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FREUDENTHAL'S WORK CONTINUES

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In this paper I address a number of projects on elementary mathematics education carried out at the Freudenthal Institute. The focus is on (a) using picturebooks to support kindergartners' development of mathematical understanding, (b) revealing mathematical power of special needs students, and (c) conducting textbook analyses to disclose the learning opportunities that textbooks offer. I discuss how these projects are grounded in the foundation-laying work of Freudenthal and his collaborators in the past and how our work will be continued.
Picturebooks, special education, subtraction, textbook analysis, didactics of mathematics

THE TITLE EXPLAINED

The title of this paper makes it likely that something is going on in Utrecht, which is indeed the case. Part of the Freudenthal Institute will move. In fact, this is the umpteenth removal of the institute in its moving history. The institute was established on January 26, 1971 as IOWO (Institute for Development of Mathematics Education), as an independent part of the State University of Utrecht with Hans Freudenthal as its first scientific director. The heart of the IOWO was the so-called Wiskobas (Mathematics in Primary School) project with Adri Treffers as one of the leading persons. On December 31, 1980, IOWO ceased to exist and was absorbed by the newly established National Institute for Curriculum Development (SLO). Only a small part of IOWO, the research part, could stay at Utrecht University and under the name OW&OC (Research of Mathematics Education and Educational Computer Center) this part became a department of the Faculty of Mathematics and Computer Science. In September 1991, one year after the death of Freudenthal, the institute was renamed *Freudenthal Institute*. Further change took place in 2005, when all science faculties of Utrecht University merged into one Science Faculty. From that moment on the Freudenthal Institute belonged to the Department of Mathematics of the Science Faculty.

A real change happened in December 2006, when we merged with the researchers from physics education, chemistry education and biology education and our name was officially changed to *Freudenthal Institute for Science and Mathematics Education*. Finally, in September 2010, we, the mathematics education part, moved from our building in Utrecht Overvecht to the University campus De Uithof where the science part was located. As a result, we were now also physically one integrated institute. So far so good.

Alas, in November 2010, the Science Faculty announced it had to make drastic cutbacks and that it also reconsidered its mission. The latter included the decision that …

research and development in mathematics education (and science education) for early childhood, primary school, special education, and vocational education were not the core business of the Science Faculty. This means that the heart of the Freudenthal Institute will be taken out, and move to the Social Sciences Faculty. The good news is that the Social Sciences Faculty has welcomed us. What this means, is that our research and development projects on mathematics education for early childhood, primary school, special education, and vocational education will not stop. Freudenthal's work continues. To articulate this, I have taken the invitation to give a Regular Lecture at ICME 12 in Seoul as an opportunity to showcase some of my present projects that will have a new future in the Social Sciences Faculty. For every project I will look back to the work of Freudenthal and his collaborators who in the past, in one way or another, laid the basis for these projects.

MATHEMATICS EDUCATION IN KINDERGARTEN: THE DIDACTICAL USE OF PICTUREBOOKS

This section addresses the role of picturebooks in kindergartners' learning of mathematics. The section is based on various sub-studies we did on this topic, each study taking a different perspective. The studies were part of the NWO (Netherlands Organisation for Scientific Research) funded PICO-ma (PIcturebooks and COncept development MAthematics) project (Project 411-04-072) and were carried out with two PhD students and with Iliada Elia of the University of Cyprus and Alexander Robitzsch of the Federal Institute for Education Research, Innovation and Development of the Austrian School System in Salzburg, Austria. All sub-studies had in common the goal of providing insight into the power of picturebooks to contribute to the development of mathematical understanding in young children. Before discussing what these studies taught us, I will return to Freudenthal and the work of his collaborators in the foundation-laying stage of the Freudenthal Institute.

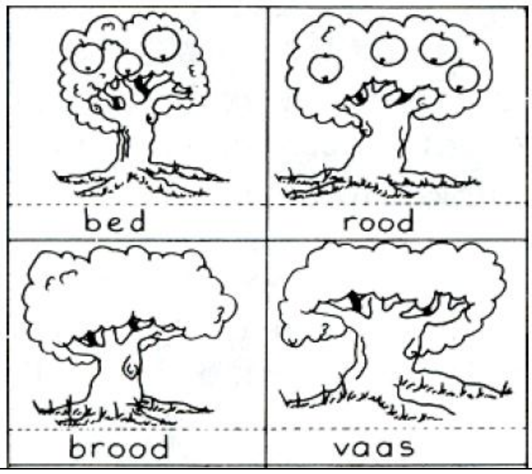
Freudenthal's and his collaborators' work in kindergarten

Kindergartners and mathematics was one of the core topics at the IOWO. People like Jeanne de Gooijer-Quint, Edu Wijdeveld, Fred Goffree, and Hans ter Heege developed many activities for prompting kindergartners to mathematical reasoning: playing with a newspaper at a magician's party in the gym, making photographs with a real polaroid camera around the sandbox in the schoolyard, looking through a set of paper binoculars, making a calendar, knocking over a pile of cans, completing two jigsaw puzzles containing the same picture but on a different scale, ordering candles in a candle shop in the classroom, discussing what the different candles may cost and comparing the difference in burning time.

Freudenthal (1984a, p. 7) himself wrote a position paper about developing education for kindergarten. Surprisingly, he started his paper by confessing that he was not familiar with kindergarten:

> "I have never been in a kindergarten class, even not when I was a kindergartner. Also, I have never dealt systematically with kindergarten education; my knowledge about the research literature in this field is restricted to what I encountered accidentally, my experience to informal meetings with kindergartners." [translated from Dutch]

However, when Freudenthal delved into the then prevailing educational practice in kindergarten, he took offence at the so-called ‘play-work sheets’ (see Figure 1) that were in use. In his eyes these sheets were meaningless assignments that could not play a role in learning.



Assignment:
In the two pictures on the top, for every letter in the word an apple is drawn in the tree. Complete this for the other words. [a]

[a] In the Netherlands, reading education only starts in grade 1.
Translation:
bed = bed
rood = red
brood = bread
vaas = vase

Figure 1. Example of (part of) play-work sheet (Freudenthal, 1984a, p. 15)

Especially the atomization, the crumbled and not integrated character of these assignments, were a real eyesore for Freudenthal. According to him, cognitive learning does not start with split-up material. The world of kindergartners is an integrated one. Therefore, instead of these paper-and-pencil sheets Freudenthal argued for offering kindergartners ‘rich contexts’ in which they could develop a first understanding of elementary mathematical structures, such as succession, repetition, cyclic-ness, and detour-ness.

Although commercial literary children’s books with stories and pictures can offer children these rich contexts and the use of picturebooks in kindergarten was not an unknown approach in the 1980s (see, e.g., Strain, 1969), Freudenthal did not mention them in his paper about mathematics education in kindergarten. In a way, this is remarkable, because Freudenthal was very interested in literature and art. Especially during World War II, when he was prohibited from pursuing his profession and was arrested for some time and had to stay in a detention camp, he wrote poems, novels, plays and short stories, including a children’s story, titled ‘Folie Antje’ [Little Ele Phant] (Mathematisch Instituut & IOWO, 1975). This story, written for his own children, is about the adventures of a young elephant and his friends. The story suffuses the spirit of ‘Winnie-the-Pooh’ and ‘Alice in Wonderland’, as may be recognized in the following passage.

> “Below the stone, there was a trap door with an iron ring. Hare Leap wanted to lift up the door, in which he also only succeeded when Little Ele Phant continued whistling. Below the trap door there was a narrow steep flight of stairs. The steps were of soil and moss, and were so slippery that Little Ele Phant and Hare Leap repeatedly slipped away. There were 144 steps. The others asked Hare Leap again and again whether it would ever come to an end! Hare Leap answered time after time that they should not be impatient. There were 144 steps, on which they could bank, and they had to finish them all.” [translated from Dutch] (Mathematisch Instituut & IOWO, 1975, p. 79-81)

Yet it is also known of Freudenthal that he was not in favor of using a world of gnomes if you could also use a real world context (La Bastide-Van Gemert, 2006). As far as I know, the only reference Freudenthal explicitly made to using picturebooks for mathematics education is in a paper about ratio (Freudenthal, 1984b), in which he discussed that picturebooks are the place where children can meet, for example, Tom Thumb and the giant.

The PICO-ma project

The rationale of the PICO-ma project that started in 2006 is the idea that stories and pictures in picturebooks can offer children rich contexts in which they can encounter mathematics-related problems, situations and phenomena which make sense to them. This learning of mathematics as a meaningful activity is one of the key principles of Freudenthal's (1973a, 1978a, 1983, 1991) approach to teaching mathematics. From a Vygotskian and action-psychological approach to learning (Van Oers, 1996) picturebooks can contribute to the process of acquiring mathematics as an activity involving meanings that are historically developed and approved. Furthermore, as Lovitt and Clarke (1992) pointed out, picturebooks can provide children cognitive hooks to explore mathematical concepts and skills. Moreover, by means of their visual images, picturebooks can give support to the initial stages of reaching a symbolic level of dealing with mathematics, which requires an ongoing semiotic activity concerning the development of meaning. Picturebooks can offer – what Van Oers (1996, p. 109) calls – "opportunities for practice with the activity of forming, exchanging, and negotiating all kinds of meaning within everyday practices." Through their interaction with picturebooks, children may be enabled to encounter problematic situations, can ask themselves questions, search for answers, consider different points of view, exchange views with others and incorporate their own findings to existing knowledge.

Our studies (Van den Heuvel-Panhuizen & Van den Boogaard, 2008a; Elia, Van den Heuvel-Panhuizen & Georgiou, 2010) that were set up to investigate children's spontaneous reactions when they are read a picturebook revealed that reading picturebooks can indeed make children cognitively active and can lead to mathematics-related utterances.

In the study by Van den Heuvel-Panhuizen and Van den Boogaard (2008a) four 5-year-old children were individually read *Vijfde zijn* [Being Fifth] (Jandl & Junge, 2000) without any questioning and probing. The story is about a doctor's waiting room in which five broken toys are waiting for their turn. The toys go into the room behind the door one by one (see Figure 2).

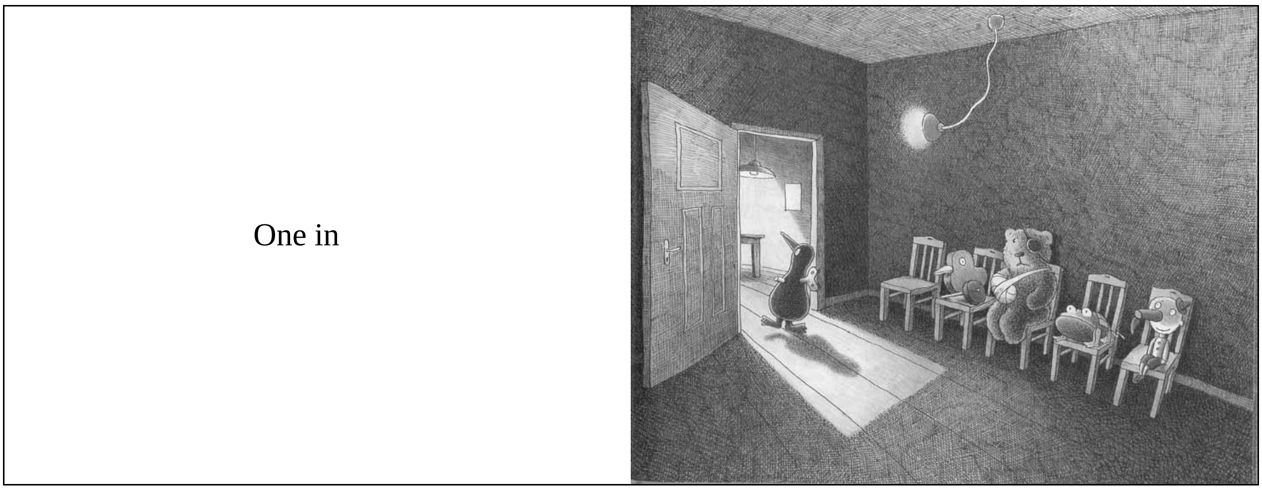


Figure 2. Page 3 of the picturebook *Vijfde zijn* [Being Fifth] (Jandl & Junge, 2000)

In total, the four children produced 432 utterances spread over a total of 22 pages, front cover, back cover and endpapers included. About half of the utterances were mathematics-related and all four children in the study were found to contribute to this result. The mathematics-related utterances were about equally distributed over the pages of the book, indicating that the book as a whole has the potential to evoke mathematical thinking.

The children's mathematics-related utterances were distinguished into two different types with respect to their content: spatial orientation-related utterances and number-related utterances. The spatial orientation-related utterances (31% of all utterances) exceeded the number-related utterances (14%). Of this latter type, most utterances referred to resultative counting, "how many there are". A closer look at these utterances revealed that in a number of cases the children structured numbers. For example, when describing a picture in which the five toys are sitting in the waiting room, a child said "two are looking at the ceiling, and three are watching television". Within the spatial orientation-related utterances, the children spontaneously took the waiting room perspective instead of the doctor's office perspective that is taken by the author of the book. As a result, there was a discrepancy between the children's utterances and the text.

Because there is evidence that picturebooks vary in the amounts and kinds of mathematics-related utterances they evoke in children (Anderson et al., 2005), some picturebooks might have more power than others to provide children an environment in which they can learn mathematics. Therefore, we set up another sub-study (Van den Heuvel-Panhuizen, & Elia, 2012) to gain more knowledge about the characteristics which picturebooks should have to contribute to the initiation and further development of mathematical understanding in young children. The study started with examining relevant academic and professional publications on views related to characteristics of picturebooks that might be supportive for developing mathematical understanding. Based on this literature review a first version of a framework of learning-supportive characteristics of picturebooks was developed. In the second part of the study, a four-round Delphi method was applied in which seven experts were asked to comment on and to work with the framework when evaluating three picturebooks. As a result of this procedure, the framework was refined and its tenability was tested. The final version of the framework is shown in Figure 3.

Apart from the power of the picturebooks themselves to elicit mathematical thinking in children, we also investigated possible ways of reading. In our view the reading style that best fits the power of picturebooks is dialogic book reading (e.g., Whitehurst et al., 1988), but with not too many questions asked by the readers of the book. To let the books do the work, we requested the teachers to maintain a reserved attitude and not to take each aspect of the story as a starting point for an extended class discussion, since lengthy or frequent intermissions could break the flow of being in the story and consequently diminish the story's own power to contribute to the children's mathematical development. In addition, we tried to enhance the power of the books and cognitive involvement of the children by having the teachers as a role model of cognitive engagement or as a person who provokes discussion with the children that brings them to mathematical reasoning as well. Therefore, we suggested to the teachers involved in our project to react to the story and pictures in the picturebooks themselves by performing behavior such as (a) asking oneself questions, (b) playing dumb, and (c) showing inquiring expressions. Classroom vignettes with examples of this behavior are provided in Van den Heuvel-Panhuizen and Elia (submitted).

Learning-supportive characteristics of picturebooks for learning mathematics

I. Supply of mathematical content

II. Presentation of mathematical content

I.1. Mathematical processes and dispositions

The picturebook shows *mathematical processes*
- Solving problems with mathematical knowledge
- Using mathematical language and representations
- Reflecting on mathematical activities and results
- Mathematical reasoning

The picturebook shows *mathematical dispositions*
- Eagerness to learn and inquiring attitude
- Tenacity in solving problems
- Sensitivity to the beauty of mathematics

I.2. Mathematical content domains

The picturebook deals with

I.2.a. *Numbers-and-counting*
- Counting sequence
- Ordering numbers
- Determining numerosity of collection (resultative counting), estimating, ordering/comparing numbers, representing numbers, operating with numbers (adding, subtracting, multiplying, dividing)
- Contextualizing numbers (giving meaning to numbers in daily life situations), positioning numbers (indicating where a number is on a numberline) or structuring numbers (decomposing or factorizing)

I.2.b. *Measurement*
- Different ways of measuring: directly measuring, pacing out units of measurement (natural units or standardized units), using measuring tools, representing and interpreting measuring results, using reference measures
- Dealing with different physical quantities such as length, volume, weight, time

I.2.c. *Geometry*
- Orienting: localizing, taking a particular point of view, rotations and directions
- Constructing: concretely constructing of objects and visualizing constructions (explaining how a building is built, reproducing a building), properties of spatial and plain shapes
- Operating with shapes and figures: geometrical transformations (shifting, mirroring, rotating, projecting, and combinations of these)

I.3. Mathematics-related themes

The picturebook deals with
- Growth
- Perspective
- Fairness
- Ratio
- Order (in time, of events)
- Cause and effect
- Routes
- …

II.1. Way of presenting

The mathematical content ...
- is addressed *explicitly* (something mathematical is happening that is explained) or is addressed *implicitly* (something mathematical is happening that is not explained)
- is *integrated* in the story (either explicitly or implicitly) or is *isolated* from the story (e.g., there is a picture of somebody wearing a dress with a nice geometrical pattern, but the story does not mention this dress)

II.2. Quality of presentation

II.2.a. *Relevance*

The picturebook ...
- contains mathematical content that is valuable for children to learn
- offers mathematical content that is presented in a meaningful context (the contexts make sense, are worthwhile, contain natural connections with other subjects)
- shows mathematics that is correct (misconceptions should be avoided, however incorrect things and inaccuracies can be learning-supportive under particular conditions)

II.2.b. *Degree of connection*

The picturebook ...
- connects mathematics with children's life and world
- connects mathematics with interests of children
- makes connections between mathematics and reality
- shows the coherence between mathematical concepts and connects different appearances and representations of mathematics
- establishes relationships between mathematics and other subjects

II.2.c. *Scope*

The picturebook ...
- makes understanding possible at different levels
- offers multiple layers of meaning
- anticipates future concept development

II.2.d. *Participation opportunities*

The picturebook ...
- offers opportunities to make children actively involved in the picture book (prompts children to do something by themselves)
- draws in children passively (makes them listen and observe)
- stimulates particular modalities (engages the children cognitively, emotionally, or/and physically)

by means of ...
- Asking questions: questioning or posing problems, asking open-ended questions, presenting challenges, conflicts, changes of perspectives, ambiguities, or mistakes
- Giving explanations: explaining mathematical content, giving hints or clues, visualizations, describing experiments, including repetition or accumulations
- Causing surprise: showing astonishment, tension, including jokes, surprising events, provocative language, offering a reward

Figure 3. Framework of learning-supportive characteristics of picturebooks for learning mathematics; from Van den Heuvel-Panhuizen and Elia (2012, p. 34)

Finally we investigated the effect of reading picturebooks to kindergartners on their performance in mathematics by conducting an experimental study with a picturebook reading program as an intervention (Van den Heuvel-Panhuizen, Robitzsch, & Elia, in preparation). In total, 384 children (4- to 6-year-olds) participated in our study: 199 children from nine kindergarten classes in the experimental group and 185 from nine kindergarten classes in a comparable control group. During three months, the children of the experimental group were read a collection of 24 literary picturebooks in which mathematical topics are unintentionally addressed by the authors of the books. The picturebooks deal with number, measurement, or geometry. All children were pretested and posttested with a project test on these topics, the so-called PICO test, and a standardized mathematics and language test.

The intervention effect on mathematical understanding was tested by conducting a regression analysis in which the PICO pretest score was used as a covariate when comparing the PICO posttest scores of the experimental and the control group (Model 1). In order to find estimates of the intervention effect with the least bias, another regression analysis was applied, in which the various variables representing children's characteristics (mathematics ability, language ability, grade, age, gender, SES, Dutch home language, urbanization level) were included (Model 2). Both models revealed a significant intervention effect (Model 1: B=.89, p=.01; Model 2: B=.71, p=.07) with the explained variance increasing only slightly between the two models: R^2=.70 in Model 1 and R^2=.73 in Model 2. The effect sizes, as defined by Cohen (1988), were calculated for each model in order to investigate the size of the general intervention effect. For Model 1 we found an effect size d=.15 and for Model 2 it was d=.12. These effect sizes can be considered as rather small (Cohen, 1988). However, comparing the size of the intervention effect with the mean increase from the PICO pretest to the PICO posttest in the control group revealed a considerable effect of the picturebook program on the children's PICO test scores. The mean increase has an effect size of d=.59, which means that the influence of the intervention amounts to 25% of this effect size (.15/.59=.25). In other words, in the experimental group the mean gain from the PICO pretest to the posttest was found to be 25% higher, compared to the control group. In Model 2, the Cohen's d is .12, indicating an increase in effect size of 20%. This finding supports the assumption that picturebook reading can contribute to the learning of mathematics.

The PICO-ma project will be continued in the PRIMAL (Picturebook Research Into Mathematical Language) project in which the influence that picturebooks can have on the development of mathematics vocabulary of young children is investigated. It is a four-year PhD research, carried out by Nathalie Martel in collaboration with the PICO-ma team and possibly a new colleague of the Social Sciences Faculty.

THE MATHEMATICAL POTENTIAL OF STUDENTS IN SPECIAL EDUCATION

This section addresses the learning of mathematics by students with special needs. In the Netherlands, about 3% of children of primary school age are in special education (SE) schools for students with mild learning difficulties. These SE students have a severe delay in their mathematical development. At the end of special primary school, SE students' scores are between one to four years behind those of their peers in regular primary schools (Kraemer, Van der Schoot, & Van Rijn, 2009). Therefore, the more advanced topics in the primary school curriculum such as ratios, rational numbers, measurement, geometry, combinatorics and data handling are often not taught in SE. The focus is mostly only on straightforwardly carrying out addition and subtraction problems with whole numbers and some multiplication and division.

The IMPULSE (Inquiring Mathematical Power and Unexploited Learning of Special Education students) project which started in 2008 aims at investigating SE students' mathematical potential through the use of dynamic, ICT-based assessment approaches that offer SE students opportunities to show what they are able to do. In this project I cooperate with my PhD student Marjolijn Peltenburg and with Alexander Robitzsch. Here, I will discuss two sub-studies of this project. The focus of the first sub-study was on SE students' performance in solving elementary combinatorics problems (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012b, submitted). The second sub-study investigated SE students' ability to apply alternative methods for solving subtraction problems up to 100 (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012a). The starting point of this study was the strong belief in circles of SE educators and psychologists in the Netherlands, but also in other countries, that students who have low scores in mathematics cannot handle different calculation methods. The idea is that it is better to teach them only one fixed method for each number operation because otherwise they get confused.

Freudenthal's and his collaborators' idea of mathematics for all, combinatorics and flexibly solving subtraction problems

The two sub-studies are in line with the idea of 'mathematics for all', which is an inalienable aspect of Freudenthal's (1968, 1991) conceptualization of mathematics and its teaching. In many observations of children's learning processes, Freudenthal (1975, 1976, 1977, 1978b) made it clear that mathematics can be done at any level. Moreover, he showed that there is hope for underachievers (Freudenthal, 1981). In this way Freudenthal guided us in revealing the mathematical potential of SE students. In particular, this also applies to Hans ter Heege (1980, 1981-1982) with his pioneering work on low achievers in mathematics.

However, there is more that guided us. With respect to eliciting young children's mathematical reasoning, Freudenthal (1978a) and Treffers (1978, 1987) emphasized the power of combinatorial counting problems (see also Van den Brink et al., 1973). These problems offer children opportunities to mathematize and discover abstract structures (combinations) in concrete materials such as different routes to travel.

Although even Freudenthal (1973b) once pleaded for one solution method when a very confusing subtraction method was suggested for low achievers, he (1991, p. 76) also warned us against inflexible instruction especially for these students: "Flexibility [referring here to the use of palpable material] should be allowed, and if need be, taught rather than fought." Regarding subtraction, Freudenthal (1982, 1983, p. 107) emphasized how necessary it is to work on a broad mental constitution of mathematical concepts. Just as addition can appear as putting together and as appending, we should also address the two appearances of subtraction: taking away and finding the difference, or in his words: "explicitly taken away suffices as little for the mental constitution of subtraction as uniting explicitly given sets suffices for addition."

The IMPULSE project

Solving combinatorics problems (see details in Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012, submitted) – This sub-study investigated SE students' mathematical potential by examining whether SE students' success rate and strategies in solving two- and three-dimensional combinatorics problems differ from those of students in regular education (RE). By designing accessible tasks in a meaningful context and presented in a dynamic ICT environment we aimed to give both groups of students the opportunity to demonstrate their abilities. The context was about dressing puppets. The students had to find all possible outfits by combining clothing items (see Figure 4).

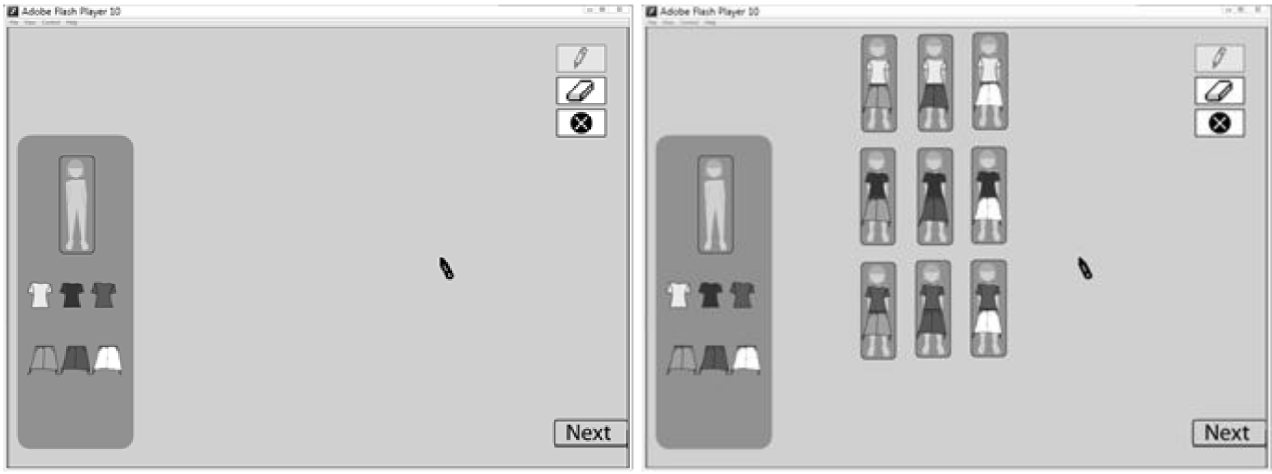


Figure 4. Screenshots of combinatorics ICT environment; left: supply of little puppets and clothing items; right: all possible outfits

In total, 84 students from five SE schools and 76 students from five RE schools participated in this sub-study. Of each school four randomly chosen students were involved who scored near the 50th percentile on the mid-grade levels of the CITO LOVS test of grades 2, 3, 4, and 5.

The SE students correctly solved the combinatorics tasks in 56% of all cases (students *x* tasks). For the RE students this applied for 57% of all cases. The difference in success rate between the two groups was not significant; $t(158)=.26$, $p=.79$. Furthermore, no significant differences were found in the use of systematic, semi-systematic and non-systematic strategies ($Phi=.051$, $Chi^2=2.485$, $df=2$, $p=.29$).

Figure 5 shows the relationships of the success rate and strategy use with the students' mathematical level for both SE and RE students. Especially remarkable is the larger 'growth' of SE students in their use of systematic strategies over the mathematics levels.

A further observation was that all students, in SE and RE, used the ICT-based assessment environment in a natural and self-evident way, with no difficulties in using the digital manipulatives. This observation strongly illustrates that digital manipulatives can be a powerful tool for eliciting mathematical problem solving.

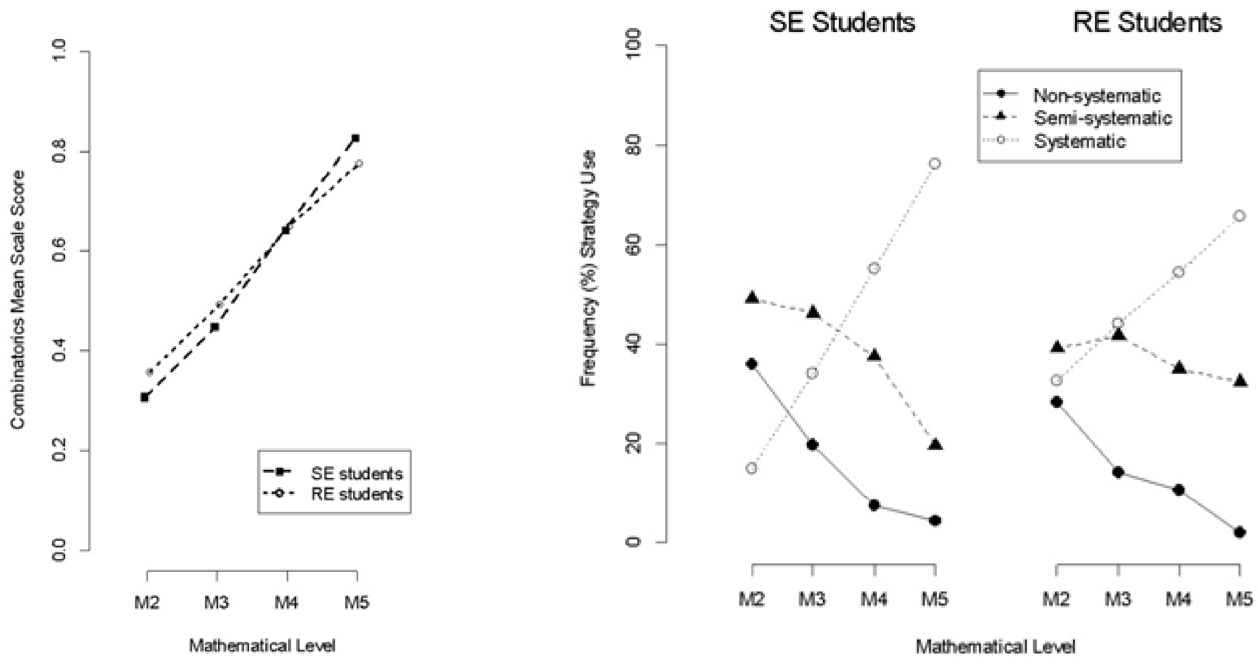


Figure 5. Relation of success percentage on combinatorics test (left) and strategy use (right) with students' mathematics level for SE and RE students

Solving subtraction problems by adding on (see details in Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012a; Van den Heuvel-Panhuizen, 2012) – The methods that can be applied for carrying out subtractions up to 100 can be described from two perspectives (see Figure 6).

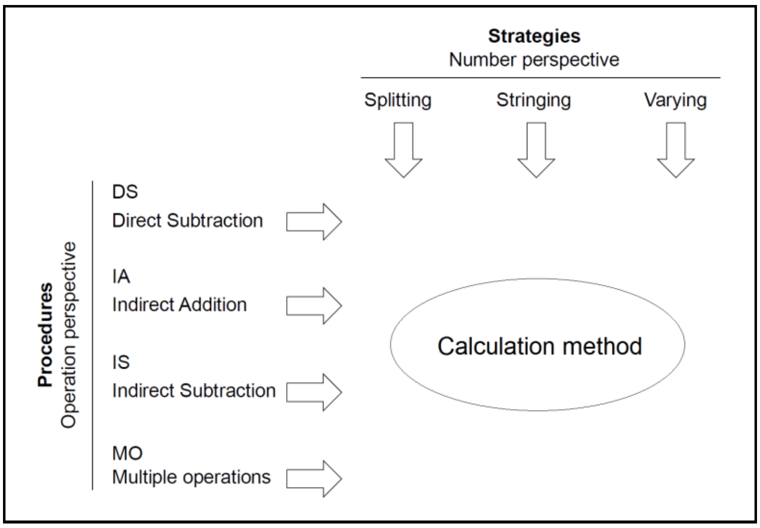


Figure 6. Calculation methods for subtraction up to 100

From the operation perspective subtraction problems up to 100 can, for example, be solved by: direct subtraction (DS), indirect addition (IA), and indirect subtraction (IS). The number perspective describes how the numbers involved are dealt with. Roughly speaking, there are three strategies: splitting (the minuend and the subtrahend are split into tens and ones and then the tens and ones are processed separately), stringing (the minuend is kept intact and the subtrahend is decomposed in suitable parts which are subtracted each after each other from the minuend) and varying (the minuend and/or the subtrahend are changed in order to get an easier subtraction problem). Although in theory all three strategies can be combined with each of the four procedures, not all combinations are that suitable.

Connected to the debate about whether or not teaching SE students one fixed method for solving number problems, there is the controversy on whether SE students should be taught IA to solve, for example, a problem like 62–58 (i.e., calculating 58+2=60, 60+2=62, so the answer is 4, instead of calculating 62–50=12; 12–2=10 and finally 10–6=4).

The present sub-study was set up to answer the question whether SE students can make spontaneous use of IA for solving subtraction problems up to 100. In total 56 students from fourteen second-grade classes in three Dutch SE schools participated in the study. The participating students were 8-12 years old, with an average age of 10 years and 6 months (*SD*=10.4 months). These students were 1 to 4 years behind in mathematics compared to their peers in regular primary school.

Data were collected with an ICT-based Subtraction test in which the item characteristics were varied systematically over the fifteen items. These characteristics include number characteristics (the size of the difference between minuend and subtrahend, whether the tens have to be crossed and whether or not minuend and subtrahend are close to a ten) and format characteristics (bare number problem or context problem). The context problems either described a taking-away situation or an adding-on situation.

Our study showed that SE students: (a) are able to use IA spontaneously to solve subtraction problems, (b) are rather flexible in applying IA to solve subtraction problems, (c) are quite successful when solving subtraction problems by IA.

The plan is to continue the IMPULSE project in the ExPo project in which informing teachers about the mathematical power of students with special needs will be used to raise teachers' expectations and consequently increase their students' achievement. If our proposal is granted, the project will be carried out together with the Leibniz Institute for Science and Mathematics Education Kiel (IPN) and the Social Sciences Faculty of Utrecht University.

TEXTBOOK ANALYSES TO SECURE RELEVANT OPPORTUNITIES TO LEARN

More than any other project I am currently involved in, the META (Mathematics Education Textbook Analyses) project discussed in this section is grounded in the didactics of mathematics. Through analyzing textbooks for mathematics education, the project examines what mathematical content is taught in primary school and how it is taught. In the META project, I collaborate with my PhD student Marc van Zanten, who also is a mathematics teacher educator of prospective primary school teachers.

Freudenthal's and his collaborators' work on textbook analyses

Because textbooks have a crucial role in determining mathematics education, the Wiskobas group at IOWO started carrying out textbook analyses (De Moor & De Jong, 1980) as early as 1975. This allowed them to give advice to teachers and school teams on choosing a textbook series. Especially in the Netherlands giving such help to teachers was (and is still) essential because of the absence of a centralized textbook design and the lack of a state authority which approves textbook series before they are put on the market. The textbook analyses by IOWO, and its successor OW&OC, resulted in a series of documents published in 1980, 1983, and 1987, which described and evaluated the available textbooks. The crowning glory of this textbook analysis work was the PhD thesis of De Jong (1986) whose study investigated the influence of the Wiskobas project on textbooks. It is interesting to mention here that, after IOWO ceased to exist, Rob de Jong became a staff member of the Department of Education of the Social Sciences Faculty of Utrecht University. Thus, in a way, he preceded us.

And what about Freudenthal's thoughts about textbook analyses? Although it is clear that he studied numerous textbooks (Freudenthal, 1973a) and De Jong (1986) mentioned him as involved in his analysis, Freudenthal (1991, p. 177) said later that he had never reviewed textbooks (which in his eyes might not be the same as analyzing textbooks) and that he was reluctant to do so, because he was "wonder[ing] how much teaching experience is required to undertake the task of reviewing textbooks before or without having used them." Moreover, "[w]hether one likes it or not, textbooks are merchandise, and in the marketplace good quality is what appeals to the needs and the tastes of prospective customers." As Freudenthal admitted, "[t]his would seem to be a gloomy perspective for change". However, he also gave us hope, when continuing "were it not that needs can be stimulated and tastes can be educated."

Considering the reform movement that has been taken place in the Netherlands (Van den Heuvel-Panhuizen, 2001) textbook analyses have unmistakably had their desired effect. In the 1980s, the market share of textbooks with a traditional, mechanistic approach was 95% and the textbooks with a reform-oriented approach – based on the idea of learning mathematics in context to encourage insight and understanding – had a market share of only 5%. In 1987, the market share of these latter textbooks was around 15%. In 1992 this had increased to almost 40%, and 75% in 1997. In 2004, the reform-oriented textbooks reached a 100% market share. However, due to the debate that has taken place in the Netherlands after 2007, in which the reform-oriented approach is criticized in favor of a return to the traditional, mechanistic approach (Van den Heuvel-Panhuizen, 2010), in their new editions some textbook series have adapted the content (more emphasis on algorithms) and teaching approach (more training of knowledge and skills). Therefore, textbook analyses are once again important to inform teachers and others about students' opportunities to learn mathematics with these textbooks.

The META project

The focus in the first sub-study of the META project was on subtraction up to 100 (see details in Van Zanten & Van den Heuvel-Panhuizen, submitted). In agreement with the idea of Freudenthal (1978a, 1983) that a didactical phenomenology is an indispensable precondition of educational research in mathematics, we started with what we called a mathedidactical analysis of the concept of subtraction (later more about this difference in terminology). This analysis, together with a literature review, resulted in an analysis framework covering three perspectives: the mathematical content, the performance expectations and the learning facilitators included in the textbooks to be analyzed (Figure 7).

Content	Types of problem	Decomposing numbers up to 10	
		Backwards counting with jumps of 10	Starting from tens
			Starting between tens
		Subtraction up to 10	
		Subtraction up to 20	Between 20 and 10
			Bridging 10
		Subtraction up to 100 [T=Tens U=Units]	T – T
			TU – T
			TU – U
			TU – TU
			T – U (from T)
			T – TU (from T)
			TU – U, bridging T
			TU – TU, bridging T
	Format of problem	Bare number problems	
		Context problems	
	Semantic structure of problem	Subtraction as taking away	
		Subtraction as determining the difference	
Performance expectations	Knowing subtraction facts	Knowing subtraction facts up to 10	
		Knowing subtraction facts up to 20	
	Carrying out subtractions	Being able to use standard methods	
		Being able to use alternative methods	
	Using subtractions in context problems		
	Understanding of subtraction	Being able to choose an appropriate method	
		Being able to give explanations	
Learning facilitators	Degree of exposure	Number of tasks	
	Structure of exposure	Sequence in types of tasks	
		Sequence in level of abstraction	
	Didactical support in exposure	Use of models	
		Use of contexts for meaning making	
		Use of textual instructions	
		Use of analogy with easier subtractions	
		Use of own productions	

Figure 7. Framework for textbook analysis regarding subtraction up to 100

Our textbook analysis was applied to two recently developed textbook series that, although they are from the same publisher, clearly position themselves in two contrasting approaches to mathematics education. The first textbook series, called ‘Rekenrijk’ (RR) (Bokhove et al., 2009), is a reform-oriented textbook series. The name refers to ‘rich arithmetic’ and ‘realm of arithmetic’. The second textbook series, called ‘Reken zeker’ (RZ) (Terpstra & De Vries, 2010), is a new textbook series that was presented as an alternative for the reform-oriented approach. The name of this textbook series means ‘arithmetic with certainty’. The analyzed materials were the Grade 2 books from which we excluded the assessment lessons and the additional tasks for students who need repetition or more advanced content.

The analysis revealed that both textbook series in grade 2 have more subtractions between 20 and 100, and less subtractions in the range up to 10 and up to 20 (see Table 1). However, in RR most subtraction tasks involve bridging a ten, while in RZ most subtraction tasks do not bridge a ten. Furthermore, attention for the prerequisite knowledge for these problems also differs in the two textbook series. For decomposing numbers up to 10, RR has a substantial number of such tasks and RZ almost none. For counting backwards with tens (e.g., 46-36-36), RR has very few tasks, while RZ has none. When examining the content in grade 1, we found that both textbooks put more emphasis on prerequisite knowledge in grade 1 than in grade 2. But again, in grade 1 there were more such tasks found in RR (418) than in RZ (167).

Table 1: Types of subtraction-related tasks in RR and RZ in grade 2

Types of tasks	RR		RZ	
	f	%	*f*	%
Prerequisite knowledge	130	11%	5	0%
Decomposing numbers up to 10	107	9%	4	0%
Backw. count. with 10 starting from tens	4	0%	1	0%
Back. count. with 10 starting betw. tens	19	2%	0	0%
Subtraction up to 10	153	13%	78	5%
Subtraction up to 20	311	27%	261	18%
Subtraction up to 20 between 20 and 10	79	7%	135	9%
Subtraction up to 20 bridging 10	232	20%	126	9%
Subtraction up to 100	572	49%	1096	76%
Subtraction up to 100 T – T	14	1%	62	4%
Subtraction up to 100 TU – T	120	10%	115	8%
Subtraction up to 100 TU – U	17	1%	197	14%
Subtraction up to 100 TU – TU	2	0%	135	9%
Subtraction up to 100 T – U	29	2%	81	6%
Subtraction up to 100 T – TU	12	1%	26	2%
Subtraction up to 100 TU – U bridging T	90	8%	234	16%
Subtraction up to 100 TU – TU bridging T	288	25%	246	17%
Total number of subtraction-related tasks	1166	100%	1440	99%*

* Due to rounding off the total does not add up to 100%

Table 2: Semantic structure in subtraction-related tasks in RR and RZ in grade 2

Semantic structure	RR		RZ	
	f	%	*f*	%
Tasks reflecting taking away	210	18%	403	28%
Tasks reflecting determining difference	53	5%	0	0%
Tasks reflecting taking away and determining difference	28	2%	0	0%
Tasks not having a distinguishable semantic structure	874	75%	1037	72%
Total number of subtraction-related tasks	1166	100%	1440	100%

The two textbooks also differ in paying attention to the semantic structure of the problems (see Table 2). However, this result only applies to the 25% of the RR tasks and the 28% of the RZ tasks which have a distinguishable semantic structure. Although both textbook series address subtraction as taking away (Figure 8a/b), subtraction as determining the difference is only dealt with in RR (Figure 9).

Another difference between the two textbook series is the degree in which they relate addition and subtraction to each other. RR explicitly pays attention to this (see an example in Figure 10), whereas RZ does not. Moreover, only RR deals with subtractions in an addition format, which elicit subtraction as adding on (e.g., 3+…=6 and 27+…=32) (see Figures 10 and 11). However, missing number subtractions (e.g., 19=20-… and 26-…=21) are only dealt with in RZ.

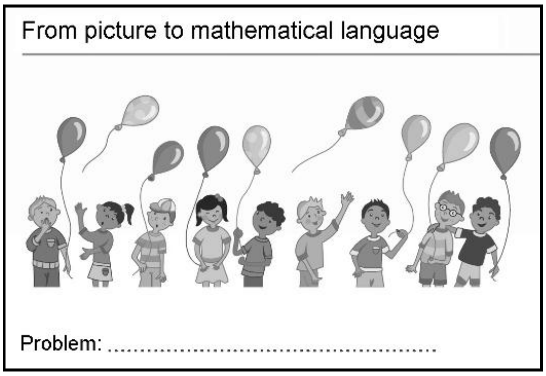


Figure 8a. RR task that reflects subtraction as taking away (RR Workbook 4a-1, p. 14)

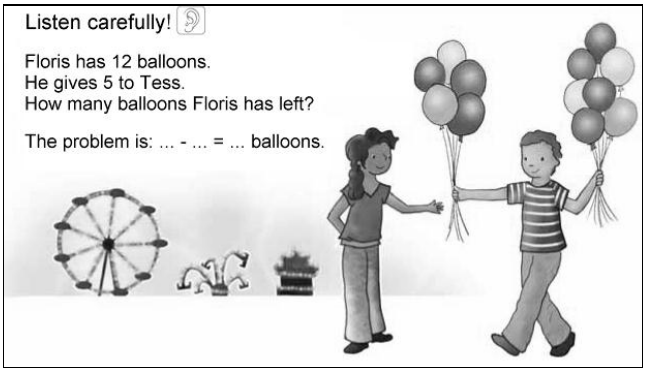


Figure 8b. RZ task that reflects subtraction as taking away (RZ Book 4a, p. 20)

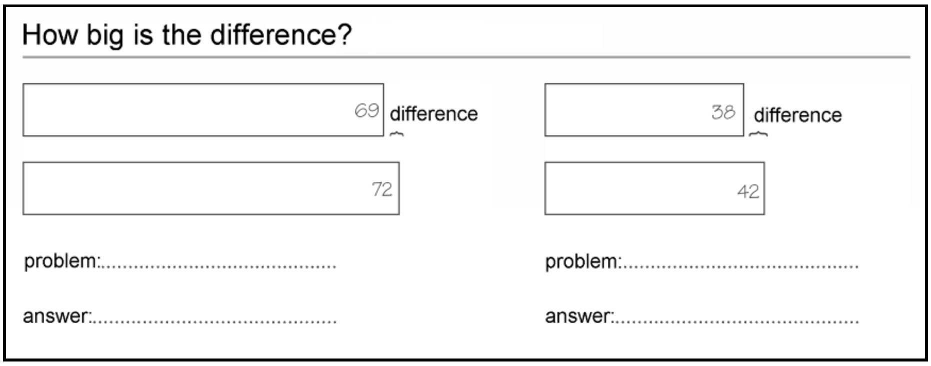


Figure 9. RR tasks that reflects subtraction as determining the difference (RR, Workbook 4b-2, p. 58)

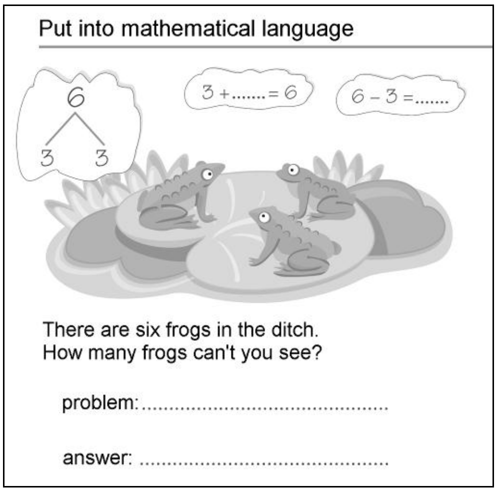


Figure 10. The relationship between addition and subtraction in RR task (RR Workbook 4a-1, p. 4)

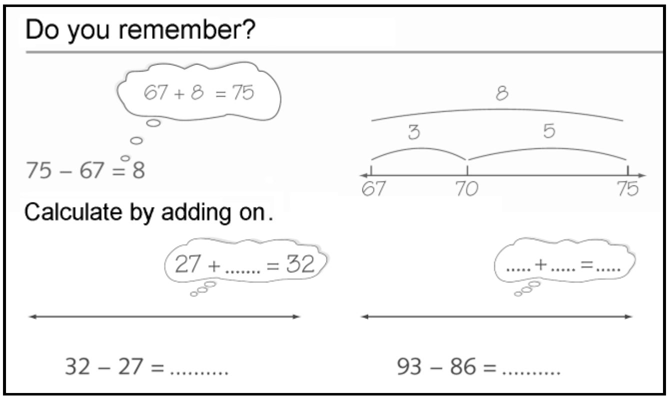


Figure 11. Subtraction as adding on in RR task (RR Workbook 4b-2, p. 61)

To measure the performance expectations regarding understanding, we determined which tasks explicitly offer directions or questions that undoubtedly have the intention to prompt students' reasoning. In RR, we found 111 such performance expectations. They include directions to students to explain their thinking, visualize their calculation method or choose an appropriate calculation method for a given subtraction with certain numbers. In RZ, we did not find clearly distinguishable performance expectations regarding understanding.

RR offers didactical support for 77% of its tasks, and RZ for 23% of its tasks. Both textbook series use models, textual instructions and analogy with easier subtractions as a form of didactical support. Contexts for meaning making and own productions are only used in RR.

Apart from not giving much didactical support, another shortcoming of RZ is the lacking match between model and strategy (Van den Heuvel-Panhuizen, 2008a). RZ uses base-10 arithmetic blocks (which is a group model that fits more to a splitting strategy) to support stringing (Figure 12a). To a certain degree, a similar inadequacy applies to RR when using a particular symbolic representation of subtraction like …-68=…, which does not match with the presentation on the empty number line that refers to 73-…=68 or to 68+…=73 (Figure 12b).

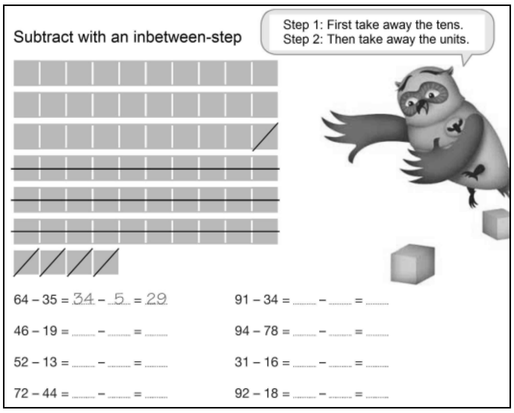


Figure 12b. RZ use of blocks combined with stringing (RZ Learn-workbook 4e, p. 12)

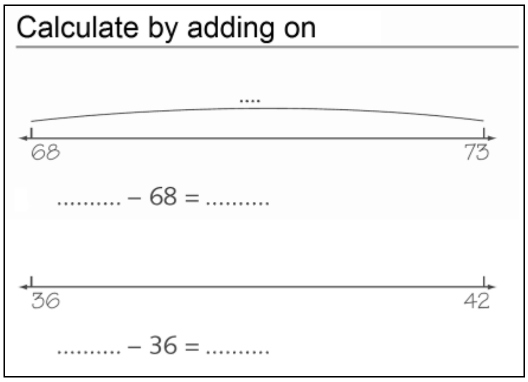


Figure 12b. RR use of empty number line (RR Workbook 4b-2, p. 78)

Our textbook analysis on subtraction up to 100 in grade 2 revealed that the two textbooks series really differ with respect to offering students learning opportunities. For example, it really makes a difference for the students whether or not they are offered a broad mental constitution of subtraction, whether or not they are presented reflection-eliciting questions and whether or not there is a match between models and symbolic representations or models and calculation methods. Of course, what is in the textbook is not necessarily similar to what is taught in class, but following Valverde et al. (2002, p. 125), there is enough evidence that "how content is presented in textbooks (with what expectations for performance) is how it will likely be taught in the classroom." Therefore, textbook analyses can give us a first inside view in how a subject is taught. As such, textbook analyses are a crucial tool that can preserve us from the danger of having a teaching practice that is not in agreement with the intended curriculum and does not offer students the desired learning opportunities. How necessary such analyses are, was shown when a textbook analysis disclosed that higher-order problem solving is lacking in Dutch mathematics textbooks (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009).

These and other examples of textbook analyses make it clear that making known the learning opportunities that textbooks offer is equally important as examining the efficacy of textbooks. When students do not encounter particular content, we cannot expect them to learn this content. Therefore the results of textbooks analyses are relevant for all involved in education: for teachers (when using textbooks), for mathematics educators (when introducing prospective teachers to textbooks), for inspectors (when controlling the quality of education), and for textbook authors (when writing or revising textbooks). Last but not least, textbook analyses are also important for the further development of the didactics of mathematics as a scientific discipline. In textbooks, as the potentially implemented curriculum, a broad variety of operationalized didactical knowledge converges – even including didactical fallacies – which can feed our thinking and understanding of how to teach mathematics.

MORE ELEMENTARY MATHEMATICS EDUCATION RESEARCH AND BEYOND

Besides the projects discussed above, other research projects in elementary mathematics education (including primary school and the adjacent areas of pre-school, kindergarten, and the beginning of secondary school) are presently carried out, just finished, or will soon start at the Science Faculty or the Social Sciences Faculty.

In the Curious Minds project, two new studies are initiated together with Paul Leseman of the Social Sciences Faculty. The first study is about the role of embodied cognition and representational redescription in children's understanding of phenomena in mathematics, science and technology. The second study is about how perception-action affordances of mathematics, science and technology tasks can elicit and guide children's exploration behavior towards discovering scientific principles embedded in these tasks.

Furthermore, we are looking for a continuation of the POPO (Problem Solving in Primary School) project that was aimed at investigating early algebra in primary school, i.e. the use of an online game to give primary school students experience in dealing with covarying quantities (Kolovou, 2011). Together with the Social Sciences Faculty a new interlinked research project is in preparation that is meant to contribute to a thorough theoretical and practical understanding of how higher-order thinking in mathematics develops and can be fostered in primary school, in particular in the subdomains of data handling, probability, and early algebra. The project will be theoretically grounded in embodiment theory and variation theory and will make use of interventions with ICT environments containing mathematical applets and with what we call 'learning movies'. Another project in which the transition from arithmetic to algebra is scrutinized is the PhD study of Al Jupri. The goal of this IISPA project is to understand and improve Indonesian students' low performance in algebra (Jupri, Drijvers, & Van den Heuvel-Panhuizen, in preparation).

Also close to the core work at the Freudenthal Institute is Ariyadi Wijaya's PhD study into the difficulties that particularly Indonesian students experience in solving context-based mathematics tasks (Wijaya, Van den Heuvel-Panhuizen, Doorman, & Robitzsch, in preparation). In this CoMTI project, the principle of teaching mathematics-in-context is revisited by investigating this principle when it is applied in another cultural context.

In the BRXXX (Basic Number Skills with Mini-games) project, in which I work together with my PhD student Marjoke Bakker and my colleague Sylvia van Borkulo, the focus is on students' learning of multiplication tables – or more formally expressed – the learning of multiplicative relationships of positive integers. As such this is a classic topic, which has been investigated many times at the Freudenthal Institute in the IOWO and OW&OC times – as well as in many other places in the world – but in the BRXXX project we situate this learning in a new learning environment. The students play mini-games and are shown 'learning movies' which explain to them the ins and outs of the games and give them hints to cleverly play the games by which they are implicitly put on the track of better understanding multiplicative relationships between numbers, and eventually will be led to an improved performance. The effect of the mini-games is examined in three conditions (playing the games in a mathematics lesson context, just playing at home, and playing at home with afterwards only a class discussion). Moreover, there is a control group that plays mini-games in class on a topic not related to multiplicative relationships. Besides the use of online mini-games and the online assessment of the students' performances – which was also applied in the POPO project – the BRXXX project also reflects another new step in mathematics education research at the Freudenthal Institute. That is its large scale. In total, approximately 1500 students are involved who we are following over more than two years (of which about 250 students in special education, who are followed for only one year). The focus will be both on their understanding of multiplicative relationships and their appreciation of mathematics as a school subject. Some first results can be found in Bakker et al. (2011) and Van Borkulo et al. (2011).

In the SANPAD-funded COCA (Count One, Count All) project, carried out in collaboration with the University of Cape Town and the Cape Peninsula University of Technology, a teaching/learning trajectory for number in primary school has been developed (Van den Heuvel-Panhuizen, Kühne, & Lombard, 2012). The project was mainly aimed at professionalization of teachers, i.e. South African teachers in the foundation phase. The new trajectory description was inspired by the TAL teaching/learning trajectory on number (Van den Heuvel-Panhuizen, 2008b) which was developed at the Freudenthal Institute. The new trajectory description was initially meant as a support for teachers involved in the COCA project, but now it is also available for other teachers, as well as for teacher advisors, teacher educators, and researchers of mathematics education.

Finally, a few words about the recently started ICA (Improving Classroom Assessment) project, again a project building on earlier research and development activities carried out at the Freudenthal Institute, in this case especially on the assessment work by De Lange (e.g., 1987, 2007) and myself (Van den Heuvel-Panhuizen, 1996, 2003). However, in contrast with this older work on assessing students' understanding of mathematics in which the focus was mostly on task design, the ICA project involves the assessment activities undertaken by the teacher. In this way the ICA project can be considered a continuation of the CATCH (Classroom Assessment as a basis for Teacher CHange) project that the Freudenthal Institute ran in the USA, and which was aimed at using classroom assessment as a means for professional development of teachers (Dekker & Feijs, 2005). In the ICA project, my PhD student Michiel Veldhuis and I work together with researchers from Cito (Central Institute for Test Development in the Netherlands) and Twente University. We started with a joint survey on how primary school teachers in the Netherlands collect data about their students' learning processes so that they can make informed decisions on how to continue teaching. In the second phase of the project, the Freudenthal Institute part of the ICA project will develop jointly with teachers a collection of informative classroom techniques to improve classroom assessment. In the third phase, the effect of the improved classroom assessment on student achievement will be evaluated in an educational experiment. Soon, the ICA team will be extended with Xiaoyan Zhao, my new PhD student from China with whom we will carry out a study about classroom assessment in China.

A BLUE PRINT FOR CONTINUING OUR WORK

In mathematics education research carried out at the Freudenthal Institute four different perspectives are included: the students' learning, the teachers' teaching and learning, the teaching/learning process and the environment – understood in the broadest sense – in which this process takes place, and the assessment to inform teachers and students (and others) about the teaching/learning process (see Figure 13). Of course, having these different viewpoints does not apply exclusively to research at the Freudenthal Institute, but can be recognized in mathematics education research in general. Furthermore, it should be clear that making a distinction between these different viewpoints does not mean that they are dealt with in isolation. In most of our projects the different perspectives are considered in close connection with each other.

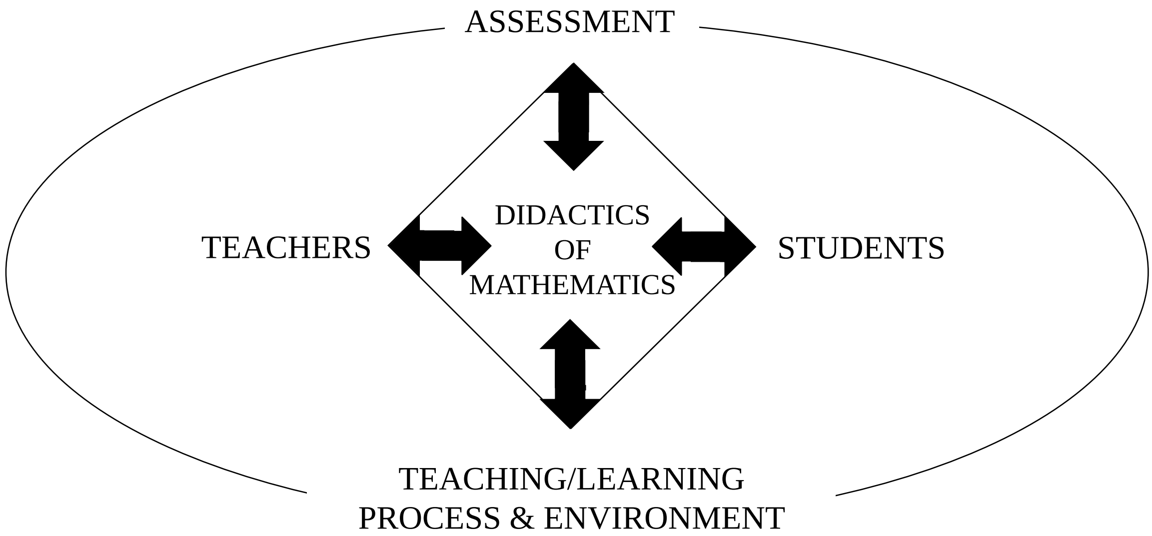


Figure 13. Different perspectives in mathematics education research

Furthermore, what all projects have, or should have, in common, is that whatever perspective is taken, research within a project should both contribute to the didactics of mathematics education as a scientific discipline and emerge from, or at least be guided by, knowledge and theories generated within the didactics of mathematics. Certainly, research of mathematics education can also be informed by other scientific disciplines within the educational and learning sciences. However, in the figure provided I left these disciplines out both for clarity reasons and to focus on the didactics of mathematics.

Depending on the research questions that are at stake within the various perspectives, research in mathematics education makes use of different research methods (Figure 14). Among these methods, design research approaches have undeniably a very prominent place, especially when the goal is to develop educational material. Nevertheless, other research methods such as quasi-experiments (including pretest-posttest designs and micro/ macrogenetic designs), surveys (including comparative achievement studies), and document studies (including textbook and software analyses) are equally significant for the further development of the didactics of mathematics. What matters is whether the research methods fit to the research questions and whether the methods guarantee robust, tenable findings – and this also applies to design research. Yet we should not forget Freudenthal's warning, expressed in a letter sent to Henry Pollak in 1977 (see La Bastide-Van Gemert, 2006, p. 284): "The greatest danger is the so-called empirical work, processed with statistical nonsense methods."

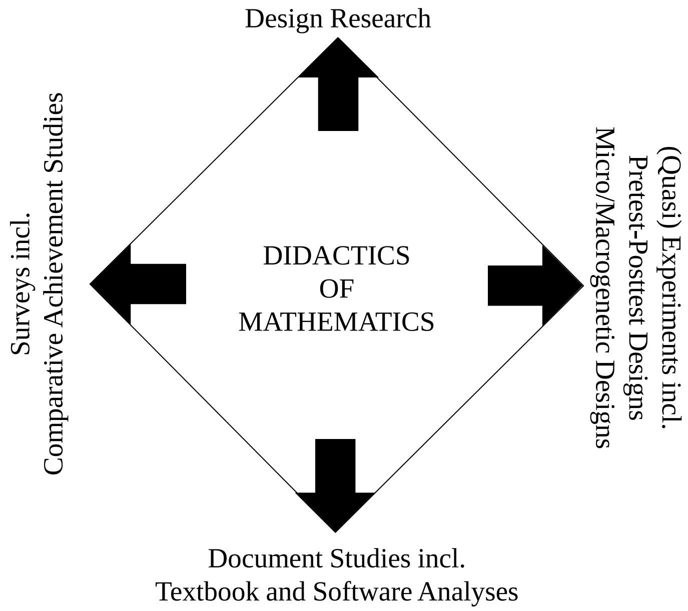


Figure 14. Different research methods

But there is more. Research in mathematics education does not only require sound empirical methods as they are generally used in the social sciences. To make research results relevant for students' learning of mathematics, the application of these methods should deal with mathematics that makes sense and is worthwhile to be learned. To achieve this, the empirical methods should be nourished by analyses that are related to mathematics. These analyses form the heart of the didactics of mathematics (see Figure 15).

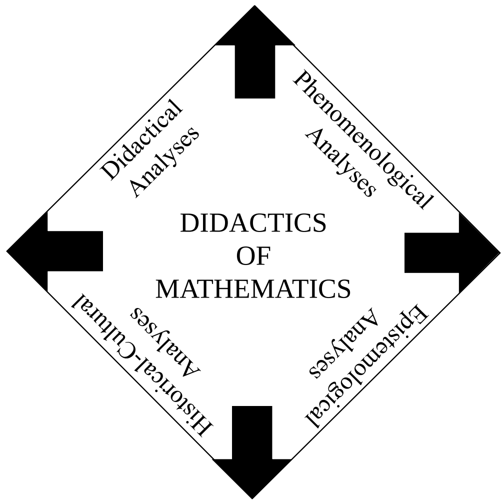


Figure 15. Mathematics-related analyses that constitute the didactics of mathematics

The didactical analyses are aimed at revealing the nature of the mathematical content as a basis for teaching this content. By identifying the determining aspects of mathematical concepts and the relationships between concepts, knowledge is gathered about, for example, the didactical models that can help students to understand these concepts. Further analyses can make this knowledge basis for research in mathematics education stronger. Phenomenological analyses disclose possible manifestations of these mathematical concepts in reality and can suggest contexts in which students can meet these concepts. Epistemological analyses focus on students' learning processes and can, for example, uncover how students in a classroom interaction can make a shift in mathematical understanding. Finally, in historical-cultural analyses, we may come across various approaches to teaching mathematics in the past and in other countries through which we can gain a better understanding of how to learn mathematics and how education can contribute to it.

To avoid misinterpreting Figure 15, I have to say that these analyses do not exclusively fit to a particular research method but can be applied in combination with any of the methods mentioned in Figure 14. Moreover, these analyses are not all required in one research project. The focus can be on one type of analysis or a combination of different approaches.

What is essential of these analyses is that they all take mathematics as their starting point. Considering these analyses as the heart of the didactics of mathematics education follows strongly the ideas of Freudenthal. Although he did not distinguish four different types of analysis, Freudenthal's DNA is firmly rooted in this heart.

In his *Preface to a Science of Mathematical Education*, Freudenthal (1978a) explained that a profoundly scrutinizing analysis of the subject matter is fundamental to educational research in mathematics. The name he chose for this analysis, which he exemplified by an analysis of the topic of ratio and proportion, was "didactical phenomenology". However, after naming it in this way, he added immediately:

> "[T]he name does not matter; nor is that activity an invention of mine; more or less consciously it has been practised by didacticians of mathematics for a long time. In various earlier books and papers I have given examples of the didactic phenomenology of mathematics, and I hope to deal with it comprehensively in another book" (Freudenthal, 1978a, p. 116).

That later book was *Didactical phenomenology of mathematical structures* (Freudenthal, 1983) in which he gave more examples of didactical phenomenologies. Although he also included in this book a short chapter about the method, it never became very clear how such an analysis should be carried out. Another characteristic of Freudenthal's didactical phenomenology is that it does not only focus on its strict meaning of describing how "mathematical concepts, structures and ideas [can] serve to organise phenomena – from the concrete world as well as from mathematics" (Freudenthal, 1983, p. 28), but that it also encompasses other ways of analyzing mathematical content for educational purposes. In fact, the mathematics-related analyses mentioned in Figure 15 can all be recognized in Freudenthal's didactical phenomenology. For us, the challenge is to keep this heart alive and equip it with a strong methodology under the umbrella of both the Science Faculty and the Social Sciences Faculty of Utrecht University.

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