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TEACHERS, STUDENTS AND RESOURCES IN MATHEMATICS LABORATORY

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This communication deals with the methodology of mathematics laboratory from two points of view: the first one concerns teacher education, the second teaching experiments in the classroom. Mathematics laboratory (described in the Italian national standards for mathematics for primary and secondary schools) can be considered as a productive "place" where constructing mathematics meanings, more a methodology than a physical place. It can be associated to inquiry based learning for students. An example of mathematics laboratory with cultural artefacts such as the mathematical machines (www.mmlab.unimore.it) is discussed.

Mathematical laboratory, instrumental genesis, semiotic mediation, mathematical machine, teacher education.

INTRODUCTION

Interest in mathematics teacher education is growing in practise and research in mathematics education (Ball et al., 2008). The aim of this work is to contribute to the discussion presenting the analysis of a teacher education program based on the methodology of mathematics laboratory.

This paper presents five parts. A first part places the idea of mathematics laboratory in the history of mathematics teaching. After specifying the idea of mathematics laboratory considered here, essential elements of a theoretical background are hinted in the second part. The third part contains the description of the education program, which is analysed in the fourth part. Concluding remarks ended this paper.

THE IDEA OF MATHEMATICS LABORATORY

Roots of mathematics laboratory

The idea of mathematics laboratory is rooted in the studies of pedagogists, psychologists and educators at the end of XIX century, as John Dewey (1859-1952), Georg Kerschensteiner (1854-1932), Edouard Claparède (1873-1940) and Maria Montessori (1870–1952), when the idea to offer spaces to learners to expound a spontaneous and constructive activity, to cultivate their own individuality and to socialize appears.

Between the end of the XIX century and the beginning of XX century, different European and North American mathematicians discussed their reflections on the ways to teach mathematics, often in opposition to the traditional lesson. In this trend, John Perry (1850-1920) proposes a new didactic method, called *Practical Mathematics* (Giacardi, in press). He thinks that mathematics should be taught "with experiment and common-sense reasoning" (quoted in Giacardi, in press).

In France, the reform of secondary education (called *humanités scientifiques*) starts in 1902. An important contribution is given by Émile Borel (1871-1956), who in a famous conference in 1904 wishes the creation of *atelier mathématiques*[1]. This idea is associated to a joiner's shop, where pupils could create models by their hands, made measures, under the supervision of teacher and in the presence of a joiner.

In Germany, the main advocate for the use of concrete models and dynamic instruments is Felix Klein (1849-1925), who also describes some tools in his *Elementary mathematics from an advanced standpoint* (Bartolini Bussi, Masami, & Taimina, 2010).

In Italy, Giuseppe Vailati (1863-1909) contributes to this movement with his innovative idea of school-laboratory (Giacardi, in press). According to Vailati, school-laboratory does not understand in the reductive sense of laboratory for scientific experiments, but as a place where the student is given the means to train under the guidance and advice of the teacher, to try and resolve issues, to test himself through obstacles and difficulties. Giacardi highlights that Vailati dreams a methodology based on problem solving, production of conjectures and argumentations, but the most important aim is the construction of mathematical meanings within the theoretical structure of mathematics. In this sense, he has a broader idea than the mathematicians considered above.

Those educational questions are also present in many discussions in "L'enseignement mathématique" (the official journal of ICMI from its creation in 1908). For instance, in the second part of an important paper (''The modern tendencies of mathematics teaching'')[2], founding the program of the ICMI, some traces of discussions among teachers in schools are presented (Maschietto & Trouche, 2010). This kind of discussion is summarized in some contribution to the Working Group 4 at ICMI Symposium 2008[3] (Bartolini Bussi & Borba, 2010).

In Italy, in the Sixties, a new relaunching is given by the work of Emma Castelnuovo (Castelnuovo, 1963), who inspires, with other mathematicians, the teachers who founded the heart of the Laboratory of Mathematical Machines (MMLab)[4] at the University of Modena e Reggio Emilia (Maschietto, 2005; Pergola & Zanoli, 2010).

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[1] http://smf4.emath.fr/Publications/Gazette/2002/93/smf_gazette_93_47-64.pdf. Accessed April 2012.
[2] http://www.unige.ch/math/EnsMath/. Accessed April 2012.
[3] http://www.unige.ch/math/EnsMath/Rome2008/WG4/WG4.html. Accessed April 2012.
[4] http://www.mmlab.unimore.it. Accessed April 2012.

Mathematics laboratory in mathematics education nowadays

Over the last ten years, the mathematics laboratory urges a renewed interest in institutional and international level. Even if the name 'laboratory' is not always used, there are several institutional positions that are consistent with the principles of mathematics laboratory. For instance, the Inquiry Based Science Education fostered by the European Commission (Rocard, 2007), or the *demarche d'investigation* in French mathematics curriculum (Maschietto, 2010a). Furthermore, the expression 'mathematics laboratory' is related to several forms of laboratory (e.g., a room for students after school time, Kahane, 2006), with different time, places and organization (Maschietto & Martignone, 2008).

In Italy, the Italian Mathematical Union (UMI) has drawn on the ancient idea of the mathematical laboratory, when the new mathematics standards from 5 to 18 years old students are prepared (AA.VV., 2004). The document reads:

> A mathematics laboratory is not considered a place (e.g., a computer classroom) but rather a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people, structures, and ideas. We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on the one side, to the use of tools, on the other, to the interactions among people working together.[5]

In 2007, the Italian Ministry of Education publishes guidelines[6] for the compulsory school curriculum, where the methodology of laboratory is presented as a key component not only for mathematics and sciences.

Tools in mathematics laboratory

A common element to different forms of mathematics laboratory concerns the reference to tools in doing mathematics, with some elements from its historical development (e.g., the rules and compass in Euclid). They can also be digital (Information and Communication Technologies, i.e., ICT) or classical tools. In our work (Bartolini Bussi & Maschietto, 2006; Maschietto & Martignone, 2008), we consider above all mechanical devices, as the mathematical machines, which are collected in MMLab (Maschietto, 2005). As argued by Bartolini Bussi and Maschietto (2006), they are part of the historical phenomenology of geometry.

Mathematics laboratory in our research work

In this paper, we consider mathematics laboratory in school, according to AA.VV. (2004). It is relevant to emphasize some basic elements of this methodology:

- group work, peer interactions, group discussions as well as interaction between students and between students and the teacher as an expert;
- presence of a question to understand, a problem to solve, an object to be discovered;

[5] http://umi.dm.unibo.it/old/italiano/Matematica2003/prima/premessa2.pdf . Accessed April 2012.
[6] http://www.edscuola.it/archivio/norme/programmi/indicazioni_nazionali.pdf. Accessed April 2012.

- every student can contribute, even and especially students that do not "make mathematics" in traditional classes;
- manipulative aspects, gestural and procedural intertwine with theoretical aspects;
- presence of tools that are not only used as technical instruments (Vygotsky, 1978).

According to (Ciappini & Reggiani, 2004), a laboratory is a phenomenological space of students' conceptualisation and reflexive thinking (Norman, 1993); it is "finalised to the construction of the experiential base which is necessary for the appropriation of the mathematical meanings" (Ciappini & Reggiani, 2004, p. 3). From the viewpoint of the didactic implementation of a mathematics laboratory, Chiappini (2007) stresses the presence of a process of transposition of mathematical knowledge.

THEORETICAL REFERENCES FOR MATHEMATICS LABORATORY WITH TOOLS

In all the research studies carried out by the team of the MMLab about mathematics laboratory with classical technologies, at least three analytical components are present (Arzarello & Bartolini Bussi, 1998):

- an epistemological component, with attention to mathematical meaning;
- a didactic component, with attention to the classroom processes;
- a cognitive component, with attention to processes of learning

Maschietto and Trouche (2010) discuss the notion of mathematics laboratory from historical and theoretical perspectives and propose to strongly connect the two frameworks, the instrumental approach (Rabardel, 1995) and the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008). But in mathematics laboratory other components can be presents, as about argumentation and proof. We hint at them afterwards.

In order to specify the didactic use of a tool in mathematics laboratory, it seems to be useful to extend the definition of "didactical functionalities", proposed about ICT tools by Cerulli, Pedemonte and Robotti (2006) in order to enhance teaching/learning processes according to a specific educational goal:

> The three key elements of the definition of the *didactical functionalities* of an ICT tool are:
>
> 1. a set of features/characteristics of the tool;
>
> 2. a specific educational goal;
>
> 3. a set of *modalities of employing* the tool in a teaching/learning process referred to the chosen educational goal. (p. 1390)[7]

The authors specify that the modalities of employment of the tool depend on the chosen theoretical framework.

Instrumental approach

The instrumental approach is founded on the distinction between artefact (a material or abstract object, already produced by human activity) and instrument (a psychological

[7] Italic in the original version.

construction, built from the artefact and utilisation schemes), and on processes (called instrumental geneses) leading to the construction of instruments from the artefact. The instrumental genesis is composed of two kinds of processes: instrumentation and instrumentalisation. The former is relative to the emergence and evolution of utilisation schemes; the latter concerns the emergence (as a first level) and evolution of artefact components of the instrument. As stressed by Trouche and Drijvers (2010), the notion of scheme can be related to Vergnaud's definition, i.e., a scheme is an invariant organisation of behaviour for a given class of situation (Vergnaud, 1990). A scheme is composed of action and anticipation rules, but also operational invariants and inferences. It is schematically represented in Figure 1.

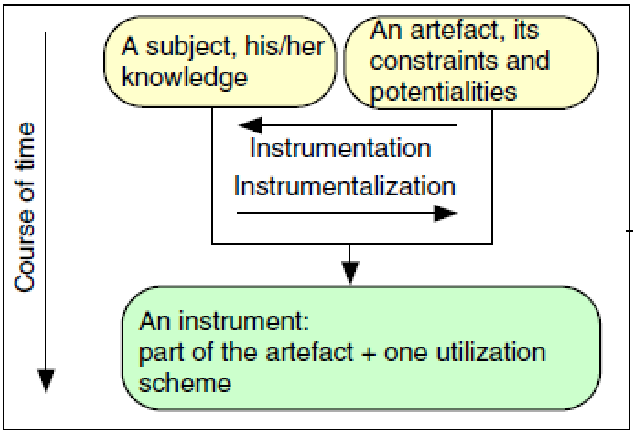


Figure 1. Instrumental approach (Maschietto & Trouche, 2010, p. 37)

Theory of semiotic mediation

The Theory of Semiotic Mediation has been elaborated and applied to mathematics education by Bartolini Bussi and Mariotti (2008) within a post-Vygostkian perspective, founded on the relevance of the use of artefacts in human activities. The process of semiotic mediation may be described schematically by means of the following drawing (Figure 2).

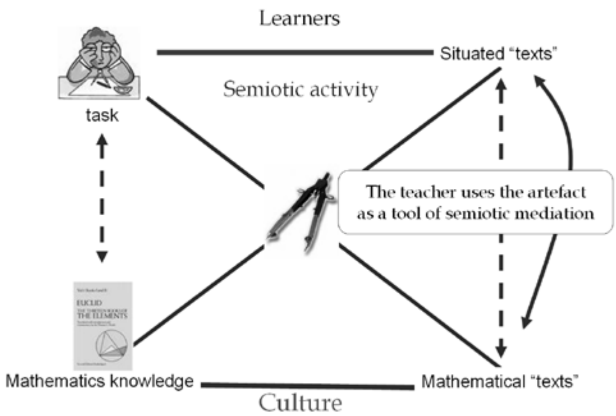


Figure 2. Theory of Semiotic Mediation (Bartolini Bussi & Maschietto, 2008, p. 192)

The essential elements of this framework are:

- The specification of the use of noun ‘mediation’ according to Hasan’s definition (quoted in Bartolini Bussi & Mariotti);
- In mental activities, human beings reach higher levels through mediation by artificial stimuli (signs or semiotic tools), that are referred to as psychological tools according to Vygotsky (1978);
- when an artefact is introduced in the process of solving a given task, a double semiotic link (named semiotic potential of an artefact) is recognizable: the first is between the artefact and the task and the second is between the artefact and a piece of knowledge;
- the activity with a specific artefact foster the production of signs (CF. Figure 2, cognitive component: the higher triangle “task – artefact – situated texts”);
- teacher guide the evolution of students’ signs produced using the artefact into mathematical ‘texts’ (CF. Figure 2, didactic component : the right triangle “task, situated texts, mathematical texts”);
- the importance of teacher’s role as cultural mediator with respect to mathematical contents (CF. Figure 2, right and left arrow bottom-up).

When the teacher uses an artefact to mediate mathematical meanings, according to the elements above, he/she uses it as a tool of semiotic mediation.

The epistemological component is present in the analysis of the semiotic potential of the chosen artefact, where some elements from the instrumental approach are considered. From a didactic viewpoint, the process of semiotic mediation is grounded in a specific structure of activities (called the “didactical cycle”): activities with artefacts usually in small groups that promote the emergence of signs (words, sketches, gestures, ...) in relation to the use of particular tools; individual written production of signs (drawings, writing, ...) and collective moments leading to social production of signs. In the latter, mathematical discussion is the fundamental didactic strategy.

An example of teaching experiment based on this framework is discussed in Maschietto and Bartolini Bussi (2009), where the meaning of prospective drawing is mediated by the introduction of a Dürer’s glass, historical texts and a model of visual pyramid realised through treads in the classroom.

In other cases, the content of the mediation is not a specific mathematical content but a fundamental process (or cultural component) of mathematics, like argumentation and proof (Mariotti, 2006; Mariotti, 2010). Activities of problem solving are also considered in ICT environment. In those cases, we would say that an artefact can be considered with different didactics functionalities.

TEACHER EDUCATION AND MATHEMATICS LABORATORY

The question of the diffusion of the methodology of mathematics laboratory could be considered as a part of research questions about the integration of tools in mathematics education (Hoyle & Lagrange, 2010), because of the presence of tools (not only mathematical machines, but also ICT). It could also be connected to teachers’ need of resources, not only material but also human and cultural resources (Adler, 2000).

The laboratory activity is a great challenge for teachers, as it requires specific professional competences, which cannot be taken for granted (Maschietto & Bartolini Bussi, 2011). Maschietto and Bartolini Bussi discuss some kinds of activity concerning a particular mathematical machine as paradigmatic examples of mathematical laboratory activities. The activities are proposed to prospective teachers in order to:

- be experienced in a mathematical laboratory session;
- provide a model that might serve for future class activity;
- make they think over the relationships between manipulative and theoretical aspects in doing mathematics, since manipulation alone is not enough to construct mathematical knowledge.

On the basis of the experiences of the MMLab research group, a training program for practising teachers on mathematics laboratory, with mathematical machines, has been considered in the 2-year project (MMLAB-ER, Laboratories of Mathematical Machines for Emilia-Romagna[8]) funded in 2008 by the Region Emilia-Romagna (with the collaboration of several policy makers) and aimed to construct a network of mathematical laboratories in the provinces of the region and a network of practising teachers implementing laboratory sessions in their own classrooms (Bartolini Bussi & Maschietto, 2010). The second part of the project is ongoing.

In this section, the theoretical references are enriched by the documentational approach (Gueudet & Trouche, 2009), in which mathematical machines are considered as resources for teacher and mathematics laboratory.

Documentational approach

Gueudet and Trouche (2009) recall the principles of the instrumental approach and proposed a generalisation concerning the specific work of mathematics teachers. Following the distinction between artefact and instruments, they introduce a distinction between resources and documents, with the notion of documentational genesis (Figure 3). Documents are developed throughout documentational geneses starting from resources (or systems of resources).

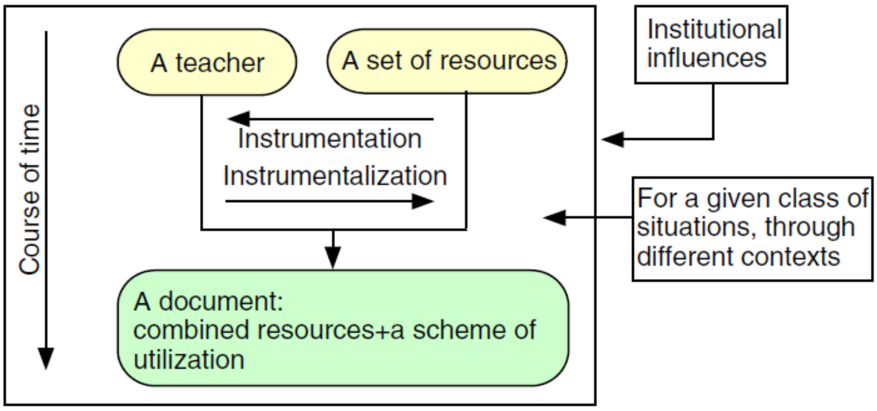


Figure 3. Documentational approach (from Gueudet & Trouche, 2009, p. 206)

[8] http://www.mmlab.unimore.it/on-line/Home/ProgettoRegionaleEmiliaRomagna.html. Accessed April 2012.

The authors retain the formula: Document = Resources + Scheme of Utilization. Schemes are related to operational invariants, which can be inferred from the observation of invariant behaviors of the teacher for the same class of situations across different contexts. As a result of the genesis, “a document is saturated with the teachers’ experience” (Ibidem, p. 205).

The structure of the teacher education program

The teacher education program of MMLAB-ER is composed of two phases (Garuti & Martignone, 2010): the first step considers six presential courses, in the second step teachers have to experiment mathematics laboratory sessions in their own classes. The participants teach at different school levels (primary and secondary school levels, 6-16 years old pupils).

In the first phase, each session concerns a specific mathematical content or activity, with different mathematical machines involved (starting with a pair of compasses, then pantographs for geometrical transformations and curve drawer and finally the arithmetical machine Zero+1[9]). It is structured following a laboratory methodology: group work about the exploration of a mathematical machine with a working sheet or geometrical construction by a ruler and compass; collective discussion on the exploration of the machine and analysis of tasks and resolution processes (Garuti, 2011). Ideas for teaching experiments are also discussed. In the second phase, teachers carry out teaching experiments in their classes. At the end, they have to hand in a logbook with their analyses of the progress of the activities.

In the second year of the project (for training in Modena and Bologna), teachers have an e-learning platform (Moodle) to support training and experimentation phases (inspired by (Guin & Trouche, 2006). They could not only download material used during the laboratory sessions, or concerning mathematical machines and teaching experiments, but also upload their own files. One goal of that implementation in our project is to provide a tool to accompany and support teachers and to foster the development of a collaborative work (Maschietto, 2010b). By means of the Wiki tool, teachers (split into groups) are asked to write a report for each meeting (each group was in charge of only one report), according to the following requests: a presentation of the topic in which teachers revised what they got from their viewpoints (called situated analysis); a reflection on what and how they made in the meeting, as well as on processes activated during the activities (analysis of shared and distributed knowledge).

ANALYSIS OF THE TEACHER EDUCATION PROGRAM

This program can be analysed at different levels of granularity: at the institutional level; at the macro level, where we see the articulation between the two phases; at a meso level, where the two phases are analysed, and at a micro level, where the individual teacher and the impact of this education in his/her professional development can be analysed, with the individual management in the classroom and the choices made. In this paper, the meso level is considered.

At a macro level, the teacher program realizes a cycle between education program and teaching practice, due to the fact that it is addressed to practising teachers and it requires an

[9] CF. Maschietto and Ferri (2007) and Maschietto (2011).

experimental phase. The return to the training happens by the way of a final session where the teachers share their experiences and reports, as logbooks and papers for the book of the project[10]. The platform supports this cycle (Maschietto, 2010b).

The training sessions are structured as laboratory sessions (Bartolini Bussi & Maschietto, 2008), where the teachers work and discuss in small groups. At a meso level, the education program is analysed by the lens of appropriation:

- Appropriation of certain mathematical machines as resources, in two levels: as instruments to make mathematics and as resources to make students do mathematics (as "vehicle of learning", Wislow, 2003);
- Appropriation of elements of the analysis of cognitive processes of subjects working with artefact and analysis of the artefact;
- Appropriation of the methodology of mathematics laboratory, combining the previous elements and teaching experiments carried out during the second phase of the training program, by the particular case of the laboratory with mathematical machines.

These terms correspond to those components that characterise mathematics laboratory as a cultural and professional challenge for teachers. The two phases of the training program will be analysed by connecting and combining instrumental and documental approaches and the theory of semiotic mediation.

Working sessions in mathematics laboratory

In our analysis of the laboratory sessions, we will refer to the reports written by group of teachers in the Wiki tool.

In the first step of the working session, the teachers are in a position of students, in front of a mathematical machine to explore by means of a worksheet (or a geometrical construction to do if ruler and compass are considered). It is important that the mathematical machines, apart from the compass, appear as new objects for teachers. As for pupils (Maschietto & Martignone, 2008), the motivation of teacher is quite high. With respect to Figure 2, this phase corresponds to the higher triangle "task – artefact – situated texts". It is the first component of a didactic cycle.

The questions asked in the worksheets (Bartolini Bussi, Garuti, Martignone & Maschietto, 2011) support the instrumental genesis of mathematical machines, as a tool for doing mathematics. It promotes instrumentalisation (how the artefact is made, which components and their characteristics) and instrumentation (how using that mathematical machine) processes. In this instrumental genesis, teachers construct an instrument to do a certain mathematical object, highlighting the mathematical knowledge that the machine evokes first in the group work and then in the collective discussion. In the worksheets there are also questions concerning the proof about the product of the machine, that is the identification of the mathematical principle at the base of its functioning. This request is strongly related to the exploration of the machine and to the processes accomplished by the teachers, because the

[10] http://www.mmlab.unimore.it/site/home/progetto-regionale-emilia-romagna/risultati-del-progetto/libro-progetto-regionale.html. Accessed April 2012.

trainer wants to bring out the invariant components of action (that is the processes allowing to have a certain result), on which the construction of utilisation schemes is based. Some conclusive questions concerns the change of some elements of the machine (instrumentalisation process) in order to study the relationships between certain changes and mathematical meanings embedded in the machine. Those questions are classified as problem solving (Garuti, 2011). Following Chiappini and Reggiani (2004), the idea of laboratory as an experiential space where mathematical knowledge is restructured through the introduction of the artefact is realised.

With respect to the framework of semiotic mediation, we can say that in the first part of education program, the mathematical machines are not chosen to mediate specific mathematical meanings by educator. Instead, the content to be mediated is represented by elements of the laboratory methodology, of the analysis of processes that can be promoted in the laboratory and analysis of exploration scheme of mathematical machines.

In their situated analysis, the teachers comment their instrumental genesis, where a schema of exploration of mathematical machines appears:

> The methodology of work seems thus established, we discuss the object as an artefact and as an instrument without being tied to the proposed worksheet. (IV session)

The teachers are experienced adults, who generally connect the mathematical machine to a precise mathematical content, that can be considered as a mix of mathematical knowledge and pedagogical knowledge. Different connections are observed depending on the grade school where teachers work: for example, the theory of conic sections is not a mathematical content taught at primary school, so those teachers' mathematical knowledge appears less available than for secondary school teachers. These differences represent richness for group work and collective discussion, even if it could become dangerous if discriminating on what to invest energy in relation to teaching contents.

During collective discussions, the teacher educator uses all the elements appeared in group works, starting from the answers to worksheets, in order to pay attention to processes of exploration of a mathematical machine (instrumental genesis), to the analysis of answers and argumentation and proof performed by the teachers themselves. In this way, the teacher experiments a form of mathematical discussion, above all a balance discussion, in the position of student. The presence of teachers from different grade levels is enriching this discussion, because they ask to specify some mathematical elements (for instance, certain steps in a proof) that seem transparent to teachers, especially secondary school teachers:

> We start working, finding and comparing the possible solutions among ourselves, split by groups, we have "animated" the group conversation, reflecting on the various educational implications and any insights / ideas, which are derived from our geometric constructions (I session).

The teachers work on mathematical knowledge and cultural aspects (Boero & Guala, 2008), which are present by the questions and for the use of mathematical machines as tools came from the history of mathematics (this is an important component of the training, because those elements are not very considered in teacher education, as stressed by Adler, 2008). In their analysis in Wiki tool, the teachers take into account all these aspects (mathematical, cultural and cognitive):

> [we] highlight the different processes, but above all the various mathematical knowledge(s) latent to the different [geometrical] constructions, detecting, among other things, a certain difficulty in transmitting orally the procedure [of construction] (II session).
>
> In our opinion, this experience can make us reflect both on the aesthetic value as well as formal [value] of a proof, and on the mental scheme of each of us, who is induced to more easily follow a type of argumentation rather than another one, and then to consider it [the first one] more good (IV session).

Based on this kind of comparison, a second genesis starts. We could call it instrumental-documentational genesis, in which a mathematical machine begins to be seen as a tool to make students do mathematics, a resource for the teacher as a professional. Following (Wislow, 2003), a mathematical machine is considered as a "vehicle of learning". Collective discussions, comparisons among different exploration strategies, argumentations but especially teachers' experiences with machines (movements, constrictions, ...) allow to detect didactic functionalities of the mathematical machines and their semiotic potentials, that is a basic elements for the use of an artefact as tool of semiotic mediation (Mariotti & Maracci, 2010). At the same time, other resources are considered, as teaching experiments previously carried out (Bartolini Bussi & Maschietto, 2006). In addition, teachers share their constructions and their proofs after the sessions, uploading files, in general dynamic geometry files, to the platform. This material represents other kind of resources, product of the collective discussions (we can see a germ of community geneses).

The analysis of written report on Wiki shows that collaborative writing is carried out in different ways by the groups, with a different participation of their members. These reports highlight several voices[11]: teacher-as-student voices in the situated analysis (when they work with artefacts), teacher-as-professional voices, in the analysis of knowledge and cognitive processes. A third voice is represented by the educator voice, that drags teachers toward didactic, cognitive and cultural analysis. But other voices are presents: students' voices (students from teachers' own classes and from proposed teaching experiments), and constructor's voice.

> We are students and teachers at the same time; these activities solicit us to make a meta-reflexion, toward a deep reflexion that reinforces the meta-cognitive teacher (aware of what he knows and what he wants to do). (II session)

During the first phase of the education program, the teachers are supported to enlarge or construct their systems of resources for teaching. The resources are not only constituted by the artefacts mathematical machines, but by the instruments (according to instrumental approach) with elements for the analysis of their didactic functionalities. In this sense, they are complex resources, as "secondary resources" for teachers (Maschietto & Trouche, 2010,

[11] The term 'voice' is used according to Bachtin, to mean a form of speaking and thinking that represents the perspective of an individual (Bartolini Bussi, 1996).

p. 45). So, this first phase supports the appropriation of the mathematical machines as resources for the mathematics laboratory.

Design and teaching experiments

Together with the laboratory sessions, teachers, according to a training contract, initiate the design of experiments to be carried out in their classrooms.

This second phase demands the conception and implementation of didactic projects of mathematics laboratory using the mathematical machines. This request forces the teachers to begin a process of documentational genesis (Gueudet & Trouche, 2009), which is supported by teacher tutors (an intermediate figure between teachers and educator) and educator, as well as by the platform, whose use allowing collaborative work is fundamental.

From the viewpoint of the documentational genesis, the teacher constructs a document for his work in the classroom, starting from the new resources (in the sense previously assumed) and existing systems of resources. He initiates instrumentalisation processes, as he fits and constructs (design for use, Folcher, 2007) worksheets for his students, and instrumentation processes, performed by *a priori* analysis, strategies for anticipating students' strategies, classroom management. These strategies can become schemes for the methodology of mathematics laboratory. It is assumed that behind these schemes there is knowledge, some arising from the training, some other that belong to the teacher outside of training, such as knowledge about the curriculum and on students, on teaching mathematics (professional knowledge, Trgalova, 2010). Concerning resources, the use of personal resources, such as dynamic geometry software, raises the question of instrumental orchestration (Trouche, 2004). According to Chiappini (2007), in this design phase, the teacher realizes a re-configuration of content to teach, in order to make it an object of investigation for students and to encourage the construction of mathematical meanings.

In the training in Modena, the teachers have decided the machines and the mathematical contents to propose to their students. They are split into four groups, each of them have a reserved space on the platform (Maschietto, 2010b). The group work allows to deep the appropriation of the new resources, to contextualise the use of the machines (as to foster argumentation and proof, or to construct specific meanings, that are related to their didactic functionalities) and, last, to support documentational genesis. For this design, a grid is proposed, whose aim is to urge into doing a priori analysis of the experiment to bring out components of laboratory methodology, in particular the kind of tasks for students and the way to manage students work and collective discussions. This grid also asks to anticipate students' difficulties and crucial points.

We consider now an example concerning teaching experiments on conic sections[12], carried out by three teachers working in three different schools (two senior high schools specialized in classical studies and one vocational school). The teachers meet twice to discuss about their projects and to define worksheets for students. During those meetings, they use the curve drawers by treads (Figure 4) to explore them again before writing tasks for students. In so

[12] http://www.didatticaer.it/cerca_didattica/macchine_matematiche_classe.aspx. Accessed April 2012.

doing, they deep their analysis of didactic functionalities in terms of didactic aims (to introduce conic section by the mediation of the mathematical machines) and semiotic potential (related to utilisation schemes and potential personal meanings of their students).

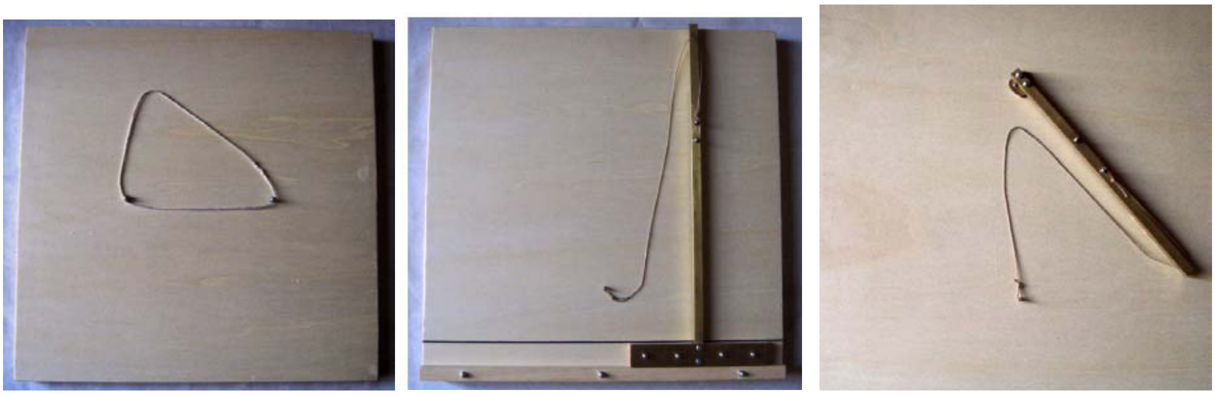
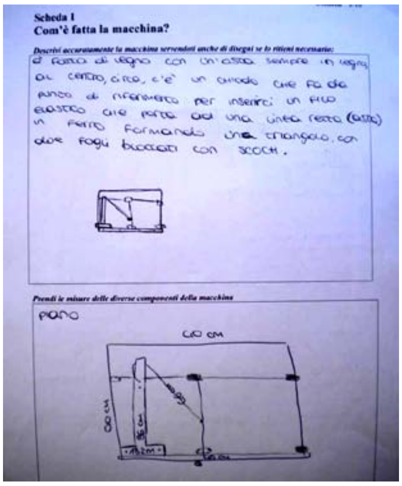


Figure 4. Mathematical machines for conics (ellipse, parabola and hyperbola)

On the basis of this collaborative work, each teacher has constructed his/her own document for the class, taking into account knowledge of students and content, knowledge of the curriculum and time constriction. So, part of the documentational genesis is highlighted by the design grid, which shows a different documentational genesis, even if there are common elements. For the three experimentations, the first session is carried out in the MMLab at the Department of Mathematics, where students are introduced to the topic through a historical perspective on conics in Greek mathematics. The ppt file used for this introduction is another resource of the training, which is changed by each teacher (some slides are deleted, other added). This detects an instrumentalisation process for each teacher. At the same time, the implementation of this session required an instrumental orchestration for teachers (orchestration of video projector, large models of cones, mathematical machines for students, paper and pencil).



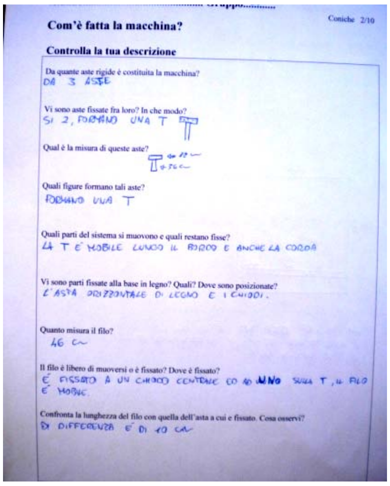


Figure 5. The first worksheet (front and back) concerning the parabola drawer

Another example of different documentational genesis concerns the content: the teacher working in a vocational school does not propose hyperbola and prefers to work on parabola as an example of function. The same teacher, in order to support his students' instrumental genesis, proposes two kinds of activities (Figure 5): on the front side of the worksheet, tasks to explore the machine are proposed; on the back, questions to control given answers are proposed.Concerning laboratory methodology and the implementation of didactic cycles, group works have had a more important place with respect to collective discussions.

This case is paradigmatic of the second phase of the training program, in which individual and collective documentational geneses take place.

At the end of class work, teachers have to present a logbook, which requires the *a posteriori* analysis of the experimented laboratory sessions. The goal is to bring out the role of mathematical machines used in the classroom, the students' cognitive processes, critical points and difficulties, but also positive aspects. In general, teachers pay attention to their students' motivation and engagement in the proposed activities. In most logbooks, a great difficulty for students lies in language and in writing or expressing their explorations, formulating conjectures and argumentations. Teachers are aware of their role during a mathematics laboratory, very different from a classical lesson, and the management of didactic time. In their analysis, teachers report also the differences between the design and the experimentation, due to time and changes depending from the answers of their students (some part to expand, new questions arose). For instance, a teacher writes:

> Each exploration proposed by the worksheets can be the beginning of an unexpected development proposed by the students: [the teacher] needs to be aware during the session and change her plan in order to seize that opportunity.

Following Folcher (2007), this teacher performs a "conception in use", resulting by user's activity with a specific aim and producing instruments that are developing during their appropriation process.

THE CONTINUATION OF THE TRAINING PROGRAM

In the documentational approach, the documentational genesis is characterised as an ongoing process rather than a process with a final step. During the training program, the teachers have constructed documents for their own use. But those documents, as well as final reports and logbooks, can be considered as resources for others teachers. When a teacher considers those as resources, he starts a documentational genesis.

In the year following the training program previously analysed, some teachers in Modena have continued to meet and work together: some primary and secondary school teachers on compass, some secondary school teachers of different level on reflection, a secondary teacher on conic sections. The platform is always available for teachers, in particular resources produced during the training.

In the first case concerning compass, primary teachers have considered the logbook written by a secondary teacher about compass as a new resources. Even if they have participated to all the sessions the year before, they need a certain time for appropriation of this new resource. Their aim is to use compass to mediate the meaning of circle (5th grade), but at the same time to work within a secondary school perspective for their pupils. The work together with the secondary teacher helps them in the instrumentalisation of the logbook. The analysis of the teaching experiment the year before pays attention to the question of time and tasks for students, in order to support instrumental genesis and construction of the mathematical meaning.

In the second case, the mathematical machine for reflection is used to mediate this meaning (Bettini, Facchetti, & Maschietto, in press). Two teaching experiments, carried out at different school levels (grades 7 and 9), are strictly intertwined and represent a good example of conception in use: the second experiment is planned on the basis of the analysis of the first one, which was modified in order to take into account students' difficulties. The analysis of the first experiment deepens the analysis of the semiotic potential of the pantograph for reflection. In these teaching experiments, instrumental orchestrations of mathematical machines, paper and pencil environment and dynamic geometry environment performed by an interactive whiteboard are taken into account.

The continuation of the training program supports teachers with respect to their appropriation of the laboratory methodology and enrich their systems of resources.

CONCLUDING REMARKS

This paper presents the analysis of a teacher education program concerning the methodology of mathematics laboratory with the lens of instrumental and documentational genesis. The aim of this training is the spread of the laboratory methodology. We try to develop the idea expressed in Maschietto and Trouche (2010), seeing a resource as an artefact, becoming, along a complex appropriation process, an instrument for a given teacher. This model of thinking resources design can be seen as a tertiary artefact for modelling teachers' development.

During this training, the teachers try laboratory sessions and construct resources for it. At a meso level, we can observe a cycle of resources-documents: from the document of the educator to resources for teacher, from teachers' documents for teaching experiments to resources for other teachers. This is a complex cycle, where a lot of elements come into play: primary and secondary resources as the mathematical machines, teacher's personal resources, knowledge on content and pupils, analysis of content and cognitive processes, etc. These geneses are supported by the structure of the training and by the use of a platform, in order to develop collaborative work.

In addition, in order to realize teaching experiment in their own classes, the teachers also have to take into account circumstances and constrictions (Bosch, 2010). According to Bosch, a great challenge for the diffusion of teaching methodology is to study in a systematic way the circumstances allowing a certain kind of activity to live in a class, in order to change blocking components into developing ones.

Systems of resources also affect educators. En effect, at the same time, teachers' documents can become resources for educator, as papers on previous teaching experiments are. For instance, the continuation of the training program starts by reading certain logbooks, in order to deepen the analysis of experiments. So, teacher training changes too. But a question arises: what is the relationship between the resources for educator and resources for teachers? When a lot of resources are available, which of them are important for the training, if any, in order to support teachers' geneses?

The construction and publication of resources on mathematics laboratory contributes to the debate about the characteristic of resources to foster their appropriation

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