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WEAVING EXPLORATION IN THE PROCESS OF ACQUISITION AND DEVELOPMENT OF MATHEMATICAL KNOWLEDGE

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This paper presents an experience of a process of gathering mathematical knowledge by exploration of artifacts of students' cultural environment – the mat twill weaving techniques and their resulting products as well. That process means starting from posing and reflecting on "why …", "how …" and "what if …" questions related to the existence and gestalt of such artifacts, when one is manipulating them either physically or mentally. This highlights the Gerdes' research approach in the new research field called ethnomathematics. Furthermore, the paper brings the context of all that process, the classic theoretical framework on research methods in mathematics education, illustrating what does mean "doing mathematics", and how mathematics teachers can make their students feel themselves mathematics producers and owners, just by exploring those artifacts. The experience was gain both in and out of school settings but always leaded to know about the process of acquiring mathematical knowledge by the involved subjects.

The context of involvement with the research problem

From my own first experience as a teacher of mathematics and physics at secondary school level and later as a secondary school teacher educator, I had the impression that one of the main reasons for weakness in mathematics education might be the approach to teaching which does not create in students eagerness for and a confidence in what they are learning. My concern on this problem led me to investigate particular aspects of that situation of students' dropout, at the end of every school year, due their marks in mathematical contents. The experience reported here is from Mozambique, where, like in other many former colonized countries, has an educational system still influenced by teaching methods that do not privilege the richness of the learner´s previous experience and culture in the process of mathematics learning.

One example which illustrates that impression, are some puzzling questions posed by students (in and out of the mathematics class) such as: does mathematics have any relationship to the physical world? How did 'mathematicians' invent mathematical formulas, theorems, and rules? In most cases these students' questions do not produce any satisfactory answers from the teachers. Students progress into higher grades with their doubts about mathematics growing even more. Few teachers at secondary school level attempt to explain the origin and development of mathematical knowledge. Very often teachers say to their students: don't worry about that, you will learn it later... In some cases, behind this response there is an authoritarian teaching style which does not allow students to ask such "crazy questions". Why does this happen?

A brief comment given by some teachers is that they rarely make connections between mathematical concepts and physical world phenomena or historical aspects in their teaching because of the time constraint: the contents of the syllabuses are too extensive for the time allocated. Other teachers do not see any reason for wasting time with matters which never come in the exams. I found students who developed negative attitudes towards (learning) mathematics. Mathematics anxiety among many students is still strong. This causes what D'Ambrosio (1984:6) and Gerdes (1991:21) call psychological and cultural blockage. Psychological blockage involves the learner in a hard cognitive conflict so that s/he, on the one hand is forced to create alternative conceptions about the (in school) learning mathematics and, on the other hand, loses control in understanding the mathematical ideas s/he brings from her/his social and cultural environment before school engagement. In Mozambique some students and even some people who have finished secondary school, manifest negative attitudes towards mathematics, calling it “the bug with seven heads” (cf. Gerdes, 1998b:35) to mean that mathematics is something irritating them. Although we can already see a great enthusiasm towards mathematics by youngsters, there still a big challenge for a desirable standard in our schools.

In the last fifteen years or so, several mathematics educators in different parts of the world share the position that a main reason for the widespread math anxiety (not only in Mozambique), and consequently for large number of dropouts, may strongly be linked with disregard for mathematics as a cultural phenomenon. According to this position, in mathematics learning and teaching, cultural aspects of the learner´s environment should be taken into consideration.

My research problem and critical question arise out of this context: How we pay more attention for this issue in our schools? Is that — disregard of cultural aspects — really a reason for our problems in mathematics learning and teaching? How can a cultural related activity be used to introduce and develop mathematical ideas in the classroom? And once a cultural activity is used in the classroom, to what extent does it changes the students’ performance and attitudes towards mathematics? Is the formulation of those questions that originated all my motivation to come engaged with the work I developed during almost the last two decades.

Preliminary concepts

The explored artefacts considered to illustrate the acquisition and development of mathematical knowledge through exploration activities are some very common basket weaving techniques used in Mozambique and in many other parts of the world. There are concepts used, which may not be immediately understandable for someone who is not familiar with weaving techniques. Two weaving techniques are treated: the plain weave and the twill weave. Both are flat weaving and have a peculiarity of being made by two layers of strands (see figure 1), generally in perpendicular directions to each other.

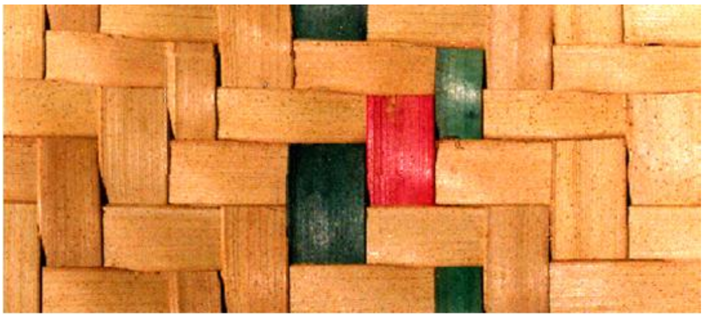


Figure 1: Part of an artifact woven by two layers of strands

In the context of the research for classroom activities, instead of using plant strips as used in real mat or basketry weaving, strips of cardboard of a constant width are used. In terms of procedures these techniques are made in a similar way to loom weaving: having one layer of fixed strips while the other layer — of loose strips — is being interwoven, strip by strip, with the fixed strips. On the other hand, the treatment used is strongly related to mat and basket weaving in terms of the designs and colouring. The basic glossary of weaving terms I extracted from different authors (cf. Grünbaum & Shephard 1980:139-161; LaPlantz 1993:141-142; Bazin & Tames 1995:42-47; Sudduth, 1999:130-133). Some terms I arranged myself in a particular way (e.g. weaving board, same phase, etc.) in order to express what I am intending to communicate.

Plain weave is an over-and-under weaving, i.e., a strip of one layer goes, successively, over only one strip and under only one strip of the other layer (see figure 2).

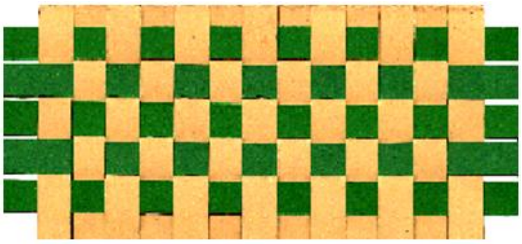
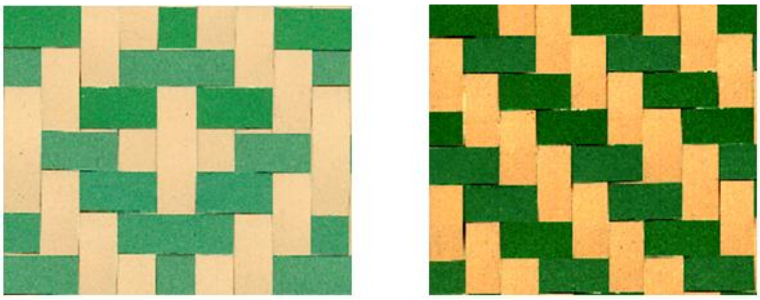


Figure 2: Plain weave

In this study I considered plain weave as a particular case of **twill** weave.

Twill weave (or simply twill) is a weave in which at least some strips of one layer pass over or under more than one strip of the other layer. The figures 3a and 3b show two variants of twill.



3a 3b

Figure 3: Twill patterns

To describe a twill we can abbreviate the language. For example, for the twill in figure 3b, instead of saying "two strips over and two strips under…" we may simply say "two by two" and use the notation: **2/2**. Note that this notation indicates the number of strips passing over and under before a repetition of the pathway, i.e., before the same "overs" and "unders" come again in the way of a strip. For this notation the first number will always indicate the strip(s) being passed over — the "overs" — while the number after the slash indicates the strip(s) being passed under — the "unders" — of the strip being described.
The "overs" or "unders" may be different along a pathway; it may be found, for example, a "two by one, one by three" twill (figure 4), and so we it is noted accordingly as **2/1-1/3**.

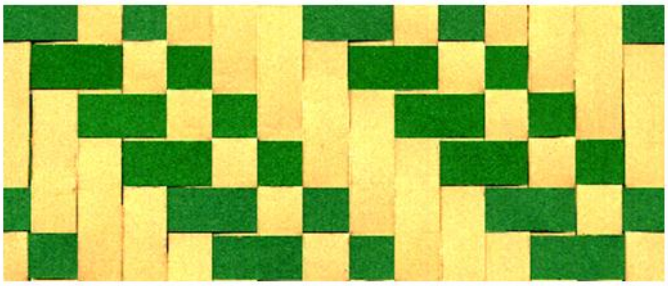


Figure 4: A twill 2/1-1/3

Since a repeated pattern is formed in the pathway of a strip, we call that repetition the **period** of a pathway. The period will be indicated by the sum of "overs" and "unders". So, the period of 2/2 is 4; the period of 2/1-1/3 is 7. A **balanced twill** is the weaving over and under the same number of strips, i.e., a twill in which the period of pathways is the same for all strips and has the same number of "overs" and "unders"; otherwise, the twill is unbalanced. Thus, for example, the twill in figure 3b is balanced, while the twill in figure 4 is unbalanced.
Looking at figure 5 it is easy to follow the pathway of a particular strip. For example, strip **A** is a 2/2 twill. Looking at the strip **B** it can be seen that it is woven exactly like the strip **A**, that is, the strip **B** has the same period and it also goes, respectively over and under the same strips. We say that two strips have the same phase when the two strips go, for the entire path, over and under the same strips.

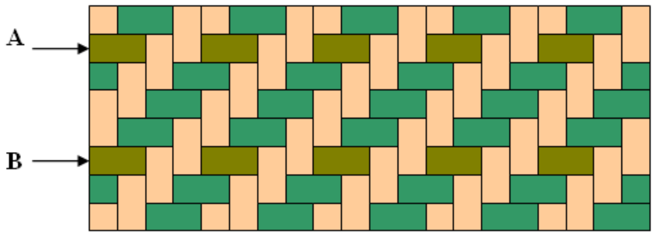


Figure 5: Example of strips with same phase (**A** and **B**)

It is important to distinguish between weaving design and colouring, although these concepts are closely dependent on each other.
I call weaving **design** the finite texture that is created by weaving. In other words, design is a portion of a weaving pattern, i.e., a pattern is infinite while a design is finite.

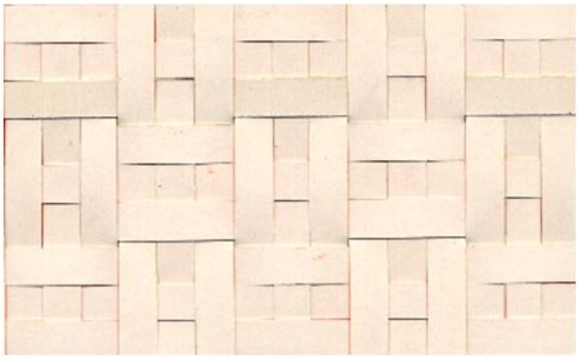


Figure 6: Design “X”

Under **colouring** I mean the combination effect that appears by using strips of different colours in the same weave. Thus, considering the design “X” in the figure 6 different colourings can be devised as shown, for example, in the figures 7a and 7b.

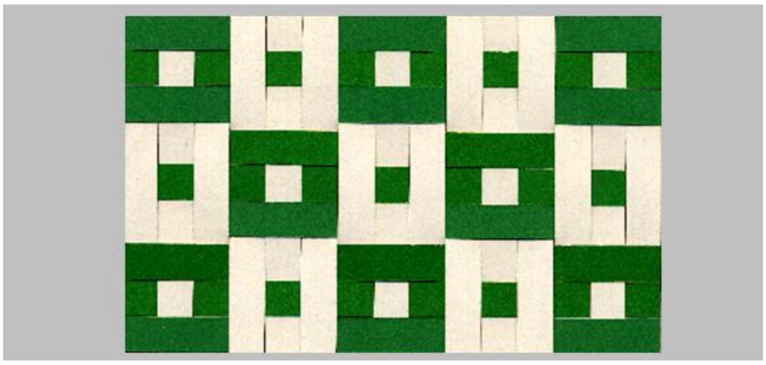


Figure 7a: A possible colouring of design “X”: All “horizontal” strips are green.

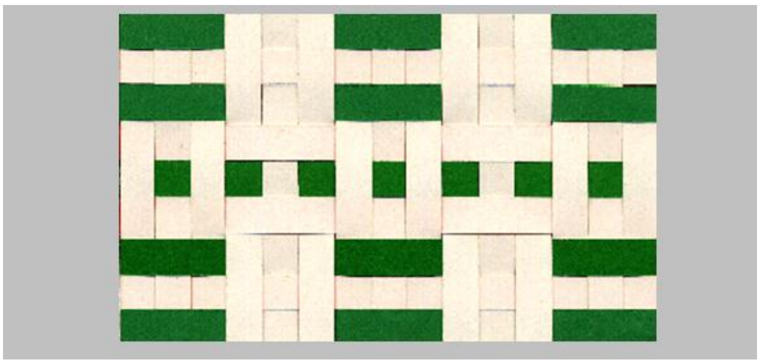


Figure 7b: A possible colouring of the design “X”:
The colour of “horizontal” strips alternates — green and white.

This shows that for the same design we can have many different colourings.

The **weaving board**, hereafter abbreviated by **WB**, (figure 8) is an apparatus I developed when challenged with the questions on how to bring my ethnomathematics results into a mathematics classroom, where the teacher has a determined topic of the programme to teach in a determined space of time. So, the WB was indeed invented to assist twill weaving for the teaching and learning of mathematics in the classroom. It consists of **f**ixed strips of cardboard (hereafter abbreviated by **F**-strips) attached to a stiff cardboard background, and of some **l**oose strips (hereafter abbreviated by **L**-strips) to interweave with the fixed strips.

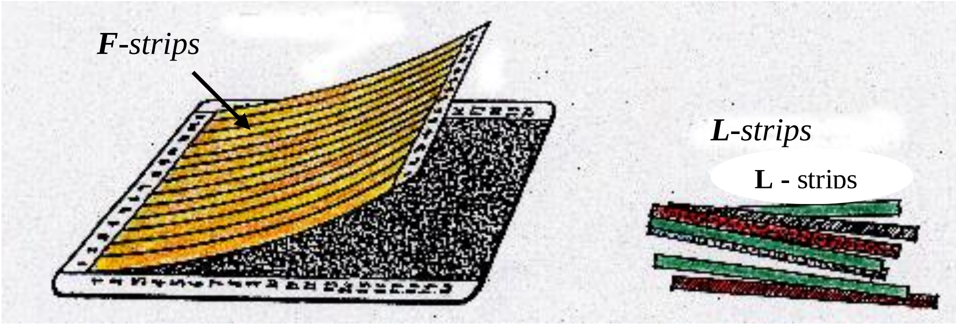


Figure 8: Weaving board (WB)

The border of the stiff cardboard is numbered to make the description of the pathway of the strips easier. Furthermore, the stiff cardboard background helps the weaving in case no flat supporting surface is available, particularly for a large weaving board. Hereafter, **WB-activity** means an activity using a weaving board.

The methodology

The methodology of the study refers to two aspects. The one aspect is related to what characterizes the research following Gerdes' approach on the exploration of basket weaving techniques, with implications to mathematics education. The other aspect of the methodology refers to the classic research methods in (mathematics) education, which include the description of the type of research design according to the research problem being addressed, the intervention study, the instruments for data collection and the approach for data analysis.

On Gerdes' research approach

The aspect of Gerdes' research approach I used, starts by observing an artifact (a mat woven by plain or twill technique in this case), raising questions such as "Why is the pattern so ...?", "What if ...?". The first question motivates and pushes us — as mathematics educators near to the weaver — to try ourselves to imitate the weaving techniques, not necessarily to produce an authentic objects of art but, at least, to produce similar woven patterns in order to try a linkage between mat weaving as a cultural manifestation (of students as of artisans) and the subjacent fact of "doing mathematics" embedded in it. The second question may lead us to a deeper mathematical exploration, making us think beyond the properties of the motivating artifact, coming to pure mathematics.

Gerdes presents this methodology as an alternative for reconstruction and exploration of mathematical traditions, "… when probably many of them have been — as consequence of slavery, of colonialism ...— wiped out. Few or almost none (as in the case of Mozambique) written sources can be consulted. Maybe for number systems and some geometrical thinking, oral history may constitute an alternative." (Gerdes, 1988a:140)

Particularly with regard to educational aims it is important that educators themselves get involved in (re)inventing or (re)discovering of the mathematical knowledge "hidden" in cultural artifacts. Besides possible (mathematical) data that weavers may provide from their praxis, mathematics educators will find this methodology as necessary. Gerdes (1988a) explains it as follows:

> *"We developed a complementary methodology that enables one to uncover in traditional, material culture some hidden moments of geometrical thinking. It can be characterized as follows. We looked to the geometrical forms and patterns of traditional objects like baskets, mats, pots, houses, fish traps, etc. and posed the question: why do these material products possesses the form [or patterns] they have? In order to answer this question, we learned the usual production techniques and tried to vary the forms [the patterns]. (...) The traditional form [or pattern] reflects accumulated experience and wisdom. It constitutes not only biological and physical knowledge about the materials that are used, but also mathematical knowledge about the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, pyramids, cylinders, etc."* (p. 140)

Answers to the two questions above, if given by the artisan who made the real artifact may differ from those given by students in the classroom. One reason would be that the weaver produces authentic artifacts, in principle, with different objectives to those of the teacher in bringing or referring to mat patterns in the mathematics class. Moreover, the material (straw, sisal, bamboos) used to produce real artifact do not have the same flexibility as the cardboard strips used in a WB. That is why, for example, we may rarely find a sleeping mat made by plain weave using bamboos, because large holes would appear in crossing strands.

In this work I used those input questions, starting by imitating the technique in order to get insight for the exploration of weaving per se, as opposed to mathematical activities for the classroom. I tried to find out what would be relevant as common features in producing plain and twill weave, by both artisan and learner in the classroom. On one hand I see the activity of the artisan as something which lies in between cultural manifestation and mathematical thought. On the other hand, I see the weaving activity of the learner, as something lying in between mathematics and cognitive process, which apparently does not have any connection with the weaver's activity. In their activities, both artisan and learner are "weavers" since they produce weaving, independent of the objective that each one has, and of the material that each one uses. The common product of both is plain and twill weave as weaving technique — and not the artifact resulting from that same technique. Considering the differences of objectives and of the material used in both cases, at the end of a weaving activity the one has artistic products and the other has mathematical ideas.

The approach can be represented as in the schema below (figure 9).

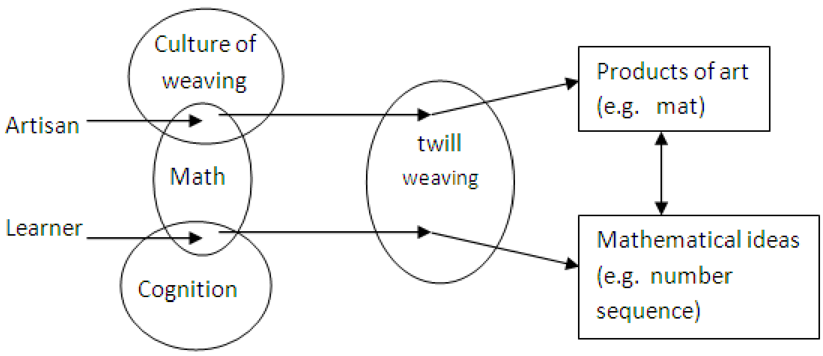


Figure 9: The artisan and learner activities

The premise is that mathematical objects (ideas) produced by the learner in the process of weaving, e.g., the pattern of figure 7a or 7b, can be applied for stimulating new mathematical ideas and for producing new patterns in the weaving of the mats as well. The schema makes it clear that there is a moment in the activities of both, artisan and learner, where it is assumed they are thinking in the same way. This is the crucial moment for my study. It is the instances where I say that both are "weavers", versus both are "mathematicians". This assumption leads to review a theoretical framework throughout a line which explores cultural, epistemological, and cognitive aspects, relating twill weaving to mathematical ideas. In short, it is evident that we are dealing with ethnomathematics research questions.

On the classic research approach

In this section I give an overview of the common research methods used in mathematics education research. The design consists of a structure which includes three different phases: the Pilot Study, the Main Study-part I and Main Study-part II.

The Pilot Study was designed to find out how the subject, as an individual, learns to weave, using twill techniques, by managing the WB. Taking the hypothesis that one can learn mathematics — at least think mathematically — by this process of weaving, I conducted case studies with five children to closely observe what happened with each child when engaged with that activity.

This method enabled me to rearrange some aspects concerned with the WB itself, and with the worksheets relating the weaving patterns to mathematical ideas specifically to sequences and series. From the conclusions drawn from these case studies I prepared the research instruments for the next stage: the Main Study-Part I.

The Main Study-Part I (MS-I) extended the Pilot Study in order to confirm and to deepen it. Instead of working with one child at a time, the MS-I was done in classroom settings. Once it was known what could emerge from the individual learner when s/he carries out WB-activities, I needed to test the effectiveness of the proposed approach when compared to traditional ways of teaching. Therefore the MS-I was set up as quasi-experimental research, involving a control and an experimental group of students. The subjects were from grade 11, since the mathematics syllabus includes a topic on numerical sequences and series, which the Pilot Study showed to have strong relationship with the WB-activities. The working hypothesis was that students being taught numerical sequences and series by the proposed approach—using WB-activities—would perform better than those taught by the usual ways. In the usual ways students are given some few number sequences in order to find the missing terms, without any concrete context. Students are urged to find formulas and almost none demand for use of verbal language is posed to them. I call this way of treating the topic as "usual method" (or traditional method). The hypothesis was that students being taught by the proposed approach—using WB-activities (as an example of students' cultural environment)—may become aware, confident and eager for school mathematics. In other words, students may gain positive attitudes towards (the learning of) mathematics.

The Main Study-Part II (MS-I) was designed to deepen the previous approach, both in "horizontal" and in "vertical" perspective. In the "horizontal" perspective, unlike the previous study (MS-I), I involved teachers as the main subjects. The teachers were expected to find more WB-tasks beyond those I presented in the source booklet "Exploring the Weaving

Board". I continued to elicit from teachers further tasks, beyond the tasks in that booklet, to discuss in order to include such tasks in later experiments in the classroom.

In the "vertical" perspective I looked for specialised literature about twill weaving, in order to find relationships between weaving patterns and mathematical ideas which could be used in the classroom. On the other hand, I researched related literature in order to discuss my theoretical framework and to situate and argue the relevance of the research in the context of mathematics education.
Briefly, the MS-II was a teaching experiment (having teachers testing their plans of lessons based on WB-activities) combined with basic research (having teachers and myself searching for new WB-activities).

Example on weaving exploration and developing of mathematical ideas

Case studies were conducted to find out how well the WB functioned. This entailed observing external evidence (manipulative operations, verbal and written language) which might help to understand the extent to which the weaving board assists the learner when carrying out twill weaving related to mathematical activities. Thus the following questions were posed as follows: Can the subject manipulate the WB, i.e., can s/he produce a twill weave using the weaving board, and how does s/he do it? Is the subject using the WB for responding to questions on twill patterns related to mathematical ideas, and how does s/he do it?

According to these questions the subjects produced different types of data. As a result of manipulative operations, the data produced were pictures (the woven designs) and procedures (the sequences of operations to produce those designs). Furthermore, verbal or written explanations for the posed questions were produced. The data presented do not constitute every design woven by each subject nor all the respective verbal or written information given. This is just a sample selected according to categories describing individual differences of the subjects' outcomes, but during the study, common features of the subjects' weaving procedures, woven designs, and discourse of their explanations were also presented and analysed. The paragraphs in the following descriptions are numbered to facilitate their reference in the analysis. In the explanations the underlined words are considered keywords implicitly revealing the mathematical ideas of the subjects. A same keyword is underlined only once in the explanation of each subject.

Let us follow the exploration done by subject **D** (D is the initial of his name) and the respective analysis for interpretation of the extent to which he developed his mathematical ideas. Subject **D**, a 13 years old boy, in grade 7, started to imitate the design of the figure 10 and ended up at the figure 11. He was asked to explain how he reproduced it.

(1) **D**: I looked at <u>the design</u> [at the booklet] and I made it here [in the WB].

(2) Researcher (hereafter abbreviated by **Rsr**): I would like to know how did you started to weave and how you continued the pathway, for example, let us consider this strip [the strip L1] (see figure 11).

(3) [**D** looked at the **Rsr**, smiled and started to explain.]

(4) **D**: This strip [he followed the pathway of the strip L1 with his finger] goes over these strips and under these strips; and then again over these and under these ... [he meant over and under the strips of the frame.]

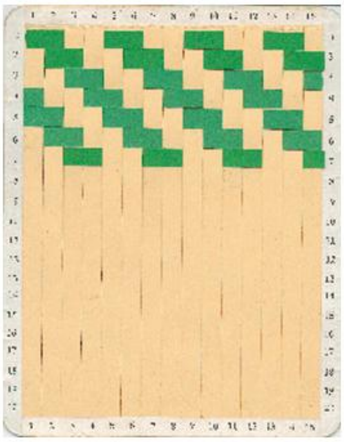


Figure 10: Design chosen by **D**

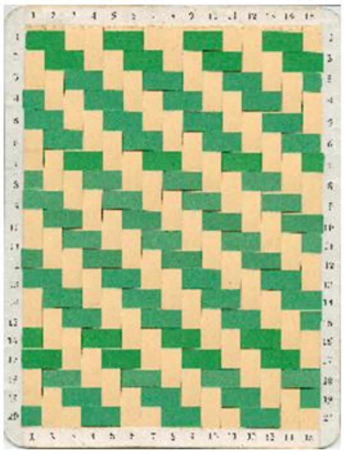


Figure 11: Design reproduced by **D**

Figure 12: “Could it [L-strip 1] go like this?”

(5) **Rsr**: Could it go like this? [**Rsr** changed the pathway of the strip L1, see arrow in figure 12]
(6) **D**: Humm... It doesn’t have a good look!

(7) **Rsr**: Why?
D smiled again before he gave the answer:
(8) You see, here [figure 10] are one, two strips over and one, two strips under... So, here [figure 12] you have to do in the same way: count one, two strips over and one two strips under...
After that, **Rsr** asked him to continue to weave the pattern of the figure 13. **D** attempted it and he did it correctly and he said:
(9) It is not so difficult... you just count the strips.

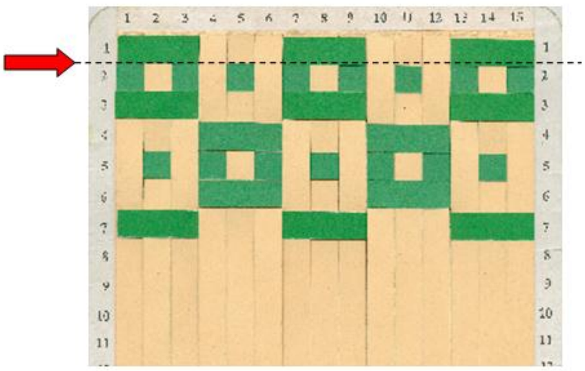


Figure 13: The “squares” design

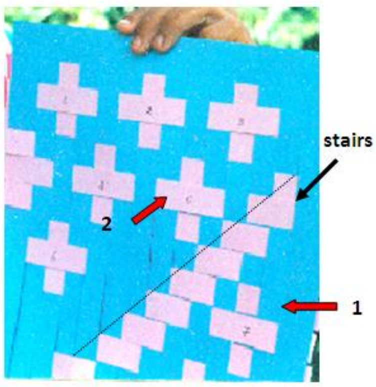


Figure 14: “Cross” design by **D**

(10) **D** explained further: For each strip you want to interweave, you have to count the number of strips [in the frame] you have to jump. To continue to weave you have to look at what was done above this strip [L7, figure 13]. To interweave further I divided the squares here [see arrow in figure 4.13].
(11) **Rsr:** Why did you divide the squares exactly here? [at the doted line]
(12) **D**: To see the part I have to complete ...
(13) [He searched in the already woven design a step identical to that of the last horizontal strip [L7] and from there he followed the pathways of the strips below.]
(14) [In the last task **D** made two designs with axial symmetry. The **Rsr** asked him to invent more designs and he came up to the design in figure 14.]
(15) **D**: This cross [see arrow 1 in figure 14] I have done on the basis of this one [see arrow 2 in figure 14] because they are ordered by the "stairs".

The figures 10 and 11 show an example in which the subject is capable of imitating, at least that particular twill. At the beginning he counts strips running his finger along their pathways but after having woven some strips he continued to weave faster and without counting with a finger any more. He automated the procedures to get the pattern of that design. According to the Serra's definition (1993:39) of inductive/[deductive] reasoning, it is found that the subject first observed the data given (to choose) in the booklet and he recognised the twill patterns, since he continued to weave, following the sequence of the given woven portion.

The generalisation from the observation — the third aspect of the Serra's definition — will came in the second feature of analysis, where the subject attempts own designs and explains his procedures and strategies. The subject manipulates the WB trying different patterns and finding his own ways to explain them. His explanation (almost all verbal) reveals his way of thinking, using the WB to put in evidence the mathematical ideas hidden in his woven designs. In paragraphs (1) and (4) it is shown that subject **D** has worked inductive/deductively—observing the data and recognising patterns:

I looked at the design [at the booklet] and I made it here [in the WB]. (1)

(...) This strip goes over these strips and under these strips; and then again over these and under these ... (4)

This fact became more evident when he was able to reject the pathway for the strip L1 [when I changed the pathway of the strip L1, see arrow in figure 12].

Rsr: Could it [L1] go like this? (5)

D: Humm... It doesn't have a good look! (6)

For subject **D**, "good look" may implicitly express what mathematically is known as symmetry, in broader sense, or translation in that particular case. Here it can be seen that subject **D** has an own conception for symmetry, that is, for him, a design with "good look" may exhibit symmetry. This fact is particularly analysed in the following feature on the links between process/product of WB-activities and mathematical ideas, given by the underlined words in the subject's explanation. Furthermore, **D** attempts to justify [(7), (8)] the reasoning in **D**'s own words, using the WB:

Rsr: Why? (7)

D: You see, here [figure 10] are one, two strips over and one, two strips under... So, here [figure 12] you have to do in the same way: count one, two strips over and one two strips under... (8)

The third aspect of inductive/deductive reasoning — the generalisation from the observation — was reached by the subject **D**. He found that [see (9)] to continue to weave a determined pattern, one needs to count the strips:

(...) It is not so difficult... you just count the strips. (9)

(...) For each strip you want to interweave, you have to count the number of strips [in the frame] you have to jump. To continue to weave you have to look at what was done above this strip [L7, figure 13]. To interweave further I divided the squares here [see arrow in figure 13]. (10)

His explanation [see (11) and (12)] makes it clear that he searched in the already woven design a step identical to that of the last horizontal strip [L7] and from there he followed the pathways of the strips bellow:

Rsr: Why did you divided the squares exactly here? [by the doted line, figure 13] (11)

D: To see the part I have to complete ... (12)

One of the invented designs by **D** is the "crosses" design shown in figure 14. In the booklet the "crosses" design (figure 15a) and the "stairs" design (figure 15b) are presented. Attempting to invent his own design, **D** may have recalled these two designs and tried to integrate them. The reflection by the line "stairs" (figure 14) is not perfectly done but the explanation of the subject [see (15)] shows what he was intending to weave.

This cross [see arrow 1 in figure 14] I have done on the basis of this one [see arrow 2 in figure 14] because they are ordered by the "stairs". (15)

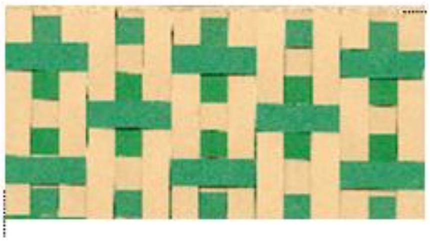


Figure 15a: "Crosses" design

Figure 15b: "Stairs" design

The linking words or expressions used by **D** are interpreted as following:

This strip [he followed the pathway of the strip L1 with his finger] goes over these strips and under these strips; and then <u>again</u> over these and under these ... [he meant over and under the strips of the frame.] (4)

The word **again** means that there is a repetition of "overs" and "unders" in the pathway of the strip. Using mathematical vocabulary it may be said that there is a **translation** of "overs" and "unders".

Humm... It doesn't have a good look! (6)

Good look may mean a harmony, a certain regularity in the pathway of the strip. The regularity in this case is a translation. So, it is reasonable to interpret "good look" as **translation** which in the more generalised sense could be related to symmetry.

(...) You see, here [figure 10] are one, two strips over and one, two strips under... So, here [figure 12] you have to do in the same way: count one, two strips over and one two strips under... (8)

The meaning of **same way** may be interpreted as **same phase** as defined in the section above: Two strips have the same phase when they go, for the entire path, over and under the same strips.

For each strip you want to interweave, you have to count the number of strips [F-strips] you have to jump. (10)

To jump means to pass over. To interweave every L-strip, one should determine the number of F-strips being passed over (being "jumped") by the L-strip. So, the number of strips to jump means the **number of "overs"**.

(...) To continue to weave you have to look at what was done above this strip [above the strip L7, figure 13]. To interweave further I divided the squares here [see arrow in figure 13]. (10)

Looking at *what was done above this strip* in order to continue to weave means that regularity must be followed to maintain [the/a?] **pattern** as asked. [To maintain "the pattern" or "a pattern" — this is particularly discussed by the analysis on subject T]. Dividing the squares here, by the doted **line**, one get the portion of the woven design which makes it possible to continue the weaving in a determined pattern, in other words, the portion to be translated. Such a line of division may indicate a **line of symmetry**.

"... the part I have to complete ..."

(...) To see the part I have to complete ... (12)

The part to be completed may be seen in different ways by different observers. In this case, since subject **D** has externalised his thinking by indicating the strip [the line] which separates the part to be completed, it became evident that the part in question is the one from the doted line down to the strip L7. At this point reflection appears again as central to the **concept of pattern**.

Subject C

Subject **C,** a 17 years old girl in grade 11, found the design of (figure 17) "very beautiful" and it was her first choice to start weaving using the WB. Several times she used her finger to count strips over and strips under, but after some steps she began to interweave faster. Her explanation was the following:

(1) I counted the number of strips going over and under. In general, from the first strip up to seventh, there is a repetition; I mean, this [she pointed the strip L7] goes like the first [L1]; this [L6] goes like the second [L2] and this [L5] goes like the third [L3] (see figure 18a).

(2) **Rsr**: Good! And what about this one? [L4]

(3) **C**: Oh, this separates [see doted line, figure 18a] those which are repeated.

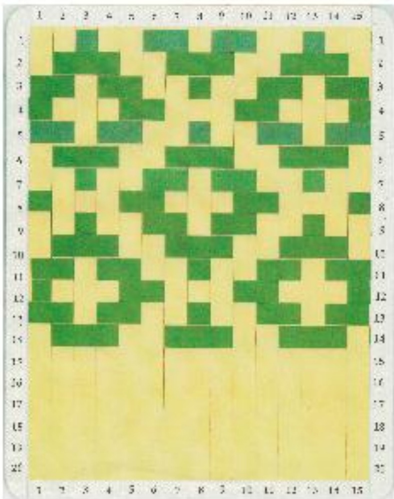


Figure 17: A “very beautiful” design chosen by **C** to start weaving in the WB

(4) [The **Rsr** asked her why she interwove the strip L15 in the way she did. She continued to explain.]

(5) **C**: Look, this strip [L14] goes like this [L6], so, the following strip here [she pointed out the 15th row in the WB] should go like this one [L17].

(6) [After interweaving the strip L20, the **Rsr** asked her to find out in the WB all L-strips which go in exactly the same way as the strip L1, i.e., L-strips with the same phase.

(7) [Responding to this task, **C** wrote down the following numbers]:

(8) 7, 9, 15, 17. (*)

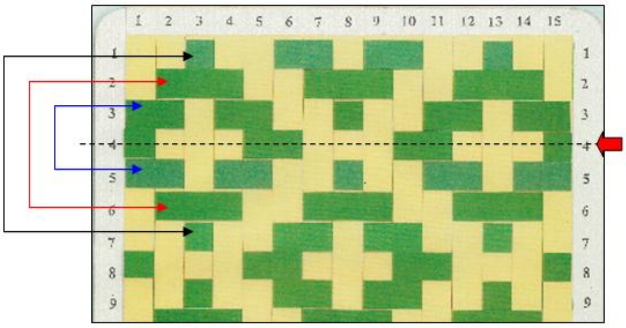


Figure 18a: “In general ... there is a repetition”

(9) [That means, L7, L9, L15, and L17 are the strips in the WB satisfying the question.]

(10) [She was then asked to say which the next L-strip with the same phase should be, supposing that the WB had a greater dimension than it actually has.]

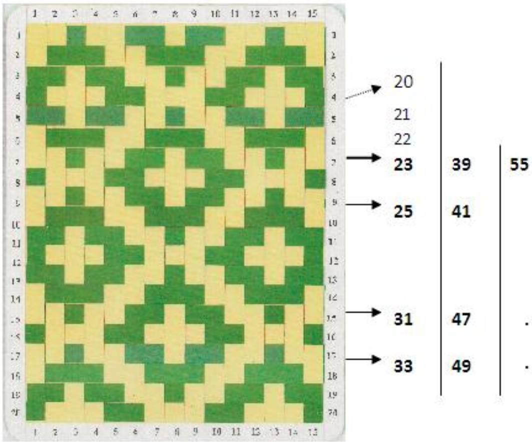


Figure 18b: "Twenty... twenty one, twenty two, **twenty three**!"

(11) [**C**'s strategy to find the correct answer was the following: She started to count the strip L4 as "20" and she ended up on the strip L7 as "23" (the correct answer).]

(12) [To find the next five L-strips with the same phase as L1, she managed the WB in the same way to find "23", so getting the following numbers: 25, 31, 33, 39, 41 (figure 18b).]

(13) [When asked to find the L-strip with that phase at the 30th position in the sequence]

(14) **C** claimed: The number at the 30th position?! ... <u>30th position is far away</u>!

(15) [However she claimed, she was convinced she would be able to get it, and even to find the number of the L-strip at any position in the sequence, only needing to add successively "6s" and "2s".]

(16) **C**: Look, starting here [by the "17" in the number sequence (*), see line 7] you should <u>add 6 to get 23</u>, and from 23 you <u>add 2 to get 25</u>; and from 25 you add 6 to get 31; and 31 plus 2 you get 33; and so <u>you continue, adding "6s" and "2s"</u>.

(17) [After this stage she worked with the worksheet only that is, with numbers, without manipulating the WB physically.]

To interweave a strip she counts first the "overs" and "unders" on the chosen design. Sometimes she stopped a little while, looking carefully at the design appearing on the WB. She is able to reproduce and continue to weave a twill correctly [a twill woven correctly means that it keeps periodicity]. Her gathering of pattern occurs by counting strips, observing their pathways, but not always by running her finger over the given design. **C** has used a pencil to make space between the F-strips, as suggested in the booklet.

Subject **C** may have appreciated the given design as a whole picture (a 'very beautiful" one), but to reproduce it, she attended to it strip by strip. By manipulating the WB, when she started to weave faster, she was already able to justify her procedures:

I counted the number of strips going over and under. In general, from the first strip up to seventh, there is a repetition; I mean, this [she pointed to the strip L7] goes like the first [L1]; this [L6] goes like the second [L2] and this one [L5] goes like the third [L3] (see figure 18a). (1)

Her generalisation from the observation occurred through considering the design strip by strip, enabling her to weave the design of figure 18b. She gave a clear explanation of her procedures, revealing evidence of knowing the (mathematical) ideas of sequences and series, within the context of WB-activity. The strategy used by C to find out which should be the next L-strip with the same phase, supposing that the WB has greater dimension than it does has, shows her intelligent way of using the WB, starting to count the L4 as "20" and going on until the correct answer. This strategy enabled her to respond to that question in terms of an "infinite" WB (see figure 18b).
Let us see possible interpretation of the underlined words from the transcript of C:

"Very beautiful"

She found the design of figure 17 *very beautiful* to start weaving on the WB. That expression may hide the recognition of a *symmetry*.

"In general, ... there is a repetition"

The expressions *in general, ... there is a repetition* and *...goes like* ... [see (1)] may mean that for each strips in the design there is a *translation* of its pathway.

"... this separates those which are repeated"

Rsr: Good! And what about this one? [L4] (2)

C: Oh, this separates [see doted line, figure 4.29] those which are repeated. (3)
The strip L4 serves as axis of symmetry.

"... 30th position is faraway!"

The number at the 30th position?! ... 30th position is faraway!. (14)

Considering p as "position number" the word faraway may be interpreted as great p (perhaps a $p \geq 30$, for the subject point of view).

"... add 6 to get 23 ... add 2 to get 25 ... you continue, adding "6s" and "2s"

Look, starting here [by the "17" in the number sequence (*)] you should add 6 to get 23, and from 23 you add 2 to get 25; and from 25 you add 6 to get 31; and 31 plus 2 you get 33; and so you continue, adding "6s" and "2s". (16)

By adding continuously "6s" and "2s" one is dealing with *sequences* and *series* of numbers.

An important outcome was that subjects could express mathematical concepts or processes "hidden" in twill weaving by using their own words or expressions (e.g., "good look" to indicate symmetry properties displayed by the woven patterns.) Those are particular words or expressions which actually carry within them the mathematical pre-knowledge that has to be taken as a starting point for teachers to help students to construct (formal) mathematical concepts and processes in the classroom.

More similar examples like those above can be presented. The explorations go beyond planar weaving, reaching complex three dimensional shapes. What was still kept is the weaving technique that children were more and more becoming "master", like artisans do, trying new shapes and colouring. The pictures below shows some three dimensional models produced using cardboard strips, just by application of the weaving technique, that is, there is no glue, staples or rubber bands to fix those models. One may imagine to what extent do the children get engaged in "doing mathematics" in process of producing such models! ...



Concluding remarks

At conclusion I want to focus on the role of the weaving – as an artefact from a cultural activity – in facilitating the acquisition and development of mathematical knowledge; in moving of the learner's understanding from a concrete treatment to an abstract level of "doing mathematics".

The fact that the subjects have manipulated the WB to certain extent and then continued thinking without it, at least physically (using numbers only, or reproducing woven pattern on squared paper to facilitate the reasoning) reveal the different representational systems that the learners used in attempting the least complex way to acquire and develop mathematical knowledge. Psychological analyses (see Behr, Lesh, Post, Silver, 1983:102-103) show that

> manipulative artifacts [such as the WB] are just one component in the development of a representational system [of mathematical ideas] and that the other modes of representation [pictures, spoken symbols, written symbols, and real world situations (see figure 16)] also play a role in the acquisition and use of the concepts. (p. 102)

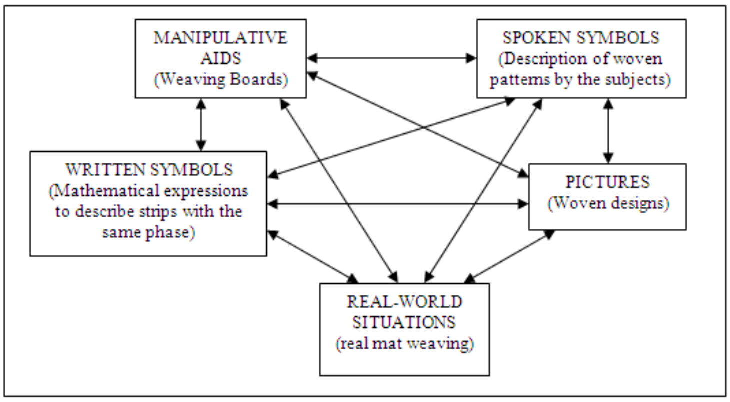


Figure 7.1: An interactive model for using representational systems

The different system of representation are interpreted as interactive rather than linear, and translations within and between modes are given as much an emphasis as manipulation of a single representational system. In this model comes in brackets the interpretation of that interaction in the case of my study.

If the "traditional" approach to teaching, for example, the topic on sequences and series is used, the interaction is mostly within the written symbols and there is little interaction between this and the spoken symbols system. This was reflected in the results obtained in the treatment phase where the subjects preferred to express their conjectures in algebraic symbols and very seldom by words. A major purpose of the research was to suggest an exploratory teaching/learning approach which will stimulates the interaction within and between all five modes of representation in our schools. This will allows teachers to recognise and explore students' cognitive skills and thus to produce different practical classroom activities to use in their teaching, and therefore, making their students feel themselves builders of the mathematics they are learning.

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