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EMOTIONS IN PROBLEM SOLVING

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Emotions are important part of non-routine problem solving. A positive disposition to mathematics has a reciprocal relationship with achievement, both enhancing the other over time. In the process of solitary problem solving, emotions have a significant role in self-regulation, focusing attention and biasing cognitive processes. In social context, additional functions of emotions become apparent, such as interpersonal relations and social coordination of collaborative action. An illustrative case study presents the role of emotions in the problem solving process of one 10-year old Finnish student when he is solving an open problem of geometrical solids. The importance of emotions should be acknowledged also in teaching. Tasks should provide optimal challenge and feeling of control. The teacher can model the appropriate enthusiasm and emotion regulation. Joking and talking with a peer are important coping strategies for students.

Emotion, problem solving, coping

INTRODUCTION

Problem solving competence is given high priority in curriculum documents (OECD, 2003) and especially in mathematics, problem solving is considered essential (Schoenfeld, 1992). In this presentation we do not consider mathematical routine tasks as problems. Instead, a mathematical task is a problem for a person only if he or she does not immediately know how to solve it.

The role of affective elements in mathematical problem solving has been widely acknowledged. Already Polya (1957) addresses determination and hope (p. 93) in his short dictionary of heuristics, mentioning also the necessity to become familiar with all emotions related to the problem solving process. More explicitly the role of affective variables was elaborated in the 1980's, when Mason, Burton, and Stacey (1982), Schoenfeld (1985), McLeod (1988), Goldin (1988, 2000) and Cobb, Yackel & Wood (1989) all gave a significant role for affect in their analysis of mathematical problem solving.

The first part of this presentation will review non-cognitive aspects of non-routine problem solving behaviour. This discussion will be divided into two sections, the first focussing on students' relatively stable emotional traits that may influence the problem solving behaviour and the other part on the continuously changing emotional states that occur during the problem solving process. The second part of the presentation will provide an illustrative example of one student's emotions and coping while engaged in problem solving in classroom setting. The last part of the presentation will focus on the different ways how the teacher can enhance the desirable emotional climate in a problem solving lesson.

In the literature, there are several definitions for emotions stemming from three distinct traditions: Darwinian, Freudian, and cognitive tradition. Yet, there is a general agreement that emotions consist of three processes: physiological processes that regulate the body, subjective experience that regulates behaviour and expressive processes that regulate social coordination. Moreover, most emotion theories agree that emotions are closely related to personal goals, that they have an important role in human coping and adaptation, and that they include a physiological reaction that makes them different from cognition. Emotion theories are different in the number of emotions they identify, the degree of consciousness of emotions, and in how they perceive the relation between emotion and cognition. (Hannula, 2004)

Some emotion theories (e.g. Lazarus, 1991) identify a large number of different emotions based on the different social scenarios and cognitive appraisals related to the emotion, while some other emotion theories (e.g. Buck, 1999; Ekman, 1971; 1992) identify a small number of basic emotions that differ in their physiology, and the different cognitive appraisals and social scenarios are seen as external (though closely related) to the emotion. Moreover, some theorists have attempted to organize emotions based on the dimensions of arousal and valence (e.g. Lang, 1995).

In this paper, we identify different emotions along the tradition of basic emotions. These basic emotions include at least happiness, sadness, fear, anger, and disgust (Buck, 1999; Power & Dalgleish, 1997, p. 110). Further basic emotions include some of the self-regulative emotions, where best candidates seem to be surprise, curiosity and confusion (Lehman, D'Mello & Person, 2008; Goldin, 2000). Furthermore, the list should include also further social emotions such as shame (Pekrun & Stephens, 2010), which has characteristic physiological expression (blushing) and attachment and submission (Buck, 1999), which have important influence on interpersonal relationships. Other emotions (e.g, gratitude, pride, contempt etc.) are basic emotions in a characteristic situation or with a characteristic target or, in some cases, blends of two or more basic emotions. For example, anxiety can be considered as fear for failing on what one is doing and hopelessness as anticipatory sadness when failure is seen inevitable. The more complex emotions are based on the basic emotions and characterised largely by the related cognitions (Buck, 1999; Power & Dalgleish, 1997). Although we adopt the approach of basic emotions, we use more specific vocabulary for these emotions when the social scenario or cognitive appraisal plays a significant role. Moreover, we consider moods as low-intensity emotions, not as a separate concept (Pekrun & Stephens, 2010).

In this paper, we adopt the view by Damasio (1999) regarding three stages of consciousness of emotions:

> I separate three stages of processing along a continuum: *a state of emotion*, which can be triggered and executed nonconsciously; *a state of feeling*, which can be represented nonconsciously; and *a state of feeling-made-conscious*, i.e., known to the organism having both emotion and feeling. (p. 37; as cited in Schlöglmann, 2003)

Student disposition and mathematical competence

It is widely acknowledged that the problem solver's overall disposition (attitudes, beliefs, values, motivational orientations) influences how successful the solver will be in their attempts to solve the problem. There are several different theories that focus on different aspects of the disposition. Usually these studies have focused on the relationship between a specific affective trait (anxiety, attitude, motivation or beliefs) and achievement in mathematics. However, there is good reason to assume that these findings apply in a similar way also to the more specific relation between these affective traits and achievement in problem solving.

Perhaps the first approach to explore the connection between disposition and mathematical performance were the numerous studies on mathematics anxiety and its negative correlation with mathematics performance (for a meta-analysis, see Hembree, 1990). Similar relationship was confirmed between attitude towards and achievement in mathematics (McLeod, 1992).

Motivation research has several theoretical approaches and use of terminology is sometimes confusing (Murphy & Alexander 2000; Niemivirta 2004, 10; Pintrich 1994). However, it is clear that motivation correlates with performance (e.g. Middleton and Spanias, 1999). The more value the student gives to solving a particular problem, the more persistent the student is in his or her effort. More specifically, it has been confirmed that intrinsic motivation or interest in the task is more productive than extrinsic motivation due to rewards (Middleton & Spanias, 1999). In the achievement goal theory, positive relationships have been found between mastery goal orientation and achievement (Friedel, Cortina, Turner & Midgley 2007; Midgley, Kaplan, Middleton, Maehr, Urdan, Anderman, Anderman & Roeser 1998). Results concerning performance goal orientation and achievement have been less consistent. Some have identified negative learning behavior, while other results indicate performance orientation to lead to positive learning behavior and achievement (Freeman 2004; Midgley et al. 1998.)

Bandura's theory of self-efficacy is looking at the connection between self-efficacy beliefs and actual performance in a specific domain (Bandura & Schunk 1981). Numerous studies have confirmed the high correlation between the two (e.g. Lee, 2009).

Although survey studies indicate a clear correlation between mathematics-related affect and achievement, it has been more difficult to confirm the direction of causality. Ma and Kishor (Ma & Kishor, 1997a, b; Ma, 1999) have summarised much of that research in their meta-analyses. In one of these studies, they synthesised 113 survey studies of the relationship between attitude towards (=liking) mathematics and achievement in mathematics. The causal direction of the relationship was found to be stronger from attitude to achievement (Ma & Kishor, 1997a). However, there has been criticism of studies that do not use a longitudinal design (see Ma & Xu, 2004). Minato and Kamada (1996) reviewed eight studies that had used a cross-lagged panel correlation technique (a longitudinal design) in order to synthesize findings on the causal relationship between attitude towards mathematics and achievement in mathematics. In most of the studies, there was no predominance of either attitude or achievement. However, in the few instances that predominance was found, the causal direction was from attitude to achievement. However, Ma and Xu (2004) found a contrasting result with a larger and more representative sample. According to their study, the dominant causal relationship is from achievement to attitude. Taken together these studies suggest a reciprocal rather than unidirectional causality between achievement and affect. Such reciprocal relationship has been identified between self-efficacy and achievement in mathematics across countries (Williams & Williams, 2010).

There are also studies that address more specifically how student's beliefs influence the choices they make. Schoenfeld (1985) concluded that students' beliefs were an important determinant of their problem solving success. He observed that most students were likely to give up if the problem was not solved in five minutes, and concluded that students possessed a belief that mathematics problems can be solved in five minutes or less.

As a summary we can conclude that there is strong evidence for the correlation between student disposition towards mathematics and their actual performance in mathematics. High achievement is related to liking mathematics and determination to do well. Moreover, the relationship is reciprocal, indicating that changes in either can lead to changes in the other. Moreover, beliefs about mathematics and problem solving may direct the problem solver to explore venues that support innovative solutions or they may suggest staying on more familiar paths.

Emotional states and problem solving

Another perspective looks more specifically the different choices the student makes throughout the problem solving process and the role of emotions in it. Regardless of the overall disposition, all problem solvers encounter positive and negative emotions that influence their solution process (e.g. Schoenfeld, 1985; McLeod, 1992; Goldin, 2000). In fact, emotions are an essential part of the problem solver's self-regulation (Goldin, 2000; Hannula, 2006; Carlson & Bloom, 2005; Malmivuori, 2006). Moreover, problem solving takes place by social beings in the complexity of the learning environment where multiple goals need to be addressed (Hannula, 2006, Goldin et al. 2011).

Next, we will review some studies that have explored which emotions are present in the learning context and more specifically in problem solving. It has been suggested that six basic emotions (anger, sadness, fear, disgust, happiness, and surprise; Ekman 1972, 1992) would be rare in learning context (Vogel-Walcutt, Fiorella, Carper, & Schatz, 2012). However, as these basic emotions relate to social coordination, they should be more frequent in collaborative learning settings. Pekrun has identified a number of emotions that appear frequently during the learning process and three dimensions in their taxonomy: valence, activation, and object focus (Pekrun & Stephens, 2010). If the variety of emotions in the twelve categories thus defined is categorized under basic emotions, the negative activating emotions represent three different emotions: anger (anger, frustration), fear (anxiety), and shame (shame) while all deactivating negative emotions (boredom, hopelessness, sadness, and disappointment) are variants of sadness. The activating positive emotions (enjoyment, hope, anticipatory joy, joy, pride, and gratitude) are all variants of happiness while the positive deactivating emotions (relaxation, anticipatory relief contentment, and relief) seem not to be basic emotions, but rather lack of any emotions or removal of a negative emotion (relief).

Emotions that have been identified to appear frequently during solitary problem solving (logic problems) are curiosity, boredom, frustration, confusion and happiness while occurrences of anxiety, contempt, eureka, anger, disgust, fear, sadness, and surprise were found to be rare. However, the study design included an incorrect feedback to 25 % of the responses, which might explain the high frequency of curiosity, frustration and confusion. (Lehman, D'Mello, and Person, 2008). In solitary mathematical problem solving, emotions identified to have importance in the process include curiosity, puzzlement, bewilderment, frustration, pleasure, elation, satisfaction, anxiety, and despair (DeBellis and Goldin 2006). When mathematical problem solving has been studied in the mathematics homework context, researchers have identified the following emotions to appear in the mother-child interaction: tension, distress/dismay, frustration, sadness, boredom/apathy, anger/disgust, contempt, positive interest, affection/caring, joy/pleasure, humour, and pride (Else-Quest, Hyde, & Hejmadi, 2008). If we search for the basic emotions that underlie the different lists of emotions above, we can expect to see most of the basic emotions, when observing problem solving: curiosity to begin with and happiness if the problem is solved. In case the student struggles with the problem or cannot solve the problem there might be sadness, confusion, fear, anger or disgust, depending the attribution the student gives for the lack of progress.

There is a general agreement that emotions are functional and that they have an important role in human adaptation to different situations and learning. This applies also to mathematical problem solving. However, dynamic theories about the role of emotions in the process of problem solving are at the moment at their preliminary stages (Lehman et. al., 2008, Goldin, Epstein, Schorr & Warner, 2011)

Emotions serve three fundamentally different functions in human self-regulation: physiological, psychological and social. The first function is physiological adaptation, where the most clear example is the 'fight of flight'-response to surprising threatening stimulus. The emotion triggers release of adrenaline, which prepares the body physically to fight or alternatively to escape. (Power & Dalgleish, 1997) This functional aspect of emotions is relevant to learning situations in the sense that most emotions have physiological reactions as a side effect. For example, fear may influence the physiology in a way that is detrimental for optimal cognitive functioning in a test situation.

The second role of emotions is in the psychological self-regulation through influence on cognitive processing. Just as fear or anger have clear consequences in the physiological adaptation; surprise and curiosity have clear influence on attentional processes and memory. It is now well established that emotions direct attention and bias cognitive processing. For example, fear (anxiety) directs attention towards threatening information and sadness (depression) biases memory towards a less optimistic view of the past (Power and Dalgleish 1997; Linnenbrink and Pintrich 2004).

It has been suggested (Forgas 2008) that the positive emotions would promote the more inductive, bottom-up thinking while the negative emotions would promote the more deductive, top-down thinking. Although there is not yet sufficient evidence to conclude that to be case for all positive and negative emotions, it seems that positive emotions would facilitate the creative aspects of problem solving, while the negative emotions would facilitate reliable memory retrieval and performance of routines, which also are essential in certain phases of problem solving (Pekrun & Stephens, 2010). More intensive levels of mathematics anxiety seem to be exclusively detrimental for problem solving and the suggested mechanism is its overloading of working memory as the subject is preoccupied with one's math fears and anxieties (Ashcraft and Krause, 2007, Rubinstein and Tannock, 2010).

Moreover, emotions not only bias memory retrieval, but also function as a 'fixative' in the storage to long term memory. A recent study has identified that activity in the amygdala during an Aha! experience is a strong predictor of which solutions will remain in long-term memory (Ludmer, Dudai and Rubin 2011).

Emotions serve also a third adapting function in the social coordination of a group, which brings forth new types of emotions. The close relation between emotions and the social interaction is well acknowledged (e.g. Forgas, 2008, Pekrun & Stephens, 2010). Previously Cobb et al. (1989) identified that students' emotions are related to two types of problems in collaborative problem: mathematical problems and cooperation problems. Hannula (2005) identified three different social functions for emotions in collaborative problem solving:

1. emotions concerning interpersonal relationship needs and goals (e.g. sadness due to exclusion),
2. emotions concerning individual learning goals when their cause is attributed to peers (e.g. gratitude for help), and
3. emotions concerning social coordination of individual goals (e.g. anger when own idea has been rejected by others).

The interpersonal relationship needs and goals are not specific to mathematics or even learning, but they should not be ignored when we analyse student emotions in the classroom. The emotions that related to individual learning goals, on the other hand, are rather straightforward extensions of self-regulative emotions in solitary problem solving. The third category, on the other hand reserves further elaboration.

In the coordination of collaborative behaviour, emotions can be expressed and interpreted unconsciously. Emotions may also be used consciously in power games or as means to solve communication problems. Furthermore, emotions may be interpreted consciously, when they become subject to reflection and re-evaluation. Emotional communication in the coordination of collaborative problem solving behaviour can be very powerful, one example being shared cognitive intimacy (Hannula 2005, c.f. Williams 2002):

> In shared cognitive intimacy, students enter an intimate interaction with each other and with a task. This intimacy is indicated by one student frequently continuing or completing the other student's utterances and occasionally by both speaking in unison. It is an example of a situation where students can achieve their cognitive and social goals simultaneously. This dual intimacy with peers and mathematics is rewarding for the students and, furthermore, it can be an extremely useful tool for enhancing the classroom climate. One

> problem with this kind of intimacy is that sometimes it may exclude other students. (Hannula 2005, p. 35)

The same study identified also a number of strategies that a student may use to ‘saving their face’ when problem solving does not seem successful (Hannula 2005). Of these strategies devaluing of the task and expressing lack of effort may be detrimental for the development of productive social norms for collaboration.

A study on mother and child emotions during mathematics homework supports the hypothesis that emotions are both an influence on and an outcome of mathematics performance. Moreover, it provides some interesting qualitative data on the dynamics of emotions while doing homework, suggesting a reciprocal relation where either mother or child may initiate important changes in the emotional state of the other. (Else-Quest, Hyde, & Hejmadi, 2008).

Emotions and problem solving in the school context

So far, most research on mathematics-related affect has been done using surveys, which has provided strong evidence for the theories regarding trait-type emotions (although good longitudinal studies are too few). Research on the role of emotions in cognitive processes has mainly been conducted qualitatively, and while such studies are good for identifying important concepts and their relations, they seldom are rigorous enough for testing theories. Quantitative studies, on the other hand, are mainly conducted in laboratory settings and the ecological validity of research findings must be tested in realistic school settings (Pekrun & Stephens, 2010).

Perhaps the first to report the role of emotions both in the cognitive and social domain were Cobb et. al (1989). Their study challenged the previous observations of emotionally neutral or ‘flat’ classrooms, illustrating instead a lively classroom:

> Children frequently jumped up and down, hugged each other, and rushed off to tell the teacher when they solved a particularly challenging problem. Significantly, the positive emotional acts occurred when the children completed personally challenging tasks or constructed mathematical relationships. (Cobb et al., 1989, p. 61)

In their framework, social norms provide a framework for interpreting the individual emotions (Cobb et al., 1989).

Another approach has been to report case studies of student’s emotions while solving problems in a mathematics classroom (e.g. Evans, Morgan & Tsatsaroni, 2006; Hannula, 2003; Op ‘t Eynde & Hannula, 2006; Williams, 2002).

However, Goldin, Epstein, Schorr, and Warner (2011) have suggested that there is a need for ‘mid-level’ concepts between in-depth qualitative studies and ‘high-level’ concepts like norms. In a qualitative study of an US inner-city middle school, they identified a number of behavioural patterns that integrate students’ affective and social interactions, which they call *engagement structures*. Each engagement structure is characterized by ten aspects, which include goal, behavioural and emotional pattern, self-talk and interaction with beliefs. These engagement structures could be also described as behavioural scripts that relate to a specific goal or desire. (Goldin, Epstein, Schorr, & Warner, 2011)

Emotions are ubiquitous in human interaction and, therefore, it is extremely challenging to extract the influence of teacher's emotional support in research design. However, in computer-based learning environments it is easier to study the influence of emotional or motivational support. Kim and Hodgen (2012) found out that even a minimal six minute video aimed at improving student emotion regulation was able to produce significant influence on university students' academic emotions in an on-line remedial mathematics course.

As said earlier, emotions serve the three purposes of physiological adaptation, psychological regulation, and social coordination. Although emotions are functional for the human species, not all emotional reactions are functional in classroom context. For example, anger provides functional physiological adaptation to overcoming obstacles when solving a physical problem (e.g. moving a heavy object). Yet, it is usually less functional when solving cognitive challenges collaboratively. Moreover, the physiological aspect of the emotion (e.g. adrenaline or endorphins) may have effects beyond the duration of the emotional event and such emotional residual may interfere with following emotional episodes. Such unwanted side effects of emotional reactions could be considered as emotional 'noise' confusing smooth emotional communication.

Therefore, especially in the schools context, emotions need to be regulated (De Corte et al., 2011; Fried, 2012). Emotion regulation refers to "the ways individuals influence which emotions they have, when they have them, and how they experience and express these emotions" Gross (1998, p. 275). There are different ways to categorize emotion regulation strategies. The target of emotion regulation can be attention, emotion-relevant knowledge, or body manifestations of emotion, their psychological function may be oriented towards needs, persons or goals, and both antecedent and response-focused strategies are possible (Fried, 2012). Emotion regulation that mainly focuses on reducing negative emotions or their effects is often called coping. The coping strategies that Flemish high-school students report using when facing difficult mathematics test, homework or lesson are (in order of frequency) active coping (i.e. effort), joking and acceptance, social-emotional coping (i.e. seeking social support), abandoning and negation, religion and – rarely – alcohol and drug use (De Corte et al., 2011). In addition to coping, it is important to consider the emotional regulation that occurs before emotional reaction through choice of goals and selective attention.

AN ILLUSTRATIVE EXAMPLE: AN 11-YEAR OLD STUDENT SOLVING AN OPEN PROBLEM

In this section, we shall analyse one student's emotions in the course of one lesson. The case will illustrate different functions of emotions.

The context

The analysis is based on a video recording of a research class that participates in a project where one mathematics lesson each month is used to solve an open-ended problem and the participating eight teachers meet once a month with project research team to discuss using open-ended problems in teaching (Näveri, Pehkonen, Hannula, Laine & Heinilä, 2011). This analysis will focus on one particular 10-year old male student (Tomi[i]) and his interaction with his teacher and his two peers sitting next to him (Arto and Eetu). One video camera was used for recording the overall progress of six students. We do not have a full coverage of Tomi's behaviour during this lesson. However, due to his position in the middle of the group, he appears on the video most of the time.

The analytical framework

For the analysis of this case, we shall use the emotion coding scheme by Else-Quest, Hyde, and Hejmadi (2008), which was developed to analyse mother-child interaction during homework. This coding scheme is based on two emotion coding schemes: Ekman's FACS system (Ekman, Friesen & Hager, 2002), which identifies six basic emotions from facial expressions and Gottman's SPAFF system (Levenson & Gottman, 1983), which identifies 16 emotions in dyadic interactions based on facial expressions as well as othe verbal and nonverbal expressions of emotions. This coding scheme was chosen for this analysis, as it has been tested and found useful in context where both self-regulative and social aspects of emotions are essential. The coding scheme includes 13 emotions: tension, distress/dismay, frustration, sadness, boredom/anxiety, anger/disgust, contempt, positive interest, affection/caring, joy/pleasure, humour, pride, and off-task. The coding scheme provides a number of markers for each emotion. Table 1 provides markers for two of the emotions. For details, see Else-Quest, Hyde, and Hejmadi (2008).

Table 1. Two examples of emotions and their markers used for the analysis.

Emotion	Markers
Joy/Pleasure	High-fives, smiling (lip corners up, raised cheeks, outer brow down), exclamations (e.g., “Wow!” or “Cool!”)
Pride	Sitting upright, ”showing off”, similar facial expression as joy/pleasure, but antecedent event is achievement

The case

The open problem of the lesson was to construct models of different solids using given manipulatives (peas and cocktail sticks) and to record the number of edges and vertices of each solid. In the first phase, the students were asked to find out different solids that they can construct using no more than 12 sticks. Later, the number of sticks was gradually increased to 14, 16 and 18 sticks when student could not find further solids with the smaller number of sticks.

Teacher gives instructions to the class and shows students one exemplary model of a shape. each student is given their own pile of sticks, and Tomi, Arto, and Eetu receive one cup of peas to share, which is located on Tomi's desk. The time starts to run from the moment students have received the peas and sticks and they begin their construction.

Time	
00:00	{Tomi, Arto, and Eetu all start building a cube as their first solid.} [ii]
01:39	{Tomi accidentally hurts his hand into a sharp stick.} Ow, sshiit! {He pulls his hand away in a reflex-like motion. Emotion: distress/dismay }

Here, Tomi experienced pain and cursed because of it, indicating that he did experience anger. However, no further consequences could be observed. It is likely that Tomi was little more careful when attaching the next pea, but the incident did not distract him from his work nor did other students react to it. However, a similar incident happened later in the lesson, when he encountered a rotten pea. Then his face indicated disgust, and his work was interrupted until he had cleaned the rotten pea away. The incident also attracted the attention of Arto and Eetu, who interrupted their own work to examine the cause of disgust.

These two incidents illustrate how the 'primitive', self-protective role of emotions can intervene problem solving. In both cases they were given priority (the first one was like a reflex), but at least for Tomi, the distraction was short. However, students who are more timid and prone to anxiety, might not overcome the emotional distraction as easily as Tomi.

01:45	{Tomi is still building the cube. Emotion (momentarily): tension. After that he speks some off-topic with Arto and Eetu.}
02:38	{The cube is more than half ready. Tomi leans close to it and points at each of the sticks (verteces) and also to each vertex that has not yet been constructed. Arto and Eetu speak off-topic past Tomi, who ignores it. Emotion: positive interest.}
02:44 Tomi:	This will be exactly 12! {The tone of voice indicates excitement, perhaps also pride.}

In this episode Tomi is effectively regulating his attention when focusing on the task and he expresses elation when realizing that the construction fulfils the requirements of the task. Specific to this episode is that these emotions relate to the individual goals of Tomi and perhaps for that reason they do not well fit the markers of the coding scheme. The tension in the beginning is not of unpleasant nature; rather it just reflects the intensity of concentration. Later, Tomi expresses his excitement (which is not verbally, perhaps wishing to share the emotion with peers, but they are focusing on their individual work and they do not react.

03:07 Tomi:	{Arto is picking pea from the cup, as he has done several times.} Why are you taking mine! {Turns his face to looks Arto directly in the eyes, at the same time moving his hand on the desk sharply towards Arto, perhaps as to mark 'his' space. Then moves his hand towards Arto's hand, as if trying to stop him from taking peas. Clearly, he is challenging Arto, although the specific emotion is not clear.}
03:10 Arto:	Because we don't have [own][iii].
03:11 Tomi:	[Ah, right.] We don't have. {Covers his face with both hands.}
03:13 Arto:	What are you freaking out! {Smiling}

03:16 Eetu:	{Tomi takes hand off his face, is blushing, smiles meekly. Emotion: shame} For real, Tomi! Laughable. Someone's insane here. One freaks out. 'Why are you taking mine!' {pitched voice}. {Arto smiles widely and giggles, also Tomi laughs shyly, still blushing}
03:26 Tomi:	I forgot that we'd have our own {in a meek voice}
03:30 Eetu:	You forgot that we'd have our own?
03:32 Tomi:	{Inaudible}

This second episode is clearly about social coordination, use of shared resources but also about the social relationships and reputation. First, Tomi reacts as defending his 'rightful territory'. However, as he realizes that peas were a shared resource he is very embarrassed, which is in this case very expressive: he gives rapidly up his challenge, covering his face, blushing, and even his tone of voice changes. Such a reaction is perhaps the most reliable apology to make, as blushing cannot be faked. Eetu is emphasizing the incident, making fun of Tomi, who in this situation is vulnerable. However, all three laugh at the end, indicating that Tomi does not feel the attack too serious. This incident was also noticed by two other students who were seen on the video, turning to look at the three boys little bit worried first, and at least one of them was smiling at Eetu's ridiculing of Tomi. Also in this case the coding scheme is insufficient, providing a category for neither of the main emotions, although the challenge, could be classified under tension.

Also this incident is passed with little effect on student problem solving. There is clearly some emotional residual for Tomi, and possibly this incident influenced his goals in the future episodes (he was ambitious, wanting to produce something special).

All three boys finish their cubes, Tomi makes swiftly also a tetrahedron.

03:36 Tomi:	{to Arto} What are you doing? {Looks at the constructions of Arto and Eetu.}
04:21 Arto:	For real, don't do exactly same as me.
04:26 Tomi:	I don't even know what you intend to do.
05:00 Tomi:	You are doing the same as me. All these [cubes] are identical.
05:12 Tomi:	{Builds a 'house' of the cube and the tetrahedron. Emotion: Humour} Haha haha, look! Ha! {Arto looks at the 'house' and smiles}
05:21 Tomi:	What to do now? {Emotion: boredom.}
05:44 Tomi:	What would I do now? {Tomi leans back, slouching on his chair, soon sits again more upright, then puffs air out with a sound and fidgets with sticks. Emotion: boredom.}
06:04 Tomi:	{to teacher} I don't get it. {Emotion: distress/dismay. Teacher's response is inaudible}
06:08 Tomi:	Can they be something totally own? {Emotion: Joy/pleasure. Tomi begins to construct a new model.}

Teacher: They can be your own. As long as there are not more than 12 sticks.

Tomi has difficulties to continue after the two first solids. He realizes that they have done identical cubes and Arto warns him not doing the same as he does. Tomi uses joking as a coping strategy, but symptoms of boredom become soon clearly visible. He does not like the situation, and seeks support from his teacher. Something that the teacher said, gave him an idea to work with.

For the sake of brevity and to keep the red line visible, only part of Tomi's interactions is reported in the next section. Tomi builds a 'flat' shape of two attached squares.

08:30 Tomi: Teacher! See, ladders! {Shows his construction to teacher, whose reaction cannot be seen, plausibly he is occupied with some other students. Arto is looking at Tomi's 'ladders' and both boys are laughing. Emotion: humour}

08:40 Eetu: What is that?

08:42 Tomi: {Emotion: changes rapidly to neutral.} I don't know. This just came out like this.

08:50 Tomi: {To teacher, showing his 'ladders', smiling. However, emotion is not joy/pleasure.} Does this count?

08:54 Teacher: That's not a solid yet. It's on a plane, in a way. Think if you could continue it.

{Tomi makes a new type of construction.}

09:40 Tomi: Look! {Emotion: joy/pleasure}

09:47 Tomi: Well, teacher! Is this one?

09:52 Teacher: See, it is still slightly open here {pointing}

09:53 Tomi: Hmph! {Emotion: frustration.}

{Tomi adds one stick to his construction, ending up with a model that is not a polyhedron. It is relatively complex model and it is ambiguous which solid it would represent. However, it could be interpreted as a model of a solid whose some surfaces are curved.}

10:02 Tomi: Well is this now? {Emotion: Joy/pleasure. Teacher is with another student.}

10:27 Tomi: {to Arto} Hey look! {Expressed amazement}. Diamond. {Emotion: joy/pleasure}.

11:14 Tomi: Teacher! Come and look. I will show.,

11:18 Tomi: {funny voice} It's a diamond. A perfect diamond.

11:28 {Teacher arrives. Tomi's emotion: looking for reinforcement?}

11:32 Teacher: Is it a solid now? {The teacher does not accept the solution and his nonverbal communication seems to converse that to Tomi.}

11:33 Tomi: No.

11:34 Teacher: Think! {Turns and leaves.}

11:35 Tomi: {Twists the model a little.} Now it is! {Smiling. Emotion not clear. Teacher turns to look, smiles gently, his expression communicating something like "you can't fool me".}

11:45 Tomi: I don't want to demolish this. {Pulls the model apart. Emotion: sadness.}

11:47 Tomi: A dog! {Arto gasps, pretending to be amazed.}

11:48 Eetu: Where's a dog? {First Arto, then also Tomi and Eetu start laughing. It is not clear, but Tomi might be slightly blushing again. Emotion: Humour.}

12:20 Tomi: Now I got it!

12:45 Tomi: Hey teacher, this is certainly a solid! {Hits his desk with both hands, fists clenched. Emotion: frustration.}

{Tomi makes a new shape, which also has 'curved' surfaces}

13:52 Tomi: Is this, then? And this? {Emotion is not clear. Although he gives a social smile, the emotional feeling is more like sadness.}

13:58 Teacher: You really got [interesting.]

13: 59 Tomi: [And look!], this is a house. No, a tent.

14:06 Teacher: It has to be closed from all directions. {Turns and leaves.}

14:10Tomi: Now I will close it from all directions. I will cram if need be. {Emotion: Anger/disgust}

Here it becomes apparent that Tomi considers his teacher as the criterion for acceptable solutions. Although he himself gets pleasure for his models, his joy proves to be repeatedly premature as his teacher's judgement rejects the attempts. Joking is the all-around coping strategy for Tomi. Although Tomi shows great resilience, his negative emotions grow stronger as rejections repeat. The expressions of anger and sadness and the related self-talk are not directed at anyone. Yet, there is a feeling of display attached to them. Perhaps the display of negative emotions in a theatrical fashion is a way to regulate the expression of emotions to be socially acceptable. Moreover, Tomi's emotional communication with his teacher seems to develop towards increasingly angling for sympathy.

This analysis provides an example of the rich variety of different functions of emotions in mathematical problem solving in classroom setting. More specifically, the task was open and the duration of problem solving was long, here we have analysed less than half of the time Tomi spent working on the task. One thing that is apparent here is the presence of emotions and student's capacity to regulate them in a productive manner.

CONCLUSIONS AND IMPLICATIONS FOR TEACHING

The main lesson to learn from the research on emotions in problem solving is that emotions are an essential part of problem solving. Some emotions direct attention and intuition and are functional, perhaps necessary, in the process of successful problem solving. Emotions play an important part also in learning from the experience of both successful and unsuccessful problem solving. Moreover, emotions influence the formation and development of mathematics-related motivation, attitudes, and beliefs. As some emotions seem to be more beneficial to learning outcomes than others, teachers and curriculum developers need to pay attention to the student emotions.

What are the characteristics of a classroom that promote optimal emotional climate? It has been shown that the teacher enthusiasm (Frenzel et al., 2009) and the chosen method of teaching (Schukajlow, Leiss, Pekrun, Blum, Müller, & Messner, 2011) can have an influence on student emotions.

Several schools have implemented programs to enhance students' social and emotional learning and aim to promote a healthy learning environment. Specific goals for the program are that students acquire core competencies to recognize and manage emotions, set and achieve positive goals, appreciate the perspectives of others, establish and maintain positive relationships, make responsible decisions, and handle interpersonal situations constructively. According to a meta-analysis, universal school-based social-emotional development programs have beneficial effects on positive social behaviour, problem behaviours, and academic performance (Durlak, Weissberg, Dymnicki, Taylor & Schellinger, 2011).

When students are engaged and face an optimal challenge, they can experience flow (Csikszentmihalyi & Csikszentmihalyi, 1992; Williams, 2002). Unfortunately present school seems to rarely provide experiences of flow. Quite the contrary, a specific and persistent problem is that classrooms are often emotionally flat, and boredom (i.e. temporary feelings of low-arousal and unpleasant emotions induced by environmental factors) is one of the most frequently experienced emotions. (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010; Nett, Goetz, & Hall, 2011; Vogel-Walcutt, Fiorella, Carper, & Schatz, 2012). Classroom environment seems to be a critical determinant of students' boredom experiences indicating that instructional approaches can influence the amount of boredom in the class (Nett et al., 2011). Specifically, subjective experience of control decreases the level of boredom (Pekrun et al., 2010).

Pekrun and Stephens (2010) claim that except for research on test anxiety, there is little research concerning the effects of the task and the environment on academic emotions. Based on Pekrun's control-value theory and results regarding test anxiety they suggest the following characteristics to have positive effects on student emotions:

- students perceive value in the task and have a feeling of control,
- there is a match between task demands and student competence,
- tasks and learning environments meet individual needs,
- teacher is enthusiastic,
- student autonomy is supported,
- emphasis on mastery goals,
- positive feedback, and
- positive consequences of achievement.

Because of the functional aspect of emotions in self-regulation and learning, there should be space for emotions in the classroom. Even the negative emotions related to failures seem to have an important role in creating intuitions (Immordino-Yang & Faeth, 2010). However, sometimes emotions need to be regulated. There are three possible perspectives to emotion regulation in the classroom: teacher regulation of student emotions; student and teacher self-regulation of own emotions and students' regulation of peer emotions (Fried, 2012).

The teacher can influence the students' emotion regulation through modelling emotion regulation strategies. Through teacher modelling students learn to recognize and understand the role of different emotions in problem solving and they also learn about the different ways to control the experience and expression of these emotions. The teacher may also provide more direct support through controlling student emotions, although a teaching style where students are encouraged to think for themselves (autonomy supportive) is more effective in helping students develop their emotion regulation. (Fried, 2012)

Perhaps more important than focusing directly on students' emotion regulation, is to develop such social norms in the classroom that encourage students to regulate their own and peer emotions. Also joking and seeking peer support serve important function in coping with frustration or other negative emotions (De Corte et al., 2011). Firstly, expressive environments have been found to support development of emotion regulation strategies. If such expressive environments have also a feeling of community, where students feel belonging to the classroom, they can more easily assimilate external regulation they observe into the self. (Fried, 2012)

Cobb et al. (1989) also emphasized the relationship between social norms and emotions. In the classroom they observed, engagement in mathematical activity was the goal and therefore, even weaker students experienced and expressed positive emotions as they participated in group activities and whole class discussions. More specifically, they did not observe a single event over the semester, where a student would have become frustrated and given up the task.

Hannula (2006) suggested that open approach (e.g. Nohda, 2000) would provide opportunities to meet student needs of autonomy, competence and belonging. An open approach would fulfil many of the criteria suggested by Pekrun and Stephens (2010). In the ongoing research project (Näveri et al., 2011) we explore the overall effect of open problems for mathematics related affect and achievement. It is clear already from our preliminary analysis that open problems bring a lot of emotions in the classroom, including flow. Moreover, we have identified that in a good open task there are different levels of complexity that the students can choose from.

One key element is to develop social norms in the class that ensure safe space for explorations and a feeling of playfulness in the lesson. The appropriate climate of a lesson can be further supported through presenting problems in a humorous form, and introducing them to the class in a humorous light (Shmakov & Hannula, 2010). The modes of working with the problems should provide opportunities for students to reach their social goals of belongingness and the teacher should be prepared to provide emotion scaffolding for those students who are not yet able to cope with moments of frustration. Finding the solution should provide a feeling of accomplishment and pride for all students. This can be best achieved through focus on the process, and by highlighting important variation in ideas and perceiving unsuccessful attempts as important learning opportunities.

Joking is a coping strategy that students use frequently (De Corte et. al, 2011).

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[i] All names are pseudonymes

[ii] Curly brackets {} indicate observations and interpretations based on the video.

[iii] Square brackets [] indicate overlapping talk.