12th International Congress on Mathematical Education
Program Name XX-YY-zz (pp. abcde-fghij)
8 July – 15 July, 2012, COEX, Seoul, Korea (This part is for LOC use only. Please do not change this part.)

CURRICULUM REFORM AND MATHEMATICS LEARNING: EVIDENCE FROM TWO LONGITUDINAL STUDIES

Jinfa Cai

University of Delaware

jcai@udel.edu

Drawing on evidence from two longitudinal studies from the LieCal project, I discuss issues related to mathematics curriculum reform and student learning. The LieCal Project was designed to longitudinally investigate the impact of a reform mathematics curriculum called the Connected Mathematics Program (CMP) in the United States on teachers' teaching and students' learning. I recommend attending to three levels of curriculum—intended, implemented and attained—when searching for evidence of the impact of mathematics curricula on student learning. A variety of evidence from the LieCal Project is presented to show the impact of mathematics curriculum reform on teachers' teaching and students' learning.

Key words: Curriculum reform; Mathematics learning; Longitudinal Studies; Liecal Project, Problem Solving

MATHEMATICS EDUCATION REFORM IN THE UNITED STATES

Education is commonly seen as the key to a nation's economic growth and prosperity and to its ability to compete in the global economy. For years, the United States of America has adopted national strategies for development and reform in education with a focus on improving the quality of individual life and the competitiveness of the nation (National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Science Board, 2010; Nie, Zheng, Sun, & Cai, 2010; Ravitch, 2000). Historically across the nations, changing the curriculum has been viewed and used as an effective way to change classroom practice and to influence student learning to meet the needs of the ever-changing world (Cai & Howson, in press; Cai, Moyer, Wang, & Nie, 2011; Cai, Wang, Moyer, & Nie, 2011; Howson, Keitel, & Kilpatrick, 1981; Senk & Thompson, 2003). In fact, curriculum has been called a change agent for educational reform (Ball & Cohen, 1994; Darling-Hammond, 1993). The school mathematics curriculum remains a central issue in efforts to improve students' learning.

The curriculum plays a significant role in mathematics education because it effectively determines what students learn, when they learn it, and how well they learn it. In recent years, some reform materials have been accepted into the curriculum and some have been rejected, leading towards more commonly accepted learning goals in school mathematics

(Cai & Howson, in press). In addition to developing traditionally accepted mathematical knowledge and skills through mathematics instruction, increasing emphasis has been placed on developing students' higher-order thinking skills. Although there are no commonly accepted definitions of such skills, the frequently cited list found in Resnick (1987) might help. According to Resnick, higher-order thinking:

- Is *non algorithmic*. That is, the path of action is not fully specified in advance.
- Tends to be *complex*. The total path is not "visible" (mentally speaking) from any single vantage point.
- Often yields *multiple solutions*, each with costs and benefits, rather than unique solutions.
- Involves *nuanced judgment* and interpretation.
- Involves the application of *multiple criteria*, which sometimes conflict with one another.
- Often involves *uncertainty*; not everything that bears on the task at hand is known.
- Involves *self-regulation* of the thinking process.
- Involves *imposing meaning*, finding structure in apparent disorder.
- Is *effortful*; considerable mental work is involved in the kinds of elaborations and judgments required.

This list clearly shows that higher-order thinking skills involve the ability to think flexibly so as to make sound decisions in complex and uncertain problem situations. In addition, such skills involve monitoring one's own thinking—metacognitive skills. In particular, mathematics instruction should ideally provide students with opportunities to: (1) think about things from different points of view, (2) step back to look at things again, and (3) consciously think about what they are doing and why they are doing it. Resnick's list does not include the ability to collaborate with others, but being able to work together with others is also an essential higher-order thinking skill. Collaborative work encourages students to think together about ideas and problems as well as to challenge each other's ideas.

The desirable aim of developing such skills is related to the view that mathematics education should be seen as contributing to the intellectual development of individual students: preparing them to live as informed and functioning citizens in contemporary society, and providing them with the potential to take their places in the fields of commerce, industry, technology, and science (Robitaille & Garden, 1989). In addition, mathematics education should seek to teach students about the nature of mathematics. In this view, mathematics is no longer simply a prerequisite subject but rather a fundamental aspect of *literacy* for a citizen in contemporary society (Mathematics Sciences Education Board [MSEB], 1993; NCTM, 1989). Education in general and mathematics education in particular have the responsibility for nurturing students' creativity and critical thinking skills not only for their lifelong learning but also for their general benefit and pleasure.

In the United States, NCTM specified five goals for students in its monumental *Standards* document published in 1989: (1) learn to value mathematics, (2) learn to reason mathematically, (3) learn to communicate mathematically, (4) become confident of their mathematical abilities, and (5) become mathematical problem solvers. NCTM also specified major shifts to achieve these goals in teaching mathematics, including movement toward: (1) Classrooms as mathematical communities--away from classrooms as simply collections of individuals; (2) Logic and mathematical evidence as verification--away from the teacher as the sole authority for right answers; (3) Mathematical reasoning--away from merely memorizing procedures; (4) Conjecturing, inventing, and problem solving--away from an emphasis on mechanistic answer-finding; and (5) Connecting mathematics, its ideas, and its applications--away from treating it as a body of isolated concepts and procedures.

THE LIECAL PROJECT

With extensive support from the National Science Foundation (NSF), a number of school mathematics curricula were developed and implemented to align with the recommendations of the NCTM *Standards*. The Connected Mathematics Program (CMP) is one of the *Standards*-based middle school curricula developed with NSF funding. CMP is designed to build students' understanding of important mathematics through explorations of real-world situations and problems. It is a complete middle-school mathematics program. Students using the CMP curriculum are guided to investigate important mathematical ideas and develop robust ways of thinking as they try to make sense of and resolve problems based on real-world situations.

The research reported here was part of a large project designed to longitudinally compare the effects of a *Standards*-based curriculum (CMP) to the effects of more traditional middle school curricula on students' learning of algebra (hereafter called non-CMP curricula). In this project, *Longitudinal Investigation of the Effect of Curriculum on Algebra Learning* (LieCal)[1], we investigated not only the ways and circumstances under which these curricula could or could not enhance student learning in algebra, but also the characteristics of the curricula that led to student achievement gains (Cai, Wang, et al.; Moyer et al., 2011).

The LieCal project was conducted in 14 middle schools in an urban school district serving a diverse student population in the United States. Approximately 85% of the participants were minority students: 64% African American, 16% Hispanic, 4% Asian, and 1% Native American. Male and female students were about evenly distributed.

By longitudinally comparing the effects of the CMP curriculum on students' learning of algebra to the effects of more traditional middle-school mathematics curricula, the LieCal Project was designed to provide: (a) a profile of the intended treatment of algebra in the CMP curriculum and a contrasting profile of the intended treatment of algebra in non-CMP curricula; (b) a profile of classroom experiences that CMP students and teachers had, with a contrasting profile of experiences in non-CMP classrooms; and (c) a profile of student performance resulting from the use of the CMP curriculum, with a contrasting profile of student performance resulting from the use of non-CMP curricula. Accordingly, the project was designed to answer three research questions:

[1] In 2006 and 2009, the CMP authors published revised editions of the CMP curriculum under the name CMP2. This article is based on the original CMP curriculum because the students in the LieCal project used CMP, not CMP2.

- What are the similarities and differences between the intended treatment of algebra in the CMP curriculum and in the non-CMP curricula?
- What are key features of the CMP and non-CMP experience for students and teachers, and how might these features explain performance differences between CMP and non-CMP students?
- What are the similarities and differences in performance between CMP students and a comparable group of non-CMP students on tasks measuring a broad spectrum of mathematical thinking and reasoning skills, with a focus on algebra?

In previous years of the LieCal Project, we compared the performance of middle school students in classrooms that used CMP with the performance of students in classrooms that used non-CMP curricula. Currently, we are following the same cohort of middle school students during their four high school years to investigate how the use of different types of middle school mathematics curricula affects the learning of high school mathematics in the same urban school district. More importantly, we are examining how students' curricular experiences in the middle grades affect their algebra learning in high school by providing empirical evidence about the relationships between the development of conceptual understanding, symbol manipulation skills, and problem-solving skills in middle school and the learning of mathematics in high school.

The findings from the LieCal-Middle School Project (Cai, Wang, et al., 2011) exhibit parallels to the findings from research on the effectiveness of Problem-Based Learning (PBL) on the performance of medical students (Barrows, 2000; Hmelo-Silver, 2004; Norman & Schmidt, 1992; Vernon & Blake, 1993). Researchers found that medical students trained using a PBL approach performed better than non-PBL students (trained, for example, using a lecture approach) on clinical components in which conceptual understanding and problem-solving ability were assessed. However, PBL and non-PBL students performed similarly on measures of factual knowledge. When these same medical students were assessed again at a later time, the PBL students not only performed better than the non-PBL students on clinical components, but also on measures of factual knowledge (Norman & Schmidt, 1992; Vernon & Blake, 1993). This result may imply that the conceptual understanding and problem-solving abilities learned in the context of PBL facilitated the retention and acquisition of factual knowledge over longer time intervals. As we described above, the CMP curriculum can be characterized as a problem-based curriculum. Analogous to the results of research on PBL in medical education, in the LieCal Project, CMP students outperformed non-CMP students on measures of conceptual understanding and problem solving during middle school. In addition, CMP and non-CMP students performed similarly on measures of computation and equation solving. Continuing the analogy, it is reasonable to hypothesize that the superior conceptual understanding and problem-solving abilities gained by CMP students in middle school could result in better performance on a delayed assessment of manipulation skills such as equation solving, in addition to better performance on tasks assessing conceptual understanding and problem solving in high school. We are currently testing this hypothesis as we follow the LieCal cohort through their high school years.

THREE LEVELS OF CURRICULUM

In the LieCal Project, we made use of a three-level conceptualization of curriculum (intended, implemented, and attained) which has been widely accepted in mathematics education (Cai, 2010). The intended curriculum refers to the formal documents that set system-level expectations for the learning of mathematics. These usually include goals and expectations set for the educational system along with textbooks, official syllabi, and/or curriculum standards. The implemented curriculum refers to school and classroom processes for teaching and learning mathematics as interpreted and implemented by the teachers, according to their experience and beliefs for particular classes. Thus, the implemented curriculum deals with the classroom level. The classroom is central to students' learning since students acquire most of their knowledge and form their attitudes from classroom instruction (Robitaille & Garden, 1989). Regardless of how well a curriculum is designed, it has little value outside of its implementation in classrooms. Finally, the attained curriculum refers to what is learned by students and is manifested in their achievements and attitudes. It exists at the level of the student, and deals with the aspects of the intended curriculum that are taught by teachers and actually learned by students.

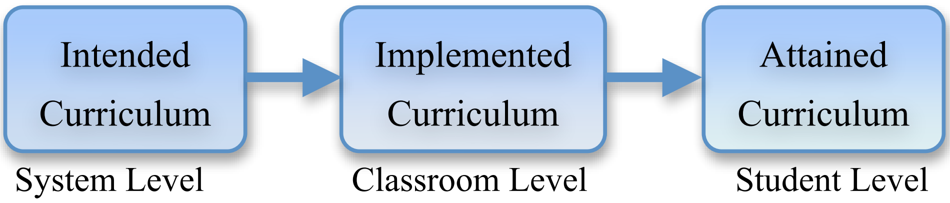

Figure 1. The conceptualization of the three levels of curriculum

As shown in Figure 1 above, conceptualization of the three levels of curriculum is quite useful for comparative studies of mathematics curriculum. It highlights the differences between what a society would like to have taught, what is actually taught, and what students have actually learned. At the same time, all three levels are related to each other, and each one supports the others in the evaluation process. In the following sections, I will specifically discuss the issues and methods of studying mathematics curricula. In this discussion, I will draw examples from the LieCal Project to discuss the theoretical and methodological issues that arise in each of these three levels.

Intended Curriculum

The intended curriculum specifies goals, topics, sequences, instructional activities, and assessment methods and instruments. The most common method of evaluating an intended curriculum is content analysis, which involves judging the quality of the content of a curriculum and the quality of its presentation. The National Research Council (2004) has proposed a list of factors to consider when conducting content analysis to evaluate the intended curriculum (see Table 1). When conducting comparative studies of curricula, we may focus on one or more factors, depending on the specific purpose of the study.

Table 1: Factors to consider in content analysis of mathematics materials (*Adapted from NRC (2004), p. 42.*)

Listing of topics
Sequence of topics
Clarity, accuracy, and appropriateness of topic presentation
Frequency, duration, pace, depth, and emphasis of topics
Grade level of introduction
Overall structure: integrated, interdisciplinary, or sequential
Types of tasks and activities, purposes, and level of engagement
Use of prior knowledge, attention to (mis)conceptions, and student strategies
Reading level
Focus on conceptual ideas and algorithmic fluency
Emphasis on analytic/symbolic, visual, or numeric approaches
Types and levels of reasoning, communication, and reflection
Type and use of explanation
Form of practice
Approach to formalization
Use of contextual problems and/or elements of quantitative literacy
Use of technology or manipulatives
Ways to respond to individual differences and grouping practices
Formats of materials
Types of assessment and relation to classroom practice

In the LieCal Project, we first searched for evidence of the impact of reform by conducting a detailed analysis of the intended curriculum. If a curriculum is to be considered a reform curriculum, it must have conceptualizations and features which distinguish it from the traditional curricula. I highlight two sets of findings from the LieCal Project that identify such distinguishing characteristics at the level of the intended curriculum: (1) the introduction of mathematical concepts and (2) the analysis of mathematical problems.

Introduction of Mathematical Concepts. A common approach in curricular comparisons is to examine how a mathematical concept is introduced in various curricula (Cai et al., 2002). In the LieCal Project, we conducted detailed analyses of the introduction of key mathematical concepts in the CMP and non-CMP curricula and found significant differences between them (Cai, Nie, & Moyer, 2010; Nie, Cai, & Moyer, 2009; Moyer, Cai, & Nie, 2012). Overall, our research revealed that the CMP curriculum takes a *functional approach* to the introduction of algebraic concepts in the teaching of algebra, whereas the non-CMP curricula take a *structural approach*. The functional approach emphasizes the important ideas of change and variation in situations. It also emphasizes the representation of relationships between variables. In contrast, the structural approach avoids contextual problems in order to concentrate on developing the abilities to generalize, work abstractly with symbols, and follow procedures in a systematic way (Cai et al., 2010). In this section, we highlight specific differences in the ways that the CMP curriculum and the non-CMP curricula define and introduce variables, equations, equation solving, and functions.

Defining and introducing the concept of variables. Because of the importance of variables in algebra, and in order to appreciate the differences between the CMP and non-CMP curricula, it is necessary to understand how the CMP and non-CMP curricula introduce variable ideas (Nie et al., 2009). The learning goals of the CMP curriculum characterize variables as quantities used to represent relationships. Though the CMP curriculum does not formally define variable until 7^{th} grade, CMP's informal characterization of a variable as a quantity that changes or varies makes it convenient to use variables informally to describe relationships long before formally introducing the concept of variables in 7^{th} grade. The choice to define variables in terms of quantities and relationships reflects the functional approach that the CMP curriculum takes.

In contrast, the learning goals in the non-CMP curriculum characterize variables as placeholders or unknowns. The non-CMP curriculum formally defines a variable in 6^{th} grade as a symbol (or letter) used to represent a number. It treats variables predominantly as placeholders by using them to represent unknowns in expressions and equations. By introducing the concept of variables in this fashion, the non-CMP curriculum supports its structural approach to algebra.

Defining and introducing the concept of equations. Given the functional approach to variables in the CMP curriculum and the structural approach in the non-CMP curriculum, it is not surprising that the concept of equation is similarly defined functionally in CMP, but structurally in the non-CMP curriculum. In CMP, equations are a natural extension of the development of the concept of variable as a changeable quantity used to represent relationships. At first, CMP expresses relationships between variables with graphs and tables of real-world quantities rather than with algebraic equations. Later, when CMP introduces equations, the emphasis is on using them to describe real-world situations. Rather than seeing equations simply as objects to manipulate, students learn that equations often describe relationships between varying quantities (variables) that arise from meaningful, contextualized situations (Bednarz et al., 1996). In the non-CMP curriculum, the definition of a variable as a symbol develops naturally into the use of context-free equations and places the emphasis on procedures for solving equations. These are all hallmarks of a structural focus. For example, the non-CMP curriculum defines an equation as "…a sentence that contains an equals sign, =" illustrated by examples such as $2+x=9, 4=k-6$, and $5-m=4$. Students are told that the way to solve an equation is to replace the variable with a value that results in a true sentence.

Defining and introducing equation solving. The CMP and non-CMP curricula use functional and structural approaches, respectively, to introduce equation solving, consistent with their approaches to defining equations. In the CMP curriculum, equation solving is introduced within the context of discussing linear relationships. The initial treatment of equation solving does not involve symbolic manipulation, as found in most traditional curricula. Instead, the CMP curriculum introduces students to linear equation solving by using a graph to make visual sense of what it means to find a solution. Its premise is that a linear equation in one variable is, in essence, a specific instance of a corresponding linear relationship in two variables. It relies heavily on the context in which the equation itself is situated and on the use of a graphing calculator.

After CMP introduces equation solving graphically, the symbolic method of solving linear equations is finally broached. It is introduced within a single contextualized example, where each of the steps in the equation-solving process is accompanied by a narrative that demonstrates the connection between what is happening in the procedure and in the real-life situation. In this way, CMP justifies the equation-solving manipulations through contextual sense-making of the symbolic method. That is, CMP uses real-life contexts to help students understand the meaning of each step of the symbolic method of equation solving, including why inverse operations are used. As with the introduction of variables and equations, CMP's functional approach to equation solving maintains a focus on contextualized relationships among quantities.

In the non-CMP curriculum, contextual sense-making is not used to justify the equation-solving steps as it is in the CMP curriculum. Rather, the non-CMP curriculum first introduces equation solving as the process of finding a number to make an equation a true statement. Specifically, *solving* an equation is described as replacing a variable with a value (called the *solution*) that makes the sentence true. Equation solving is introduced in the non-CMP curriculum symbolically by using the additive property of equality (equality is maintained if the same quantity is added to or subtracted from both sides of an equation) and the multiplicative property of equality (equality is maintained if the same non-zero quantity is multiplied by or divided into both sides of an equation). This approach to equation solving is aligned with the non-CMP curriculum's structural focus on working abstractly with symbols and procedures.

Defining and introducing functions. Consistent with their approaches to variables and equations, the CMP and non-CMP curricula once again use functional and structural approaches, respectively, to introduce the concept of functions. Their respective approaches can be seen quite clearly in the differences between their stated learning goals for the function concept. CMP's learning goals for students are (1) that they be able to understand and predict patterns of change in variables, and (2) that they be able to represent relationships between real-world quantities using word descriptions, tables, graphs, and equations. In contrast, the non-CMP curriculum's stated learning goals are (1) that students explore the use of algebraic equations to represent functions, and (2) that they be able to identify and graph functions, calculate slope, and distinguish linear from nonlinear functions.

The CMP curriculum informally introduces the concepts of function and variable at the same time in 6^{th} grade, identifying a function as a relationship between real-world quantities (variables). At the beginning of 7^{th} grade, when the concept of variable is formally introduced in the *Variables and Patterns* unit, coordinate graphs are used as a way to tell a story of how changes in one variable are related to changes in another. In an introductory investigation, students graph how many jumping jacks they can do in successive 10-second intervals for two minutes. Then they analyze the graph to determine whether a relationship exists between time and the number of jumping jacks. At the same time, students are exposed to the concepts of independent variable and dependent variable. This occurs well before the concept of function is formally introduced during the second half of 7th grade in the *Moving Straight Ahead* unit. Although the concept of function is introduced in this unit, the term "relationship" is almost always used instead of the word "function." Furthermore, in the teacher's guide, the term "function" is explicitly identified as nonessential. In fact, the term "function" is not given any importance in the CMP curriculum until the introduction of quadratic functions in the 8th grade unit *Frogs, Fleas, and Painted Cubes*.

The non-CMP curriculum informally introduces the concept of function in the preview to Lesson 9-6 in 6th grade by having students make a function machine out of paper. The function machine has three key elements: input, output, and operation. The operation, or rule, lies at the core of the function machine, while input and output are external to it. Immediately after the introduction of the function machine, the non-CMP curriculum formally introduces the concepts of function, function table, and function rule in Lesson 9-6. This formal introduction begins with the following situation: "A brown bat can eat 600 mosquitoes an hour." The student is then asked to write expressions to represent the number of mosquitoes a brown bat can eat in 2 hours, 5 hours, and *t* hours. Finally, the terms function and function table are illustrated, and the term function rule is defined. The function rule is characterized as a rule giving the operation(s) that will transform an input into an output. The non-CMP curriculum defines a function as a relationship where one thing depends on another. However, it treats a function as a process of starting with an input number, performing one or more operations on it, and getting an output number. The main purpose of the function machine and the function table seems to be for students to experience the process of computing the output values from given input values and vice versa. That is, the development of the concept of function in the non-CMP curriculum emphasizes operations on input variables rather than the relationship between two variables.

Analysis of Mathematical Problems. Comparative studies of intended curricula must take into account the quality of activities, their use in instruction, and their frequency of use. Indeed, a number of researchers have analyzed problems and worked examples in mathematics curricula (e.g., Cai et al., 2002, 2010; Fan & Zhu, 2007; Li, 2000). In the LieCal Project, we compared both the types of mathematical problems involving linear equations in the CMP and non-CMP curricula and the level of cognitive demand of the problems in the two curricula.

Types of Problems Involving Linear Equations. In both the CMP and non-CMP curricula, the vast majority of the equation problems involved linear equations. Thus we further classified problems involving linear equations in the CMP and non-CMP curricula into three categories:

- One equation with one variable (1eq1va)--e.g., $2x + 3 = 5$;
- One equation with two variables (1eq2va)--e.g., $y = 6x + 7$;
- Two equations with two variables (2eq2va)--e.g., the system of equations $y = 2x + 1$ and $y= 8x + 9$.

Table 2 shows the percentage distribution of these categories of problems involving linear equations in each of the two curricula. The two distributions are significantly different (χ^2 (2) = 1262.0, p< .0001). The CMP curriculum includes a significantly greater percentage of one equation with two variables problems than the non-CMP curriculum (z = 35.49, p < .0001). However, the non-CMP curriculum includes a significantly greater percentage of one equation with one variable problems than the CMP curriculum (z = 34.15, p < .0001). These results resonate with the findings reported above. Namely, the CMP curriculum emphasizes an understanding of the relationships between the variables of equations, rather than an acquisition of the skills needed to solve them. In fact, of the 402 equation-related problems in the CMP curriculum, only 33 of them (about 8% of the linear equation-solving problems) involve decontexualized symbolic manipulations of equations. However, the non-CPM curriculum includes 1,550 problems involving decontexualized symbolic manipulations of equations (nearly 70% of the linear equation solving problems in the non-CMP curriculum).

Table 2: Percentage distribution of problems involving linear equations in the CMP and non-CMP curricula

	1eq1va	1eq2va	2eq2va
CMP (n=402)	5.72	93.03	1.24
Non-CMP (n=2,339)	86.19	11.67	2.14

The non-CMP curriculum not only incorporates many more linear equation-solving problems into the curriculum, but it also carefully sequences them based on the number of steps required to solve them. Of the 2,339 problems involving linear equations, over 50% are one-step problems like, $x + b = c$, $ax = c$ or $x = a * b$. About 30% of the problems are two-step problems, like $ax + b = c$ or $x/a = b/c$. Only a small fraction of the linear equations involve three steps or more, like $ax + bx + c = d$ or $ax + b = cx + d$. Each grade level of the non-CMP curriculum includes one-step, two-step, and three-plus-step problems involving linear equations. As the grade level increases, however, the curriculum provides increasingly more comprehensive procedures, suitable for solving all forms of linear equations.

Cognitive Demand of Mathematical Problems. If an intended curriculum claims to be problem-based, we should expect to see it contain a large proportion of cognitively demanding tasks. In the LieCal Project, we analyzed the cognitive demand of mathematical problems in both the CMP and non-CMP curricula (Cai et al., 2010). We classified the problems into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics (Stein & Lane, 1996). As Figure 2 illustrates, the CMP curriculum had significantly more high-level tasks (procedures with connections or doing mathematics) (χ^2(3, N = 3311) = 759.52, p < .0001) than the non-CMP curricula. This kind of analysis of the intended curriculum provides insight into the degree to which different curricula expect students to engage in higher-level thinking and problem solving.

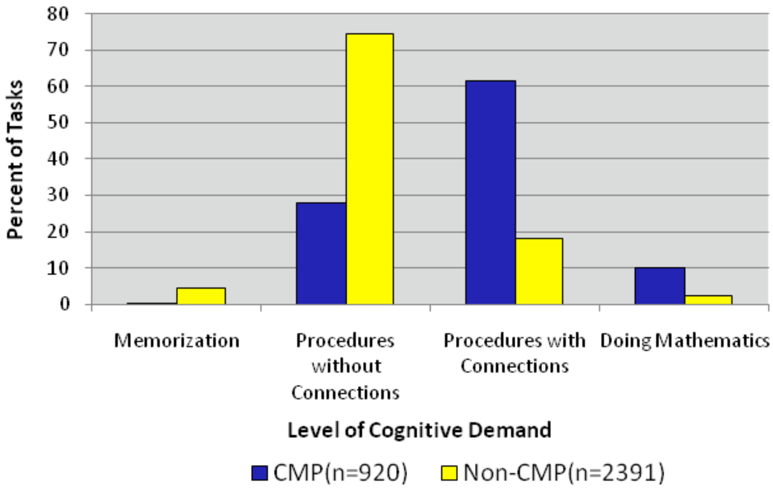

Figure 2. Percentages of various types of tasks in CMP and non-CMP curricula

Implemented Curriculum

The implemented curriculum is concerned with *what* mathematics is actually taught in the classroom and *how* that mathematics is taught. Therefore, a key issue for the implemented curriculum is the recognition that what teachers teach may or may not be consistent with the intended curriculum. When the implemented curriculum, as seen in teachers' instruction, is congruent with the goals of the intended curriculum, we may say that there is fidelity of implementation. Teachers may vary widely in their commitment to the intended curriculum. Therefore when evaluating the implemented curriculum, it is important to determine whether, how, and to what extent teachers' instruction is influenced by the intended curriculum.

In the LieCal Project, we collected data on multiple aspects of implementation. We conducted 620 detailed lesson observations of CMP and non-CMP lessons over a three-year period. Approximately half of the observations were of teachers using the CMP curriculum, while the other half were observations of teachers using non-CMP curricula. Two retired mathematics teachers conducted and coded all the observations. The observers received extensive training that included frequent checks for reliability and validity throughout the three years (Moyer et al., 2011).

Each class of LieCal students was observed four times, during two consecutive lessons in the fall and two in the spring. The observers recorded extensive information about each lesson using a 28-page project-developed observation instrument. During each observation, the observer made a minute-by-minute record of the lesson on a specially designed form. This record was used later to code the lesson. The coding system had three main components: (1) the structure of the lesson and use of materials, (2) the nature of the instruction, and (3) the analysis of the mathematical tasks used in the lesson.

The analyses of the data we obtained from the classroom observations revealed striking differences between classroom instruction using the CMP and non-CMP curricula. In this paper, we briefly discuss the differences related to three important instructional variables: (1) the level of conceptual and procedural emphases in the lessons, (2) the cognitive level of the instructional tasks implemented, and (3) the cognitive level of the homework problems.

Conceptual and Procedural Emphases. The second component of the coding section included twenty-one 5-point Likert scale questions that the observers used to rate the nature of instruction in a lesson. Of the 21 questions, four of them were designed to assess the extent to which a teacher's lesson had a conceptual emphasis. Another four questions were designed to determine the extent to which the lesson had a procedural emphasis. Factor analysis of the LieCal observation data confirmed that the four procedural-emphasis questions loaded on a single factor, as did the four conceptual-emphasis questions.

There was a significant difference across grade levels between the levels of conceptual emphasis in CMP and non-CMP instruction ($F = 53.43$, $p < 0.001$). The overall (grades 6-8) mean of the summated ratings of conceptual emphasis in CMP classrooms was 13.41, whereas the overall mean of the summated ratings of conceptual emphasis in non-CMP classrooms was 10.06. The summated ratings of conceptual emphasis were obtained by adding the ratings on the four items of the conceptual-emphasis factor in the classroom observation instrument, which implies that the mean rating on the conceptual-emphasis items was 3.35 (13.41/4) for CMP instruction and 2.52 (10.06/4) for non-CMP instruction. That is, CMP instruction was rated 0.40 points above the midpoint, whereas non-CMP instruction was rated 0.5 points below the midpoint. Thus, on average, CMP instruction was rated about 4/5 of a point higher (out of 5) on each conceptual emphasis item than non-CMP instruction, which was a significant difference ($t = 11.44$, $p < 0.001$).

In contrast, non-CMP lessons had significantly more emphasis on the procedural aspects of learning than the CMP lessons. The procedural-emphasis ratings for the non-CMP lessons were significantly higher than the procedural-emphasis ratings for the CMP lessons ($F = 37.77$, $p < 0.001$). Also, the overall (grades 6-8) mean of summated ratings of procedural emphasis in non-CMP classrooms (14.49) was significantly greater than the overall mean of the summated ratings of procedural emphasis in CMP classrooms, which was 11.61 ($t = -9.43$, $p < 0.001$). The summated ratings of procedural emphasis were obtained by adding the ratings on the four items of the procedural-emphasis factor, which implies that the mean rating on the procedural emphasis items was 3.62 (14.49/4) for non-CMP instruction and 2.91 (11.61/4) for non-CMP instruction. On average, non-CMP instruction was rated about 7/10 of a point higher (out of 5) on each procedural emphasis item than CMP instruction, which was a significant difference.

Instructional tasks. As we did with the mathematical problems in the intended curricula, we again used the scheme developed by Stein et al. (1996) to classify the instructional tasks actually used in the CMP and non-CMP classrooms into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics. Figure 3 shows the percentage distributions of the cognitive demand of the instructional tasks implemented in CMP and non-CMP classrooms (note that Figure 2 referred to problems from the intended, not the implemented, curricula). The percentage distributions in CMP and non-CMP classrooms are significantly different ($X^2(3, N = 1318) = 219.45$, $p < .0001$). The difference confirms that a larger percentage of high cognitive demand tasks (procedures with connection or doing mathematics) were implemented in CMP classrooms than were implemented in non-CMP classrooms ($z = 14.12$, $p < .001$). On the other hand, a larger percentage of low cognitive demand tasks (procedures without connection or memorization) were implemented in non-CMP classrooms than were implemented in CMP classrooms. Additional analyses showed that, not only did CMP teachers implement a significantly higher percentage of cognitively demanding tasks than non-CMP teachers across the three grades, but also within each grade (z values range from 6.06 – 11.28 across the three grade levels, $p < .001$).

Over 45% of the CMP lessons implemented at least one high level task (involving either procedures with connections or doing mathematics), but only 10% of the non-CMP lessons did so ($z = 14.12$, $p < .0001$). Nearly 90% of the non-CMP lessons implemented low-level tasks involving procedures without connections, whereas only 55% of the CMP lessons did so ($z = 14.12$, $p < .0001$)..

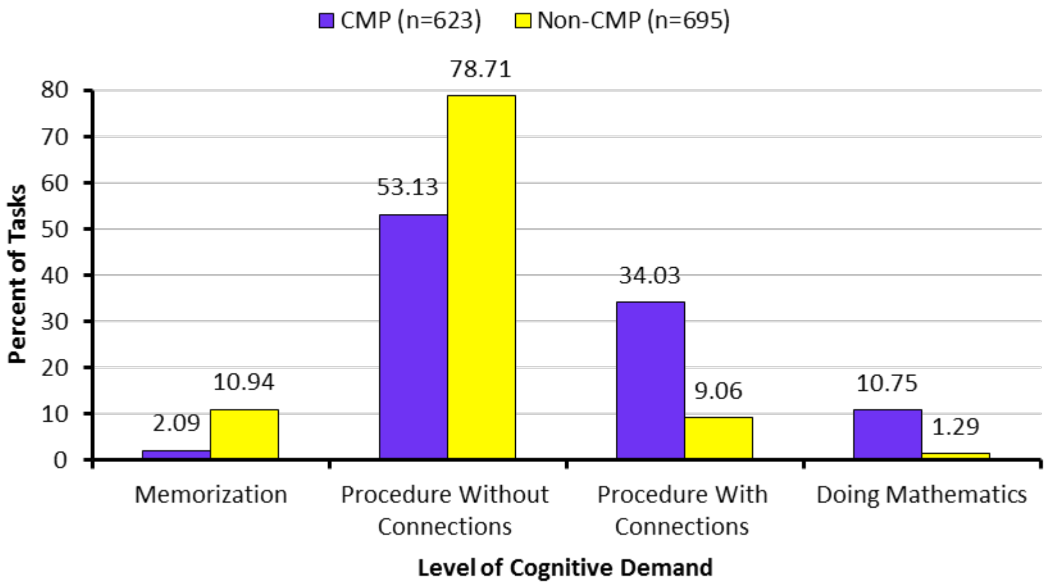

Figure 3. The percentage distributions of the cognitive demand of the instructional tasks implemented in CMP and non-CMP classrooms

Homework Problems. Each of the participating teachers was asked to keep logs and submit all of their assigned homework problems as part of the logs. The analysis of homework problems was based on all of the homework problems recorded in the logs of the CMP and non-CMP teachers. Each homework problem was coded in terms of its source, contexts, representations, and cognitive demand.

We randomly sampled half of the homework problems in each grade. A total of 10,310 of the homework problems assigned by middle school teachers during the three years were included in the analysis. Most of the homework problems (about 90%) came from the respective textbooks for each curriculum; only a small proportion of the assigned homework problems (about 10%) was supplemented by teachers. Overall, the profile of representations used in CMP homework problems was significantly different from the profile of representations used in non-CMP homework problems ($\chi^2(1, N = 10310) = 34.95, p < 0.0001$). Of note, a larger percentage of non-CMP homework problems (39%) than CMP homework problems (20%) involved symbolic representations ($z = 19.90, p < 0.0001$). In contrast, a larger percentage of CMP problems (45%) than non-CMP problems (22%) involved a table, picture or graph ($z = 24.49, p < 0.0001$). However, nearly all homework problems, CMP or non-CMP, involved written words (97.7% of the non-CMP problems and 99.8% of the CMP problems).

We examined the contexts of the homework problems using the following categories: no context, context without tables or pictures, context with tables and pictures, and context with manipulatives. Overall, the distributions of homework problem contexts for CMP and non-CMP students were significantly different ($\chi^2(3, N = 10310) = 431.43, p < 0.0001$). Non-CMP teachers assigned a larger percentage of homework problems without contexts than CMP teachers (56% and 37%, respectively) ($z = 18.30, p < 0.0001$). CMP students were assigned a larger percentage of homework problems involving contexts with tables or pictures than non-CMP students (39% and 22%, respectively) ($z = 18.92, p < 0.0001$). In both the CMP and non-CMP groups, about one quarter of the homework problems involved contexts without tables or pictures. There were very few homework problems in either group with contexts involving manipulatives.

Our analysis of the cognitive demand of the homework problems produced similar results to the instructional tasks. The levels of cognitive demand in the CMP and non-CMP homework problems were significantly different ($\chi^2(3, N = 10310) = 793.08, p < .0001$). A larger percentage of CMP homework problems (29%) than non-CMP homework problems (9%) were high cognitive demand problems (procedures with connections or doing mathematics) ($z = 26.08, p < 0.0001$). However, a larger percentage of non-CMP homework problems (91%) than CMP homework problems (71%) were low cognitive demand problems (memorization or procedures without connections) ($z = 26.08, p < 0.0001$).

Attained Curriculum

The ultimate goal of educational research, curriculum development, and instructional improvement is to enhance student learning. Thus the evaluation of a mathematics curriculum at the student level—evaluation of the attained curriculum—is of critical importance. In studies of the attained curriculum, we must address multiple facets of mathematical thinking (Cai, 1995; Sternberg & Ben-Zeev, 1996). Therefore, mixed methods such as observing students doing mathematics, performing tasks, and taking tests, should be used to collect information to evaluate the attained curriculum. Special attention must be paid to the selection of assessment tasks and methods of analysis when conducting comparative studies of attained curricula.

Assessment Tasks. Even though various methods can be used to measure students' learning, the heart of measuring mathematical performance is the set of tasks on which students' learning is to be evaluated (National Research Council, 2001). It is desirable to use various types of assessment tasks, thereby measuring different facets of mathematical thinking. For example, different formats of assessment tasks (such as multiple-choice and open-ended tasks) may be used to measure students' learning. Multiple-choice tasks have many advantages. For example, more items can be administered within a given time period, and scoring can be done quickly and reliably. However, it can be difficult to infer students' cognitive processes based on their responses to such items. To that end, open-ended tasks may be used to supplement multiple-choice tasks. In open-ended tasks, students are asked to produce answers, but also to show their solution processes and provide justifications for their answers. In this way, open-ended tasks provide a better window into the thinking and reasoning processes involved in students' mathematics learning. Of course, a disadvantage of open-ended tasks is that only a small number of these tasks can be administered within a given period of time. Also, grading students' responses is labor-intensive. To help overcome the disadvantages of using open-ended tasks, we recommend using a matrix design with samples of students' responses to the administered open-ended tasks. This reduces both testing time and grading time while maintaining a good overall estimate of students' learning of mathematics.

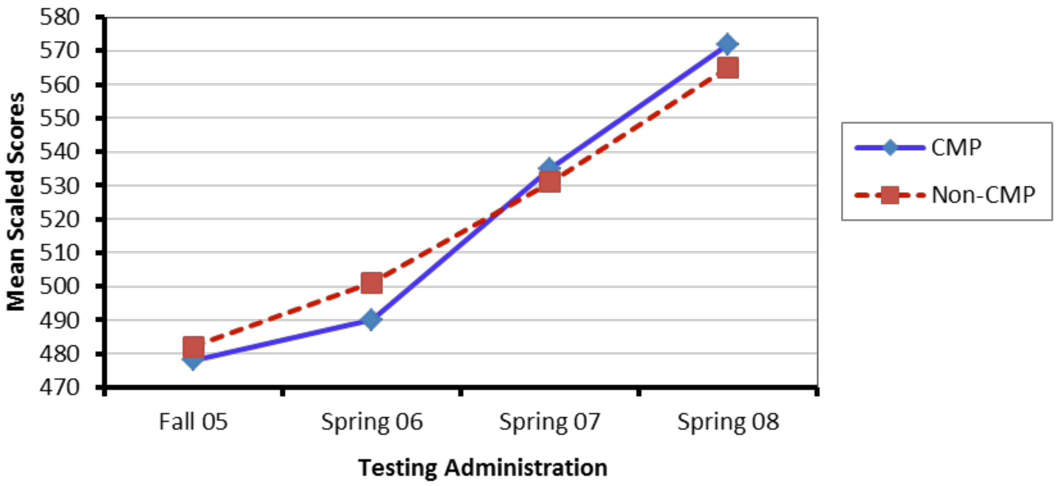

Figure 4. Mean for CMP and non-CMP middle school students on the open-ended tasks.

In the LieCal Project, we used both multiple-choice tasks and open-ended problems to assess student learning. On the open-ended tasks, which assessed conceptual understanding and problem solving, the growth rate for CMP students over the three years was significantly greater than that for non-CMP students (Cai, Wang, et al., 2011). Figure 4 shows the mean scores for CMP and non-CMP students on the open-ended tasks. In particular, our analysis using Growth Curve Modeling showed that over the three middle school years the CMP students' scores on the open-ended tasks increased significantly more than the non-CMP students' scores ($t = 2.79$, $p < .01$). CMP students had an average annual gain of 25.09 scale points whereas non-CMP students had an average annual gain of 19.39.

An additional analysis using Growth Curve Modeling showed that the CMP students' growth rate remained significantly higher than non-CMP students on open-ended tasks even when students' ethnicity was controlled (t = 3.61, p < .01). Moreover, CMP and non-CMP students showed similar growth over the three middle school years on the multiple-choice tasks assessing computation and equation solving skills.

These findings suggest that, regardless of ethnicity, the use of the CMP curriculum was associated with a significantly greater gain in conceptual understanding and problem solving than was associated with the use of the non-CMP curricula. However, those relatively greater conceptual gains did not come at the cost of basic skills, as evidenced by the comparable results attained by CMP and non-CMP students on the computation and equation solving tasks. Thus, by using both multiple-choice and open-ended assessment tasks in the LieCal Project, we were able to obtain a more comprehensive comparison of the attained CMP and non-CMP curricula.

Performance Beyond the Middle School. In the 2008-2009 academic year, the CMP and non-CMP LieCal middle school students entered high school as 9th graders. We followed about 1,000 of these students who were enrolled in 10 high schools in the same urban school district. In these high schools, the CMP and non-CMP students were mixed together in the same mathematics classrooms and used the same curriculum.

As we noted above, the results of the LieCal middle school project presented parallels to the results of research on the learning of medical students using the PBL approach. CMP students outperformed non-CMP students on measures of conceptual understanding and problem solving during middle school. In addition, CMP and non-CMP students performed similarly on measures of computation and equation solving. Thus, we hypothesized that the superior conceptual understanding and problem-solving abilities gained by CMP students in middle school could result in better performance on a delayed assessment of manipulation skills, such as equation solving, in addition to better performance on tasks assessing conceptual understanding and problem solving in high school. We therefore examined several student learning outcome measures to examine the impact of middle school curriculum on students' learning in high school. In general, all of the student learning outcome measures, CMP students performed better than or as well as non-CMP students in high school. Here, we present evidence from three outcome measures.

Ninth grade achievement. An analysis of covariance (with middle school achievement as the covariate showed that 9th graders who used CMP in middle school performed as well as or significantly better than 9th graders who used non-CMP curricula in middle school (F = 4.69, p < .05) on open-ended tasks in the district assessment.

Tenth Grade State Math Test. A series of analyses of covariance were conducted using the 10th grade state math test scaled score as the dependent variable along with various covariates. We found that CMP students had significantly higher 10th grade scaled scores than the non-CMP students, regardless of covariate, as shown in Table 3.

Table 3: Analysis of co-variance on 10th grade state math scaled score

Covariate	F-Value	Significant Level
PI-Developed 6th Grade Multiple-Choice (MC) Tasks	5.13	< .05
PI-Developed 6th Grade Open-ended (OE) Tasks	3.90	< .05
Both PI Developed 6th Grade MC and OE tasks	7.76	< .01
6th grade State math scaled score	9.58	< .01
7th grade State math scaled score	9.57	< .01
8th grade State math scaled score	11.79	< .001

Eleventh Grade Problem Posing. In the 11th grade, we used 13 open-ended tasks to measure students' conceptual understanding and problem solving. Two of these tasks were problem-posing tasks, where students were given problem situations and were required to pose mathematical problems based on the situation. We divided the students into thirds based on their performance on baseline exam tasks taken in the 6th grade. We then compared the performance of the CMP and non-CMP students in each third on the 11th grade problem posing tasks. When comparing the problem posing performance of the CMP students in each third to the non-CMP students in the same third, the CMP students performed as well as or better than their non-CMP counterparts. For example, when grouped into thirds using the baseline equation solving scores, the CMP students in the top third were more likely ($z = 2.01$, $p < .05$) to generate a problem situation that matched at least one of the graph conditions (slope and intercept). Similarly, the CMP students in the top third were more likely to generate a problem situation that reflected the linearity of the graph ($z = 2.40$, $p < .05$).

CONCLUSION

Curriculum reform is often seen as holding great promise for the improvement of mathematics teaching and learning. However, the realization of that promise requires careful attention to the different levels on which curriculum exists and functions. In the LieCal project, we have analyzed the nature and impact of the intended, implemented, and attained levels of the Connected Mathematics Program curriculum as compared to more traditional middle-school mathematics curricula. Our goals have been correspondingly threefold: to characterize the intended treatment of algebra in the CMP curriculum and identify how it is different from the intended treatment of algebra in non-CMP curricula; to understand how the intentions of the CMP and non-CMP curricula are implemented and embodied in the classroom experiences of students and teachers; and to understand how these distinct experiences may translate into different levels of student attainment.

With respect to the intended curriculum, CMP paints a distinctly different picture from traditional curricula of what middle school mathematics, and particularly the learning of algebraic concepts, should be. The stated goals and their embodiment in texts and mathematical problems indicate that the CMP curriculum intends for students to take a functional approach to algebra, focusing on the understanding relationships between quantities in contextualized, real-life problems. This stands in contrast to a more traditional, structural approach to algebra that puts the focus on decontextualized operations and procedures with symbols and mathematical objects. These two approaches to the learning of algebra are evident in the ways that the two types of curricula introduce concepts such as variables and equations and in the kinds of problems the curricula provide.

The implementations we have observed of the CMP and non-CMP curricula strongly reflect the intentions embedded in the curriculum materials. As teachers take each curriculum and shape it into actual instruction in their classrooms, the underlying functional and structural approaches continue to be evident in the choices that teachers make in balancing the conceptual and procedural aspects of the mathematics. In addition, the types of instructional tasks that teachers choose to use and the homework problems they assign to their students further illustrate that the implementation of the CMP curriculum looks very different from the implementation of the non-CMP curricula.

Finally, when we look to the results of the implementation of the CMP and non-CMP curricula in terms of student attainment, we see that CMP students experience greater growth in their conceptual understanding and problem-solving abilities than their non-CMP counterparts without having to sacrifice procedural skills. It would appear that the intentions that guided the development of the CMP curriculum materials, combined with classroom implementations that reflect those intentions, are associated with student learning along the intended lines. In other words, students using the CMP curriculum experience instruction that emphasizes a conceptual understanding of algebra as a way to represent and solve problems involving relationships among quantities, and they learn accordingly. In addition, they continue to develop procedural skill on par with other students, even when those students use more traditional curricula that have very different intentions and implementations. The advantages of CMP students continue as they enter high school. In fact, in various learning outcome measures, CMP students performed better than or equally well as the non-CMP students.

Thus, the three-level construct of curriculum we have used to examine the CMP curriculum affords us a powerful mechanism for understanding how curriculum reform can have an impact. In addition, we suggest that this conceptualization of curriculum provides a fruitful structure for curricular comparisons, both within and across nations. Indeed, as cross-national comparisons of curricula continue to be conducted, it is important to recognize and remember the relationships between the levels. Understanding how curriculum can be used to improve student learning requires an understanding of the goals of the curriculum and their embodiment in instruction.

Acknowledgments

Research reported in this paper has been supported by grants from the National Science Foundation (ESI-0454739 and DRL-1008536). Any opinions expressed herein are those of the author and do not necessarily represent the views of the National Science Foundation. This research was conducted in collaboration with John Moyer, Ning Wang, and Bikai Nie. We are very grateful for the support from a number of researchers and research assistants, including, Maria Alyson,Pat

Bolter, Darlene Boyle, Carole Bryan, Janis Freckmann, Tony Freedman, Tammy Garber, Joan Grampp, Yuichi Handa, Patrick Hopfensperger, Stephen Hwang, Connie Laughlin, Victorial Robison, Mark Roche, Maxwell Scheinfield, Chelsey Schwander,Carly Toth, Matt Wells, Courtney White, Libby Wenner, and Yue Zeng.

References

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is—or might be—the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher, 25(9)*, 6-8, 14.

Barrows, H. S. (2000). *Problem-based learning applied to medical education*. Springfield, IL: Southern Illinois University School of Medicine.

Bednarz, N., Kieran, C., & Lee, L. (Eds.). (1996). *Approaches to algebra: Perspectives for research and teaching*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cai, J. (1995). A cognitive analysis of US and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. *Journal for Research in Mathematics Education Monographs Series,* 7, Reston, VA: National Council of Teachers of Mathematics.

Cai, J. (2010). Evaluation of mathematics education programs. In P. Peterson, E. Baker, & B. McGraw (Eds.), *International Encyclopaedia of Education* (Vol. 3, pp. 653-659). Oxford: Elsevier.

Cai, J. & Howson, A. G. (in press). Toward an international mathematics curriculum. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & K.S. F. Leung (Eds.), *Third International Handbook of Mathematics Education Research*. Springer.

Cai, J., Lew, H. C., Morris, A., Moyer, J. C., Ng, S. F. & Schmittau, J. (2005). The Development of students' algebraic thinking in earlier grades: A Cross-Cultural Comparative Perspective. *ZDM-International Review on Mathematics Education*, *37*(1), 5-15.

Cai, J., Lo, J. J., & Watanabe, T. (2002). Intended treatment of arithmetic average in U.S. and Asian school mathematics textbooks. *School Science and Mathematics, 102*(8), 391-404.

Cai, J., Moyer, J. C., Wang, N., & Nie, B. (2011). Examining students' algebraic thinking in a curricular context: A longitudinal study. In J. Cai & Knuth, E. (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 161-186). New York, NY: Springer.

Cai, J., Nie, B., & Moyer, J. C. (2010). The teaching of equation solving: Approaches in *Standards*-based and traditional curricula in the United States. *Pedagogies: An International Journal, 5*(3), 170–186.

Cai, J., Wang, N., Moyer, J. C., & Nie, B. (2011). Longitudinal investigation of the curriculum effect: An analysis of student learning outcomes from the LieCal Project. *International Journal of Educational Research*, *50*(2), 117-136.

Darling-Hammond, L. (1993). Reframing the school reform agenda. *Phi Delta Kappan, 74*(10), 752-761.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: a comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics, 66*(1), 61-75.

Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? *Educational Psychology Review, 16*, 235-266.

Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development for school mathematics*. Cambridge: Cambridge University Press.

Husen, T. (1967). *International study of achievement in mathematics: A comparison of twelve countries*, Volume I & II. New York: John Wiley & Sons.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education*, *3*1(2),

234-241.
Lloyd, G. M., & Frykholm, J. A. (2000). How innovative middle school mathematics materials can change prospective elementary teachers' conceptions. *Journal for Research in Mathematics Education, 21*, 575–580.
Mathematical Sciences Education Board. (1993). Measuring what counts: A conceptual guide for mathematics assessment. Washington, DC: National Academy Press.
Moyer J. C., Cai, J., and Nie, B. (2012). *Developing function sense in middle school: approaches in Standards-based and traditional curricula*. Paper presented at the ICME-12, South Korea, July 8-15, 2012.
Moyer, J. C., Cai, J., Nie, B., & Wang, N. (2011). Impact of curriculum reform: Evidence of change in classroom instruction in the United States. *International Journal of Educational Research*, *50*(2), 87-99.
National Commission on Excellence in Education (1983). *A nation at risk. The imperative for education reform.* U.S. Government Printing Office.
National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common core state standards: Mathematics*. Retrieved fromhttp://www.corestandards.org/assets/CCSSI_Math Standards.pdf
National Research Council. (2001). *Knowing what students know: The science and design of educational assessment.* Committee on the Foundations of Assessment. Pelligrino, J., Chudowsky, N., Glaser, R., Editors. Washington, DC: National Academy Press.
National Research Council. (2004). *On evaluating curricular effectiveness: Judging the quality of K-12 mathematics evaluations*. Washington, DC: National Academy Press.
National Science Board (NSB). (2010). *Preparing the next generation of STEM innovators: Identifying and developing our nation's human capital*. Arlington, VA: National Science Foundation.
Nie, B., Cai, J., & Moyer, J. C. (2009). How a Standards-Based mathematics curriculum differs from a traditional curriculum: With a focus on intended treatments of the ideas of variable. *ZDM-International Journal on Mathematics Education, 41*(6), 777-792.
Nie, B., Zheng, T., Sun, W., & Cai J. (2010). *Current mathematics education reform in the United Stated of America*. Beijing, China: People's Education Press.
Norman, G. R., & Schmidt, H. G. (1992). The psychological basis of problem-based learning: A review of the evidence. *Academic Medicine, 67*, 557-565.
Ravitch, D. (2000*). Left back: A century of battles over school reform*. New York: Touchstone.
Resnick, L. (1987). *Education and learning to think*. Washington, DC: National Academy Press.
Robitaille, D. F. & Garden, R. A. (1989). *The IEA study of mathematics II: Contexts and outcomes of school mathematics*. New York: Pergamon Press.
Senk, S. L., & Thompson, D. R, (Eds.). (2003). *Standards-Based School Mathematics Curricula: What are they? What do students learn*? Mahwah, N.J.: Erlbaum.
Stein, M.K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80.
Sternberg, R. J., & Ben-Zeev, T. (Eds.) (1996). *The nature of mathematical thinking*. Hillsdale, NJ: Erlbaum.
Vernon, D. T., & Blake, R. L. (1993). Does problem-based learning work? A meta-analysis of evaluative research. *Academic Medicine, 68*, 550–563.