATICAL LITERACY IN JAPAN TO DATE

It was in the latter half of the 20th century that discussion of mathematical literacy began in Japan (Abe, 2010). Here I will give you a summary of trends in mathematical literacy in Japan with a focus on activities involved in and resulting from the Project "Science for All".

3.1 The latter half of the 20th century

In Japan, the term "mathematical culture" was used in a national course of study of mathematics education in upper secondary school published in 1955. In the course of study, ways of mathematical thinking were shown as specific examples of mathematical culture. Since then, there has been almost no discussion of mathematical culture or mathematical literacy in mathematics education. In the 1980's, discussions about computer literacy, information literacy, and media literacy arose in response to changes in upper secondary school education, as well as in society. These discussions were continued until the mid 1990's. During the latter half of 1980's, there were some discussions on mathematical literacy, but not many. From the mid 1990's, there were various discussions on mathematical literacy (including NCTM's views), matheracy, numeracy, mathemacy, quantitative literacy, and statistical literacy. Since the beginning of 2000 until now, mathematical literacy as defined by PISA has been the core of various discussions.

Among these, there are three types of literacy that seem to be unique to Japan. They are: 1) mathematical literacy for fostering intelligence in the majority of upper secondary school students, 2) matheracy, and 3) mathematical literacy for ordinary citizens in a highly information-oriented society.

Mathematical literacy for fostering intelligence in the majority of upper secondary school students was advocated by a mathematician, Professor H. Fujita, in 1983. According to Professor Fujita, it is important to foster mathematical intelligence in upper secondary school students. He believed that mathematical intelligence was composed of mathematical thinking and mathematical literacy. He felt that mathematical thinking was necessary for those students who wanted to become specialists in the future, whereas mathematical literacy was necessary for the remaining majority of students. He assumed that mathematical literacy included mathematical laws, mathematical logic, and the ability to apply mathematical concepts for talking about science at the ordinary citizens' level.

Matheracy, which was derived from mathematical literacy, was advocated by then-president of Japan Society for Mathematical Education (JSME), Professor T. Kawaguchi, in 1983. Matheracy was an idea for building curricula based on mathematical cognitive activities. It also split mathematical content into four categories: objects, activities, inferences, and problem solving activities. Problem solving activities integrated the other three elements. Here, the aims and objectives of the curriculum were explained based on the existing premises, and activities were described as the core of educational content and methods.

A report on mathematical literacy for ordinary citizens in a highly information-oriented society was made by an ad-hoc committee of the JSME in 1987. In the report, discussions of mathematical literacy were expanded into discussions of computer literacy in response to the growing importance of computers and technology.

As for discussions on mathematical literacy in Japan, although some attempts were made to pursue something unique to our country, the discussions became influenced by NCTM's and PISA's definitions of mathematical literacy. After all, in those days, ideas on mathematical literacy in Japan were understood in terms of school education, and there was no consensus that the mathematical literacy was for "each and every" student. Also, the discussions did not include societal context. Thus, the discussions were unable to truly embody mathematical literacy.

3.2 Entering into the 21st century: Project "Science for All"

In 2005, a body of scientists and educators used governmental funds to start Project "Science for All" to outline scientific and technological literacy. The term of this project was divided into three periods and is still going on now (Nagasaki, 2010a).

From 2005 to 2006, the study "Research on Building Scientific and Technological Literacy" laid the groundwork for scientific and technological literacy. About seventy scientists and educators took part in this study and examined the current states of scientific, mathematical, and technological literacy in Japan, several foreign countries, and international institutions. In addition, they delved into the significance and necessity of literacy, as well as the organizational systems for developing literacy (Kitahara, 2006; Nagasaki, 2006). The foreign countries included the USA, Canada, England, and China. World organizations such as UNESCO and OECD were also examined. In particular, Project 2061 by the American Association for the Advancement of Science (AAAS) and its great fruits, "Science for All Americans" (AAAS, 1989) were investigated in detail, as well as research on mathematical literacy (Jablonka, 2003). Findings were published in special issue of the Journal for the JSME (Vol.89, No.9, 2009) and in other publications.

In 2006 - 2008 the Project "Science for All" defined scientific and technological literacy as "the knowledge, skills, and ways of viewing science, mathematics, and technology that all adults are expected to acquire" (Project "Science for All", 2008a). About 150 members, including scientists, engineers, educators, science museum personnel, members of the media, and various NPO's took part in the project. The seven special subcommittees to define literacy were composed; Mathematical Science, Material Science, Life Science, Informatics, Earth, Space and Environmental Science, Human and Social Science, and Technology.

The Project "Science for All" drew an image of what we wanted Japan to be like in the future. According to this image of the future Japan, 1) every individual would be recognized as an irreplaceable member of our society, 2) every member of our society would care for planet Earth, share wisdom, and take action toward the realization of sustainable society, and 3) our society would be equipped with an effective system in which young people can entertain hope for the future and inherit a culture. We decided that we must make scientific and technological literacy a reality for the enrichment of the Japanese people and for a sustainable and democratic society.

We believe that the significance and the need to define scientific and technological literacy lie in the following four objectives: a. To make judgments concerning science and technology, b. To transmit scientific knowledge over generations, c. To provide a coherent, long-term perspective for primary and secondary school education in science, mathematics, and technology, d. To convey the sense that learning, including the learning of science, mathematics, and technology, is a lifelong activity. Recent advancements in science and technology, as well as characteristics of Japanese culture, were incorporated into defining literacy. In preparing the framework for the reports, we made an effort to build its framework with human society at the core, and to include perspectives from the present and the future.

In 2008, the Project published one volume of the "*Integrated Report on Scientific and Technological Literacy*" and seven volumes of "*Summary Reports*" on scientific and technological literacy (Project "Science for All", 2008a; 2008b).

The Integrated Report on Scientific and Technological Literacy is composed of six sections as follows: Section 1. Towards Science for All and Lifetime Enrichment in the 21st Century; Section 2. The Essence of Science & Technology; Section 3. Science for All – Seven Doors; Section 4. Perspectives on Science for All; Section 5. The Application of Science for All; and Section 6. The Future: Succession and Sharing of Science for All

The Summary Report of the Mathematical Science Subcommittee is composed of five sections as follows: Section 1. The Essence of Mathematics; Section 2. The World of Mathematics (A): Objects and Important Concepts of Mathematics; Section 3. The World of Mathematics (B): Methods of Mathematics; Section 4. Several Topics; and Section 5. The Relationship Between Mathematics and Humanity. This report was mainly developed by mathematicians and centred around mathematics as language (Namikawa, 2009).

The results of the Project have been published in various magazines, such as the Journal for the Japan Society for Science Education (JSSE; Vol.32, No.4, 2008), which includes discussions on mathematical literacy.

Since 2009, the Project has shifted its emphasis to publicity and revision of scientific and technological literacy. In 2011, meetings for the revision were held in which the project members explained scientific and technological literacy to elementary school teachers who in turn critically investigated the concepts. Based on their exchange, the members and the teachers held a dialogue and worked together to prepare a report for the revision.

3.3 Update on mathematical literacy

Researchers on mathematics education involved in the Project "Science for All" have continued to engage in ongoing research focused on mathematical literacy (Iwasaki (ed.), 2010; Nagasaki (ed.), 2011). Among their research, there are several of works on mathematical literacy, such as research on the construction of a framework for mathematical literacy that includes "the nature of mathematics," "mathematical concepts," "mathematical abilities" and "mathematical application" (Nagasaki, 2009), curriculum research focused on fostering critical thinking in regard to mathematization and demathematization as advocated by Jablonka et al. (Iwasaki et al., 2010) and PISA's key-competencies for school mathematics (Shimizu, 2010). The framework also drew heavily on research on the significance of learning mathematics in mathematical literacy (Nagasaki, 2010b), and mathematical modelling as one of mathematical methods (Abe, 2010). Also, the researchers discussed the collaboration of science education, technology education, and mathematics education for scientific and technological literacy (Nagasaki, 2011).

4. FRAMEWORK FOR CONSIDERING MATHEMATICAL LITERACY IN THE FUTURE

The previous chapters outlined the context and significance of mathematical literacy in Japan. This chapter deals with the mathematical literacy that all adults will be expected to acquire in the future.

4.1 Addressing individuals in contemporary society

Mathematical literacy, as mentioned above, must address both individuals and contemporary society as a whole. Mathematical literacy must be meaningful for people and society (Senuma, 2002). That is to say, mathematical literacy must be relevant to daily life. Moreover, the connection must continue over the course of a lifetime.

People today must deal with a wide range of information. That information, which utilizes mathematics, not only expands quantitatively but also changes qualitatively over time. Furthermore, computers continue to evolve and process more and more massive amounts of information with no difficulty - information that would bury humans. As a result, there is the increasing danger that humans will be left behind by computers.

The mathematics embedded in these vast amounts of information exists in the context. Abstract mathematics that is isolated from the context is not needed, but mathematics appropriate for the context is needed. You might think that as long as you memorize mathematical contents isolated from the context, the rest should not matter. However, that is not the case. Suppose mathematical contents are to be memorized and treated as contexts increase, mathematical contents will be as infinite as the volume of information. As mathematical literacy, basic mathematical contents are needed to some extent, but it is clear that it is impossible for abstract mathematical contents alone to cope with the present situation.

4.2 Focusing on mathematical methods

As compared to mathematical contents, mathematical methods are applicable to a wider range of information. For example, the inductive method can be used for finding rules or laws in various situations and the deductive method is needed to verify rules or laws that were found. In this sense, it is natural to focus on mathematical methods as mathematical literacy. Even if information multiplies immensely as times change, mathematical methods do not change so much. Mathematical methods are always used, intentionally or unintentionally, beginning with learning various mathematical contents in elementary school.

In connection with mathematical methods, it is important to duly appreciate the role played by computers. As computers were indispensable for solving the four-colour problem, it must not be forgotten that computers demonstrate their great power in numerical calculation and processing of geometrical figures.

Mathematical methods can be divided into two categories. One is the category of general methods in science that can be obtained through not only learning mathematics, but also learning other subjects such as science, and the other category is that of intrinsic methods to mathematics.

4.2.1 General methods in science

The most typical of general methods in mathematics is logical thinking. From early times, logical thinking has been considered related to mathematics. But logical thinking is not intrinsic to mathematics. Rather, it is a method of thought for science in general (Todayama, 2011). However, needless to say, mathematics education involves a great deal of logical thinking.

A more general method that involves logical thinking is critical thinking. In critical thinking, we reflect upon various matters through interactions with others. Since mathematics is a tool for communication, communication is also a general method. When an individual’s subjective knowledge becomes objective knowledge, collaboration is needed as a general method. In mathematics, there are several general methods that are useful in society.

4.2.2 Methods intrinsic to mathematics

There are some methods that are intrinsic to mathematics. Typical of such methods is a method of mathematization (Freudenthal, 1983) that transforms phenomena to mathematics or transforms one form of mathematics to another for solving problems. In Japan, mathematization was emphasized in 1940’. With respect to abstract mathematics, there are methods of expressing concepts as symbols and methods of generalization and extension of concepts. On the other hand, with respect to the connection between mathematics and the real world, there is the method of mathematical modelling.

Further, there are several methods of thought that are more deeply dependent on mathematics. They include analytic and geometrical methods that relate mathematical expressions and geometrical figures, methods to find rules or laws in mathematics, methods to think functionally, methods to think statistically, and methods to think based on a limit and others.

4.3 Re-examination of the meaning of learning mathematics

When we consider mathematical literacy, we need to focus on mathematical methods. At the same time, in Japan, we need to place special focus on the meaning of learning mathematics. The goals of mathematics education are usually discussed from three points of view, namely: disciplinary, practical, and cultural (Nagasaki et al., 2007). Japanese students, however, as I already mentioned in my introduction, seem to have lost their motivation to learn mathematics, as they focus on nothing but entrance examinations when they study mathematics. This means that Japanese society as a whole seems to have lost the real meaning of learning mathematics.

If we understand as a society why we learn mathematics as part of mathematical literacy, it will encourage students to study not only what mathematics is, but how it applies to the real world. In order for Japanese mathematics education to break away from thinking "mathematics as a sieve" and in order for adults and students to enjoy learning mathematics, we need to build a society that shares a true understanding of learning mathematics, even though we have a long way to go to achieve such society.

5. DESIGNING SCHOOL MATHEMATICS FOR MATHEMATICAL LITERACY

As I mentioned in the previous chapter, mathematical literacy is mainly concerned with mathematical methods and the importance of learning mathematics. Keeping in mind that mathematical literacy is for all adults who have been through school, school education must change. One big reason for this is that adults do not retain the results they learn after they finish school, as I mentioned in my introduction. Now, let us review Japanese mathematics education up to this point and consider how to design school mathematics so that adults can retain and even further develop mathematical literacy.

5.1 General structure of school mathematics in regard to mathematical literacy

Mathematics education in elementary, lower secondary, and upper secondary school gives a span in both time and space, if we think in terms of mathematical literacy. Here, the span in time and space is considered as general structure of school mathematics.

5.1.1 Life-long education

When we think about mathematical literacy, we need to formulate a plan that looks at learning as a life-long activity. This clearly differs from the "mathematics for entrance examination" or "mathematics as sieve" mindset. We must break the vicious circle of thinking that "the harder we work on mathematics education, the less people like mathematics." Since mathematical literacy takes life-long learning into account, it is our hope that children will learn mathematics of their own free will. When we teach them, we must help them connect with the real world and continue to guide them as they grow.

Life-long education means that we should not think of education as finished at school. People continue to learn even after they go out into society. Educational systems must be changed in accordance with this fact, and individual awareness must be raised.

5.1.2 Integration of various academic cultures

Mathematical literacy integrates general mathematics education for all students and mathematics education for students who go on to pursue science and technology or advanced mathematics. The mathematical methods and the meaning of learning mathematics, which are important elements of mathematical literacy, are useful not only for all students, but also for students who go on to pursue science and technology because it gives them an overview of their future career.

Thus, the cultures of science and technology and the humanities (Snow, 1964) can be integrated by mathematical literacy. Furthermore, since the target of mathematical literacy is all adults in a democratic society, cooperative learning among heterogeneous groups of students is imperative. This idea is connected "Interacting in Heterogeneous Groups" in OECD’s concept of key-competencies (Rychen et al, 2003). Mathematical literacy will integrate these diverse cultures.

5.1.3 Enjoyment of learning and preparation for future endeavours

With an awareness of mathematical literacy, we can see that there are two aspects of the curriculum for general education. One is enjoyment of learning in the present and the other is preparation for future endeavours.

Enjoyment of learning means that all students can collaboratively engage in appropriate mathematical activities with common mathematical content for the greatest possible development. Students think and express themselves freely in their groups.

Preparation for future endeavours is composed of two types of learning. One is for all students to jointly learn common mathematical methods and the meanings of learning mathematics. The other is for different students to learn different mathematical content.

5.2 Addressing diversity in the design framework of school mathematics

If we keep mathematical literacy in mind when revising the general structure of school mathematics, we see that many practices and studies conducted in Japan should not be changed altogether. The clarification and renewal of concepts in some practices and studies yield a design framework relevant to improving the general structure. Here the central concept for the framework is diversity. I have been saying that although mathematical literacy is for all adults, it includes diversity. In Japanese mathematics education, we have a long tradition of giving consideration to students’ diverse ideas. In Japan, mixed-abilities classes have been conducted for a long time. Though consideration for individual instruction is occasionally lacking, in a sense, our mathematics education has been addressing students with diverse backgrounds. In this part of my lecture, I would like to consider the design framework of school mathematics in relation to this diversity and give some concrete examples.

5.2.1 Collaborative research by elementary, secondary school, and university teachers

In Japanese mathematics education, results of a collaborative research project by a team of elementary, secondary school, and university teachers has ongoing for some time. The team began to discuss the framework based on a certain idea. While many teachers make plans and conduct experimental classroom lessons based on the framework, other participants observe the lessons. The team discusses findings and issues based on lesson records. The subjects of the research projects are diverse in their inclusion of elementary, lower and upper secondary students. The researchers are also diverse, including teachers from elementary and secondary schools, as well as those from universities.

Such collaborative research among various teachers made it possible for us to put mathematics education in long-term perspective. The findings of practical research in each school level become more objective through researchers' viewpoints of different school levels. And this type of collaborative research leads participating teachers to common understanding of not only mathematical development throughout elementary to university levels, but also students' ways of mathematical thinking. The collaborative research is appropriate for taking people's lives into account, which I elaborated on when I spoke about the general structure of school mathematics with viewpoints of mathematical literacy. In addition, as a matter of course, these research results will become relevant in society beyond school.

5.2.2 Integration of mathematical content and methods in educational curriculum

The mathematics curriculum for each grade in Japan is known internationally as a large unit curriculum. In a given mathematical unit, the following processes are aimed. Students start with a problem prepared by a teacher. The problem should be relevant to society or daily life. The students must determine how the new mathematics can solve the problem, examine the properties of newly introduced mathematics, summarize the procedures and properties, and tackle new problems using the new mathematics (Shimada, 1970). This type of integration of mathematical content and methods not only allows us to see mathematical culture integrated, but also to put both enjoyment of learning and preparation for future endeavours in perspective.

Problem solving plays a vital role here. Students understand the content and acquire the methods through problem solving. This idea became clear in elementary school mathematics in the 1930's, and in upper secondary school mathematics curriculum in 1950's. But the idea evolved in elementary school mathematics, just as in other educational reforms in Japan.

5.2.3 Setting of learning situation based on students' mathematical activities

Though mathematics learning situations are different depending on teaching and learning content, it is thought in Japan that students' mathematical activities are central to understanding concepts and acquiring methods. Also it is thought that in order to better evaluate mathematics education, planning situations in which students can freely display their own abilities is essential.

Mathematical activities are conducted based on situational mathematical problems. The mathematical contents include pure mathematical problems, as well as problems in the real world. The mathematical activities include solving mathematical problems, finding mathematical rules, and making new mathematical problems. In these kinds of learning situations, the following processes are commonly found: a) thinking individually, b) presenting various ideas, c) whole class discussion. The last process (c) is called "*neriage*," which means to “polish up”. In this process, students and teachers talk together about differences and similarities or relations among various ideas so that they can understand concepts more deeply as a whole class.

5.3 Design of school mathematics

Based on the design framework of school mathematics in regard to diversity, let us consider several findings from research on mathematics education practices, as well as future possibilities in Japan. Two recent publications are helpful. “*Handbook of Research in Mathematics Education*” (JSME, 2010) is a comprehensive summary of research in mathematics education over 50 years in Japan, and “Toward the Problem-Centered Classroom: Trends in Mathematical Problem Solving in Japan” (Hino, 2007) summarizes research in problem solving since 1980 in Japan. In this part of my lecture, I will mainly discuss the results of collaborative research by elementary, secondary school, and university teachers that are still valuable in terms of mathematical literacy.

5.3.1 Classroom lessons that make students think

Mathematics education in elementary school in which children think mathematically based on concrete problems was explicit in elementary mathematics textbooks in 1930’s in Japan. Before these textbooks were published, there were two books on reforms for secondary mathematics education published in 1924. They are “*Fundamental Problems for Mathematics Education*” (Ogura, K., Idea-shoin) and “*Fundamental Investigation on Elementary Mathematics Education*” (Sato, R., Meguro-shoten).

The idea that children should use mathematics by themselves to solve problems given by their teachers remains at the core of teaching and learning. For example, there was a study that investigated various aspects of children’s mathematical thinking in elementary and secondary schools (Matsubara, 1971). “Classroom lessons that make students think” was also conducted in elementary and secondary schools (Matsubara, 1987; Handa, 1995). The authors claim that “The essence of the classroom lesson is not to teach. It is to have children be active. In order to realize that, the classroom must become a forum for thinking.” They also say, “The most important thing that teachers should do to hold good lessons is to know children well.” They discuss how the lesson should be introduced, how teachers should bring up questions, and how to work out ways to make children think for themselves. In this study, mathematical thinking was thought as functional thinking. And it seems for me that it is mathematization.

This type of teaching and learning that explicitly accept diversity of children’s ideas was developed into “Teaching that appreciates students' various ideas” (Kotoh, 1992). In the lesson, mathematics is thought to offer multiple ways to solve a problem with only one correct answer. The ideas, which reflect the students’ various perspectives, are summarized at the conclusion of the lesson. This concept showed us a course of action to treat all children with diversity important in whole class teaching of a mixed-ability class.

5.3.2 Open-end approach

In 1960, the IEA First International Mathematics Survey found that Japanese students thought of mathematics as something to memorize. In response to this finding, development of mathematical thinking became an issue to be tackled. So ways of thinking mathematically were emphasized as an objective of the official mathematics curriculum of elementary and secondary school, and "integrated and developmental" became a slogan in 1960's-70's (Nakajima, 1981). The slogan means development of mathematics with awareness of integration. It is similar to the idea that subjective knowledge becomes objective knowledge in society (Ernest, 1991).

Under these circumstances, elementary, secondary school, and university teachers developed a project to evaluate ways of thinking mathematically. For this purpose, they thought the priority should be to prepare a situation in which children could conduct comprehensive mathematical activities. Thus, the open-end approach was advocated: ways of teaching and learning that present an open-ended problem with multiple correct solutions (Shimada, 1977; Becker & Shimada, 1997). It was the first book on mathematics education in Japan that explicitly advocated mathematical modelling to explain mathematical activities.

The open-end approach is a form of instruction that "sets an open-ended problem as a task, develops the classroom lesson positively using a variety of correct solutions of the problem, and gives students experiences in which they combine learned knowledge, skills, and ways of thinking in various ways to discover something new in the process." This instruction had a big impact on mathematics education, since the prevailing idea in those days was that mathematics had only one correct solution. The project developed open-ended problems that required three types of mathematical activities. They are: how to measure, how to classify, and how to find rules. Later on, the project included problems with excessive information or insufficient information in open-ended problems.

5.3.3 Developmental approach to mathematical problems

The creation of mathematical problems was focused on expanding the open-end approach, or as one type of mathematical activities, or as an extension of activities to think in various ways. Another team of elementary, secondary school, and university teachers started up a project. The team came up with the idea of teaching and learning focused on a learning situation in which students first solve a problem and then formulate new problems based on the one they have solved. They proposed teaching and learning through a developmental approach to mathematical problems (Takeuchi et al., 1984).

Instruction through the developmental approach to mathematical problems involves "engaging students in spontaneous learning activities such as starting from a given problem and having students make up new problems which they solve by themselves by changing elements of the problem for similar or more general ones, or by thinking the converse etc." This is "instruction through problem creating." In the beginning of the 1900's in Japan, other type of instruction through creating problems was conducted. Then, elementary school children created mathematical problems in class by looking at their surroundings. The developmental approach to mathematical problems is based on mathematical problems. It has real difference from problem solving method. Students who were thought to be unable to participate in problem solving were now involved in mathematical activities.

5.3.4 Connection between society, culture, and mathematics

The IEA Second International Mathematics Study conducted in 1980, as well as subsequent international surveys, found that Japanese students had little awareness of a connection between mathematics and society. Based on the findings of these studies, another elementary, secondary school, and university teachers began to study ways to put mathematics instruction into societal context. They started thinking about learning situations that focused on solving problems relating to children's lives, society, and culture. In other words, they began to think about "connecting mathematics to society and culture" (Nagasaki, 2001).

Connecting mathematics to society and culture means "teachers in the classroom focus on mathematical activities and interactions that help to prepare children to tackle problems in society or problems related to mathematical culture. It means helping those children acquire competencies and attitudes to improve society and culture."

The ability to connect mathematics with society includes four domains, namely, understanding of quantities and shapes in society, ability to solve social problems mathematically, ability to communicate using mathematics in society, and ability to use approximation. There, mathematization of phenomena was the necessary method first and mathematical modelling was the central method. Though this kind of mathematics education was carried out during the 1950's in Japan, it failed due to the weakness of the mathematical aspect. Here our instruction was conducted having objectives as mathematics appropriately against the failure. It has the same idea as mathematical enculturation (Bishop, 1988).

5.3.5 Future possibilities

Research on the design of school mathematics in Japan has gradually been introduced into Japanese mathematics textbooks and spread widely. Recently, research has expanded further.

Let me give you some examples of these research works. Concerning open-end approaches, there is a study on fostering the ability to make decisions by actively treating societal problems as open-ended problems (Shimada, 2011). On the subject of developmental approaches to mathematical problems, they conducted a study on problem creating in which university students used computers (Shimomura et al, 2009). As for connecting mathematics with society, there are study on development of teaching materials and instruction (Nishimura et al, 2010), study on the ability to perform mathematical modelling (Nishimura, 2010), and study on the ability to think statistically (Matsumoto, 2010).

Also, there are several studies on mathematical activities, including a research project focused on ways to handle mathematical methods in elementary and lower secondary level textbooks (Japan Textbook Research Centre, 2006), studies on systematizing mathematical competencies (Nagasaki et al, 2008), and a study on teacher education focusing on types of classroom teaching such as problem solving (Kubo, 2012), and a study on teacher education focusing processes of thinking mathematically (Ohta, 2012).

The design of school mathematics was based on practice and research in mathematics education. "Mathematics for all" was implicitly understood. Above all, we asked, "can there really be mathematics for all?" Based on the findings of cognitive science (Inagaki et al, 1989), mathematics education for all children which employs the concept of collaborative and shared learning is starting to become the subject of research (Matsushima, 2012).

Furthermore, it seems that the main goal of mathematics education so far has been to build a system of mathematical knowledge by proving true propositions deductively. However, it seems to be equally important to find false propositions in a society flooded with information that is interwoven with both truths and falsehoods. Expressions that are intended to make us misconceive are discussed in statistical teaching. By the same token, it is necessary to find false propositions in other area of mathematics as a mathematical activity. That is to say, it is necessary to actively and intentionally introduce problems that enable refute by counter-example (Lakatos, 1976). Deductively proving true propositions and relating false propositions leads to deeper understanding of concepts, as well as better use of methods.

6. MAINTENANCE AND EXTENSION OF MATHEMATICAL LITERACY OUTSIDE OF SCHOOL

Most of our lives are spent outside of school. People must maintain and extend what they learned in school. Ways of doing this include conversations at home, traditional media such as books, newspapers, radio, TV, magazines etc., and internet. However, in addition to these informal means, some formal activities are needed. There are some sorts of formal activities related to scientific literacy.

6.1 Lessons from diffusion of scientific literacy

In the beginning, scientists diffused scientific literacy by giving lectures on it. But it became clear that a mere outpouring of knowledge was not sufficient. Later on, concepts such as science communication, science interpretation, and consensus conferences for all adults have been implemented. Here, I will mention four activities for the diffusion of scientific literacy.

First, there are activities in science museums or science centres that families can visit (Ogawa, 2011). These were developed as out-of-school educational institutions in the past, but today they emphasize working in coordination with school education. Though most of their business activities are related to non-mathematical science and technology, there are some activities linked to mathematics. Second, there are activities in science cafes and science pubs run by NPOs, universities, and other organizations. In these establishments, scientists and the general public discuss science on the same plane. Third, there are consensus conferences (Kobayashi, 2004) at which scientists and ordinary citizens exchange opinions about important policies related to science and technology and prepare reports. Fourth, there is a science communication movement. In this movement, scientists do not communicate scientific literacy to citizens unilaterally, but use two-way communication to share scientific literacy. These activities are all steps forward in communication based on science.

6.2 Reconsideration of mathematical literacy

For diffusion of mathematical literacy, there are lectures given by mathematicians in open schools for adult education, as well as upper secondary classroom lessons given by mathematicians.

Looking at the diffusion of scientific literacy from the standpoint of mathematical literacy, science museums, science centres, and science cafes could also be used for mathematical literacy. And consensus conferences offer one answer to the question of who develops ‘mathematical literacy for all adult’. Consensus conferences show us that mathematical literacy should not be developed by specialists alone, but by the collaborative efforts of specialists and non-specialists. And science communication reminds us that mathematics is a “powerful means of communication” (Cockcroft, 1982).

Mathematical literacy must be geared toward all adults and take people’s lives into account. Every day, adults face social, cultural, and family events that compel them to think and judge. Though systematic learning is sufficient in school, as adults, individuals must think for themselves in the different situations they face. Formal activities are important, but real problems are even more important. Today, for example, adults are compelled to make judgements about global warming and nuclear power plants, mathematical literacy is vital. In such situations, information from the media, as well the ideas of each individual adult would enhance mathematical literacy.

Ultimately, mathematical literacy must be organically connected and integrated within each adult. It is not enumeration of mathematical content and methods, but a representation in language and figures that allow us an overhead view of the whole of the connection; connection between mathematics and the problems of individuals, society, and culture, and connections among various areas of mathematics (Project ‘Science for All’, 2008a).

7. CONCLUSION

Mathematical literacy varies from time to time and from society to society. In this paper, mathematical literacy has been discussed keeping Japan’s present and near future in view. I have attached importance to taking people’s lives into account and making connections with society when mathematical literacy is considered.

These ideas about mathematical literacy still leave us with challenging issues. For example, what happens to curriculum development (Howson et al., 1981) for fostering mathematical literacy in school education? Though mathematical methods are focused on as part of mathematical literacy, how would mathematical methods be structured by age group? What other types of teaching and learning could be developed by practice and research of elementary, secondary, and university teachers? How can mathematical literacy be maintained and developed in society? How can mathematical literacy be evaluated?

Also, ideas about mathematical literacy seem to raise several issues for mathematics education. For example, why is mathematics taught as a core subject in school? Are screening tests compatible with ideas of mathematical literacy? How should mathematics and mathematics education be connected with society?

By raising such serious issues, ideas of mathematical literacy re-examine mathematics education at its foundations.

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