ICMI STUDY 24

CONFERENCE PROCEEDINGS

School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities

November 25-30, 2018

Tsukuba, Japan

University of Tsukuba

Editors: Yoshinori Shimizu and Renuka Vithal

International Commission on Mathematical Instruction

筑波大学
University of Tsukuba

The Twenty-fourth ICMI Study

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PREFACE

It was timely, some might even say overdue, when the International Commission on Mathematical Instruction (ICMI) announced the next ICMI Study 24 was going to be on School Mathematics Curriculum Reforms at ICME-13 (2016, Hamburg, Germany), given the large number of countries, states or regions around the world who have or are undertaking school mathematics curriculum reforms. Soon after the International Programme Committee (IPC) was finalized, work began on developing an ICMI Study Discussion Document, which was finally released in December 2017 (see the end of these Proceedings) and disseminated internationally. The Discussion Document served as a call for papers for this ICMI Study 24 Conference, which was hosted a year later in Tsukuba, Japan.

As scholars have noted here and elsewhere, school mathematics curriculum reforms are complex. So, it comes as no surprise that five broad themes were identified in the Discussion Document and each theme in turn, referred to a diverse range of questions, which provided the basis for inviting papers. The themes attempted to attend to the study topic from different perspectives – historically, in terms of the subject of mathematics, issues of implementation, globalization and internationalization and the agents and processes of curriculum development – and they drew attention to different aspects of school mathematics curriculum reforms. Perhaps, it was to be expected that the largest number of papers were submitted in the theme on “Analysing school mathematics curriculum reforms for coherence and relevance”, which examines issues of curriculum goals, content, pedagogy, assessment, resources and technology (to name but a few aspects). It will be noted that inevitably many parts of the themes do indeed overlap.

We thank all the authors who responded to the call, submitted papers and participated in the conference. These conference proceedings includes 68 papers from diverse countries: Algeria, Australia, Chile, China/Hong Kong, Costa Rica, Denmark, France, Hungary, Indonesia, Iran, Ireland, Israel, Italy, Japan, Lebanon, Malaysia, Mexico, Netherlands, Peru, Philippines, Portugal, Serbia, South Africa, Spain, Thailand, United Kingdom, Unites States of America and Vietnam.

Each paper was reviewed by IPC members and then appropriately amended (where necessary) by authors, before being accepted for publication in this electronic conference proceedings. In addition, we were very honoured that select scholars in the topic accepted our invitation to present keynote lectures and participate in panel discussions. The three keynotes and two panels are aligned to the five themes and we are very grateful that each speaker submitted a paper for inclusion in the proceedings for deliberation by the conference participants. One special contribution was the privilege we had to conduct an interview with renown scholar Jeremy Kilpatrick and have included the transcript of the interview in these proceedings.

The ICMI Study Conferences are unique in that they focus less on each participant formally presenting their paper but rather serve primarily as a platform for discussion of papers in the context of the identified themes with their associated questions; and these intense deliberations are directed towards the preparation of a published volume. For this reason, the conference proceedings were disseminated prior to the conference so that delegates would have time to read the papers and the conference can truly serve for delegates to confer. Hence, for much of the time during the conference, the 94 conference delegates met in working groups related to the five themes under the leadership of IPC members.

We are very pleased that the ICMI Study 24 Conference was successfully hosted in the beautiful "science city" of Tsukuba, Japan; and excited that our joint work towards producing a much anticipated ICMI Study 24 volume has begun.

We have set ourselves the ambitious goal of having the ICMI Study 24 volume prepared for launching at the next ICME-14 in Shanghai, China in 2020, where we hope to all meet again.

Y. Shimizu

Co-Chairs: Yoshinori Shimizu and Renuka Vithal

13 December 2018

MAKING SENSE OF MATHEMATICS AND MAKING MATHEMATICS MAKE SENSE

William McCallum

University of Arizona

> I [want to] emphasize the practices, because from my point of view that's where the content lives.
>
> — Alan Schoenfield, 3 April 2013
>
> . . . at first I thought no, that's wrong, the practices live in the content standards, and then I realized we were both saying the same thing, namely that having this separate free-floating set of practices that are independent of the content is a bad idea.
>
> —William McCallum, 4 April 2013

VIEWS OF MATHEMATICS

The exchange above is from a meeting that Alan Schoenfeld and I attended at the Mathematical Sciences Research Institute in Berkeley, CA in 2013 [7]. The content and practices referred to are the Content Standards and Practice Standards in the Common Core State Standards in Mathematics (CCSSM), a collaborative effort of the 50 US states to write common standards, which came out in 2010 [14]. I will return to a discussion of CCSSM later in this paper, but first I would like to use the exchange to lay out a dichotomy in views of mathematics.

In [19] Schoenfeld describes a spectrum of views of mathematics:

> At one end of the spectrum, mathematical knowledge is seen as a body of facts and procedures dealing with quantities, magnitudes, and forms, and relationships among them; knowing mathematics is seen as having "mastered these facts" and procedures. At the other end of the spectrum, mathematics is conceptualized as the "science of patterns," an (almost) empirical discipline closely akin to the sciences in its emphasis on pattern-seeking on the basis of empirical evidence.

A casual internet search on "mathematics as facts and procedures" does not find anybody advocating it as a complete definition, but finds many saying that mathematics is more than that. It is true, however, that this view of mathematics seems embedded in the culture of US classrooms. Writing in 1999 Stigler and Hiebert [20] said

> In the United States, . . . the level is less advanced and requires much less mathematical reasoning than in [Germany and Japan]. Teachers present definitions of terms and demonstrate procedures for solving specific problems. Students are then asked to memorize the definitions and practice the procedures.

Despite efforts to reform this state of affairs going back to the 1989 NCTM standards, this culture remains prevalent today.

Schoenfeld associates one end of his spectrum, the view of mathematics as facts and procedures, with what he calls the content perspective [19]:

> A consequence of this perspective is that instruction has traditionally focused on the content aspect of knowledge. Traditionally one defines what students ought to know in terms of chunks of subject matter, and characterizes what a student knows in terms of the amount of content that has been "mastered." . . . From this perspective, "learning mathematics" is defined as mastering, in some coherent order, the set of facts and procedures that comprise the body of mathematics. The route to learning consists of delineating

> the desired subject matter content as clearly as possible, carving it into bite-sized pieces, and providing explicit instruction and practice on each of those pieces so that students master them.

Note there are really two perspective here, one on what mathematics is, and another on how it is learned. One could in principle hold the first and not the second. In contrast to the content perspective, and by preference, Schoenfeld proposes the process perspective. In writing about Everybody Counts, a 1989 report of the National Research Council [15], he says:

> . . . . there is a major shift from the traditional focus on the content aspect of mathematics . . . to the process aspects of mathematics—to what Everybody Counts calls calls doing mathematics. Indeed, content is mentioned only in passing, while modes of thought are specifically highlighted in the first page of the section.

The process perspective has taken various forms over the years: the NCTM process standards [16], the focus on problem-solving as a core activity in reform curricula, and the practice standards of CCSSM [14]. Again, one might hold a content perspective on what mathematics is and a process perspective on how it is learned; for example, problem-solving could be a way of learning facts and procedures. Schoenfeld's own version of the process perspective is described in [19] as a view of mathematics as pattern-seeking.

The last sentence in the second quotation above captures a danger of the process perspective: "content is mentioned only in passing." The danger is that mathematics content is a backdrop to the action, a backdrop that can be inaccurate or forgotten.[1] For example, curricula written from the process perspective might be organized around large projects that pull different mathematical tools in at different times. Without careful planning there is the danger that mathematical dependencies get mixed up. Some curricula are organized around "big ideas," lists of overarching themes that recur throughout the curriculum. This can work well if done judiciously; but some ideas in mathematics are not well-described as "big": rather they are small but consequential. Completing the square is an example of such an idea (see [11]).

Approaches from the process perspective—mathematics as pattern seeking, mathematics as problem-solving, big ideas–have in common what I call the sense-making stance. In this stance, mathematics is a source of material for important processes such as problem-solving and communication. It is an important stance, but it carries risks. If mathematics is about sense-making, the stuff being made sense of can be viewed as some sort of inert material lying around in the mathematical universe. Even when it is structured into "big ideas" between which connections are made, the whole thing can have the skeleton of a jellyfish.

I would like to propose a complementary stance, which carries its own benefits and risks.

THE MAKING-SENSE STANCE

Where the sense-making stance sees a process of people making-sense of mathematics (or not), the making-sense stance sees mathematics making sense to people (or not). These are not mutually exclusive stances; rather they are dual stances jointly observing the same thing. The making-sense stance is related to the content perspective described by Schoenfeld, without the unappetizing

[1] To be clear, that is not what Schoenfeld is advocating; indeed, at the same conference mentioned above he explicitly said that he intends neither to ignore nor downplay mathematics.

"carving content into bite-sized pieces." It views content as something to be actively structured in such a way that it makes sense.

That structuring is constrained by the logic of mathematics. But logic by itself does not tell you how to make mathematics make sense, for various reasons. First, because time is one-dimensional, and sense-making happens over time, structuring mathematics to make sense involves arranging mathematical ideas into a coherent mathematical progression, and that can usually be done in more than one way. Second, there are genuine disagreements about the definition of key ideas in school mathematics (ratios, for example), and so there are different choices of internally consistent systems of definition. Third, attending to logical structure alone can lead to overly formal and elaborate structuring of mathematical ideas. Just as it is a risk of the sense-making stance that the mathematics gets ignored, it is a risk of the making-sense stance that the sense-maker gets ignored.

Student struggle is the nexus of debate between the two stances. It is possible for those who exclusively take the sense-making stance to confuse productive struggle with struggle resulting from an underlying illogical or contradictory presentation of ideas, the consequence of inattention to the making-sense stance. And it possible for those who exclusively take the making-sense stance to think that struggle can be avoided by ever clearer and ever more elaborate presentations of ideas.

The work entailed in the making-sense stance is mathematical work, so it is not surprising that much of the work of mathematicians in mathematics education falls under this heading. Wu [22] has written about "textbook school mathematics" as a degraded subject that is not faithful to mathematics as it is understood by mathematicians. Howe and Epp [6] have written about the mathematical ideas behind place value. Baldridge [2] has constructed a vast edifice of grade-level-appropriate, internally consistent definitions of ideas that arise in school mathematics.

An important strand of research in mathematics education is composed of work where the two stances are taken simultaneously, often by pairs of mathematicians and education researchers. For example, Ball and Bass argue in [3] that

> Making mathematics reasonable is more than individual sense making. making-sense refers to making mathematical ideas sensible, or perceptible, and allows for understanding based only on personal conviction. Reasoning, as we use it, comprises a set of practices and norms that are collective, not merely individual or idiosyncratic, and rooted in the discipline.

Another example is the work of Iszák and Beckmann [8], who propose a unified definition of multiplication that applies to the many situations modeled by multiplication. In their definition a product is measured simultaneously by a base unit and by a group, which is itself measured by base units. Their work provides a nice example of coordinating the making-sense stance with the sense-making stance. On the one hand their work is an attempt to make the diverse array of multiplication situations make sense through a unified definition. On the other hand, it recognizes the role of the sense-maker, the person who must make the choice of base unit and group in order to make sense of a multiplication situation.

> We think that mathematics education as a field should seek more completely worked out coherent approaches to the [multiplicative conceptual field] based on consistency and logical interconnection. The absence of such articulation may be constraining our capacity to help students and teachers use prior knowledge and experience to effectively relate topics and construct interconnected bodies of knowledge. It is one thing to know that multiplication can be used to model a variety of situations and another to perceive a common underlying structure.

COHERENCE

Coherence is the sine qua non of the making-sense stance. Schmidt et al [18] talk about coherence of standards:

> We define content standards . . . to be coherent if they are articulated over time as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject-matter comes. . . . This implies that, for a set of content standards to 'to be coherent', they must evolve from particulars . . . to deep structures.

This definition was elaborated by Cuoco and McCallum [4] to include coherence of curriculum and coherence of practice. Iszák and Beckmann argue for a coherent view of multiplication in mathematics education research [8]. Attempts to bring coherence to school topics also underly the work of mathematicians mentioned above.

Coherence was a guiding principle in the writing of the Common Core State Standards in Mathematics (CCSSM) [10] in 2009–2010. An important precursor was the report in 2008 of the National Mathematics Advisory Panel, which laid out the following principles [17]

> A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. . . .
>
> By the term coherent, the Panel means that the curriculum is marked by effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones.

Standards have an inherent tendency to interfere with focus and coherence, in that they attempt to reduce a subject to a list, Schoenfeld's "bite-sized pieces." The pieces can lose connection with each other, breaking coherence, and there is a danger that everybody's favorite pieces get added to the list, breaking focus. Maintaining focus in CCSSM was a matter of resisting temptation. Maintaining coherence was a matter of building structures that transcended the bulleted list. See [5], [23], and [10] for more detail on the process.

One important way of maintaining coherence was to build the standards on progressions: narrative descriptions of how the mathematical ideas in a particular domain evolve over a sequence of grades [21]. These were the first documents produced in the writing of the standards. For example, there was a progression for Number and Operations in Base Ten (NBT) in grades K–5, which told the story of that domain over the grades. Different progressions were tied together by cross-domain connections. For example, it makes sense that the place in the NBT progression where students learn about multiplication should come in the same grade where the geometry progression talks about area of rectangles. These connections tied the different stories together into a coherent whole.

A particularly knotty area in mathematics curriculum is the progression from fractions to ratios to proportional relationships. Part of the problem is the result of a confusion in everyday usage, at least in the English language. In common language, the fraction $\frac{a}{b}$, the quotient $a \div b$, and the ratio $a:b$, seem to be different manifestations of a single fused notion. Here, for example are the mathematical definitions of fraction, quotient, and ratio from Merriam-Webster online [13]:

> Fraction: a numerical representation (such as $\frac{3}{4}$, $\frac{5}{8}$, or 3.234) indicating the quotient of two numbers.
>
> Quotient: (1) the number resulting from the division of one number by another (2) the numerical ratio usually multiplied by 100 between a test score and a standard value.

> Ratio: (1) the indicated quotient of two mathematical expression (2) the relationship in quantity, amount, or size between two or more things.

The first definition says that a fraction is a quotient; the second says that a quotient is a ratio; the third one says that a ratio is a quotient. Thus it would appear that these words all mean the same thing. The definitions are not wrong as descriptions of how people use the words. For example, people say things like "mix the flour and the water in a ratio of $\frac{3}{4}$."

From the point of view of the sense-making stance, this fusion of language is out there in the mathematical world, and we must help students make sense of it. From the point of view of the making-sense stance, we might make some choices about separating and defining terms and ordering them in a coherent progression. In CCSSM the following choices were made:

1. A fraction $\frac{a}{b}$ as the number on the number line that you get to by dividing the interval from 0 to 1 into b equal parts and putting a of those parts together end-to-end. It is a single number, even though you need a pair of numbers to locate it.
2. It can be shown using the definition that $\frac{a}{b}$ is the quotient $a \div b$, the number that gives a when multiplied by b. (This is what Beckman and Iszák call the Fundamental Theorem of Fractions.)
3. A ratio is a pair of quantities; equivalent ratios are obtained by multiplying each quantity by the same scale factor.
4. A proportional relationship is a set of equivalent ratios. One quantity y is proportional to another quantity x if there is a constant of proportionality k such that $y=kx$.

Note that there is a clear distinction between fractions (single numbers) and ratios (pairs of numbers).

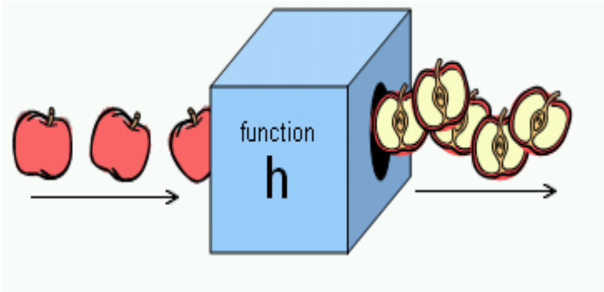
This is not the only way of developing a coherent progression of ideas in this domain. Zalman Usiskin (private communication) prefers to start with (2) and define $\frac{a}{b}$ as the quotient $a \div b$, which is assumed to exist. One could then use the Fundamental Theorem of Fractions to show (1).

There is no a priori mathematical way of deciding between these approaches. Each depends on certain assumptions and primitive notions. But each approach is an example of the structuring and pruning required to make the mathematical ideas make sense; an example of the making-sense stance.

FIDELITY

Another principle of the making-sense stance is fidelity. In [12] I define fidelity as "the extent to which a curriculum, or a collection of curriculum materials, faithfully presents the underlying mathematical concept as it is situated in the discipline of mathematics." I go on to say that "mathematical fidelity is not the same as mathematical formality; a mathematical concept can be presented in a way that is appropriate for the age of the students, while still being presented with fidelity."

Examples of lack of fidelity abound on the internet. Consider, for example, this representation found at [9].



The caption on the figure is

Fruit Halving Function: This shows a function that takes a fruit as input and releases half the fruit as output.

The image would seem to violate the condition that a function have one output for each input, since an apple has two halves. Or, if we take the caption to mean that the machine is throwing away one of the halves, there is still the question of which half. A function does not randomly choose outputs from two possible choices.

Fidelity is to some degree a matter of taste. Consider, for example, the distinction between order of operations—the set of rules for how to read arithmetic expressions, such as giving precedence to multiplication over addition—and the properties of operations—the set of rules governing how operations work, such as the distributive property. In school mathematics these topics are often given equal salience. However, most mathematicians would regard the first as merely convention and the second as fundamental law. The order of operations could be changed; there is nothing mathematically wrong with saying that addition takes precedence over multiplication, in which case the distributive property would be written $a{\cdot}b{+}c{=}(a{\cdot}b){+}(a{\cdot}c)$. But the distributive property itself is fundamental, and has the same meaning no matter how it is notated. Although it would not be mathematically incorrect in a curriculum to present order of operations and properties of operations in a flat list with the same degree of emphasis, it would be a little tone-deaf.

This subjective aspect of fidelity means that there can be reasonable disagreements about it. A making-sense stance takes seriously the task of discussing those disagreements with evidence from the professional norms of the discipline.

CONCLUDING THOUGHTS

One might take the point of view that the distinction between the sense-making stance and the making-sense stance is artificial or unnecessary. A complete view of mathematics and learning takes both stances at the same time, with a sort of binocular vision that sees the full dimensionality of the domain. An example of this is Arcavi's paper on symbol sense [1], which shifts beautifully back and forth between the two stances. However, this coordination of the two stances does not always happen. Rather than provide examples, I invite the audience to think of their own examples where one stance or the other has become dominant. This has been particularly a danger in my own work in the policy domain. I hope that spelling out the two stances will contribute to productive dialog in mathematics education, such as the one that started this article, allowing for conscious recognition of the stance one or one's interlocutor is taking and for acknowledgement of the value of adding the dual stance.

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OECD 2030 LEARNING FRAMEWORK: FUTURE OF EDUCATION AND SKILLS

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This paper summarises the key messages of the OECD's project Future of Education and Skills 2030." It focuses on introducing the key concepts being developed for the OECD 2030 Learning Framework, which sets out the types of competencies, including mathematical competencies, today's students will need to thrive in and shape the world towards well-being in 2030. It also introduces the key challenges OECD countries are commonly facing when redesigning curriculum, including mathematics.

THE OECD NEW PROJECFT: FUTURE OF EDUCATION AND SKILLS 2030

We are facing unprecedented challenges – social, economic and environmental – driven by accelerating globalisation and a faster rate of technological developments. At the same time, those forces are providing us with myriad new opportunities for human advancement. The future is uncertain and we cannot predict it; but we need to be open and ready for it. The children entering education in 2018 will be young adults in 2030. Schools can prepare them for jobs that have not yet been created, for technologies that have not yet been invented, to solve problems that have not yet been anticipated. It will be a shared responsibility to seize opportunities and find solutions.

To navigate through such uncertainty, students will need to develop curiosity, imagination, resilience and self-regulation; they will need to respect and appreciate the ideas, perspectives and values of others; and they will need to cope with failure and rejection, and to move forward in the face of adversity. Their motivation will be more than getting a good job and a high income; they will also need to care about the well-being of their friends and families, their communities and the planet.

Education can equip learners with agency and a sense of purpose, and the competencies they need, to shape their own lives and contribute to the lives of others. To find out how best to do so, the Organisation for Economic Co-operation and Development (OECD) has launched *The Future of Education and Skills 2030* project. The aim of the project is to help countries find answers to two far-reaching questions:

- **What** knowledge, skills, attitudes and values will today's students need to thrive and shape their world?
- **How** can instructional systems develop these knowledge, skills, attitudes and values effectively?

THE OECD 2030 LEARNING FRAMEWORK (OECD LEARNING COMPASS 2030)

This OECD Learning Framework 2030 offers a vision and some underpinning principles for the future of education systems. It is about orientation, not prescription. The learning framework has been co-created for the OECD Education 2030 project by government representatives and a growing community of partners, including thought leaders, experts, school networks, school leaders, teachers, students and youth groups, parents, universities, local organisations and social partners.

Education 2030: A Shared Vision

The members of the OECD Education 2030 Working Group are committed to helping every learner develop as a whole person, fulfil his or her potential and help shape a shared future built on the well-being of individuals, communities and the planet.

Children entering school in 2018 will need to abandon the notion that resources are limitless and are there to be exploited; they will need to value common prosperity, sustainability and well-being. They will need to be responsible and empowered, placing collaboration above division, and sustainability above short-term gain.

In the face of an increasingly volatile, uncertain, complex and ambiguous world, education can make the difference as to whether people embrace the challenges they are confronted with or whether they are defeated by them. And in an era characterised by a new explosion of scientific knowledge and a growing array of complex societal problems, it is appropriate that curricula should continue to evolve, perhaps in radical ways.

Need for new solutions in a rapidly changing world

Societies are changing rapidly and profoundly.

A first challenge is **environmental**: e.g.

- Climate change and the depletion of natural resources require urgent action and adaptation.

A second challenge is **economic**: e.g.

- Scientific knowledge is creating new opportunities and solutions that can enrich our lives, while at the same time fuelling disruptive waves of change in every sector. Unprecedented innovation in science and technology, especially in bio-technology and artificial intelligence, is raising fundamental questions about what it is to be human. It is time to create new economic, social and institutional models that pursue better lives for all.
- Financial interdependence at local, national and regional levels has created global value chains and a shared economy, but also pervasive uncertainty and exposure to economic risk and crises. Data is being created, used and shared on a vast scale, holding out the promise of expansion, growth and improved efficiency while posing new problems of cyber security and privacy protection.

A third challenge is **social**: e.g.

- As the global population continues to grow, migration, urbanisation and increasing social and cultural diversity are reshaping countries and communities.

- In large parts of the world, inequalities in living standards and life chances are widening, while conflict, instability and inertia, often intertwined with populist politics, are eroding trust and confidence in government itself. At the same time, the threats of war and terrorism are escalating.

These global trends are already affecting individual lives, and may do so for decades to come. They have triggered a global debate that matters to every country, and call for global and local solutions. The OECD Education 2030 contributes to the UN 2030 Global Goals for Sustainable Development (SDGs), aiming to ensure the sustainability of people, profit, planet and peace, through partnership.

Need for broader education goals: Individual and collective well-being

Unless steered with a purpose, the rapid advance of science and technology may widen inequities, exacerbate social fragmentation and accelerate resource depletion.

In the 21st century, that purpose has been increasingly defined in terms of well-being. But well-being involves more than access to material resources, such as income and wealth, jobs and earnings, and housing. It is also related to the quality of life, including health, civic engagement, social connections, education, security, life satisfaction and the environment. Equitable access to all of these underpins the concept of inclusive growth.

Education has a vital role to play in developing the knowledge, skills, attitudes and values that enable people to contribute to and benefit from an inclusive and sustainable future. Learning to form clear and purposeful goals, work with others with different perspectives, find untapped opportunities and identify multiple solutions to big problems will be essential in the coming years. Education needs to aim to do more than prepare young people for the world of work; it needs to equip students with the skills they need to become active, responsible and engaged citizens.

Learner agency: Navigating through a complex and uncertain world

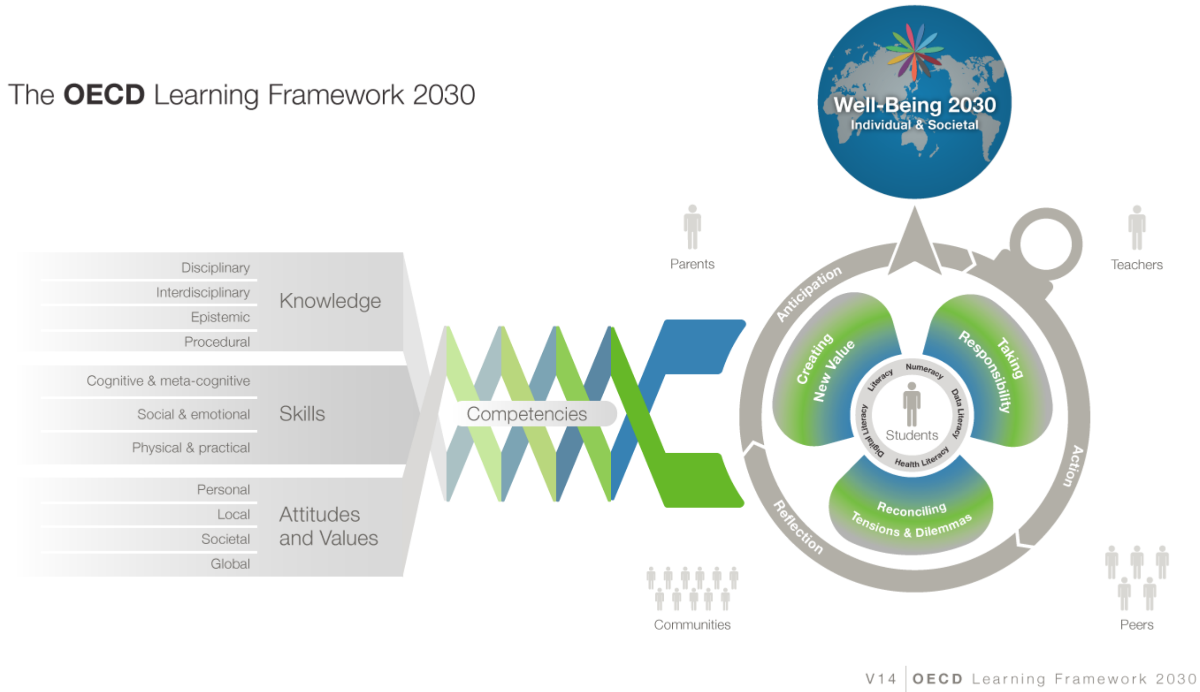
Future-ready students need to exercise agency, in their own education and throughout life. Agency implies a sense of responsibility to participate in the world and, in so doing, to influence people, events and circumstances for the better. Agency requires the ability to frame a guiding purpose and identify actions to achieve a goal.

To help enable agency, educators must not only recognise learners' individuality, but also acknowledge the wider set of relationships – with their teachers, peers, families and communities – that influence their learning. A concept underlying the learning framework is "co-agency" – the interactive, mutually supportive relationships that help learners to progress towards their valued goals. In this context, everyone should be considered a learner, not only students but also teachers, school managers, parents and communities.

Two factors, in particular, help learners enable agency. The first is a personalised learning environment that supports and motivates each student to nurture his or her passions, make connections between different learning experiences and opportunities, and design their own learning projects and processes in collaboration with others. The second is building a solid foundation: literacy and numeracy remain crucial. In the era of digital transformation and with the advent of big data, digital literacy and data literacy are becoming increasingly essential, as are physical health and mental well-being.

OECD Education 2030 stakeholders have co-developed a "learning compass" that shows how young people can navigate their lives and their world (Figure 1).

Figure 1. The OECD Learning Framework 2030: Work-in-progress



Need for a broad set of knowledge, skills, attitudes and values in action

Students who are best prepared for the future are change agents. They can have a positive impact on their surroundings, influence the future, understand others' intentions, actions and feelings, and anticipate the short and long-term consequences of what they do.

The concept of competency implies more than just the acquisition of knowledge and skills; it involves the mobilisation of knowledge, skills, attitudes and values to meet complex demands. Future-ready students will need both broad and specialised knowledge. Disciplinary knowledge will continue to be important, as the raw material from which new knowledge is developed, together with the capacity to think across the boundaries of disciplines and "connect the dots". Epistemic knowledge, or knowledge about the disciplines, such as knowing how to think like a mathematician, historian or scientist, will also be significant, enabling students to extend their disciplinary knowledge. Procedural knowledge is acquired by understanding how something is done or made – the series of steps or actions taken to accomplish a goal. Some procedural knowledge is domain-specific, some transferable across domains. It typically develops through practical problem-solving, such as through design thinking and systems thinking.

Students will need to apply their knowledge in unknown and evolving circumstances. For this, they will need a broad range of skills, including cognitive and meta-cognitive skills (e.g. critical thinking, creative thinking, learning to learn and self-regulation); social and emotional skills (e.g. empathy, self-efficacy and collaboration); and practical and physical skills (e.g. using new information and communication technology devices).

The use of this broader range of knowledge and skills will be mediated by attitudes and values (e.g. motivation, trust, respect for diversity and virtue). The attitudes and values can be observed at personal, local, societal and global levels. While human life is enriched by the diversity of values and attitudes arising from different cultural perspectives and personality traits, there are some human values (e.g. respect for life and human dignity, and respect for the environment, to name two) that cannot be compromised.

Competencies to transform our society and shape our future

If students are to play an active part in all dimensions of life, they will need to navigate through uncertainty, across a wide variety of contexts: in time (past, present, future), in social space (family, community, region, nation and world) and in digital space. They will also need to engage with the natural world, to appreciate its fragility, complexity and value.

Building on the *OECD Key Competencies* (the DeSeCo project: Definition and Selection of Competencies), the OECD Education 2030 project has identified three further categories of competencies, the "Transformative Competencies", that together address the growing need for young people to be innovative, responsible and aware:

- Creating new value
- Reconciling tensions and dilemmas
- Taking responsibility

Creating new value

New sources of growth are urgently needed to achieve stronger, more inclusive and more sustainable development. Innovation can offer vital solutions, at affordable cost, to economic, social and cultural dilemmas. Innovative economies are more productive, more resilient, more adaptable and better able to support higher living standards.

To prepare for 2030, people should be able to think creatively, develop new products and services, new jobs, new processes and methods, new ways of thinking and living, new enterprises, new sectors, new business models and new social models. Increasingly, innovation springs not from individuals thinking and working alone, but through co-operation and collaboration with others to draw on existing knowledge to create new knowledge. The constructs that underpin the competency include adaptability, creativity, curiosity and open-mindedness.

Reconciling tensions and dilemmas

In a world characterised by inequities, the imperative to reconcile diverse perspectives and interests, in local settings with sometimes global implications, will require young people to become adept at handling tensions, dilemmas and trade-offs, for example, balancing equity and freedom, autonomy and community, innovation and continuity, and efficiency and the democratic process. Striking a balance between competing demands will rarely lead to an either/or choice or even a single solution. Individuals will need to think in a more integrated way that avoids premature conclusions and recognises interconnections. In a world of interdependency and conflict, people will successfully secure their own well-being and that of their families and their communities only by developing the capacity to understand the needs and desires of others.

To be prepared for the future, individuals have to learn to think and act in a more integrated way, taking into account the interconnections and inter-relations between contradictory or incompatible ideas, logics and positions, from both short- and long-term perspectives. In other words, they have to learn to be systems thinkers.

Taking responsibility

The third transformative competency is a prerequisite of the other two. Dealing with novelty, change, diversity and ambiguity assumes that individuals can think for themselves and work with others. Equally, creativity and problem-solving require the capacity to consider the future consequences of one's actions, to evaluate risk and reward, and to accept accountability for the products of one's work. This suggests a sense of responsibility, and moral and intellectual maturity, with which a person can reflect upon and evaluate his or her actions in light of his or her experiences, and personal and societal goals, what they have been taught and told, and what is right or wrong. Acting ethically implies asking questions related to norms, values, meanings and limits, such as: What should I do? Was I right to do that? Where are the limits? Knowing the consequences of what I did, should I have done it? Central to this competency is the concept of self-regulation, which involves self-control, self-efficacy, responsibility, problem solving and adaptability. Advances in developmental neuroscience show that a second burst of brain plasticity takes place during adolescence, and that the brain regions and systems that are especially plastic are those implicated in the development of self-regulation. Adolescence can now be seen as a time not just of vulnerability but of opportunity for developing a sense of responsibility.

DESIGN PRINCIPLES FOR MOVING TOWARD AN ECO-SYSTEMIC CHANGE

The OECD Learning Framework 2030 therefore encapsulates a complex concept: the mobilisation of knowledge, skills, attitudes and values through a process of reflection, anticipation and action, in order to develop the inter-related competencies needed to engage with the world.

To ensure that the new learning framework is actionable, the OECD Education 2030 stakeholders have worked together to translate the transformative competencies and other key concepts into a set of specific constructs (e.g. creativity, critical thinking, responsibility, resilience, collaboration) so that teachers and school leaders can better incorporate them into curricula. Such constructs are currently under review.

They have also built a knowledge base for curriculum redesign. Curriculum change assumes that education is an ecosystem with many stakeholders. Students, teachers, school leaders, parents, national and local policy makers, academic experts, unions, and social and business partners have worked as one to develop this project. In its work across different countries, OECD Education 2030 has identified five common challenges.

1. Confronted with the needs and requests of parents, universities and employers, schools are dealing with curriculum overload. As a result, students often lack sufficient time to master key disciplinary concepts or, in the interests of a balanced life, to nurture friendships, to sleep and to exercise. It is time to shift the focus of our students from "more hours for learning" to "quality learning time".

2. Curricula reforms suffer from time lags between recognition, decision making, implementation and impact. The gap between the intent of the curriculum and learning outcome is generally too wide.
3. Content must be of high quality if students are to engage in learning and acquire deeper understanding.
4. Curricula should ensure equity while innovating; all students, not just a select few, must benefit from social, economic and technological changes.
5. Careful planning and alignment is critically important for effective implementation of reforms.

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INTERVIEW WITH DR. JEREMY KILPATRICK

Interview by Yoshinori Shimizu and Renuka Vithal

Dr. Jeremy Kilpatrick is emeritus Regents Professor of mathematics education at the University of Georgia, USA. He is a winner of the Felix Klein Medal for 2007 for his sustained and distinguished lifetime achievement in mathematics education research and development. He is an internationally renowned researcher and has published groundbreaking papers, book chapters and books in many area - many of which are now standard references in the literature - on problem solving, on the history of research in mathematics education, on teachers' proficiency, on curriculum change and its history, and on assessment. In particular, his publications include the seminal work, "Curriculum Development in Mathematics" (Howson, Keitel, and Kilpatrick, 1981).

This special interview session, invites him to reflect on one of the themes from the ICMI Study 24 on school mathematics curriculum reforms - Theme A: Learning from the past; and to share his perspectives with the audience (See ICMI Study 24 Discussion Document in these Proceedings). He addresses key questions about the driving forces and barriers shaping mathematics curriculum reforms.

Y: 1. In your seminal book *Curriculum Development in Mathematics*, you, Keitel and Howson identified several mathematics curriculum approaches in the period leading to the 80s such as the New Math approach, Behaviourist approach, Integrated approach and several others. How would you characterize the main curriculum development approaches (or some might say movements) since the 80s until the present day?

J: Well, I guess there are two, I would say two major directions in which curriculum development has gone. One is in response to the New Math approach. There have been, since the 1980s, a number of projects to build curriculum around the more applied parts of the subject matter, including statistics and other ways of looking at representations of Mathematical problems, especially looking at how children can approach practical problems. One of the big arguments against the New Maths was that the pure math didn't have applications or at least the students were not introduced to applications. And so, in response to that, a number of projects, in a number of countries worked on applications. Today we have many, many applications for the earlier grades, which we didn't have during the New Math era.

When I was teaching in Berkley, California, I took a summer school course at Stanford and one of the instructors of that course was Morris Kline, (that's different from Felix Klein). But Morris Kline was, in the US, probably the most critical person of his day of the New Math. He eventually wrote a book "*Johnny can't add*" (Kline, 1973), which was an attack really on what the New Math had tried to do because Morris was a professor of applied mathematics at New York University. I knew him after I had had him as an instructor. He wanted us to get, he wanted to build, if he could, a curriculum of applications of mathematics for students, because he considered that a better way to get into the subject matter. There were a number of back and forth discussions of that sort of thing in the US at that time, and so clearly one of the approaches that came after the New Math was looking at applications.

What I'm thinking about the school mathematics curriculum is to say that it really has two foci. That it has two poles, it's bipolar. It's bipolar in the sense that originally the elementary curriculum did not have many pure aspects to it. It was mostly applied, arithmetic with some simple geometry. But over time that changed and during the New Math era, some abstraction and some pure mathematics were introduced into the earlier grades. The other pole, the other part of the bipolar thing is that pure mathematics had always dominated the secondary curriculum. Now that's because the secondary curriculum wasn't for every child, at least originally, it wasn't. That's what happened, during this prior century, was that more and more children were studying secondary mathematics, all around the world. But before that time, the secondary curriculum was really just for people who were going on to universities. Therefore, it was rather pure and rather removed from the real problems. Over time, what happened was that these two poles - the pure mathematics and the applied mathematics - became more mixed and that's what we see today. In fact, we have many more applied mathematics topics in the curriculum today, then we did back in the 1980s. So that's one of the big differences.

Another difference that one finds in curriculum projects today, has to do with, what has been called the *social turn* in mathematics education. Rather than just looking at how does the individual child learn, projects and curriculum developers are looking at how do classes of students learn and how can we treat the social aspects of mathematics learning. That has been a big focus in recent years, and there are a lot of projects that deal with that sort of thing, because people recognize that the situation in which you learn mathematics affects the mathematics that you learn. And that was not really well-understood and thought about in the 1980s. So, I would say that the *social turn* and the *applications of mathematics* are the big changes. There are other changes too. Technology, we will talk later about that I guess, technology has certainly helped with both of those things actually.
So, technology by itself, has made quite a difference in the school curriculum.

R: Jeremy where would you include the cultural aspect in that description that you just offered. Or do you think culture has not made enough in-roads yet. In the description when you talked about the social turn and you gone on to talk about technology, I just wondered whether you would insert culture anywhere along that.

J: Certainly, I think that's one of the things we've seen now, is that we cannot think about developing curriculum without taking into account the culture of the classroom, and that was one of the things that happened during the New Math era. People thought if we just write new textbooks and gave them to teachers, everything will happen and that there would be a change and so forth. One of the hardest lessons, I think that came out of the New Math movement (and we do talk a little bit about it in the last chapter of this book – Li and Lappan, 2014), the hardest lesson was to recognise the teacher was the critical person in curriculum reform. That is, if the teacher didn't understand why the change was being made or understand what the change was, it didn't matter what materials you gave to the teacher. The teacher had to be part of this process of understanding what is going on here and fitting it into the culture of the classroom. Because, that's another thing that we learned, is that every country has a different classroom culture when it comes to the teaching of mathematics. There are some that have some connections to each other. But around the world, there are lots of different cultures. In some cases, the teacher is expected to pose all of the problems, and in the other cases, the book is supposed to have the problems and all the teacher does is help the students work. Countries differ quite a bit on that question.

The other part of the social turn is whether the teachers work together on mathematics instruction. In some countries, each teacher just closes the door and does what she or he wants to do. In other countries, teachers, at least in principle, work together and help each other change. We did some studies in this in the US on curriculum development and found that it was only when there were groups of teachers working together that we got good curriculum change, because when teachers try to do it on their own, there were so many barriers and so many problems making the change, that it was not successful. It was teachers working together that made the difference.

Y: So Jeremy has mentioned bipolar…

J: Yes, I want to stress that bipolarity because I think that's an important quality of the school curriculum and every teacher and every country has to deal with: - how much attention do we give to the purer side of mathematics. The New Math thought that it should be entire but that didn't work really as well as people thought. So how much attention do we give to the pure part of mathematics and how much to the applications and how much do we engage together. Because it turns out if the applications are well-chosen and can be understood by the children then that helps them move toward the purer parts of the field. But if you just ask pure mathematicians about what the curriculum should be, they tend not to recommend applications. There are problems with applications. Teachers don't necessarily know them and they don't know how to handle them in the class if they have not seen that done. But when it works, it works well because the kids can say, "Oh! Now I understand where I would use this mathematics", which is one of the big problems with pure mathematics. "When will I ever use this?" is the natural question students ask.

R: In this bipolar situation of pure mathematics and applied maths, as much as both elements appear in the curriculum, would you say the shift has been more towards the applications, especially given by the fact that more learners go into secondary and the big focus, for example, on maths literacy. Do you think that's pushed curriculum reforms more into the application part of the bipolar…

J: Yes. I would say in general the stronger force at the moment, at least over the past decade, has been in that direction, toward applications. And it's been difficult, as I said for the teachers, because their own training doesn't necessarily include much exposure to applications. So, they are having to deal with applications they may have not studied in their own preparation. And so, if the curriculum reformer is going to try to get teachers to do work on this, they are going to have to explain some of these applications. The whole idea of trying to organize the applications into a coherent curriculum is a special problem of its own. In a sense, pure mathematics is easy to organize into a curriculum because everything is sort of logical and connected and so on. But the question of: what order do we take these applications; where do we start with applications; and which ones do we use. Those are big questions. Nonetheless, as I say, the experience we had, but this is in the upper secondary course that we studied in several places in the US, the teachers told us that the students loved those examples of applications of mathematics, that it really helped them understand why they were doing this mathematics. And they understand much more about functions, for example, then they would have from just a pure mathematics approach. So, I think there are pedagogical values in working with applications even though it's difficult to put together a sensible curriculum made up largely of application. That's the problem. How do we weave together the pure mathematics and the applied mathematics? But I would argue that whatever we do, it's going to be some kind of coalescence of pure and applied. We can downgrade the applied part and we have done that in the past. But I think for pedagogical reasons, there are good pedagogical reasons for raising the level of the applications and the number of applications. It's just that we have to be careful about how we choose those.

R: Can I just follow with one more question, it's a little bit on the side. Do you think Jeremy, this point that you have just made on application, this move towards more applications, explains to some extent, the lack of students in some countries moving into mathematics in the post-school era?

J: Yes, I think it is a problem. And there are lots of problems associated with bringing applications into the curriculum. Parents may say, "why is this in here; I didn't study this when I was in school, why are you having students to do this; this is not mathematics". Mathematician will tell you, "this is not mathematics, these are applications, they are not part of mathematics”. And so, for some mathematicians, it is ruining the subject, to bring in applications; even if it makes students happy, it is not staying true to what mathematics really is. As you suggest Renuka, if we stick with pure mathematics, with no application, what students cannot see, “when will I ever use this?”, it’s not surprising that they don’t go on to take more mathematics. So, I think for self-preservation, mathematicians and mathematics educators should work on the question of: how do we orchestrate the curriculum so that applications play a good role?

There is even a problem with the word applications, because it implies first you do the mathematics, then you apply it. And actually, it can go the other way. You can start with a good application, with a situation where mathematics can be applied, and then you can show students or they can learn how mathematics comes in, is brought into the situation; and helps them see, what good is this, “I’m learning quadratic functions” and “what good does that do me”. Well, if you have a good application then you can convince people that it does work and people do need to know this.

Y: And the emphasis on mathematical modeling in recent years, may be related to this issue and the authenticity problem.

J: Exactly. I didn’t mention that mathematical modeling and statistics and other types of applications are a part of this movement towards applications

R: Just to conclude this point, maybe to say, I think it will be interesting in our study, to what extent, as we look across countries, this has in fact happened.

J: I think it will be interesting and it will probably be dependent, in part, on the balance between the pure and applied mathematicians who are working on curriculum development as well as the mathematics educators; how comfortable are the mathematics educators themselves with applications of mathematics. It’s something new for all of us.

Y: I think this is a very important point, of having these two interwoven in a very nice way.

J: Well, that is one way of characterizing a country's curriculum. I think it will, in your study, be very interesting to show us different ways in which countries have done this or are doing it.

Y: 2. Historically and as evidenced in the New Math era of the last century, the discipline of mathematics and mathematicians played a strong and influential role in shaping school mathematics curriculum reforms. Is this still the case? Why yes or no?

J: This is certainly connected to what we were just talking about. During the New Math era, there were a lot of mathematicians, as you say, played a strong and influential role in shaping the curriculum. And some of them, in a way, got burnt. They thought that they knew what primary school children should learn, and they wrote books on that. Teachers had trouble with it and the students had trouble with it. It didn't turn out the way they thought it would be. It's one thing for mathematicians to talk about the secondary curriculum, because the connection between that and what's happening in the colleges are clear. But what mathematicians had to say about elementary or primary school curriculum, that is a different story. There are some mathematicians who had stayed with this topic. But in general, there are not a large number of mathematicians who feel comfortable working on school curriculum. In general, it's not a rewarding thing for them to spend time on school mathematics, because they have their own area to work in and they get their rewards from proving theorems and doing other things like that. In mathematics, there is not much rewards for mathematicians to spend time on this.

In the past, individual mathematicians, like Felix Klein and some more people like that, they looked at the school curriculum and said that it needs to be made more like the university curriculum, and now that was a part of what their contribution was. Felix Klein probably did the best job by introducing functions as a concept and making calculus the end point of secondary education. Klein really had an impact on the school curriculum. So, throughout history, we have had mathematicians who helped us understand how the secondary curriculum could be made more like what the university curriculum was becoming. But the question of what kind of help mathematicians could provide the primary curriculum, that proved to be much more difficult and we had fewer people working on that. The question of modelling and statistics and that sort of thing, again more mathematicians did not want to work. They don't consider statistics as mathematics. They don't really see the point of it. And yet, it's something students need to know; and most countries want to make it part of school mathematics. So, we have to get more statisticians to help us understand what mathematics of statistics should be in the schools. So, mathematics and mathematicians have played strong roles, but again, as I said earlier, it's now the applied parts of mathematics. And it's the applied mathematicians who, I think, have more to offer us than what the pure mathematicians were during 1980's.

R: So Jeremy one of the areas we are interested to look at – the idea for this study also came from a colleague in Costa Rica who was involved in curriculum reforms – one of the points we discussed was that in more recent curriculum reforms, for example, teacher unions have a strong say in how a curriculum may (or may not) unfold, given the challenges that a new curriculum may demand on teachers. I wondered what your thoughts on that was. If we look at the period of the New Math era, it was also the time that mathematics education as a discipline was coming into its own. Also, at this time, different groups of researchers in different parts of the world, were researching primary mathematics and beginning to influence mathematics curricula a lot more. But it feels like, in more recent times, mathematicians and even mathematics educators, perhaps for the reasons you have already mentioned, are not really involved or participating in curriculum policy. Would you agree with that or do think that is not yet the case?

J: I think there are not as many people, mathematicians who are working in the area of curriculum, for some of the reasons as I said. But I think they also discovered during the New Math era that it was more work than what they thought it was going to be, and they didn't necessarily have much to offer. But if we learned anything from that period, we learned that teachers have to be a part of the conversation. If the teachers are in a union and not interested in pursuing curriculum development, then we are going to have a hard time. So, what we have to do is to convince teachers that it is in their interest to participate in the curriculum development. Again, one of the lessons we learned is that it is simply not enough to prepare materials for teachers. What one has to do is professional development with the teachers - professional development which the teachers themselves conduct and work together, and again we come back to the social turn. The teachers have to work together to change the curriculum. And they can do this in their own schools. The teachers in the school can be a team to be changing the curriculum in the school. But they need help. So, we need to figure out ways to organize teams of mathematicians, mathematics educators and math teachers to work together on changing the curriculum. And helping the teachers see what changes are needed and how to make those changes.

Y: In the case of Japan, we have a national committee of mathematicians, math educators and math teachers, and people from outside areas of the school contexts. All the committee has discussion to do the curriculum development and has been done by such a mixture the people…

J: I think that is the way a country needs to do it. Bringing the people from outside as well as those sorts of the things.

R: In fact, we have a panel which is made of math educators who have participated in a national or major curriculum reform. I think it is going to be very interesting to hear about that from different country perspectives. How this has changed, and is it the same across different country contexts or how is it different. And who are the main agents of change. Who has become the main agent of the changes that eventually, after very contested processes, settle into the curriculum, and about the kinds of math content and what gets agreed about the kinds of approaches, assessments and so on.

J: You mentioned earlier about the researchers, and I think that one of the lessons that has been learned is that people who want to research the curriculum cannot do it without engaging with the people in the classroom. And those that are going to be doing the reforms and creating the materials and creating the teacher development plans, that research can't be separated from all of that and has to be tied into that. I think some researchers have made the mistake of going to study the curriculum as if it was out there. But they need to be a part of the change, in order to study it.

R: In fact, Jeremy would you say that we have not really studied that curriculum making process, you know, in how curriculum reforms are motivated and then actually happen and the policy making space – it is not well-understood how the contestations play out. That it is an intensely political process as much as it may have debates about content and pedagogy and so on.

J: Yes, that's right. Actually, this book, *Mathematics curriculum in school education* (Li and Lappan, 2014), one of the themes of this book is, we haven't really, even though people in here make some effort to talk about it, we haven't really done a good job of studying how this process works or could work in schools around the world. We just don't know, and this is a sort of a first step. But it's clear, despite an enormous amount of curriculum development work, we do not have an enormous amount of curriculum development research. And that is what your project is going to be working on.

Y: In the context of the United State it has been much more complicated…

J: The United State offers the same thing as a number of different countries, that's right.

R: Expand a bit more on the US context for us Jeremy. I know the process may be very different across different states but what are your reflections on how you see this aspect in the US.

J: Well, first of all, the United States is almost unique in the fact that we don't have a ministry of education that choses the curriculum. And one of the articles of faith in the US public is, we don't want Washington telling us what our curriculum should be, and what we should be teaching. So, all of our efforts in recent years is to bring some structure into the school curriculum across the country, and having to face up to a public that says "we don't want this", and "who are you to tell us what to do".

The fact that we have a National Council of Teachers Mathematics setting up the standards programme. That's very unusual. I don't know of any other country that has something like that happening. So, I've heard from people saying, who chose the NTCM to do this work. Well, they may have decided to do it themselves, and the government didn't set it up. But the government has in some cases embraced it. That's one of the problems we've had. We've had political problems attributed mostly to the fact that we don't have a national curriculum. And some people think we should have one, and other people say no. We have never had a national curriculum. So, there are a lot of divisions about that, and if you start offering something as a core curriculum that everybody should work on, you'll get a lot of politicians who say "no, don't you" and parents, and others "don't do that". We have some kind of a special situation. Elsewhere around the world, I think there is more acceptance of a national curriculum. There is a wonderful quote in the book I remember all of the time. Essentially it goes back to the time when the UK didn't have a national curriculum. Before the UK had the national curriculum, one minister said about the UK, everyone is supposed to be going his own way, nobody is. In the France, everyone is supposed to be doing the same thing, nobody is. So that sums up the difference between what politicians say and what teachers do.

Y 3. With the rise and dominance of technology, what is your prediction about the nature, role and place of mathematics in school curricula in the next decades.

J: My prediction is that mathematics will become more applied as teachers learn more about how to handle the application of mathematics. So, I would expect that programmes based on modeling, on statistics, or other applications of mathematics will grow as soon as teachers can learn what they want to do with that. But I think the focus of the push will be in that direction, because the technology is allowing us to deal in the classroom with the applications that were never possible.

When I was teaching 16 years ago, we could not do a lot of the applications, because we didn't have computers, and the students couldn't do the calculations that were needed, in order to figure out these applications. So, it was not possible, even if we had those good applications, we could not handle them very well in our classroom because the students would get bogged down in the calculations. Today we can use the computer, and let the computer do the calculations. Then they can go much farther, and I think that we are moving in that direction. I would guess that school mathematics is going to become a much more applied subject. But again, I think that one of the things that will hold us back is the teachers are not sure what to do with that and they don't necessarily know the applications. Although, I think, a lot of them are out there, they can look online for some things too but they may not be comfortable with that. So, it will be a slow process.

Y: And again, teachers should be key players for these advanced lessons.

J Absolutely, because they are the ones who know the kids in front of them, they are the ones who know what these kids can do or cannot do, and we need to trust the teachers to bring in the applications that these kids will be able to learn from.

R: Do you think that if we move toward more applications that in some ways that could lead to a kind of splintering of mathematics, or in a way the mathematics disappearing into the application. So that we have modelling maybe or different areas of applications kind of emerging in their own right.

J: That could happen. It depends on the culture and the country and the circumstance. That could happen, but I would guess that because people who become mathematics teachers are attracted by the mathematics, they will always preserve a certain part of their teaching to paying attentions to mathematics. They know what the mathematics is or they should know. And they want to convey to the students the interest that they find in mathematics. Mathematics is interesting in and of itself as well as in its applications. I think that teachers understand that, and I think they should prevent mathematics from being taken away.

R: Except Jeremy, the students of today who are attracted to mathematics by the applications, may not have loyalty to the mathematics. The future teachers may not have loyalty to the mathematics because they are attracted to the applications.

J: Again, that depends how the curriculum is orchestrated. Because what should happen, in my opinion, is that the applications should be a stimulus for looking at the mathematics. In other words, I would say that if the teachers are not showing the mathematics behind or allowing the students to discover the mathematics behind the applications, then yes, the mathematics is going away and it's only the application that's interesting. But if the teacher is well prepared and understands what is going on, they will help the students see the mathematics that is in the application. It's ridiculous to do applications of mathematics and not at all look at the mathematics which you are applying. You need to consider how has the mathematics been used in this application, and let's talk about that. So, the curriculum has to be orchestrated, so that the applications and the pure parts are connected.

R: Do you think mathematics will continue to be valued and regarded as important into the future?

J: Yes, I do. I think it will. I think all of us went into it because we saw something in it that has not gone away, even as the applications have been modified. There are still important ideas of pure mathematics that need to be understood and that are important for society. We cannot just throw that away. So, I think that mathematics will continue to have a good place in the academy. It goes all the way back to Plato and still today, maybe mathematics departments are not seen in the same way today as they were in Plato's time, it's still a subject that has a lot of respect, I think.

R: Let's come to the technology, how do you see the technology changing how we teach and learn maths, and what do you see technology doing in the maths curriculum.

J: The main point I see is that the technology allows the teacher a way of getting into the mathematics they could not have done before. I am so impressed by some of these applications that are out there, used to illustrate mathematical ideas. I could never have done that when I was teaching. I didn't have computers in my class. I couldn't have done it. As I go back to the point Morris Kline made that these applications have to be found and brought into the curriculum. At the time, I heard him as an instructor, we couldn't do it because we couldn't find the applications. But today, the applications are all over. They can be handled by students if curriculum developers can put them in the right frame and help teachers teach the mathematics behind the applications.

Y: Is there any chance of a much more related connection to science and mathematics through applications? The applications of mathematics which you are talking about.

J: Science and mathematics are too different fields, at least here in the US. And I'm in a department of mathematics and science education. We like our colleagues in science education, but they do different things than we do, and they have a different culture than we have. Partly that's because, I guess I am not sure how much this is specific to the US, mathematics is a required subject all the way through school, and science is not. So, they have a different job than we do. What mathematics educators do is different from what science educators do. Even though we are working in the field that are very close together, I think they are coming a little bit closer. But I don't expect them ever to join completely.

Y: I think in some countries just like a close disciplinary curriculum development might be on going.

J: Well, we have an example of that in this book (Li and Lappan, 2014) - unified mathematics and science. But that has not been sustained. We haven't been able to keep alive, the connections between mathematics and science. So, I don't know where that's headed.

Y: So now we have listened to the voice of STEM education around the world, which makes a lot of issues.

J: A lot of people who work on STEM are worried about the mathematics parts coming out of that. I think maybe there is something to be worried about. It seems to be the direction we are headed.

Y: 4. It could be argued that the School Mathematics Standards developed by the USA have influenced national mathematics curricula in many countries, but continue to appear to be controversial within the USA - Do you agree with this statement and if so, what is your explanation for this in the USA.

Yes, it's partly because it became involved in political matters. At the time of the New Math era, when we wrote our book (Howson, Keitel and Kilpatrick, 1981), politicians did not have any connection to the school mathematics curriculum. There were no cases of politicians saying "vote for me and we will have this curriculum in the schools", with the one exception of Germany. I think Germany was one country where there were different plans. Politicians who took different sides on the German school mathematics curriculum. But Germany was the only case I've ever heard of. But I think in the US today, there are politicians who say, "if you elect me, we will go back to that curriculum, we will not follow this curriculum". And in particular the Standards and the Core have been debated. We have a movement to privatize school education and that movement is caught up with some politicians on one side and other politicians on another side. And somehow the mathematics curriculum gets connected with that. It started largely with the idea, should we teach mathematics to everyone, or shall we teach it just to the people who deserve it, or shall we have different curricula for different pupils. And politicians have gotten into that to say “well, these people are trying to teach the same mathematics to everybody, they are ruining mathematics”. There are mathematicians who say that. So somehow politicians, mathematicians and mathematics educators are involved in discussions today, in the US, that they were never involved in during the New Math era - it was not a political issue at the time.

R: So, Jeremy, are you saying that what we see now is, where mathematicians, for example, disagree maybe with the curriculum reforms being proposed, then actually involve politicians because that would be one of the ways they could effect the change, that they may not be able to do with just their voice.

J: Yes, that is how I see it. It’s connected with the idea of should we teach the same mathematics to everyone, can everyone learn the same mathematics. One of the ideas during the New Math was that we ought to have a constant standard curriculum, which may take some students longer to cover that math materials. But it ought to be the same for everybody. This was the general idea in the New Math. But that idea is not widely accepted in the US. We have lots of cases where students are given a test at the end of a certain grade and if they don’t do well in the test, they are put into one set of classes; and if they do well, they are put into a another set of classes. So, we have layers of mathematics, if you do well on a test, you get a certain mathematics, and if you don’t do well, you don’t get it.

R: Is that happening in the compulsory phase of schooling or is it happening in the post compulsory. I’m assuming, is it happening in the elementary or junior secondary…

J: It depends. It happens in different ways in different parts of the country. There are schools which have different primary courses in mathematics for different students. If you do well, you are put into one course, and if you don’t do well, you are put into another course.

R: In primary?

J: Typically, it comes in the middle grades. Typically, a line is drawn around grade eight, and if you pass, you go into one program, and if you don’t, you go into another. But in some cases, it happens earlier than that, in the primary grades. It almost never happens that students are kept together as a group all the way through to the 12th grade. That almost never happens. So, we haven't figured out what we want to do. I mean there is a lot of rhetoric that says "we should keep kids together in the same class to learn mathematics regardless of what mathematics we are teaching". And there are others who say "no, we have to separate them because some of them are going to do well, and others who are not going to do well, and we shouldn't put those people into the same class". So that's a political issue in many places.

R: In the senior secondary, I recently read that students do have choice. This was an interesting article kind of showing that more students are beginning to say, take statistics compared to other areas, like algebra, and so on. In the later years it looks like the selection is by topic areas.

J: Yes, it is different for different topic areas, that's true. But I think each country has to deal with the question of: when do we start differentiating the curriculum, and how do we give students' choices, how do we give anybody choice, and who chooses, the teachers, the parents, the students, and what are the paths that students can take? When do they start taking mathematics and do they have to take it every year? Those are all questions that each school system, or each nation has to decide. Are we going to teach the same mathematics all the way through school? Most places say no, we should not.

R: Yes, that's right. In most places, somewhere along grades 9 or 10 or generally in the post-compulsory school era there is differentiation, either by kind or by content or by combination of topics. This is also a very topical issue in the South African context, where coming out of the Apartheid era we have a stronger view to wanting everyone to do the same but because of the inequalities in resourcing and so on, the outcomes are in fact very inequitable. So, the idea that everyone does the same, results, in fact, in quite starkly different outcomes. But it's one of the ongoing debates in South Africa. I suppose in Japan, where it is more homogeneous, it might be different.

J: I think this is an issue that every country has to solve somehow. Different countries have done it differently.

Y: For example, in Singapore the differentiation is much earlier.

R: So, I think what will be very interesting out of this discussion, would be to see what shapes different approaches in different countries and what drives it - how politicized it is, what drives the eventual decision about how far a single curriculum is taken by students and then if it differentiates, in what ways. I don't think we have studied it across a range of countries and studied how young people are treated after a particular age or grade level.

J: Yes, and in particular, what is the role of the mathematics teacher is in these decisions, and what is the role of mathematicians, what is the role of the public and the politicians.

Y: 5. We may have some other topics, but the final question is related to theme D actually, the globalization and internalization of mathematics curriculum in terms of a society that is changing. What is your opinion about the rise of international comparative studies in mathematics performance and their impact on mathematics education? For example, the OECD's PISA or TIMSS have strong impact on math curriculum reform, I think. Would you regard their impact as mainly positive or negative on school mathematics curriculum reforms?

J: That's the most difficult question, because I can see positive outcomes, impacts. I can see the negative impacts. The positive thing, I think, is that it has made some countries at least, aware of what's happening in other countries, and what their curriculum looks like. And it has for all of us, allowed us to see across the world what kids can do and what kids cannot do. So that part, I think, has been positive for people in their own country to see what their own kids can do, and then to compare that with kids in other countries.

But the negative aspect is a problem, because these are all artificial frameworks that have been drawn up for different purposes. I have made criticisms of efforts by American educators to try to use the data to make points about the US schools, because TIMSS is one thing and PISA is another. You can't mix the two, that's the one issue. But the other is that these are pretty arbitrary. PISA is trying to get a picture of how 15-year olds can deal with applications of mathematics, largely. And whereas TIMSS is trying to give a picture of how well kids at different levels, now 8th grade, come out of the math programme, what can they do, and what cannot they do. And all this is pretty arbitrary.

I very much remember a conference in Malaysia, where I heard someone from Singapore say that they were going to look at how the Singapore kids did on the different kinds of questions in PISA, and then they were going to change their curriculum to deal with the places where the students were not doing so well. That struck me as completely backwards, because you don't want to use the framework to say this is how our curriculum should be. You should decide what your curriculum is and if it doesn't match what PISA has, "Okay it doesn't match it". But the idea that the people from Singapore were going to be taking the PISA framework as the gold standard, I don't buy that. I worked with some of these people in putting these frameworks together. These are just opinions of some people that this should go in there, and that should not go in there. I remember in TIMSS, at one point there was a question on conversion from Fahrenheit to Centigrade. For people in the US that makes a certain amount of sense. We don't have or we haven't gone completely metric. But for the rest of the world, it didn't make any sense, so they threw those questions out of the TIMSS frameworks, because it only applied to one country that we can figure out. So, it wasn't good enough to put on the TIMSS. These are arbitrary constructions of experts. OK experts. But who says they should be what the people in a given country are using as their gold standard, as their framework. That's a problem I think in these international comparative studies are being misused when the framework is taken as the thing which we want kids to be able to do. It's helpful. It gives some general idea of how your kids are doing on this topic or that topic. But to use it as an overall evaluation of what your country is doing is, I think, a big mistake, because a lot of the things you doing in school mathematics may not be on the test. They are not there. They don't show up. But they are important things that your kids are learning. So why not keep them there. I think it's terrible that these frameworks are becoming the gold standard. If they are teaching something out of the frameworks, then we shouldn't be teaching it.

R: Are you agreeing, in some ways, the backwash effect from these international studies is resulting in more and more convergence around the kinds of curricula, the frameworks that are emerging across very different countries, notwithstanding, very different contexts, very different cultures, social situations, and so on.

J: Exactly, that's what bothers me the most about it. I mean I understand that in order to make comparison you have to have a common measuring stick, but you don't have to take that measuring stick as the goal for your curriculum. That's where I think is the problem. If you using the measuring stick as this is what we want, you haven't solved the curriculum problem for your country, because these things are a kind of consensus documents, these frameworks. As I say, you and your country may be teaching something very important and very good, and getting good outcomes. But it's not measured on TIMSS or PISA. Does that mean you should throw it away? I don't think so.

Y: But the TIMSS and PISA can be used for the political talk in the education communities still. It is a strong influence on the communities.

J: Yes, and that has its down sides as well. I agree. In the US we don't do well in these things, and we very seldom look closely at the PISA results. TIMSS seems to dominate our attention, PISA doesn't or doesn't get as much. That's kind of crazy too, because I think both have something to tell us. It's just that the message doesn't come through very clearly. As I said, people get into comparisons between states, for example, or between school systems on the basis of these tests. It's not a good idea.

R: Would you also say that the rise of these tests and the way the TIMSS and the PISA results are announced and played out in the media, results in a greater politicization of the curriculum, more than it needs to be, because then people speak about the maths curriculum, and about maths and maths education almost all the time with reference to these tests.

J: Yes. We haven't learned yet to put a distance between ourselves and these results, but I think as the results power up, and as people get use to these situations, it may get better, because then people stop being attracted by the … Well for one thing, they stay relatively constant, so there isn't much to be gained from the way the results are being reported. But I think there is a kind of lack of attentions that's happening. That's probably a good thing.

Y: Just come back to the issue of the PISA literacy concept. Mathematical literacy tells us something like to be reflective as a citizen, an effective citizen. Mathematics could be used for that. This message sounds quite attractive in a sense for educational purposes. In some countries including Japan, Korea, and some other countries, the Danish curriculum, is based on competencies – not content but process aspects…

J: That's the point, I should have made when you asked me about the changes since 1980's curriculum development, because I think that's another direction that curriculum development has gone - away from content toward process. Maybe that's connected to the application idea, but it does seem to be the case. As you mention, Denmark, Japan, Korea, these are countries, by looking at mathematical literacy rather than knowledge and specific content, are looking for other outcomes from school mathematics. And that's a good thing, I think. So, to the extent that PISA gives us some ideas of mathematical literacy, I see that as very good. But unfortunately, what happens when the results are reported, at least in the US, all we get are these numbers in the newspapers. Japan was here, and the US was here. We don't get any discussions of the mathematical literacy of the US students or Japanese students.

Y: I just am reminded that Mogens Niss from Denmark might be arguing that the Danish curriculum was prior to the PISA, or something like that.

J: Yes, that true. PISA is very close to the Netherland's curriculum, because the people from the Netherlands were very influential in setting up the PISA framework.

R: I know that was the last question. I just wanted to pick up one point we discussed earlier. Jeremy do you think the mathematics curriculum reforms as a topic, has been under-researched and under-theorized.

J: Yes, absolutely, both. Under-researched certainly, that as I said, is one of the issues discussed by almost every chapter in this book (Li and Lappan, 2014). But under-theorized too. It's a really complicated subject as we have indicated, differs from school to school, from country to country, from grade to grade, and so forth. So, the idea that you can have a comprehensive theoretical structure that can cover all of those differences, and we know it's based on culture, it's based on knowledge of teachers, teacher's knowledge, and it's based on what the public wants from education, it's based on how education is structured in the country. It's got many, many influences on it. So, the idea that you can come up with a theoretical structure of curriculum, it's very difficult to imagine. And if you think of how we dealt with this in the book (*Howson, Keitel and Kilpatrick, 1981*) it was really case studies, and the various frameworks that we came up were mostly tied to individual cases of curriculum development or comparisons of curriculum development. It's because it's really hard to think of a structure that would allow you to investigate all of the school mathematics curricula around the world. It's just too big a topic.

R: Would you say that besides it being a big problem, it's not really being taken up by maths education researchers, not only because it is difficult but because it's so far from the frame of their experience, so what is easier, what is in your field or your view - the classroom research, the content research and so. Is it because maths education researchers are generally people not involved in reforms and so on, that is resulting in that?

J: I don't know about it. I think it's possible that mathematics educators do get involved in reform. But I think what happens is that they get involved with very small pieces of reform. They may study what is happening with a particular topic at a particular grade level in a particular situation, and therefore the theories that they have are micro theories, they are small theories of what's happening in these situations. And I think actually we have people who are doing that kind of work, but we don't think of it as curriculum research, because it's not looking at the whole curriculum. It's looking at a fraction, a small piece of the curriculum in a particular situation because that's the only way people know how to do research on it. So, we have a lot of researchers now, and we have a lot of people who are studying the teaching of various topics, the learning of various topics, how such things as lesson study are being conducted, ideas like that. But the process ….

R: Not the big picture.

J: No. If you think about the studies that we report in here (Howson, Keitel and Kilpatrick, 1981), during the New Math era, there was lots of money spent on big projects. And today people are not doing that. So, it's expensive to do a kind of large-scale curriculum study. The national study that I worked on, cost millions of dollars, and really came up with very little, almost no theoretical contribution.

Y: In the 1960's right? There was a huge amount of money.

J: Yes, it was in the 60s. That was a big amount of money for the 60's. I don't know how we got the money. It was from the National Science Foundation, that was giving a lot of money for that. They wanted evidence on what worked and what didn't work but they didn't get it. Anyway, these are big ticket, big money items and we don't have that kind of research going on. So, it's not surprising what we not doing it.

R: So, Jeremy the point then is that, that kind of money is going into the PISA and TIMSS and so on, who are doing that research, who come from statistics, economics, development studies - all of those areas. They do make these big pronouncements from their studies?

J: Yes they do.

R: So we, in a way, math educators, are kind of on the outside looking in on that and that's sort of, in a way, driving the big reforms. Do you agree with what I'm saying?

J: I guess you are right. I think there are some mathematics educators involve in working on the PISA and TIMSS. It's just not enough to command respect because so much of what they do is measurement studies. It involves measurement, and it doesn't really involve mathematics education. There's a new book from the NTCM on exactly just this. People in the US who are asking how can we do research into the kind of measurement, which is going on the PISA and TIMSS, to get a better idea for mathematics educators of what's happening. We may get to it again, I don't know.

Y: I just wanted to say that ICMI Study 24 participants may be very much interested in: what if you write a second edition of this particular book (*Howson, Keitel and Kilpatrick, 1981)* nowadays. What does it look like?

J: We talked about this, we three authors talked about revising this. None of us had the energy at that time to do it. It's too bad that we didn't. But I think the circumstances were right for us to do it at that time. I don't know that I would want to try it today. The situation is so much more complicated. There are so many more countries that are working on their curriculum. That's good. There so many more researchers and curriculum developers working on them. That's good. They are not just borrowing from each other and they are doing their own things. That's good. But to try and put it all together in one package, I wish you luck with your Study 24.

R: Maybe Jeremy to pick up on one question. In a curriculum reform there is always reference to values and goals. Do you see the values and goals of curricula as having changed over time and have they in a way converged or diverged, how do you see it? Normally school maths curricula would be prefaced with particular values and goals. In recent times there has been some discussion around the values and goals of school mathematics curricula. As a kind of overarching perspective on a curriculum, beyond what we have been talking about content, and pedagogy and so on. Would you have any comment on how that has remained the same or changed?

J: I think we have touched on it in some ways. This switch or this movement from content to process is an example of how the goals have changed. For example, the book that I worked on "*Adding it up*" (Kilpatrick, Swafford and Findell, 2001) talks about what is mathematical proficiency, and offers a framework for mathematical proficiency. Now, I don't want to say that's the end. But that is an attempt to say, if you aiming for mathematical proficiency, you need to think about more than just content and process, you need to think about other dimensions that are being dealt with. And this metaphor of a braid, strands that are being developed along the way. It is a different metaphor for how the curriculum could work, then the metaphors we discussed in this book. So, I think the idea of curriculum as a process, and one that needs to be shaped by the situation in the school, the situation in the country, the situation in the classroom - all of that has changed from what it was before. So, today I would say the goals are much more towards recognizing that the goals may be the different across different school systems, across different countries, across different situations. And that each country has to figure out what are the goals for us, and what are the directions that we want to go.

R: In some of the European countries there has been this development around a competency-based framework. I wondered if you had seen that and how would you see the notion of proficiency that you have developed in that book compared to this notion of 21st century competency-based framework.

J: I don't know enough about the competency-based framework to really respond to that. As I say, I think the notion of proficiency worked for us to have chosen because we wanted to be able to talk about something that teachers in any grade could work on, and that work could go on all the way through the primary grade, through the senior or high school. I think talking about mathematical proficiency allows you to focus on different content of mathematics and different processes.

R: In terms of an output would you say, in terms of kind of expressing it in an output of proficiency?

J: As a process, a process to be monitored. As long as output is only at the end, I guess. The idea of proficiency is not that you become proficient when you leave 12th grade. It's that all the way along, you should be asking how proficient is this child, given what the child has done so far, and we look at the different components of that. And that is a much better way, I think, of looking at the curriculum, saying, how far have we got with this child, and where are the strengths and where are the weaknesses. And we need a detailed way of inspecting that child's knowledge and competence and proficiency at doing. Proficiency at doing mathematics. We want people to be able to do mathematics and not just remember it. So, I don't think that we've really changed the goals for school mathematics. I'm not sure that we have. But I think, we think about them differently maybe. Maybe that's what's happening.

Y: I think it is impressive to me that you included productive disposition among the five strands.

J: Yes, that was the most controversial choice that we made. There were people in the committee, including some mathematicians, who said we shouldn't include it. And there were people, actually who deal with assessment, who said that we cannot assess it, so we shouldn't include it. But we had teachers on the panel who produced that, who said we have to put something like productive disposition in there, because we have so many problems with students who learn about mathematics, but don't like it. So, we need to say the disposition you come out with is an important part of your mathematics learning. And if we don't look at that affective part of proficiency, we are not doing our job.

Mathematics is a subject the kids start out liking, around third grade they like it. Around 6th or 7th grade they don’t like it anymore. There is a productive disposition problem, so we were finally able to convince the mathematicians that we needed to put that in, even if we can’t measure it very well. Teachers know what it is, and teachers can tell you whether this child has a productive disposition or not.

R: Maybe one area, we did touch on, on curriculum resources, materials and so on. Jeremy did comment on that. I think, Jeremy you indicated that you can give teachers the texts that doesn’t mean they will be able to implement a curriculum reform.

J: That’s right. The SMSG started out that “we will write new textbooks and the teachers will use them, and that will change the curriculum”. We discovered that is not the case. It will not happen, especially in primary school. They wrote primary textbooks and the teachers didn’t know how to use them; they didn’t understand what was in there; they had not been educated in the ideas in there. So, it didn’t work. The New Math had an effect, had some positive effects. But in the case of elementary school, with elementary teachers, giving them a new book, is not going to change their teaching.

R: I think that has, that recognition, maybe that learning has resulted in the growth of this whole area of teacher professional development.

J: Yes, that’s right. So, in a way it is artificial for us to think of the curriculum as being separate from the teacher’s professionalism, because it completely depends on that, and we cannot talk about reforming the curriculum, getting it in a new form, if the teachers are not with us, if they don’t understand it. They have to both understand the changes that they are being asked to make, and they have to agree that those are good ideas and they have to try them out to see if it works for them. So, there are a lot of conditions there on curriculum development. You can develop on paper what is the best curriculum ever seen, but if the teachers don’t understand it or don’t agree with that, or don’t know how implement it, you won’t get any change.

R: This of course, has led to new challenges, we haven’t talked that much about when major curriculum reforms are implemented, which require big changes in the content, or the pedagogy that teachers are then expected to teach, the whole problem of effecting a change in what teachers do in their teaching, assessments, etc. then raises other big problems about how to bring about changes through teacher professional development at a national level and the different models that may or may not work.

J: That's right. Every country has to figure out how do we reach teachers, what can we do to help teachers, because they need our help. And some countries, I guess, have mechanisms for getting teachers educated, for teachers to learn about change, but other countries don't have the resources. So, when we are talking about resources, it's not just the materials which we give the teachers, it's also the education that the teachers get.

Y: It's maybe in the system as the whole that the teachers are working in.

R: I think we've covered most of the questions.

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IMPLEMENTING CURRICULAR REFORMS: A SYSTEMIC CHALLENGE

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This contribution addresses the challenge of implementing curricular reforms. I first briefly present the approach I propose considering education systems as complex dynamic systems and the main theoretical elements I will rely on, offered by the Anthropological theory of the didactic and the ecological perspective underlying it. Then I use this approach to discuss the challenge raised by the implementation of curricular reforms using the case of French curricular reforms and the outcomes of a symposium at the EMF 2012 Conference comparing the situation in six Francophone countries.

INTRODUCTION

As highlighted in the Discussion Document for this ICMI Study, curriculum reforms are transformations that generally affect education systems "as a whole at a national, state, district or regional level". They modify the conditions and constraints of their functioning to cause changes in the state of these systems. Their *raisons d'être* situate at different levels, *raisons d'être* concerning the content of teaching, the balance and relations between school disciplines, pedagogical methods, or *raisons d'être* concerning more generally the social contract between a society and its School, which are, more and more, the expression of supra-national visions. Their design mobilizes a diversity of institutions and agents, and their implementation an even greater number. Design and implementation are processes that take place over time and whose dynamics depend on a multiplicity of factors and their interaction. At the very moment when a curriculum reform is eventually finalized with the corresponding texts adopted by authorities, and is ready to be implemented, these factors and their possible interactions are only very partially identified and even less controlled, if even controllable. The texts, however constraining they may appear, give a certain degree of freedom to all those involved in the implementation for expressing their agency, opening up a range of possible dynamics whose regulation is a crucial issue. To question the implementation of curricular reforms, which determines their success or failure, is therefore to try to understand the functioning of these particular dynamic systems, in the face of the ecological disruption that is always a curriculum reform, and the means used to regulate these dynamics. In this contribution to the ICMI 24 study, I adopt this ecological approach in terms of dynamical systems. In the following section, I introduce the main concepts from the anthropological theory of the didactic (ATD) I use in this reflection, before trying to draw lessons from two case studies. They respectively concern the French education system and a group of Francophone countries from the EMF (Espace Mathématique Francophone) network affiliated to ICMI.

ELEMENTS FOR AN ECOLOGICAL APPROACH SUPPORTED BY THE ATD

As mentioned above, I consider curriculum reforms as ecological disruptions of education systems and the analysis of their implementation and effects as the study of the responses to these disruptions. Such an ecological perspective being central to the theory of didactic transposition and to its ATD extension, I use these theories to approach the dynamics of curriculum reforms. In this section, I briefly introduce the main elements of these two theories supporting my reflection. For more details, I refer the reader to (Artigue, 2011).

Didactic transposition: niches, habitats and trophic chains

The theory of didactic transposition developed in the early 1980s to overcome the limitation of the prevalent vision at the time, seeing in the development of taught knowledge a simple process of elementarization of scholarly knowledge (Chevallard 1985). Beyond the well-known succession offered by this theory, which goes from the reference knowledge to the knowledge actually taught in classrooms (see Figure 1 extracted from (Bosch & Gascón 2006)), ecological concepts such as those of *niche*, *habitat* and *trophic chain* (Artaud 1997) are also essential in it.

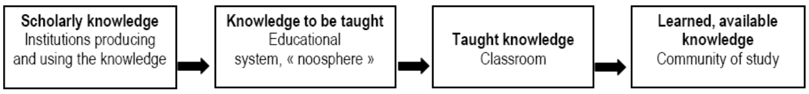

Figure 1: The didactic transposition process

The habitat of a specie (here a mathematical object, type of task, technique...) refers to the environment in which it lives, while its niche refers to the function(s) it has in this habitat. This ecological vision invites us to pay attention to the action of curriculum reforms on habitats and niches, and their consequences. In addition, it invites us to consider the objects at stake as elements of trophic chains, being fed by some objects while feeding others. Even apparently minor curricular changes can break existing trophic chains generating learning difficulties in topics a priori not directly concerned by these changes. As pointed out in (Artigue 2011), this is not independent of the fact that the official time of teaching is distinct from the time of learning. The teaching of a new object is thus an opportunity for consolidating the relationship with old objects and its zone of influence on learning is an area with fuzzy contours, difficult to identify.

Anthropological theory of the didactic: institutions and institutional positions, praxeologies, hierarchy of levels of codetermination level

The extension of the theory of didactic transposition within the framework of TAD has provided new conceptual tools for approaching curriculum reforms. As highlighted in (Chevallard, 2018), key concepts here are those of institution and institutional position. Indeed, as already mentioned, a curriculum reform, whether at the level of its conception or its implementation, mobilizes a diversity of institutions; it also mobilizes agents who occupy different positions in these institutions (the position of teacher is not that of pupil, nor that of school principal or parent) to which are associated different relationships to the knowledge recognized by the institution. During curriculum reforms, these positions are modified intentionally but also unintentionally. Understanding these moves and their possible, actual effects, is important for understanding curricular dynamics.

Another essential tool provided by ATD is the notion of praxeology, which is used to model all forms of human activity, thus mathematics and didactic practices. At its most elementary level, a praxeology (called punctual) is a quadruplet $[T/\tau/\theta/\Theta]$ where T designates a type of task, τ a technique or way of processing this task, θ a technology defined as a discourse making this technique intelligible and justifying it, and Θ a theoretical discourse which in turn makes θ intelligible and justifies it. Types of task and techniques constitute the practical block of praxeologies (praxis), while technology and theory constitute their theoretical block (logos). In a given institution, punctual mathematical organizations do not live in isolation; they are embedded in structures. As Chevallard (2002) points out, for the professor, the unit of account is a local praxeology, an amalgam of punctual praxeologies sharing the same technology θ, and corresponding to a *theme of study*. Local mathematical praxeologies sharing the same theory or piece of theory are grouped into regional organizations corresponding to *sectors of study*, and the latter in turn are grouped into global organisations corresponding to *fields of study*. Studying the dynamics of praxeological organizations, both mathematical praxeologies and the didactical praxeologies with which they are in co-determination relationship, is a means of gaining an understanding of curriculum dynamics.

The last conceptual element provided by the TAD that I will mention here is the hierarchy of co-determination levels. This hierarchy introduced in (Chevallard, 2002) and gradually refined nowadays comprises ten levels: *subject - theme - sector - domain - discipline - pedagogy - school - society - civilization - humanity*. The lower levels: *subject - subject - theme - sector - field* are, as shown in a previous quotation, closely related to the different levels of the curriculum organization of the subject, here mathematics. But the constraints and supports that condition the praxeological organizations and their curricular dynamics are not limited to these levels, hence the introduction of higher levels: *pedagogy - school - society - civilization - humanity*. At each level different agents intervene, new power relations, new rules of legitimacy are established. These different conceptual tools support the analyses presented in the next sections.

A FIRST CASE STUDY: THE 2000 REFORM OF HIGH SCHOOL EDUCATION

Main characteristics of the 2000 reform

This reform of high school general education from grades 10 to 12 offers an interesting case. This was not a curricular revolution, but it introduced some substantial changes still in effect today. To analyze the challenges posed by its implementation, I should briefly describe them. For more details, the reader may refer to (Artigue, 2003). At the level of school structures, unlike the reform currently under way, there were no major changes, and in particular the three orientations that organized the differentiation of teaching in general high school from grade 11 (L for literature, ES for economic and social sciences, S for sciences) were maintained. Continuity was stressed, as shown by the following sentence in the introduction of the grade 10 programme (DESCO 2000):

> This programme essentially retains the objectives of the previous programme (decree of 25 April 1990): the introduction and the accompanying documents reproduce them in a sometimes new wording.

At the pedagogical level, continuity is also evident. The curriculum discourse remained a constructive discourse and the place to be given to problem-solving in the construction of knowledge and its use was reaffirmed. But it was also stated that the school institution was challenged by scientific, technological and cultural developments and should regularly rethink its objectives in the light of these developments. For example, in the introduction of the programmes for grade 10, it reads:

> The constant evolution of our society, both socially and economically as well as scientific and technological, constantly challenges the educational institution. The latter, depending on the choices of its leaders and its various actors, takes this evolution into account to a greater or lesser extent. It is with this in mind that the programme published in 1999 is part of this approach.

This consideration led to substantial changes. In mathematics, the main ones were: the strengthening of the statistic domain, a differentiation according to the three orientations of study more sensitive to their specificities and for instance the introduction of graph theory in ES, the consideration of technological evolution, an increased emphasis on the interaction between scientific disciplines and more generally on interdisciplinarity. The major upheaval was undoubtedly the importance given to the teaching of statistics with the ambition to introduce grade 10 students to statistical thinking through the experience of sampling fluctuations allowed by the use of computer simulations. It is no coincidence that the person chosen to lead the group of experts in charge of preparing the mathematics curriculum was the researcher in statistics Claudine Schwartz. The accompanying documents (DESCO, 2000) specify that:

> The statistical mind is born when one becomes aware of the existence of sampling fluctuations [...] The pedagogical choice here is to go from observation to conceptualization and not to introduce probabilistic language first and then to see that everything happens as predicted by theory.

The attention paid to the articulation between scientific disciplines and interdisciplinarity more generally was also a strong point of this reform. The joint work of the expert groups in charge of the scientific disciplines on radioactivity resulted in an introduction of the exponential function as solution of the differential equation $y'=y$ and no longer as reciprocal of the logarithm function. However, the most important change was the introduction of interdisciplinary projects called TPE (Travaux personnels encadrés) in grade 11. TPE involve at least two disciplines, one of these concerning the students' orientation, and their preparation is supervised by teachers from the disciplines at stake. Two hours per week are allocated to TPEs in the students' schedule. The assessment takes into account the students' production as well as their written document and oral presentation. The curricular texts specify that this work aims to facilitate a multidisciplinary approach to non strictly academic issues and to help students mobilize their knowledge in such a context, to broaden their intellectual curiosity, to develop their autonomy, to help them acquire working methods and group work competencies, to develop their capacities for documentary research using the Internet, the selection and critical analysis of documentary resources, and finally to establish other relationships with their teachers.

The changes were thus substantial. The example of the exponential function shows that niches changed and that some trophic chains were certainly broken. New praxeological organizations had to be built for the new domains introduced as well as their progressive structuring over the years of high school. This was all the more demanding as most teachers had not encountered either graph theory or inferential statistics in their academic preparation. And even for those with a university culture in statistics, there was a didactic inversion between statistics and probability, as made clear by the quotation above. Moreover, teachers were asked to base the teaching of each domain on a certain number of study themes selected among those proposed according to their students' interests, which also required praxeological reorganizations. Multidisciplinary work, project pedagogy on subjects chosen by students involving the critical use of Internet resources, were also new for most teachers.

The implementation of the reform

This reform could have been rejected. The work of the group of experts had given rise to strong tensions with the General Inspectorate of Mathematics, a key institution for the implementation of curriculum reforms in France. The emphasis on statistics was considered exaggerated by many professionals, especially since it occurred at the expense of other domains, particularly geometry. Many also wondered about the possibility of making sense of inferential statistics without any probability background, and questioned the sense that students would make of the experimental work based on computer simulations proposed to them. There was also great concern about TPEs, especially among mathematics teachers who wondered whether they would find a role for their discipline in these.

The reform generated vivid and at times hard debates. The change of political majority in 2002 resulted in a number of changes and in particular to a rewriting of the programme for the L orientation under the control of the General Inspectorate. However, globally the reform resisted. TPEs still exist; the importance given to interdisciplinarity projects and modelling, the place of the statistical domain and probabilities have maintained or even strengthened in the next reform, that of 2010. Several factors undoubtedly favored this resistance, and I list a number of them below, by lowering the levels of the didactic codetermination hierarchy. The announced ambitions of the reform and most of the changes introduced aligned with international perspectives, which contributed to their legitimacy. At the national level, the work carried out by the CREM (Commission de réflexion sur l'enseignement des mathématiques), set up at the request of the mathematical community in 1999, chaired by the mathematician Jean-Pierre Kahane and including several members of the group of experts, contributed to legitimize its global vision. The reform was carefully prepared by the groups of experts appointed by the CNP (Conseil national des programmes) and bringing together a diversity of expertise. The CNP guidelines ensured coherence at a global level. The expert groups had a substantial amount of time, two years, to prepare the programmes. They also produced consistent accompanying documents, covering all new domains and showing how the proposed themes of study could be exploited. A specific website *Statistix* was created offering teachers the possibility to download dynamic simulations and access statistical data. The IREM network (Instituts de recherche sur l'enseignement des mathématiques), which contributes in an essential way to in-service teacher education in France, also mobilized, and especially the inter-IREM Commission on statistics and probability. Locally, IREM groups built situations and progressions, experimented, proposed training sessions, produced a number of paper publications and online resources, some in collaboration with the APMEP (Association des professeurs de mathématiques de l'enseignement public). The IREM network and APMEP journals devoted many articles to these innovations. TPE working groups were also created in various IREMs. They supported and analyzed the implementation of TPEs in the high schools of their members who were secondary school teachers, and proposed training sessions based on this experience as part of the professional development activities offered in the regional plans. French didacticians contributed to these IREM activities. Moreover, which is not frequent in France, a pre-testing of TPEs was organized, and when the reform was implemented, its results and a number of tools were made available to teachers by the Ministry (DESCO, 2001). Finally, teachers adapted to the mathematics innovations proposed in the ES orientation quite easily, in particular to graph theory and the proposed associated thematic work, with the help of the resources and training activities offered. There is no doubt that this new domain resulted more accessible than the statistical domain.

These factors certainly helped the implementation of the reform and, after a few years, the training demand decreased in the teacher community. However, this does not mean that the implemented curriculum eventually aligned with the intended curriculum. Still today, teaching inferential statistics, finding a niche for mathematics in TPE projects, remains a challenge for a number of teachers.

A SECOND CASE STUDY: RECENT CURRICULUM REFORMS IN THE FRANCOPHONE SPACE

In 2012, as part of the EMF conference in Geneva, two round tables were organized on how recent curriculum reforms were designed and implemented in French-speaking countries. Six countries or regions were considered: Federation Wallonia-Brussels in Belgium, Burkina Faso, Quebec in Canada, France, Romand Switzerland and Tunisia. The round tables were prepared by a two-year collaborative work. The perspective adopted was to conceive curriculum reforms as changes in the social contract between School and Society, at a time when the tercentenary of the birth of Jean-Jacques Rousseau was being celebrated in Geneva (Artigue & Bednarz 2012). The work carried out considered recent curriculum reforms from their conception to their implementation, specifying the educational and curricular contexts, identifying the institutions involved in the reforms and their respective roles, describing the global curriculum dynamics, before focusing on a dimension particularly important in each case study. Given the theme of the panel, I focus on the implementation of reforms, and due to space limitations I just contrast three case studies, regarding respectively Wallonie, Quebec and Tunisia.

The case of Federation Wallonia-Brussels

The Belgian contribution concerns the French-speaking part of Belgium. Although it concerns a small population, the education system is complex, combining three distinct educational networks. The study conducted (Baeten & Schneider, 2012) focused on the curriculum reorganization in terms of competencies started in 1997. As the authors point out, this curriculum reform was part of a global plan for equal opportunities, social integration and citizenship education, but it went along with a policy of centralization and increased control of the education system. It was indeed accompanied by a standardization of the curriculum with the drafting of competency frameworks for all levels of education and the creation of assessment tools to serve as external references common to the three education networks. The emphasis was put on transversal competences, valid for all disciplines. In mathematics, it was more particularly placed on problem-solving competencies, described in very general terms (asking questions, formulating hypothesis...) without taking into account the specificities of particular domains.

The contribution presents a critical analysis of this reform. According to the authors, the formulation of general competences was poorly coordinated with the mathematical content that remained nearly the same, and the resources provided to teachers, the training offered, were not very helpful. Ten years after the implementation of the reform in 2008-2009, the General Inspection Service produced a critical report, pointing out a number of inconsistencies, discontinuities, omissions and repetitions, and the fact that planning the progressive development of competencies was still a major challenge for most teachers. The report suggested defining, for each discipline, the "unavoidable" knowledge, that "really useful for the exercise of competencies and which can reasonably be considered as the foundations of a citizen culture in the disciplinary field at stake" (ibidem, p. 64). As explained by the authors, this led to a revision of the definition of terminal competencies in mathematics, according to a new framework. The authors mention the main ideas underlying it: insistence on disciplinary work concepts; revalorization of "knowing" understood with a certain level of reflexivity to which is granted the status of competence; and the idea that the development of transfer competencies requires specific teaching enabling students to construct homologies and thus identify classes of problems. There is no doubt that such ideas can be interpreted as a serious reconsideration of the role given to general competencies in the curriculum.

The evolution towards curricula organized in terms of competencies is an international movement as highlighted in the Discussion Document. The preparation of the EMF round tables made clear that we were all concerned by this evolution and had to face the difficulty raised by the duality competence/content. However, depending on how the reforms were designed, implemented and regulated, one could observe different dynamics. The case of Quebec is particularly interesting from this point of view and we present it briefly in the next sub-section.

The case of Quebec

This case study (Bednarz, Maheux & Proulx 2012) shows a long process of curriculum development beginning with the "Etats généraux sur la qualité de l'éducation" in 1995 and ending in 2008, mobilizing and coordinating the action of a multiplicity of actors, coming from various horizons. From the outset, as the authors point out, there was a visible shift in the orientation of the education system, moving from a policy of 'accessibility for all' to a policy of 'success for all', organizing the curriculum in terms of both knowledge and know-how that would become competencies, and stressing the active role that teachers should play in curriculum design and regulation. The "top-down" logic that had prevailed until then in the design and implementation of reforms was rejected. More specifically with regard to implementation, some interesting characteristics can be highlighted:

- large scale implementation was prepared by previous work in pilot schools with support in context, responding to local needs and ensuring that each school developed its expertise and autonomy.
- implementation was supported, throughout all its duration, thus more than a decade, by substantial training activities organized both at national level and regional levels. National activities targeted educational advisers, resource persons and managers, focused on the global elements at the heart of the reform (concept of competence, transversal competence, culture, socio-constructivism, evaluation, etc.) and favored appropriation of the reform through small group work. Disciplinary issues were addressed at regional level, targeting teachers and pedagogical advisors. In the specific case of mathematics, the emphasis was placed on the concrete construction of situations by teachers in relation to the core elements of the curriculum, with as much as possible experimentation of the situations collectively designed in classrooms and a posteriori joint analysis.

In addition, a permanent process of regulation was planned by the Commission des Etats Généraux. Thus, in 1997, the Minister of Education officially established the Curriculum Commission, which later became the Advisory Committee on Curricula, to which a mission of continuous regulation was entrusted, until the end of its mandate in 2010.

This case shows a coherent global process of design, implementation and regulation, conceived as a continuous process obeying a participatory logic. This logic is intended to be:

> transparent and rigorous, so as to allow it to be adjusted as new needs or knowledge emerge, and to avoid piecemeal changes under partisan pressure from professional associations or political pressures (ibid., p. 81).

The curriculum development model is "hybrid" combining "top-down" and "bottom-up" aspects. The study explores them in depth, through interviews capturing the points of view of a diversity of actors.

The evolution towards a curriculum structured in terms of competencies took place in this context. The authors present this evolution as an inversion:

> Previously, mathematics was defined by its contents, and these were to be achieved through mathematical activity. Now, mathematics is defined by its activities, and these activities are mobilized in work on various contents: numbers, algebra, statistics, geometry, etc. (ibidem, p. 101)

Disciplinary competences (solving a problem situation, reasoning mathematically, communicating mathematically) have become the central objects of teaching. The authors do not minimize the difficulties raised by this inversion and the accompanying and regulation work it required. It is clear, however, that this inversion has taken place under a system of conditions and constraints very different from the case of Wallonia, and the move towards competences was not reconsidered. As the authors point out in the conclusion of their study, what the case of Quebec shows is the case of a curriculum that is constantly developing, a "living" curriculum that leaves room for teachers and other school stakeholders to make it their own. This is a demanding but visibly productive vision.

The case of Tunisia

The case of Tunisia is quite different (Smida, Ben Nejma & Khalloufi-Mouha 2012). The authors describe the five curriculum reforms having taking place since the independence, the last one at the time of EMF 2012 being that of 2002. Due to space limitation, I focus on this last one. As the other reforms evoked in this text, it reflects the influence of international trends: the desire to build an inclusive school for citizenship, the emphasis put on transversal competences. The aim is to build the "School of tomorrow", which must "train a citizen who learns to learn, to act, to be and to live with others" (ibidem, p. 132), and the teaching of mathematics, like that of science, has the task of developing competences in reasoning, problem-solving and modelling. The organization of the reform obeys a new structure with:

- a first commission bringing together inspectors, university academics and various personalities responsible for defining the aims of the education system and preparing specifications for the disciplinary commissions, and for setting curriculum structures.
- multidisciplinary commissions (science, languages, humanities, art) composed of inspectors and university academics, which delimit transversal competences (for instance for the Science Commission, applying a scientific approach, communicating in appropriate language, solving problems, organizing and analyzing information, integrating ICT, understanding the contribution of science).
- and finally, disciplinary commissions headed by a university academic and composed solely of inspectors, at the request of the latter, due to the profound differences between the views of the two communities revealed by previous reforms.

After a study of a selection of foreign mathematics curricula and to promote the development of the competencies mentioned above, in mathematics the emphasis was placed on probability and statistics, approximate calculation and orders of magnitude, articulation of semiotic registers, resolution of problems related to social life and the environment, and the integration of digital technologies. As with previous reforms, the implementation of the reform was taken in charge by the inspectorate and there was a single official manual. On the other hand, there was no longer any unified accompanying documents, each inspector being responsible for identifying specific local needs and for adapting training to them. Inspectors' coordination meetings were however held three times a year, but the authors point out the heterogeneity of the body of inspectors, the number of inspectors having tripled in five years, and the impossibility of accessing documents identifying local needs or describing the training offered. They also point out that, as was the case with the previous reform in 1993, the lack of clear training strategies and resources led to significant resistance among teachers. The new features of the reform in terms of links with social life and the environment, the place to be given to approximate calculation in a context where many pupils did not have access to scientific calculators, for instance, were hardly taken into account.

We therefore see a process that contrasts with the one described above. The process remains completely top-down but with a desire for decentralization. A predominant role is given to the inspectorate in both design and implementation, and teacher support in term of accompanying activities and resources seems limited. According to the authors, these conditions result in significant resistance and implementation difficulties, what the specific study they conduct on the teaching of algebra illustrates well.

CONCLUSIVE COMMENTS

The case studies briefly reported in this contribution clearly show that recent curriculum reforms express rather close visions of what our respective societies expect from mathematics education. Common trends are observed in the proposed curricular changes, such as the move towards curricula structured around competences transversal to mathematical domains, the increased importance attached to showing the role of mathematics for addressing societal and environmental issues, to the connection between STEM disciplines and to interdisciplinary practices, the increasing space given to the stochastic domain, to modelling activities, and the attention paid to students' specific interests and needs. They confirm that conditions and constraints situated at the highest levels of the hierarchy of didactic co-determination influence these reforms. However, these case studies also show the specificities of each context and the diversity of curricular dynamics that result from them. They also show us that the success of a curriculum reform highly depends on the strategies developed for its implementation, the long-term support provided to those who have to implement it, the production and accessibility of appropriate resources, the combination of top-down and bottom-up processes in a productive way. They also show us that no matter how carefully a reform is designed and implemented, the dynamics it generates remain partly unpredictable. Regulatory mechanisms are necessary and must be designed with all actors involved. However, it seems that too often most of the efforts are still focused on the design of reforms, much less on their implementation, monitoring over time and regulation.

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CHINESE MATHEMATICS CURRICULUM REFORM IN THE 21ST CENTURY

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We examine the development and implementation of Chinese mathematics curriculum standards, with a focus on the development mechanism and characteristics of curriculum policy and its impact on public schools as well as the educational systems in China during the early 21st century.

Social and economic development in China (especially the development of information technology, digital technology, life-long learning, and democratization (The Research Group of Mathematics Curriculum Standard, 2002) have raised the bar for mathematics literacy. New demands for modern citizens have required corresponding changes in public schools, especially in mathematics curriculum and instruction (Ma, 2001).

MATHEMATICS CURRICLUM FOR COMPULSORY EDUCATION (GRADES 1-9)

The Development of a New Standard for Compulsory Education

The Mathematics Curriculum Standards for Full-time Compulsory Education (draft) was completed and put forth for extensive comments from the community in March of 2000. The mathematics standards research group mentioned above consisted of mathematics and mathematics education scholars, researchers and staff members from local provinces (cities), and school teachers. About 70 percent of the research team members worked in higher education institutes and about 30 percent of them worked in public schools.

The development of the mathematics curriculum played an important role in this round of curriculum reform in fundamental education, which provides the idea of basic value, the mechanism of implementation, and the way to develop the standard for other subjects in fundamental education. The Ministry of Education formally promulgated and implemented Mathematics Curriculum Standards for Full-time Compulsory Education (Trial version)(MCSFCE) in June 2001.

The Features of Standards for Compulsory Education

In addition to focusing on additions and deletions of some content topics, the MCSFCE differed from the products of previous curriculum reform in several fundamental aspects, such as the basic curriculum ideas, curriculum objectives, curriculum implementation (including guidance on textbook development), teaching suggestions, evaluation recommendations, and even curriculum management. It provided detailed descriptions in some dimensions. For example, the traditional syllabus only provided a brief description of teaching content and objectives. Most of the descriptions of teaching objectives were included in the textbook developed by the state. MCSFCE changed both the scope and depth of the role that the state plays in the curriculum by providing descriptions of learning content, learning processes (special attention), and teaching recommendations (including several cases for some content). This provided a standard for the transformation from one single national textbook policy to a policy of diversity; a national committee certificated and authorized the different versions of textbooks, according to the curriculum standards.

To examine some of the differences between the old Syllabus and MCSFCE in more detail, consider the following descriptions of how students and teachers should approach the Pythagorean Theorem.

The old Syllabus:

Master the Pythagorean Theorem. (Students) know how to use the Pythagorean Theorem to solve for the third side given the measurement of the other two sides. (Students) know how to use the converse of the Pythagorean Theorem to determine if a triangle is a right triangle. Conduct patriotic education by introducing the research on the Pythagorean Theorem done by ancient Chinese mathematicians.

The MCSFCE included some dimensions not covered in the previous Syllabus, such as suggestions for evaluations and recommendations for textbook development:

Explore the proof process of the Pythagorean Theorem. (Students) know how to use the Pythagorean Theorem to solve simple problems. (Students) know how to use the converse of the Pythagorean Theorem to determine if a triangle is a right triangle. The recommendations for textbook development suggest introducing several well-known proofs (such as the Euclidean proof, Zhao Shuang1 proof, etc.) and some well-known problems so that students are aware that mathematical proof can be flexible, beautiful and sophisticated. Students should also be aware of the Pythagorean Theorem's rich cultural connotations. At the same time, some teaching suggestions include guidance on the teaching activities and teaching process of the Pythagorean Theorem.

As mentioned above, the MCSFCE proposed a basic reform idea: "Mathematics for All." In other words, "Everyone can learn valuable mathematics; everyone can learn the necessary mathematics; different people benefit from different mathematical development" (Ministry of Education of the People's Republic of China, 2001). This concept was totally different from the underlying idea of the old Syllabus (Zhang & Song, 2004). The MCSFCE suggested following the psychology of learning mathematics and using real-life experience to motivate student development. Students were to experience the process of mathematical modeling, which would allow for the interpretation and application of the problem-solving process. Thus, as was the hope of mathematics education reformers elsewhere in the world, students would be enabled to grow in mathematics understanding, mathematics thinking ability, attitudes towards mathematics, and appreciation of mathematics (NCTM, 1989, 2000).

The Implementation of Standards for Compulsory Education

The Ministry of Education started a national curriculum reform conference to convene the implementation of the new curriculum in July 2001. Several decisions were made at the conference. First, the overall objectives and strategies for the implementation of the new curriculum in public schools were determined. Second, the strategies to spread the curriculum reform to all Chinese public schools were developed. Third, professional development and teacher training programs were set up. The positioning of the trial version of the curriculum standards necessitated a multi- stage process for spreading the new curriculum. The first stage was to set up the goals, then to conduct preliminary experiments before the nationwide implementation, and finally to broaden the experiment gradually.

The Revision of Mathematics Curriculum Standards for Full-time Compulsory Education

Since the implementation of the MCSFCE (Trial Version), the work of developing it has never been interrupted. After the first round (3 years) of mathematics curriculum reform, the revision process began. Based on the experience, account was taken of the problems arising from the implementation of the standards, as well as comments from society (including severe criticism from some mathematicians). In May 2005, the Ministry of Education organized the revision group for mathematics curriculum standards for compulsory education, and officially began the revision process.

There were 14 members in the revision group, from different backgrounds including universities, coaching offices and primary and secondary schools. About half of them had worked on the design of MCSFCE (Trial version). Through the process of surveys, situation analysis and discussions of special issues, the Mathematics Curriculum Standards for Compulsory Education (2011 Version) (MCSCE2011) were finished in 2010, and approved in May 2011. The standards were published officially in December 2011. (Ministry of Education of the People's Republic of China, 2012, p. 34).

MCSCE2011 was developed from the trial version; several revisions were made (Zhu, 2012), such as the basic curriculum ideas, curriculum objectives, content standards and suggestions for curriculum implementation.

With the base established by the implementation of the MCSFCE (Trial Version), MCSCE2011 was implemented at one time. Since the autumn semester, all beginning grades (for primary and middle schools) began to implement the new curriculum standards (not only mathematics).

Some changes appeared in the high-risk examinations. For example, the entrance examination to high school in Beijing adapted the concrete content and new rubrics were introduced focusing on the Mathematical View, Mathematical Activity Experience and Mathematical Ability (Wang, 2013).

Some scholars thought that the issues of assessment, hardware and the teachers' views were still obstacles to the implementation of the new curriculum (Zhu, 2013).

MCSCE2011 discussed the relationship between plausible and deductive reasoning, and the relationship between the real-life world and systems of knowledge. Its objectives highlighted the development of students' creative and application abilities, and added the ability to discover and raise problems (Ministry of Education of the People's Republic of China, 2012, p. 84).

The two versions of standards consolidated and perpetuated the achievements of the the new century mathematics curriculum reform and played an important role in giving impetus to the healthy and continuous development of mathematics education in China.

MATHEMATICS CURRICULUM FOR HIGH SCHOOL EDUCATION (GRADES 10-12)

The Development of Standards for High School Education

In the process of standards development, the research team studied mathematics curricula in several developed and developing countries around the world. The study included the following topics: trends of current research in mathematics, current demands on public education, learning in secondary school, international comparison studies, and current teaching and learning in China (Song & Xu, 2010). The team conducted surveys, interviews and classroom observations in several provinces. The participants in these initial studies were teachers, students, principals and guidance officers in secondary schools.

The research team formalized the reform theory, curriculum objectives and corresponding high school mathematics curriculum standards based on the research results of previous studies. The development process for the secondary school curriculum standards was similar to that for compulsory education. The research team solicited suggestions from all parties including mathematicians, mathematics education experts, scholars from research institutes, secondary school teachers, and experts from related disciplines such as educational psychology (National High School Mathematics Curriculum Standard Group, 2002). At the same time, the research team conducted several studies, in more than 30 high schools, of some newly added content (such as algorithms) and mathematical investigations (including curriculum design and pilot teaching). These research results provided both evidence and experiences for the later development and revisions of the standards (Song & Xu, 2010, p. 123).

After 30 revisions, the draft version of Mathematics Curriculum Standards for Secondary Education came out at the end of 2002. The final version, MCSSE (Trial version), was formally published and promulgated in April 2003, after the Ministry of Education completed the document review.

The Features of Standards for High School Education

MCSSE was fundamentally different from the curriculum guidelines developed in previous reforms. It shared similar characteristics to the MCSFCE, including the outline of structural changes. It deepened and specified some dimensions (e.g. curriculum content descriptions). MCSSE also included teaching suggestions, teaching materials, suggestions and recommendations (Ministry of Education of the People's Republic of China, 2003).

MCSSE proposed "student-centered" curriculum ideas, such as cultivating mathematics literacy, increasing active learning, mastering basic knowledge and basic skills, integrating mathematics and information technology, developing critical thinking skills, developing application and mathematics modeling skills, and the significance and values of a mathematics culture. MCSSE advocated that the high school mathematics curriculum should include a mathematics culture, through which mathematics literacy could be achieved.

MCSSE also advocated a modular structure (36 classes per module), with each module mutually independent, but also with logical connections. The new curriculum offered a variety of selections to meet the needs of individual students. The old curriculum only provided two elective courses at the high school level—mathematics for liberal arts majors and mathematics for science majors. The new curriculum provided more choices. Students needed to take five required modules before the elective courses. There were four elective series, where Series 1 (targeting students majoring in humanities and social science), and Series 2 (targeting students majoring in science, engineering, and economics) were basic elective courses. Students could continue to choose Series 3 or Series 4 after finishing courses in Series 1 or Series 2. Series 3 and Series 4 had a number of topics, with each topic requiring 18 classes. They were designed for students who were interested in mathematics and hoped to learn more. They involved several topics aimed at some important mathematical ideas, scientific value, application of mathematics, and the understanding of a mathematics culture, which reflected some important mathematical ideas, hoping to provide a mathematical base for students' life-long development. Selective topics in Series 3 included the history of mathematics, information security and passwords, and spherical geometry (six topics). Selective topics in Series 4 included geometric proofs, matrices and transformations (10 topics).

The intention was to expand these elective topics gradually, with careful monitoring of the quality of these courses. MCSSE encouraged schools to set up certain topics in Series 3 and 4. Schools also had opportunities to enrich and improve various additional elective courses based on the school-based curriculum and faculty resources (18 classes for each credit).

In addition to the new electives (which mostly appeared in Series 3 and 4), the MCSSE also contained several new topics, including orthographic views, spatial coordinates, algorithms, block diagrams, random numbers, and statistics. It also presented this new content using new ways of representation. For example, in three-dimensional geometry, the new textbook took the whole-part approach, rather than the traditional logical approach of point, line, plane and solid. In terms of geometry objectives, the new textbooks followed a cognitive order from overall perception to the details of point, line, and plane. The new curriculum also presented probability and statistics in the order of statistics, probability and counting techniques, rather than the traditional order of counting techniques, probability and statistics (Cao, 2008, p. 34).

In addition to the curriculum based on mathematical knowledge, the MCSSE designed the series of Mathematical Exploration, Mathematical Modeling and Mathematical Culture, which was required to be integrated into the regular curricula.

The Implementation of Standards for High School Education

With the promulgation of the MCSSE, the high school curriculum reform entered an experimental deployment stage. The high school curriculum policy was promoted under a step-by-step experimental expansion model. Different from the compulsory education case, the experimental deployment of the high school reform began in large regions such as provinces, self-regulated regions, and municipalities. In fall 2004, four provinces, self-regulated regions and municipalities became the first experimental zones of the high school curriculum. The curriculum reform received strong criticism and even opposition in 2005, which slowed down the deployment process (Cao, 2005; Wang, 2005; Zhang, 2000). By fall 2012, the high school curriculum had been adopted at entry-grade level in all high schools.

A survey was used to summarize the implementation of MCSSE 10 years after it was published. The sample size was 13 provinces, 446 mathematics teachers, and 5685 students (Lv et al., 2015). The results showed that the implementation of the multi-objective was good. The students' problem-solving and creative-thinking abilities and the ability to collect, clean and analyze information had increased gradually. As well, some teachers thought that the skills of operation, logical reasoning, and spatial imagining had decreased in varying degrees. The teaching method had changed in a positive direction, but the space left for the students' self-learning was still not enough. The learning method tended to be diverse, but the loading of learning was still heavy. The limitations of the examination system were still obvious, especially for the selective Series 3 and 4. For example, for Series 3, since it was not included in the college entrance examination, 70.6% of teachers reported that their schools had not set this series, and for series 4 only 6.8% teachers thought their students could select curricula freely. Furthermore, the examination system limited the development of a multi-assessment system.

The Revision of Mathematics Curriculum Standards for High School Education (Trial Version)

With the publication of MCSCE2011, the revision work of the MCSSE was started in November, 2014, 10 years after it was first published.

The revision raised a new central concept of "key competencies," which was one of the trends of the international curriculum reform. The model of key competencies was applied to promote the curriculum reform (Xin, Jiang & Liu, 2013). Mathematical key competencies formed the most fundamental component, which decided the main line of the curriculum. The key competencies at high school level included mathematical abstraction, operation, deductive reasoning, mathematical modeling, intuitive imagination, and data analysis.

Based on the existing published literature (Hong et al., 2015), the "Curriculum Plan of High School (Revision)" was based on subjects, and did not distinguish the students according to science or the social liberal arts. The requirement for graduation credit was 144, and 88 for the essential curriculum, with no less than 42 for selective series 1, and no less than 14 for selective series 2.

It was intended that the new curriculum would include Essential Series, Selective Series 1, and Selective Series 2. The Essential Series consisted of "Preparing Knowledge" (set, logic language, equivalence and inequality, etc.), "Function and Sequence" (the concept of function and the principles, fundamental functions, sequences, and the application of functions), "Vector and Geometry" (solid geometry, two-dimension vectors, and the application of vectors: solving for triangles), "Statistics and Probability" (random sampling , error modelling, estimation, classical probability, and geometric probability, which emphasized the fundamentals and modernization of the content.

Series 1 included "Function and Derivative" (derivatives and applications, optimizing, inequality), "Vectors and Geometry"(solid vectors and solid geometry, analytical geometry, conics, etc.), "Statistics and Probability"(counting principles, conditional probability, discrete random variables, Bernoulli model, and linear regression, which emphasizing the fundamentals of the content.

Series 2 was divided into five categories, A, B, C, D and E. Category A was for students who chose a science direction, including calculus with one variable, three-dimensional geometry, three-dimensional linear algebra, and models of statistics and probability. Category B included calculus, linear algebra, and statistics and probability, which had less content than A, and emphasized application and mathematical modelling. Category C (social science) included logic, social surveying, and mathematical modelling, which emphasized application. Category D was "Beauty and Mathematics", which included mathematics in sport, in music, and in art. Category E was the school-based curriculum, an adaptation of the Advanced Placement Curriculum, including calculus with one variable, integration with one variable, linear algebra, and statistics and probability.

Based on the existing published literature, the revised high school standard was changed a lot from the MCSSE, including the organization of the curriculum, the division between science and liberal arts, and the introduction of AP. The teaching method and other directions were changed according to the change of curriculum.

It needs to be noted that the revision was still ongoing, so systematic revisions still needed to be made after the publication of the final version.

In early 2018 the MCSSE2017 was released and the official examination of related textbook was ongoing. Revision of the MCSCE2011 was initialed and it was planned to be finished in 2020.

CONTENTIONS WITH THE MATHEMATICS CURRICULUM REFORM

The curriculum reforms of the early 21st century led to deep changes in ambitions, curriculum content, teaching methods, textbooks and assessment methods. These changes had prompted the development of mathematics education in China. As well, both the preparation and deployment processes of the curriculum reform had caused various theoretical and practical contentions in the mathematics education community. All aspects of the curriculum reform were subject to some contention (see, e.g., Cao, 2005; Wang, 2005; Zhang, 2000), especially the requirements for the objectives and content of the curriculum, such as the issues of “Calculation”, “Mathematical Systems”, “Geometry”, and “Uniformity and Diversity”.

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IMPLEMENTING THE *K TO 12* MATHEMATICS CURRICULUM IN THE PHILIPPINES: MODELS AND PROCESSES OF TEACHER DEVELOPMENT

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The implementation of the K to 12 Basic Education Program is a major reform in the educational landscape of the Philippines. In particular, the intended K to 12 mathematics curriculum is designed based on the spiral progression approach where five learning domains; namely: Numbers and Number Sense, Measurement, Geometry, Algebra and Patterns, and Probability and Statistics, cut across the grade levels with increasing complexity. With the goal of developing critical thinking and problem solving while anchored on constructivist pedagogical approaches, the reformed mathematics curriculum poses challenges in closing implementation gaps through more responsive and sustained teacher development programs. With the important role of school mathematics teachers as key implementers of the reform, this discussion paper examined the models and processes for professional teacher development that have been carried out in the Philippines to address the needs for school mathematics teachers in expanding their knowledge bases and enhancing their capacities for implementing the K to 12 mathematics curriculum.

THE PHILIPPINE EDUCATIONAL SYSTEM AND THE CONTEXTUAL REALITIES OF CURRICULUM REFORM

The Philippines, an archipelago in Southeast Asia with a population of over 100 million as of 2015 Census, has a school system that is considered one of the largest in the region in terms of student enrolment. The Philippine Statistics Authority (2017) reported that combined enrolment size in basic education system (elementary and secondary) is 21.6 million as of School Year 2016-2017. Moreover, the country's educational system is dynamic as it has undergone dramatic changes amidst various social, economic and political forces through various historical periods of its educational evolution with a mix of Spanish, American and Asian influences (De Guzman, 2003). Gaerlan and Bernardo (2013) further claimed that educational reform is necessary as continuous improvement in education will have "large social returns, in health, wealth and well-being of a nation's citizenry" (p. 1).

For the past decades, the Philippine basic education cycle was considered one of the shortest in Asia with only 10 years of pre-university education comprising 6 years of elementary and 4 years of secondary level. In 2012, the Department of Education launched the *K to 12* Basic Education program which is a major curriculum reform in the educational landscape of the country aimed at expanding the basic education cycle from 10 to 12 years and, at the same time, enhancing the quality of educational outcomes (Department of Education, 2012). This education reform was enacted into law as Republic Act No. 10533 otherwise known as the *Enhanced Basic Education Act in the Philippines* (Congress of Philippines, 2013). From a national perspective, this educational reform primarily reflects the shared experience of change of a country's educational system as it adopts to changing contextual realities of the 21st century, national priorities and emerging global standards. From a global perspective, more recent curriculum reforms are characterized by current shift from subject-centered models to outcomes-based, standards-based and competency-based integrative curriculum models towards improvement of educational outcomes (Sahlberg, 2006).

With the goal to improve education outcomes in terms of achievement, participation and completion rates, the Department of Education (2012) further rationalized the *K to 12* Basic Education reform as a measure to enhance the quality of basic education in the Philippines which was deemed urgent and critical considering the dismal performance of Filipino students in the National Achievement Tests (NAT) and in international assessments like the Trends in International Mathematics and Science Study (TIMSS) where the Philippines ranked 23rd in performance out of 25 countries in Grade IV Math and Science and 34th out of 38 countries in high school mathematics in TIMSS 2003, and 10th among 10 participating countries in TIMSS 2008 for Advanced Mathematics (cited in Department of Education, 2012).

Teachers as Key Implementers of Curriculum Reform

The challenge of implementing the intended *K to 12* Basic Education curriculum lies in the hands of the teachers who are the key actors in any curriculum reform. Leung (2008) contended that teachers should be the major focus of analysis and source of evidence in the introduction of curriculum reform. Given that it is largely the responsibility of the teacher to manage the teaching-learning environment in order to attain the desired outcomes, there is a need to assess teachers' capacity to implement the reformed curriculum. Consequently, it is of vital concern to look into teachers' professional development needs as they cope with the demands of this reform.

THE K TO 12 MATHEMATICS CURRICULUM REFORM IN THE PHILIPPINES

With the overarching goal of "producing holistically developed Filipino citizens with 21st century skills," the national *K to 12* curriculum for basic education comprised four cluster of subjects that cuts across the grade levels from Kindergarten to Grade 12 to nurture the learner's holistic development. These subject clusters are: (1) Languages (Mother Tongue, Filipino and English), (2) Mathematics and Science, (3) Arts and Humanities, and (4) Technology and Livelihood Education (Department of Education, 2012).

Some salient features of the reformed curriculum which has substantial impact in the teaching of Mathematics and Science include the use of spiral progression approach to ensure mastery of knowledge and skills age each level and the use of pedagogical approaches that are constructivist, inquiry-based, reflective, collaborative and integrative (Department of Education, 2012). These features have profound implication on the training of both preservice and inservice mathematics teachers. This paper discusses the implementation of the reformed *K to 12* mathematics curriculum in the Philippines with particular focus and context in in-service teacher development.

The Intended *K to 12* Mathematics Curriculum

The intended *K to 12* mathematics curriculum encompasses five learning domains; namely: (1) Numbers and Number Sense, (2) Measurement, (3) Geometry , (4) Algebra and Patterns, and (5) Statistics and Probability. Further, the mathematics curriculum framework identified the development of problem solving and critical thinking as the twin goals of mathematics teaching, and the pedagogical approaches are grounded on the underlying learning principles and theories of

Constructivism, Experiential and Situated Learning, Reflective Learning, Cooperative Learning, and Discovery and Inquiry-based Learning (Department of Education, 2012). Inspired by Bruner's model of the spiral curriculum, the adoption of the spiral progression approach to curriculum design in the *K to 12* Mathematics curriculum implies that the same concepts are developed and taught from one grade level to the next in increasing complexity and sophistication (Tan, 2012).

The challenge of implementating the reformed *K to 12* mathematics curriculum is centered on how the mathematics teacher will address the attainment of the twin goals of teaching mathematics within the five content domains, skills and processes, diverse contexts, effective use of mathematical tools along with development of a set of values and attitudes within various pedagogical approaches anchored on Constructivism as a philosophy and theory of learning. This discussion paper examines the important role of mathematics teachers as key implementers of this reformed *K to 12* Mathematics curriculum and the challenges in teacher professional preparation and continuing development. In the context of ICMI Study 24 Theme C: *Implementation of reformed mathematics curricula within and across different contexts and traditions,* this discussion paper will focus on the question:

> *What models or processes for professional teacher preparation and continuous development have been carried out in the Philippines in the implementation of the reformed K to 12 Basic Education curriculum, and what are there influences, effectiveness, successes or failures?*

TEACHERS' PROFESSIONAL PREPARATION AND DEVELOPMENT FOR IMPLEMENTING THE K TO 12 MATHEMATICS CURRICULUM

Mathematics teacher preparation and development is essentially viewed as comprising two stages, the pre-service and in-service stages which are generally regarded as a continuum rather than discrete phases. While there may be gaps between what is taught in pre-service teacher education programs and what beginning professional teachers need in implementing the curriculum, studies on how teacher education graduates manage the transition from being pre-service student teachers to beginning professional teachers may inform both schools and teacher education institutions on the need for coherence and alignment, particularly in the case of mathematics and science teacher education programs (Reston, Rosaroso, Capistrano, Japitana, 2012). Moreover, with the *K to 12* Basic Education reform in 2012, implementation gaps may be wider when considering how in-service mathematics teachers will cope with the demands of the curriculum reform when their pre-service teacher education preparation was based on the 2002 Basic Education Curriculum that preceded *K to 12*. Thus, continuing professional development of in-service teachers was an urgent need following the beginning years of implementation of the *K to12* basic education reform.

In 2011, the Science Education Institute of the Department of Science and Technology (DOST-SEI) and the Philippine Council of Mathematics Teacher Education (MATHTED), Inc. published the *Framework for Philippine Mathematics Teacher Education* which provided a set of standards that could guide Teacher Education Institutions (TEIs), universities and colleges, professional organizations of mathematics teachers, schools and other educational groups involved in the educational and professional development of school mathematics teachers in the Philippines. With the ultimate goal of raising the quality of mathematics education in the Philippines to world standards, the framework also presented a vision of a competent mathematics teacher as follows:

> A fully competent mathematics teacher possesses a strong mathematical content knowledge, is armed with mathematical pedagogical knowledge as well as general pedagogical knowledge and management skills, displays an appropriate mathematical disposition and values one's own professional development (SEI-DOST & MATHTED, 2011, p. 11).

With this characterization of the professional knowledge bases of a competent mathematics teacher, the framework also mapped out a professional development continuum for mathematics teachers and outlined performance expectations at each growth level of teacher development; namely: novice, emerging, accomplished and expert (SEI-DOST & MATHTED, 2011).

PROFESSIONAL DEVELOPMENT MODELS AND PROCESSES FOR K TO 12 MATHEMATICS TEACHERS

Different stakeholders of Philippine education from both government and private sectors responded to this need for teacher development in relation to the reform. For the Department of Education, the professional development of teachers are planned at the national office and primarily consisted of mass trainings by geographical regions and by academic subjects. The *Cascading Model* was applied where in-service trainings and seminars move from the national, regional, division, then school level with decreasing duration at each lower level (Bentillo, et al, cited in Lomibao, 2016). These in-service trainings and seminars usually span for 2 to 5 days and conducted twice a year, during midyear break and summer break. Bentillo et al. (2003) commented that there was much dilution in the in-service trainings as they reach the school or division level using this top-down one-shot model. Further, there are rarely any documented evidences on how these trainings impact teaching practice and led to improved student outcomes.

Another model of professional development used in the Philippines is the *Cluster-based training* which involves teachers from several schools attending the same training program conducted by invited subject specialists as trainers with the content determined by the master teachers and the department coordinator of the schools in consultation with the teachers (Ulep, 2006). Ulep further claimed that dilution may be avoided in this model, however, if the trainers are not fully aware of the schools' situations, the relevance of the training may not be well appreciated by the teachers.

In practice, it has been observed that professional development efforts for teachers in the Philippines are primarily episodic and training-oriented. The most popular approaches are in these forms of short-term seminar-workshops and mass trainings which are usually "one-size-fits-all" form of training where teachers are passive consumers of knowledge with little or no opportunity to reflect on the connections and applications of their learning to their own teaching practice (Reston and Canizares, 2018). Hawkes & Romiszowski (2001) contended that many educational reform efforts targeting improved student outcomes have been unable to produce the kind of desired learning outcomes and they attributed this failure to the lack of sustained, serious, systemic investments in the knowledge base of individual teachers.

Currently, there is an increasing number of mathematics educators and researchers who explored more progressive models of teacher development such as the *Lesson Study* approach. The University of the Philippines National Institute for Science and Mathematics Education

Development (UPNISMED) has advocated the Lesson Study approach for science and mathematics teacher development. In 2013, UPNISMED organized the National Conference in Science and Mathematics Education with the theme " *Empowering Teachers of the K to 12 Curriculum through Lesson Study.*" Originally a Japanese practice of enhancing teaching practice, the Lesson Study is described as process wherein teachers work collaboratively in small groups to conduct a systematic inquiry into their pedagogical practice by closely examining their lesson (Fernandez, 2002). To date, UPNISMED has conducted several lesson study groups in various schools within Metro Manila and nearby provinces for both elementary and high school lessons (UPNISMED, 2017). Several groups of mathematics teachers and teacher educators have also turned to Lesson Study as a school-based professional development model for improving mathematics teacher quality and student outcomes. As examples, Elipane (2011) promoted the use Lesson Study as a teacher development model in pre-service mathematics teacher education. Lomibao (2016) applied Lesson Study as a professional development approach to enhance teacher capacities on implementing the Grade 10 mathematics lessons on Polynomial Functions and Baroja et al (2017) integrated the history of mathematics in teaching Grade 7 lessons on Measurement and documented the processes and outcomes through a lesson study.

More recently, the Department of Education (DepEd) issued DepEd Order No. 35 series of 2015 on institutionalizing the Learning Action Cell (LAC) as a school-based continuing professional development strategy for improving teaching and learning in the K to 12 Basic Education program (Department of Education, 2016). A Learning Action Cell is a group of teachers who engage in collaborative learning sessions to solve shared challenges encountered in the school facilitated by the school head or a designated LAC leader. The LAC shared some commonalities with the lesson study as it promotes teacher collaboration and the growth of professional learning communities or school-based communities of practice, though there are marked differences in focus of the collaborative learning sessions and group structure. Furthermore, this development indicates DepEd's willingness to embrace more progressive teacher development models beyond the traditional training models. Moreover, the success and challenges of implementation of LAC in the DepEd school system still need to be documented.

The evolution of professional teacher development models to address teacher needs in implementing the *K to 12* basic education reform, particularly for the reformed mathematics curriculum, is indicative of the openness and flexibility of various institutions and professional teacher groups to embrace a wide range of options to improve teaching quality and learning outcomes. Further, this discussion will present a specific case of a mathematics teacher development initiative using a needs-based professional development model, and winds up with some suggested future directions towards bridging gaps between pre-service teacher preparation and continuing professional development of in-service mathematics teachers.

A Needs-based Professional Development Model for K to 12 Mathematics Teachers

In response to the challenges of the *K to12* Basic Education reform in the Philippines, the Science and Mathematics Education Department of the University of San Carlos in Cebu City, Philippines conducted in SY 2014-2015 a needs assessment survey participated by 98 Mathematics teachers across 17 randomly selected public and private schools in Metro Cebu, Philippines. Following the SAEDIR Professional Development model by Arome and Levine (2007) which emphasized the importance of needs assessment as a starting point in planning for professional development, we conducted a cross-sectional survey using a researcher-developed questionnaire to identify teachers' professional development needs in terms of developing knowledge bases and capacities for implementing the *K to12* mathematics curriculum. Further, guided by the professional development model by Loucks-Horsley, Love, Stiles & Hewson (2003), the needs assessment considered the range of knowledge bases that teachers need for teaching, including teachers' mathematical content knowledge and beliefs about teaching and learning as well as their professional contexts. The results were validated with a one-day workshop which engaged volunteer teacher-respondents in activities that assessed their pedagogical content knowledge for teaching in the five learning domains of the K to 12 mathematics curriculum.

In terms of teachers' self-assessment of their mathematics content knowledge for teaching across the 5 learning areas of the K to 12 Mathematics curriculum, the findings revealed that *Probability and Statistics* was ranked 1st by majority (63.3%) of the teachers as the area where they are least confident to teach and in which they need more professional development. This was followed by *Geometry* , *Measurement*, *Algebra and Patterns* and *Numbers and Number Sense,* respectively (Reston & Canizares, 2018). These results confirmed with our previous efforts on training school mathematics teachers to teach statistics (Reston and Bersales, 2008).

In response to this need, we embarked on a five-year teacher development project entitled *Improving Statistics and Probability among K to 12 Mathematics Teachers in the Philippines*. Now on its fourth year, this ongoing project is a collaboration of the University of San Carlos with expert support from *Academics without Borders* (AWB), a non-governmental organization based in Montreal, Canada, and the Department of Education Region 7. The project aimed to: (1) enhance teachers' pedagogical content knowledge (PCK) for teaching Statistics and Probability across the *K to 12* Basic Education curriculum; (2) assist in the development of materials that can be used in workshops for practicing teachers; (3) develop a support structure for practicing teachers which could include online support. The project is implemented in three phases. Phase 1 comprise one year capacity building of workshop facilitators along with the development of activities and learning resources for the workshops. Phase 2 consisted of the on-going implementation of workshop-based courses held in parallel sessions for elementary, junior and senior high school mathematics teachers. Phase 3 includes the development of a support structure with e-learning and communication platform for participating teachers to access additional resources, share best practices and participate in a professional learning community of teachers. Results of pre-and post workshop evaluations showed positive results on teachers' reaction and learning (Reston and Loquias, 2018). Future directions include expansion to a *Certificate Program for Teaching K to 12 Mathematics* to cover the other learning domains, starting with Geometry as the next identified area of need. Following Kirkpatrick's model of evaluation of training programs, there is still need to evaluate program results/outcomes and impact.

IMPLICATIONS AND FUTURE DIRECTIONS

The need to improve the quality of inservice teacher development has led to emergence of newer models of professional development beyond traditional approaches of episodic trainings in seminar-workshop format. Addressing gaps in the implementation of the *K to 12* Mathematics curriculum require the need to focus on the teacher preparation and development in both pre-service and in-service stages of teacher development. Future directions will consider various modes of program delivery to include the optimal utilization of technological platforms for teaching and learning, and the integration of reflective practice and research within the teacher development model.

Further, in response to the *K to 12* Basic Education Reform, the Commission on Higher Education (CHED) has recently released the Programs, Standards and Guidelines (PSG) for Teacher Education as CHED Memorandum Order No. 75 Series of 2017. This PSG stipulated the "shift to learning competency-based, standards and outcomes-based education" in response to 21st Century Philippine Teacher Education Framework and anchored on the salient features of the *K to 12* Enhanced Curriculum and the Philippine Professional Standards for Teachers. This document articulates the core competencies expected of teacher education graduates, including Mathematics majors. Finally, it is important to consider the necessary connections and implications of pre-service teacher preparation and the continuing in-service teacher development to close implementation gaps in the reformed *K to 12* mathematics curriculum.

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NATIONAL AND INTERNATIONAL CURRICULAR USE OF THE COMPETENCY BASED DANISH "KOM PROJECT"

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This paper begins by offering a conceptualisation of "curriculum" as a vector consisting of six components ("goals", "content", "materials", "forms of teaching", "student activities" and "assessment"). A curriculum is then determined by specifying each of these components and is implemented by enacting them. Then the paper presents the Danish competency-based KOM Project and discusses the extent to which this project has actually been implemented in curriculum reforms in Denmark. The conclusion is "only partly", the main reason being that no official measures were instigated to ensure the much needed professional development of authorities and teachers. Nevertheless, the KOM Project had quite an impact on Danish mathematics education, albeit "from bellow", but had an even greater impact internationally, both indirectly via PISA and directly as a consequence of personal contacts between mathematics educators in Denmark and a number of other countries. Finally, the paper offers some reflections on the conditions for successful implementation of novel curriculum ideas.

TERMINOLOGICAL CLARIFICATION OF KEY CONCEPTS

The title of the panel to which this paper is a contribution is "Implementation of reformed mathematics curricula within and across different contexts and traditions". In addition to "mathematics", this title contains some key words such as "curriculum", "implementation" and "reform(ed)" that are in common use around the world, yet carry a lot of different meanings. I therefore find it necessary to begin this paper by proposing some clarification (I hope!) of these and some related terms.

The key word "curriculum" means rather different things in different places (Niss, 2016). Thus, the Collins Cobuild dictionary (1999) offers the following definition: "A curriculum is all the different courses of study that are taught in a school, college or university" (p. 401). Kilpatrick (1994), in contrast, focuses on a single subject rather than on an entire collection of subjects and writes "The curriculum can be seen as an amalgam of goals, content, instruction and materials" (p.7). A somewhat different definition, focusing on the mathematics teacher and on what is actually happening in the classroom, is put forward by Stein, Remillard, and Smith (2007): "…we use the term curriculum broadly to include mathematics curriculum materials and textbooks, curriculum goals as intended by the teacher, and the curriculum that is enacted in the classroom" (p. 319, footnote).

Irrespective of what definition of curriculum we adhere to, any curriculum is situated and lives within an *educational setting*, i.e. the institutional, structural and organisational entity within which the teaching and learning addressed by the curriculum take place. A prime example of an educational setting is the entire public school system of a given country or political sub-unit. As other examples we may think of a particular school, a particular tertiary institution, or a particular course, say in a university.

In (Niss, 2016) I proposed, along the lines of Kilpatrick's definition, to define a (mathematics) curriculum with respect to a given educational setting as *a vector with six components*, as follows:

- *Goals*

  (the overarching purposes, desirable learning outcomes, and specific aims and objectives of the teaching and learning taking place under the auspices of this curriculum);

- *Content*

  (the topic areas, concepts, theories, results, methods, techniques, and procedures dealt with in teaching and learning);

- *Materials*

  (the instructional materials and resources, including textbooks, artefacts, manipulatives, and IT systems employed in teaching and learning);

- *Forms of teaching*

  (the tasks, activities and modes of operation of the teacher in this curriculum)

- *Student activities*

  (the activities of, and the tasks and assignments for, the students taught according to this curriculum);

- *Asssessment*

  (the goals, modes, formats and instruments adopted for formative and summative assessement, respectively, in this curriculum).

Specifying a curriculum in a given educational setting then amounts to specifying each of these six components. Furthermore, *implementing* a given curriculum amounts to specifying it, as well as to *carrying it out*, i.e. putting all the six components into practice.

The agency that determines a curriculum and has the power to implement it within some educational setting is the *curriculum authority* for that curriculum (Niss, 2016). It may happen that a curriculum authority chooses to leave some of the six components unspecified. Then these components are open for others, e.g. teachers, to specify, for instance by way of enactment. In some countries national curriculum authorities specify only a few of the components, typically "goals", "content" and "assessment", whilst the remaining ones are left to be decided upon by, say, local governments, institutions, or teachers.

What, then, do we mean by *reformed* mathematics curricula, as hinted at in the title of the panel? Well, the term "reform" suggests some desired changes of a rather fundamental nature, which are likely to affect several components of the curriculum, probably all of them. Usually, one wouldn't use the term "reform" unless at least "goals" and "content" are explicitly affected. However, even though the primary reform target may be "goals" and "content", the other components are likely to be affected as well, by derivation, even though this may not be explicitly intended.

COMPETENCY-BASED MATHEMATICS CURRICULA – THE CASE OF DENMARK

In the late 1990's, the Danish Ministry of Education, on the advice of the then existing Council for Mathematics and Science Education, saw a need for reforming the mathematics (and other) curricula in Denmark across all educational levels. This need was spurred by a number of issues and problems that became more and more manifest and visible within and outside the education system. These included that too many students didn't benefit enough from the mathematics instruction they were offered, and that there were serious transition problems and severe academic and socio-cultural discontinuities when students moved from one segment of the education system to the next, from primary to lower secondary education, from lower to upper secondary, and from upper secondary to tertiary education. These transition problems went hand in hand with insufficient progression in students' mathematical learning within and across these segments, which led to "consumer" complaints about the decrease in students' mathematical capabilities. Moreover, it was a widely held perception that not all teachers were adequately prepared for offering high quality mathematics teaching to their students. These – and several other – problems were seen as (co-)responsible for the fact that students opted away from further education programmes in science, mathematics and technology, i.e. the so-called STEM programmes, which was (and is) considered a serious societal problem.

Against this background, the Ministry, assisted by the Council, in 2000 established a commission (a task force), composed of mathematicians and mathematics educators (researchers, teachers, and ministerial inspectors) and a few representatives from society at large. The Commission was chaired by me, whilst Tomas Højgaard (Jensen) was the Commission's academic secretary. The task of the Commission was to (1) identify, uncover, chart, and analyse the entire set of *problématiques* pertaining to mathematics education at all levels of the Danish education system, and (2) to propose measures and tools that were likely to be effective in improving the state-of-affairs by counteracting the problems identified and by remedying (some of) the deficiencies observed. The Commission worked for two years in what became known as the *KOM Project* ("KOM" is an acronym for "Competencies and the Learning of Mathematics" in Danish) , and ended up publishing a comprehensive report, known as the KOM Report (Niss & Jensen, 2002; Niss & Højgaard 2011), which was presented and debated widely in several places and quarters in Denmark and soon after in a number of other countries as well (e.g. Germany, Norway, Sweden).

The brief for the KOM Project was far from solely focused on proposing new curricula, whatever that word meant in those days, but had a much wider scope. In other words, the KOM Project was *not* meant to be a curriculum project. However, it was assumed, also by the members of the Commission, that the design of mathematics curricula could be substantially supported by the outcome of the work. I shall return to this issue below.

The KOM Project took its point of departure in the need for creating and adopting a general conceptualisation of mathematics that goes across and beyond educational levels and institutions. Only then would it be possible to deal with mathematics in a manner that was neither tied to nor dependent on particular levels and types of institutions., which was necessary in tackling the transition problems in the education system. We also wanted to avoid being locked into the specifics of particular mathematical subject matter domains or topics such as algebra, geometry, functions, calculus etc., the place and content of which vary greatly across levels and institutions.

We therefore decided to base our work on an attempt to define and characterise mathematical competence in an overarching sense that would pertain to and make sense in any mathematical context. Focusing - as a consequence of this approach - first and foremost on the *enactment* of mathematics means attributing, at first, a secondary role to mathematical content.

We then came up with the following definition of mathematical competence:

Possessing *mathematical competenc*e – mastering mathematics – is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve actual or potential mathematical challenges of *any kind*.

In order to identify and characterise the fundamental constituents in mathematical competence, we introduced the notion of mathematical competenc*ies*:

A *mathematical competenc*y is an individual's capability and readiness to act appropriately, and in a knowledge-based manner, in situations and contexts that involve *a certain kind* of mathematical challenge.

A metaphor may illuminate the relationship between competenc*e* and a competenc*y*: If we think of mathematical competence as a huge, complex molecule (say a polymer), the competencies represent much smaller building blocks (atoms or monomers) in this molecule.

Eight competencies were identified, in the beginning on theoretical and experiential grounds only. Later on they became corroborated empirically. These competencies are:

- Mathematical *thinking* competency

  Mastering mathematical modes of thought

- *Problem handling* competency

  Being able to pose and solve mathematical problems

- *Modelling* competency

  Being able to analyse and construct mathematical models

- *Reasoning* competency

  Being able to reason mathematically in the context of justification of mathematical claims

- *Representation* competency

  Being able to handle different representations of mathematical entities

- *Symbols and formalism* competency

  Being able to handle symbol language and formal mathematical systems

- *Communication* competency

  Being able to communicate, in with, and about mathematics

- *Aids and tools* competency

  Being able to relate to the material aids and tools for mathematical activity

Since the competencies are meant to go across all mathematical subject matter domains, in a given educational setting it neither makes sense to consider deriving the competencies from such domains, nor to consider deriving domains from the competencies. Even though the competencies can, of course, only be developed and exercised in dealing with subject matter, the relationship between competencies and mathematical domains should be perceived as constituted by two independent, yet interrelated dimensions, as depicted in the matrix in Table 1:

	Topic 1	Topic 2	…	Topic n
Thinking				
Problem handling				
Modelling				
Reasoning				
Representation				
Symbols/formalism				
Communication				
Aids and tools				

Table 1: The competencies by topics matrix

Each cell in this matrix represents the relationship between the competency in the corresponding row and the topic in the corresponding column. More specifically, it allows one to specify the ways in which this competency plays out in dealing with Topic j, and the ways in which Topic j plays out in exerting the competency at issue.

KOM-referenced curriculum reforms in Denmark in the 21st century

As mentioned above, the KOM Project was not established as a curriculum project. However, it was certainly intended and expected that the outcomes of the project, including the eight mathematical competencies, would be instrumental in designing new curricula that would help counteracting some of the problems identified prior to and within the project.

Even if the notion of curriculum introduced at the beginning of this paper wasn’t in place at the time of the KOM project, the project actually adopted a similar notion of curriculum, which was also partly reflected later in the Danish curriculum reforms of the 21st century. This means that all the above six components of curriculum were addressed in the curriculum design, albeit with varying degrees of specification. It follows from what was said above that the “content” component had to be specified independently from the competencies, whereas the competencies were paid attention to in shaping the other components. The “goals” component, in particular, was typically formulated in competency terms.

In a number of different ways, the KOM Project was a great challenge to traditional conceptualisations of mathematics teaching and learning in Denmark. With the project's primary emphasis on the enactment of mathematics, across education levels and mathematical topics, rather than on mathematical content, curriculum authorities – the official Danish education system, governed by the Ministry of Education – as well as teachers experienced difficulties at coming to grips with how the outcomes of the KOM Project could in fact guide the design and implementation of new curricula that weren't (to be) defined in terms of classical content strands.

This implied that the new curricula of the first two decades of the century continued to be primarily based on subject matter domains, whereas the competencies were presented in the general sections of the curriculum documents accompanied by general requirements that the teaching of those domains should pursue competency-oriented goals and that competencies should be paid attention to "throughout" the teaching activities.

In Denmark national exams at the end of grade 9 and again at the end of grades 10, 11 or 12, (the latter depending on which of several possible mathematics streams the individual student is in at the upper secondary level) are high stakes exams organised by the Ministry of Education. Without going into details with the somewhat complex exam structure and organisation, suffice it to say that the written component of those exams ended up paying almost no attention to the competencies. In the oral component, which is mainly dealt with locally within the individual institution, there is room for paying attention to the mathematical competencies, if the teacher so wishes, which is also the case when it comes to formative assessment. In other words, the crucial curriculum component "assessment" was never markedly influenced by the competency approach, and since "what you assess is what you get" this partly jeopardised the competency approach and made it largely rhetorical at the official level.

However, other curriculum components, such as materials, including textbooks, forms of teaching, and student activities were oftentimes pretty much influenced by the competency thinking of the KOM Project. The same is true of pre-service teacher training and in-service professional development.

So, whilst the competency approach mainly had a rhetorical impact on the official curricula, especially as regards the components that are somewhat tightly controlled by the Ministry of Education ("goals", "content", and "assessment"), it wouldn't be correct to say that this approach has had no impact on the implementation of these curricula in everyday practice. As a matter of fact, the competency approach and the associated terminology substantially influences the *discourse* amongst mathematics educators in Denmark, who readily express themselves and explain their activities in terms of the KOM competencies.

Ironically, then, we may say that what from the point of view of the Ministry of Education should have provided a top-down platform for an entirely new approach to mathematics teaching and learning never became such a platform, primarily due to inertia in the different segments of the official system, whereas the approach and the thinking of the KOM Project gradually, in a bottom-up process, crept into significant – but certainly not all – aspects of everyday mathematics education.

This development begs an answer to the question: Why did things happen in this way?

Well, this is a highly complex issue, which involves a combination of universal as well as national features of curriculum design and implementation. I shall focus on the national features.

It is clear that the thinking in and behind the KOM Project and the competency approach taken were novel – if not outright radical – ambitious and demanding for the Danish education system and the teachers to come to grips with. So, it was far too optimistic on the part of the system to expect that the KOM Project ideas could be transposed into curriculum design and implementation without further ado, by just reading the KOM report. Neither the curriculum authorities nor the teachers were exposed to a systematic, thorough introduction to the ideas and their consequences, or were offered professional development activities beyond the written report itself. This is typical of Denmark, in which political unwillingness to spend public money on human resources in combination with anti-elitism has got a strong foothold during the last fifty years. In retrospect it would have been absolutely necessary for a much more forceful and effective implementation of the competency approach in Danish curricula to have had large-scale, systematic in-service activities within all layers of the system. In the absence of such activities, the competency ideas had to enter the system mainly by osmosis.

Against this background it is remarkable that the KOM Project thinking and the competency approach have in fact influenced mathematics teaching and learning in Denmark as much as they have. This can only be explained by the existence of serious needs amongst educational authorities and mathematics educators for conceptual innovation in mathematics education. The policy lessons that can be learnt from this case are primarily two: (1) You cannot effectively pursue goals and aims unless you are willing to invest and apply material and immaterial means that are conducive to the aims and goals, and (2) Only very rarely are top-down measures successful. If you really want to achieve change, it is essential that those who are to bring that change about have ownership to not only the need for change but also to the means to achieve it. If not, you might be able to see changes on the surface of things, but they won't really affect the substance the way you desired and expected.

THE COMPETENCY APPROACH IN OTHER COUNTRIES

During the first two decades of this century, many countries and quarters took an interest in the KOM Project and in the competency approach to mathematics education (Niss et al., 2016). This was partly, but not exclusively, stimulated by the fact that competency ideas were involved in shaping all the PISA mathematics frameworks between 2000 and 2012 (Niss, 2014) by underpinning and developing the notion(s) of *mathematical literacy*. However, due to direct personal contacts between mathematics educators in Denmark and in countries such as Germany, Norway and Sweden, these countries early on adopted and adapted aspects of a competency approach as well as some of the related KOM Project ideas in their curriculum development. Especially the German *Länder*, in the first decade of this century, agreed to take an explicit competency approach when reforming their curricula, leading to the so-called "Bildungsstandards" (see, e.g., Kultusministerkonferenz, 2012). Many countries in Latin America and Spain were also inspired by the competency ideas, primarily via PISA.

The most important thing to observe here is that it was never a matter of direct translation and adoption into other countries of the KOM Project ideas, let alone the documents, in curriculum development, design and implementation. Rather, it was a matter of modification and adaptation of (some of) these ideas so as to suit national circumstances, needs and traditions. Oftentimes, the eight competencies of the KOM Project were amalgamated or modified in various ways, typically into fewer than eight competencies. In some cases, adaptations were not even in conformity with "the spirit" of the KOM Project but were, nevertheless, inspired by some of its features.

Once again, there are lessons to be learnt from these developments. Firstly, one should never aspire to directly translating, transferring and adopting curricula or curricular ideas from one setting to another, especially not from one country to another. Such import, even of curricula that were highly successful in their original setting, is almost doomed to failure because the socio-cultural environments, as well as the economic, technological, structural and institutional boundary conditions vary so much within and across countries.

Secondly, the lesson just mentioned should not be taken to suggest that inspiration from others is likely to fail. On the contrary, and this is the second lesson worth mentioning, thoughtful and careful consideration of what others have accomplished, whilst paying attention to the conditions and circumstances under which the accomplishments were achieved, is likely to stimulate positive innovation (and innovation always comes with a "sign") in new places, provided those who are to implement this innovation are genuine shareholders in it.

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PROCESSES AND AGENTS OF CURRICULUM DESIGN, DEVELOPMENT AND REFORMS IN THREE DECADES OF SCHOOL MATHEMATICS IN CHILE

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During the last three decades, school mathematics curriculum in Chile has experienced continuous change. Reforms have followed previous, recently implemented, reforms. Driving forces of these changes are of a social, economic and political nature. After a brief characterization of these processes, some of the lessons learned by this prolonged period of reforms are described, as they may give some insight into questions like: what are the processes? Who are the agents? And, what are roles those actors have in the generation and implementation of reform in school mathematics curriculum? Curriculum reforms in the country evolved and were nested in broader processes. During the last three decades the country has experienced a deep transformation. The very structure of the educational system has been questioned and been deeply modified. Some of the new emerging institutions are also described because of their impact in the process of generating and putting into action the new curriculum. We are dealing with an issue –curriculum reform- where generalization is hazardous, especially considering that the reformed curriculum and the new institutions have been recently implemented or are in process of being created. Present analysis may be valid considering that perhaps the only way to understand curriculum reform is, precisely, by analyzing these phenomena where and when they occur.

Introduction

Since the beginning of the nineties, Chile has experienced continuous economic and social growth. This process has been slow but sustained. There has been a significant improvement in economic and social development indicators. Reduction of poverty and a substantial improvement in the quality of life are unmistakable signs of a positive change. The continuous clamor for a better education, "*quality education for all*", has forced the above-mentioned period of repeated reform efforts. National and international tests show little progress in learning. These small gains are not compatible or sustainable when compared to the development of the country in other areas. Another driving force is the pervasive and perverse gap between the haves and the have not's. A single and driving force is inequity as shown by learning results. Evidence shows that learning outcomes in public schools are significantly inferior to the ones in private educational institutions. This gap has shown to be the most difficult barrier to trespass in the Chilean educational system. The search for more equitable educational outcomes may be the most important driving force behind a thirty-year effort to reform the national educational system in the country.

Some milestones[1]

The reform of school mathematics curriculum is to be understood as embedded in a broader process: the reform of the educational system. The following are some of the milestones in the reform process which are major decisions that might impact the educational system as a whole: the creation –as a result of a multi-sector consultant committee- of the National Council of Education (CNE), (1996-1998); the extension of compulsory education up to 12 years of schooling (2003)[2]; a major reform of the framework defining the education for the country[3] (2009); the creation of the Q*uality Agency* (2011), responsible for the national test as applied in various school levels; a new definition for elementary, secondary and technical education[4] and the creation –in process- of regional entities responsible of the administration of public schools which are accountable for the implementation of the national curricula, a policy that promotes decentralization of the educational system. In a minor scale -but significant because they are some of the major results of reform efforts- the following can be mentioned: new infrastructure for schools throughout the country; new standards for teacher selection and teacher preparation; an improvement, although still insufficient, of working conditions and professional development for teachers; the almost universal access to digital technologies; free, newly designed, textbooks for all students in public schools; the extension of school schedules; and, especially relevant to the subject of this analysis, a renewed and more demanding school curriculum. National tests applied to the entire system, at various school levels, are mentioned separately because, although considered to be a guaranty of quality control, have become, at the same time, the operational definition of school curriculum and the latter competes with the official national curriculum.

Tendencies in the process of reform of the national mathematics curricula

Before discussing the role of different agents, institutions and driving forces in the process of renewal of the mathematics school curriculum in the country, some of the most salient tendencies of the reform of the school mathematics curriculum are now summarized.

There has been a remarkable effort to bring the national curriculum closer to international standards. Simultaneously, ideas, themes or content, before reserved for the last two years of schooling or the beginning of university courses, are now included in lower levels. This tendency can be observed in the treatment of functions, previously reserved for grades 11 and 12, now initiated in grade 7 or 8. The same occurs with probability and statistics or patterns and algebra, beginning now in first grade. Geometry includes, now, coordinate geometry and vectors. Another tendency is the emphasis of skills over content. The national curriculum in Chile promotes modeling, problem solving, communication and argumentation, and multiple representation skills. Mathematical reasoning has been of major concern among policy makers of the mathematics curriculum. The new curriculum points to classroom management that encourages the formulation, analysis and verification of conjectures. Modeling skills are emphasized throughout the curriculum. The proposed intense use of digital technologies is another new emphasis. It will be discussed, when analyzing the implemented curriculum, that the above-mentioned emphases on mathematical skills and digital technologies are the most difficult for school teachers to put into practice.

[1] See the appendix: An itinerary of three decades in the mathematical curriculum in Chile.

[2] A reform that implied reforming the constitution (Nº 19876, May 22, 2003), it was complemented in 2013, including kindergarten as mandatory.

[3]LGE, *Ley General de Educación* (General Education Law, the national framework for education in the country.

[4] The old structure had eight years of elementary education and four of secondary. The new structure –in process of being implemented- assumes six years of elementary and 6 of secondary. Furthermore, secondary is divided into four years common for all schools and two differentiated, including Technical Schools.

Agents, institutions and driving forces

Society and culture evolve. Students, their parents, newspaper editorials, researchers, political agents, and other public media, call for better schools and better learning results. Students have gone to street protests and teachers have also used this media to air their grievances. How are these tendencies and energies channeled? Who takes charge?

In practice, a combined action led by committees appointed by the division of the Ministry of Education responsible for curriculum and evaluation (UCE), and especially appointed committees, have been responsible for interpreting those voices demanding new ways of implementing education in the country. The above-mentioned division, which is responsible for school curriculum, has specialized teams in different areas of the curriculum, particularly in mathematics.

A team of five or sometimes six professionals with some modifications of its members, has been at the center of the reform process of the national mathematics curriculum. Who are they? Included are mathematics educators, professional mathematicians, researchers, some of them, with successful and significant experience as school teachers and also some recently graduated, promising professionals holding a degree in mathematics education or in mathematics. The present leader is a professional mathematician with recognized experience in the field of mathematics education and is the creator and former director of a graduate program in teaching.

What is the role of the mathematics team at the Ministry of Education? When involved in a reform process, the main responsibilities are the analysis of existing curriculum, the compilation and analysis of evidence about curriculum implementation, the search and analysis of the demands and proposals of specific leading actors, the search for significant results of research and, in the field and international experience in mathematical curriculum, the interpretation of general directives as generated by educational authorities within the Ministry of Education. Moreover, there is the formulation of proposals for the new curricula, the participation in different consultations and validation processes and the incorporation, to proposed curriculum, of the results of the consultation process. Once the new curriculum has been approved, several other tasks are in order: textbook specifications; the search for and the evaluation of different resources including digital ones and digital support; the participation both in the process for the diffusion of new curricula and the implementation of several actions related with diffusion and teacher preparation. Also, there is participation in actions related to the impact of new curricular proposals in teacher preparation and national tests which include the university entrance procedures and their corresponding exams.

The consultation process, its major contributors and the role of the National Education Commission

Proposed new curriculum, in the form of a curriculum framework, is the result of a process led by and developed by the team in the also referenced division of the Ministry of Education responsible for curriculum and evaluation (UCE). After different consultation actions and internal reviews, a first complete version of the school mathematics curriculum is ready for an approval process. This is when the National Council of Education acts. It is a regulatory body, with the necessary attribution to make ultimate decisions.

Several consultations precede the presentation of the curricular proposal to the National Council. The consultation process and the action of the National Council are the mechanism that seek to balance or counterbalance the action of the technical teams of the Curriculum Unit.

Consultation has been shown to be a powerful instrument in the definition of new curricula. Who is addressed in the process of consulting on the new proposals and how consultation instances are organized, are important issues subject to analysis and improvement. Teachers, research centers, researchers, mathematics and the mathematics education associations, leader private educational organizations and general public have been consulted. Consultations have been done, mostly, in the modality of focus-groups, also with small groups of experts and public web questionnaires. Face to face feedback was effective in all the consultation meetings that were organized. Public consultations on the web proved to be more effective in making the proposals be known than in generating a specific contribution. The fact that a reform has been consulted and has received more than 15.000 public reactions is a powerful factor for face validity and acceptance. A generalized statement can be made for both faces to face and web consultations. Most of the feedback and sometimes the whole of it were about teaching methods or teacher preparation. In a smaller proportion, reactions focused on teachers' abilities needed to put into practice what was proposed and also on the necessary conditions for implementation. A generalized reaction was: “what is proposed is too much; the amount of content exceeds what is possible in the time available to treat it”. Those responsible for the proposal often agreed with this evaluation. When authors of this comment where asked about what to remove from the proposal, the most frequent answer was “nothing” and in many opportunities, “nothing, but there are many things missing”. It is clear that the entire process of curriculum innovation and the way it has been implemented in the country lead very naturally to a growing curriculum. This is one of the questions to be addressed in the next section.

The role of the *National Council of Education* is now mentioned because it addresses two important needs of a reform that leads to a new formulation of the curriculum: the decision-making regulation and necessary institutional counterweight. The national curriculum in Chile is law enforced. Before a new curricular proposal becomes compulsory, a complex -also a matter of law procedure- needs to be implemented. Proposals are generated in the Unit of Curriculum previously mentioned. Once the design has been approved within the Curriculum Unit, they are subject to approval by de National Council. This process is a guaranty of quality, pertinence and proper formulation. Two additional consequences of this process are mentioned later as open questions: one is –and this is a statement that reflects only the author´s point of view- the exaggerated weighing that has the opinion of one or very few experts when summoned as reviewers by the Council. This delicate situation has generated distortions or imbalances in the curricula that it has acted on. It is a question to be analyzed. The second issue to be considered is the excessive rigidity that the whole reform process gives to the curriculum. Once constituted by law, a change, an improvement, no matter how minor, must go throw the same procedure. The result is unnecessary rigidity.

Main social, cultural and technical factors shaping school mathematics curricula, new questions and pending issues

The gap factor shows that there is a significant, odious and until now permanent difference between the learning outcomes of students attending public and private schools. This non-solved situation poses the question of who we are formulating the curriculum for. During decades, national curricular requirements have been growing. Results, in national tests, show that students attending public schools, close to de 85% of school population, are not fulfilling those standards. How does mathematical school curriculum contribute to this gap? How might mathematical curriculum be a factor in the reduction of these differences? Topics such as function, systems of inequations or homothetic figures are increasingly lower in the curriculum. Is it advancing topics that make a curriculum be better? Does the maturity of the student matter when deciding these advances? There is tremendous and extremely valuable talent diversity. Can we justify the existence of only one curriculum and only one way to evaluate it through standardized tests?

Testing gives solid information and has impact on the gap between stated and actual curricula. From one point of view, national tests [5] are very much valued as indicators of learning outcomes. Simultaneously, they act as an operational definition of the mathematics curriculum. Teachers, schools, local educational authorities and parents give high value to SIMCE results. In consequence, what is measured ends up being a guide for teachers when making subject matter decisions. As it is very simple to guess, higher-level learning and skills as promoted by reformed mathematics curricula, therefore, are often not covered by classroom teachers. This is an unsolved dilemma: to test or not to test. Mathematical modeling, argumentation skills, guessing and testing of one's own ideas or those of peers are difficult to measure and, thus, they lose importance for the teachers. What are adequate relations between national curriculum and national tests? How may skills in argumentation, modeling and enquiry be evaluated?

Globalization has influenced national mathematics curriculum in several ways: media generates access to news, cultural issues, tendencies and frequent expert opinions on educational results; international tests have proved to be very influential. Another factor is the almost universal and instant access to any nation's curriculum, including those of leading countries and economies. Are we moving to one internationally accepted curriculum? As there are values in both local and global knowledge's and skills, then, what is an appropriate interaction between local and global values, practices, traditions and expressions of culture?

New technologies have influenced in several ways school mathematics in the country. Since 1996 digital technologies are strongly required by Chilean national curriculum. During fifteen years a national project was in place to provide schools with digital resources and teachers with needed knowledge and skills to use them. Presently, reformed curriculum includes the requirement to use digital resources. Results in these matters are modest. What are effective and efficient strategies to introduce these technologies to the mathematics classroom? What are the skills teachers need to have to use them effectively? There are strong questions we have not yet addressed in designing the national mathematics curriculum: what is it that mathematics students need know in order to do mathematics in an environment where technology offers the capabilities to do so? What are the skills a person needs to learn to take all the advantages of existing digital technologies when doing mathematics? Information and communication technologies have shaped our culture. The second

[5] SIMCE is a national testing process responsible for the evaluation of learning outcomes. Tests in language arts and mathematics are applied, with some exceptions, every year in grades 4, 6, 8 and 10. With a variable frequency, other subjects are also evaluated. This test plays an important role when school performances are evaluated. Oher influential test is the university entrance examination: PSU.

derivative of this change grows. Is computational thinking a necessary knowledge for everyone? What should a mathematics teacher know about computer science?

Currently there are new social and cultural requirements: gender, the inclusion of those showing physical or learning disabilities and personal and environmental care. All of these pose new questions. How is curriculum worded to promote inclusion? How does one formulation for the curriculum take care of the diversity in talent? How is personal and environmental care included in the school mathematics curricula? How is the mathematics classroom organized and monitored, if handicapped students are to be included?

New practices, in old packages, or school culture vs. new needed classroom practices generate their own questions: content vs. skills, more vs. depth, expository vs. participative-collaborative classroom practices. These are also dilemmas in reform efforts. How do we move from a tradition that focuses in content to one that emphasizes skills? The same can be asked if depth is to be preferred to the amount of content covered. There is also a dilemma when new practices are resisted by important actors as school leaders and teachers.

An important issue, only to be mentioned here, referrers to the instruments that end up being the concrete manifestation of a reform. In terms of Robert Gass (1972), "*media are a subtle but powerful expression of the values and priorities of those who created them*". An example to be analyzed, a expression of school curriculum format called *Progress Maps*[6], was used in 2007 in Chile. They showed to be very effective as teachers, textbooks creators and researchers reported. Also, starting with the first public consultation in knew school curricula, Internet potentiality was confirmed. Many throughout the country new about the reform, had opinions and had the means to communicate them to the teams responsible for the reformed curriculum. Consequently, what are effective ways to communicate new curriculum? What are effective ways to use web options when implementing a curriculum reform? Are, e-learning courses one of those effective uses of the web? Are we using all the potentiality of the means we use to express and communicate curriculum?

Another issue to be analyzed refers to when and why reforms are initiated. These have begun in a casuistic, not predictable agenda, the opposite to a planned systematic process. Search for long term, periodically evaluated curriculum proposals has been an issue in Chile. A one-year educational committee was appointed (2016) to deal with this issue. Nationally recognized educational authorities were asked to generate proposals to create a "*National policy of curriculum development*". The purpose of the Committee was to make reforms of school curricula less vulnerable to political or conjectural factors[7]. These are important questions: What is an appropriate-long- term policy in school mathematics reforms? What are the conditions that make a reform needed? Is there a way to apply significant and defensible school curriculum diagnosis? How is a new reform decided?

There is a fundamental role played by researchers, and research and development centers and institutions. The period of school mathematics curriculum considered in these pages is the first in Chile where researchers –both in mathematics and in mathematics education or didactics- have had significant influences on school mathematics. In another publication, (Rojas and Oteiza, 2014) the authors refer to this as "new actors". There are three graduate programs in Chile in didactics of mathematical education and four university research and development centers which have had a significant influence in the school mathematics' curriculum. This is an unmistakable sign of progress. However, questions remain: How does the knowledge generated by the research reach the classroom?

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[6] With the assistance of a curriculum team of *New South Wale, Australia*

How do the questions that originate in the classroom reach a research center or a graduate program? "*Publish or perish*" has led our researchers to publish in prestigious international journals, but, are the problems and local questions addressed by those publications?

There are new conditions, new actors and new possibilities. New conditions challenge some success indicators and show new tendencies. One of these indicators is the arithmetic average, the mean score. The success of a student, class, school or country is measured using -almost exclusively- averages as indicators. Some signs showing a shift in this practice, are in place. These are: the use of standard deviation to show an aspect of equity: a smaller distance between the higher and lower outcomes. Another emergent indicator is the proportion of women in top places of school mathematics tests or contests. Integration of students with disabilities also has been used to express success, challenging prevailing competitiveness in our society. There is a number of national universities among the 100 or 50 international best; there is also a singular result of an astronomer, a woman in this case, recognized, among the "discoverers of planets", included among the arguments of better results in national education. Moreover, the number of scientific divulgation books of national authors that are successful in bookstores, the number of programs leading to the doctorate, the proportion of the national budget invested in education, are all examples of the shift in priorities. Attention on those indicators point to new national education trends. The mathematical curriculum is to be understood as part of that shift both as an effect and a cause.

CONCLUSION

School mathematics curriculum reforms are complex processes; this is only a short communication, however, most of the issues addressed in the above sections require and deserve a deeper analysis; in fact, almost all of them are research theme candidates. In this opportunity an effort was done to analyze actors, processes, institutions and curriculum design needs in a country from a point of view that is desirably valuable from an international perspective.

The process of creation, the required efforts to implement it and the multiple and somehow discrepant reaction to a reformed school of mathematical curriculum, generate their own questions and force a new formulation of the old ones. Questions remain about the causes, the actors, and the context of each reform. National efforts are unique and different. We all learn from sharing knowledge and new questions. The matter of how to improve the reform processes from its inception to the next moment, when new needs force a new curricula formulation, continues to be a valid and open question. Thanks, for the opportunity to participate in a cooperative effort on the search of better education for boys, girls and young people around the world!

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Appendix

An itinerary of three decades in the mathematical curriculum in Chile

1990. A reform period is initiated. A national program is implemented to accompany and eventually improve conditions and learning results in the 900-school showing the least results in the country (*P-900*).
1990. MECE Básica[8] y MECE Rural. A major reform of the first 8 grades of schooling.
1992. MECE Media[9], the "*11 studies*", specially appointed research teams are responsible for a set of studies to document needed secondary school curriculum reforms.
1993. *Enlaces*, a 15 year project to introduce digital technologies in education was initiated.
1994. MECE Media, the reform of secondary education is initiated.
1996. A new curriculum is in place: Functions are introduced at levels 9 to 12, Probabilities and Statistics are introduced al levels 11 and 12; the *National Council of Education* is created.
2002. First distance course for teachers in the web: *funciones.cl*. To be followed in 2004, with b-learning courses in geometry, probabilities, statistics and algebra.
2003. The extension of compulsory education up to 12 years of schooling.
2006. A new curriculum is approved for grades 1 to 10. Four strands, numbers, algebra, geometry and probabilities and statistics. Algebra and pattern recognition and probabilities and statistics from grade 1 to 12. A special emphasis is given to *mathematical reasoning* -conjecturing and the verification of own ideas are emphasized. "*Progress maps*" are published, becoming a new standard in mathematics education.
2007. A major reform of the framework ruling the national system of education is approved (The *General Law of Education, LEG*).
2011. The creation of the *Quality Agency*, responsible for national tests and the evaluation of school performances. "Bases Curriculares", a new curriculum for grades 1 to 6. The strand of *measurement* is introduced, special emphasis is given to concrete, pictorial and symbolic representation of mathematical ideas and procedures; also, skills on *problem solving, modeling, representation, communication and argumentation* are emphasized.
2013 - 2015. "Bases Curriculares" are extended until grade 10, a new emphasis in *functions, representation, argumentation and modeling*, are in place.
2015. Preschool education becomes compulsory.
2016. A *national policy in curriculum development agenda*, is announced.
2020. A differentiated curriculum for grades 11 and 12 is to be implemented; as a result, students will be able to choose between courses on *the initiation to calculus, 3D geometry, computer thinking and programming, and a first course in inferential statistics*.

[8] Grades 1º - 8º (Elementary School level).
[9] Grades 9º - 12º (Secondary School level).

ICMI Study 24
SCHOOL MATHEMATICS CURRICULUM REFORMS: CHALLENGES, CHANGES AND OPPORTUNITIES
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THE ASPIRATIONS OF THE AUSTRALIAN CURRICULUM IN PROMPTING REFORM OF SCHOOL MATHEMATICS

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Among many steps that might result in effective mathematics teaching and learning in classrooms, one is the documentation of the intended content and processes. In the preparation of such documentation there are a number of decisions, described here as dichotomies, that are made. In the creation of the Australian Curriculum: Mathematics, in the period leading up to 2010, explicit decisions were made about some of these dichotomies. The following outlines the decisions that were taken and what has happened subsequently. It is not argued that the choices made are the only possibilities, but the intention is to explain what the issues were and what has happened subsequently. The aim is to inform debates about curriculum documentation and to indicate what has worked in this case.

Introduction

This contribution and the associated presentation provide an opportunity to reflect on the intention and processes for the design and writing of the Australian Curriculum: Mathematics (AC:M) and to reflect on subsequent developments. The argument is that curriculum reform can be an agent and process for prompting teacher professional learning but whether this happens or not depends on whether the structure of the curriculum documentation and associated support foster such knowledge creation. Whatever the rationale for curriculum reform, whatever are the processes for resolving documentation, whatever are the methods for dissemination of the documentation, the critical agent in the reform process is the teacher, and so success of the reform is dependent on the support for teachers.

In any curriculum reform process there are many dilemmas or dichotomies about which active decisions are taken. One of the meanings of *dichotomy* is that there are two mutually exclusive, opposed, or contradictory positions. This contribution outlines some of the dichotomies in any curriculum reform process and reflects on ways that such dichotomies were and are being resolved in the Australian Curriculum.

As with any central or system decision making process, especially when there are jurisdictional complexities, there will be vested interests who make decisions more in the interest of the jurisdiction or their agents than in the interest of end users. This has happened in the case of the Australian Curriculum.

The process of development

Even though there are broader definitions of curriculum, including terms such as intended, planned and enacted (see, for example, Porter, 2004), this discussion focuses on documentation associated with centrally developed curriculums and decisions on the form and substance of that documentation. Of course, the real curriculum results from the ways that such documentation is interpreted, implemented and experienced in schools and classrooms, but the main opportunity for governments to intervene meaningfully is at the level of documentation.

Prior to the creation of a single national curriculum, there were eight Australian jurisdictions that each had their own curriculums and associated supporting resources. The responsibility for such curriculums was jealously guarded. In most cases the curriculums were informed by earlier national profiles so there was substantial overlap in the substance of the content specifications in the various jurisdictions but the extent of collaboration on aligning the documentation was minimal. Some of the jurisdictions are quite small with limited resources that made the earlier development of high quality curriculum documentation difficult.

The motivation for creating national curriculums in all domains was essentially political but the rationale was related to efficiencies in the development of text and online resources for teaching, the potential for better and cheaper text books, more aligned professional language, and better and more comparable assessments. The Australian curriculum started from four domains, Mathematics being one.

The first step was the development of a discussion paper that set the goals and processes of the curriculum. This was described as the Shape Paper (Australian Curriculum Assessment and Reporting Authority (ACARA), 2009) and outlines the principles, the aims, the terms used, the focus of the respective levels of schools, various issues such as connectedness and clarity, and a discussion of pedagogy and assessment especially as they related to equity and inclusion. The paper was developed by a broadly based writing team and sought online and face to face feedback nationally. I was invited to lead the process, presumably because of my experience as a teacher educator and researcher and the initial discussions included a range of researchers ensuring that, where relevant, research findings were considered.

The following discussion describes some of the dichotomies and is intended to raise some of the considerations in the documentation of curriculums generally.

Dichotomy 1: Teacher proofing or teachers as learners

Curriculum reform and associated teacher learning are integrally connected to views that curriculum developers and system decision makers have of teachers. There is a clear dichotomy of perspectives apparent in the ways that the initial curriculum was designed and has been interpreted.

On one hand, if teachers are seen as unreliable and unable to interpret curriculum documents then the curriculum will be written and supported in a particular way. On the other hand, if teachers are viewed as thinking, flexible and creative agents, then the curriculum documentation and associated support can reflect those perspectives. The Shape Paper and the initial curriculum design opted explicitly for the latter position. The underlying assumption is that if systems place trust in teachers, they will come to see the underlying principles of the curriculum. In this process, teachers can become better educators.

While there was always an expectation that there would be some jurisdictional customisation, it turns out that at least some of this rewriting has been arguably counterproductive and certainly minimizes the impact of some of the reform initiatives, especially seeing teachers as learners and creative agents. For example, one of the content descriptions in the Australian Curriculum (ACARA, 2018) is:

> Choose appropriate units of measurement for length, area, volume and mass

In at least one jurisdiction, this has been broken up into five separate statements creating the impression that the intention is the learning of the individual attributes (length, area, etc.) rather than "choosing appropriate units". The focus of the original descriptor is lost when the original statement is compartmentalised. In other words, increasing the detail of the documentation can be counterproductive to the mathematical intention and also to the learning of teachers.

Another decision taken was to seek to reduce the breadth of the specified content so that the more important aspects were presented. Each time jurisdictions increase the level of detail and breath of expected content, they reduce teacher decision making and the potential for teachers to learn about the broader goals of mathematics learning. The same is true for management (meaning "compliance") processes that insist on breadth of coverage at the expense of depth. In the ICMI 24 conference, it seemed that a common concern was that curriculums, worldwide, are crowded and this is counter-productive to building understanding, problem solving and reasoning.

Dichotomy 2: Documenting everything possible vs including just enough information

One of the initial decisions in the creation of the AC:M was that the curriculum should be described clearly and succinctly. Indeed, the intention was that the content for any one year be presented on a notional single "page", described parsimoniously and presented flexibly via a dynamic web based environment to emphasise the need for teachers to make active decisions (ACARA, 2009). The dichotomy is that, on one hand, comprehensive documentation would provide teachers with guidelines of what to teach, while on the other hand it would have the effect of restricting teacher decision making, causing it to be harder for teachers to see the "big picture".

The early consensus in the creation of the AC:M was that mathematics is much less a set of isolated micro skills to be learned independently of each other than it is sets of connected concepts and processes and that it is better for teachers to see the connections. To explain this, imagine that students are asked to complete a set of exercises from a mathematics text. If students complete the questions and exercises one by one without considering the bigger picture, they are less likely to appreciate the intentions of the designer of the exercise. If, on the other hand, teachers prompt students to consider, for example, "in what ways is question 15 more complex than question 1", "which of the earlier questions are likely to help you to answer question 15?" students can be encouraged to consider the intentions of the designers and consider the purpose of the exercise as a whole. But teachers are unlikely to pose such prompts if they themselves are not anticipating the overall intentions of the learning. In other words, excessive compartmentalization and documentation can reduce the possibilities of teachers seeing connections. The tendency in some jurisdictions in Australia, subsequent to the initial publication, has been to increase the level of detail in and complexity of curriculum descriptions which has the effect of limiting the extent to which teachers can imagine the bigger picture or even consider seeing the broader perspective as important.

A related aspect is the ways that the curriculum fosters connections between and within strands and substrands. A key international perspective which emphasises the importance of connections is Variation Theory (see Kullberg, Runesson, & Mårtensson, 2013). Watson and Mason (2006) outlined the importance of thoughtfully constructed sequences of learning experiences out of which the underlying concepts can be extracted. Similarly, Dibrenza and

Shevell (1998) described number strings as an example of the ways that sequences of related exercises can emphasise number properties. Sinitsky and Ilany (2016) explained that considering both change and invariance illustrates not only the nature of the mathematics but also the process of constructing concepts. In other words, finding ways to support teachers in seeing and using connections between and within concepts can support teacher learning and effective teaching. To achieve this, the curriculum needs to be clear and concise.

This connects directly to classroom implementation. It goes without saying, regardless of the educational context, teachers are better able to support students when they know what they hope the students will learn. Hattie and Timperley (2007), for example, reviewed a large range of studies on the characteristics of effective classrooms. They found that feedback was one of the main influences on student achievement. The key elements identified were that students should receive information on "where am I going?", "how am I going?", and "where am I going to next?" To advise students interactively, it is important for teachers to know their goals.

One of the disadvantages of having the content determined by a student text is that teachers are less required to think about their own broader purposes. The same is true for curriculums in which the teachers are "told" which tasks to teach without having to appreciate the goals, both content and processes, associated with the tasks. One of the critical foci for teacher learning is to enhance their capacity to make their own decisions using the curriculum documents and other resources to which they have access.

A further central aspect that relates to the nature of the documentation is the expectations that teachers will collaborate with colleagues in their planning of sequences of learning. It seems that in some countries the textbook serves as the curriculum and teachers need only to turn to the next page in planning their lessons. In Australia it is common for groups of teachers to plan sequences of lessons together. Not only does this allow teachers to learn from each other but also planning together encourages them to anticipate how students might respond, identify potential blockages and misconceptions, share the development of supporting resources, and so on.

Dichotomy 3: Practitioner vs specialist writers

Another early dichotomy relates to whose voice should be heard. One of the initial considerations was whether the curriculum should be written by experts or by practitioners, with the latter option being chosen. The process for creating the curriculum and associated documents was collaborative involving extensive, indeed exhaustive, consultation. Subsequently curriculum writers, predominantly classroom teachers, were employed and an advisory committee formed. There were extensive consultations around successive drafts, piloting in schools across the nation, mapping of the drafts against the various state and international curricula, and many other actions as well. The advantage of this process is that a curriculum was developed which was familiar to many teachers. The disadvantage is that the writing was informed by many and diverse contributions. In other words, there is a tension between seeking consensus and maximising coherence that is not generally acknowledged by commentators.

Dichotomy 4: Mathematics as preparation for later study or mathematics as experience

One of the key dichotomies in determining a mathematics curriculum is related to the nature of the mathematics to be described. One perspective refers to the structure and content of many mathematics curricula that create the impression that the main goal of learning mathematics is preparation for study in a subsequent year level. An alternate perspective is that curricula should inform an experience of learning that is like being a mathematician, in which the learning about and using mathematics is the primary goal. Of course, a balanced curriculum will consider both perspectives but the intention in the AC:M was to move away from a curriculum that focused only on the former.

In describing perspectives on teaching mathematics, Ernest (2010) categorised the desired goals as being:

- functional numeracy, which is the mathematics adequate for general employment and functioning in society;
- practical knowledge, particularly work readiness and the mathematics used by various professional and industry groups; and
- advanced specialist knowledge, which is the mathematics that forms the basis of science, technology, engineering and mathematics courses and professional preparation.

In these terms, the competing perspectives are numeracy/practical mathematics on one hand and specialised mathematics on the other. These different emphases are evident in ACARA's (2009) statement of the aims of the mathematics curriculum as being on one hand:

> to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help form their predictions and decisions about personal and financial priorities. … In a democratic society, there are many substantial social and scientific issues raised or influenced by public opinion, so it is important that citizens can critically examine those issues by using and interpreting mathematical perspectives.

And on the other hand:

> mathematics has its own value and beauty and it is intended that students will appreciate the elegance and power of mathematical thinking, experience mathematics as enjoyable, and encounter teachers who communicate this enjoyment — in this way, positive attitudes towards mathematics and mathematics learning are encouraged.

The AC:M took an explicit stance that the mathematics and numeracy that should be experienced by school students is much more than the emphasis on procedures and computational processes that seemed to constitute much of the teaching of mathematics in Australia at the time (see Hollingsworth, Lokan, & McCrae, 2003; Stacey, 2010). It is unfortunate that much of the subsequent discussion of the curriculum starts from the perspective that the primary rationale for the inclusion or emphasis on an aspect of content is that it will be used in subsequent study. This tendency is especially evident at senior levels with the pressure from interest groups being to increase the emphasis on procedures and routines and to include additional topics exacerbating the already crowded curriculum.

At ICMI 24, some of the discussions centred around the nature of the precision with which the content should be described. This perspective is that mathematical fidelity is a function of the readiness of the students to appreciate the purpose of such mathematical focus.

Dichotomy 5: General vs specific descriptions of expected mathematical actions

The first aspect of the AC:M that teachers access is the descriptions of the concepts or content that form the focus of learning experiences. There are achievement standards available that give advice to teachers of the expected standards of performance. The key device for broadening teacher focus to encourage them to value specific mathematical actions was described as proficiencies.

ACARA (2009) proposed that the content be arranged in three strands that can be thought of as nouns, and four proficiency strands that can be thought of as verbs. The content strands, Number and algebra; Measurement and geometry; and Statistics and probability, represent a conventional statement of the "nouns" that are the focus of the curriculums worldwide.

These four proficiency or process strands (the "verbs") were adapted from the recommendations in Kilpatrick, Swafford, and Findell, (2001). The first of these, Understanding, (the Kilpatrick et al. term was conceptual understanding) was described as follows:

> Students build a robust knowledge of adaptable and transferable mathematical concepts, they make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

A second proficiency is fluency (the Kilpatrick et al. term was *procedural fluency*) was described as:

> ... choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly used facts, and when they can manipulate expressions and equations to find solutions.

A third action is problem solving (*strategic competence*) which was described as:

> ... the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify their answers are reasonable.

The fourth proficiency, reasoning (*adaptive reasoning*) includes:

> ... analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

The proficiencies are represented as intersecting with each of the three sets of descriptions of content, illustrating that the proficiencies are not only a focus of learning of all aspects of mathematics but can be the vehicle for that learning. There was an explicit intention to support teachers in seeing mathematics learning as incorporating all of these actions. In previous Australian curriculums, the metaphor of "working mathematically" was used to prompt teachers to incorporate processes into their teaching. Unfortunately this seemed to communicate to teachers that working mathematically was an additional content strand, so the broader process actions were somewhat hidden.

It is noted that while the first two proficiencies, understanding and fluency, can be prompted by explicit teacher instruction, problem solving and reasoning require student centred approaches, further communicating to teachers about the breadth of pedagogies needed and the nature of learning experiences that they can create.

Some jurisdictions have sought to complicate the issue by introducing additional proficiencies, which seems to overlap substantially with at least one of these four, making assessment of the proficiencies more complex and thereby reducing teacher flexibility.

Dichotomy 6: Mathematics for elite or mathematics for all

A further key element of the AC:M, which was intended to inform teacher learning is related to the challenge of equity. In various reports on international assessments (e.g., Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2010) and in other analyses (e.g., Sullivan, 2011) the diversity of achievement of Australian students is noted. In particular, it seems that low SES students as a group perform substantially below other students. This is connected to the curriculum in various ways.

ACARA (2010) argued that all students should experience the full range of mathematics in the compulsory years. Mathematics learning creates employment and study opportunities and all students should have access to these opportunities. This is both an equity and a national productivity issue. The curriculum makes the explicit claim that all students should have access to all of the mathematics in the compulsory years.

> A fundamental educational principle is that schooling should create opportunities for every student. There are two aspects to this. One is the need to ensure that options for every student are preserved as long as possible, given the obvious critical importance of mathematics achievement in providing access to further study and employment and in developing numerate citizens. The second aspect is the differential achievement among particular groups of students. (ACARA, 2009)

An explicit goal of education in Australia is the intention to build an inclusive society in which all citizens can participate. The connection to inclusive processes for learning mathematics is obvious.

The prevalence of achievement grouping in many schools is a major threat to equity in that students, even from the earliest years, can be offered a restricted curriculum. If mathematics, using the terms above, is seen as accessible by only some students, with numeracy being the focus for others, this reduces learning opportunities of some students. The implication in achievement grouping is that teachers do not have the repertoires to address the diverse needs of learners, whereas the documentation around the AC:M implied that professional learning around such pedagogies should be a priority.

Conclusion

The claim here is that the initial intentions of the AC:M were that the curriculum should be seen as an agent of reform with the emphasis being on documentation that both assumes and creates a focus for teachers being active learners about curriculum and pedagogy. This intention was also evident in the processes used to communicate to teachers that doing mathematics is as important as skill development, and that not only is it possible to structure classrooms to be inclusive of all students but also that that is an expectation.

In some ways, the debates around curriculum documentation have been conducted without any attempt to considers the dichotomies around the decisions taken and have moved in the direction of limiting teacher agency, restricting inclusiveness and reducing relevance of the experience of learning mathematics.

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THE MATH CURRICULUM REFORM IN LEBANON: ACHIEVEMENTS, PROBLEMS AND CHALLENGES

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The purpose of this paper is to conduct a reflection on the mechanisms of development and enactment of the reformed Lebanese Mathematics Curriculum. In particular, it aims to discuss the internal coherence and mutual influences between the declared theoretical and pedagogical foundations of the curriculum on one hand, and some of the implementation tools and practices on the other. The paper is based on results of several research works that investigated, over the past years, different aspects of the reformed Lebanese curriculum, but goes beyond those results to present a more comprehensive view of the curriculum. Results of studies on the curriculum foundational documentation, textbooks, and national examinations, have converged to uncover inconsistencies among the different curriculum components. In the absence of suitable resources, such inconsistencies act as obstacles to change and put practitioners at a higher risk of reverting back to old practices.

INTRODUCTION

It is widely agreed that, throughout the processes of its development and implementation, a curriculum does not remain a static entity. As it is conceptualized, framed, developed and applied, the curriculum is reshaped by the agents involved – e.g. stakeholders, curriculum developers, school administrators, teachers, students – and may take, at each level, a different form. A large scope of research works have coined terms to characterize different representations of a curriculum, such as the *intended*, *implemented*, *attained, tested or assessed* curricula. McKenny et al. (2006) assert that internal consistency and harmony among curricular representations is an important condition for a successful and coherent curriculum (Schmidt, Wang & Mcknight 2005; Schmidt & Prawat 2006).

The purpose of this paper is to discuss the internal coherence of the Lebanese Mathematics Curriculum (LMC) and the extent of alignment between the declared theoretical / pedagogical foundations of the curriculum and some of its implementation and assessment tools. The paper is based on results of several research works that investigated, over the past years, different components of the reformed LMC; but it goes beyond those results to present a more comprehensive and synthetic view of the curriculum.

BACKGROUND AND MAIN HYPOTHESIS

The educational system in Lebanon is characterized by a high level of centralization and a national curriculum that is binding to both, public and private schools. Decision making and developments are exclusively under the jurisdiction of the Center for Educational Research and Development (CERD), overseen by the Ministry of Education (MoE). While public schools apply only the national curriculum and textbooks, private schools may apply other programs and may use different series of textbooks, local or foreign. They are, however, bound to cover the national curriculum. A major tool of governmental control is the national examinations, referred to as official exams.

The Lebanese Ministry of Education proposed, in 1994, a project for overhauling the educational sector, as stipulated by the Taïf Agreement (1989), which has put an end to the 15-year-long war. In October 1995, the government approved a plan for developing the new curricula. Starting 1995, a reform process of the educational system and national school curricula began, after a stagnation that lasted more than 25 years, partly because of the war that hit the country. The older national curriculum initially created in 1946, just after the independence of Lebanon, was partially revised in 1968 and 1971 to include instances of the worldwide “New Math” wave, such as the set theory. An extremely abstract, procedural and directive spirit has always characterized the old, long lasting math curriculum, setting up an educational culture guided by, and revolving around stereotypical national examinations (Osta, 2007). In those curricula, conceptualization was neglected and students were seen as passive receivers of information and executors of algorithms.

Between 1995 and 1999, the reform efforts mobilized politicians, educators, teachers, textbook developers, and other constituents of the Lebanese society. The educational ladder has been organized into two main levels: Basic Education (BE) and Secondary Education (SE). The BE consists of three cycles, three years each – Elementary cycle 1 (grades 1 to 3), Elementary cycle 2 (grades 4 to 6) and Intermediate cycle (grades 7 to 9). Secondary Education includes grades 10 to 12. The main curriculum document, delineating general objectives and objectives of the cycles, as well as the scope-and-sequence and contents to be taught in every grade level, was issued in 1997. The national textbooks were gradually developed and applied over three years thereafter (every year, the new curriculum and textbooks were implemented in one more grade level of each cycle), till the year 2000 that witnessed full implementation at all grade levels, and culminated into the first national exams under the new reformed curriculum.

After a long period of adoption of an old traditional curriculum, the reform of the LMC constitutes a revolution. It changes the ways the nature of mathematics and its teaching are perceived by the educational community. The intention was to align the new curricula with the worldwide curricular trends at that time. The methods adopted are defined as constructivist and active, the learner being the "center of the teaching/learning operation", and the capacities of “reasoning and problem solving” outweighing algorithmic procedures and memorization of facts. Compared to the old curricula, a real revolution was announced and expected.

The major question remains: Has this revolution been maintained throughout the curriculum development and implementation processes? An essential claim of this paper is that, with the marginal role of teachers, absence of internal coherence of the curriculum, and lack of suitable resources, the high-stake national exams determine, to a large extent, the orientations of the curriculum enactment and make it revert back to the deeply rooted old practices.

REFLECTION ON THE LEBANESE MATH CURRICULUM

In the rest of the paper, four of the main components of the LMC will be discussed, namely: 1) the foundational documentation of the curriculum. The role of this documentation was to act as a guiding roadmap for the development of textbooks and an interface between the curriculum philosophical / pedagogical foundations and the educational community; 2) textbooks as the main guiding resource for teachers; 3) teachers as the main agent for the enactment and reshaping of the curriculum; and 4) the national examinations as the central focus and determinant factor of the curriculum development, implementation and reorganization.

Foundational documentation

The foundational curriculum documentation consists of: 1) the main curriculum document issued by an official governmental decree (CERD, 1997) delineating the aims of the curriculum, its pedagogical recommendations, general objectives (GOs), and objectives of cycles (OCs); 2) the *details of content*, published gradually in three volumes over three years (1997 for the first year of every cycle, 1998 for the second years, and 1999 for the third years). They include the specific objectives (SOs) and detailed information about the contents of the mathematics subject for each grade-level year.

Osta (2003) investigated the internal coherence of the LMC documentation using mapping tables and text analysis of the curriculum documents above. The analysis of the main curriculum document showed a high level of coherence between the general objectives GOs and the philosophical and pedagogical foundations announced in the introduction. They both use a language focused on the development of cognitive abilities, the importance of problem solving, and the appreciation of mathematics as a practical tool related to everyday life. Following are a few examples:

Mathematics is defined in the introduction as “a fertile field for the development of critical thinking, for the formation of the habit of scientific honesty, for objectivity, for rigor and for precision. It offers to students the necessary knowledge for the social life and efficient means to understand and explore the real world”. As for the recommended teaching methods, they “consist of starting from real-life situations, lived or familiar, to show that there is no divorce between Mathematics and everyday life”. As described, the recommended teaching methods are clearly constructivist and focus on problem solving; “the stress is mainly on the individual construction of Mathematics; it no longer consists of teaching already made Mathematics but of making it by oneself. Starting with real-life situations in which the learner raises questions, lays down problems, formulates hypotheses and verifies them, the very spirit of science is implanted and rooted”.

The General Objectives (GOs) are clearly consistent with this approach; they insist on the importance of "the construction of arguments" and on "developing critical thinking, and emphasizing mathematical reasoning", the latter being presented as the first GO. Problem solving is presented as the second GO and described as “perhaps the most significant activity in the teaching of mathematics. On the one hand, every new mathematical knowledge must start from a real-life problem. On the other hand, students must learn to use various strategies to tackle difficulties in solving a problem”. The student must also "encode and decode messages, formulate, express information orally, in writing and/or with the help of mathematical tools", which makes mathematical communication a third main OG. We will refer to these three objectives by "cognitive objectives", to distinguish them from objectives purely related to the factual and procedural mathematical content.

The curriculum therefore proposes a progressive teaching approach. A constructivist approach, focused on reasoning, problem solving and communication, is reflected in the teaching method and general objectives advocated in the first curriculum declarations. It is to be noted that the three highlighted cognitive objectives are mostly in line with the American "Standards" (NCTM, 1989) which have profoundly affected modern international trends in mathematics education at that time.

However, only partial consistency is found between the cycles' objectives COs and the GOs, with a deviation in the discourse that reflects a beginning of separation from the pedagogical foundations above. Indeed, the COs continue to reiterate the importance of the three cognitive objectives, which systematically appear as the three first objectives for every cycle, followed by content-related, factual and procedural objectives.

One example, where we can touch upon the deviation of discourse, is found in the objectives cited under "Problem solving" for the secondary cycle: "Find the solution of a problem following a given algorithm". Requiring that solving the problem should follow a "given algorithm" is in opposition to the very meaning of problem solving. It also defeats the purpose stated in the GOs, delineating the traits of the learner as being "an individual with a critical mind who questions, doubts, proposes solutions", and who "must learn to use different strategies".

The deviation from the curriculum's foundations and GOs increases and becomes more serious at the level of the specific objectives in the SOs in the *details of content* volumes. The three cognitive objectives are not maintained in the SOs. Not only have they disappeared as independent objectives, but they are also very rarely reflected in the contents. The analysis of the SOs shows that they mostly represent declarative knowledge and procedural skills related to formal mathematical content, emphasizing the execution of predetermined and automated steps and overlooking conceptual understanding. Very few SOs are linked to the cognitive GOs, which are supposed to perpetuate the link to the constructivist intentions of the curriculum. In an analytic quantitative study of the coherence between the GOs and SOs of the intermediate grade levels, Shatila (2014) found that the percentages of SOs that reflect reasoning in grade-7, 8 and 9 textbooks are 7.03%, 7.19% and 10.81% respectively. Those that reflect mathematical communication are 8.59, 9.35 and 7.43; while those that reflect real problem solving do not exceed three SOs out of the number of SOs in each grade level.

A spirit of "drill-and-practice", rather than conceptualization, is remarkable in the *Details of Content*. The phrase "to train the student" is frequently used. The learner is seen as a passive receiver of information and executer of algorithms, and the teaching style that is detected from the teaching tips is extremely directive. Consider for example the case of problem solving: Even though the GOs insist on problem solving as a context "from real, lived or familiar situations" for both, learning and applying concepts, we find in the *details of content* clear reluctance to actual situations and mistrust of learners' abilities as problem solvers.

The *details of content* were later used as the main basis for the development of the subsequent documents and tools, including the student textbooks, pedagogical guides and evaluation guides.

Textbooks

School textbooks are the main interface between teachers and the curriculum foundations, as well as the main tool for their educational practices. The question raised here is: considering the fact that the *Details of Content* drifted away from the innovative spirit of the intended curriculum, and the fact that school textbooks are the main tools in the hands of teachers, how can teachers maintain the link between the tools available to them in their professional practice, and the GOs and OCs which ensure the true reflection of the intended curriculum's foundations?

Shatila (2014) analyzed the textbooks of the intermediate level (grades 7, 8 & 9). She mapped all exercises and problems in those chapters against the three main GOs – problem solving, reasoning and mathematical communication. Results showed surprisingly low levels of coherence, reflected by the low percentages of exercises that target the three cognitive GOs. The study also showed that grade 9 textbook is remarkably less compliant with the curriculum change than grade 7 textbook.

Knowing the fact that the textbooks for the first year of each cycle (grades 1, 4, 7 & 10) were authored just after the development of the foundational documents in 1997 and that the textbooks for the third year of each cycle (grades 3, 6, 9 & 12) were authored two years later in 1999, it may be legitimate to assume that the textbook authors have gradually deviated from the reformed curriculum's foundations and reverted back to the old approaches. It is worthy to mention that all members of the curriculum committees had been taught math under the old curriculum and have taught that curriculum for many years as well.

With this question in mind, many discrepancies can be found in the national textbooks. For instance, *proportionality* is addressed in two chapters of the grade-6 textbook (authored in 1999) and one chapter of the grade-7 textbook (authored in 1997). It is noted that the Objectives stated at the beginning of grade-6 textbook chapter to introduce the topic reflect a completely numerical and abstract approach. The objectives are to "recognize and construct proportional chains" and "calculate the proportionality coefficient and the fourth proportional term in a proportion". Though some word problems in the chapter do touch upon real-life situations, they are not used as a context for developing the concept. They come after a series of purely numerical exercises. It is however in the grade-7 textbook that more meaningful problem situations are provided and better connection to everyday life is reflected in the introduction and objectives of the *Proportionality* chapter. At the beginning of the chapter, a short introduction highlights such a connection: "Proportionality is one of the most useful mathematical concepts; it applies in many fields of everyday life". The objectives of the chapter state that students should be able to: "identify a situation of proportionality, recognize a proportion, calculate a new proportion starting from a given one, calculate the fourth proportional term, and use calculations of the fourth proportional to solve problems".

Teachers and the reform

A radical reform requires involving and preparing the teachers for the enactment of the intended change. It also offers opportunities for teachers' professional development in view of modifying their beliefs about the nature of mathematics they will be teaching and the approaches to its teaching. Educators agree that teachers are main agents for any educational change. Their attitudes towards, and perceptions of the reform influence their teaching practice (Fullan, 1982). They do not just transmit the curriculum or execute its guidelines; they develop it as they teach, interpret it and redefine it. Teachers' role becomes even more important in a general and radical reform of curricula that brings new methods and techniques, a new paradigm.

The MoE and CERD have conducted "training" workshops in the new curriculum, involving a large number of mathematics teachers, especially in the public sector. These workshops proved to be too directive. They mainly revolved around providing information on the new content, as well as the recommended pedagogical approaches. Paradoxically, while the new curriculum advocates active methods of learning mathematics and development of critical thinking and reasoning, the training was rather characterized by a "patriarchal" spirit (Osta, 2006), with a tendency towards training in procedures and ready-made techniques almost imposed on teachers. There was general malaise in the ranks of teachers, reflecting resistance to change and negative attitudes towards the new guidelines.

A radical reform offers a valuable opportunity for the professional development of teachers, in a sense that goes beyond guidelines for implementation, to reach a serious reflection aimed at developing the abilities of teachers as "professionals" working in complex environments and to which they must constantly adapt (Artigue et al., 2003). They must have a more active role in the processes of their own learning and professionalization. Proulx (2005) takes this idea even further by developing the notion of *objectives to work on* as opposed to the notion of *goals to be achieved*. He criticizes the tendency to organize mathematics teacher education programs around convergence toward, or compliance with, "best practices" or other idealized conceptions of mathematics education.

In a study that solicited teachers' reflections on the reformed Lebanese curriculum and their feedback about the workshops (Osta, 2006), all participants reported that they were not sufficiently prepared to apply the recommended teaching methods. They requested more practice on techniques such as group dynamics, group work management, active methods, design of didactical situations, development of students' autonomy, use of calculator and computer for teaching / learning purposes. Teachers expressed their belief that the educational authorities which "impose" such methods should provide support to teachers up to the classroom level, such as providing "model lessons", activity sheets or additional exercises to respond to certain learning problems that may arise.

The synthesis of all teachers' feedback reflects an image that teachers have of themselves, under such a radical change of the educational system. In the absence of a serious dialogue that would involve them in the conception and comprehension of the reform, and in the absence of needed resources and tools to help them in the enactment of the curriculum, they see themselves as ready (because obliged) to execute the new teaching methods, provided they are shown exactly what to do in class. Such a teacher self-image would deeply hinder achieving the intended change and would make the teachers even more prone to revert to their old, long lasting practices.

National exams

In Lebanon, national (known as official) exams take place every year at two grade levels: the end of the intermediate cycle of study (grade 9), for the "Brevet" certificate, which gives access to secondary school, and the end of secondary level (grade 12), for the "Baccalaureate" certificate and graduation from pre-college education. They are high-stakes exams and have an imposing power. In the Lebanese culture, a major goal for schools is to raise their students' test scores in the official exams. It is as well an indicator of school improvement. Teachers whose students pass the official exams gain in reputation and receive good offers with high salaries from private schools. This leads to the observed fact that teachers tend to teach to the test, and that school administrators shape their school policies and focus their academic activities around that goal. As a result, the official tests determine the valued mathematics that should be taught, and the ways it should be taught.

Osta (2007) conducted a study that aimed at developing and piloting a methodological framework to investigate the alignment between the Lebanese official Brevet tests and the math curriculum, during the transitory period of the curricular reform, while the old curriculum was still in effect. The intention was to use that methodological framework later to study the alignment under the new reformed curriculum.

This study showed that the official exams under the old curriculum kept a stable structure and addressed a specific body of mathematical content. It was noticed that many topics in the curriculum were never addressed in the official exams. The topics frequently occurring in test items defined a "mini-curriculum" that gradually replaced the original one, and was reinforced every year and in every test. This "mini-curriculum" fosters memorization of answers to stereotyped test items, through drill and practice rather than conceptual understanding.

The study led to a hypothesis expecting that the extremely procedural nature that has always characterized the old math official exams, has established a deeply rooted testing "culture" focused on direct procedural skills. Consequently, the new official exams could not, over the years, reflect the real change intended by new curriculum. This "culture" was nurtured by the long-lasting old curriculum, and its official exams are still influencing the new official exams. The hypothesis above was confirmed by three studies that used the framework developed by Osta (2007) to investigate the extent of alignment between the official exams over 10 to 12 years, with the reformed curriculum. One of the three studies (Shatila, 2014) analyzed the Brevet tests, while the other two analyzed the Baccalaureate tests, one (Sleiman, 2012) for the *Literature and Humanities* track, and the other (Safa, 2013) for the *Life Science* track. The results of the three studies converged to confirm the hypothesis above. A mini-curriculum was identified, and low levels of alignment are found between the exams and the curriculum guidelines, especially as pertains to the cognitive general objectives. They found, however, that the alignment improved gradually over the years. Global alignment remained, however, lower than enough to reflect actual change in the testing culture.

The nature, scope and structure of the official tests send a clear message to the educational community (teachers, administrators, parents and students) over the years. This implicit "contract" among all involved parties binds, in return, the committees in charge of constructing the tests. Even if they want to include modifications or additions, they find themselves bound to the "mini-curriculum". This closed cycle is sustained by the "doctrine of no surprise" that English and Steffy (2002, p. 46) explain as being the idea that students should not be taken by surprise by any test question.

It is widely agreed that assessment approaches widely affect teaching practices. Many educators have discussed these types of relationships. Boud (2000), for example, asserts that assessment achieves the "double duty" of judging achievement and transmitting what we value. Problems occur particularly when the national tests do not align in a balanced way with the curriculum, such as in this case. The national exams then act as obstacles to the intended change in the educational practice.

CONCLUSION: FROM DESIGN TO IMPLEMENTATION

In general, while teachers are in direct contact with the implementation tools, among which the textbooks and the official exams, they are at a distance from the other foundational components, including the pedagogical foundations and general objectives of the curriculum. Those are particularly absent from their direct perception and their day-to-day professional practice, if they are not actively implicated in the reform movement. Even an attentive reading by the teachers and their participation in informational workshops are not enough to guarantee a modification of their professional practices that they developed over many years according to the old curriculum.

It therefore becomes very important to control the consistency between the curriculum’s philosophical / pedagogical foundations and the implementation tools made available to teachers.

Mechanisms and controls should be established that ensure the preservation of the curriculum's foundations throughout the development process. Actively involving teachers in the development process would, on the other hand, increase the chances of alignment between the intended and implemented curricula.

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SCHOOL MATHEMATICS REFORM IN SOUTH AFRICA: A CURRICULUM FOR ALL AND BY ALL?

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Mathematics curriculum reform in South Africa has shifted from a focus on provision at the dawn of democracy twenty four years ago, to its current focus on the quality of the mathematical experiences of learners and teachers. Throughout this period the reform took its bearings from the underpinning debates in the Mass Democratic Movement in the decade prior to the new democratic era. There has been several iterations of school mathematics reforms since 1994 and this paper provides a broad-brush account of the context in which these reforms have been made. While there is considerable concern about the level of mathematical proficiency of South African learners, there is at the same time a deep concern about how the mathematics curriculum will contribute to the kind of people we want to see coming out of our school system as the citizens of a transformed and admirable society and what kind of teacher and educational ethos will be needed in order to produce such people?

Introduction

The term *curriculum reform,* like any other concept, always has a contextual *ancestry.* It also has a *career* that needs to be recognised and understood within a particular setting. But while there is general acceptance that curriculum reform grows into its own career and takes shape within a context, we often need to be reminded that this evolution is not bound by some transcendent, universally applicable set of laws which are independent of people. The political aspirations and ideological commitments of the drivers of the reform and the social forces that shaped the reform cannot be ignored and omitted from its ancestral *biography*. I see the purpose of this discussion as an attempt to understand how we can influence the development of these contextual careers of mathematics curriculum reform by understanding how choices were made within the various contexts and to what extent there was a willingness to embrace the complexity and ambiguity for the greater public good.

Curriculum inertia occurs when we choose to ignore the complexity inherent in making educational choices and retreat to the false safety of the universality of mathematics. Behind this wall we see our task as creating access to fixed, universally accepted ways of knowing and learning mathematics, stripped of all the clutter of ideological and cultural expectations.

South Africa is a society in transition. We have moved away from what was a stable but cruel past to a new and dynamic present. The conventional signposts have been swept away and we have been travelling on largely unchartered waters since 1994. One way of describing the new, democratic, educational reality in South Africa is that of celebrating the chaos and turbulence of a new beginning. It has been exciting to be part of this wonderful and dynamic period of our history and for me it has been particularly rewarding to be asked by both the previous and present Ministers of Education to play a key part in educational reform in post-apartheid South Africa.

Challenges Facing Curriculum Reform in South Africa.

Over the many years of apartheid two education systems coexisted - one predicated on the goals of a first world education, the other intended to be merely reproductive. The one was seen to be sufficient to produce enough high-level skills to support the larger-economy, the other to reproduce people who were just sufficiently functional to serve the low-level skills demand of the extractive-metals economy. Race was the main determinant of educational access, provision and quality. Throughout the years of apartheid, there was a continuous groundswell of resistance to "Bantu education" culminating finally in the 1976 Soweto uprising. Since that time the Mass Democratic Movement (MDM) and the politics of confrontation in education, became increasingly organised until it established the National Education Crisis Committee (NECC) in 1980.

The failure of the then government to respond to the crisis in education led the MDM to resolve to strive for *People's Education for People's Power* at its first Education Crisis Conference, in December 1985. People's Education (PE) would lead to educational practices that would enable the oppressed to understand and resist exploitation in the workplace, school and any other institution in society. It would also encourage collective input and active participation by all in educational issues and policies, by facilitating appropriate organisational structures. These ideals found expression in the work of three commissions, one each in the fields of History, English and Mathematics. When it became clear that PE would be introduced in schools by mid-1986, the apartheid government moved in very quickly to restrict its impact. The momentum for PE, during the years after the restrictive measures, was sustained for a while in large part, by the work of the **Mathematics Commission**, but this momentum also finally ground to a halt for a variety of reasons.

An underlying assumption in educational policy in South Africa is that the achievement of democracy requires a (national) curriculum to realise its goals. Curriculum change in post-apartheid South Africa thus started immediately after the election in 1994. So the genesis of new curriculum thought in South Africa finds its roots in the debate within the Mass Democratic Movement over previous decades. The first major curriculum statement of a democratic South Africa was known as Curriculum 2005 launched in 1997. It signalled a dramatic break from the past with its narrow visions and concerns for the interests of limited groupings at the expense of others. But it was also bold and innovative in its educational vision and conception. It introduced new skills, knowledge, values and attitudes for all South Africans and stands as the most significant educational transformation framework in South African education.

At the dawn of democracy in 1994, South Africa had nineteen different educational departments separated by race, geography and ideology. While these were merged into nine provincial departments, there was also a need for a single core syllabus. It did not touch the core content since a part of its brief was not to necessitate new textbooks. So beyond the rationalisation and consolidation of the existing syllabi, the process could at best sanitise the syllabus by removing overtly racist and other insensitive and offensive content forms from the syllabi.

After the completion of the syllabus revision process in late 1994 the national Department of Education set in place a new vision for education through a series of policy initiatives in 1995. This included a vision for curriculum development and design. At the same time South Africa adopted **a** National Qualifications Framework (NQF) as the focus for systematic transformation of the *education and training* system. Some of the objectives of the NQF are to create an integrated national framework for learning achievements and to accelerate the redress of past unfair discrimination in education, training and employment opportunities.

Furthermore, an **outcomes-based education** approach was chosen as the vehicle to implement the objectives of the NQF at all levels and sectors of education and training in the country. When the Minister of Education announced the introduction of a new curriculum framework in 1995, there were plans to introduce it into all grades by 2005. In line with this timetable the new National Curriculum Statement (NCS) became known as Curriculum 2005 (C2005). At a broader level, eight critical outcomes have been chosen to ensure that learners would be prepared for life in a global society. These generic, cross-curriculum outcomes also reflect the aims of the Constitution.

C2005 was inspired, not so much by the theories of others, nor on experiences elsewhere, but was an attempt to respond in an authentic manner to the realities facing the South African classroom. But it was also flawed in several ways. Some of these were design flaws while others were directly attributable to the rate and scope of implementation. None of these however, outweigh the significance or detract from the impact of C2005 as the curriculum policy that would forever change the landscape of education in South Africa.

The development of a National Curriculum Statement (NCS) was seen as a key project in the transformation of South African society. The thrust of the project is towards achieving a "prosperous truly united, democratic and internationally competitive country with literate, creative and critical citizens leading productive, self-fulfilled lives in a country free of violence, discrimination and prejudice." (Department of Education, 1997)

Curriculum reform since 1994 faced several challenges**.** These include

- The post-apartheid challenge: to provide awareness and the conditions for greater social justice, equity and development. This is the challenge of developing new values and attitudes.
- The global competitiveness challenge: to provide a platform for developing knowledge, skills and competences to participate in an economy of the twenty first century.
- The challenge of developing critical citizens: Citizens in a democracy need to be able to examine the many issues facing society and where necessary to challenge the status quo and to provide reasons for proposed changes.

The view taken by the curriculum designers was that the best route to greater social justice and development is through a high-knowledge and high skills curriculum and that mathematics education can play a vital role in the realisation of this vision.

The general expectation was that the NCS would result *in learners who are literate, numerate and multi-skilled, but who are also confident and independent, compassionate, environmentally respectful and able to participate in society as critical and active citizens.*

Review Committee on Curriculum 2005 recommended major changes to the NCS (C2005) in May 2000 and the Revised National Curriculum Statement (RNCS) was implemented immediately thereafter. The vision adopted by the Review Committee in 2000 keeps in focus the dual challenge for C2005 of addressing the legacies of apartheid on the one hand and preparing learners to participate in the global village on the other - these two are taken as indivisible. The RNCS has been further refined in 2011 through a new statement called the Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education, 2011) that specifies content and assessment criteria in a more integrated manner.

Mathematics Curriculum pre-1994

During the apartheid period the canonical syllabus for mathematics, although compartmentalised by race, had remained roughly invariant for everyone over decades. In a sense, the content was almost immaterial and by itself, made very little difference to the way mathematics, as a school subject, was used as a means of control and social stratification. Some attempt was made to revise the mathematics syllabus every eight years or so, but this rarely made any substantive change to the core content. Even in the current South African curriculum parlance, mathematics is referred to as a “gateway subject” precisely because it provides access as a gatekeeper. More than any other subject, mathematics will decide who will stay behind and who will go ahead. Although some may feel that mathematics has only been able to assume this central position in the curriculum because it is over-admired and over-privileged, very few will question the need for all learners to be “mathematically literate”.

In fairness it must be acknowledged that a feature of school mathematics during the late 1980s and early 1990s was a concerted effort by some mathematics educators to adopt a different approach to the teaching and learning of mathematics at school. The impetus for this change came largely from the world-wide swing towards a *constructivist perspective* that was implemented mainly in white primary schools in South Africa. Euphemistically called the *"problem-centred approach*", this perspective came across in the South African context as a prescriptive methodology, a new orthodoxy, which dismissed and replaced any set of ideas mathematics teachers may have had about the teaching of the subject. Nevertheless, few will deny that where this constructivist approach was piloted, it made a significant change to the classroom culture. Pupils at these schools developed very positive attitudes to mathematics and there is strong evidence that they also developed powerful ways of learning mathematics. It would therefore be unfair to say that this "socio-constructivist" approach to mathematics did not have a beneficial effect on classroom practice. It is however the case that the classroom of majority population in South Africa, where the teacher typically has to cope with a large class and poor resources, was left virtually unreached and therefore unaffected by this approach.

During the pre-1994 period People's Mathematics developed independently and indigenously rather than an attempt to embrace the “loudest fad from the West”. In addition to facilitating discourses around mathematics in the communities, People's Mathematics also developed a unique emphasis and character. Cyril Julie (1991) argues that the four major distinguishing features of People's Mathematics were:

- Its ability to reveal how school mathematics can be used to reproduce social inequalities;
- Its rejection of absolutism in school mathematics and its contribution towards seeing mathematics as a human activity and therefore necessarily fallibilist;
- Its incorporation of the social history of mathematics into mathematics curricula and
- Its belief in the primacy of applications of mathematics.

Julie (1991) acknowledges that People's Mathematics did not have the desired effect on the development of a mathematics culture at the time. This he claims, is partly due to the preoccupation of the advocates of Peoples Mathematics to design mathematical activities that had a direct bearing on the day-to-day political struggles of the people. Another reason for its lack of efficacy was the sense of scepticism and even distrust about the notion of People's Mathematics as a poor substitute for the “real mathematics”. Peoples Education failed to re-direct its focus away from *a struggle in the streets to a struggle within the classroom*. While it may be the case that it was too overtly political or even woolly at times, the People’s Mathematics Movement did provide a focus for mathematics curriculum debate and indeed for PE itself and it was encouraging to how the spirit and core ideas of PE became mainstreamed in the National Curriculum Statement.

Mathematics Curriculum Reform post-1994

In the post-apartheid era, mathematics curriculum reform continues to be influenced by two main considerations namely, a call for mathematics **for all** and the need to ensure mathematics **by all.** The first deals with the legacy of the past and considerations of equity, while the second is response to a renewed focus on quality of provision and global economic challenge of participating in a global village.

Mathematics for All

In a country where there has been a neglect of provision for decades, the need for massification of provision remains a major challenge for the future of education in general, and of mathematics in particular. The legacies of gross discrimination of the past meant that blacks were actively discouraged from taking mathematics as a subject. Historically between 30% and 40% of secondary schools in the country simply did not offer any mathematics beyond grade nine. We now have a policy that requires that everyone must take some form of mathematics. "*Mathematics for all*" is fundamentally a statement of policy, and as such it is a statement of provision. Of course it is a statement about curriculum, but essentially it signals that every learner should have the opportunity to learn mathematics.

But mathematics for all does not necessarily mean the same content for all. It is a truism that what content is used must be tied to purpose. It is therefore perfectly reasonable to assume that while all learners need mathematics, not all need the same mathematics. Mathematics for all however, must mean the same **quality** of mathematics for all. Although this seems to be an educationally defensible position, the idea of a differentiated approach to subject offerings at school (including mathematics) was rejected in favour of a single undifferentiated approach to mathematics. This decision should be seen within its historical and political context. During the pre-democratic era and up until 2007, more than 10 years into the new democracy, mathematics, like all other subjects was offered at two levels namely Higher Grade (HG) or Standard Grade (SG). At the dawn of democracy only twenty percent of blacks were taking HG mathematics while seventy percent of whites took mathematics at the same level. A Ministerial Committee on Differentiation (Department of Education, 2003a) recommended that curriculum reform in South Africa move away from differentiation at subject level.

In order to comply with the new policy that all learners to take some form of mathematics, Mathematics Literacy was introduced as a high-school subject from Grade Ten level in 2006 as part of the field of mathematics. Although seen as part of the "field of Mathematics" it had a very different purpose to that of Mathematics. While mathematics is important as a foundation for those with an interest to pursue work and further study in fields that require mathematics (such as business, science and engineering), mathematical literacy is about helping people to participate more fully in the choices that affect their lives. Mathematical literacy may help individuals to engage in discussion with employers over what constitutes fair wages and conditions of service, make sense of even participate in national debates on issues such as health, crime etc., particularly where quantitative arguments are used. Generally Mathematical Literacy was intended to assist learners to take charge of their own experiences as self-managing individuals and critical citizens in a democracy, crucial for nation-building and the strengthening of the new democracy. However it was never meant to be a dead-end low-level subject that represents a kind of watered-down mathematics in the same way that SG mathematics differed from HG mathematics. In short, the difference between Mathematics and Mathematical Literacy is a difference in kind rather than level or degree. Initially, many more learners opted for Mathematical Literacy but in recent years there has been a more even split with 56% of the 617 982 Grade 12 candidates enrolled for Mathematics Literacy in 2018.

One of the points of departure is that the South African school curriculum is composed of "learning areas" rather than subject disciplines. Integration within and across learning areas is another important building stone of the curriculum.

In the learning area of mathematics there are five learning outcomes. (Department of Education, 2003b) They are:

1. **Numbers, Operations and Relationships**: The learner is able to recognise, describe and represent numbers and their relationships and can count, estimate, calculate and check with competence and confidence in solving problems.
2. **Patterns, Functions and Algebra**: The learner is able to recognise, describe and represent patterns and relationships, and solves problems using algebraic language skills.
3. **Space and Shape**: The learner is able to describe and represent characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions.
4. **Measurement:** the learner is able to use appropriate measuring units, instruments and formulae in a variety of contexts.
5. **Data handling**: The learner is able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, as well as interpret and determine chance variation.

As in the case of the other learning areas, the mathematics learning area is based on the principles of high knowledge, high skills and integrates within mathematics and with other learning areas. It infuses concerns of human rights and inclusivity throughout the assessment standards.

There is however always a danger that there would be a lack of fit between the intended curriculum and the actual or implemented curriculum. This danger is of course very great in South Africa where the biggest challenges for implementation are the lack of resources and adequate teacher training, infrastructure and leadership capacity. Teachers implementing C2005 indicated that although they believed it to be beneficial to their learners and were eager to implement it, they were undermined in their efforts to do so in the absence of the necessary support.

Mathematics by All.

While mathematics for all is a statement of provision, *mathematics by all* is a statement of participation and a statement of mathematical engagement. If we are concerned only with provision of opportunity and the construction of mathematics curricula, without considering who is engaged in mathematics and how they are engaged, we will be giving ourselves a false sense of comfort. There is very little point in laying a table with the best food without inviting those around the table to participate in the eating and enjoyment that goes with it. There is a recognition that if we are going to effect change in South Africa, we have to accept that both "mathematics for all" and "mathematics by all" are essential ingredients of a transformation agenda. The focus in education generally has been shifting from provision and access to quality.

At the same time the educational measurement industry both locally and internationally has, with its narrow focus, taken the attention away from the things that matter and has led to a traditional approach of raising the knowledge level. South Africa performs very poorly on the TIMSS study. In the 2015 study South Africa was ranked 38th out of 39 countries at Grade 9 level for mathematics and 47th out of 48 countries for Grade 5 level numeracy. Also in the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), South Africa was placed 9th out of the 15 countries participating in Mathematics and Science – and these are countries which spend less on education and are not as wealthy as we are. South Africa has now developed its own Annual National Assessment (ANA) tests for Grades 3, 6 and 9. In the ANA of 2011 Grade 3 learners scored an average of 35% for literacy and 28% for numeracy while Grade 6 learners averaged 28% for literacy and 30% for numeracy.

Although these performances are pertinent in assessing educational quality of mathematics in the country, we have become pre-occupied with the political pressure to 'do better' and to improve our relative standing in relation to other countries using the comparative construct provided by these studies. In this process our focus has been fixed on the 'knowing of mathematics' instead of the 'doing of mathematics'. In our attempt to get teachers and learners to demonstrate knowledge we forget sometimes that teaching and learning are actions and that people rather than knowledge must be at the center. Mathematics by all is about changing the focus away from provision and compliance towards engagement and taking charge of our own mathematical experiences. This is not being reckless about the importance of knowledge but to see the key challenge facing mathematics teachers and learners as that to engage with the subject and to get them to believe that mathematical engagement could be part of their "possible selves".

Mathematics by all means that everyone is engaged in a quality mathematical experience. Quality of mathematical teaching and learning depends on whether the teacher can select cognitively demanding tasks and plan the learning experiences by encouraging learners to go beyond the "answer" to seek elaborations and generalisations whenever appropriate to do so through these tasks. This will require learners and teachers alike to commit to extra time on task and be engaged cognitively, socially and mathematically.

Allocating sufficient time for the learners to engage in and spend time on mathematical tasks in an already overcrowded curriculum presents a significant challenge. To address this challenge policy makers are currently in engaged in developing a new "*Mathematics Teaching and Learning Framework for South Africa: Teaching Mathematics for Understanding*" (Department of Basic Education, 2018). It is not intended to be a new curriculum but supports the implementation of the existing CAPS curriculum by introducing a model to help teachers change the way they teach. Taking its bearing from the work of Kilpatrick et al, (2001), the model of teaching mathematics has four dimensions: *conceptual understanding*, *mathematics procedures*, *learners own strategies* and *reasoning* while each of these takes place in a *dynamic classroom culture*. In addition the topics in the existing mathematics curriculum will be re-sequenced and even where necessary, removed to make space and time for deeper mathematics engagement.

While it is recognised that one of the major problems in mathematics education in South Africa is the level of teacher knowledge, it is felt that there has been too much emphasis on "teacher blame" when trying to explain the poor level of learner proficiency in mathematics. While teachers with strong content knowledge are more likely benefit from high level interventions and they therefore are more likely to lead their learners into richer mathematical experiences, strength in content knowledge does not always transfer to pedagogical knowledge. However we need now to go beyond this and ask what we can do within the current reality. To wait until teachers' knowledge has all radically improved would drive us into paralysis. Transformation of the classroom practice must begin with an enabling framework. Teachers' re-socialization into the new mathematics landscape envisaged in the new framework would have to start with unfreezing and deconstructing existing notions of working mathematically. The work Leone Burton (1999, 2009) and Jo Boaler (1998, 2002) illustrate how important it is for teachers to themselves be immersed in mathematical experiences that will give them an insight into the practice of mathematicians.

Conclusion

South Africa has a new set of values: *democracy, social justice and equity, equality, non-racism and non-sexism, ubuntu (human dignity), an open society, accountability (responsibility), the rule of law, respect, and reconciliation* are the ten fundamental values of our Constitution. The promotion of these values is seen as important, not only for the sake of personal development, but also for the evolution of a national South African character. These values have been infused in all learning areas and school mathematics in particular is expected to respect these values. The need is to develop a mathematics curriculum that will not only recognise the global competitiveness challenge by providing a platform for developing the knowledge, skills and competences to participate in an economy of the twenty first century, but also to show how our fundamental values can be lived out in our everyday experience while at the same time illuminating and exposing violations of these values. The mathematics curriculum reform in South Africa holds in tension the need to provide mathematics for all on the one hand, while creating opportunities to ensure that mathematics achievement is seen and experienced as part of the 'possible self' of every learner.

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MATHEMATICAL LABORATORY IN THE ITALIAN CURRICULUM: THE CASE OF MATHEMATICAL MACHINES

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This paper addresses the theme A (Learning from the past: driving forces and barriers shaping mathematics curriculum reforms). The focus is on the Italian curriculum reform, designed,at the beginning of the 21st century, by the Italian Mathematical Union – Italian Commission of Mathematical Instruction (UMI-CIIM). The emphasis on the Mathematical Laboratory, elaborated in the intended curriculum, which biased the institutional documents ("IndicazioniNazionali" or National Guidelines NG) issued by the Ministry of Education in the following yearsuntil now. After presenting the general features of the Mathematical Laboratory, the case of the Laboratory of Mathematical Machines is reported. The difficulties met for the implementation of this methodology all over the country are discussed.

INTRODUCTION: THE MATHEMATICAL LABORATORY AS A DRIVING FORCE FOR REFORM

> In mathematics, as in other scientific disciplines, the laboratory is a fundamental element, understood both as a physical place and as a moment in which the student is active, formulates his hypotheses and monitors the consequences, designs and experiments, discusses and argues his own choices, learn to collect data, negotiate and build meanings, leads to temporary conclusions and new openings the construction of personal and collective knowledge. In primary school it possible to use the game, which has a crucial role in communication, in the education to respect shared rules, in the development of strategies suitable for different contexts (translated by the authors, IndicazioniNazionali[i], 2012, p. 60).

The quotation above is taken from the Italian National Guidelinesfrom pre-primary to grade 8, issued in 2012 by the Ministry of Education. Laboratory activity is considered a general methodology not only for the scientific disciplines but for every subject matter as it is

> the working method that best encourages research and planning, involves pupils in thinking, creating, evaluating shared and participated experiences with others, and can be activated both in the different spaces and occasions within the school and by enhancing the territory as resource for learning (translated by the authors, IndicazioniNazionali[i], 2012, p. 35).

A recent (2018) document ("National Guidelines and New Scenarios") prepared by the Committee for the implementation of the National Guidelines[ii]has focused again the importance of the laboratory and, in particular, of the Mathematical Laboratory:

> the laboratory can also be a gym to learn how to make informed choices, to assess the consequences and therefore to assume responsibility, which are central aspects for the education to an active and responsible citizenship (translated by the authors, Indicazioninazionali e nuoviscenari[ii], 2018, p. 12).

In the NG for grades 9-13[iii], the wording "laboratory" is in general referred to the teaching of scientific disciplines: the spirit of the laboratory activity is maintained, with reference to ICT(Information and

Communication Tchnologies), to the history of mathematics, to mathematical modeling and, in general, to students' agency. In particular, ICT Laboratory has become popular thanks to the diffusion of the project M@tabel[iv]

This paper aims at reporting the features of the Mathematical Laboratory in the intended mathematical curriculum and one of the implementations with concrete materials, realized by the authors and acknowledged in the national context and in the international literature.

THE MATHEMATICAL LABORATORY IN THE UMI-CIIM CURRICULUM: HISTORY, GOALS AND VALUES

The history of the Mathematical Laboratory in the European culture is long and interesting (for a review, see Maschietto 2015) and witnesses the importance of the laboratory as a driving force towards the constitution of ICMI in 1908 (Borba and Bartolini Bussi, 2008). In Italy, at the beginning of the 21st century, a committee chaired by Ferdinando Arzarello and appointed by the UMI-CIIM (Italian Mathematical Union, Italian Commission on Mathematical Instruction) prepared a Curriculum for Mathematics for primary and secondary schools. Some volumes were produced and are available (in Italian) in the UMI-CIIM website[v]. A synthesis of these documents (in English) was prepared for the ICME10 (2004). In the methodological part a special emphasis was given to the Mathematical Laboratory. A webpage (in Italian) on the Mathematical Laboratory was created in the institutional website of UMI-CIIM for all the Italian mathematics teachers from primary to secondary schools[vi].

The short excerpts mentioned in the introduction from the NG draw on the larger text by UMI-CIIM (*Matematica 2003*[vii]), that reads

> A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together (without distinguishing between teacher and students). It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning cannot be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. The construction of meaning, moreover, requires also to think individually of mathematical objects and activities (Matematica 2003[iv], p. 26, translated by the authors).

Different examples of instruments are offered in *Matematica 2003*: e. g. "poor" materials, mathematical machines, dynamic geometry software, and original sources from the history of mathematics. Hence, in this vision, a Mathematical Laboratory is neither reduced to modelling nor to experiments; moreover it does not contain only computers. It is rather related to meaning construction, hence to genuine experiences of mathematical reasoning (e. g. formulation of definitions, production of conjectures, and construction of proofs). It aims at exploring the cultural values of mathematics by means of historical sources and problems.

In the quoted documents some methodological indications are offered, mentioning the importance of mathematical discussion and, generally, of semiotic activity, that are carefully described in the document produced for ICME10[v]. The document does not contain specific references, but the description hints at the theoretical frames developed at the beginning of the 21st century by research groups chaired by Bartolini Bussi (Bartolini Bussi & Mariotti, 2008) and Arzarello (2006).

In the following, an example of a Mathematical Laboratory for all grades (1-13) is illustrated. In the concluding remarks, some difficulties in the dissemination of such a model all over the country are reported and shortly discussed.

THE CASE OF THE LABORATORY OF MATHEMATICAL MACHINES IN MODENA: CONTENTS AND PROCESSES

The origin of the laboratory of mathematical machines

A specific Mathematical Laboratory had been implemented since the 1980s in Modena by a group of secondary school teachers who were collaborating with the local University in the *Laboratory of Mathematical Machines* (MMLAB) and constituted the no-profit *Association of Mathematical Machines*[viii].

The MMLAB contains a collection of some hundreds of wooden geometrical instruments (mathematical machines), that have been reconstructed in Modena drawing on original historical texts (Bartolini Bussi, 2017; Maschietto, 2017). A mathematical (geometrical) machine is a tool that forces a point to move or to be transformed according to a given mathematical law. Most of instruments date back to the 16th – 19th centuries (e. g. perspectographs, curve drawing devices, pantographs). There are also instruments from ancient Greece, linked to the theory of conic sections and to geometrical problem solving.

The MMLAB exploits the presence of both material artefacts and historical sources, as stated by the UMI-CIIM curriculum[iv]. Several activities are carried out: didactical research on the teaching and learning of mathematics by means of instruments; support for schools; pre-service and in-service teacher education; popularization of mathematics.

The diffusion of mathematical machines internationally

The first author of this paper was in the IPC of the ICMI STUDY 16th on "Challenging Mathematics in and beyond the Classroom". The first IPC meeting of the Study was held in Modena in the MMLAB (November 2003, Barbeau and Taylor, 2009). The first and the second author of this report took part in the Study Conference in Trondheim, Norway, in 2006. A case study on the Laboratory of Mathematical Machines, coauthored by them, was published in the Chapter 5 of the Study Volume (Bartolini Bussi et al., 2009). The activity of the MMLAB was also mentioned in the Chapter 2 of the volume, when the importance of raising public awareness about innovative educational practices was highlighted. Exhibitions in different countries were invited (a map is available[ix]). In 2004 the project Hands on Maths (presented by Bartolini Bussi and Turrini) was one of the six finalists in Paris at the Altran prix *Innover pour découvrir, comprendre et aimer les sciences* (Raichvarg, 2005). Journals for the general public in Italy and abroad published dossiers on the MMLAB. Also research collaboration on didactics of mathematics and teacher education crossed the borders. It is worthwhile to mention at least the collaborations with French colleagues (in Lyon[x]and at La Reunion Island[xi]) and with U.S. and Japanese colleagues (Bartolini Bussi, Taimina and Isoda, 2010; Isoda and Bartolini Bussi, 2009).Besides geometrical machines, nowadays the Laboratory of Mathematical Machines has been extended to include also arithmetical machines (e.g. pascaline Zero+1, Maschietto and Soury-Lavergne, 2013).

The diffusion of mathematical machines in Italy

Several travelling exhibitions have been realized in Italy by the MMLAB. Moreover small collections of mathematical machines have been built for schools and cultural institutions. A collaboration was realized with the "Piano Lauree Scientifiche"[xii], a national project funded by the Ministry of Education and Confindustria, the main association representing manufacturing and service companies, addressing 11th 12th 13th graders, with the aim of increasing the number of students enrolled in the scientific Departments of the Universities: a travelling exhibition was realized and allowed to organize Mathematical Laboratories in several secondary school in Southern Italy.

As far as teacher education is concerned, it is worthwhile to mention a 5 year long project funded by the Region Emilia Romagna aiming at creating a regional network of Mathematical Laboratories and of expert teachers in each province (Bartolini Bussi and Martignone, 2013; Bartolini Bussi et al., 2011; Maschietto,2015).

Two examples of systematicuse in secondaryschools

In this section we report two examples concerning 7-grade students (13-years old students) and 11-grade students (17-years old student). The former focuses on the Pythagorean theorem (Maschietto, Barbieri and Scorcioni, 2017), the latter deals with approaching conic sections (Maschietto and Turrini, 2012).

The two didactical projects, planned by a team of teacher-researchers and researchers, are developed within the theoretical framework of semiotic mediation (Bartolini Bussi and Mariotti, 2008); tasks for students are organized in didactical cycles with activities on mathematical machines in small group (GW – group work), individual tasks (IW – individual work) and collective mathematical discussions (CW – collective work).

The project on Pythagorean theorem, started in 2013, is based on the use of two mathematical machines: the first machine M1 (Figure 1, on the left) is composed of a square frame and four right triangles that can be moved inside the frame in order to form one square hole or two square holes; the second machine M2 (Figure 1, on the right) is a dynamic mathematical machine based on Leonardo's proof of the theorem.

The activities are structured as follows:

Phase A: 1) Exploration of the first mathematical machine M1 (GW); 2) sharing of the description of the M1 (CW); 3) construction of the M1 by paper (GW); 4) study of the possible configurations of the four triangles of M1 (GW); 5) representation of M1 on workbook (IW); 6) identification of relationships (invariants) between the components of M1 (CW).

Phase B: 7) History of the Pythagorean theorem and Pythagorean triples; 8) Generalization of the theorem by different puzzles (GW).

Phase C: 9) Exploration of the second mathematical machine M2 and its reproduction with paper (CW); 10) Preparation of posters on the two mathematical machines (GW).

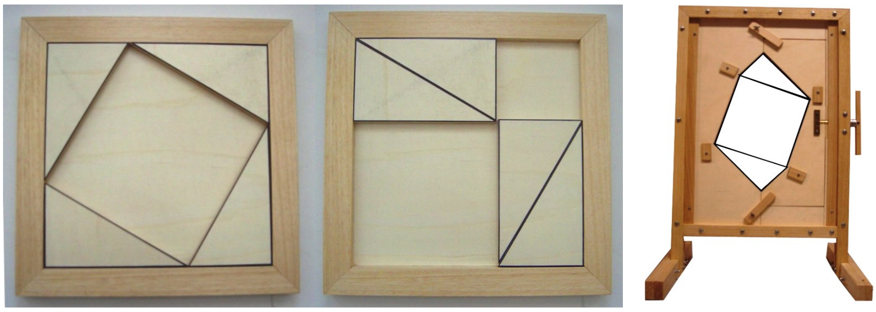

Figure1: The mathematical machines for the Pythagorean theorem[xiii]

The analysis of the laboratory sessions, carried out in several classes, allows to better inserting this laboratory project in the mathematical curriculum. For instance, a previous activity concerns the notion of equivalence of area by the use of the Tangram game. It also takes into account the institutional request, for instance the use of ICT: the teachers use the interactive whiteboard during the collective work.

The second example concerns the conic sections. The laboratory project was initially proposed in the second period of the school year to some classes of 17-years old student of scientific secondary school ("liceo scientifico"). Since it has become a project of the school, it is proposed to all the 11-grade students at the beginning of the school year. It is carried out by two teachers of the school, one of which is the third author of this paper.

Two important elements characterized this project. The first element is its coherence with the NG for secondary school[iii], thatread:

> The conic sections will be studied both from a synthetic and analytic geometrical point of view. Moreover, the student will deepen the understanding of the specificity of the two approaches (synthetic and analytic) for the study of geometry. He/she will study [...] the concept of locus, with some significant examples (translated by the authors, IndicazioniNazionali per ilicei, 2010, p. 87).

The second characteristic corresponds to the didactical choice of proposing all the conic sections from a unified perspective. This represents a kind of discontinuity with the classical Italian teaching based on the study of each curve in a kind of 'exhaustive' way (i.e., metric definition, problems, determination of tangents lines) in the analytical frame.

The project on conic section is structured as follows (Dondi, 2018):

Phase A (2 hours): Introduction to the mathematical machines by the exploration of Van Schooten's compas (composed of two bars connected by a pin at one end); some terms as parameter, variable, degree of freedom are presented.

Phase B (13 hours): didactical cycles with the use of two kinds of mathematical machines: curve drawers with tightened threads for ellipse (corresponding to gardener's methods to draw ellipse, Figure 2 on the left), hyperbola and parabola (Figure 2, on the right); curve drawers with crossed parallelogram for ellipse (Figure 3, on the left) and hyperbola (Figure 3, on the right).

Phase C (4 hours): lesson in which the definition of conic section using the director circle is also given; final test.

Our working hypothesis is to propose two kinds of mathematical machines[xiv] for conic sections: the curve drawers with tightened threads are used to support the definition process of the curves, while the drawers with the crossed parallelogram are used to foster argumentation and proof, and to introduce new definitions.

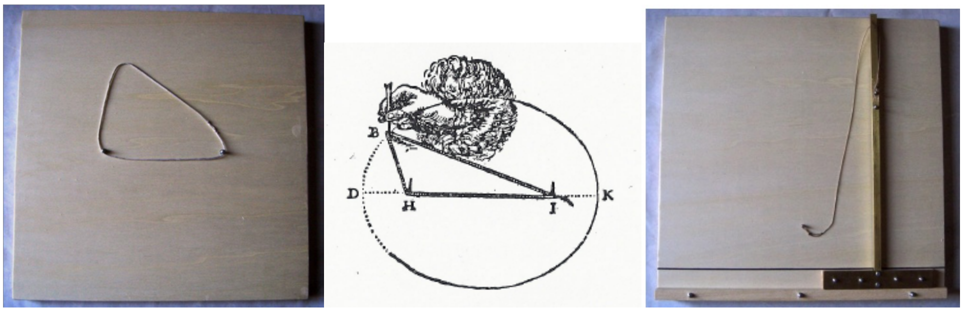

Figure 2: Conic sections drawers (ellipse and parabola)

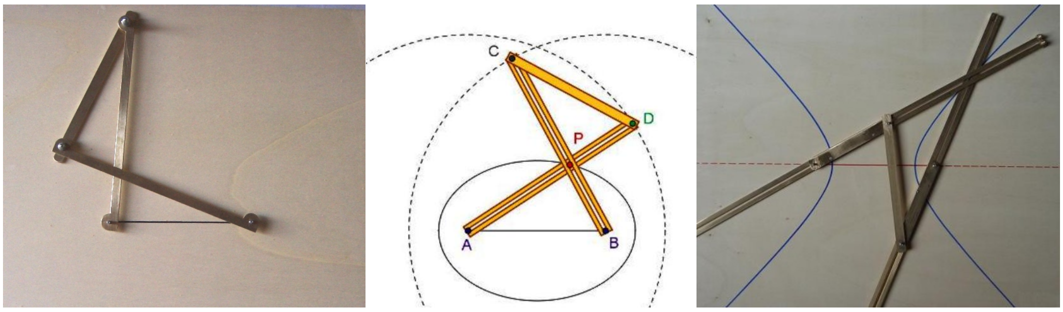

Figure 3: Conic sections drawers with crossed parallelogram (ellipse and hyperbola)

The analysis of the sessions shows students' difficulties in the defining process, and in particular, about the distinction between parameters and variables and the identification of the invariants useful for the definition. It also highlights students' conceptions of locus between static and dynamic conception, punctual and global points of view. Finally, the richness of the study of the curves drawn by the mathematical machines consists in involving other geometrical concepts, as reflection, sum and difference of segments.

CONCLUDING REMARKS: BARRIERS IN IMPLEMENTING THE MATHEMATICAL LABORATORY ALL OVER THE COUNTRY

In the previous sections we have reported the features of the Mathematical Laboratory in the Italian intended curriculum, as stated in different documents (the curricula prepared by the Italian Mathematical Union and the Italian Commission on Mathematical Instruction; the official documents issued by the Ministry of Education), together with an example of implementation in the MMLAB, that had surely some effects at a larger level. A question arises: to what extent is the idea of Mathematical Laboratory implemented all over the country, at all school levels. A report[ii] of the effects of the NG[i] in a large sample of schools (grades 1-8) has been prepared in December 2017 by the National Committee in charge of monitoring the experiments for all subjects. The conclusions are realistic and strongly support the need of a deep investment in teacher development:

> The National Guidelines have been accompanied by three years of assisted testing, with specific budget, that allowed the production of a document about the certification of skills. The school networks participating in the testing have produced meaningful reflection on the curriculum, on the didactical instruments, on the learning contexts. The national reports give an image of a lively research and debate,

> together with virtous innovative examples. Yet they give also an image of a persistence of situations of disorientation and uncertainty and of resistance to abandoning traditional didactic models of a prevalently transmissive type(translated by the authors, Indicazioni nazionali e nuovi scenari, 2018, p.3).

This document is just the starting point of a needed reflection on teacher development in the Italian schools. Teacher development had not been compulsory but realized on a voluntary base in the Italian system of instruction for decades. Only recently, for the first time in all schools, a mandatory three-year programme (2016-19) of teacher development has been issued[xv]. The issue of laboratory (including Mathematical Laboratory) needs to be a major focus of teacher development to overcome the transmissive attitude and to foster students' agency in the near future.

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[xv]https://labuonascuola.gov.it (accessed 2018/04/04)

THE EMERGENCE OF MEANINGFUL GEOMETRY: A REFORM CASE FROM THE NETHERLANDS

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This paper is about a change in geometry education that took place in the last century. We discuss the emergence of meaningful geometry in the Netherlands. Of course, this was not an isolated reform. Worldwide, mathematicians and mathematics educators came up with alternatives for the traditional axiomatic approach to teaching geometry. In the Netherlands, the pioneers were Tatiana Ehrenfest and Dieke van Hiele-Geldof. Freudenthal was a great promoter of their ideas and supported that from the 1970s on experiments were carried out to develop a new intuitive and meaningful approach to geometry education, with the focus on spatial orientation. How big the change in geometry education that resulted from these experiments was, is illustrated by comparing geometry problems from two Dutch mathematics textbooks: one from 1976 and one from 2002.

FROM AN AXIOMATIC APPROACH TO TRANSFORMATION GEOMETRY

Euclid's Elements is the stereotype source of inspiration for many textbooks on geometry. Its structured and axiomatic approach starts with defining points (which have no part) and lines (breadthless lengths) and problems with increasing complexity, such asproving that the angles at the base of an isosceles triangle are equal (Book 1, Proposition 5). However, for many students suchformal definitions and problems did not make much sense, since you can immediately 'see' properties like these. As an alternative, an approach to geometry was introduced based upon transformations.At the end of the 19th century, Klein started with this approach and inspired secondary schools in Germany to replace Euclidean geometry with so-called 'motion geometry' (see Botsch, cited by Barbin and Menghini, 2014), a simplified version of transformation geometry. In the transformation geometry students were involved in constructing and transforming shapes instead of onlyanalyzing given angles and triangles and reasoning about congruency. Although the importance of building on students' intuitions was emphasized, the formal axiomatic structure still played an important role.

PRECURSORS OF MEANINGFUL GEOMETRY EDUCATION

New developments towards a meaningful approach to geometry education with building on students' intuitions and paying attention to the development of spatial insighthad already been proposed early in the 19th century. In particular Fröbel and Montessoriwere important driving forces for a meaningful approach to geometry education. In the Netherlands, it was Tatiana Ehrenfest-Afanassjewa (1876-1964) who contributed significantly to introducing an approach to geometry education with attention for the development of spatial insight. She was originally from Russia and lived in the Netherlands for a long time from 1912 on. Ehrenfest had a great interest in teaching and education and gave this interest a practical expression by organizing monthly mathematical-didactical colloquia for teachers at her house. Here, spirited discussions were held about the, in her view, fossilized mathematics education in the Netherlands (La Bastide-van Gemert, 2015). Among other things, she developed an introductory geometry course with exercises in spatial geometry. In this so-called *Übungensammlung zu einer Geometrische Propädeuse*(Ehrenfest-Afanassjewa, 1931) she took geometrical phenomena as a starting point for developing geometrical concepts. With this course, she enriched the domain of geometry with how we experience space. Ehrenfest considered activities of looking along two objects, identifying parallel lines in a classroom and lines as light beams and determining angles, basic for an intuitive understanding of the straight line as a mathematical object. In her introduction of the course she motivated the importance of such a phenomenological introduction by contrasting it with the geometrical method that emphasizes a logical-deductive approach.

Halfway through the 20th century, a further impetus to change geometry education in the Netherlands came from the couple Pierre van Hiele (1957) and Dieke van Hiele-Geldof (1957) who proposed introductory activities with concrete materials like folding, cutting, gluing, and paving. Through these activities, students became acquainted with the geometrical objects and with fundamental notions of concepts such as right angle (defined by folding). During subsequent analyses of the objects, other characteristics, patterns and symmetries were identified and relationships were constructed (Van Hiele-Geldof, 1957). This means a learning process which completely differed from starting with a deductive structure of mathematics.

Freudenthal was also fond of Ehrenfest's Übungensammlung. For him the relevance of her work was her plea for a resource-based approach to teaching geometry and for the need for an explorative and student-oriented approach to geometry which can be described as 'watching, acting, thinking and seeing'. Geometrical experiences start with the observation of a phenomenon in the surrounding environment. After that a model or a drawing is made to describe the phenomenon with geometrical means. Reasoning about these means is meant to understand the modelled phenomena. Freudenthal labelled the research underlying these activities as didactical phenomenology, and he summarized the resulting geometrical experiences and activities more concisely with the term 'grasping space' (Freudenthal, 1973).

EARLY EXPERIMENTS FROM 1970 TO 1980: THE FOCUS ON SPATIAL INSIGHT

From the 1970s on, experiments were carried out to develop a new intuitive and meaningful approach to geometry education (De Moor & Groen, 2012; Groen & De Moor, 2013). These experiments were carried out in educational practice through working with teachers and students in real classrooms. Initially, the plan was not to build a learning pathway for geometry, but to look for themes and problems that result in meaningful mathematical activities.The designers of this new approach to geometry were focused on developing the students' understanding of and skills in working with traditionally familiar subjects such as angles, area, symmetry. The intention was to find empirical support for a phenomenological approach to these subjects. To highlight the new character of the geometrical activities, the term 'vision geometry' was used. The experiments in class were aimed at the development of reasoning with vision lines, vision angles, sighting, rays of light, projecting, shadowing and perspective. In particular, this latter subject, perspective, was considered to play a central role in learning basic geometrical concepts and reasoning and the development of a deeper spatial insight. The setup of the designs was not axiomatic but based on phenomena and experiences in daily life.The two examples that follow now are tasks designed and tried out during the years 1970-1980. They reflect the importance of starting with three-dimensional problem situations to evoke and further develop meaningful geometrical reasoning.

The singer

The task 'The singer' (Figure 1) was developed, tried out and finally published in a geometry unit for lower secondary education (Schoemaker, 1980). The task is about a singer whose performance is filmed by four cameras. Students can explore the way an object is seen from a certain viewpoint, which is one of the core ideas of the geometry of vision. Because the task fits well within the range of student's daily experiences and intuitions, there was not much need for further explanation. The experiment confirmed that the task is easily accessible for students. Not even answering one question was needed to put students to work. They started connecting cameras with the images displayed on the four screens in the control room and they easily determined which camera saw the back of the singer and which had the slightly less decent look at the armpit. Deciding which of the cameras were responsible for the two other images required more advanced reasoning, but the two easy images gave the students a good basis to find which cameras went with the remaining images.

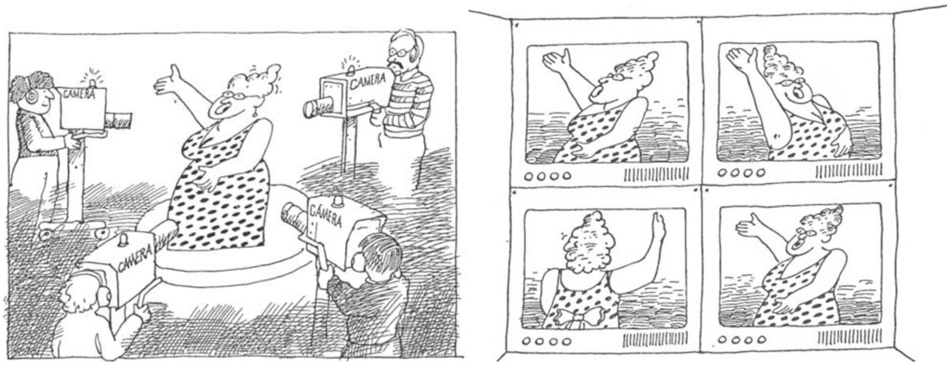


Figure 1: Which image comes from which camera? (Schoemaker, 1980, p. 24)

Tower and bridge

The 'Tower and bridge' task (Figure 2) further elaborates the need for constructing vision lines and reasoning with these lines when a situation is shown from another view. This task was used in an experiment for introducing scale and geometrical reasoning in a 3D context (Goddijn, 1979; Schoemaker, 1980) and was meant to create opportunities for students to recognize the connection with situations in reality, question them, and use geometry to explain phenomena.

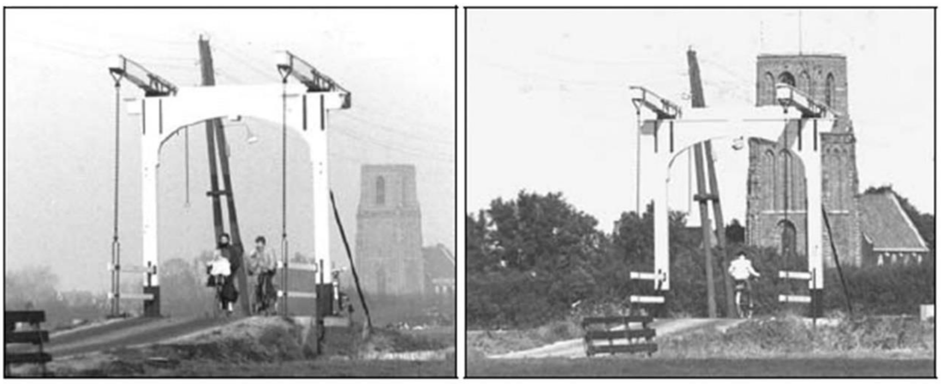


Figure 2: What is higher, the tower or the bridge? (Goddijn, 1979, p. 2)

In the left picture, the bridge seems higher than the church, while in the picture on the right the church is higher than the bridge. By constructing a side view of this situation and drawing triangles based on vision lines students can explain this phenomenon and argue that the church must be higher than the bridge. This example shows again how the teaching and learning of geometry can be a constructive and creative activity and that the geometry that focusses on grasping space starts with looking, analyzing and creating drawings like top views or side views and the vision lines as tools for explaining phenomena of vision.

What these examples have in common

Both examples show an approach to geometry education in which fundamental geometrical insights are strongly connected to phenomena that students can experience in everyday life. The tasks that were used for developing these insights are characteristic for what can be called 'Realistic Geometry Education'. More specifically this approach to geometry education implies:

- Starting with 'realistic' problems
- Considering students as active and creative explainers of problems
- Giving students opportunities for explorative activities through which they can further develop their geometrical intuitions and by which preliminary constructions can emerge
- Eliciting mathematization in students by focusing on the development of 'situation models' like vision lines which bring the students from the informal to the more formal geometry.

We can conclude that these characteristics are in line with the ideas of Ehrenfest-Afanassjewa (1931). In her introductory geometry course with exercises in spatial geometry she also tried to have students develop geometrical concepts from their own living experiences and to prevent that students would work with names and drawings that do not refer to something they know.

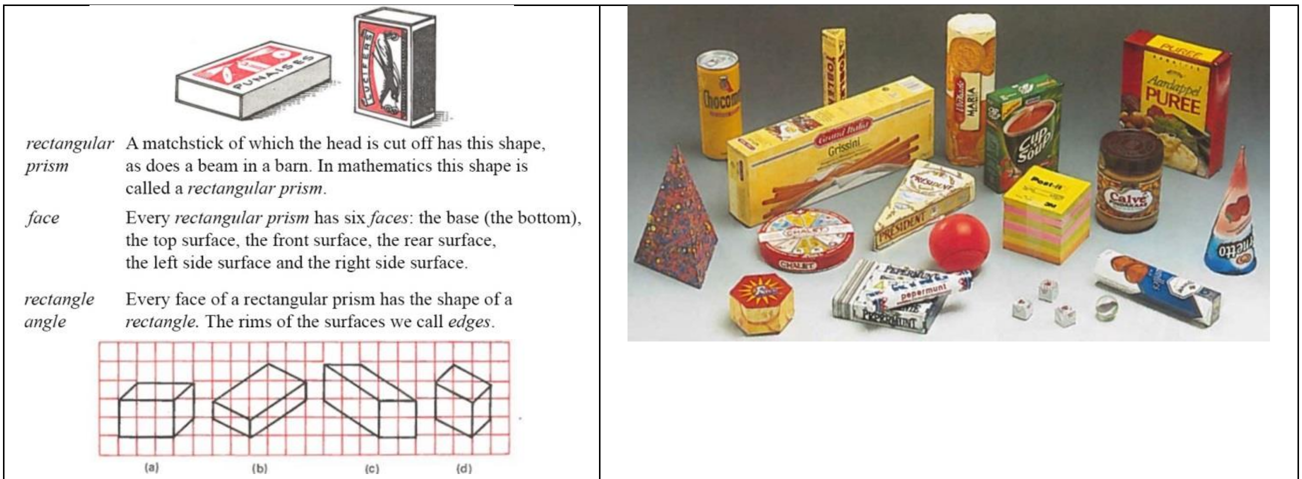
A CHANGE IN GEOMETRY EDUCATION: GEOMETRY PROBLEMS IN 1976 AND 2002

The experiments that have been carried out since the beginning of the 1970s gradually brought about a big change in geometry education. Instead of the then prevailing approach to geometry that started immediately in the world of mathematics by defining geometrical properties right from the beginning, geometry became now a discipline that was no longer isolated from the real world. Connections were made to the daily life situations of students and definitions were put at the end. For example, in the 1990s, an important attainment target for the lower grades of secondary school was formulated as: students can interpret, describe, spatially imagine and create two-dimensional representations of spatial situations, such as photos, sewing patterns, maps, plans, and blueprints (Ministry of Education, 1997).Only for students in the science-oriented track in upper secondary education the attainment targets includedmore formal geometry. These students should be able to understand the difference between a definition and a theorem, and proof conjectures using properties of straight lines, circles, triangles and quadrilaterals.Although, these attainment targets gave guidance to the geometry reform and the experiments provided indications for how to reach these targets, in some cases it was difficult for textbook authors to distinguish between the goals in the curriculum and didactical tools. Therefore, it could happen thatin some textbooks reasoning with vision lines was introduced as a didactical tool, while in others the concept of a vision linewas understood as a goal of the curriculum.

Despite differences in the interpretation of the spirit of the reform intentions, the changes inthe teaching of geometry come clearly to the fore when a textbook series from1976 is compared with a more recent one from 2002. The first textbook series is *Moderne Wiskunde voor Voortgezet Onderwijs* written by Jacobs et al. (1976). The second one is the series *Moderne Wiskunde*written by

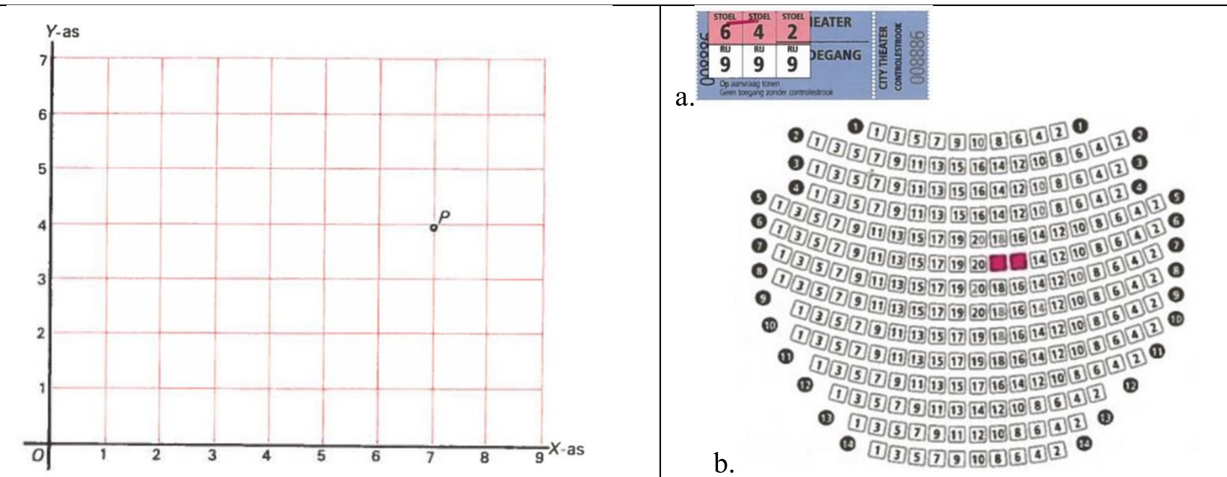
Van de Eijk et al. (2002). For the comparison, we took the books for Grade 7, which are meant for the first year of secondary school, and we chose the topics: (a) introduction to 3D shapes, (b) location, in particular the introduction of coordinate systems, and (c) reasoning with lines and angles. Due to space limitations, we can only give a few examples which never will fully do justice to the two carefully designed textbook series. Nevertheless, the three examples we provide give a clear impression of the changes that have taken place at the end of the 20th century in the Netherlands.

The first example is about 3D shapes. As a start for this topic, in the 1976 textbook, the students are shown drawings of two kinds of boxes (Figure 3). The drawings are used to introduce the mathematical terms that describe the elements of 3D shapes (faces, vertices and edges) and their characteristics. One of the assignments for the students is to list the edges that are parallel to each other and draw the mathematical shapes on grid paper. In contrast, the 2002 textbook focusses on providing opportunities to students to explore and analyze shapes that they can see in daily life. Students are stimulated to figure out all kinds of characteristics of the shapes. For example, which objects can roll and what are the similarities and differences between the sides of each of the shapes?



Jacobs et al., 1976, p. 7 | Van de Eijk et al., 2002, p. 166

Figure 3: Introduction to 3D shapes



Jacobs et al., 1976, p. 107 | Van de Eijk et al., 2002, p. 64

Figure 4: Introduction to coordinate systems

The second example is about the topic of location. Figure 4 shows how differently coordinate systems are introduced to students in 1976 and in 2002. In 1976, the idea of a coordinate system is posed as a way to organize a plane presented as a grid.

The accompanying textbook text introduces the students to the language of a coordinate system:

> Start counting from the origin: first seven lines to the right, then four up. We arrive at point *P*. […] The pair of numbers (7 , 4) are called the 'coordinates' of *P*.

Next, they have to locate other points following a similar recipe of counting lines to the right and up starting at the origin. In the 2002 textbook, the introduction to coordinate systems is preceded with activities that are connected to the need for such systems. Students are provided with problems in which they can use a coordinate system for reasoning about locations in daily life situations. The context of seating people in a theatre is used. The students are asked (a) to figure out where the seats are when you have bought tickets that tell you the chair number and the row number, and (b) to determine what information will be on the tickets when you are seated on the two colored locations on the floor map of the theatre.

The third example illustrates the differences between the introduction in both textbooks of reasoning with lines and angles. In the 1976 textbook (Figure 5, left), the students have to explain that triangles ABC and CDA are congruent. In the 2002 textbook (Figure 5, right), the topic of reasoning with lines and angles has changed into reasoning about vision lines and angles starting in 3D contexts. Doing geometry is not limited to reasoning with lines and angles in the plane but can also start with spatial situations that refer to reality. The students are provided with a picture showing the top view of a room in which a boy and a girl are sitting and showing a garden where there is a cat and two birds are flying around. In the room, there are two windows. The girl who is sitting on a sofa warns that the birds are in danger, but the boy does not understand her. The students are asked to explain this. The purpose of the problem is to introduce students to a situation which they can 'organize' with geometrical means. The students are asked to construct top and side views and to draw vision lines and angles in them that can be used to explain what is seen and how it is seen in reality.

Jacobs et al., 1976, p. 126 | Van de Eijk et al., 2002, p. 14

Figure5: Reasoning with lines and angles

The 1976 and 2002 textbook also differ in how the topics are ordered. The 1976 textbook starts with teaching the names of 3D shapes on page 7 (Figure 3). Many pages later, on page 107 (Figure 4), this is followed with introducing coordinate systems and finally, from page 126 on (Figure 5) reasoning with lines and angles in the plane is addressed. The sequence in the 2002 textbook is the other way around. Here, the introduction to reasoning with vision lines and angles is situated in the beginning of the textbook, on page 14 (Figure 5). Later, on page 64 (Figure 4), coordinate systems are introduced with reference to coordinate systems in various real situations. Only in the end, on page 166 (Figure 3), spatial shapes are explored and geometrical terms for describing these shapes are introduced.

Although in the 2002 examples many of the original ideas for a more meaningful approach to geometry education that were developed in the years 1970-1980 can be recognized, the ideal of geometry as a real constructive activity appeared to be difficult to implement in textbooks. Working with rather closed tasks in textbooks is more feasible than having open tasks that ask for classroom experiments and discussion. Take, for example, a task that deals with the concept of vision angle. Getting a good understanding of this concept requires that it is really experienced through a whole-class activity and interactive discussion in which so-called 'why-questions' are asked. However, such questions are often missing in textbooks. Also, in class, attention is seldom paid to reasoning with vision lines and demonstrating their use.

The task that mostly reflects the ideas behind the experiments that started in the 1970s is the task on the right in Figure 5, where the students are provided with a top view of a room and an adjoining garden where birds seemed to be in danger. The power of this task is that the students are offered the opportunity to geometrically organize the situation to understand what is going on. According to Freudenthal (1971), this so-called 'local organization' is the way to develop the concepts and reasoning schemes and has the potential to create the need for axioms, definitions and a logic-deductive system.

FINAL REMARKS

In this paper, we have tried to shed light on the change that took place in the Netherlands in which an axiomatic approach to teaching geometry was gradually superseded by an intuitive and meaningful approach focused on spatial orientation. Characteristic of the reformed approach, that in the Netherlands later became known under the term 'Realistic Geometry Education', is that students are introduced to the world of geometry (the language, the objects and the constructions) by providing them with tasks in 3D contexts that can elicit their intuitive geometrical reasoning. Starting geometry education by developing spatial intuition and 'grasping space' was very much supported by Freudenthal (1973) and is exactly at the heart of the ideal of Ehrenfest-Afanassjewa (1931). The result of this reform is that in the Netherlands geometry education nowadays mostly starts with an intuitive introduction in primary school (see, e.g., De Lange, 1986; De Moor, 1991; Van den Heuvel-Panhuizen & Buys, 2008), which continues with a context-rich course for 12 to 16-year olds in which geometry is a bit more formalized (see, e.g., Goddijn, 1991), and ends in working with definitions and axioms, that is, reflecting on geometry as a deductive system, by the end of secondary school (see, e.g., Goddijn et al., 2014).This means that the traditional deductive structure of the geometry trajectory which started with formal definitions and basic axioms has been reversed. Instead of taking the final state of the work of mathematicians as a starting point for teaching geometry – what Freudenthal (1973) called an 'anti-didactical inversion' of learning sequences – these definitions and axioms come now at the end of the trajectory.

Acknowledgements

The topic of this paper is inspired by the work of Ed de Moor, Wim Groen and Sieb Kemme about the development of geometry education in the Netherlands.

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AN OVERVIEW OF CHANGES IN SCHOOL MATHEMATICS CURRICULUM IN IRAN

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In the latest wave of curriculum change in Iran, changes occurred in school mathematics curriculum was the most salient one. This change not only includes the underpinning theoretical approaches but also the content and pedagogy. The full implementation of this new mathematics curriculum took eight years. In this report, we first give a big picture of education system in Iran, and the background of the mathematics curriculum within it. The study shows there is no single driving force behind the mathematics curriculum changes during the last 100 years; however, political determination is more visible. Also, any extremism for curriculum change and ignorance of local values and cultural context may lead to big challenges.

INTRODUCTION

Starting from 2011, school mathematics curriculum in Iran, has undergone radical change, including approach, content organization, and context. This sudden decision for change caused the Iranian education system to face a number of unexpected challenges; those that might explicitly or implicitly effect school mathematics curriculum in other situations as well. The forces behind these challenges do not necessarily educational in nature, yet have enough power to distract the direction of change.

To better understand the change process in recent years in Iran, we first give a short overview of the formal education system in this country and then, examine the driving forces behind the several mathematics curriculum changes that occurred until now. Next part devotes to the characteristics of the new curriculum. Finally, the paper ends with the concluding remark about the place of theory in curriculum design and the necessity of taking into account the cultural, societal, and values at the local level, to modify global theories to fit the local situations.

AN OVERVIEW OF THE FORMAL EDUCATION SYSTEM IN IRAN

The formal education system in Iran was established in 1920 (Gooya, 2010a). To serve the purpose of this paper, we only highlight some of the main characteristics of the formal education system in this country from its beginning to the present.

The first education system in Iran adapted from French education system that was highly centralised. The centralization comprised of all aspects of mathematics curriculum and since 60's, there is only one single national textbook for each school subject, which mainly could be considered as curriculum guide as well (Gooya, 1999.)

In general, schools are segregated in Iran from Grade 1 to 12; with some exception including rural and nomad schools, due to the teacher shortage and difficult geographical accessibility to resources, to name just a few.

In addition, the schooling in Iran has always been divided into a number of branches and different strands within each. During the time, various reforms brought about different changes to this division, but the essence had remained in place to the present. Further, in curricula of all divisions, mathematics has always had a central role (Gooya, 1996.)

A HISTORY OF MATHEMATICS CURRICULUM CHANGES IN IRAN

Each education system has been and will be driven by various forces that are not necessarily rooted in education per se (Furinghetti, Matos & Menghini, 2013). On the contrary, there are inevitably a number of different obstacles—especially at the intended and implemented curriculum levels—that need more attention. What follows is a glance of the mathematics curriculum change in Iran from the beginning to the present to help us extract the main driving forces in mathematics curriculum changes in this country (Gooya, 1993.)

First Mathematics Curriculum

Along with the establishment of the education system of 1920, there was a document with description of purposes and scopes of mathematics subjects, suggested teaching approach, and "limited examples" to help teaching and assessment of the subject. In addition, that document emphasised the syllabi for every grade (Gooya, 2010b). Another important feature was adapting the "social utility" approach for selection of the content as well (Bidwell & Ciason, 1970). As an example, in the very first mathematics curriculum, some content were included specific skills that the traditional workforce and the new bureaucracy, asked for. For instance, traditional Iranian accounting system called "siagh[1]" arithmetic and teaching base 10 abacus was included in the mathematics (arithmetic) curriculum, as well as simple concepts of banking and book keeping (Gooya, 2010b). Finally, the pedagogy was mainly teacher centred, based on written work, and drill and practice. Overall, the major driving force of the first mathematics curriculum was political; believing that joining the international community and moving towards modernisation, is not possible with illiterate society at large.

First Major Mathematics Curriculum Change

The first mathematics curriculum of the formal education system went through various contextual and content modifications, to accomplish the societal needs and political wills (Gooya, 2010b). After the World War II and the appearance of the "new math era" at late 1950's, many education systems, despite their different cultural and social backgrounds and needs, adapted "new math" curriculum. The approach of the "new math" was to move towards "internationalised" mathematics curriculum or as Bishop (1990) and Clemens and Elerton (1996) have pointed out, an implicit form of modern "colonisation". Nevertheless, the history of mathematics curriculum has gathered many convincing evidences to show that "internationalised" school mathematics curriculum is more at the theoretical level than real world of schools which means, "neutral" or "value and culture free" mathematics curriculum cannot exist in practical world of schooling (Chevallard, 1988; Bishop, 1997). For instance, despite the "internationalised mathematics" of "new math", the new reform in Iran was not limited to the imported approach of the "new math" and instead, the content, content organization and pedagogy adapted were influenced by the traditional and national style to some degree. To give an example, within the "new math" curriculum, still two parts of the Iranian mathematics curriculum,

[1] "Siagh" is a special counting system that instead of numbers or symbols uses "words" and until 20 to 30 years ago still was used by some professions such as traditional trading.

namely "Euclidian geometry" and "trigonometry", remained in the same traditional manner, with separate national textbooks for each. To conclude this part, the major driving force in this reform, was international hegemony for that and Iran was not an exception.

Mathematics Curriculum Change after Revolution

One of the most visible factors behind school mathematics curriculum change after the revolution of 1978 was the limited number of students who chose the mathematics-physics strand of the theoretical branch at the upper secondary level. Many factors contributed to the low percentage of students at this branch that the chief factor was shortage of mathematics teachers.

In 1984, the education system announced that the low percentage of students (6.2%) entering into mathematics- physics strand in schools is alarming (Parvaresh, 1984). To improve this situation, a number of measures were taken and among them, a special program was launched for gifted students in mathematics from 80's, to prepare the "elites" for national and later, for the International Mathematics Olympiad (Hadadadel, 1984; Parvaresh[2], 1984). The history showed that this program has been extremely successful and served its purpose (www.irysc.com.)

Numerous extra evidences convinced the decision makers that in addition to the shortage of mathematics teachers, the chief force behind this situation has been "new math" curriculum that its target population has not been "all" students, and was mainly designed for "elites" (Clements & Elerton, 1996). Therefore, the majority of students, showed resistance to that and many of them avoided mathematics- physics strand, at the senior secondary. On the other hand, the political situation of post revolution brought new expectations for change, including education in general and curriculum in particular.

The social readiness and the new political establishment were two main driving forces for another major mathematics curriculum change. By analysing the situation, the mathematics community came to the conclusion that the "new math" curriculum, was not suitable for "all" students; noting that the country was in "baby blooming" stage and its student population was fast growing (Statistical Center of Iran). Also, for the first time in the formal education in Iran, mathematics education as an academic field came to the stage to take responsibility for the mathematics curriculum along with those mathematicians who were concerned about school mathematics[3]. This opportunity, created an environment for more meaningful collaboration between officials, mathematicians and mathematics educators, with the assistance of experienced mathematics teachers. In this atmosphere, the expectation was to design a more meaningful curriculum by looking at the cultural, societal and national needs from the one hand, and try to make a well- rounded integration between new findings in the mathematics education field at the global level, and having better understanding of the local potentials to design a whole new curriculum, on the other hand.

With this vision, a number of study groups was organized in annual mathematics conferences to study the above- mentioned issues. These activities helped the mathematics community, to become

[2] The Minister of Education in that time.

[3] The school mathematics council of the Ministry of Education, consisted of mathematicians, mathematics educators, mathematics teachers and mathematics curriculum developers . The council's agenda and decisions could be retrieved from the formal site of the Ministry of Education.

sensitive about the important issue of mathematics for all and mathematics for elites. Meanwhile, because of the political urgency and population explosion of 80's, the officials decided to start mathematics curriculum change from senior secondary.

The first round of change started in 1992 covering 10% of volunteer students as first trial of the new curriculum, while the current program was in place for remaining 90%. The process of change of the secondary mathematics curriculum took place during seven year period, until the current curriculum replaced by the new one in 1998.

The approach of the new secondary mathematics curriculum was based on the ideas taken from constructivism that required pedagogical change from teacher-centred to student- centred. It was also vital to use the potential of "integrated approach" to mathematics curriculum from different perspective; both local and global. Taking this approach, the mathematics curriculum of the first year senior secondary was designed and developed for all students and included two national textbooks; Mathematics and Geometry1. After the first year of secondary, students had to choose their branch and strand. So the mathematics textbooks[4] of each strand were written based on the essence of that.

The major change happened in the mathematics curriculum/ textbooks for Human Science strand. The focus of this curriculum was mathematics for those, who had not much experience of enjoying and seeing the usefulness and applicability of mathematics to their own field. Two mathematics textbooks were written for Grade11 and pre- university (Grade12) of Human Science strand. In specific, the Grade12 textbook started with different kinds of reasoning and continued with selected topics including sequences, logarithm, mathematical modelling, and probability. The content organisation was based on the applicability of these concepts. The main purpose of this curriculum was to provide students with opportunities to experience the beauty and usefulness of mathematics in practical sense. For instance, in mathematical modelling, the focus was on optimisation, marketing, growth and decay problems, without using calculus. In addition, most of the topics were presented in forms of activities within cultural context.

The first year of mathematics- physics and strand (Grade10), had two mathematics textbooks as Mathematics2 and Geometry2. The 2nd year (Grade11), had two textbooks including "Pre-calculus" and "Algebra and Probability". The integrated approach in the latter one showed that how deterministic and stochastic aspects of mathematics are related. The curriculum design of this level led to preparing students to slowly move towards more concentration at the pre- university (Grade12) level. At the exit year, three mathematics textbooks were written for mathematics – physics strand as "Differential and Integral Calculus", "Linear Algebra and Analytic Geometry" and "Discrete Mathematics". The discrete mathematics textbook was the meeting place of some modern mathematics topics into the secondary program. This was the first place that students formally presented with graph theory, number theory, combinatory and probability.

These changes required many training sessions for mathematics teachers nationwide. The new approach of integrating different mathematics fields together, focusing on problem solving, and asking for students' involvement in teaching - learning process, needed a great endeavour from teachers. The new generation of mathematics teachers - both male and female – welcomed these

[4]In this paper, we intentionally use "curriculum" and "textbook" as two names for one. Since by that time, there was no formal mathematics curriculum and national textbooks, served this purpose.

changes more than older generation of teachers, who resisted it more strongly (Gooya, 2007). It is worth to mention that the majority of the secondary mathematics teachers had BSc. Degree in mathematics.

Latest Mathematics Curriculum Change

At the beginning of the 21st century, the Ministry of Education decided to prepare a series of documents to declare its new policies for education at the national level. This decision led to production of "national curriculum" in early 2011 (Gooya, 2010c). The new structure of education system[5] came along with this document. These new developments, paved the way for revising all school curricula including mathematics.

In mathematics curriculum change, one of the driving forces was the mathematics performance of the Iranian students at the TIMSS- 2007[6] that was much lower than what the education system expected. Along with this, a new tendency was shaped at the policy- making level to look at the factors contributing to the school mathematics curriculum of the "successful" countries as well. For this purpose, the great effort was made to combine almost all elements of different successful mathematics curricula from around the world, including theoretical foundations, research findings, national innovations and international trends. Therefore, the approach to presenting mathematics content has had drastic change from Grade1 to Grade12. The new pedagogical and curriculum approach is extremely gear towards students' activities and using "real world" contexts for almost every mathematics concept and skill. The organisation of the curriculum in the new mathematics textbooks consists of "activity", "seatwork" and "exercise". However, in many cases, there is no clear distinction between each of the three.

In fact, the main characteristics of the new curriculum reform as declared by writers of new mathematics curriculum and new mathematics textbooks is the emphasis on learning mathematics by activities and using various representations and real – life contexts. In addition, in Grades 2 to 7, "problem – solving strategies" is included as separate sections in the textbooks.

Implementation of the New Mathematics Curriculum

As was explained earlier, in education system in Iran, usually the textbook is used as curriculum guide. Thus, the new change began with writing new mathematics textbooks. The process of writing took eight years from 2011 to 2018. As shown in Table 1, the change process has not been linear. In 2011, the first draft of the new textbook for Grade1 was implemented; in 2012, Grades 2 and 6; in 2013, Grades3 and 7 textbooks and so on. It is worth mentioning that at the senior secondary level (Grades10 to Grade12) there are 14 different mathematics textbooks for three strands of the theoretical branch; one national textbook for every mathematics subject (course). As a result of this change process, 23 new mathematics textbooks have been designed, written and implemented.

2011	2012	2013	2014	2015	2016	2017	2018

[5] The new educational structure replaced the old one that was five years elementary, three years Guiding Cycle equivalent to junior secondary and three years secondary and one-year pre- university. The new structure is six-year elementary, three years junior secondary and three years senior secondary.

[6] Iran has constantly participated in TIMSS since 1995 to 2015.

G1	G1	G1	G1	G1	G1	G1	G1
	G2	G2	G2	G2	G2	G2	G2
		G3	G3	G3	G3	G3	G3
			G4	G4	G4	G4	G4
				G5	G5	G5	G5
	G6	G6	G6	G6	G6	G6	G6
		G7	G7	G7	G7	G7	G7
			G8	G8	G8	G8	G8
				G9	G9	G9	G9
					G10	G10	G10
						G11	G11
							G12

Table 1: Timetable for implementation of newly written mathematics textbooks

Because of the rush for full implementation within a short period of time, due to a political will, it was decided to do the parallel evaluations of Grade1 (Kabiri, 2011), Grade2 and Grade6 (Kabiri, 2012), Grade3 (Kabiri, 2013) and Grade 7 (Gholamazad, 2013) at the intended and implemented curriculum levels. This decision was made to partially compensate for the lack of the trial implementation of the newly written textbooks. The findings of these evaluations were used to revise the first drafts of these textbooks.

The overall findings of these evaluations identified major challenges for school mathematics education regarding this change. Some of the identified challenges are as follows (Gholamazad, 2015):

- The lack of a comprehensive curriculum document for Grade 1 to 12 mathematics, to guide school mathematics activities in a coherent and consistent manner;
- The inconsistencies caused by combining different and even sometimes contradicting theories or paradigms;
- Over-emphasis on problem solving and problem posing approach to curriculum and devotion of a separate chapter or section to it, in most of the textbooks;
- Superficial use of research findings in producing textbooks;
- The inconsistency between text books‘ activities and students‘ real experiences;
- The lack of attention to the great cultural, ethnical and societal diversities within Iranian context;
- Individual preferences of some authors in choosing new approaches that are not supported neither by research findings, nor by teachers‘ experiences;
- Teachers‘ distrust towards new curriculum/ textbooks;
- The national implementation of the newly written textbooks with no trials at all.

Considering these challenges, there is an speculation that the Ministry’s offficials are willing to stop changing mathematics curriculum and textbooks for a period of time. Instead, asking researchers to follow up on the above and other related studies, and to take these challenges seriously.

Conclusion

Since the establishment of the formal education system in Iran, mathematics has been an indispensable part of school curriculum across all grades. Although mathematics curriculum during the last century, have gone through various rises and falls due to the different driving forces. In this study, among main driving forces for mathematics curriculum reform, we have addressed political determination, international waves, international studies (TIMSS[7]), theories of learning, and new research findings in mathematics education and mathematics per se.

The results of the latest document analysis show that the mere political determination is not enough to have sustainable, authentic and implementable new mathematics curriculum. In addition, we have envisioned that in education, there is no single theory that could be adapted as one entity and expected to produce reliable result. Theories help to have more clear vision, but it cannot pave a royal road to success. We need to modify theories and approaches based on cultural, values and facilities of each locality. The message of this paper is clear that there is no single driving force to change; however, political determination is more visible. Nevertheless, the important message for the international community is that the global perspective and local production is different from "internationalisation" of mathematics curriculum in which, considers school mathematics as "culture" and "value" free. However, this does not mean that a mathematics curriculum should excessively use local contexts for presenting every concept and building skills in students. Any sort of extremism in mathematics curriculum design brings about a heavy and sometimes very costly load on education system of every country. Therefore, our clear message is to avoid "radicalism" or "extremism" for curriculum change and choose a moderate approach to include local culture and tradition, as well as being connected with the global scene and international research findings.

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VARGA'S "COMPLEX MATHEMATICS EDUCATION" REFORM: AT THE CROSSROAD OF THE NEW MATH AND HUNGARIAN MATHEMATICAL TRADITIONS

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In this article we present a historical and didactical analysis of the Hungarian Complex Mathematics Education reform led by T. Varga in Hungary during the 1960s and 1970s. We will show how this reform was formed by the international New Math movement and by local conditions, especially by a local mathematical culture. We will also emphasize the internal coherence of the reform and describe some main elements of the underlying conception on mathematics education that we call "guided discovery" approach.

INTRODUCTION

In the Hungarian mathematics education community, the "complex mathematics education" reform led by Tamás Varga during the 1960s and 1970s is considered as one of the most important milestones of the history of mathematics education in Hungary. Varga's conception is viewed as a representative of the Hungarian tradition of mathematics education focused on problem solving and mathematical discovery that we will name here "guided discovery" approach. However, this conception was never developed in a theoretical level: it can be understood from the documents of the reform (the curriculum, the textbooks, teacher's handbooks etc.) and from some articles of Varga, but most of the time, these texts present his approach on very concrete examples and give only limited, indirect access to the conceptual basis of his didactical conception.

The conception lives until today in a narrow circle of teachers, mainly direct disciples of Varga's colleagues, and, according to a general agreement amongst Hungarian specialists of mathematics education, practiced with considerable success. However, its dissemination in larger circles of teachers was never successful. An ongoing research project, supported by the Hungarian Academy of Sciences[1] aims to revisit Varga's reform, describe and situate its conception in the context of current didactical theories, draw the conclusions of the reform's experiences, and adapt it to current educational needs.

In this paper, we present a historico-didactical analysis of Varga's complex mathematics education reform[2]. Our analysis is inspired by research in the history of mathematics education (e.g. Karp & Schubring, 2014) as well as some systemic models of mathematics education as the TIMSS *SMSO-model* (Schmidt, 1996) and Chevallard's (2002) *levels of codeterminacy*. In the didactical analysis, we also built on Brousseau's *Theory of Didactical Situations* (1998): we will emphasize common points and differences between Brousseau's and Varga's approach in the conclusion.

[1] MTA-ELTE Komplex Mathematics Education in the 21th Century project supported by the Hungarian Academy of Sciences (ID number 471028)

[2] The prezent analysis is based principally on the PhD thesis of K. Gosztonyi (2015). Its annex contains several extracts from Varga's reform documents in French and in English. https://hal.archives-ouvertes.fr/tel-01766902

In the first part, we present some influencing elements of the political, institutional, scientific and cultural context of the reform. In the second part, we analyze the content and the structure of the curriculum, the characteristics of the associated textbooks and teacher's handbooks and the expected teaching practices described in this resources. In the third part, we briefly summarize what we know about the impact and the reception of the reform.

From a historical point of view, Varga's case offers an interesting example for the interference between international movements and local traditions: namely the international New Math movement and Hungarian culture of mathematics and mathematics education. Furthermore, the didactical analysis presented below contributes also to theme B of the conference, by showing the profound internal coherence of Varga's reform project.

THE HISTORYCAL CONTEXT OF THE REFORM

The reform process and its main actors

Born in 1919, Varga was a mathematics teacher and employee of the National Pedagogical Institute. He was inspired by a series of conferences of Z. Dienes and a UNESCO conference organized in Budapest in 1962 to start experimentations in primary school (Halmos & Varga, 1978). He started experimentations in 1963 in two classes of a school in the capital; in the following years, the experiment was expanded to other schools and to the lower secondary school level. The project was conducted by a group within the National Pedagogical Institute, but collaborated closely with another group which worked in the Hungarian Mathematical Research Institute on the preparation of the newly created (high school level) special mathematics classes curriculum. In the early 1970s, a ministerial commission evaluated different experimental projects led in the country concerning mathematics education. They choose Varga's project as the basis of the planned new curriculum. From this moment, the number of experimental classes grew very quickly; an optional version of the reform curriculum was introduced in 1974; the reform became obligatory in 1978, in the framework of a general reform of Hungarian curricula.

Political and institutional context

In the period in question, Hungary was a socialist country, under the influence of the USSR. However, the reform started after an important political turn. In the 1950's, the hardest period of the dictatorship led to the revolution in 1956 and the following retribution, but a consolidation began from 1962. The 1960s and 1970s were the period of softer authoritarianism with restricted oppression, some liberalization of the communist system and some opening toward the Western world (Romsics, 1999). Although we don not have any proof, it is likely that the possibility to organize an international UNESCO conference in Hungary in 1962, and to start experiments inspired by this conference are related to this political turn.

The frames of the educational system in which this reform arrived were established since 1946. Compulsory education was provided by the 8-grade single-structure "basic schools", comprising elementary (grade 1-4) and lower secondary (grade 5-8) education. Upper secondary education was provided by general and vocational secondary schools. During the 1950s and '60s, the regulation of the educational system was extremely centralized, with detailed curricular instructions. Soviet influence and the communist ideology were quite apparent in instructions as well as in teaching materials in this period. From the late 1960s however, a slow liberalization was launched (Báthory, 2001): the influence of the ideology was pushed into the background, pedagogical and psychological considerations were taken into account, differentiation as well as teachers' autonomy and liberty was emphasized. This turn played a crucial role in the preparation of the 1978 reform, and—as we will see below—Varga's project can be considered as a pioneer in this sense.

Thus, contrarily to some other reforms of the New Math era which fitted in the frame of a unification process of the educational system, like the creation of the "collège unique" in France (d'Enfert & Kahn, 2011), the Hungarian movement was set out from a unified, centralized system and fitted into a liberalization process. The impact of this context is perceptible on many aspects of Varga's reform, even in less evident characteristics as the role of mathematical language: while the above mentioned French "mathématiques modernes" reform insists on the unifying role of formal mathematical language, Varga's complex mathematics education reform emphasizes the importance of working with the diversity of students' personal expressions.

The Hungarian reform in the context of the international New Math movement

The international New Math movement is often considered as being developed in the context of the cold war's scientific and technological competition. Thus, it would be an obvious hypothesis that the New Math was a Western movement, without relevant contributions from the "Eastern bloc" or with two parallel movements in the two "blocks". However, the Hungarian reform is a good example illustrating that it is not the case. Varga always declared being influenced by the New Math; from the 1960s, he actively participated in the work of different international organizations of the movement like the UNESCO or the CIEAEM[3], he was invited to and published in the US, Canada, France and Italy, among others. According to his doctoral thesis (Varga, 1975) as well as his colleagues' memories expressed during interviews, Eastern influence was much less important on his work, although he also published in several countries of the Eastern block; his only important partner from these countries was Krygovska, the leader of the Polish reform—also an active and recognized contributor of the international New Math movement.

Many impact of the New Math movement can be observed on the Hungarian reform: the introduction of a coherent subject named "Mathematics" instead of "Arithmetic and Measurement"; new mathematical domains introduced in early ages like sets or logic; the reference to Piaget's psychology and Dienes's mathematical games; the important role of manipulative tools; etc. However, Varga was also critical with some aspects of the New Math reform, especially with the excessive emphasis on mathematical formalism—as we will see below.

Epistemology of Mathematics: a Hungarian "heuristic" tradition

When his colleagues evoke Varga's reform movement, they usually underline that, while being inspired by the New Math, it was also a specifically Hungarian conception fitting into the local traditions of teaching mathematics by discovery. This tradition exists indeed in the teaching of young mathematical talents[4] and goes back at least to the beginning of the 20st century. Varga himself was in intensive personal contact with some representative mathematicians of this tradition (L. Kalmár, R. Péter, A. Rényi, J. Surányi among others) since the 1940s; and they all supported, more or less actively, Varga's later reform movement. These mathematicians, together with well-

[3] See eg. the hommage to Varga on the site of the CIEAEM: http://www.cieaem.org/?q=system/files/varga.pdf

[4] Nowadays its most important representative is L. Pósa. See http://agondolkodasorome.hu/en/

known thinkers like G. Pólya or I. Lakatos, represent a quite coherent, “heuristic” epistemology of mathematics, which is closely related to questions of mathematics education and published mostly in texts popularizing mathematics and lectures about mathematics education (Gosztonyi, 2016).

They see mathematics as a constantly developing creation of the human mind, this development being guided by series of problems. According to them, the source of mathematics is intuition and experience; mathematical activity is basically dialogical and teaching mathematics is a joint activity of the students and of the teacher, where the teacher acts as an aid in students’ rediscovery of mathematics. They discourage excessive formalism, seeing formal language also as a result of a development. They describe mathematics as a creative activity close to playing and to the arts.

Pedagogical and psychological background: a complex situation

The pedagogical and psychological background of the reform is quite complicated to reconstitute. The reference to Piaget is obvious, but not the only influence on Varga’s conception. His wife, Á. Binét was also a psychologist and worked together with the most influential Hungarian psychologist of the period, F. Mérei. Varga refers to some soviet pedagogues too, but only a few times in his politically relevant writings: so, it is difficult to know if these are real or only politically motivated references. However, Vygotsky is almost missing from his references, although Varga’s conception shows some similarities with Vygotsky’s socio-constructivism. The socio-constructivist approach, as well as the importance of visual intuition in the learning process can also be inspired by views of S. Karácsony, who was a Calvinist pastor, pedagogue and philosopher, also in contact with most of the mathematicians mentioned above (Máté, 2006). According to Varga’s colleagues and family, Karácsony had a great influence on him—but he couldn’t be referred in Varga’s writings, again because of political reasons. In summary, pedagogical and psychological influences seem to be quite complex and their more detailed identification would need further research.

DIDACTICAL ANALYSIS OF THE REFORM

The curriculum

Similarly to other reforms of the New Math period, Varga aims to integrate new topics in mathematics education, and to present mathematics as a coherent science, organizing the curriculum in accordance with modern mathematics. It involves basing notions on sets and relations, or the strengthened role of algebra, as in a number of other reforms of the same period—but, for Varga, it also means introducing logic, combinatorics, probability or algorithmic thinking in primary and lower secondary school education. Varga was internationally recognized for his work on teaching of logic, combinatorics and probability (Varga, 1972)—the specific domains studied by the Hungarian mathematicians supporting his movement.

The internal coherence of the curriculum is ensured by the parallel, spiral presentation of 5 big domains, all being present throughout the whole curriculum, with frequent and various internal connections amongst them: 1) sets and logic 2) arithmetic and algebra 3) relations, functions and series 4) geometry and measure and 5) combinatorics, probability and statistics.

Another significant characteristic of Varga’s curriculum is its flexible structure: “suggested” and “compulsory” topics are distinguished from “requirements”. As he explains: “many concepts and skills not appearing as requirements in the school year where they are first mentioned in the syllabus, get enrolled to them in subsequent years when they are supposed to become ripened" (Halmos & Varga, 1978).

This organization gives important liberty to teachers, allows differentiation amongst students, provides a rich and varied experimental basis to the progressive generalization and abstraction of mathematical notions, and supports a learning process based on mathematical discovery while elements of mathematical knowledge can emerge as tools during problem solving situations.

Conceptions on the teacher's work

Varga's conceptions on teaching practices were reconstructed first of all from the reform's written sources, especially from teacher's guides. We also took into consideration interviews with Varga's colleagues and observations of his collaborators' and disciples' present practices. Varga's conception takes into account the constructivist approach, but distances itself from radical constructivism. In the same time, it is also inspired by a dialogic approach, characteristic of the above mentioned Hungarian tradition of learning mathematics by discovery.

In Varga's conception, teachers are supposed to organize lots of small problem situations for students. Students can work individually or in group, but collective classroom dialogue is also a very typical form of work, while the teacher acts as an experienced guide in the collective research process. Handbooks offer numerous advices to teacher's question and intervention in order to react efficiently to students contributions: to help the advancement of the collective research project while leaving an important responsibility to students in the problem solving process and in the construction of mathematical knowledge.

According to the handbooks, teachers have important responsibility in the construction of long term teaching processes, which is based principally on ordered series of problems. Mathematical concepts are constructed on a large experimental basis, by discovering links and analogies among apparently different problems and by generalizing progressively the knowledge related to concrete problem contexts. A variety of manipulative tools and representations plays crucial role in this process (some of these tools being wide-spread in the period as the Dienes blocks or the Cuisenaire rods). Thus, abstraction is a slow, progressive process in this method; it often takes several years to formulate explicit mathematical knowledge after the first experiences (Gosztonyi, 2017).

In summary, we can say that teachers' work is defined on two main levels: the construction of long-term teaching processes in form of series of problems, and the management of problem solving situations by classroom dialogues.

Resources

This expected work of teachers is supported by a series of textbooks and teachers' guides. In the period under question in Hungary, only one collection of textbooks and teacher's handbooks was available, prepared by the same team as the curriculum. For the primary school, similarly to other countries in the New Math period, worksheets were available, meant to be used only as partial resources beside various activities. Official teacher's guides served as main resources for teachers. These teacher's guides follow a special structure: their main part contains quasi-continuous text mixing examples of tasks with mathematical, didactical and pedagogical commentaries. They are organized in thematic chapters, following the above mentioned five big domains of the curriculum.

The tasks, small problems are described with several possible variations, suggestions for inventing new tasks and ideas for their realization: the guide often describes possible student reactions (based on the experimentations) and advise teachers how to deal with them.

After this main part, the books present a *possible* syllabus for the year, emphasizing that it is only an example and encouraging teachers to elaborate their own teaching progression for the year. In fact, following the offered syllabus is not easy: as the main domains are treated in parallel, most of the lessons might contain activities from several domains which located in different thematic chapters of the book. And as the thematic chapters are poorly structured and contain many internal and implicit references, teachers have to know them quite well to use them.

For middle-school, there are textbooks provided, with (much less detailed) teacher's guides. One unusual characteristic of these textbooks are the way they introduce new knowledge: they present fictive dialogues of students, discovering new knowledge while they discuss some mathematical problems. The teacher's guide encourages teachers to provoke similar discussions in classrooms.

The internal coherence of the reform's conception

In summary, we can say that the different elements of the reform, the curriculum, the task design, the resources, the indications about expected teaching practices are conceived following a coherent conception. This conception is partly related to some international trends of the New Math period ("modern mathematics" in the curricula, students' participation in the construction of mathematical knowledge, usage of manipulative tools, etc.). In the same time, it corresponds to the above mentioned "heuristic" epistemology of mathematics, represented by Hungarian mathematicians. Problem solving and mathematical discovery is in the focus of the conception: it is supported by the flexible structure of the curriculum, the parallel, dialectic presentation of different mathematical domains, the use of various material tools and representations, the construction of long-term teaching processes in form of series of problems, and a dialogic guiding of the class. These elements allow students to advance in their own pace, to have enough time and occasion to gain various experiences and to follow a slow abstraction process through progressive generalization. Matching these characteristics, we can call Varga's conception *guided discovery* approach.

THE IMPACT AND THE RECEPTION OF THE REFORM

Similarly to many other reforms of the period, Varga's complex mathematics education reform provoked vivid public debates and was followed by an important correction during the 1980s. Varga's former colleagues interpret this as a failure, and they consider the obligatory introduction of the reform as the main reason of its rejection. According to them, Varga's approach should have been disseminated progressively in the frame of a bottom-up process, as it happened during the (generally successful) experimentations—but this kind of slow diffusion was not politically supported. While a narrow circle of teachers (mostly colleagues of Varga and their disciples) followed the *guided discovery* approach with success, the majority of Hungarian teachers did not adopt it, or integrated only partial elements of the approach in their practice.

Despite of that, we have to underline that Varga's work remained influential in Hungarian mathematics education until today. Pálfalvi (2000) shows continuity in the curricula's conception: the main structure of the Hungarian curricula and several of its organizing principles are inspired by Varga's conception. Despite of numerous modifications, the main structure and the content of the curriculum remain quite stable until today. Some of the textbook authors from Varga's team were active until the 2010s and their textbooks show also important continuity with the original versions of the 1970s—although other textbooks are also available now. Most of the teacher trainers consider Varga's "guided discovery" conception still relevant and find inspiration in it, especially for primary level in-service teacher training.[5]

CONCLUSION AND DISCUSSION

In this paper, we shortly presented an analysis of Varga's complex mathematics education reform from a mixed, historical and didactical point of view. From the point of view of the history of mathematics education, Varga's reform can be seen as an interesting example of the dialectic influence of international dynamics and of local specificities. We also underlined how the different elements of context—political circumstances, specificities of the educational system, scientific, epistemologic and pedagogical context—interacted in the formation of the character of the reform. The contradictory reception of the reform—its long-lasting influence on textbooks and on the curricula itself, the recognition of the approach by researchers, teacher trainers and a small circle of teachers, and its massive rejection by other teachers in the same time—can offer relevant lessons about the dynamic of reforms in mathematics education.

From a didactical point of view, we emphasized the internal coherence of the reform, taking into account its epistemological background, its curriculum, resources and the expected teaching practices, all supporting teaching focused on mathematical discovery. Although Varga's "guided discovery" approach remained a theory in act, his reform project is an ambitious and highly coherent realization of the organization of mathematics education based on problems and mathematical discovery. In this sense, Varga's project offers also interesting lessons for the current reflections on the possibilities of Inquiry Based Mathematics Education.

Artigue and Blomhøj (2013) discuss in their article the questions of the theorization of IBME, treating a number of related didactical theories. Historical links and parallelisms can be identified between Varga's approach and several of those theories, especially those of Pólya, Freudenthal and Varga (Varga, 1975). We compared Brousseau's and Varga's conception in (Gosztonyi, 2017): while several common points can be identified, like the focus on students problem solving, the reflection on the *milieus* and the critical use of constructivism, there are also important differences, especially in the conception of teaching processes as an alternation of *adidactical situations* and *institutional phases* in Brousseau's case, in form of classroom dialogues and *series of problems* in Varga's case. Varga's *guided discovery* conception offer several elements which could fruitfully enrich the international reflection on IBME: first of all the structure of his curriculum, the conception of teaching processes by series of problems and his reflections on the management of classroom dialogues. One of the aims of our MTA-ELTE Complex Mathematics Education project is to situate Varga's approach in this international network of didactical theories.

Other aims of our project concern the above mentioned paradoxical situation of Varga's work in the present Hungarian context: its recognition among a narrow circle of specialists and its problematic dissemination. We think about its actuality and necessary adaptations to present-day context, and

[5] Most of the authors of this paper are teacher trainers and some of us participate in the elaboration of the following curiculum.

seek for new and more efficient possibilities of its dissemination amongst teachers. As most of us work as teacher trainers, conceiving teacher trainings is a natural direction of our reflection. We also put important emphasis on the analysis and conception of resources: as we showed above, the resources of the reform are quite complex documents and require high level knowledge and autonomy from teachers. Our hypothesis is that the difficulty of the usage of the resources could contribute to the rejection of Varga's reform. Thus, in the spirit of recent research on resources in mathematics education (Gueudet & Trouche, 2010), we try to develop new and more efficient resources to support teaching by "guided discovery".

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HISTORICAL ASPECTS OF MATH CURRICULUM REBUILDING

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The purpose of this paper is to discuss past and present practices of reforming mathematics school curricula in Serbia. The pathway of preliminary developments of mathematics curricula in Serbia is briefly discussed. Next, the foundations of past reforms with the starting position dated approximately 1960 are analyzed. Contributions of internationally distinguished scientists, predominantly mathematicians and psychologists are discussed as they present some of significant breakthroughs which significantly influenced the shaping of math curriculum in Serbia. Afterward, analysis of the ongoing reform process shows that the achievements on PISA and TIMSS assessments influence designers of curriculum to propose particular changes. I close up by turning up to my personal experience of being part of the curricula designing processes in the USA and Serbia. The argument of the paper implicates that character of curriculum reforms significantly reflects cultural context. Therefore, the needs for permanent rebuilding of math curriculum originates from ever-changing cultural context.

INTRODUCTION

The goal of teaching mathematics can be presented as a synthesis of the general cultural, scientific (including mathematical) and applied objectives. Various current theories of mathematical education lead to different strategies for improving mathematical education in different countries (Sriraman & English, 2010). Mathematics curriculum comes from cultural context and therefore it cannot be looked upon independently from culture. Jablonka stressed that it is not possible to promote a conception of mathematical literacy, a concept shaping up math curriculum in all times, without at the same time -implicitly- promoting a particular social practice (Jablonka, 2003).

This is why there is no one ideal mathematics curriculum. This is also why there is no possibility to create an ideal curriculum for everyone and forever. Yet we need to acknowledge that contemporary national cultures have much in common. They share common civilization, more or less similar economy, philosophical framework, scientific knowledge (particularly mathematics). Yet, there are factors brining variability in mathematics curricula and effecting their implementations. Stanic and Kilpatrick make a cautionary notice that:

> Math curriculum is strongly influenced by general curriculum policy and cultural context, today more than in the past. "…the story of mathematics curriculum reform is not the story of continual progression toward a curriculum that is best for students, teachers, and society nor even the story of different ideologies cyclically replacing each other's influence on school mathematics; instead, it is the story of a developing community preoccupied with a limited and ill-defined agenda" (Stanic & Kilpatrick, 2004, p.13).

Math curriculum can be built segmentally but there should be one guiding principle over-arching all these segments. The question is who should define the guiding principle.

Grains of history of mathematics education in Serbia

After medieval independent principalities, kingdom and imperia and centuries of living under Osman imperia, Serbian population met the beginning of the nineteen century. It was time of deliberation and restoration of the state of Serbia, which was followed by strong effort toward educational and scientific development in the principality of Serbia (1815-1882) and the Kingdom of Serbia successively. Mathematics education was built under significant influence of scientific centers where Serbia mathematicians have been educated (Budapest, Wien, Paris, Sankt Petersburg, Graz, etc.). Mathematical curriculum has been created as a result of influences of those centers, therefore we cannot speak about any particular identity of Serbian mathematics curriculum. Mathematical content has been present on all levels of education but under different names: računica (Serbian *račun*, calculation), čislenica (Russian *čislo,* number) and arithmetic. When the "Velika skola" (a predecessor to the first University) was founded, all students had to take a subject named Mathematics.

During the first half of the 20th century there was a relatively well established educational system. Mathematics curriculum was formed under dominant influence of Austro-Hungarian, French and German math schools.

As it is well known, after the II World War, the World was divided by Iron curtain, and Yugoslavia (Serbia was part of it) until 1948 belonged to the Eastern Block. Consequently, Serbian educational system at that time have been under strong influence of Soviet school, particularly in the domain of mathematics. From then on, Yugoslavia (with Serbia in it) was one of the founders and leaders of the non-aligned movement. As a result, although Russian influence in mathematics education remained, mathematical community followed developments in mathematics education research and practice in other parts of the World as well (Micic & Kadelburg, 2018).

Math curriculum reforms in the last 60 years

We may distinguish some important moments which had influenced the path of math curricula reforms in Serbia. The first was the International Symposium on the Coordination of Instruction in Mathematics and Physics, held in Belgrade, September 1960, with distinguished participants: Marshal Stone (ICMI's president at that time), Richard Courant, Giovanni Sansone, Gustave Choquet and others. Expertise of these participants contributed to accepting and spreading the ideas of modern mathematics and successful implementation of them. This event happened at the time when New Math movement was underway in the USA and worldwide.

From the 60's, in Serbia dominated movement for systematic formalization of Mathematics Sciences (influenced by the Bourbaki's school). This perspective had also reflection in reforms in mathematics school curriculum. The approach which we may call "pure mathematics oriented" was focused on presentation of mathematics as a system of knowledge. Concept of set, elements of logic, graph theory, combinatorics were introduced as early as in first grade of elementary school. Mathematics knowledge has been decontextualized with limited consideration of possible applications. As an example, a unit *Arithmetic operation: Subtraction* from 1974 is presented opposite to the same unit from a textbook published in 2018. (Figure 1). In the 1974 textbook, mathematical language related to subtraction is accented including a visual, set representation of concept of subtraction. In the 2018 textbook, concept of subtraction is introduced via pictures of various real context situations and iconic representations (Figure 1, right)

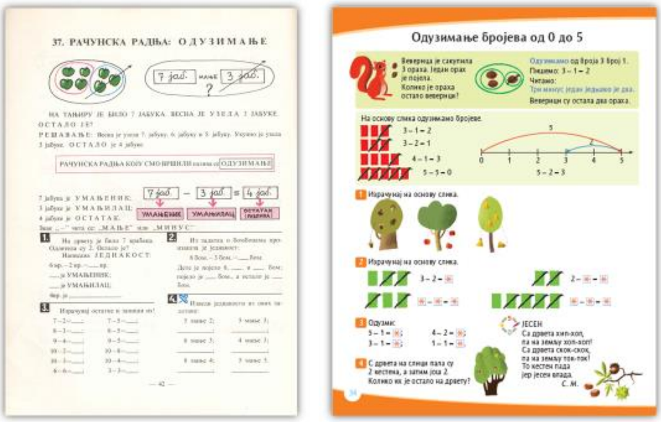

Figure 1: Subtraction in 1st grade textbooks (Nikodijevic, 1974, Milinkovic, 2018).

Examples of problems regularly found in textbooks for 1st grade pupils at that time is presented in Figure 2. In the problem on the left pupils are asked to write number of elements complementing the given subset. The problem reads: 1) *Each arrow says:"… is equally potent…" Fill in what is missing in the picture*. Problem on the right concerning simple graph theory application is given in picture without context. It is supposed that teacher would pose questions related to the given picture. Both problems are detached from real context.

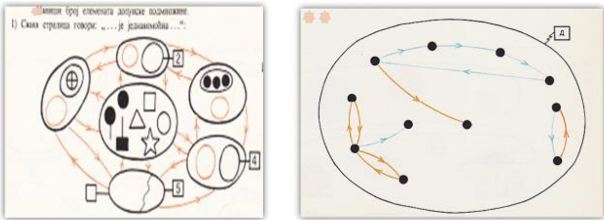

Figure 2: Examples of 1st grade problems (Prvanovic, 1974, p.19 & p.43)

Exemplary for the educational approach is also, a senseless formulation of Pythagorean Theorem worded in the following way: Set of points in the square above the hypotenuse is equal to the union of sets of points in the two squares above the legs.

New Math curriculum was established a bit later than in the USA, United Kingdom, and France, etc. and dominated in our educational system longer than in others. It gradually subsided in reforms in mid-80's. So, Serbian curriculum followed global trend with small time delay. This delay happened with other reforms as well.

At that moment, mathematicians had a strong support from the authorities in the country. Mathematics and mathematicians had a special status in educational system. But implementation of the curriculum failed since teachers were not adequately trained to carry out the curriculum. Finally, it was recognized that curricular approach and objectives were not appropriate for majority of pupils. Then and there, there was a limited access to international scientific journals. Because of that, the scientific papers which were chosen to be translated and published in professional journals in Serbia had a major impact on mathematics community. Similarly, as people did not travel too often, if a scholar did get opportunity to visit a distinguished educational center abroad, his/her experience was appreciated and fresh ideas were shared and discussed among Serbian math academic community.

A global influential event of that time was the Second ICME 1972 in Exeter, United Kingdom. It raised questions about New Math approach, resulting in strong critical reconsideration of the New Math. In Serbia, incentive for reconsideration of the curricular approach was a translation of Rene Tom's paper "Modern mathematics – does it exist?" presented at the ICME held in Exeter. As a result, slow changes in the math curriculum have taken place in direction of making mathematical content appropriate for the vocational profile of learners. The content was didactically transformed to fit certain profile so e.g. perspective medical assistant did not learn the same algebra as perspective accountant and hairdresser did not learn the same geometry as mechanists, etc. This reform of math curriculum was part of the reform happening on the level of whole educational system in 80's with the idea of promoting vocational high school education. Similar shift happened elsewhere and school mathematics turned to be more of "everyday mathematics" with extended distance from scientific mathematics discipline (Stanic & Kilpatrick, 2004).

Between two key reforms in 60s and 2017's there were ongoing small scale reforms of curriculum in 70' and 2007'. The first of them, called *directed education*, had effected primarily curriculum of secondary education. It was a reform attempting to achieve ideally conceived connection between schools and factories. The reform was a failed attempt of specialized education, as it now exists in developed countries such as Germany and Sweden.

The event preceding the next mid-reform in 80s' was a conference Problems of Contemporary Mathematical Education in 1980, held in collaboration of the Institute for Education Research with the Society of Mathematicians, Physicists and Astronomers of Serbia with the following themes: Modern mathematics and its role in building and cultural point of view, Intensification of Mathematics Teaching, and Psychological-pedagogic aspects of the modern mathematics teaching. That curriculum was designed on the ground diverging from the "directed education". It was a step backward, toward more formalized, structure oriented approach in teaching mathematics and it was implemented for primary and secondary education.

After that time, math curriculum has not been essentially reformed until 90's. Stronger critical reflection against New Math came from different sides: from prominent mathematicians: Rene Thom (France), William Thurston (USA), Vladimir I. Arnold (Russia) and Hans Freudenthal (Netherlands), a leading researcher in mathematics education, and others. Arnold (1995) for example, commented:

> "At the beginning of this century a self-destructive democratic principle was advanced in mathematics (especially by Hilbert), according to which all axiom systems have equal right to be analyzed, and the value of a mathematical achievement is determined, not by its significance and usefulness as in other sciences, but by its difficulty alone, as in mountaineering" (Arnold,1995, p.6).

New approach has been initialized upon critical notes from the academic community. Freudenthal had believed that mathematical ideas should not be presented in a way they were discovered. For mathematics instruction, he stated it is important to find "balance between freedom of inventing and the force of guiding" (Freudenthal, 1991, p.48). It is noticeable that in most European countries and the United States, the constructivist view prevailed, according to which mathematical ideas are "re-discovered" in the process of mathematics instructions based on solving realistic problem situations (Cobb et al., 2008). The formation and presentation of mathematical concepts are directed toward realistic representations of them rather than to their abstract (symbolic) representations and to understanding (versus to the automatization) of mathematical procedures. Some of influential articles well enlighten shift in basic principle toward the Dewey's idea of reflective inquiry and a move from behaviorism to constructivism (Hiebert et al., 1996, Romberg, 1992, Romberg, 1995). The new direction is based on the idea students should problematize subject, whereas problem solving rather than mastering and applying skills should be core objective. Serbia has followed similar path in the process of reforming math curriculum.

Ongoing curriculum reform process in Serbia

Currently, Serbia has a national curriculum for preschool (3-6), primary (age 7- 14) and secondary education (age 15-18). It is proposed and imposed by the Ministry of education, science and technology development in the whole country.

First phase of the current reform happened at the beginning of 21^{st}century. Educational community, as a reflection of public critics, has raised concern about overloaded curriculum and came to the decision of reforming curriculum in direction of relaxing curriculum demands and number of math lessons per week in some grades. Primary objective of the reform was to unload curriculum. Regardless, mathematical curriculum was entirely and substantially reformed under direction of Milosav Marjanovic. The ideas of Skemp, Bruner and Freudenthal were implemented in recommendations of didactical approach to particular topics. He asserts that whole arithmetic process in primary grades has to follow the Skemp's triangle (Marjanovic, 2003, Marjanovic, 2000).

The following reform of math curriculum is currently underway as a part of reforms effecting whole educational system. The plan is that the design of curriculum finishes in four years (each year three grade levels (e.g. in year 2017, new curriculum is designed for $1^{st}$and 5^{th} grades of primary school and 1^{st}year of secondary school).

To a significant level, this reform of the curriculum comes as a result of pressure from the authorities who were not satisfied with achievements of Serbian pupils on PISA and TIMSS assessments. Since 2003, Serbia has participated in four PISA research cycles (2003, 2006, 2009 and 2012). Almost 40% of students did not reach the level of functional literacy. Compared to the results of the OECD countries, mathematical literacy among students in Serbia is on average 45 points lower, which means that our students are one year of schooling behind, compared to their peers in the OECD countries (Pavlović-Babić & Baucal, 2013). On the other hand, our 4^{th} grade students were above the average scale on TIMSS 2015 with 518 points. The analysis of achievements at TIMSS 2015 shows that, the sample of students from Serbia recorded a slight increase in comparison to the achievement in 2011 (Milinković et al., 2017). In comparison with other countries, it is significantly higher than many European countries but at the same time it is 100 points lower than the most successful countries on TIMSS 2015. The analysis showed relatively high achievement of students in number sense and arithmetic operations whereas low achievement was achieved in domain of geometry. Comparative analysis of TIMSS problems' content and Serbian curriculum in primary grades showed that some topics are missing: addition and subtraction with simple fractions, rational numbers (decimal notation), use of an informal coordinate system, three-dimensional figures and their two-dimensional representations, axial symmetry and rotation (Milinkovic, 2015). These topics were part of math curriculum for primary grades in seventies but were extracted in the succeeding reforms. In upcoming curriculum some of these topics are going to be present again, this time with non-formalized approach, the emphasis is on understanding in context and applications in real context problem solving.

In the latest reform the main theoretical framework from the previous curriculum has been preserved in upcoming curriculum but some new topics are introduced, while others which were already present are dislocated or studied more extensively or earlier in school. Strong attempts has been made to introduce financial math and data analysis on all levels of school. Reading data in tables and graphs and picturing data in graphs are activities which are expected to be visible present although blended into other themes. Isometric transformations are introduced in curriculum as early as in 5th grade on the level of perception. Note that the first attempts to introduce formally isometric transformations were also in 70's. (The topic was abandon in the succeeding reforms.) Starting from the first grade the curriculum proposes exploration of ideas of translation and symmetry.

Across domain changes are related to usage of educational technology and introduction of project method and interdisciplinary thematic instructions. Electronic textbooks are mandatory in ongoing reform.

The curriculum is spirally-shape organized - the same topics are taught in different grades with different scope and depth. For example, exploration of idea of measurement with non-standard units is introduced in 1st grade, while metric system and concept of perimeter are placed in 2nd grade curriculum.

Lastly, the focus and the language used in the document has been changed in direction of less formal explanations. The policy makers have general agenda in writing curriculum document to change focus from content to achievements of students; the curriculum enlists precisely what should students know at the end of each school year.

Being a part of math curriculum reforms

I had a rare privilege to be involved in curricular reforms in the USA and in Serbia with time delay of a decade. Prior to my stay in the USA, I have been mathematics teacher in Mathematical Gymnasia, a High school for talented pupils. The Wisconsin Center for Education Research (WCER) is placed in Madison, Wisconsin, in the USA and I happened to be there in the time of curricular reforms. As I joined the international team on the research project *Mathematics in Context,* designing a comprehensive math curriculum for the middle grades, I learned about this innovative, unexperienced approach to teaching math. The curriculum was developed in 1991 through 1997, and revised 2003 to 2005 in collaboration with the WCER, School of Education, University of Wisconsin-Madison and the Freudenthal Institute at the University of Utrecht, The Netherlands. The curriculum was built on the idea that math concepts can be introduced within realistic contexts that support mathematical thinking, modeling skills and abstraction. Through exploration of (often ill defined) complex problems and discussion with support of teacher students develop their own models, strategies and procedures. Novel ideas for that and this time.

Upon return to Serbia, I started to work at Teacher Education Faculty. I found that educational community in Serbia does not rush into changes in school. But, few years from then, the reform of mathematics education started to evolve. Just before that the Standards of Achievements for the First Cycle of School document has been created (ZUOV, 2011). In 2017, a new cycle of curriculum reform has begun. We are still in the process of reconstruction of the curriculum. I am a part of it.

Concluding remarks

Globally, essential difference in math curriculums over the last decades is seen in the emphasis on the formation and presentation of mathematical concepts as abstract (symbolic) or as realistic representations. Another principal difference is related to the importance that is attached to understanding (versus to automatization) of mathematical procedures and their applications. Over time curriculum reforms happened for various reasons. I touched upon some of them in this paper.

The fact is that scientific and other knowledge in almost all areas is permanently expanded and multiplied. At the same time, society expects rapid implementation of advances in knowledge and positive effects on the welfare of the community. This results in high public scrutiny of the school system and excessive expectations about responsiveness and adjustability of school curricula.

On the other hand, educational systems on all levels are large and very complex, with a numerous participants, executors, executives, exposed too many constraints and influences. The characteristic of such a system is inevitably a significant measure of entropy, which results in the need of a large amount of energy for moving it from the current state into desired. These opposing faces of school systems, dynamics and idleness, must be balanced. For this reason, it is important to make changes in rational and cautiously manner. It is my opinion that math curriculum must be permanently reconstructed (instead of comprehensively refuted and built from ashes), be harmonized, and constantly controlled by field's experts. Thurston's remarks that "policymakers often do not comprehend the nature of mathematics or of mathematics education." (Thurston, 1990, p. 844). Although mathematicians still play role in creating math curriculum in Serbia their participation is evident but constrained. Design of mathematical curriculum is considered to be result of collective effort of mathematicians, teachers and other educational experts under supervision of stakeholders (general policy makers, parents). Finally, critical phases of any reform, once design of new curriculum is finished, are creation of textbooks, teacher preparation and control of implementation. Influence of "free market" is evident. It is my opinion that involvement of multiple publishers in production of educational materials contributes to rising quality of teaching materials but it also brings factors, other than quality in process of selection. Besides, quantity of textbooks does not assure significant creative diversity.

Historically, character of math curriculum reforms have been essentially reflections of the surrounding cultural contexts. Mathematics curriculum in Serbia has been shaped by different forces: educational research, development in mathematics and global historical context but the struggle to achieve global standards in education has been a leading force in all reforms. In modest level past reforms have been adjusted to the cultural contexts and this was perhaps one of the reasons of their transience.

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TIME TO REFLECT: LESSONS LEARNT FROM AN IRISH CURRICULUM REFORM

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A new mathematics curriculum was introduced in Ireland in 2010. This was the first mathematics curriculum reform effort since 1992 and was a significant milestone in Irish education. This paper reports on three studies that were conducted by researchers based in EPI·STEM, the national centre for STEM education in Ireland that examined this curriculum reform. The three studies sought to obtain teachers' views on enablers and inhibitors of curriculum reform and draws on Memon's (1997) framework for curriculum change. An extensive number of teachers were surveyed and a range of barriers were revealed, many of which fit into the underpinning framework. However, this paper reports on a missing paradigm in the Memon (1997) framework and the authors ascertain that instruction time must be included in all future curriculum reform frameworks. The Irish reform has shown that failure to take this curricular factor into account can have detrimental effects on all reform efforts. If time is not explicitly considered as a factor affecting curriculum reform then it may, as was the case in Ireland, be inadvertently overlooked by curriculum designers and policy makers, which may have damaging consequences for the successful implementation of the curriculum.

INTRODUCTION

This paper aligns with Theme A: and addresses the following question: *What potentially crucial aspects of mathematical curricula have not been considered, and even less, touched upon?* In this paper the authors utilise Memon's framework for curriculum change to investigate the barriers to a mathematics curriculum reform in Ireland. The authors highlight the lessons learned from this curricular reform and discuss how these findings can contribute to the development and evaluation of future curriculum reform internationally.

The Irish education system comprises eight years at primary school (age 5-13) followed by five or six years at post-primary school. Post-primary education consists of a three year Junior Cycle, which culminates in the Junior Certificate examination. This is proceeded by a two or three year Senior Cycle (students have the option of completing a bridging year between cycles, called Transition year, where students participate in a range of activities including work experience and entrepreneurial opportunities (Prendergast & O'Meara, 2017), which ends with students completing the Leaving Certificate examination at approximately 17/18 years of age. The Leaving Certificate is the gatekeeper to higher education and for it students typically complete examinations in 6 or 7 subjects, with mathematics taken by approximately 95% of the cohort. A student is not permitted to enter university unless they have passed mathematics in this examination, regardless of whether their choice of degree programme has a mathematics component or not.

Prior to the introduction of the new mathematics curriculum in 2010 much of the research conducted on post-primary mathematics education in Ireland demonstrated that mathematics was being taught in a procedural fashion, with very little emphasis on problem-solving (Gill, 2006; Conway & Sloane, 2006). The mathematics curriculum and the assessment (100% terminal State Examination) received much of the blame for this. In the State Examinations the questions asked were largely focused on the mastery of procedural skills and were highly predictable based on previous examination papers (NCCA, 2006; Liston & O'Donoghue, 2010). This had a direct, adverse effect on the teaching of mathematics. Teachers largely focussed on procedural approaches to teaching as they were able to predict, with high levels of accuracy, what questions would be required of students in their examinations (NCCA, 2006). Additionally, parts of the curriculum were often omitted due to the choice of questions offered on the examination papers (Gill, 2006). The application of concepts to real-life scenarios was almost non-existent on examination papers, therefore students were given little exposure to applying their mathematical knowledge and skills to solving real world problems in the classroom (NCCA, 2006). The impact of this teaching and assessment was evidenced in students' work, as reported by the Chief Examiner's report (2005). This analysis of student examination scripts revealed a lack of conceptual understanding, and poor problem solving and decision making skills among students (State Examinations Commission (SEC), 2005). Students were confounded when questions required anything more advanced than the routine application of memorised rules. While their procedural skills were deemed adequate, when a deeper understanding was required to complete a problem, the students exhibited deficient levels of understanding (SEC, 2005).

In response to these criticisms a new syllabus, entitled 'Project Maths', was introduced. The new curriculum was in line with many syllabi globally including Australia and the UK , as it consisted of five strands (Statistics and Probability, Geometry and Trigonometry, Algebra, Number and Functions). These strands were selected in an effort to improve the alignment between the existing primary school curriculum and the new post-primary one (Table 1). The new curriculum was developed as part of a collaborative consultation process between a number of educational stakeholders including the National Council for Curriculum and Assessment (NCCA), the Department of Education and Skills (DES), and the State Examinations Commission (SEC). The reform was piloted in 24 schools in 2008 and then implemented on a phased basis nationally between 2010 and 2014. It was a significant reform of the post-primary mathematics curriculum for Junior and Senior Cycles, not just in terms of content, but also in terms of methodologies and assessment. More active learning and problem solving methodologies were encouraged under the new curriculum. In many areas, but not all, the level of mathematical content on the old curriculum was reduced, for example Linear Algebra was removed entirely and the Calculus content was significantly reduced. This was done in an effort to facilitate the adoption of these new methodologies, which were deemed to be more time consuming than the old traditional teaching methodologies.

Pre- Project Maths Junior Cycle Syllabus	**Primary Maths Syllabus**	**Project Maths**
•Sets •Number systems •Applied arithmetic and measure •Algebra •Statistics •Geometry •Trigonometry •Functions and graphs	•Early mathematical activities •Number •Algebra •Shape and space •Measures •Data	•Number •Algebra •Geometry & Trigonometry •Functions •Statistics & Probability

Table 1. Primary and Post Primary (Pre and Post Project Maths) Mathematics Syllabi

In 2011, Lubienski conducted an investigation into the initial implementation of the new curriculum. The purpose of her research was to investigate if the reformed curriculum had been implemented as planned, and what it was like in practice. She interviewed both curriculum developers and teachers from the pilot schools. She found that these teachers felt that they were inhibited by a lack of curricular guidance, as it was not immediately apparent to them that they would be required to collaboratively develop their own resources as part of the pilot study, since no text books were provided at the outset. Encouragingly, Lubienski found that over 80% of teachers surveyed had partaken in three out of 10 national instructional workshops[1] at the time of her research. While teachers were positively disposed to the new curriculum in general, she found there was still a large emphasis being placed on the Leaving Certificate examination, which tended to determine how teachers taught within the privacy of their mathematics classroom. She stated that this led to teachers playing the role of "exam coach" (p.38) with students, in a system where examinations were traditionally very predictable.

Further research, conducted by Jeffes et al. (2012), explored the impact of the new curriculum on student achievement and attitude. Their findings indicated that while students were performing well in many areas of the new syllabus, and were positively predisposed to the new teaching methodologies, a large proportion of students were still being exposed to more traditional 'chalk and talk' teaching practices and an over-reliance on textbooks.

ENABLERS & INHIBITORS OF SUCCESSFUL CURRICULUM REFORM

In order for any reform, such as the changes to the Irish mathematics curricula, to be implemented and evaluated effectively, it is critical that all stakeholders are cognisant of the enablers and inhibitors that impact on curricular reform. Teacher resistance to mathematics education reform can adversely impact on its execution (Memon, 1997). Memon's (1997) research details the factors affecting curriculum change in, but not limited to, Pakistan. He constructed an extensive list of factors affecting curriculum change, classifying them as curricular, instructional and organisational, in order to derive a theoretical framework to reduce teacher resistance and improve the process of curriculum change. According to Memon (1997), curriculum related factors include a lack of clarity around the changes to the curriculum, incompatibility between the intended and the implemented curriculum, and the curriculum users' needs not being addressed. Deficiencies in content and pedagogical knowledge, lack of professional development, and examination led teaching are categorised by Memon (1997) as instructional factors, whereas physical resources that may act as barriers to change are deemed to be organisational factors. A full breakdown of his framework is provided in Table 2.

Curriculum Factors	Instructional Factors	Organizational Factors
Change is not responsive to curriculum users' needs	Importance attached by teachers to old practice	Lack of supportive mechanism
Lack of curriculum users' participation	Inadequate knowledge of subject matter, method and assessment	Lack of coordination
Non-clarity of curriculum changes	Examination dominated teaching	Lack of communication
Mismatch between official curriculum and actual curriculum	Mismatch between teachers, belief system and curriculum goals	Lack of classroom materials
Externally imposed innovation	Lack of detailed planning	Lack of physical facilities

[1] These 10 workshops were part of the Continuing Professional Development (CPD) offered to all mathematics teachers in the country following the introduction of the reform.

Imported innovation	Lack of motivation, incentives and rewards	Lack of resources
Unplanned change	Lack of professional development	Lack of INSET
	Lack of classroom interaction	Lack of community participation
	Lack of students' interest	Influences of political leaders
		Influence of bureaucracy

Table 2: Factors affecting the success of curriculum reform [Memon's (1997: p61)]

METHODOLOGY

The authors of this paper utilise Memon's framework on curricular change to frame the barriers to a recent national mathematics curriculum reform in Ireland. The barriers were identified in three studies conducted locally to evaluate the implementation of Project Maths. In the subsequent part of this paper the authors will highlight the lessons learnt from this Irish curricular reform and identify how these studies can contribute to the development and evaluation of future curriculum reform internationally.

EPI·STEM is the national centre for STEM education in Ireland, and it has a duty to conduct research into critical curricular issues. A research team from EPI·STEM, along with other researchers, conducted a number of studies to investigate the newly introduced mathematics curriculum in Ireland. This paper reports on three such studies that addressed elements of the three pillars of Memon's (1997) framework. The Mind the Gap study investigated components of the *organisational factors* pillar; the Time in Mathematics Education (TiME) study explored aspects of the *curriculum factors* pillar; while the Teachers' Perceptions of Curriculum Reform study investigated issues from the *instructional factors* pillar.

The aim of the Mind the Gap study was to investigate issues surrounding the transition from primary to post-primary mathematics education in Ireland and to analyse the levels of co-ordination between primary and post-primary schools at this key stage of a students' educational trajectory. For this national study, the authors used stratified sampling to select a sample of 700 primary school teachers and 400 post-primary schools[2].

Time is seen as a key element of the curriculum (Glatthorn et al., 2012) but very little research has been done into the quantum of time allocated to curricular subjects in Ireland. The aim of the TiME study was to quantify the time allocated to mathematics in Ireland. Using a representative sample of 400 schools, the authors distributed surveys to deputy principals and qualified mathematics teachers to determine the profile of time allocated to mathematics in post-primary schools and to investigate the impact it had on curriculum reform.

The Teachers' Perceptions of Curriculum Reform study set out to investigate teachers' perceptions of the recently reformed mathematics curriculum and identify any misalignments that exist between the beliefs held by teachers and the goals of the reformed curriculum. Online questionnaires were distributed to all teachers who subscribed to the Irish Maths Teachers' Association, the national association representing and supporting mathematics teachers in post-primary schools in Ireland.

[2] The primary teachers in this study were teaching students in their final year of primary school while the post-primary sample consisted of teachers who taught mathematics to students in first year of post-primary school.

A brief overview of key methodological aspects of each study is provided in Table 3. This Table indicates the link between these studies and the underpinning theoretical framework. It also shows how the TiME study unearthed a missing dimension in Memon's framework.

	Mind the Gap	TiME	Teachers' Perceptions
Researchers	O'Meara, N. Prendergast, M., Harbison, L. & O'Hara, C.	O'Meara, N. & Prendergast, M.	Freemayer, J., Johnson, P. & Fitzmaurice O.
Methodological Approach	Mixed Methods	Mixed Methods	Mixed Methods
Research Instrument	Survey Research	Survey Research	Survey Research
Nature of Sample	Primary & post-primary teachers	Post-primary teachers & deputy principals	Post-primary teachers
Sample Size	173 post-primary teachers 298 primary teachers	540 post-primary teachers 182 deputy principals	147 post-primary teachers
Link to Memon's (1997) Framework	Lack of coordination		Mismatch between teachers' belief system and curriculum goals

Table 3. Overview of Studies

BARRIERS TO EFFECTIVE CURRICULUM REFORM: FINDINGS FROM IRELAND

Mismatch between teachers' belief system and curriculum goals

The literature on educational innovation has frequently identified mismatches between curricular goals and teachers' beliefs as a barrier to the successful implementation of change (Memon, 1997; Orafi & Borg, 2009). As part of the Teachers' Perceptions of Curriculum Reform study a survey of 147 teachers in Ireland was conducted 5 years after the implementation of the mathematics curriculum reform, to ascertain their level of support for the reform, and also to gain insight into their beliefs regarding the reform. When asked about their level of agreement with the goals of the new curriculum 84% of surveyed teachers said that they somewhat, mostly or totally agreed with the goals. A follow up question sought teachers' opinions on whether they believed there was an alignment between the State Examinations and the goals of the new curriculum. Just under 50% (n = 73) of the teachers said that they believed, or somewhat believed, that there was alignment. Contrastingly, when asked how effective they believed a traditional, teacher-directed instructional approach, which was reportedly utilised extensively prior to the introduction of the new curriculum (Conway & Sloane, 2006), would be in preparing students for the new examinations, over 91% (n = 134) of the teachers responded that they felt it was a somewhat, mostly or totally effective teaching approach.

> T19 [Junior Cycle Teacher]: Some questions really assess goals very well, some encourage me as a teacher to return to older approaches.

> Other teachers offered a more critical appraisal of the alignment between the examinations and the goals of the curriculum reform and emphasised the disconnect that exists between the two.

> T54: There is a disconnect between the student-centred ideology espoused by Project Maths and the over whelming nature of the compulsory totality displayed in the exam papers.

Lack of coordination

An organizational factor identified by Memon (1997) that can have a detrimental impact on curriculum reform was lack of coordination. This too was investigated in an Irish context, via the Mind the Gap study. In this study, 173 second level teachers responded to a survey question which investigated if they felt there was a fluid transition between primary and post-primary education in Ireland since the introduction of the new curriculum. 45% of post-primary teachers felt that there was not a smooth transition despite curriculum reform in Ireland resulting in significant changes to the first-year post-primary mathematics programme, to ensure that it aligned with the primary mathematics curriculum, introduced in 1999. On the other hand only 1% of all teachers surveyed strongly agreed with the statement "There is a fluid transition between primary & post-primary mathematics education". Many potential reasons why teachers hold these beliefs have been detailed in the literature. These include the repetitive nature of the first year curriculum as opposed to a progressive curriculum that builds on students' prior knowledge (Ryan, 2018) and deficient levels of horizon knowledge (Ball et al., 2008) among post-primary teachers (O'Meara et al., 2017).

REFINING THE FRAMEWORK: ADDING A NEW DIMENSION

When analysing Irish teachers' views of curriculum factors which affect curriculum reform, the authors found that the one factor most frequently raised by teachers was not explicitly stated in Memon's (1997) extensive framework. Time was initially mentioned by teachers as a barrier to curriculum reform in the Teachers' Perceptions of Curriculum Reform study, and this key curricular issue was further investigated by O'Meara & Prendergast (2017). They found that 83.18% of the 495 Junior Cycle teachers that responded believed that the new curriculum had impacted on the time required to teach mathematics at this level, while the corresponding figure for the 495 Senior Cycle teachers was 92.5%. Despite such findings, 88% (n = 447) of Junior Cycle, along with 81.2% (n = 403) of Senior Cycle teachers, reported that the time allocated to mathematics had not changed since the revised curriculum was introduced.

Failure to revise the time allocation in tandem with curriculum change has resulted in teachers believing that it is not feasible to teach the revised curriculum as intended. For example, 83.02% (n = 440) of Junior Cycle teachers stated that the aims and objectives of the new Junior Cycle curriculum could not be achieved under the current time provisions, while 94.88% (n = 482) of Senior Cycle teachers were of a similar disposition. This is despite the majority of teachers, at both levels, agreeing, in principal, with the aims and objectives of the new curriculum, as discussed earlier. Furthermore, 62.29% (n = 317) and 82.12% (n = 418) of Junior and Senior Cycle teachers, respectively, felt that there was currently insufficient time allocated to mathematics. This finding was supported by the qualitative findings of this study:

> T283 [Junior Cycle Teacher]: Not enough time to do the course if teaching in a school with moderately bright students, not even speaking about teaching for understanding, practical work etc.
>
> T102 [Senior Cycle Teacher]: More hours needed per week for both higher level and ordinary level. Very rushed to finish the syllabi and always end up giving extra classes, outside of school time. [3]

[3] The TiME study revealed that over 68% of Senior Cycle mathematics teachers and over 54% of Junior Cycle mathematics teachers offered classes, outside of school hours, on a weekly basis, without pay to counteract the perceived lack of time allocated to mathematics.

These findings suggest that time is a critical and defining factor in the successful implication of curriculum reform. Teachers in the TiME study, clearly indicate that time is impacting on their ability to implement the curriculum as intended, thus adversely impacting on students' opportunities to learn. Where 'opportunities to learn' once solely related to content taught and assessed, its definition now encompasses the quality of the curriculum and teaching they experience (Banicky, 2000), and such factors are often impacted by the time available for teaching and learning. According to Burkhardt et al. (1990), if teachers perceive barriers to exist, such as time, then it is inevitable that low take up, dilution and corruption of the curriculum reform will occur. Due to such findings the authors contend that there is a need to include this missing paradigm in Memon's 1997 framework. While many see time as an integral part of the curriculum (e.g. Glatthorn et al., 2012), and as such may perceive it to already be included in Memon's framework (1997) as part of the *Mismatch between official curriculum and actual curriculum* dimension, the ramifications of oversights in relation to time, outlined here, indicate that there is a need to include this as a core factor to ensure that it is considered in tandem with all future curriculum change.

LESSONS TO BE LEARNT

Curriculum developers need to be cognisant of the link between time and successful curriculum reform. They must ensure that the time recommended for mathematics is realistic and allows for the curriculum goals to be achieved. In an Irish context some topics were removed in an attempt to reduce the volume of content in the revised curriculum so as to provide more time for the new teaching methodologies but this reduction did not alleviate all time pressures, as stated by many teachers in two of the three studies discussed. Consultation with teachers, the main drivers of any curriculum reform, is key in achieving this objective. The concept of time also needs to be recognised as an important enabler in all future curriculum reform frameworks, so that it gets the attention required by government bodies and curriculum developers alike. Failure to recognise and address this key barrier results in a lack of sufficient teaching time in mathematics which in turn impacts negatively on students' opportunities to learn (Banicky, 2000).

Another key enabler of successful curriculum reform is co-ordination (Memon, 1997). Curriculum developers in Ireland considered this when designing the revised curriculum and as a result strands were introduced in the post-primary mathematics curriculum that better aligned with the primary school curriculum. However, further work was needed in this regard in terms of educating teachers in relation to presupposed (post-primary teachers) and horizon (primary teachers) knowledge. Without explicit professional development in this regard, any efforts to align revised curricula with existing curricula will result in reform efforts not realising their full potential.

The Teachers' Perceptions of Curriculum Reform study discussed in this paper showed that teachers are in favour of reform but the barriers identified across all three studies influenced their teaching style in a negative way and led them to revert to traditional, less time consuming, teaching methods. The main barriers identified by Irish teachers were time and the breadth of the curriculum. Going forward, curriculum developers need to be conscious of these key barriers and any future framework for curriculum reform need to reflect this awareness.

The authors recognise that work is still needed to improve the ongoing reform efforts in Ireland. By listening to teacher's voices they were able to identify steps that can be taken to improve the standard of mathematics education in Ireland. Simultaneously, these teachers' voices have allowed the authors to pre-empt barriers that can limit the success of curriculum reform internationally and bring these to the attention of future curriculum developers. It is only when such barriers are identified and addressed that curriculum reform will bear the desired fruit.

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REALISTIC MATHEMATICS EDUCATION IN THE NETHERLANDS:TEXTBOOKS AS CARRIERS AND BARRIERS FOR REFORM

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From 1968 on, Realistic Mathematics Education (RME) has evolvedinto the dominant approach to mathematics education in the Netherlands. This paperdescribeshow this reform came into being and furtherdeveloped,and what were decisive factors in this process.The choices that were made about the mathematics curriculum and didacticscompletely changed the textbooks that were on the market thirty-five years later. Of course, this change in the textbooks had consequences on students' learning outcomes. In 2004, students had improved their understanding of number butdeclined in algorithmic calculation. Thisresulted in newrevisions of textbookswith more attention to calculation procedures. At the same time, however, research had shown that in RME-based textbooksmathematical reasoning and problem solving, which are prominent parts of RME,only play a very minor role. To investigate how the Dutch primary school mathematics curriculum could become more mathematical the Beyond Flatlandproject was set up. The paper ends with discussing thetwo different movementsthat are currently taking placein the ongoing reform process.

WISKOBAS: DEVELOPING REALISTIC MATHEMATICS EDUCATION

The development of Realistic Mathematics Education (RME) started in 1968 with the Wiskobas project.The main goal of this projectwas to elaborate an alternative for the –at that timein the Netherlands–prevailing mechanistic mathematics education. Characteristic of thisapproach was its focus on teaching fixed procedures in a step-by-step manner. Real-world problems were only used for applying earlierlearned calculation procedures, and little or no attention was paid to developing insight in the underlying mathematics. Moreover, mathematics was taught in an atomized way, with the teacher demonstrating how to solve each type of problem (see, e.g., Van den Heuvel-Panhuizen & Drijvers, 2014). The then popular New Math was not seen as a suitable alternative for the mechanistic approach. In the words of Freudenthal (1981, p. 141), New Mathrelied on "the wrong perspective of [...] replacing the learner's insight by the adult mathematician's insight".Instead, Wiskobas emphasized that students should get the opportunity to realize what happens in a (mathematical) situation and should be supported to imagine what could happen (De Jong, Treffers, & Wijdeveld, 1975). To this end, students should be presented context situations based on which they could develop new mathematical concepts. For example, in the context situation of the bowling game in Figure 1,first graderswere asked to interpretthe picture,explain in their own words what is happening,and then write this down using mathematical (arrow) language. In this way,students could not only learn to connect different representations of the situation (the pictorial, the verbal, and the symbolic), but they couldalso build up understanding of the relationship between addition and subtraction. This was a break with the mechanistic approach, in which addition and subtraction were initially taught separatelywith first addition and later on subtraction.

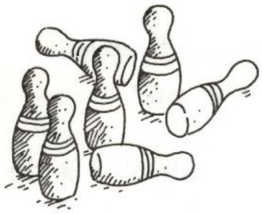

Figure 1:De Jong, Treffers, & Wijdeveld, 1975, p. 40

This Wiskobasapproach of providing students opportunities to come up with their own interpretations of situations and organizing them in their own way, is in line with Freudenthal's ideas of mathematics as a human activity and offering students tasks containing occasions for mathematization instead of transmitting ready-made mathematics to students (Freudenthal, 1968, 1973; Wijdeveld, 1980). Treffers (1978, 1987), one of the leading persons of Wiskobas, later on distinguished horizontal and vertical mathematization, with the first referring to transforming a real-world problem into mathematical terms and the latterto the process of reorganization within the mathematical system resulting in a further generalization of solution processes and formalization.

From 1971 to 1980, the Wiskobas team developed rich material for teaching mathematics,including challenging problems, thematic projects, and a number of outlines for a new primary school curriculum. These materials were developed in close collaboration with teachers, teacher educators, and other professionals in mathematics education. Materials were piloted in a so-called 'design school' and then made available for other schools and mathematics education working groups, and discussed in coursesand at conferences for teachers and for teacher educators. Most of the materials were published in the professional journal *Wiskobas Bulletin*.Over the years, the Wiskobas team published a total of eleven curriculum documents in which a detailed view was presented of what primary school mathematics education according to Wiskobas should imply. All these publications were open for discussion. In 1980,the Wiskobas project formally came to an enddue to a government-determined reorganization, which meant that the research and development of mathematics education were accommodated at different institutions. Nevertheless, the Wiskobas' way of working continued in other forms of cooperation.

FURTHER DEVELOPMENT OF REALISTIC MATHEMATICS EDUCATION

In 1983, the Dutch Association for the Development of Mathematics Education (NVORWO), in which all the relevant players in the field were reunited, initiated the development of a national program for primary school mathematics. The first step consisted of a nationwide consultation of professionals about proposed key points for this program (Treffers & De Moor, 1984).These points, based upon the work of Wiskobas and the further development of RME since then, reflected a real shift in the mathematical curriculum content and didactics. Among other things,the general idea was that context situations should have a central role in mathematics education, not just for application at the end of a learning process, but also in the beginning as a source for developing mathematical concepts.Further, it was proposed to spend less time on algorithmic digit-based calculation in favor of insightful whole number calculation and estimation.Of the almost 300 respondents – teachers, teacher educators, school counselors, and school inspectors – a large majority of 91 percent agreed with the proposal as a whole. For almost all proposed key points, the agreement was well above 90 percent (Cadot & Vroegindeweij, 1986).

In the next years,Treffers, with the help of many mathematics didacticians, teacher educators, developers, and researchers of mathematics education worked on the elaboration of the intended national program. Following the tradition of Wiskobas, provisional parts were published and open for discussion, through the newly established journal*Tijdschrift voor Nascholing en Onderzoek van het Reken-wiskundeonderwijs* [Journal for Professional Development and Researchof Mathematics Education]. At the end of the 1980s, this work resulted in the publication of the *Proeve van een Nationaal Programma voor het Reken-wiskundeonderwijs op de basisschool* [Design of a National Program for Mathematics Education in Primary School] (Treffers, De Moor, & Feijs, 1989). This*Proeve*was the culmination of two decades of development, research, theory building, and discussion, resulting in a description of the learning goals and didactics for primary school mathematics.A few years later, the government, for the first time in Dutch history, established statutory Core Goals for the end of primary school (OCW, 1993) and for mathematics these Core Goals were almost completely based on the*Proeve*. Now, also the government statedthat “education in mathematics aims at students being able to make connections between the education in mathematics and their experiences from daily life,acquire basic skills, understand simple mathematics and apply it in practical situations, [...] and can use research and reasoning skills and describe these in their own words” (OCW, 1993, p. 19). The official establishment of these goals was a kind of confirmation that the bottom-up curriculum reform started by Wiskobas in 1968 had become legitimized by the government in 1993.In 1997, this support was continued when the Ministry of Education commissioned the Freudenthal Institute to develop the *TAL*teaching-learning trajectories for primary school mathematics (e.g., Van den Heuvel-Panhuizen, 2008).

CORE IDEAS OF REALISTIC MATHEMATICS EDUCATION

In the *Proeve*, RME was characterized a reconstruction-oriented didactics with the following five learning and instruction principles:constructing and making concrete; levels and models; reflection and own productions; social context and interaction; structuring and intertwining (Treffers, De Moor, & Feijs, 1989). Later on, Van den Heuvel-Panhuizen (2000, 2001) revisited this description and identified the following six principles of RME: the activity principle, the reality principle, the level principle, the intertwinement principle, the interactivity principle, and the guidance principle. In these principles, the main ideas of mathematizing and the importance of contexts are incorporated.

The *activity principle* refers to interpreting mathematics as a human activity. It also emphasizes that mathematics is best learned by doing mathematics. In RME, students are active participants in their learning process.The *reality principle* means starting from situations that are meaningful to students. Students can imagine these situations that are therefore real in their mind, which offers them opportunities to attach meaning to the mathematical concepts they develop understanding of while solving problems. The *level principle* underlines that learning mathematics implies students passing through various levels of understanding: from the ability to invent informal context-related solutions, to the creation of various shortcuts and schematizations, to the acquisition of insight into how concepts and strategies are related. Models serve as an important device for bridging the gap between informal context-related mathematics and more formal mathematics.The *intertwinement principle* means that mathematical domains such as number, geometry, measurement, and data handling are not considered as isolated curriculum chapters but as heavily integrated. This principle also applies within domains. For example, within the domain of number, mental arithmetic, estimation, and algorithms are taught in close connection to each other.The *interactivity principle* signifies that learning mathematics is not only an individual activity but also a social activity. Whole-class discussions and group work offer students opportunities to share their strategies and inventions. In this way, students can get ideas for improving their strategies. Moreover, interaction evokes reflection, which enables students to reach a higher level of understanding. Finally, the *guidance principle* means that students are provided with a guided opportunity to re-invent mathematics. This implies that teachers have a pro-active role in students' learning and that educational programs should contain scenarios that have the potential to work as a lever to shift students' understanding to a higher level. To realize this, the teaching and the programs should be based on coherent longitudinal teaching-learning trajectories, which are provided by *TAL*.

FROM REFORM IDEAS TO REFORM-BASED TEXTBOOKS

Textbook development in the Netherlands is a kind of free enterprise. The government nor any other authority is ordering the productions of textbooks, is involved in designing them or has to approve them before they are put on the market. This means that there are hardly restrictions in developing and publishing textbooks other than concerns about market share, since all textbooks are put on the market by commercial publishers. Under this condition of "freedom of design" (Van Zanten & Van den Heuvel-Panhuizen, 2014) a change took place towards reform-based textbooks. Already around 1980, when most textbooks were still mechanistic, in several new textbook series Wiskobas'ideas were incorporated (Treffers, 1980). Over the decades, the market share of RME-based textbooks increased steadily and in 2004 all available textbook series wereRME-based (Figure 2). Publishers also presented their textbooks explicitly as 'realistic' as a marketing strategy. Of course, that alone does not indicate to what degree these textbooks actually include RME characteristics. However, despite the differences between textbook series, in all textbooks on the market in 2004and in almost all current textbook series, core ideas of RME are present.

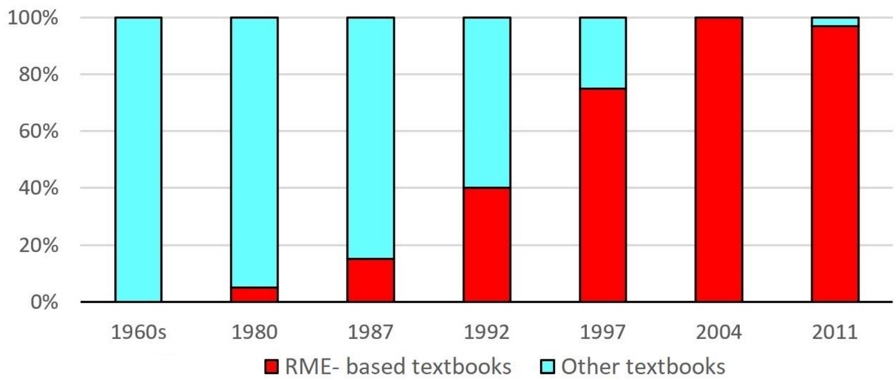

Figure 2:Proportion of market share of RME-based textbooks and other textbooks

A characteristic present in all RME-based textbook seriesis the use of context situations that can be mathematized and help students to come up with representations and strategies that contribute to the development of their mathematical knowledge and understanding. This characteristic unmistakably reflects the reality principle. Moreover, in all these textbook series the level principle can be recognized in the use of models that form a bridge from more context-connected informal solutions to more formal solutions and thus the understanding of mathematics. Figure 3 shows a task from anRME-based textbook for Grade 2, in which both principles are used to support students to develop a broad understanding of the subtraction operation; not only meaning taking away but also determining the difference, and solving subtraction problemsby an adding-on strategy. The context situation and picture make that the model of the number line comes up in a natural and self-explaining way. Similar to what was suggested thirty-five years earlier by Wiskobas (see Figure 1) students are asked to describe the situation with mathematical language, and again, by use of the number line, the relationship between addition and subtraction is emphasized.

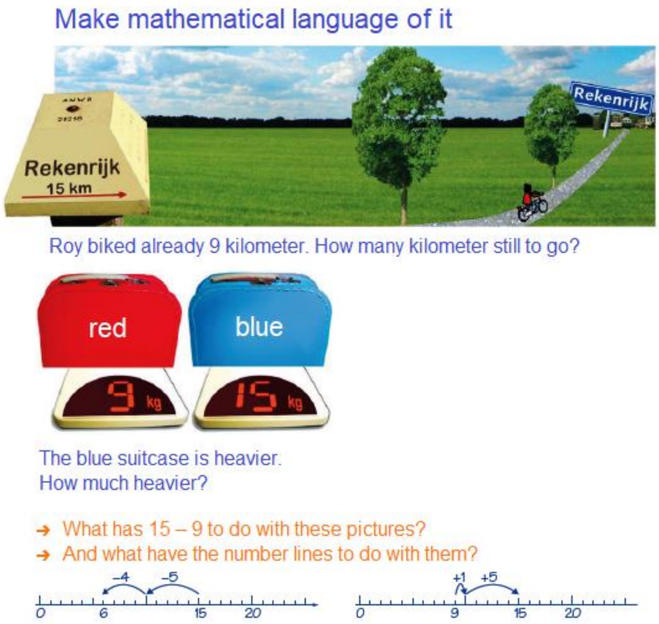

Figure 3:Subtraction problems from the textbook series*Rekenrijk*, Grade 2 (2009)

Also, the shift in mathematical content that was proposed as a result of the consultation carried out halfway the 1980s, is from 2004on clearly reflected in the RME-basedtextbooks. In agreement with the *Proeve* and *TAL*, in these textbooks there is less emphasis on digit-based algorithmic written calculation and more attention for whole-number-based written calculation, mental calculation and estimation.

NEW MOVEMENTS IN THE REFORM

More attention to calculation skills

Of course, these changes in the textbooks had consequences for students' learning outcomes. This was revealed by the PPON studies of Cito, the Netherlands national institute for educational measurement. Until recently, in these studies, every so many years the educational outcomes of were assessed. In the 2004 PPON it was found that students' proficiency in number, number relations and estimation had significantly improved since the first PPON in 1987, while achievement in written calculation had significantly decreased (Janssen, Van der Schoot, & Hemker, 2005). Although not surprising, the latter result was cause for public debate about the quality of mathematics education. Especially in the media, this debate became quite heated and some opponents of RME suggested a return to traditional mathematics education (see, e.g., Van den Heuvel-Panhuizen, 2010). Although the Royal Netherlands Academy of Sciences after an investigation concluded that there was no evidence for qualifying one approach better than the other (KNAW, 2009), the debate did have an influence on textbooks published since then. Again, more emphasis is put on written calculation, including digit-based algorithmic procedures. However, in most of these textbooks, the calculation trajectory that is followed globally reflects the structure as described inthe *Proeve* and *TAL*, starting with a phase of transparent whole-number-based written calculation. This means that RME characteristics are upheld in most current textbooks. Yet, publishers refrain from using the term 'realistic' for marketing purposes.

Beyond Flatland

Apart from this movement back to a more mechanistic approach instigated by the declining scoresof students on procedural calculation, there was also another movement backwards. This was the movement towards the origins of RME in which there was much attention to mathematical reasoning and problem solving. At about the same time when the 2004 PPON study was published a small-scale study (Van den Heuvel-Panhuizen & Bodin, 2004) was carried out with non-routine puzzle-like problems. The problems turned out to be very difficult, even for high-achieving students. This worked as a wake-up call for didacticians. Thetextbook analysis (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009) that was carried out hereafter, revealed that the RME-based textbooks in use then,largelycontained straightforward problems. This meant that students were hardly offered the opportunity to learn problem solving. A recent replication of this study (Van Zanten & Van den Heuvel-Panhuizen, 2018)showed that the situation in current RME-based textbooks has not changed since then and that mathematical reasoning and problem solving are even not mentioned any more in the most recent statutorygoals for mathematics education. Therefore in 2015, the Beyond Flatland project was set up to investigate how the Dutch primary school mathematics curriculum can be made more mathematical, that is, by including more mathematical reasoning and genuine problem solving.

In 1980, when discussing the textbooks that were in use and being developed at that time, De Moor and Treffers reflected upon the mostly traditional (calculations) content that was included in these textbooks and the more mathematical content they envisioned to be included. In their discussion, they explicitly stated that the "newer" contents of "relations and functions, probability and statistics, […] [and] using graphs" (Wiskobas team, 1980, p. 230) should also become part of the textbooks, possibly in separate sections, but preferably in an integrated manner with the traditional content. Notwithstanding this advice, the three content domains of (early) algebra, probability, and graphs (dynamic data modelling, graphical reasoning) were mostly neglected in the primary school mathematics textbooks that appeared in the ensuing decades.In fact, the focus was mainly on plain calculation problems. Therefore, now in the Beyond Flatland project,lesson series have been developed inwhich these content domains play a central role. At the same time this project offers a new arena to explore how in vein of the RME tradition, the RME principles can be enriched with ideas from recent theories about learningby incorporating insights of embodied cognition (grounding mathematical concepts in bodily experiences), representational redescription (making implicit understanding more explicit by verbal-symbolic representations), and variation theory (acquiring understanding of key aspects of concepts by experiencing variation).

TO CONCLUDE

A lesson that can be learned from the history of the reform towards RME isthat it was a long-lasting process, which is still going on.Even today experiences from practice call for new research and the development of new local RME instruction theories. Thus, contrary to what is sometimes thought, RME is not a fixed and finished theory of mathematics education (see Van den Heuvel-Panhuizen, 2001).

A further characteristic ofthe Dutch reform process, which certainly also appliesto reforms in other countries, is its complexity and the many and iterative steps that have to be taken: from feeling the need to innovate, getting ideas for reform, trying them out in practice and receiving new ideas while doing this, discussing them, again trying them out in practice, discussing the materials and teaching methods with other professionals including textbook authors, and in the meantime continuously working on the implementation through teacher education and offering opportunities for professional development. These steps reflect the many influential players in the field of reform, not to mention the many determining factors such as the circumstances in education practice, the knowledgeof teachers,changes in student population and the educational policy.

Professional development is indispensable to achieve a successfully developed and implemented reform. However, in the Netherlands, until now in-service training for teachersis not compulsory. Therefore, textbooks played and still play a crucial role in bringing the reform to teachers and eventually to classrooms. The steady increase of RME-based textbooks from 1980 on made it possible forteachers to becomeacquainted with RME. In this sense, the textbooks were the carriers of the reform. Yet, instead of carriers they can also work as barriers. Due to the freedom publishers in the Netherlands have to determine what kind of textbook they bring to the market,the publishers could, most likely motivated by commercial motives, recentlymake the move back to a more mechanistic approach. Remarkably, this same barrier function also applied to the RME-based textbooks with respect to the absence in these textbooks of offering opportunities for problem solving and mathematical reasoning. In addition, also for other aspects of RME the implementation fidelity in some RME-based textbooks can be questioned.

Finally, this paper only focuses on the role of textbooks, but of course the full story of a reform cannot be told without taking the actual classroom practice into account. Looking at the RME reform from that perspective would show that it is hard to bring ideas and principles to life. More work has to be done to really implement RME in practice.

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THE IMPACT OF HISTORY EVENT AND GLOBAL TENDENCY ON MATHEMATIC CURRICULUM REFORM IN VIETNAM: TWO CASE STUDIES

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In education and training, the improvement of curricula, textbooks and teaching methods is important for the whole Vietnam's society as it directly affects people—the human resource for the country. Some questions to discuss are: What mathematical curriculum reforms from the last decades are considered the most important in Vietnam? What mathematical contents were chosen to be added/removed? What are their characteristics? What is the effect of adding or removing to teaching and learning? What is the effect on the behaviour of the student in his personal relationship to a certain concrete mathematical object? From the point of view of didactics of mathematics, our paper focuses on two specific mathematical objects: the concept of integral and the concept of probability. These objects point out some of the highlights for Vietnam's educational picture under the intrinsic effects of the historical event of the country and, from the outside, of the world's educational orientation in the past forty years.

1. CONTEXT

1.1. Some historical landmarks to point[1]

Throughout its history, Vietnam had a time of separation in two large regions (between 1954 and 1975): The North of Vietnam whose regime was communist and the South of Vietnam which followed capitalist. After the great event called *Liberation of the South* (dated April 30th, 1975), Vietnam reunited by becoming a fully communist country. Looking at the image of Vietnamese education at that time, we could see that there were two programs in the general curriculum with two different sets of textbooks until 1990: a 10-year program for the North and another 12-year for the South. During these years, there were always two different set of exams for the national baccalaureate exam according to the two different programs.

There is not only this difference in general training duration. The political difference involved two distinct programs and we would like to cite an example in the case of mathematics: the presence of the notion of integral in the program and manual of grade 12 (terminal class) in the South (influenced by the French program), against its absence in grade 10 (terminal class) in the North (close to the Russian program).

This fact made a lot of difficulties for mathematics teachers who worked in northern Vietnam and then continued their job after immigration to the South because of their lack of experience in the teaching of integrals. We will return to this issue later on.

The level distribution of the different programs could be presented such as:

[1] In this paper, we concentrate on the Vietnam system of education since 1975.

Level	Nord of Vietnam			South of Vietnam
	1975-1981 10-years duration	1981-1992 11-years duration	From 1992 12-years duration	From 1975 12 years duration
Primary	4 years (grades 1^{st}, 2^{nd}, 3^{rd}, 4^{th})	5 years (grades 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th})		
Upper school	3 years (grades 5^{th}, 6^{th}, 7^{th})	3 years (grades 6^{th}, 7^{th}, 8^{th})	4 years (grade 6^{th}, 7^{th}, 8^{th}, 9^{th})	
High school	3 years (grades 8^{th}, 9^{th}, 10^{th})	3 years (grades 9^{th}, 10^{th}, 11^{th})	3 years (grades 10^{th}, 11^{th}, 12^{th})	

Table 1: Comparison of level distribution within different programs (from 1975)

1.2. Some main points of Vietnam mathematical curriculum

For more than forty years (from 1975), Vietnamese mathematics curricula and textbooks had changed twice:

- In 1990, the first curriculum reform for general education was applied entirely in Vietnam. As the country was reunified, this reform produced a unique and compulsory curriculum for 12-years general education. It existed two sets of textbooks compiled by two different authors groups with a weak difference on their contents (one set is used in the North and the second in the South). The highlight of the mathematic program at this period is that the *integral concept* was introduced in grade 12^{th} for the whole Vietnam. Since this reform, the concept of integral had appeared in the Baccalaureate's mathematics examination and also in the entrance examination to the university. The emergence of the integral concept in the Vietnamese mathematics program is an illustration of the influence of historical factors on the change of the mathematics curriculum: the unification of the country leads to the unification of the curriculum and the contents. We will present it as a first case study with its evolution.

- The second reform[2] was in 2006 with two new programs and two sets of textbooks for the two sections called *basic section* and *advanced section*. The choice of the programs and textbooks was left to the students and their parents. Therefore, the students having the same choice have to be organized in the same class. The highlight of the mathematic program at this period is that the *probability concept* is inserted in the school mathematics curriculum in grade 11^{th} for the first time (until now). Why this choice? The answer found in the guide book for teachers shows that the new aims are to perfect the knowledge, to apply mathematic knowledge to solve real life problems, to link to other subjects (interdisciplinary) and a special reason is to be close to the international mathematics curriculum. We know that the probability concept has been in the international math curriculum for a long time, and it only recently appeared in the math textbooks in Vietnam (12 years ago). Therefore, we could consider it as the effect of the global trend on Vietnamese programs, even late. The probability concept will take its part in second case study.

[2] In 2003, there was a experimental phase to apply the new high school curriculum and textbooks for three years including two sets of textbooks: one for the section natural sciences and the other for section social sciences.

There are also others new concepts in statistic (such as frequency, median, variance, standard deviation, 10th grade) and complex numbers (12th grade) which also appeared for the first time in the curriculum too but we let them for another research.

2. CASE STUDIES

2.1. Integral concept: evolution in contents and teaching methods

According to the historical characteristics of Vietnam, the integral concept was studied in four periods:

- The pre-reform period 1975-1990: the two concepts *indefinite integral* and *definite integral* are present (exclusively for the South Vietnamese mathematics curriculum).
- The first reform period 1990-2000: the curriculum and textbooks have been changed, and the two concepts of primitive and integral have been officially present for the whole country until now.
- Preparatory period for the second reform 2000-2006: the program remains the same and only the textbook is revised.
- Second reform period 2006-present: The curriculum and textbooks have been renovated.

The following table summarizes the definition, naming, and symbol of the concept of integration with the presence / absence of the preceding proposition in the four phases above:

<table>
<tr><th>Period</th><th>1975-1990</th><th>1990-2000</th><th>2000-2006</th><th>2006 to now</th></tr>
<tr><td rowspan="2">Concept of primitive</td><td rowspan="2">primitive on (a; b) or [a; b]
Notation: $\int f(x)dx$</td><td>primitive on (a; b)</td><td>primitive on
(a; b) or [a; b]</td><td>primitive on an open /close interval K</td></tr>
<tr><td colspan="3">Notation : none</td></tr>
<tr><td>Concept of indefinite integral</td><td>indefinite integral: set of all primitives
Notation: $\int f(x)dx$</td><td>None</td><td>family of primitives
Notation: $\int f(x)dx$</td><td>family of primitives
Notation: $\int f(x)dx + C$</td></tr>
<tr><td>Concept of definite integral</td><td>definite integral
Notation : $\int_a^b f(x)dx$</td><td colspan="3">integral
Notation : $\int_a^b f(x)dx$</td></tr>
<tr><td rowspan="2">Definition</td><td rowspan="2">Common limit of sums of Riemann $\int_a^b f(x)\mathrm{d}x = \lim_{\mathrm{d}\to 0}\sum_{i=1}^{n} f(\xi_i)(x_i\text{-}x_{i\text{-}1})$</td><td colspan="3">Newton-Leibniz's formula $\int_a^b f(x)dx = F(b)\text{-}F(a)$</td></tr>
<tr><td>F: a primitive of f on (α; β) containing [a; b]</td><td>F: a primitive of function f on open interval K (K contains $[a; b]$)</td><td>F: a primitive of function f on $[a; b]$</td></tr>
</table>

Table 2. Introduce of primitive, definite integral and integral concepts in term of periods

According to Tran Luong (2002), from the perspective of the didactic transformation, the concept of integration has been presented as the generalization of the problem of the trapezoidal area and this type of problem becomes the motor to form the integral concept. In other words, the integral becomes a type of "algebraic" area (we know that the integral theory of Riemann serve to develop a function having an infinite number of discontinuous points in a Fourier series).

Another remark is that the textbooks of two intermediate periods do not respect their definitions by considering that all continuous functions on any interval are always integrable. In particular, although the following integral does not exist (according to the concept definition of integral in the manual of the third period), in a textbook of the first reform period, it is asked to calculate:

$$I = \int_0^2 \sqrt{4 - x^2}\, \mathrm{d}x$$

And the solution given by the textbook is:

> $\mathrm{I} = \pi$ although I does not exist as we cannot find any interval $(\alpha; \beta)$ containing $[0; 2]$ so that the function $y = \sqrt{4 - x^2}$ is continuous on it.

The absence of verification of the integral's existence condition leads to the existence of a didactic contract rule (that has been verified by an experimentation by Tran Luong (2002)):

> Students do not have the responsibility to verify the integrability condition of a function when calculating its definite integral on an interval.

This result also provides a partial answer to the research question: "In the teaching and learning of integral notions, what difficulties do teachers and students encounter? Are these difficulties the consequence of the teaching choices or of the presentation of the integral concept itself?"

The case of the non–existence of the integral of the exercise above was noted by the authors of the current grade 12 mathematics textbooks so that the same error disappears in the current version.

2.2. Probability concept: choice of Vietnamese institution

Referring to the four previous phases, the notion of probability was only introduced in the mathematics curriculum in Vietnam in the second reform, from 2006 to the present. Vu Nhu (2005) [3] shows that there are three approaches to the notion of probability:

> Laplace approach: the probability of an event is "the ratio between the number of cases and the number of possible cases". The calculation of probabilities reduces to counting and so combinatorics takes the main role in this calculation.
>
> Statistical approach: the probability of an event is a number that the frequencies of this event oscillate stably around it during a great number of random experiments. It is called objective probability because its value is approximated only after experimentation.
>
> Axiomatic approach: probability is defined as "a bounded positive measure defined on an abstract set that models possible outcomes of a random experiment" by satisfying a system of axioms.

[3] This research was conducted on pilot probability statistic and pilot textbooks that are very close to the current textbook.

This study has also highlighted that the Vietnamese institution has chosen the classic approach (Laplace approach) to present this concept (Algebra and Analytics 11, Text book, p. 66):

> Suppose that the experiment T has a set of outcomes Ω which is a finite set and the outcomes of T are equiprobable. If A is an event in relation to the experiment T and if Ω_A is the set of outcomes describing A, then the probability of A is a number, signed by $P(A)$, and determined by the following formula:
>
> $$P(A) = \frac{|\Omega_A|}{|\Omega|}$$

According to this choice of the Vietnamese institution, all the random experiments mentioned in the manuals are experiments with equiprobable events. Therefore, students do not check if the outcomes are equiprobable when they have to calculate the probability of an event. This result is expressed as a didactical contract and proven by an experimentation for students in a class 11^{th} grade (see Vu Nhu T.H. (2005)). In another experimentation, done by Tran T.A. (2007), when the teacher asked to find the probability of event "get the 5-point face" when launching a 6-sided die that is cut in a corner and becomes a die of "seven faces", there were some students who answered that the probability was 1/7. This shows that the student applied the Laplace formula for the calculation, although this dice is not balanced.

Figure 1. Dice of "seven faces" is used in the experiment

Another experiment in the form of didactic engineering was developed and experimented for the 11^{th} grade students for the purpose of modifying the student's personal relationship to the notion of probability (see Vu Nhu, 2005, p. 68-74).

2.3. Lack of links between the two concepts. And what's the need for the future?

Regarding statistics and probability, the Vietnam mathematics program does not introduce the concept of standard normal distribution, so there is no probability density function nor bell curve. This absence of standard normal distribution makes a lost the opportunity to link these two concepts, integral and probability, although the concept of probability would be defined later through the concept of infinity integral in university.

The new suggested research question is: what is the candidate object which can be chosen by Vietnamese institution to make a link between the concepts of integral and probability?

3. CONCLUSIONS AND PERSPECTIVES

The two topics selected in this paper as a representative example of the above study show the impact of historical factors and the problem of globalization orientation, which has a real impact on changing the math curriculum in Vietnam. Positive points are the presence of new mathematical objects in the mathematics curriculum:

- The reunification of Vietnam led to the introduction of the integration concept in mathematics curriculum: after a 40-year inclusion in the general mathematics curriculum, the integration concept has been modified and is now relatively stable in the current program.

- Globalisation gave the occasion to introduce the concept of probability: this concept has been for 12 years in the mathematics curriculum in Vietnam and has not been changed so far.

In particular, as discussed above, the two concepts of integration and probability are taught independently, without any articulation between them.

New perspectives and new challenges:

In January 2018, the Ministry of Education and Training (MOET) announced the project of the new education program. This program will apply from the 2019-2020 school year. According to the details of the mathematics program, we find out an emphasis that is introduction of statistics and probabilities from primary to high school. The statistical and probabilistic elements are arranged consecutively from 2^{th} grade to 12^{th} grade. An interesting point is the presence for the first time of the Bernoulli distribution and the binary distribution in the probability section. It means that they are introduced in same level as the concept of integration(12^{th} grade). The drafting of the new manual has not finished, so it is still too early to draw a complete conclusion. But this seems to offer an opportunity for the expected connection between the two objects studied above, in particular, and other mathematical knowledge, in general.

So new challenges for the authors of mathematics textbooks are suggested:

- connecting the concepts of integration and probability through standard distribution, which problems can be the candidates?

- linking two concepts of integration and probability is the intention or desire of the institution teaching mathematics in Vietnam?

We hope to have a chance to return to discuss these questions in the next time.

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THE EVOLUTION OF THE KNOWLEDGE TO BE TAUGHT THROUGH EDUCATIONAL REFORMS: THE CASE OF PROPORTIONALITY

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We use the theory of didactic transposition to explain the evolution of the knowledge to be taught around proportionality from classical mathematics up to the present time, paying special attention to the effects of the New Math reform. What happened to be an important element at the core of old school mathematics, the "Theory of Ratios and Proportions", was intended to be replaced by the new language of sets, variables and functions. After the failure of the New Math reform, some elements of the classical curriculum were partially restored. This gave rise to a hybrid organization where proportions remain disconnected from equations and functions, and where the notion of quantity has trouble finding its place. Through the case of proportionality, we show how didactic transposition appears as a key tool to explain the decisions taken in the constitution of new curricula and their effects on the structure and coherence of the knowledge to be taught, that is, on the efficacy of the knowledge tools students are required to learn.

INTRODUCTION: PROPORTIONALITY AS KNOWLEDGE TO BE TAUGHT

In our societies, the educational contract is usually established around a set of knowledge objects and activities structured in disciplines, subject areas, blocks of contents, notions and tasks, which have recently been enriched with new entities such as skills, competencies and values. By adopting a broad notion of knowledge, all these entities form what has been called the *knowledge to be taught* in the theory of the didactic transposition (Chevallard, 1985; Chevallard and Bosch, 2014). Research in mathematics education has to pay attention to the conditions under which such a process of elaboration of the knowledge to be taught is performed—usually over long periods of time—, to the main agents taking part in it, and to the criteria and assumptions underlying the decisions made. To do otherwise would mean considering the knowledge to be taught as an unquestionable object; as if there was only one possible way to select, organize and name the specific content and activities proposed to be learnt at school.

Our aim is to use the didactic transposition methodology to explain the evolution of the knowledge to be taught over a long period in the case of a given mathematical content: proportionality. We will show how what is taught today as "proportionality" and the position of this content in the global mathematical curriculum can be explained as the result of choices and elaborations based on different historical constructions of the scholarly mathematical knowledge. This highlights the importance for research to take into account, not only the process of didactic transposition, but the different entities it brings into the open and that provides the raw material of curriculum reforms.

Proportionality or proportional reasoning has been the object of numerous investigations in mathematics education since the 1970s (a short summary can be found in Adjiage & Pluvinage, 2007). Most of this research deals with students' difficulties with proportional reasoning, especially with regard to discriminating multiplicative from additive situations. Given the increasing importance attributed to the way mathematics is taught to explain student performance, investigations from different countries have started focusing on the analysis of textbook material related to proportionality. For instance, Shield and Dole (2002, 2013) analyze different Australian textbooks to explore how knowledge connections and proportional reasoning are promoted. Their work is followed by Ahl (2016) in the case of Swedish textbooks. Da Ponte and Marques (2011) present a description of proportion tasks in mathematics textbooks for middle school students in Portugal, Spain, Brazil, and the US. The French case is analyzed in detailed by Hersant (2005), covering a long period of time, from the beginning of the 19th century up until now. Enlarging the mathematical domain considered, in her analysis of Indonesian textbooks, Wijayanti (2015) includes the treatment of the theme of proportion in three content blocks: geometry (similarity), arithmetic (ratio and proportion), and algebra (linear functions). She examines the way these themes are linked to each other, as well as the connections that remain absent, essentially due to the underdevelopment of the algebraic and functional tools at this level.

At the risk of generalizing, we can say that nowadays the mathematical knowledge to be taught at lower secondary school usually includes proportionality (or proportion) in three different domains or blocks of contents. Its more direct presence is in arithmetic, including the theme of ratios and proportions and the related theme of percentages; it also appears as the first and simplest relationship between quantities in the form of the function of proportionality (or linear function); and finally, it is also part of the geometrical work with the study of similarities between figures.

This knowledge organization is relatively stable and shared in different countries. It is the result of different educational reforms over a long period. How can the didactic transposition process explain it current form? Where do the current knowledge to be taught about proportionality come from? Why does it have the form it has? How has it been selected, designated, shaped, organized and arranged? What is its role in relation to the other pieces of mathematical knowledge? A curriculum reform is an intention of strongly modifying the way a didactic transposition process takes place. Better knowing the mechanisms and constraints of this process thus appears as a crucial issue in any research about curriculum reform.

DIDACTIC TRANSPOSITION TO ANALYSE CURRICULUM CONSTRAINTS

The core of the educational contract is the result of complex historical processes where different agents (members of the educational system and scholars from different fields of knowledge) elaborate bodies of knowledge and resources that concretize what has to be taught and learnt at school. The notion of didactic transposition was introduced to take into consideration these type of processes. These agents are part of what we call the *noosphere*, a layer surrounding the educational system, a kind of "membrane" of those who think (*noos*) about the educational system. The elaboration of the knowledge to be taught, by (and within) the noosphere, is not a creation that starts from scratch. What students have to learn is not an invention of the school. It comes from knowledge—always in the broad sense that includes activities, skills and values—existing outside the school, within what is called the *scholarly* institution. Therefore, the educational contract proceeds by the designation of some pieces or bodies of *scholarly knowledge* students have to learn. The selected scholarly knowledge is then *transposed* to fulfil the requirements to be taught at school. This is what we call the *external* didactic transposition. The term "transposition" includes the assumption that the bodies of scholarly knowledge are not just disseminated into school but need to be transformed into something "teachable", adapted to the school conditions. More specifically, they have to be decontextualized, depersonalized and sequentialized (Chevallard, 1985), but always preserving a certain similarity with the original scholarly knowledge they claim to be. We will not consider here the *internal* didactic transposition that corresponds to the transformations followed by the knowledge to be taught inside the educational system until it becomes *knowledge actually taught*.

As explained in (Bosch & Gascón, 2006), the analysis of didactic transposition requires a certain general epistemological model to describe the different types of knowledge in a unified way, so as not to introduce value distinctions between them. Even if the scholarly knowledge provides the knowledge to be taught with legitimacy, both should be questioned, scrutinised and analysed in the same terms. In our case, we describe knowledge and activities using *praxeologies*, that is, organisations of types of tasks and techniques—the *praxis*—and theoretical elements to describe, justify and structure this praxis—the *logos*.

The didactic transposition analysis poses a methodological problem about the kind of evidence that has to be gathered and the scope of the object of study considered. The entity "knowledge to be taught" is not easy to understand since it is not an official entity in the educational system. Therefore, it has to be delimited and shaped by researchers, and it has to be proposed as a model to study didactic phenomena, not as an empirical reality in itself. In this study, the unit of analysis used can be broadly defined as the mainstream of the knowledge to be taught in relation to proportionality, at least considering the countries we have information about: Indonesia as well as Spanish, French, Portuguese and English-speaking countries. The following discussion relies on data of books from different periods and countries, distinguishing "scholarly" books and "textbooks", even if in practice there might sometimes be a combination of both—as many scholarly books also have an educational purpose. Because of space limitations, only a very small sample of empirical evidence will be given. More can be found in Bosch (1994), Hersant (2005) and Wijayanti & Winsløw (2017).

We will only consider three main periods of time that we match with three main types of curriculum organizations. We will call the first one "classical mathematics", using the distinction proposed by the mathematician Atiyah (2002) when he presented his vision on how the world of mathematics changed around the 20th century. The second period corresponds to the New Math (or Modern Math) reform that took place between 1960- and 1980, depending on the country (Kilpatrick, 2012). This international reform that affected many countries was led by the conviction that "mathematics has to act as a driving force in the development of hard sciences and of human and social sciences as well, in citizens' daily lives, and, beyond that, in the modernization of society and particularly at school" (Gispert, 2014, p. 238). The era that began with the "modern mathematics" reform was abandoned in the early 1980s "in favor of a teaching method that, envisioning mathematics in the diversity of its applications, placed the accent on problem solving and favored 'applied' components of the discipline" (Gispert, 2014, p. 239).

PROPORTIONALITY IN CLASSICAL MATHEMATICS

Scholarly knowledge. In classical mathematics, ratio and proportions were an important tool in all domains. Mathematicians used to describe the (mathematical, physical and social) world in terms of ratios and proportions between quantities. Proportions appear as the "basic language" to work with relationships between arithmetical, geometrical and physical entities. For instance, Newton's second law of motion was formulated in 1687 as "The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed". Two centuries later, it is still the main language of mathematicians and physicists:

> The increments of volume of bodies are in general proportional to the increments of the quantities of heat which produce the dilatations (Fourier & Freeman, 1878, p. 28)

In classical scholarly mathematics, proportion also appears as a structured body of knowledge. For instance, Euler's *Elements of Algebra* (first published in German in 1765 and translated into English from its French version in 1828) is organized in two parts: "Analysis of Determinate Quantities", covering from arithmetical operations with numbers to the resolution of algebraic equations with one unknown, and "Analysis of Indeterminate Quantities", addressing more complex equations with more than one unknown. In the first part, a whole section III (50 pages) is devoted to the theme "Ratios and Proportions" and contains 13 chapters:

> (1) Arithmetical ratio, or the difference between two numbers (2) Arithmetical proportion (3) Arithmetical progressions (4) The summation of arithmetical progressions (5) Figurate, or polygonal numbers (6) Geometrical ratio (7) The greatest common divisor of two given numbers (8) Geometrical proportions (9) Observations on the rules of proportions and their utility (10) Compound relations (11) Geometrical progressions (12) Infinite decimal fractions (13) Calculation of interest.

This is the structure of what can be called the "classical organization" around proportionality. It was the basis of the mathematical organizations found in textbooks for many years, till the arrival of the New Math reform in the 1960s. The definition of different types of (arithmetical and geometrical) ratio were included, together with their main properties and transformations. A rather developed work including the transformation of geometrical proportions followed, using some proper terminology (the "extremes" and "means" of the proportion, the first and last members, etc.). The work showed a particular way to solve practical problems regarding different social and commercial situations through the establishment and resolution of one or several proportions between quantities of a different nature. This brief description aims at providing some evidence on the fact that there existed an elaborate body of knowledge called "Theory of ratios and proportions", which played a central role in the school and scholarly mathematics of the classical period. It appeared as the main tool to describe relationships between quantities, in a way similar to how we use functions today.

Knowledge to be taught. When we look at the knowledge to be taught at that same period, we find a transposed version of the Theory of Ratios and Proportions, usually under the same name. This organization appears in Arithmetic books in a simplified version, with few theoretical elements and many practical cases to be solved using different versions of the rule of three (simple direct or inverse, multiple or compound): commission, brokerage, and insurance; discount; equation for payments; stocks; bankruptcy; partnership; exchange (Hotson, 1842). In also appears in Algebra books to provide a more general presentation, using letters and further developing the theoretical elements (properties of the ratios and proportions) to obtain, in certain cases, highly developed calculations that could compete with the current work with equations (figure 1). The theory of ratios and proportions was a core part of classical mathematics related to the arithmetical and algebraic work. This kind of content organization is found in many school and college textbooks of arithmetic and algebra in the late 19th and the first half of the 20th century, until the New Math reform.

Find two numbers, the greater of which shall be to the less as their sum to 42, and their difference to 6.

Let x be the greater, and y the less: then, $x : y :: x + y : 42$ and $x : y :: x - y : 6$.

$$\therefore \text{(Art. 146) } x + y : 42 :: x - y : 6$$
$$\therefore \text{alt}^{\text{do.}},\ x + y : x - y :: 42 : 6 \text{ or } :: 7 : 1$$
$$\therefore \text{comp}^{\text{do.}} \ \& \ \text{div}^{\text{do.}}\ 2x : 2y :: 8 : 6$$
$$\text{or } x : y :: 4 : 3$$

$$\therefore x = \frac{4y}{3}$$
$$\text{and } 4 : 3 :: \frac{y}{3} : 6$$
$$\text{whence } y = 24, \text{ and } \therefore x = \frac{4y}{3} = 32.$$

Figure 1. Problem from Hotson (1842) using the "algebra of proportions"

In summary, what we call the classical organization of proportionality can be described in praxeological terms as follows:

- The *praxis* was mainly based on three important types of problems and the corresponding techniques: direct, inverse and compound proportions or rules of three. These problems were aimed at addressing a variety of applications in daily life, commerce and trade. The complexity of the problems approached and the techniques used could vary depending on the educational level, period of time or author, but they were all based on the consideration of a co-variation between quantities and the preservation of rations between these quantities.
- The *logos* contained a "Theory of ratios and proportions" to give rationale to the techniques and to calculate with proportions in a similar way as we operate with equations nowadays (figure 1).

Typically, the notion of ratio and proportion and the techniques of the rule of three (reduction to the unit and "cross product") were introduced at the primary level in arithmetic courses, while the theory of ratios and proportions addressed more generally corresponded to algebra, which was taught at a higher level. Even if we are not developing this point here, ratios and proportions also played an important role in geometry, the third domain of classical mathematics.

THE EFFECTS OF THE NEW MATH REFORM

At the end of the 19th century, in scholarly mathematics, the language of functions started to replace the language of ratios and proportions. Mathematicians worked with functions, variables, sequences, etc. and they would soon start to talk about structures. The classical organization of ratios and proportions was considered as obsolete, since functional relationships could perfectly replace it. This is what happens in today's scholarly mathematics. For instance, if we search in the Encyclopaedia of Mathematics (https://www.encyclopediaofmath.org), there is no entry under the term "proportion". Only a short definition of "Arithmetic proportion" is proposed as "an equation of the form $a - b = c - d$" and, in the entry "Arithmetic" to explain Euclid's work, it is assimilated to "the theory of fractions": "Euclid also studied the theory of proportion, i.e. the theory of fractions".

If we look at the mathematical curriculum that was elaborated during the New Math reform, the organization around proportion disappeared. The table of contents of a book like (May, 1959), addressed to high school students and teachers, proposes a sequence of chapters titled:

> 1. Elementary Algebra, 2. Elementary Logic, 3. Elementary Theory of Sets, 4. Plane Analytic Geometry, 5. Relations and Functions, 6. Numbers, 7. Calculus, 8. Probability, 9. Statistical Inference, 10. Abstract Mathematical Theories.

Proportionality appears in chapter 5, together with the study of linear functions. The way it is dealt with is rather extreme, but it illustrates how radical the reform was:

> When in scientific discourse it is said that "y varies directly with x" or that "y is proportional to x", the meaning is that there exists a number m such that $(x, y) \in mI$ whenever x and y are corresponding values of the variables […]. For example, for constant speed, distance is proportional to time. Letting y = distance and x = time, $y = mx$ for some m. Here m = the speed. If we know one pair of values, we can calculate m and so determine the function. We call m the *constant of proportionality.* (May, 1969, p. 271)

When we look at other textbooks like (Murphy, 1966), we see that proportion is not addressed at all. It is only used in geometry to define similar triangles: "Two triangles are similar if corresponding angles are congruent and corresponding sides are proportional" (p. 183). And a similar situation appears in (Peterson, 1971).

In summary, the New Math reform produced a complete transformation of the mathematical knowledge to be taught in order to make it more compatible with the scholarly knowledge, "so as not to risk the denial of mathematicians, as it would undermine the legitimacy of the social project, socially accepted and supported, of its teaching" (Chevallard, 1985, p. 26). The idea of structure and structure building was the driving force of the organization of contents; the ill-defined notion of quantity was replaced by the construction of sets of numbers, and the notion of map or function was introduced to represent relationships between variables. In this new organization of contents, the main goal was to provide students with updated tools. However, these news tools were not to be applied to the old problems. The "practical applications" that were at the core of the classical organization were also completely put aside. What prevailed was the logic of the construction of mathematical notions, not the set of questions those notions were supposed to help address.

PROPORTIONALITY IN TODAY'S MATHEMATICAL ORGANIZATIONS

In the 1980s, a "counter-reform" started to be applied in many countries. As Kilpatrick (2012) says:

> In no country did school mathematics return to where it had been before the new math movement began […] [M]any of the ideas brought into school mathematics by the new math have remained. For example, textbooks still refer to sets of numbers and sets of points […]. Pupils encounter and solve inequalities along with equations. Numbers are organized into systems that have properties […] Terms such as *numbers*, *numeral, unknown, inverse, relation, function*, and *graph* are given reasonably precise definitions and used to clarify notions of quantity, space, and relationships. (Kilpatrick, 2012, p. 569.)

However, the counter-reform also restored many elements of the (old) praxeologies and inserted them into the renewed mathematical organization. In the case of proportions, the old techniques of the rule of three and their theoretical environment reappeared, but now have to coexist with the praxeologies structured around the notions of function and equations. The mathematical "ecosystem" is not the same anymore. The role played by quantities in classical mathematics has now been replaced by a rigorous construction in terms of real numbers and functions of numerical variables that, although it is not explicitly introduced at school, remains in the background of the whole curriculum organization. As Hersant (2005) noticed, since 1977, the return to the study of specific situations did not mean the return of a real work on proportional quantities but rather on sequences of measures that end up being only numbers. We are thus in front of blurred or hybrid organizations made up of pieces taken from different mathematical periods, mixing elements from different praxeologies that maintain redundancies and some incoherence in the kind of tools used. For instance, functions or relationships between variables can be introduced to define proportionality—even function $f(x) = a/x$ is called "inverse proportionality"—but only the old techniques of the rule of three (more or less modernized in terms of tables and cross-products) are used to solve the problems. Moreover, students are asked to solve "proportionality problems", while the rest of the situations presented to be modelled by other relationships between variables does not receive a specific name.

In summary, the elements of the process of didactic transposition described in the previous sections show a diversity of relationships between pieces of knowledge. The curriculum is formed with elements that come from the updating of both the scholarly knowledge and the old elements of the knowledge to be taught (table 1).

Period of time	Scholarly knowledge	Knowledge to be taught
Classical mathematics	Proportions as the main tool to describe and establish relationships between quantities	Ratios and proportions between quantities. Practical problems (especially in commerce) solved using proportionality between quantities
New math reform	Functions as the main tool to describe relationships between variables. The construction of the set of real numbers avoids the use of the notion of quantity, which is relegated to its use in sciences	Sets, numbers, maps, numerical variables functions (no quantities)
Counter reform (current situation)		Proportionality between numerical variables, coexistence of ratios and proportions with equations and linear functions Quantities are not properly addressed

Table 1. Summary of the didactic transposition process related to proportionality

The analysis of the didactic transposition processes and their evolution in time show a hybrid organization of the praxeological elements that constitute the knowledge to be taught about proportionality. Some of them come from the classical organization of ratios and proportions—the old knowledge to be taught—and others from the modern organization of functions between numerical variables. The outcome leaves many disconnections between techniques, problems and theoretical elements, as well as incoherencies and a certain arbitrariness in the types of problems considered and the selection of tools to address them.

CONCLUDING REMARKS

We have seen that proportionality remains as a piece of the current knowledge to be taught in spite of its disappearance from scholarly mathematics. Considering that the scholarly knowledge is supposed to provide legitimacy to the knowledge to be taught (and to the taught knowledge), their permanence in the curriculum indicates that there must exist other sources of legitimacy to replace—or at least compensate—the influence of the mathematicians' scholarly institution. The first one can be located in the noosphere under the impact of educational research: cognitive education (from the tradition of genetic psychology) has considered "proportional reasoning" as one the most important mathematical school learnings for the past 30 years and appears as a strong influence in curriculum decision making. Another source of legitimacy might come from the natural sciences as scholarly institution. There, the language of ratios and proportions is still used as a way to refer to relationships between quantities in the case where the multiplicative constant is of no interest. For instance, to deduce from $F \propto m/d^2$ (F proportional to the quotient m/d^2) and $m \propto L$, that $F \propto L/d^2$. However, this last case remains far from the specific tools—praxeologies—that define the knowledge to be taught around proportionality at secondary level (in mathematics as well as in science).

The analysis of curriculum reforms needs to scrutinize and question not only the general principles that determine what should be taught and learnt at school, but also consider the choices made with respect to the more specific praxeological ingredients that form the knowledge to be taught, together with the criteria adopted for these choices. In the teaching of relatively new subjects such as statistics or algorithmics, the knowledge to be taught is more regularly contested or, at least, put under question, with a periodic renewal of the transpositive work. When considering traditional domains, the knowledge to be taught appears as an entity that tends to become transparent, invisible because assumed as obvious, natural and unquestionable. The analysis in terms of didactic transposition aims at bringing it back to the debate.

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THE INTENDED INTERMEDIATE-LEVEL GEOMETRY CURRICULUM FOR ISRAELI HIGH SCHOOLS

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This paper outlines the intended intermediate-level geometry curriculum that is part of a reform of high-school mathematics teaching in Israel. The main points of reference of the curriculum are integration of analytic geometry, trigonometry, and synthetic geometry; linking mathematical rigor to the development of intuition and valid visualization-based reasoning; the characteristics of the intended students include their possible academic aspirations; possibilities created by DGEs; and ideas of experimental mathematics applied to high-school geometry. The paper describes the curriculum reform, the rationale behind the geometry curriculum, an excerpt of the tenth-grade outline with examples, an example of a problem for use in the eleventh-grade matriculation exam, and deliberations and dilemmas related to implementation.

INTRODUCTION

Historically, geometry has provided a basis for mathematical intuition and clarity, visual thinking in many mathematical fields, logical inference and reasoning, applicability and closeness to nature, philosophical depth, and mathematical aesthetics. Among the many sources that refer to geometry's place in mathematics, science, and human culture, See, e.g., Artstein (2014), Glaeser (2012), and Moise (1990), *inter alia*. Some developments in math and math education in the past century have impaired geometry teaching in schools, particularly in respect of theoretical geometry, sometimes favoring its empirical versions. (Sinclaire 2008, p. 77, e.g., addresses the processes of decline and revival of theoretical geometry in schools; see also May & Hoyles, 2001; and Schoenfeld, 1986). Some of the problems and obstacles are "natural," e.g., developmental-cognitive factors, and are being extensively researched and investigated. Others are man-made, by-products of the quest for the best way to teach geometry. Prime among the latter impediments is the artificial antagonism between empirical and theoretical approaches to geometry, as noted by, e.g., Niss (1998): "Any meaningful teaching of geometry will have to strive to create or at least help […] reconciliation between empirical and theoretical geometry […]." Schoenfeld (1986) elaborates:

> There is some fairly clear evidence that the students' separation of deductive and empirical mathematics is learned behavior. Moreover, this behavior seems to have been learned in the students' geometry classrooms. [… The] persistent false dichotomy, which stipulates that a logical, deductive approach to geometric knowledge is antithetical to an empirically based inductive approach to the subject, seems to be one of the man-made obstacles to study and to overcome.

Visual reasoning and, in turn, application of geometry to other fields of mathematics remain on the map of mathematics education. Thus, e.g., Rivera's book on visually-oriented curricula (2011), while not including geometry explicitly in its contents or its index, is (as expected) interspersed with geometric patterns and visualizations, beautiful geometrical interpretations of various analytical results, etc. Some visual patterns in the book are purely schematic but others entail mastery of a corresponding geometric apparatus. "Since [...] visual thinking must be learned, we predict that the interest in basic geometry will grow" (Hansen in Mammana & Villani, 1998, p. 261).

No twenty-first-century high-school curriculum can disregard the potential of Dynamic Geometry Environments (DGEs). Deep theories and ideas back the didactical approaches that DGEs apply, going back to the Erlangen program, invariance vs. properties varying under transformations embodied by dragging, etc. (See, e.g., Bonotto 2007; & Leung, Baccaglini-Frank, & Mariotti, 2013). These ideas make it possible to experiment with geometry as a source for apposite inductive conjecturing, leading to logically valid deductive reasoning. Moreover, the use of DGEs

> opens up a type of geometrical reasoning […] that possibly suggests a different type of pedagogical process […]. Such a process should bestow on the teaching and learning of geometry an explorative and experimental nature that is complementary to deductive and inductive approaches (ibid.).
>
> For some reason, mathematical experimentation in schools is developing mostly in analytic mathematical domains. Thus, for example, Abramovich (2014) devotes most experimentation to analytic topics, including analytic geometry, overlooking theoretic (synthetic) geometry. His few purely geometric appearances are not of experimental nature (e.g., p. 56).

Niss (1998) asserts that teaching and learning geometry serve the same goals as teaching and learning mathematics in general, "even if, in modern times, considerable skepticism towards the justification of the underlying belief in its effectiveness has gained momentum" (p. 267). This implies that geometry is as intrinsic to mathematics as ever and should be incorporated into school curricula as such.

THE SITUATION IN ISRAELI HIGH SCHOOLS AND THE REFORM POLICY

High-school mathematics studies in Israel takes place at three levels:

- The lowest level acceptable for a matriculation certificate, known as the three-point level. (In 2016, about 62 percent of students entitled to the matriculation certificate took this level);
- The intermediate four-point level (23 percent of students in 2016);
- The highest level: five points (17.8 percent in 2016) (Perl, Neria, Segal, & Sion, 2018).

In recent decades, the Israeli high-school (grades 10–12) math curriculum has evolved *de facto* into a matriculation program,[1] with implications for mathematics studies generally and geometry studies especially. In particular, textbooks consist primarily of exercises and teaching in grades 10–12 is largely exam-oriented. During this time, however, Israel's math curricula from primary school upward have been undergoing review and revision. The overhaul of the high-school math curriculum began in 2014–15 and is continuing today.

Presented here is an intended four-point geometry curriculum, part of a new high-school curriculum planned to replace the existing one.

In Israel, curricula are designed by program committees that are assembled *ad hoc* for each level separately, comprised of mathematicians, math educators, Ministry of Education subject representatives and curriculum specialists, and experienced math teachers. The results of each committee's work are subject to the approval of the Ministerial Mathematics Subject Commission, which also includes mathematicians and mathematics educators, the Ministry's authorities on the

[1] See http://cms.education.gov.il/NR/rdonlyres/A12ECCF0-2F89-4C00-AD04-E8266B837E8A/157128/4yechdotNoseim1.pdf (in Hebrew).

subject, and math teachers. Once approved, the curriculum is presented to textbook writers, subjected to pedagogical experiments, and revised as warranted.

For several reasons, those in charge of mathematics at the Ministry decided to redistribute the levels of high-school mathematics studies before designing the new curricula for all levels. The population intended for the new four-point curriculum is supposed to include the stronger 50 percent of students at today's three-point level plus those at today's four-point level, which renders about 50% of graduates eventually entitled to the matriculation certificate.

The guidelines for the new four-point curriculum include the nature of geometry and its role in mathematics as reviewed above; integration of analytic geometry, trigonometry, and synthetic geometry; linking mathematical rigor to development of intuition and visualization-based valid reasoning; the Ministry's policy (particularly the intended students' characteristics), technological innovations, possibilities created by DGs, and experimental mathematical ideas that support systematic inductive reasoning.

Characteristics of the intended population for whom the curriculum is intended, plausible estimations of their difficulties and of their future academic intentions

Hansen (in Mammana & Villani, 1998) recommends:

> a distinction […]between those who will attend science and technology faculties and those who will attend all other faculties. When selecting the geometrical content in the curriculum at the secondary school level, it will therefore become increasingly important to choose such units of geometry which foster the right skills, abilities and attitudes for a meaningful and useful tertiary education (p. 260)

The new intermediate level of mathematics is intended to provide the basis for subsequent academic studies in economics, social science, life sciences, etc. Although the mathematical requirements for these fields are less challenging than, say, for engineering, academic and professional success in these fields requires high critical thinking abilities, mastery of diverse apparatuses and the ability to integrate them and/or pick the right tool for the problem being solved, flexibility and creativity, and inferential and conjecture-testing ability. By the same token, most students who take the three-point level today have considerable difficulties in multistep deductive proofs, spatial vision, etc.

PRINCIPLES, GOALS, SPECIFICATIONS, AND EXAMPLES OF THE NEW CURRICULUM

If so, the main principles and goals of the new four-point geometry curriculum are as follows:

Principles: 1. Synthetic geometry, trigonometry, and analytic geometry shall be integrate; 2. Problems should be solvable in no more than 2–3-steps of deduction. Problems needing a longer solution will be split into hierarchical problems and guiding questions that lead to the desired result; all problems involving computation and analytic representation will appear with numerical data and not it a general form. Problems will appear with diagrams. 3. It is recommended that digital platforms should be used to enhance inductive conjecturing followed by deductive testing (proof or refutation) of hypotheses thus formulated.

Goals: Students will develop abilities in: 1. deductive reasoning, including awareness of the need for proof and justification, understanding the nature of proof and ability to write proofs, including proof by contradiction; 2. inductive thinking based on generalization, formulating hypotheses and testing them by logical methods of deductive geometry; 3. appropriate merging of visual, symbolic, and verbal representations; 4. formulating and testing statements opposite to a given one and reasoning in terms of necessary and/or sufficient conditions; 5. applying knowledge and skills acquired and selecting an appropriate tool for each situation; 6. intelligent use of calculator, including adjusting the form of a numerical result to the type of the problem at hand ($\frac{\sqrt{2}}{2}$ vs. 0.707); 7. awareness of the need to subject results to critical evaluation and appraise their correctness; 9. applying geometry in real-world situations.

Specifications of the expected modes of teaching and learning geometry

In view of these challenging expectations, the new geometry curriculum also includes recommendations on teaching and learning methods—especially important given the deficiencies of today's curriculum–as also appear, e.g., in NCTM2000 standards for grades 9-12 (Sinclaire, 2008, p.79) and other countries' curricula. The list of topics for each grade comprises a small part of the curriculum document; the rest contains detailed examples of recommended teaching sequences and examples of problems and learning sequences. ("Teaching sequences" specify a possible order of topics in a teaching plan and their interrelations; "learning sequences" are structured activities that students are advised toapply in their further learning and exercises. To illustrate this general description, an abridged version of the tenth-grade geometry curriculum follows: list of topics, two examples of problems / learning sequences, and an example of a problem illustrating the level and spirit of the eleventh-grade matriculation exam, which summarizes the topics of plane geometry.

Concise outline of topics in geometry in the 10th grade curriculum

Synthetic geometry: theorems on special lines in triangle: medians, heights, angle bisectors, perpendicular segment bisectors, including locus properties of both bisectors; proofs of concurrence of special lines (without mentioning incircles / circumcircles, which are postponed to eleventh grade when most circle theorems are studied); theorems related to the similarity of triangles; trigonometry: trigonometry of acute angle based on right triangle; computational problems on figures composed of right triangles (deltoids, rectangles, perfect polygons, etc.); analytic geometry: points, segments, straight lines in coordinate plane; line equation by the slope and a point and by two points; mutual location of lines in plane; perpendicularity. Since only acute-angle trigonometric functions are included in the tenth-grade curriculum, slope is defined as the tangent of the acute angle between the line and the x-axis, with a "+" sign if the line represents an increasing function and "-" sign if it represents decreasing function." In eleventh grade, sine and cosine definitions are extended to obtuse angles for the sake of sine and cosine theorems in the triangle.

In addition to the list of topics, clarifications are added to enhance the spirit of the document, such as: "equation of a straight line by slope and a point on it, as an analytic implementation of the axiom of parallels"; "equation of a straight line by two points on it, as an analytic implementation of the axiom claiming the existence and uniqueness of a straight line passing through two given points";

Prototypical examples of learning sequences and exercises

The examples below are prototypical because they reflect many features described in "Principles and Goals."

Example 15. This guided learning sequence yields new geometrical knowledge to be applied in subsequent activities. It is preceded by formulae for distance and the midpoint between two points, the slope of a line by two points on it, and slopes of perpendicular lines.

<u>Introductory part</u>: *The edges of segment AB are points whose coordinates are A(_,_), B(_,_).* (The coordinates must be numerical, in compliance with the complexity level outlined in “Principles.”)

a. Compute the slope of AB.

b. Compute the coordinates of C – midpoint of the segment AB.

c. Write the equation of the perpendicular bisector of segment AB.

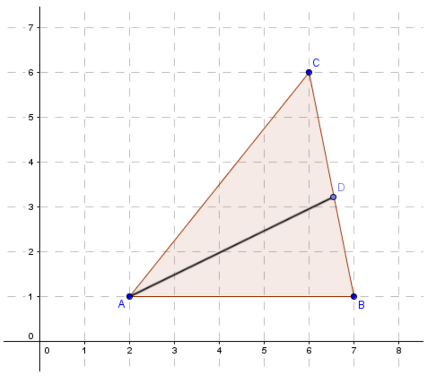
<u>Part 1.</u> *Choose any point D on the perpendicular bisector and find the length of segments DA and DB. What do you observe? Choose another point E and repeat the computation. Do you observe a similar result? Choose more points if you wish and perform similar computations. Formulate a conjecture referring to points on the perpendicular bisector of a segment. Prove it.*

<u>Part 2.</u> *Choose a point Q not on the perpendicular bisector and compute the lengths of segments QA, QB. What do you observe? Formulate and prove a conjecture for this case (choosing more points if you wish).*

This exercise implements the experimentation leading to inductive and deductive reasoning, the interplay of synthetic and analytic geometry, and differentiation between a conjecture and its reverse, virtually leading to formulation of the properties of a geometrical object in terms of locus. Obviously, all the computational stages should preferably be performed using a digital platform, dragging the points in a DGE environment; this is an explicit recommendation in the curriculum. This gives students a real opportunity to experiment as much as they consider necessary before formulating a conjecture.

Exercise 16 implements the properties of the perpendicular bisector as a locus for proving the concurrence of perpendicular bisectors to triangle sides. Thus, students are guided and encouraged to apply the task that they performed instead of regarding it as a mere exercise. Later, a similar sequence is proposed for angle bisectors, also aiming eventually at the concurrence of angle bisectors in a triangle.

Example 11. This example involves analytic geometry, trigonometry and, desirably, synthetic geometry.



The vertices of ΔABC are gridpoints on a coordinate plane (see sketch; AB is parallel to the x-axis). AD is the angle bisector of $\angle BAC$, so that $\angle BAC = 2\angle BAD$. Is it true that the slope of AC is twice that of the slope of AD?

This exercise is meant above all to put to critical testing a wrong supposition that inexperienced students might find correct. Although functional linearity is not a concept ”attributed” to geometry, this visual representation is an example of linkage between geometry and other fields of mathematics.

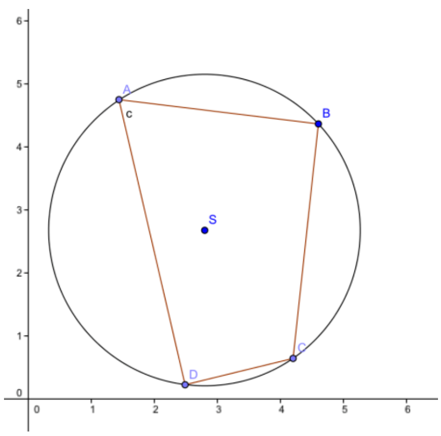
The triangle sits on the coordinate plane in a way that allows evaluation of slopes by oral calculation, given the existence of grid points on the angle bisector. An observant student may ignore the exact coordinates and just count the squares; less confident students may perform the rather simple calculations using the coordinates of appropriate points. Also, students may (and probably should be advised to) apply the angle bisector theorem to the right triangle formed by the gridlines and AC as a hypotenuse, if they have already been taught it. The exercise should elicit a discussion of whether this special case suffices to answer the general question posed in the problem and the issue of counterexample as a way of refuting conjectures, in contrast to supporting examples that may lead only to conjectures that are tested by deduction.

Exercises 12 and 13 use another triangle located correspondingly in the coordinate plane for a similarly guided exercise leading to the conclusion that $tg(\alpha - \beta) \neq tg\alpha - tg\beta$. The recursive appearance of such questions guides a student toward the habit of doubting and testing "self-evident" beliefs. For the same triangle, the exercise includes angles and sides calculations by trigonometric and/or analytic methods; it is completed by the area calculation in at least two different ways and the explicit instruction is to make sure the results are equal. An instruction to perform one calculation by different methods has a dual goal: searching for different solutions and comparing them, and taking a critical approach toward the obtained results and testing them.

Evaluation

As an obvious part of curriculum planning, assessment and, in particular, matriculation exams must reflect the attainment of defined goals for a defined category of learners. The Ministry of Education's current intention concerning matriculation exams in mathematics is to hold two exams: at the end of eleventh grade and upon the completion of twelfth grade. The evaluation is a vector of several components: the student; the subject matter appropriate for him/her; the items; the occasions; and the procedure including who does what, etc. (Niss p. 268). Therefore, planning the geometry curriculum includes, as an integral part, its "proof of examinability," i.e., plausible examples of problems that are solvable within defined time restrictions by the students for whom the curriculum is intended and reflecting the spirit, contents, and level of what has been learned. Such an example with appropriate commentary appears below.

Example of a problem for the eleventh-grade matriculation exam



The four vertices A(_,_), B(_,_), C(_,_), D(_,_) of a quadrilateral ABCD belong to circle S (diagram). Comment: The coordinates of the points should be given numerically and values should be chosen so that the computations are not too messy. The attached sketch is an illustration; the more precise sketch attached to the exam must fit the coordinates' numerical values.

a. What can you claim about angles BCD, BAD? Expected answer: The sum of these angles is 180°. Comment: Question (a) demands plain knowledge of the geometric theorem. If the student has forgotten it, he/she may successfully answer the ensuing questions anyway.

b. One angle of ABCD equals 90º. Find the radius of circle S. Expected answer: It suffices to check the slopes of one pair of adjacent sides for perpendicularity. If, e.g., DA⊥BA, then BD is the diameter. Otherwise, angles B, C equal 90º and AD is the diameter. Comment: Student who forget that the sum of opposite angles equals 180º or misses the logical "shortcuts" may have to do extra calculations but can pass this part anyway.

c. Find all angles of ABCD. Expected answer: Given (a, b), it suffices to find just one more angle; the angle opposite to it will complete it to 180º; the cosine theorem will do the job. Comment: Again, students who forget the property of inscribed quadrilaterals will have to make extra computations but may pass anyway.

FROM INTENDED CURRICULUM TO IMPLEMENTATION: DILEMMAS AND PITFALLS

In elaborating the new geometry curriculum, the perennial dilemma between teaching theory and exercising and "teaching to the test" arose. The inclusion of a theoretical and perhaps less-"examinable" topic concerning circle area, circumference, and π was mulled. According to the geometry curriculum for all grades, students encounter these issues only in sixth grade and receive it as a more-or-less computational piece of knowledge. The program committee discussed whether once in their scholastic lives, students at the intermediate mathematical level should learn more about π given its centrality in mathematics, its history, and its culture. The committee decided to place a theoretical outline in a handbook and add optional learning activities in which students may actually observe the appearance of digits of π as a result of computation performed using an appropriate technological platform, based on a series of inscribed and circumscribed regular polygons. The second example concerns the computation of volumes of solids in the twelfth-grade curriculum. Again, the only occasion when students hear an explanation (if any) of the "$\frac{1}{3}$ " in volume computation of cones and pyramids is in sixth grade, when they fill a cylinder or a prism three times with sand or water using a cone or a pyramid of the same height and basis. The program committee agreed to include a theoretical elaboration of the volume formulae for cones and pyramids and for cylinders and prisms, with plausible proportion maintained between rigor and intuition based on decomposition, similarity of structures, and Cavalieri's principle as outlined, for example, in Moise (1990, pp. 353-366), supported by DG and other digital demonstrations and supplemented by appropriate exercises. This approach made it possible to include prisms and pyramids that are not necessarily right, although obvious limitations dictated by the prescribed level of the curriculum are needed.

Above I presented part of the intended intermediate-level geometry curriculum for Israeli high schools. The next steps toward implementation, after the Ministerial Commission gives its approval, are textbook-writing, DGE applications, organizing teachers' PD activities, piloting the use of the new materials, elaborating and adopting didactic approaches that implement DGE-based experimentation, work with textbooks, review of existing paradigms for the allocation of teaching / learning time, and the aforementioned reconciliation of theoretical and empirical geometry involving mathematical experimentation. The success of these activities is far from assured; it will entail productive collaboration among all stakeholders: textbook writers, Ministry supervisors, and, first and foremost – mathematics teachers. The first meetings of the committee members who represented the geometry curriculum vis-à-vis leading teachers were rather encouraging. The teachers were asked to design tasks in the spirit of the new curriculum and most of them did so, suggesting that the immense work to be done has already begun.

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THE EXTERNAL TRANSPOSITION OF INQUIRY IN MATHEMATICS EDUCATION: IMPACT ON CURRICULUM IN DIFFERENT COUNTRIES

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In this paper, we analyse the conditions and constraints that exist when implementing inquiry-based mathematics education defined and framed by school mathematics curricula. Within the theoretical framework of the anthropological theory of the didactic, we organize our analysis using two main tools. On the one hand, we focus on the external didactic transposition of inquiry in mathematics education. That is, we focus on analysing how the "knowledge to be taught" addresses inquiry, how it is selected, adapted and declared to be taught from international studies to local curriculum reforms. With this purpose in mind, we present an analysis of curricula of four European countries (Denmark, France, Portugal and Spain) to explore which kind of conditions (and constraints) each curriculum establishes for inquiry-based mathematics education. We use the levels of didactic codeterminacy to identify where these conditions and constraints for inquiry come from. It will allow us to compare relevant situations between aforementioned countries.

INTRODUCTION

In recent years, a paradigm shift in education encouraging inquiry-based education (in contrast to the transmission of knowledge) has been promoted by international policy-makers and educational institutions. An example is the report by the European Commission on trends in pedagogy in the educational systems across Europe emphasising "the alarming decline in young people's interest for key science studies and mathematics" (Rocard et al., 2007, p.5) and the encouragement of changing teaching into more inquiry-based approaches. Furthermore, PISA results and an orientation towards developing competencies together with thinking and reasoning processes, rather than standard routines and isolated concepts, promote favourable conditions for the adaptation towards Inquiry-Based Mathematics Education (IBME) in educational systems. Dorier and Maaß (2014) refer to IBME, as a student-centred paradigm of teaching mathematics and science, in which students are invited to work in ways similar to how mathematicians and scientists work. This means they have to observe phenomena, ask questions, look for mathematical and scientific ways to answer these questions, and to interpret, evaluate, communicate and discuss their solutions effectively.

Research in mathematics education aligns with this transition and a number of IBME projects are being promoted both nationally and internationally. European projects such as PRIMAS and MASCIL foster the use of inquiry-based mathematics and science education in everyday practice through the design of actions in terms of professional development (Maaß & Artigue, 2013). These reports and studies are assumed to have an impact on national curricula reforms, acting as driving forces and unifiers of school mathematics curricula. However, the way IBME is included in mathematics curricula differs both in formulations and pedagogical approaches in each country. In this paper, we focus on analysing the following questions: what new discourses and in what terms have IBME been introduced in school mathematics curricula? Which conditions (and constraints) are established by curricula to facilitate inquiry to exist in the mathematics to be taught? How similar are these conditions and constraints if we look at different countries?

We present a comparative study on the incorporation of IBME in the latest curriculum reforms at primary and lower-secondary school levels in Denmark, France, Portugal and Spain. These countries were chosen by proximity of researchers working within the framework of the Anthropological Theory of the Didactic (ATD), but this research can be scaled to integrate more countries and curriculum reforms. Our analysis addresses the above questions from the institutional perspective provided by the ATD (Dorier & García, 2013), in which we base our curriculum analysis on identifying the institutional conditions and constraints that are established by the latest curricula reforms for the local and large-scale dissemination of IBME.

EXTERNAL TRANSPOSITION OF INQUIRY IN MATHEMATICS EDUCATION

One of the main contributions of the theory of didactic transposition (Chevallard, 1991) is taking into account that, in order to analyse what knowledge can be taught and learnt, it cannot be detached from its institutional origin. This knowledge undergoes transformations from its production (as *scholarly knowledge*) to *knowledge to be taught* and beyond (Figure 1). The incorporation of IBME into national curricula may thus be understood in the process of didactic transposition where different "agents" are involved. In this paper, we focus on the *external didactic transposition* (see blue rectangle in Figure 1) that involves the institutions producing knowledge and the "noosphere" selecting and adapting what has to be taught.

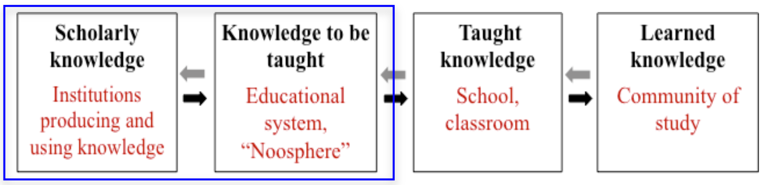

Figure 1. The didactic transposition process

In particular, reforms of curricula have integrated changes proposed by international reports (such as Rocard et al., 2007) or the formulation of the competence approach (Niss, 2015). This process can be seen as the educational community acting as the *noosphere* setting the conditions to stimulate a transition of the prevailing teaching paradigm still close to transmission of knowledge into more inquiry-based approaches. Furthermore, analysing this transposition process can bring forth diverse *conditions* and *constraints* defining and delimiting how *knowledge to be taught* (in particular, concerning IBME). As a result, the transposition is delimiting what can (and cannot) happen in schools, classrooms and eventually what can effectively be learnt by students. To develop this "ecological analysis", we use the *levels of didactic codeterminacy* (Chevallard, 2002) (see Figure 2) as a common methodological tool to illustrate at which level these *conditions* and *constraints* appear in different national curricula.

Civilizations ↔ Societies ↔ Schools ↔ Pedagogies ↔

Disciplines ↔ Domains ↔ Sectors ↔ Themes ↔ Questions

Figure 2. Scale of levels of didactic codeterminacy

In the next section, we focus on analysing each curriculum proposal of the four European countries. In order to unify the analysis and to be able to compare the situation between countries at a transnational level, each country focuses on the last curriculum reform affecting mathematics at primary and lower-secondary school levels.

We present analyses in relation to (a) what new terms and discourses concerning IBME have been introduced into curricula at these school levels, and (b) which *conditions* and *constraints* are (explicitly or implicitly) stated for the teaching of IBME. Methodologically, it is important to place at which level of generality do these conditions and constraints appear (Chevallard, 2002, p.9). That is, placing them in the didactic codeterminacy scale, which moves from the more *specific level* (from the disciplinary level, here mathematics, and how the teaching is structured in themes or questions, skills or competences) to the *generic levels of codeterminacy*, referring to how civilizations, societies, schools or pedagogies delimits and organise the study of disciplines. This common methodological framework is different from e.g. qualitative analysis drawing on grounded theory (e.g. Frejd, 2013), but in line with those presented in Bosch and Gascón (2006).

SETTING UP CONDITIONS THROUGH LOCAL CURRICULUM REFORMS

The case of the Danish curriculum

In 2003, the notion of mathematical competencies (Niss, 2015) was introduced in mathematics curricula. Further reform efforts were made in 2009, 2014 and the latest reform took place in 2018. The main change in 2018, compared to the 2014 reform, is that "skills and knowledge goals" are now guidelines instead of requirements for the teaching of any discipline (DME, 2018). There is a set of goals be reached after grade 3, grade 6 and grade 9. The curriculum is divided into 4 areas of competencies. For grade 3 and 6, the first one is mathematical competency, which is divided into six categories: problem solving; modelling; communication; aids and tools; reasoning and mathematical thinking; representing and symbols. The second competency is numbers and algebra, divided into: numbers, calculations and algebra. The third competency is geometry and measures, divided into geometrical properties and relations, drawings, placing & translation and measures. The last competency is statistics and probability, divided into the two domains it delimits. If we count the appearance of the notions inquire, investigate, examine relations, discover relations, interpret results in a real world context in the goals, we find 13 sub-goals out of 76 for grade 3, 10 out of 80 for grade 6, and 6 out of 88 for grade 9. Though in grade 9, there are also 5 sub-goals addressing phases of the modelling cycle or problem-solving competency, which are interpreted as close approaches for IBME (Artigue & Blomhøj, 2013, p.806).

In grade 3, we find formulations of the disciplinary goals such as "the student can investigate simple everyday situations" or "the student may know the features of inquiry-based work". For grade 9, the discourse of the goals is closer to tasks, e.g.: "the student may know methods to investigate the relation between graphs and equations, including digital tools" or "the student may know elements of modelling processes and digital tools to support simulations". At first glance, the presence of these goals reveals favourable conditions in the *pedagogical-disciplinary level* for IBME implementation. However, the conversion of requirements into guidelines may lead to IBME goals disappearing from classrooms in favour of the non-IBME. The mandatory written exit exam at the end of lower secondary school fosters traditional "teaching to the test", though more explorative problems have been introduced lately (e.g. pattern recognition). But written exit exams' represent important constraints for the realisation of inquiry-based activities at the levels of *school* and *pedagogy*.

If we move to the *society level*, we do not find any formulation regarding inquiry as a method for teaching in the School Act. It is stated that the school must prepare students to be responsible citizens and this through methods that nurture students' creativity and enthusiasm for acting (DME, 2016). More generally, the debate on school mathematics evolves around the lack of skills and content knowledge. As an answer to this, emphasis has been put on "learning goal-oriented teaching" in the ministerial guidelines for teachers, as a means to secure progression and expected positive learning outcomes of using clear learning goals. At the same time, researchers have opposed the idea of learning goal-oriented teaching, arguing that it delimits situations where students engage in IBME and limits learning potentials of the activities offered to the students (Misfeldt & Lindenskov Tamborg, 2016).

The case of the French curriculum

In France, school is a major political issue. This condition, which comes from the *society* level, has turned into an important constraint that is reflected in frequent curriculum changes. The primary school curriculum (children aged 3 to 11) was changed in 2002, 2007, 2008 and 2016 and the lower secondary school curriculum (children aged 11 to 15) in 2005, 2008 and 2016.

Current curricula are moving towards a new educational paradigm, including inquiry-based teaching going beyond science teaching. In the 386-page document (MEN, 2015) presenting the curriculum of all primary or secondary schools' subjects, the words "research" or "search" are used nearly 120 times, including expressions such as information search, documentary research, bibliographic search, Internet search, search the validity of information, free search (trial and error), individual or collective search for arguments to support a point of view, presenting the result of a search, search for (personal or original) answers, search for solutions. Concerning the mathematics curriculum, since the beginning of primary school, *problem solving* is considered as one of the core school activities (MEN, 2015, p.5). It is regarded as "the crucial point of students' mathematical activity" (p.73) and the "principal criterion to master knowledge in all domains in mathematics, but it is also the means for ensuring its appropriation, which guarantees its meaning." (p.197). Problems can be "internal to mathematics, or related to situations in everyday life or other subjects" (p.365).

The mathematics curriculum begins with the presentation of six competencies: *research, modelling, representation, reasoning, calculate and communicate*. The research competency is always associated to certain key words: methods, observe, question, manipulate, experiment, and formulate hypotheses. In relation to the contents, they are organised in three domains at primary school level: numbers and calculations; magnitudes and measure; space and geometry. Two new domains are added at secondary school: data organization analysis and functions (including probability); algorithm and programming. There is a last section in the mathematics curriculum devoted to the "crossing between disciplinary teachings" which evokes the potential role of mathematics in studying topics in other subjects.

All these conditions introduced in the mathematics curriculum to facilitate inquiry and research in mathematics education, which remains at the level of the *pedagogy-discipline*, differ from certain constraints that appear at the *society level*. The evolution of family life and the professional activity of both parents have generated new social demands to schools, which led the government to authorise the reduction of school time to 4 days per week in primary schools without any reduction to the teaching content. Teachers are thus confronted with a paradoxical injunction: using inquiry-based teaching that requires more time in order to stimulate students' initiatives in fewer days of school. However, to support the introduction of this new pedagogical paradigm, in the past few years the diffusion on the website of documents providing teachers with examples of inquiry activities has increased. Still, most of the teachers are not aware of the existence of these documents and there is little or no in-service professional development (only 18 hours per year for primary school teachers and not required for secondary teachers), which is far from sufficient. What is important is that schools and teachers have the pedagogical freedom to choose the resources they use. Schools therefore allow teachers to ignore the documents and curriculum guidelines made available to promote change.

The case of Portuguese curriculum

In Portugal, as in the previous cases, there have been several curriculum changes in the past years. We consider the last two curricula reforms (ME, 2007; ME, 2013) at primary and lower-secondary school levels, which are divided into three cycles: 1st cycle (students aged 6-10), 2nd cycle (aged 10-12) and 3rd cycle (aged 12-15).

The 2007 mathematics curriculum introduced three main skills to be developed through the teaching of mathematics: *problem solving, mathematical reasoning,* and *mathematical communication*. It also requested to replace a more transmission-based teaching approach by an exploratory teaching-learning approach to lead students to create their own strategies to solve tasks. The 2013 curriculum pursues three main aims: the structure of thought, the analysis of the natural world and the interpretation of society, regardless of the fact these aims are not connected to the previously described skills. The evaluation of students' progression in these aims focuses on verifying if the student is able to identify, designate, extend, recognize, and explain the result of a problem, but without requiring any kind of justification (ME, 2013). In the 1st cycle, the contents domains are: *numbers and operations*; *geometry and measurement*; *data organization and analysis*. In the 2nd cycle, *algebra* is added to the previous domains and, in the 3rd cycle, a new domain appears: *functions, sequences and successions*.

It may be observed that IBME is completely absent in the current Portuguese curriculum, both in the aims and in the content description. In this sense, there is a clear setback if we compare the current curriculum description with the previous reform in 2007, which integrated some of the OECD recommendations. More concretely, in the 75-page document (ME, 2007) there appear 46 times words related to inquiry or research and about 64 times words linked to exploration. In the current curriculum reform, however, the terms related to inquiry or exploration are absent. Furthermore, the current curriculum stipulates: "*problem-solving should not be confused with vague activities of exploration and discovery, which, being motivation strategies, are not adequate for the effective realization of such a demanding purpose. Although students can begin by presenting more informal resolution strategies using schemas, diagrams, tables or other representations, they should be encouraged to resort to more systematic and formalized methods*." (ME, 2013, p.5).

We can conclude that the current Portuguese curriculum is organized in terms of a set of contents without any mention of inquiry, and is essentially based on content-based description and evaluation. While certain terms related to IBME (such as research tasks, modelling, exploration) appeared more often in the 2007 curriculum, currently they are absent. In addition, this curriculum is strongly prescriptive. It enormously restricts the autonomy of schools and teachers looking for alternative curricular pathways or less content transmission approaches. In 2018, the Mathematics Teachers Association called for an urgent revision of the curriculum, by which they feel strongly conditioned. After all these complaints, the ministry of education has created a commission, which is now in charge of elaborating a new curriculum for primary and lower-secondary Mathematics.

The case of Spanish curricula

In the case of Spain, each autonomous community has its adaptation of the curriculum, although a common structure in terms of contents, learning standards and evaluation items is respected. Our analysis uses the case of the Catalan curriculum as the reference document. The latest reforms took place in 2011, and there was a supplementary document about competencies in mathematics in 2013 and in 2015 when the competencies approach became central. There are some general competencies, among which the *mathematical competency* is considered as a cross-disciplinary general competency. Its definition is exactly the same as the one included in the OECD (2016, p. 25) about *mathematical literacy*. It also underlines its cross-curricular character and the reason why it is recommended: “schools have to promote interdisciplinary work. This fact has to be taken into account for the school organisation of time and space”. However, more details about how to develop this interdisciplinary work except some general recommendations beyond the *disciplinary level* are not found.

If we focus on the mathematics curriculum in primary and lower-secondary school, it distinguishes four “dimensions”: *problem solving, reasoning and proof, connections and representations-communication*. These dimensions are used to organize the specific competencies in mathematics (10 in total in primary school, 12 in lower-secondary education). In parallel, both curricula include the content description, which are organized around five domains: numbers and operations; change and relations; space and shape; measure; randomness and statistics. Even though the last curriculum reform assumes a competency-oriented approach, the concepts remain static if one compares with previous curricula reforms.

In primary school, nothing is explicitly said about inquiry. However, several similarities linked to the dimension of *problem-solving* are found. One example explains that “solving a problem always invites to *inquiry* and, through its resolution, a spark of discovery appears that allows students to experience the charm of reaching the solution” (DOGC 2015a, p.62). Moreover, in the description of the specific competencies, there are several that refer to the different stages of the inquiry process: “Pose questions and generate mathematical problems”, “Give and check solutions according to the initial questions”, “Formulate hypotheses appropriate for the situation and checking them”, “Identify the mathematics involved in everyday situations and school situations”. In contrast, in the lower-secondary curriculum, linked to the problem-solving dimension, a new competency appears: “Keep a research attitude when facing a problem, trying different solving strategies” (DOGC 2015b, p.88). A deeper analysis of this competency shows that its main aim is to enable students to use different strategies to tackle a problem and have an open attitude to propose, try, discuss and defend different strategies. These guidelines also emphasize emotional aspects of the research competency and highlight the need to promote self-esteem and creativity through problem solving. Methodological and evaluation guidelines such as promoting teamwork and encouraging students to propose different ways to tackle a problem could be applied without major concern to other specific competences. It is also necessary to observe how many constraints derived from the *traditional didactic contract* may be opened.

In contrast to the conditions that all these competencies could generate, the stagnation of contents, the rigid boundaries between disciplines and the generic methodological and evaluation guidelines that are far from being specific guidelines about how to develop inquiry activities and far from delimiting the new roles to assume by teachers and students, make us think that all these conditions remain at the *pedagogical level*.

CONCLUSIONS ABOUT THE COMPARATIVE STUDY

In this paper, our aim is not to provide a precise analysis of each country, but to detect the major types of *conditions* and *constraints* that exist in the countries we focus on, facilitating or limiting the implementation of IBME. Curricula resulting from a complex process of external didactic transposition are a productive means to detect and situate the diverse conditions established and also to compare what is shared or specific in the different countries involved.

Through the analysis of four different curricula in Europe, we illustrate the different levels of incorporation of IBME at the level of *pedagogy*, expressed in different terms and discourses (ranging from rather positive experiences to the Portuguese case where the last reform seems to hinder its incorporation). Moreover, all four countries introduced new elements into the most recent reforms to describe the curriculum in terms of "skills" or "dimensions" to promote and integrate the competencies approach rather than just focusing on a content-based approach. In relation to inquiry in mathematics, the most common reference is to *problem solving* together with *modelling* and *research* (as underlined in the cases of Denmark, France and Spain). Other terms such as observing, analysing, questioning, exploring, formulating hypotheses, experimenting, modelling, solving problems, reasoning, communicating, are terms identified in the analysed curricula (including also the case of 2007 Portuguese curriculum) which, according to Artigue & Blomhøj (2013), can be associated with IBME.

Apart from these *conditions*, several *constraints* for IBME have been detected. First, in all the countries, curricula reforms happen more often than some years ago and new changes are introduced into the more generic levels of the *society*, *school* or *pedagogy*. So far, they have had little impact on the *specific levels of the discipline*, that is in the way mathematics, as a school discipline, is delimited and organised within certain domains, sectors, themes and questions That is the reason why we have underlined the stagnation of mathematical contents, the impassable barriers established between mathematics and other disciplines, the silent questioning of the necessary changes to be made to the traditional didactic contract (besides some general recommendations) or the rigidity of content-evaluation as important *constraints* observed from this curriculum analysis. This "ecological study" will be at the core of our future research. Apart from analysing different curricula, it is necessary to include more "agents" involved in the whole didactic transposition process and how to engage teachers practices fostering inquiry approaches to teaching and learning.

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GLOBALIZATION OF MATHEMATICAL INSTRUCTION: TOP-DOWN PROCESSES MEET DIFFERENT RATIONALITIES

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With reference to the Discussion Document (p. 5, and B5), a framework, derived from Habermas' elaboration on rationality, is proposed to deal with the relations between the universal character of today mathematics, and the cultures of where mathematics is taught and of those who are taught. Some examples are presented (as regards mathematical modeling) to illustrate how teaching may meet students' conceptions and local cultures in the perspective of developing related rationalities. The problem of standardization of the teaching of pure mathematics is discussed, with reference to the variety of present rationalities within mathematics at all levels, and to the cultural resources in different social environments.

INTRODUCTION

If we consider the panorama of changes of mathematics programs and curricula in the last sixty years, we may observe cultural domination phenomena, leaded by international organisms (like OECD) and/or depending on political dominance relationships (Wagner & Lunney Borden, 2012). The New Mathematics case is typical and well known: in that case OECD plaid a crucial role for OECD countries; while in several African francophone countries the major influence came from France, due both to the language and past colonial links, and to the French roots of the New Mathematics movement. Another case concerned NCTM Standards, which from the ninetieths on became reference standards for several Latin American and Asiatic countries. More recently, OECD-PISA definition of Mathematical Literacy (OECD, 2016) is going to play a major role in influencing national programs and guidelines for curricula all over the world, thanks to the impact on public opinion and governments of the results of PISA tests. And Common Core State Standards have a growing impact in several countries (specially those who aligned in the past with NCTM standards).

A motivation brought to justify this tendency to top-down standardization of mathematical instruction in the world is the need of unifying mathematical competencies at the global level, taking into account the globalization in the economic, technologic and scientific fields. Another motivation is inherent in the present functioning of the world community of researchers in pure and applied mathematics. Such motivations are reasonable and strong; but a delicate problem is that they may result in ignoring local cultures and students' cultures. With reference to the Discussion Document (p. 5), till now reform movements have not taken into account:

- The relationships with local cultural traditions, school mathematics, local historically rooted mathematics, street mathematics and mathematics of handcraft professions, and the meaning of mathematics for the local culture (we find a very different situation in Hungary and in Italy, concerning the diffusion of mathematical games magazines!).
- The relationships with pupils' reality (their ways of thinking, their experiences – which may be different in different countries and in different social environments).

Successful students in a situation of cultural dominance are not necessarily those who have the best intellectual resources; more frequently they are those who more easily adapt themselves to what is proposed by the school for family reasons, or more intimate reasons (facility to integrate in a discourse, which is far from their ideas and ways of thinking). The same happens with most teachers.

These remarks pose a political problem (how to conciliate the need of a cultural preparation suitable to live in a globalized world, with the links to be kept with personal and cultural reality), a cultural problem (inherent in the cultural orientation of mathematics education – which mathematics to teach?), and a theoretical problem (concerning the choice of the toolkit to deal with the present rapidly evolving situation). My contribution concerns the last problem, with links with the other ones.

MATHEMATICAL ACTIVITIES IN THEIR CULTURAL CONTEXT

At the school level we may consider school mathematics as a cultural offer aiming at a kind of *local universalism* in front of students' *personal* conceptions and cultures. An obliged, more or less faithful reference to the country programs (or guidelines for curricula) drives teachers into being the agents of that universalism; while in many countries programs and guidelines for curricula implicitly or explicitly refer to *global universalism* through NCTM Standards, or Common Core, or OECD-PISA "Mathematical Literacy". The main political and educational problem concerns the relationships that should be established between the push towards universalism (from the school to the world level), and students' culture (better: cultures), and the culture (better: cultures) of the social context, where school operates. I think that the available theoretical toolkit, even if useful, is not sufficient to deal with that problem: for instance, the classical distinction between "acculturation" and "enculturation" (H. F. Wolcott, quoted in Bishop, 2002, pp. 193-194) is useful to describe and distinguish the cultural normalization of students according to the dominant culture ("enculturation") and the continuous dialogue (particularly inside school) between different cultural traditions ("acculturation"). But the existing toolkit is not sufficient to deal with the complexity of a productive, original dialogue (necessary, in my opinion) between a globalized scientific culture (particularly in the field of mathematics) offered by school, and maturation and development of local cultures stimulated by that offer, through the mediating role of the teacher. A wider theoretical toolkit is needed, aimed at:

- Putting salient characters of different traditions and cultural practices into evidence, to identify contact points and differences among them, particularly within the field of mathematics (mathematicians' mathematics, school mathematics, street mathematics…);
- Establishing relationships between disciplinary culture of mathematics, and other cultures, particularly in the case of mathematical modelling.

In the next subsection I will present the Habermas' construct of rationality, which might satisfy the above requirements. In the following sections I will show how it can be used to deal with some delicate problems inherent in the globalization of mathematics instruction.

Knowing, doing and communicating mathematics: the rationality construct

Many cultural activities (including mathematical ones) may be described as discursive activities sharing some common features:

- Criteria to establish truth and falsity of propositions, and validity of reasoning;
- Strategies to attain the goal of the activity, which can be evaluated;

- A specific language for social interaction and self- dialogue.

The *rationality* construct elaborated by Habermas (1998) may be exploited to move from such a superficial description to a deeper treatment of discursive activities. According to Habermas' construct, rational behavior is characterized by: conscious taking in charge of truth and validity criteria (*epistemic rationality*), of strategies to attain the goal (*teleological rationality*), and of communication means (*communicative rationality*); and by dynamic links between knowing, doing and communicating in the rationality perspective. In particular, links are inherent in the "generative" (of new ideas and problem solutions) feature of rational behavior – so relevant in an "expansive learning" perspective (Engestrom & Sannino, 2010):

> Of course, the reflexive character of true judgments would not be possible if we could not *represent* our knowledge, that is, if we could not express it in *sentences*, and if we could not correct it and expand it; and this means: if we were not able also to *learn* from our practical dealings with a reality that resists us. To this extent, epistemic rationality is *entwined* with action and the use of language (Habermas, 1998, p. 312; emphasis in original).

For a discussion of potential and limitations of Habermas' construct as it was adapted to mathematics education, see Boero & Planas (2014). According to the rationality construct, through different zooms corresponding to different interests we may compare:

- different mathematical rationalities within today mathematics (school mathematics in different grades, university mathematics, professional mathematics);
- salient characteristics of past and present mathematicians' mathematics, with present school mathematics, professional mathematics and everyday mathematics;
- mathematics with other disciplines (like physics, or the grammar of a language);
- mathematics with systems of knowledge and practices in local cultures: "other" rationalities, with potential opposition or congruency with mathematical rationality.

The rationality construct allows a detached, critical vision of the relationships between different discursive practices within mathematics, and between mathematics and other cultural domains. As a consequence, the rationality frame may be used in teacher education to prepare teachers to become decision makers in a rapidly changing world (Guala & Boero, 2017; Boero, Fenaroli & Guala, 2018) and in the design and analysis of teaching sequences and situation (see Douek & Morselli, 2012). It will be used here to deal with some problems of the teaching of mathematical modeling and of pure mathematics related to the cultural context, in the age of globalization of mathematical instruction.

THE CASE OF MATHEMATICAL MODELING

In the last decades the teaching and learning of mathematical modeling (according to Norman's definition of model – see Norman, 1993; Dapueto & Parenti, 1999) has become more and more important in national programs, standards and guidelines for curricula of many countries; its role is crucial in the definition of OCDE-PISA "Mathematical Literacy". I will consider three examples, for which different relationships may be established between mathematical rationalities and students' ways of thinking. Differences pose several problems but offer interesting cultural opportunities (in a perspective in which school – and curricular reforms – take in charge students' rationalities).

Money and purchases

In this case mathematical modeling agrees with usual economic transactions: in terms of rationality we may say that pragmatic validity criteria agree with the organization and results of arithmetic calculations. If we need to pay 8 €, the fact that 2+2+2+1+1=8 perfectly agrees with the fact that the payment of the price of 8 € may be performed with three coins of 2€ and two coins of 1 €. One of the origins of arithmetic calculations was in the field of economic transactions. But in the real context of the use of money we may identify three types of situations that suggest reflections about some limitations of modelling and its results, which might introduce young children to delicate aspects of mathematical modeling. They concern the relationship between the rationality of modeling and the rationalities, according to which the context works and human decision making takes place:

- the fact (very simple, in the reality, but interesting for elementary school children) that the seller, or the machine, may refuse a payment that is legitimate from the point of view of the mathematical model: a machine which sells drinks, as well as the person who sells drinks in a shop, may not accept the payment of an orange juice which costs 1,50 € through 150 coins of 1 cent each; criteria of mathematical validity do not fit with context-related validity criteria (epistemic rationality);
- the fact that "street" arithmetic strategies are often different from strategies taught in the school, but also from strategies belonging to past time mathematicians' traditions. Well known research carried on in Brazil during the eighties put into evidence relevant differences, which concern the teleological component of mathematical rationality but also other components. Particularly as concerns epistemic validity, research has shown how pragmatic criteria - related to effectiveness of strategies and validity of results – allow to validate algorithms and reasoning without needing validation within school arithmetic;
- the more complex fact of possible seller's decisions to encourage fidelity from the buyer.

Sun shadows

Geometric modeling of the sun shadows phenomenon may be considered as one of the most important intellectual constructions for the development of mathematics in the Mediterranean and Middle-East areas (Serres, 1993). The geometric model of sun shadows offers a reliable description, interpretation and forecasting of that phenomenon (at least at the macro-level). Results coming from a correct modeling process well fit how the phenomenon happens during the day and during the year. Validity of propositions (and of graphic representations that support them) agree with functioning of reality. But if we choose sun shadows as a subject for classroom work in primary and even in lower secondary school we easily realize that rationality of geometrical modeling is not the students' spontaneous way of organizing knowledge about the sun shadows phenomenon. The idea of the shadow as a living image of the object which casts the shadow (particularly in the case of the human body: shadow as "another person", or a personal attribute, like the soul) is very frequent among young children (Boero, 1994). The idea of the shadow as a carpet-entity (i.e. the shape that we see) is still frequent among secondary school students, who seem to ignore that what we see on the ground is a section of the shadow space (Boero et al., 1995). It is true that it is easy to put into question those conceptions through well-chosen observations and problem situations, but eradicating the conception of shadow as a "double" of herself may result in the loss of an important element for the affective growing up of young children. Rationality inherent in that conception belongs to the construction of the child's identity. Putting that conception into evidence and giving value to it (through stories to read and/or to invent) may contribute to the development of a rationality which is different from geometric rationality and may open links with literature and visual arts. And even the eradication of the carpet conception may compromise the necessary relationship with perception in the construction of the complex relationships between what the child sees and what she thinks.

In my personal approach to Habermas' construct of rationality, the most important experience concerned the treatment of the sun shadows phenomenon by children in a 7th-grade class in Asmara (Eritrea): in that case, the students' conception of sun shadows was rooted in the local culture, and it was connected to complex ways of thinking natural phenomena. The inherent rationality was enough sophisticated to allow students to solve a lot of interpretation and prediction problems in a correct way, and then they tried to justify the same solution within the frame of the geometric modeling of the phenomenon ("because we need to learn geometry"). The conception of sun shadows of many students relied upon a dynamic equilibrium between light and darkness: in the morning, darkness looses strength, in comparison with light, and shadows reduce their length till noon, then the strength of light diminishes, and the darkness increases…In each moment, the extension (length and width – a bi-dimensional entity) of shadows depends on the extension of the obstacles. Even the speed of decrease in the morning (and increase in the afternoon) of the extension may be interpreted that way (a mathematical interpretation is much more complex and difficult to reach for 7th-grade students).

That way of thinking a natural phenomenon according to a dynamic equilibrium mechanism is shared also by other ancient cultures (voir Cheng, 1997, as concerns Chinese culture); it is important for two inter-related reasons: it may offer a general perspective to deal with phenomena of interest for other disciplines (like ecology); and it may represent a root to enter a perspective of advanced mathematical modeling of complex natural and social phenomena (like that of the dynamic equilibrium predator – prey according to the Lotka-Volterra's model).

In an *enculturation* perspective, the sun shadows conception of some Eritrean children should be considered as an obstacle, a conception to be eradicated. In an *acculturation* perspective, it might be compared with the geometrical model, and finally the latter would emerge as superior ("often one of the contact cultures is dominant, regardless of whether such dominance is intended" – Wolcott cited in Bishop, 2002, p. 194). In the perspective that I propose according to the rationality construct, the rationality of dynamic equilibrium might be the starting point for an important cultural development towards some forms of advanced mathematical thinking and some relations to be established with present problems in the fields of economy and ecology.

The transmission of hereditary characters

In this case, the rationality of probabilistic modeling (according to Mendel's laws) meets other robust rationalities, deeply rooted in past and present cultures, which emerge in the classroom as well as in its cultural environment. In particular in the case of hereditary illnesses we find the rationality which establishes links between what happens and the will of a superior entity (a decision-maker); and we find also the rationality of a subject inherent in stochastic phenomena, who rules them- for instance, by ensuring that after four consecutive "heads" in the tossing of a coin, the probability of "tail" must increase (to equilibrate the resulting lack of equilibrium). Note that most Italian newspapers publish each week the data of Lotto cold numbers.

In an *enculturation* perspective, those conceptions must be eradicated because they are anti-scientific and even dangerous (in the field of health care); in an *acculturation* perspective they must be taken into account in comparison with probabilistic modeling in order to show (through experimental and theoretical considerations) how it represents a superior, more comprehensive and reliable way of thinking stochastic phenomena (but students may resist it: "Even if God obeys probability laws, He may take a punishment decision concerning a single case"). In the perspective of rationality we may go in depth and consider which needs originate the non-probabilistic conceptions, and make them evolve. For instance in a pilot experience with 5th-grade students at the end of a 30 hours itinerary on stochastic phenomena the teacher dealt with traffic accidents, and gradually students moved from the idea of a superior entity who decides the destiny of a person, towards the idea of another superior entity (the state), which establishes laws (and security rules) as an answer to the protection need felt by us, also including the responsibility of the subject - without excluding the reference to other protective entities, deeply rooted in the students' cultural environment.

THE CASE OF PURE MATHEMATICS

The case of pure mathematics might look easy to deal with in the perspective of globalization of mathematical instruction: a superficial analysis would put on the fore only problems inherent in the relationships with students' and teachers' conceptions of mathematics and mathematical entities. Such problems should be considered, according to the necessity of a dialogue with what students and teachers think. On the contrary, in the perspective of rationality we need to consider two different sides of the problem of the relationships between globalized mathematics, and students' and teachers' cultures. First, the variety of rationalities within school mathematics and today mathematicians' mathematics, and the question of which rationality should prevail (or not) in a global perspective of teaching and learning mathematics, also keeping local traditions and historical backgrounds into account. Second, the relationships between the need of consistent, universal acquisitions in the field of mathematics, and the local cultural resources, particularly as concerns mastery of those linguistic tools, which enable access to crucial, basic forms of mathematical rationality – like mathematical argumentation.

In the next two subsections I will try to put into evidence difficulties inherent in these two sides of the problem of globalization of mathematical instruction, as a contribution to deal with them from the perspective of rationality. I must say that I have no answer for the questions at stake, which in my opinion need further collective work in the community of mathematics educators.

Today mathematics: a variety of rationalities

In today mathematicians' products (scientific papers) we may identify many differences as concerns the three components of rationality: not only different languages (verbal expressions and symbolic systems) in different fields, but also different problem solving strategies, and even different evidence criteria and inference rules to establish truth of propositions (e. g. in the case of graph theory and in the case of algebra). The panorama is still evolving, according to the maturation of new fields and the emergence of new links between well established fields. The situation is even more complex if we consider the functioning of discursive practices of mathematicians in the phases of conjecturing and proof construction, and of validation and communication of results among them (Thurston, 1994); and if we read what philosophers and mathematicians write about the nature of mathematics.

As concerns school mathematics, we may identify important differences related to the three components of rationality in different countries (depending on local traditions, local cultures, and cultural inheritance from foreign influences). For instance, on the side of communicative rationality the degree and the kind of formalization of mathematics in textbooks is different (at the same school level) in Italy, in France and in the UK, while relatively homogeneous features may be found in francophone countries, on one side, and in some Commonwealth countries, on the other. And in each school system there are relevant differences between different school levels, as concerns rationality components (e.g. truth criteria are usually different at the primary and at the high school level).

These differences within mathematicians' mathematics and within school mathematics, together with the differences between mathematicians' mathematics and school mathematics, have left in the last century, and still leave today, a great space to different epistemological (and even ideological) options that try to influence mathematics teaching and learning at the global level. Two opposite examples (as concerns the epistemological and ideological choices) concern the New Mathematics movement, and the OCDE-PISA "Mathematical Literacy". Differences concern deep aspects of rationality inherent in mathematical activities and not only content or abilities lists. It is not easy to answer the question: how to avoid that one exclusive epistemological or ideological option prevails and try to inform curricular reforms according to a top-down model? Past experiences suggest that the long term success of a reform movement is compromised, if the variety (and potential richness) of mathematical rationalities (within mathematics, and in the schools of different countries) is ignored.

Linguistic and cultural background, and globalization of mathematical instruction

I would like to recall here an episode of forty years ago – which strongly influenced my decision to move from research in pure mathematics, to an engagement in mathematics education. During the years 1969-70 I was teaching at the Nice University; in those years the implementation of the New Mathematics reform (with a specific French approach called "Mathématiques modernes") was run in France. I was curious about it, thus I tried to understand what happened in the schools by taking part in teachers' meetings. I realized that very different things happened in the schools attended by children of manual workers and North African immigrants, and in the schools of affluent Nice quarters, particularly regarding the impact of set theory and logic activities in early grades. Some teachers of the former schools put into evidence how first grade students were unable to cope with the (relatively) high level of logical-linguistic skills needed to deal with sets, correspondences, etc., in spite of the fact that they had already a good level of operative mastery of numbers and arithmetic operations in everyday situations. This episode came several times to my mind as a "warning" concerning the students' cultural resources to tackle mathematical tasks, and some possible, different educational options: to promote students' logical-linguistic skills development through high level mathematical instruction, or to change the nature of the educational offer and lessen its logical-linguistic level in order to cope with students' limited resources, or to move from students' mathematical competencies and try to integrate their development with the empowerment of their logical-linguistic skills? During the ninetieths I met a similar problem concerning the approach to mathematical proof and proving in grades 8 to 10 with students who had difficulties to manage hypothetical reasoning, "if… then…" clauses, logical implication and disjunction even in practical situations, etc. (cf Luria, 1976). At present, I realize how (in the perspective of rationality) the problem of logical-linguistic skills concerns basic requirements for mathematical literacy if we want to move beyond the level of purely executive performances – in particular, if we want that students exercise mathematical rationality. This problem concerns not only students from poor quarters of Western big towns, but also cultures for which the *written* verbal expression of logical relationships is less easy than in *alphabetic* written languages (with implications for the presentation of proofs in textbooks — a subject of discussion at the CERME-10 Conference, during the TWG-1 sessions — see Wong, 2017).

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PLANNING THE FAMILIARIZATION WITH FRACTIONS

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The literacy about fractions that pupils achieve during primary school should be the result of a process of familiarization. This process differs from the usual process of teaching and learning and involves a specific way of designing and managing the curriculum. In our curriculum we work out the idea of fraction as megaconcept. This idea is the result of the interaction between two distinct groups of practice: a group of teaching practice and a group of "reflective philosophical practice".

INTRODUCTION

"The long persistence of unsatisfactory results in teaching and learning fractions"

As mentioned in the Discussion Document, the writing of curricula is at the center of the international debate and object of analyses that proceed from different perspectives. We take part to this debate highlighting a special need: the long persistence of unsatisfactory results in teaching and learning fractions requires some adjustments both in some didactic principles and in structuring of the content. This involves a specific way of designing and managing the curriculum.

Is it appropriate to introduce the teaching and learning of fractions in primary school?

The long persistence of unsatisfactory results, notwithstanding the efforts over decades both in research and in practice, raises questions about the opportunity to introduce the teaching and learning fractions in primary school. "One reaction to the prolonged history of poor results in rational number instruction … is the postponement of rational number instruction until the secondary school" [Kieren, 1980].

Looking for alternatives to postponement: interaction of two distinct groups of practice

Our instructional choice has been to find alternatives to this hypothesis of postponement. To this aim we have proceeded by the interaction of two distinct groups of practice: a group of teaching practice and a group of "reflective philosophical practice". The first group carried out an enquiring activity, practicing directly in the classes, from the third to the fifth of the primary school. Interacting with the children, this group of teachers has evaluated times and methods of enacting the proposals and has realized to what extent the proposals were effective. The second group has carried out both a process of exploration and a process of reflection on the progress of the "dialogical" interaction that involves teacher, children and the proposed idea of fraction .

Davydov and Tsvetkovich: the idea of familiarisation

In our exploration we have found in the paper of Davydov and Tsvetkovich on fractions [1991], many indications for our classroom practice. In particular their use of the term "familiarization" raised the need to partially renew some didactic principles in proposing fractions to primary school children: we have tried to give an interpretation of the term “familiarization” by enacting the actions in the spirit of Vygotsky's ZPD.

The most common idea of fraction: the fraction-of-something

The most common idea of fraction that pervades the ordinary curricula for primary school is the one of fraction-of-something. This idea is excellently summarized in Bobos and Sierpinska [2017]. “Fraction of a quantity is a mathematical theorization of the visual and intuitive idea of fraction of something. … The idea of fraction-of-something stays in its primitive, intuitive state and functions as an obstacle to the construction of a systemically connected knowledge about fractions.” The idea of fraction as fraction-of-something remains the primitive and intuitive representation on which the knowledge of fractions of nearly all people is built.

The formation of obstacles: a challenge

The question of the formation of obstacles is a very challenging one: it conflicts with some common didactic principles. From the one side, the question of the postponement of teaching and learning fractions is just dictated by the purpose of avoiding the formation of obstacles. On the other side, according to Besos and Sierpinska, the idea of fraction-of-something stays in its primitive, intuitive state and functions as an obstacle; that is, the presence of this obstacle is inevitable. Our challenge consists in believing and implementing in primary school a didactic process that prevents the crystallization of the representation of fraction as fraction-of-something. This should be the fundamental step in order to prevent the formation of obstacles.

“Intuitive representations” versus “primitive, intuitive state”

Our challenge involves the idea of “intuition”. The fraction-of-something as a primitive and intuitive state is strongly connected to the idea that intuitions are firmly correlated with the primitive feeling. To this we oppose the ideas of Fischbein that “intuitions are deeply rooted in our previous, practical and mental, experience” and intuitive representations are “manifestations of highly articulated and very complex structures”. Referring to Fischbein, we have guided children in practicing in order to construct intuitive representations related to fractions.

To construct an intuitive representation that allows avoiding the a priori formation of obstacles, we have explored and practiced the idea of fraction as megaconcept. This idea is suggested by Wagner (1976): "... for the person rational numbers should be a megaconcept involving many interwoven strands" [in Kieren, 1980]. The idea of fraction as megaconcept contrasts with the idea of fraction-of-something exposed by Bobos and Sierpinska. These two ideas of fraction correspond to different "bases of belief" [Bell in Fischbein, 1982].

Fraction as fraction-of-something.

The idea of fraction as fraction-of-something remains in its intuitive state; this fact constrains the way of thinking fractions and acting with them, favoring situations of division associated with the part-whole subconstruct [Kieren, 1980]. [1] In this way, the subconstruct part-whole becomes the only scheme of action that innervates the entire teaching process. Scientific literature has now largely confirmed that these situations have limited teaching effectiveness; furthermore, the didactics of fractions that remains blocked on the only scheme of action framed in the scheme of part-whole subconstruct, involves difficulties in transferring to other types of situations. As a consequence, the idea of the fraction-of-something, associated with the part-whole subconstruct, is one of the causes of the long persistence of unsatisfactory results in teaching fractions.

Fraction as megaconcept.

Considering the fraction as a megaconcept constitutes a different basis of thought: all strands /subconstructs contribute to the determination of the nature of the megaconcept and constitute its structural elements. This point of view hides a challenge for those who are about to build an educational project: on the one hand, the articulated structure of the megaconcept of fraction, if not properly embraced, can be one of the causes of the persistence of unsatisfactory results in the didactics of fractions; however, at the same time it provide the opportunity of an educational path that does not accept the a priori presence of obstacles as inevitable. We have developed the idea of fraction as megaconcept by practicing a wide variety of activities, within the structure generated by the interweaving of the different strands.

Didactic principles: "familiarization", "leggerezza" (lightness), flexibility.

We have developed the teaching practice by putting at the center the didactic principle of familiarization: differently from the usual process of teaching and learning, each activity is practiced without using pre-established formal rules. Instead of knowledge or application of rules, is the practice itself that is structured by a linguistic process of recording, reading, discussing. Just this practice should become the foundation of the subsequent teaching / learning process.

With the familiarization we have developed the lightness/ la leggerezza. The "dialogic" interaction, that involves the teacher, the children and the contents, guides the children to practice with serenity, quiet and confidence, achieving adequate results despite the complexity of the theme. All this implies a continuous rethinking of the practice and a special sensibility of the teacher.

The flexibility is another important principle: in our practice we have favored the flexibility. We have tried to choose the most appropriate manipulative to better introduce the specific idea or concept; we have varied the conditions under which the practice has been developed; we have made ever stronger the link with real demands; by discussion, we have outlined how some concepts find different realizations in different contexts. Proper implementation of the purposes have required attention and reflection.

Practice in primary school needs of a "trace": the "interweaving rhythm"

[1] The strands of the megaconcept of fraction are called by Kieren "sub-constructs of the rational number construct". Kieren identifies five subconstructs: part-whole, quotients, measure, ratios, operators; but there are other possible subconstructs: proportionality, point on the number line, decimal number and so on.

Teaching practice is inserted in the structure generated by the interweaving of the different strands. In building our curricular project we have accepted this challenge trying to articulate the didactics of the different strands into a cohesive and effective structure. The starting point is the key of this process of articulation: it has been derived by a reflection of historical and philosophical type. Searching for the "originary" content of fraction, for its "essence" (Davydov), we have made recourse to Pythagorean ideas. The Pythagorean comparison hides an extraordinarily modern act of mathematisation: the comparison between two quantities is a pair of natural numbers. This act is not only the starting point of the process of interweaving; most importantly it keeps its "trace"[2]. Following this trace we have developed an appropriate "interweaving rhythm", assuming the times and the ways of involving the different strands of the megaconcept, taking into account the specificity of the actualization process, and adapting ourselves to the peculiarities of the context. [Longoni et al., 2016].[3]

The structure the interweaving process is cohesive.

- *The starting point is the act of mathematisation: the comparison between two quantities is a pair of natural numbers, a logos, a ratio. The subconstruct ratio becomes the foundation of the interweaving.*
- *This starting point leads the children to practice the concept of common unit.*
- *Practice with appropriate manipulative takes the children to a special comparison, the measure: measure is an ordered pair of numbers, obtained not by juxtaposition but by comparison. The subconstruct measure is so linked to the subconstruct ratio.*
- *The practiced concept of common unit compels a reconceptualisation that involves the terms "whole", "unit", and "quantity".*
- *Practice with appropriate manipulative brings children to discover the link with the division by themselves.*
- *The teacher reinforces this link and proposes activities that lead pupils to practice the Euclidean division.*
- *The interweaving of the subconstructs, ratio, measure and division is achieved.*
- *The Euclidean division becomes now the core of the subsequent interweaving process that involves other subconstructs: point on the number line, decimal number and so on.*

The use of the adjective "cohesive" outline how the different strands fit together well and form a unit whole, thanks to the initial act, the trace that it keeps, and the appropriate practice.

[2] The term "trace" must be understood in the sense given to it by Lévinas. See ESU8 proceedings; in preparation.
[3] In this paper there is the theoretical basis on which our proposal concerning the fractions is built.

Differences compared to the ordinary approaches to the teaching fractions in primary school

- *Part-whole subconstruct. In many curricula the part-whole subconstruct becomes the only scheme of action that innervates the entire teaching process; in our plan the part-whole scheme is no longer a subconstruct but rather it is an important instance.*
- *Two universes. Often the basic idea is constructivist: teaching / learning is a construction of schemas. The basic scheme is that of integers. The scheme concerning the fractions is constructed starting from the scheme of integers and making the appropriate modifications. We practice instead the construction of two distinct universes. The universe of fraction is a new universe, distinct from the universe of integers, with its own rules and properties.*
- *Properties and rules of natural numbers. In our classroom practice, the children had no leaning to use concepts (such as the consecutive number) and rules that are proper to the universe of natural numbers.*
- *The "1". The reconceptualisation of the concepts of "whole", "unit", and "quantity", takes away "the question of the 1" which often affects the common teaching of fractions.*
- *Ratio. While usually "ratio is a complex concept, which demands a long-lasting learning process" (Streefland), in our approach ratio is an elementary concept: elementary because easy for children, and elementary because it is the foundation of the entire process of interweaving of the strands.*
- *Euclidean division. Children have arrived "naturally" to write and practice the Euclidean division, already in the third grade. Euclidean division gives back unity to the interweaving process and becomes the core of its continuation.*

THE PROJECT

We now present the curriculum for the familiarization of primary school children with fractions. The project is divided in two parts. The first part concerns the third class; the second part, is addressed to the fourth and fifth classes.

FIRST PART: From the comparison to the Euclidean division

The first part is divided into four steps: (a) comparison; (b) measure - reconceptualisation; (c) division; (d) Euclidean division.

Comparison. *The mathematisation of the comparison is the base the whole activity of familiarization with fractions.*

Teacher prepares activities of comparison both between discrete quantities and between continuous quantities. Pupils actively participate in comparing, playing the games and comparing the results, or searching and applying strategies of comparison. Then they collect data. Each activity is discussed. Teacher leads the discussion and highlights above all that each comparison produces a *pair of numbers*, and that in all comparisons there is a *common unit*.

Each comparison is represented and recorded in the exercise books. Teacher indicates three types of representation: (a) objective representation, (b) representation by squares or segments, (c) representation by a pair of numbers, (A;B = 12;7).

At the end of each activity, teacher takes care to discuss representations with pupils. In particular: (a) he emphasizes the presence of the common unit; (b) he focuses on the representation by means of a pair of numbers and asks pupils to describe the comparison starting from it; (c) he teach the pupils to read this representation, taking account of the common unit. In our experience these activities of comparison lasted two - three weeks.

Measure and Reconceptualisation. *The presence of the common unit in the comparison works as guide in the introduction of measure and in the reconceptualization; this latter involves the concepts of "whole", "unit", "quantity".*

Teacher introduces comparisons in which an asymmetry in the function of the compared quantities is evident: for example, in the comparison between eggs and egg packs. During the discussion and the representation, he introduces tools to highlight the asymmetry: (a) to always symbolize the whole by the letter W; (b) in the representation by squares or segments, the whole is highlighted with a certain color; (c) the common unit is always symbolized by the letter U.

Measure and fraction. *The comparison between the quantity Q and the whole W is the measure of Q with respect to W. Then the measure is an ordered pair of numbers: the fraction .*

Teacher introduces a new symbol: Q/W = 7/5. The oriented pair 7/5 is called fraction.

Teacher insists that pupils read these relationships. "The comparison between the quantity Q and the whole W is the fraction 7/5, i.e. the quantity Q contains 7 times the common unit, while the whole W contains it 5 times".

Teacher proposes many activities in which he emphasizes that the fraction is the "number of packs", and discusses this fact with pupils.

Teacher introduces each feature by proposing activities in which a particular manipulative is used (for example, eggs and egg packs). Then he proposes reinforcement activities in which other manipulative are used, either discrete or continuous. In the discussion, teacher repeatedly highlights the characteristics of each of the fractions used to indicate unit, whole, and quantity. In the recording of each activity, pupils always explicitly indicate the unit, the whole, and the quantity.

Division. *The division is one of the subconstructs that participate in the formation of the megaconcept fraction.*

Teacher organizes numerous and various activities in which the quantity is an integer multiple of the pack. In these cases the fraction is equal to an integer number. By repeating these activities, pupils, who in the same period are practicing with divisions, gain awareness of the fact that the fraction is a division. Teacher reinforces this awareness by proposing other activities, by putting together all the similar results, and by focusing the attention of children on this common feature.

Euclidean division. *Euclidean division is the "icon" of the process of familiarization.*

When pupils are familiar with the idea that the fraction is a division, teacher introduces quantities that are not multiples of the "whole" (Q = 20, W = 6). Teacher proposes numerous activities of this type and pupils perform them. Teacher gives form to the discussion through relations of the type: $20/6 > 3$; $20/6 < 4$; $22/6 = 3 +$ Remainder.

In these activities teacher guides pupils in passing from the representation by segments or squares to the representation on the number line. He asks them: to use a squared sheet; to represent a line and to indicate the zero; to determine how many squares each whole is made of; to write the unit under each square (1/6); to highlight the integers on the line and to write them, at least up to the first integer that is greater than the fraction; to explicitly indicate each whole (6/6); to highlight the point on the line that corresponds to the fraction; to put in evidence the remainder (2/6).

Teacher makes sure that each activity is summarized by pupils in the relation $A/W = 20/6 = 3 + 2/6$. This is the Euclidean division.

Part-Whole. *In the proposed interweaving, the part-whole scheme is no longer a subconstruct; it is rather an important instance.*

SECOND PART: Euclidean division as core of the instruction about fractions

Referring to the Euclidean division, teacher introduces a series of activities related to description, recognition, interpretation, understanding, representation, and esteem of properties of fraction. This part is addressed to the fourth and fifth classes.

To write a fraction as whole plus remainder. *These activities reinforce the link between fraction and Euclidean division*.

In front of a fraction, teacher asks pupils the following questions and leads the discussion: What is the unit? What is the Whole? Is the fraction greater or smaller than the whole? What are the integers closest to the fraction? What's the remainder? Teacher asks pupils to write the fraction as an integer + remainder. Pupils discuss, recognize properties and systematically describe them in the exercise book.

To represent the fractions on the number line. *The previous discussions concerning fractions are always finalized to the representation of fraction on the number line.*

Teacher pays attention to the representation of the line and takes care that pupils make use of the strategies already used during the third degree in representing the number line.

Comparison of fractions. *In the comparison of fractions the discussion is the core of the activity.*

Teacher introduces problems in which fractions with the same denominator are compared. In these problems there are simple operations of sum, of subtraction, of multiplication. Pupils discuss solutions by working both on objective representations, and on representations on the number line. Teacher also proposes problematic situations in which fractions with different denominator are compared. He emphasize the properties and strategies that pupils use, reviving the discussion on them; he suggests other strategies; he checks and submits for discussion the correctness of the arguments that pupils present and so on. As an extension of this activity, teacher proposes to place some fractions in ascending / descending order.

An extremely interesting case is the representation of equivalent fractions. Teacher pay close attention to the discussion and justification of this representation.

Decimal Numbers. *The teaching path toward the megaconcept of fraction in primary school culminates in the interweaving with decimal numbers.*

The first step toward decimals numbers is the confidence with decimal fractions. During the activities of representation of fractions on the number line, teacher introduces numerous instances concerning decimal fractions.

The role of decimal fractions in real life, is pointed out by the practice with the decimal subdivisions of the different units of measure, giving so concreteness to the whole activity. Teacher emphasizes the presence of decimal fractions of the whole in many types of measuring instruments: of length, weight, capacity and so on.

Practice with Euclidean division turns into practice with decimal numbers.

Problematic situations. *Teacher favors the search of specific strategies.*

In problems with equivalence, the rule to compare two fractions is left out: in primary school, the activity on equivalence is practice of disclosure.

Concerning the percentage, objective is not a systematic knowledge; it is rather to recognize and describe, how this concept works in some cases.

Especially in the fifth grade, to promote flexibility in interpreting the concepts related to fractions, assumes a central teaching value. Teacher proposes different types of problematic situations: (a) problems in which the habit seems to impose the whole; (b) problems in which the choice of the whole depends on the context; (c) problems in which the concrete situation directs the choice; (d) problems in which the choice is completely arbitrary.

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THE CURRICULAR CONUNDRUM IN THE LIGHT OF THE ATD

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The paper focuses on the notion of curriculum from the perspective of the anthropological theory of the didactic (ATD) whose basics are concisely summarized. The key concept is that of institutional position, with its companion notion of praxeological equipment. This study purports to make an approach based on the genuine assessment of praxeological conformity explicit.

FROM THE CLASSROOM TO SOCIETY

Far from referring to some established scientific concept, the word *curriculum* belongs to the vernacular of schools and universities. When treated as a would-be scientific notion, it turns out to be a polysemous word, with a plethora of intricate, particular definitions. We shall therefore start with shorter, straightforward dictionary definitions. According to the online Macmillan Dictionary, the word *curriculum* means “the subjects that students study at a particular school or college”. The WordNet database defines a curriculum to be “an integrated course of academic studies.” One could multiply such “carefree” examples, but they are enough to start with.

Too often, people who regard themselves as in charge of the future of some institution restrict their attention to those factors or variables over which they believe to have some practical, if not complete, control. From the point of view of a teacher, for example, the curricular problem may be said to legitimately boil down to the questions “what shall I teach them next, and how can I do so?” and “what will they have to learn next?”, while a curriculum-designer may contemplate the question “Will the teachers involved be able (and willing) to teach the chosen contents and to appropriately use the recommended ways of teaching them?” By contrast, it seems that this last question remains mostly out of scope for education policy-makers when working out a new curriculum—in such a context, teachers are usually “taken for granted” and therefore tacitly ignored.

In order to gain perspective on the elusive notion of curriculum, we have to take a step back from these legitimate but parochial viewpoints and look at the curricular conundrum from the point of view of *society* as a whole. The anthropological theory of the didactic (ATD) offers a model of human societies on which we shall draw here (Chevallard, 2018). A society is first regarded, in the ATD, as made up of *persons* and of *institutions*. Both notions are to be understood in a broad sense: an infant is a person, for example, and an institution is anything “instituted”—a family, a couple, a class, a school, a shop, a gang of youths, a football team are institutions. A third notion has to be introduced to bridge the gap between persons and institutions. All institutions are made up of a number of *positions*. A family comprises at least the position of parents and the position of children. In a class, there are the position of student and the position of teacher, sometimes also the position of teaching assistant, of special needs teaching assistant, etc. A school may offer a variety of positions, including those of teacher, student, principal, teacher’s aide, counselor, nutritionist, nurse, computer technician, etc. In a town or city, the position of citizen and the position of mayor are two main positions. There are thus persons and (institutional) positions. The main mechanism whereby a society is continually built and rebuilt is the *dialectic between persons and institutions*, that is *between persons and institutional positions*. From the cradle to the grave, all persons x simultaneously or successively occupy a great number of institutional positions p_k. As an infant, for example, a person first occupies the position of "prospective native speaker". We shall say that the person x occupying a position p in an institution I is "subjected" to p, or, more precisely, to the *praxeological equipment* of p, which is essentially what a (good) subject of I in position p is supposed to be able to do and think about it. An infant is first subjected to the languages spoken in its family or social surroundings. As a general rule, all persons in contact with an institutional position p see their personal praxeological equipment change, in order to conform, at least partially, with p's equipment. Obviously, this is the case when the person x comes to occupy the position of student in a school class. However, any position p inevitably has a formative effect on its "subjects".

Persons are, therefore, the changing outcomes of the array of institutional subjections they have experienced or are currently undergoing. In other words, persons are shaped by institutional positions and the persons who occupy them. This is, however, only one branch of the dialectic. This branch, it is fair to say, is usually given overriding importance by educators. Still, the other branch of the persons/institutions dialectic is no less decisive: institutions and their positions are not givens, but evolving constructs. If a person x has to acquire some knowledge, x will have to track down an institutional position p (or a series of positions $p_1, p_2, \ldots, p_n$, with $p_n = p$) the praxeological equipment of which includes that very knowledge. The education of persons, which is ultimately geared toward supplying institutional positions with "well-behaved" subjects, is subordinate to the creation and development of specifically "educational" positions. Institutional positions are thus the alpha and the omega of the curriculum issue.

FROM SOCIETY TO THE CLASSROOM

When analyzing curriculums and their making, one must start from the institutional positions that exist or are emerging in society together with their praxeological equipment. Let us have a look at a delightful example that we extracted from a letter by Sydney Smith (1771-1845), an English wit and Anglican cleric (Holland, 1855):

> It made me a very poor man for many years, but I never repented it. I turned schoolmaster, to educate my son, as I could not afford to send him to school. Mrs. Sydney turned schoolmistress, to educate my girls, as I could not afford a governess. I turned farmer, as I could not let my land. A manservant was too expensive; so I caught up a little garden-girl, made like a milestone, christened her Bunch, put a napkin in her hand, and made her my butler. The girls taught her to read, Mrs. Sydney to wait, and I undertook her morals; Bunch became the best butler in the county. (p. 159)

Here we see a multitude of positions: the positions of schoolmaster, schoolmistress, governess, farmer, manservant, garden girl, (ordinary) reader, butler, and waitress. Now the main question that arises is: what guarantee does the society have that these positions, as occupied extempore by Sydney Smith, Mrs. Sydney, their daughters, and the little garden girl, deserve their names? In this respect, we have one piece of evidence, which relates to the position of butler. Little Bunch, who is subjected to that position in the service of her master, is proof that the position of butler in Sydney Smith's house compares favorably with the position of butler elsewhere in the county: being a (good) butler means the same, according to the author's judgment, everywhere in the county—there seems to be no exception to the rule. In this case, it is noteworthy that a *person*, Bunch, is testament to the genuineness of a *position*, the position of butler established in Sydney Smith's house. Let us highlight too that the "little garden girl", duly renamed as Bunch—which was the first step in her new career as a butler—was methodically educated to be a butler, the educators being Sydney Smith, Mrs. Sydney and their daughters. In that case, the "host" family is functioning like a *school*, that is to say, an institution allowing educational "gestures" to take place in full legitimacy. The persons occupying a position of teacher are Sydney Smith, Mrs. Sydney and their daughters, while the persons in the position of student are Sydney Smith's son and daughters, and the little garden girl.

The situation just analyzed has a generic significance that extends to any situation involving the person/position dialectic. What exactly goes on behind the scenes? According to the common, undialectical view that focuses on persons and obliterates positions, Sydney Smith seeks to have at his service a "good butler" and, to this end, decides to train the garden girl in the art of being a butler. In contradistinction to this standard interpretation, the analysis propounded by the ATD focuses on the dialectic between persons and institutional positions. Here, Sydney Smith has to simultaneously build the position of butler p in his house and to turn a young person x into a butler, that is to say to educate her to become a good subject of the position p being built. In more technical terms, the master of the house has to define at the same time the praxeological equipement $\pi(p)$ of the position p and to help x progressively conform her personal praxeological equipement $\pi(x)$ with $\pi(p)$. The fact that, at the end of this process, Sydney Smith opines that $\pi(x)$ is in conformity with $\pi(\tilde{p})$, for all position of butler $\tilde{p}$ throughout the county, shows that "his" position p is indeed a position of butler "in the usual sense of the word" (at least in the county). Persons, who are what education aims at changing, are also the means to create and develop positions, which in turn are the wherewithal of the education of persons.

In truth, Sydney Smith also had to create two more "domestic" positions, with a view to educating Bunch as a butler: the first position, p_t, is the one to which he will subject himself in order to train Bunch; the second position, p_s, is the position to which Bunch will have to subject herself in order to learn the art of being a butler under the guidance and supervision of Sydney Smith. Although in this case the positions p_s and p seem quite close to each other, they are definitely distinct—for example, p will normally continue to exist long after p_s has fallen into disuse. Speaking more generally, we shall say that a position p_s is an *antecedent* of a position p if p_s is a position which is supposed to prepare persons x to occupy the position p, which can be denoted as $p_s \rightsquigarrow p$. (Whether or not the position p_s is declared to be a *student* position by some "authorized" institution, one can think of it as a de facto student position.) Still more generally, we can define a *positional path from* p_1 *to* $p = p_n$ to be a (finite) sequence of positions $p_1, p_2, \ldots, p_n = p$ so that $p_i \rightsquigarrow p_{i+1}$ for $1 \leqslant i \leqslant n - 1$. To a positional path from p_1 to p_n, denoted by (p_1, p_n), corresponds a *curricular trail* (p_1, p_n), defined as the sequence of praxeological equipments $\pi(p_1), \pi(p_2), ..., \pi(p_n)$. Note that, for the sake of brevity, we shall leave implicit, for each position p_i of (p_1, p_n), the potential position $\bar{p}_i$ of "teacher" (in an extended sense of the word).

We now define a *curriculum* as the curricular trail $(p_1, p_n) = (\pi_1, \pi_2, ..., \pi_n)$ associated with some positional path $\mathcal{P}(p_1, p_n) = (p_1, p_2, \ldots, p_n)$, where $\pi_i = \pi(p_i)$, for $1 \leqslant i \leqslant n$. This definition has several consequences. The first one is that a curriculum is an existing social reality not to be confused with a curricular *project*. In the second place, and consequently, "working on the curriculum" almost always means changing parts of it, while preserving its essentials. Of course, one can ignore the existing curriculums and their underlying positional paths, which are usually the result of a long and complicated history, and decide to build up a full-fledged curriculum from scratch. In that case, a number of problems are likely to arise. One of them is the tendency to restrict one's attention to a small subsequence of the positional path (p_1, p_n). For example, it can be that $p = p_2$, so that the path is reduced to $p_1 \rightsquigarrow p$, in which case an important issue arises: what will the prerequisites be for accessing the "initial" position p_1 on the newly opened trail to p? In other words, if a person x wants to occupy the position p_1, what should this person's praxeological equipment $\pi(x)$ look like? Empirical observation shows that two opposite situations may occur. In the first one, admission to the starting position p_1 can be highly selective, which solves the initialization problem "elegantly", though to the detriment of less well-prepared candidates, often already affected by restricted education opportunities. The second situation is, in a sense, the reverse of the first situation: the admission to the position p_1 is based on nonstandard criteria, which allows persons alien to the standard curricular trails leading to p to eventually reach that position. Both situations raise many questions, not the least of which is the question—that we shall not pursue here—of their social significance in terms of education inequalities.

WHAT IS THE MATTER WITH CURRICULUM DEVELOPMENT?

To what extent does what has been said about positional paths of the form $p_1 \rightsquigarrow p$ extend to other types of positional paths? In most cases, curriculum development is generally concerned only with a subsequence of (p_1, p), where $p = p_n$, of the form $p_i \rightsquigarrow \dots p_j$, with $i \geqslant 1$ and $i{+}1 \leqslant j \leqslant n - 1$. Although the path (p_1, p) ends with the position p, the end position p has often been lost sight of when p is a "non-educational" position. How is that possible, since the aim in this educational schema is to gradually transform a person's praxeological equipment $\pi(x)$ so that, in the end, it conforms with $\pi(p)$? Too often, "developers" free themselves from the burden of considering what exactly $\pi(p)$ is. How can this happen? The components of $\pi(p)$ are typically of a mixed nature: $\pi(p)$ combine elements of diverse origins. When seen from the vantage point of scholarly fields of knowledge, these coalesced praxeological entities are broken down into "pure" components assigned to mathematics, biology, physics, chemistry, technology, economy, law, management science, history, sociology, etc. The praxeological matter $\pi(p)$ consists of is thus hypostatized into fragments of "pure" knowledge abstracted from $\pi(p)$. The curriculum resulting from this hypostatization process is brought forth by two opposed facts. The first one has an attractive force which is the main driving force in the classical curriculum design: there are institutions that clearly support, not the genuine ingredients of $\pi(p)$, but the "pure" elements—be they mathematical or otherwise—that can be hypostasized from $\pi(p)$. The second fact is that inquiring precisely about $\pi(p)$ takes a toll, both psychologically and materially—some "developers" will even exclaim, "Heavens, we are mathematicians, not anthropologists!" It just so happens that we have a testimonial of the hardships that may await the curriculum "explorer" who dares to go into the "wild". It dates back to the year 1751 and is included in the "preliminary discourse" (*Discours préliminaire*) that opens the *Encyclopedia* of Diderot and d'Alembert. Its author first distinguishes between "the sciences and the fine arts", on which "too much has been written", and the "mechanical arts", on which "not enough has been written well". He concludes: "Thus everything impelled us to go directly to the workers." Here is an excerpt from d'Alembert's account:

> We approached the most capable of them in Paris and in the realm. We took the trouble of going into their shops, of questioning them, of writing at their dictation, of developing their thoughts and of drawing therefrom the terms peculiar to their professions, of setting up tables of these terms and of working out

> definitions for them, of conversing with those from whom we obtained memoranda, and (an almost indispensable precaution) of correcting through long and frequent conversations with others what some of them imperfectly, obscurely, and sometimes unreliably had explained. There are some artisans who are also men of letters, and we would be able to cite them here; but their number would be very small. Most of those who engage in the mechanical arts have embraced them only by necessity and work only by instinct. Hardly a dozen among a thousand can be found who are in a position to express themselves with some clarity upon the instrument they use and the things they manufacture. We have seen some workers who have worked for forty years without knowing anything about their machines. With them, it was necessary to exercise the function in which Socrates gloried, the painful and delicate function of being midwife of the mind, *obstetrix animorum*.

It is true that it was Diderot, the son of an artisan, who accomplished the work, with the help of many collaborators. This is, however, what curriculum development must start from: the analysis, over and over again, of the final target $\pi(p)$, as well as of the intermediary targets $\pi(p_i)$ for $i = 1, ..., n - 1$.

Before going any further, let us introduce a simple model of a country's population with respect to mathematical knowledge. The total population P can be divided into three subpopulations, P_1, P_2, and P_3. P_1 is the subpopulation of (professional) mathematicians—in this author's country, for example, P_1 can be estimated to be about 4000. P_2 comprises those persons who, though they are not professional mathematicians, have pursued, or are pursuing, higher studies in mathematics to become teachers of mathematics (or of physics), at least at the secondary level, or engineers of all types—all of them are, in a certain sense, potential users of non-elementary mathematics. In the same context as before, the subpopulation P_2 represents about 5% of the total population P. Finally, the subpopulation P_3 can be defined as $P_3 = P \setminus (P_1 \cup P_2)$. In many modern societies, when a position p is typically occupied by persons belonging to $P \setminus P_3$, the mathematics used in p are generally "managed" appropriately and efficiently along the curricular trails that lead to p, even if there is much room for improvement. In what follows, we shall focus on the subpopulation P_3, which represents a substantial majority—about 95%— of the total population P.

The members of P_3 are often unaware of the mathematics implicit in the situations they have to cope with. One reason for this shortcoming is that the praxeological analysis of the positions p they may come to occupy, as reflected in the traditional elementary mathematics curriculum, has not been careful enough to identify some not unimportant types of tasks they may be confronted with. Here are two easy examples, relating respectively to subtraction and division. A published paper goes from page 77 to page 102. How many pages does this paper cover? Because of their past experience with arithmetic, most non-math persons will be tempted to calculate the difference between 102 and 77, which misses the target. In the second example, a number of eggs have to be placed in half dozen egg-boxes. How many boxes will be needed if there are 352 eggs? Students aged 13 to 15, not to speak of adults, generally answer that the number of boxes required is the (exact) quotient of 352 divided by 6, that is 58.33 (or even 58.333)—which, of course, is absurd.

As a general rule, the insufficient attention given to $\pi(p)$ does not warrant the relevance of the existing curriculum with respect to real situations of daily life, which can generate unnecessary conflicts and misunderstandings. Here are two more examples. During an interview with a well-known politician, who had emphasized that the unemployment rate had decreased, a well-respected journalist replies, "No, sir. According to the figures I have, the number of unemployed persons has increased." The interviewee politely rejoined that, although the unemployment frequency had increased, the unemployment rate had decreased—a simple arithmetical fact (not an economic fact) that ought to be crystal-clear to a political journalist.

The second case seems even more problematic. Not long ago, a leading French feminist argued that, since “it is admitted that one in two women has been the victim of rape, assault or harassment”, we are forced to conclude that “one in two or three men is an aggressor”. For the mathematically inclined reader, let us state that, if a woman crosses paths with a thousand men during her lifetime, she has a fifty percent chance to come across one aggressor or more, provided there are, not one aggressor out of every two or three men, but seven in 10,000 men! The probability goes up to 99% if five out of 1,000 men are aggressors. It is needless to point out the danger of mathematical illiteracy in the general public and, more particularly, in activist publics.

What conclusions can we draw from the above considerations? The ATD distinguishes between two main *study paradigms*. The first one, which is declining but still dominant in most educational institutions, is the *paradigm of studying works*. A “work” is anything purposely created by human beings. Subtraction and division of “natural” numbers are examples of mathematical works. In the paradigm of studying works, one studies works like subtraction and division in and of themselves, which is often reduced to the study of formal properties (subtraction is anti-commutative and non-associative, division is right-distributive and left-associative, etc.), some of which will prove to be of little avail to general users. In contrast to this classical paradigm, another paradigm is currently emerging: the *paradigm of questioning the world*. In this paradigm, the works that stand at the forefront are of a peculiar nature: they are *questions* that arise in the institutional position p which is aimed at by the curriculum. Two easy mathematical examples of such questions have been considered above. In the first case, the question raised was about the unemployment rate, which is the ratio of the number a of people who are unemployed to the number b of people in the labor force. The correct understanding of the arithmetic fact alluded to above does not necessarily call for an in-depth study of “ratios and fractions”, but simply requires being aware of the following fact: given positive numbers a, b, c, and d, with $a \leqslant b$ and $c \leqslant d$, if $\frac{c}{a} < \frac{a}{b}$ then $\frac{a+c}{b+d} < \frac{a}{b}$. Here, one should think of $\frac{a}{b}$ as the “old” rate of unemployment and $\frac{a+c}{b+d}$ as the “new” one. This key result is rarely incorporated into studies of fractions at an early level. It is easily intuited in terms of urns and balls and can be proved outright by simple algebra.

The second case was about male sexual aggressors of women. It relies on two easy pieces of mathematics. The first one centers on the notion of mutually independent events, the second on a basic fact about limits: if $0 < a < 1$, then a^n decreases when n increases and tends to 0 when n tends to infinity. These last two examples, taken together, are a reminder that a curriculum—a “course of study”—unfolds in time. It is not unusual among teachers to strictly believe that the subject matter S must “come before” the subject matter S', as if there existed a natural, intrinsic sequencing of the contents to be taught. In the paradigm of questioning the world, the logic is different: the focal criterion to decide whether the question Q will be studied before or after a question Q' is to what extent students need an answer to the question Q on the curricular trail they are following. This entails that, in the paradigm of questioning the world, a question Q which, in the paradigm of visiting works, would be regarded as a high-level question, can be studied, if need be, at a much earlier stage of the curricular path. How is that possible? In fact, the new study paradigm leads students to adopt the epistemological style of “science in the making”, in which, at a given time, one makes headway in spite of the many items of knowledge that are still lacking. The clue to this riddle is that any study of a question Q can be resumed at a later stage, where sufficient (mathematical) means are available, thanks to the study of auxiliary questions which contributes to laying better foundations. This foundational work creates the mathematical infrastructure on which the mathematical superstructure rests: mathematical work is therefore made possible by the dialectic between infrastructure and superstructure. To take just one example, here is what John Stillwell writes in his book *Elements of Mathematics: From Euclid to Gödel* (Stillwell, 2016):

> The humble geometric series, which we have used as a foundation for much of this chapter, itself depends on a fundamental fact about limits: that $x^n \to 0$ as $n \to \infty$ when $|x| < 0$ [*sic*]. For beginners, this fact seems obvious enough, and it was assumed in landmark works on the foundations of calculus, such as Cauchy (1821) and Jordan (1887). However, Hardy (1908), in his famous *Course of Pure Mathematics*, thought it worthwhile to probe more deeply, because his hope for his students was that "accurate thought in connection with these matters will become an integral part of their ordinary mathematical habit of mind. It is this conviction that has led me to devote so much space to the most elementary ideas of all connected with limits, to be purposely diffuse about fundamental points, to illustrate them by so elaborate a system of examples, and to write a chapter of fifty pages without advancing beyond the ordinary geometrical series. (Hardy, 1908, p. vii)" So Hardy embeds his discussion of the geometric series in a long chapter about basic properties of limits. These include some properties of increasing sequences that depend on the completeness of R […]. However, the fact that $x^n \to 0$ when $|x| < 0$ [*sic*] is proved in elementary fashion. Hardy offers two proofs…" (pp. 238-239)

Stillwell refers here to a piece of work for the benefit of Hardy's students, which is typically of a foundational nature. Such work had long been delayed, he says, so that some of our outstanding mathematical forebears had to work, in this particular respect, within a not entirely secure historical framework. This is the rule, not the exception, and there is no reason why it should be otherwise in educational institutions—educationally as well as historically, the foundation problem remains indefinitely open to exploration.

The study of a question Q generates auxiliary questions Q_1, Q_2, … Q_j. In the paradigm of questioning the world as we understand it, students inquiring into any question do not count only on themselves and on their teacher. They can draw on the sum total of all the works available to them. Students who inquire into the behavior of x^n can use a calculator to check that, if $x = \frac{9993}{10000}$ (there are seven aggressors in 10,000 men), then $x^{1000} \approx 0.4965 < 50\%$ and also $x^{7000} \approx 0.0074 < 1\%$. The same students can also turn to Hardy's book for a proof that $x^n \to 0$ as $n \to \infty$ when $0 < x < 1$. The first proof offered by Hardy goes as follows (Stillwell, 2016, p. 239): (a) Write x as $\frac{1}{1+h}$ with $h = \frac{x}{1-x}$; (b) Prove (by induction on n) that, for all n, $(1 + h)^n \geqslant 1 + nh$; (c) Deduce the inequality $x^n = \frac{1}{(1+h)^n} \leqslant \frac{1}{1+nh}$; (d) Conclude that $x^n \to 0$. Needless to say, given the position occupied by the students on that particular curricular trail, one or many mathematical works involved in the proof can be beyond their reach. However, they can get some relevant understanding of these items and make the most of it. Therefore, such tools are neither black boxes nor white boxes—they are *grey* boxes, with varying shades of grey. More generally, when inquiring into a question, one has to browse through books, articles, and the Internet. When working on the case of male aggressors of women, students may come upon the martial example expounded by B. V. Gnedenko and A. Ya. Khinchin in their book *An Elementary Introduction to the Theory of Probability* (1962):

> Under certain definite conditions, the probability of destroying an enemy's plane with a rifle shot equals 0.004. Find the probability of destroying an enemy plane when 250 rifles are fired simultaneously. For each

> shot, the probability is 1 – 0.004 = 0.996 that the plane will not be downed. The probability that it will not be downed by all 250 shots equals, according to the multiplication rule for independent events, the product of 250 factors each of which equals 0.996, i.e., it is equal to $(0.996)^{250}$. And the probability that at least one of the 250 shots proves to be sufficient for downing the plane is therefore equal to $1 – (0.996)^{250}$. A detailed calculation […] shows that this number is approximately equal to 5/8. Thus, although the probability of downing an enemy plane by one rifle shot is negligibly small—0.004—with the simultaneous firing from a large number of rifles, the probability of the desired result becomes very significant. (p. 24)

Starting from this passage, students will have to engage in a "backward" study, in order to find out the authors' answers to the questions "What is the multiplication rule?", "What are independent events?", etc. The most important point, however, may be the last sentence, which reminds us that a very low number of aggressors may have disproportionate consequences.

Curriculum development is known to be a long-term undertaking (Malhotra & Bazerman, 2007, p. 146). For a curricular trail (p_1, p_n) to lead effectively to the end position $p = p_n$, a constant survey and analysis of $\pi(p)$ is needed to unendingly adjust the equipment $\pi(p_i)$ $(i < n)$ to a changing $\pi(p)$. In this sense, curriculum development is first and foremost curriculum *updating*. A key criterion in this respect is that, at any stage p_i, the study of a type of tasks T_i can be justified by genuine observations made at some later stage p_j (with $i < j \leqslant n$) of the curricular trail. In the paradigm of studying works, updates are few and far between, because the authority of tradition mostly prevails. The transition to the paradigm of questioning the world, in which a curriculum consists first and foremost of *questions*, is likely to expose the arbitrariness of old-time curriculums founded on a selection of revered works. Although we cannot ignore that this transition will require tenacity and fortitude in pressingly questioning our societies not only about their past, but, daringly, about their real present and their potential future, the choice is laid down plainly and simply.

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A REFORM OF THE BRAZILIAN MATHEMATICS CURRICULUM: SELECTION OF BASIC MATHEMATICAL COMPETENCES FOR THE FINAL YEARS OF ELEMENTARY SCHOOL

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The present article aims to analyse the final version of the National Curricular Common Base (BNCC) for Elementary School, Mathematics, which emerges in Brazil and was consolidated after a process of public consultations and debates. The theoretical contributions adopted for the analysis were based on the ideas of A. Bishop, who proposes that the required basic mathematical competences are developed according to the symbolic, social and cultural components. We also use the criteria for curriculum organization proposed by W. E. Doll Jr. and M. A. Silva: wealth, reflection, reality, responsibility, recursion, relationships, rigor and resignification. Such references were used to analyse the consistency of the prescription for the Final Years of Primary Education, relating their indications in the document, possible impacts for the implementation of the reform and consequently for curricular development.

INTRODUCTION

For Sacristán (2000), the curriculum is defined as a "selective cultural project, cultural, social, political, and administratively conditioned that fills school activity and becomes reality within the conditions of the school as it is configured" (Sacristán, 2000, p. 34). From this definition, it is considered that "in a curriculum very different components and determinations intersect: pedagogical, policies, administrative practices, productive of diverse materials, control over the school system, pedagogical innovations, etc." (Sacristán, 2000, p. 32). Thus, a curriculum cannot be understood apart from the context in which it forms and independently of the conditions in which it develops, characterizing itself as a social and historical object. In relation to the prescribed curriculum:

> In any educational system, as a consequence of the inexorable regulations to which it is subjected, taking into account its social significance, there is some kind of prescription or orientation of what should be its content, especially in relation to compulsory schooling. They are aspects that act as reference in the ordering of the curricular system, serve as starting point for the preparation of materials, control of the system, etc. (Sacritán, 2000, p. 104).

We will consider this definition to name the National Curricular Common Base[1] (BNCC) as a prescribed curriculum, because it is an official document proposed by the Brazilian government, which establishes competencies and guidelines that will guide the curricula of Basic Education[2].

About reforms in the prescribed curriculum of Mathematics, recent studies in the international literature like that of Kanbir (2016, p. 5) identified and discussed controversies in curricular reforms in the United States over the past 60 years. The research revealed that sometimes curriculum changes

[1] Movement for the Common National Base Portal: http://movimentopelabase.org.br/a-base/

[2] Regarding Basic Education, among the tasks prescribed by LDB to the States and the Federal District, is to ensure Elementary Education and to offer, with priority, Secondary Education to all who demand it. And to the Federal District and the Municipalities it is necessary to offer the Infant Education in Kindergartens and Preschools, and, with priority, the Elementary School.

and research in Mathematics Education were closely related but, for the most part, the change was the result of a perception that a challenging curriculum could be ideal "for all students" and regulation maintained by external evaluation regimes. Hoyles & Ferrini-Mundy (2013) underline that researchers in Mathematics Education are led to think that their work was perceived and had a positive influence, but they emphasize that it is doubtful that this actually happens in the curricular proposals. Clements et al. (2013) pointed out that "we need to make sure that the kind of math that is presented in the curriculum proposals as suitable for all is really suitable for everyone" (p. 33).

Thus, recent international studies on reforms in prescribed curricula lead to problems about the type of mathematics that is presented as ideal, curriculum theories, the influence of research in Mathematics Education on documents and questions related to evaluation (mainly external). Our contribution will be given by proposing the use of components and criteria for the analysis of competences from the objects of knowledge and skills prescribed in the BNCC to be implemented in Brazil.

The National Curricular Common Base: Legal Aspects, Processes and Structure

The National Basic Curricular Document (BNCC) was envisaged in the Constitution of Brazil and promulgated in 1988, Article 210, for Elementary School and extended in the National Plan of Education (PNE) for High School, with the aim of re-elaborating Basic Education. With the homologation, public and private schools will have before them the task of building curricula, based on the essential learning established (Brasil, 2017, p. 20). Based on these milestones, the Law on Guidelines and Bases of Education (LDB), in Section IV, Article 9, states that:

> it is the responsibility of the Union to establish, in collaboration with the States, the Federal District and the Municipalities, competences and guidelines for Early Childhood Education, Elementary Education and Secondary Education, which will guide curricula and their minimum contents, (Brasil, 1996).

In this article, LDB clarifies two concepts that are decisive for the curricular question in Brazil. The first establishes the relationship between what is basic-common and what is diverse in curricular matters: competencies and guidelines are common, and curricula are diverse. The second is that, in saying that curricular content is at the service of the development of competences, LDB guides the definition of essential learning, not just minimum contents to be taught.

For the elaboration, autonomous teams were formed and after a short period of time for appreciation and discussion, suggestions were sent for analysis and the promotion of state debates. Before the consolidation, the Ministry of Education disclosed that the analysis would be according to criteria clarity, relevance and pertinence and the second version was sent to the CNE. In 2017, the third and final version for Elementary School was approved and published, and it will be implemented in the year 2019. In figure 1 we have the BNCC structure of Mathematics for Elementary School:

Figure 1 – BNCC Mathematics structure for Elementary School.

PRESENTATION	• **The Mathematics Area.** • **Specific Mathematics Skills for Elementary Education.**
MATHEMATICS IN ELEMENTARY SCHOOL	• **Thematic Units (Geometry, Greatness and Measurements, Statistics and Probability, Numbers and Operations, Algebra and Functions).**
CONTENTS FOR EACH LEVEL PER YEAR	• **Thematic Units.** • **Knowledge Objects.** • **Skills.**

Source: The author, from BNCC (Brasil, 2017).

BNCC proposes five correlated thematic units that guide the formulation of skills to be developed in Elementary School. Competence in the BNCC is defined as the mobilization of knowledge (concepts and procedures), skills (practical, cognitive and social-emotional), attitudes and values to solve complex demands of everyday life, the full exercise of citizenship and the world of work (Brazil, 2017, p.8). In this sense, knowledge objects and skills represent the key points in the proposal for the development of essential mathematical competences.

Theoretical Contributions

When questioning about the basic mathematical skills necessary for student education to live in a society that requires the use of technology in different contexts, Bishop (1988) proposes that this formation takes place according to the components: symbolic, social and cultural.

The symbolic component highlights essential mathematical tools in any culture for learning. This component is organized around six universal activities present in different cultures. According to Bishop (1988):

> I do not see these concepts as "subjects" in the sense that they are given in examination programs. They are offered as organizing concepts of the curriculum that provide the knowledge framework. They should be the core of interest and should be addressed through tasks carried out in rich contexts related to the environment, they should be explored for their meaning, their logic and their mathematical connections, and should be generalized to other contexts do exemplify and validate their explanatory power (Bishop, 1988, p. 132).

For the author, the symbolic component highlights concepts that are worth knowing, through activities related to rich contexts for student learning. The social component conveys the fundamental ideas about the power of mathematical knowledge in a social context, proposing that students work on projects. Bishop (1988) considers that:

> They would allow a teacher to develop student' awareness of the power and limitations of mathematical representation and explanation, and of the relative importance of the values of control and progress (Ibidem, p. 140).

In this way, student and teacher have well defined roles in the educational process, when proposing projects to reach the concepts established by the symbolic component of Mathematics Education. The social component requires learning to think of the perspective of how mathematical ideas are used in social situations. The cultural component is concerned with expanding the student's repertoire in relation to the internal criteria of mathematics, essential knowledge in any culture. Being so,

> This component aims to demonstrate the nature of Mathematics as a culture, the type of relationship with abstractions that mathematicians have and the fact that Mathematical ideas have been invented. (...) Therefore, part of it is included to initiate the students in the technical level of the Mathematical culture, insofar as it is possible to do this with young students in an accessible way. (...) Instead of looking for an "external" perspective of Mathematics, here we will deal much more with internal criteria (Bishop, 1988, p. 149).

For Bishop, this component indicates how, or perhaps why, mathematical ideas were generated and allowed to reflect on what mathematics is. The author emphasizes the need for balance between these three components of the curriculum. For him, activities related to the environment, projects on societies of the past, current and future, as well as the creative aspects of research, are important for Education and for the formation of future generations. All curricular reform should serve this model as it allows the construction of knowledge in a more meaningful way, unlike a linear and cumulative conception. Converging with Bishop's (1988) reflections are the contributions of Rico (1998, p.21) on the importance of Mathematics Education, which affirms that it is necessary for the curriculum to offer teachers concrete proposals for understanding the knowledge and interpretation of the message (RICO, 1998). In addition, when putting into practice what is in the curriculum, it is necessary to demonstrate the usefulness of the contents. In this way, it will be possible to establish priority dimensions in the curricular organization, which allow the structuring of Mathematics Education and its purposes, so as to be able to list the curricular innovation programs, defining the different goals for each country. About the curricula, Pires (2008), highlights that:

> [...] in a kind of "eternal coexistence" with prescriptive curricula (the official documents) and the real curricula (those of the classroom, which teachers do). Thus, a phenomenon common to different levels of the education system (federal, state, municipal) is the introduction, in certain periods, of curricular changes that do not have the support of previous concrete experiences nor the involvement of the teachers, protagonists of its implementation (PIRES, 2008, p.40).

The idea proposed by Bishop (1988) points out that, together with Rico (1998) and Pires (2008), the teaching of Mathematics needs to be more significant and qualified as an integral part of a socially constructed culture. Both underline the importance of the teacher in the implementation of the curriculum. It is this professional who, through methodological and didactic choices, means the contents and qualifies them in the process of teaching and learning.

We understand that school content must be meaningful and closely linked to the development of competencies. In this sense, Silva (2009) suggests the selection and organization of content through eight criteria; four of them based on Doll Jr. - wealth, recursion, relationships and rigor - and suggests others - reflection, reality, responsibility, and resignification. Doll Jr. (1993, p.180) states that a postmodern curriculum will require "being rich in diversity, problematic, and heuristic. It proposes criteria for a curriculum designed to promote a postmodern view:

> What criteria could we use to evaluate the quality of a postmodern curriculum - a curriculum generated, not pre-defined, indeterminate, but limited, exploring the "fascinating imaginative realm of God's laughter," and consisting of an ever-growing network of "local universities"? I suggest that the four Rs of Wealth, Recursion, Relationship, and Rigor could serve this purpose (Doll Jr., 1993, p. 192).

Silva (2009) suggests a deeper reflection on the four Rs of Doll Jr. applied to Mathematics Education, proposing four other criteria. Thus, according to the specialist:

> (1) "Wealth" privileges the choice of contents that show how rich the mathematics itself is and how the theory-practice relationship can be efficiently dosed (...); (2) the "reflection" the selection of subjects that serve the interest of a certain community, chosen only after the election of local problems; (3) the "reality" the option for themes that can be modeled by means of a real situation. (...); (4) the "responsibility" the priority of mathematical contents that can be used to analyse, compare, estimate and solve social problems (...); (5) the "recursion" proposes that the contents must be arranged so that they can be resumed as the students progress; (6) "relationships" raise concerns when we organize a curriculum such as time management and reflection on issues common to all through projects (...)(7) "rigor" is linked to procedures, evaluations and the interpretation of results inserted in a new context linked to indeterminacy and interpretation. (8) the "resignification" gives the History of Mathematics its due importance in a curricular proposal as articulating and clarifying the process by which the knowledge was constructed. (Silva, 2009, p. 223-225).

Bishop's (1988) were adopted to verify how the contents and skills for the final years of elementary school in the BNCC highlight principles for mathematical competences. They are closely connected to the principles of selections and content organizations proposed by Doll Jr. (1993) and Silva (2009).

Methodological Procedures

A qualitative and documentary analysis will be carried out, considering as a document the final version of BNCC for the Final Years of Elementary School. According to Sharma (2013),

> Analysing documents is a form of collecting qualitative information from a primary or original source of written, printed and recorded materials to answer the research questions in interpretive case studies. The documents provide evidence of authentic or real activities undertaken by human beings in social organisations and human thinking (Sharma, 2013, p.3).

By means of the analytic category, principles of selection of basic mathematical competences will be selected and discussed, sections of the document referring to the symbolic, social and cultural components present in the proposal and the eight R's proposed by Doll Jr. (1993) and Silva (2009) related to content selection and curricular organization. These references were used to analyse the consistency of the BNCC prescription for the Final Years of Elementary School, listing their indications in the document, possible impacts related to the implementation and suggestions for improvements in the proposal, aiming at reflections for curricular development in Mathematics.

Principles of Mathematics Skills Selection at BNCC

BNCC for Elementary School emphasizes that mathematical processes such as problem solving, research, project development and modeling can be cited as main forms of mathematical activity throughout this stage. These learning processes are potentially rich for the development of fundamental skills for mathematical literacy (reasoning, representation, communication and argumentation) and for the development of computational thinking (Brasil, 2017, p.264).

Considering these presuppositions, and in articulation with the general competences of Basic Education, the document points out that the curricular component should guarantee the students the development of specific competences for this stage of education:

> the importance of communication in mathematical language with the use of symbolic language, representation and argumentation. In addition to the different didactic and material resources, such as checkered meshes, abacuses, games, calculators, spreadsheets and dynamic geometry software, it is important to include the history of Mathematics as a resource that can arouse interest and represent a meaningful context for learning and teaching Mathematics. However, these resources and materials need to be integrated into situations that foster reflection, contributing to the systematization and formalization of mathematical concepts. It is also important to consider that, in order to learn a certain concept or procedure, it is fundamental to have a meaningful context for the students, not necessarily of the everyday, but also of other areas of knowledge and the history of Mathematics itself (Brasil, 2017, p.296).

Competence refers to the "symbolic" component because it highlights essential mathematical tools in any culture for learning (resources) in contexts that are significant for the student. It is also referred to the criterion of curricular organization "wealth" when privileging the choice of contents that show how rich the mathematics itself is and how the theory-practice relationship can be carried out effectively. The organizational criterion "resignification" was identified in conferring the History of Mathematics as fundamental to mean concepts constructed in different contexts.

For the Final Years (6th to 9th grade), a search for excerpts of the document was made based on the questioning of the basic mathematical competences necessary for the formation of the student in our contemporaneousness of Bishop (1988), identifying in the thematic units, objects of knowledge and skills required and correlated for each year indications of the presence of the symbolic, social and cultural components. They were related to the content selection criteria (R's), proposed by Doll Jr. (1993) and Silva (2009). Table 1 below shows the list of components and criteria selected in the prescribed curriculum:

Grade	Thematic Unit	Knowledge Objects	Skill(s)	Components	Criteria (R`s)
7º	Probability and Statistics	Sample research and census research. Research planning, collection and organization of data, construction of tables and charts and interpretation.	(EF07MA36) Plan and conduct research involving social reality, identifying the need to be census or to use sample, and interpret the data to communicate them through written report, tables and charts, with the support of spreadsheets. (p.309)	Social.	Reality and Responsibility.
	Algebra	Algebraic language: variable and unknown.	(EF07MA14) To classify sequences into recursive and non-recursive, recognizing that the concept of recursion is present not only in mathematics but also in the arts and literature.(EF07MA15) Use algebraic symbology to express regularities found in numerical sequences. (p.305)	Symbolic.	Reality.
9º	Probability and Statistics	Planning and execution of sample research and reporting.	(EF09MA23) Plan and execute sample research involving social reality theme and communicate the results by means of a report containing evaluation of measures of central tendency and amplitude, adequate tables and graphs, built with the support of spreadsheets. (p.317)	Social.	Rigor and Responsibility.
	Algebra	Directly and inversely proportional magnitudes.	(EF09MA08) To solve and to elaborate problems that involve relations of direct and inverse proportionality between two or more magnitudes, including scales, division into proportional parts and rate of variation, in sociocultural, environmental and other contexts contexts. (p.315)	Social and Cultural.	Wealth, Reflection, Reality, Responsibility and Rigor.

Table 1: Components and criteria of organization and selection of contents identified in BNCC Final Years of Elementary School.

According to Table 1 above, in the BNCC prescription for the Final Annals of Elementary Education, the absence of these components in the 6th and 8th years and the presence of the cultural component in the 9th year were observed, representing gaps in the proposal in relation to the specific competencies announced by the document. The base prescribes that in this stage the student must face problem situations in multiple contexts, using different registers and languages and develop and/or discuss projects that mainly address issues of social urgency, based on ethical, democratic, sustainable and solidary principles, valuing the diversity of opinions and social groups (Brasil, 2017, p. 265). Since these dimensions are related to both the teaching and learning process and the curriculum organization of school mathematics, the absence of these can have implications for the development of basic skills listed in the document.

The absence of the organizational criterion "recursion", which seeks in the classic spiral curriculum model of Bruner (1960) the inspiration to propose that the contents must be arranged so that they can be resumed, jeopardize the progress of students' studies, once which may not be addressed in other contexts, being reviewed as simple repetition. Such a gap in the proposal for the final years deserves special attention so that it does not negatively impact the development of competences as "ability to organize, combine, inquire, use things heuristically" (Doll Jr., 1993, p. 195), which, once developed in a reflexive and convenient manner, can bring perspectives to the solution of scientific and technological problems and to support discoveries, including impacts on the world of work, situations highlighted in the specific competences for the area of Elementary Education (Brasil, 2017, p. 265). The presentation of the area also highlights that it is extremely important to consider the heuristic role of experiments in mathematical learning (Brasil, 2017, p. 263). Regarding the thematic units, no components were found in Geometry, Quantities and Measurements and Numbers, and only Algebra and Probability and Statistics were identified in the 7th and 9th years, which also refers to certain plaster and linearity that could be revised in the proposal, considering that concepts and skills should be approached from socio-cultural problems that involve all thematic aspects of the document.

In the 7th year "Probability and Statistics" unit, the required skill presupposes planning and conducting research involving the social reality, identifying typology of samples and interpreting the data to communicate them. Such a prescription refers to the "social" component, since it proposes to the curricular development to think about how to use tools and mathematical ideas in a social context. The criteria of selection, "reality", when dealing with research that can be modeled from a real situation and "responsibility", for prioritizing analysis of data that may have impacts on society, were also contemplated.

In "Algebra," skills refer to the "symbolic" component as they require situations to be exploited by their meanings, logic, connections and regularities, and which can be generalized to other contexts or areas. The criterion "reality" when pointing out that in real situations the algebraic language allows to describe, represent and present results accurately and to argue about their conjectures, establishing relations between it and different representations.

In the 9th year for the unit "Probability and Statistics" the "social" component was identified since the ability suggests the planning of a research project that allows the development of critical awareness from the tools of organization and analysis and statistical data of subjects that should be chosen from a social context. Regarding curricular organization, the criterion "rigor", because sample research is intrinsic to procedures, evaluations and interpretations that will take into account variables related to a social context. The criterion of selection "responsibility" stands out because such statistical surveys allow analysis, comparison and estimates that can co-operate with the resolution of social issues.

For Algebra, the listed ability aims to broaden the student's repertoire in relation to the knowledge essential in any culture, referring to the "cultural" and "social" components, emphasizing that fundamental ideas about mathematical knowledge in a social and cultural context can bring work perspectives with projects about the society of the past, the current and the future. In relation to the content selection criteria, we identify the "wealth", because relations of proportionality and variation rates allow us to perceive how rich Mathematics is, the "reflection" favoring the selection of subjects in socio-cultural, environmental and other contexts; the "reality" because they are objects of knowledge that allow the treatment of real situations and the "responsibility" for allowing them to be used to analyse, compare, estimate and solve social problems. On the organizational criterion "rigor", the prescription suggests the work with rates of variation allows the modeling in contexts linked to indeterminacy and interpretation, including in decision-making processes and future predictions.

Final Remarks

The analysis of the BNCC for the Final Annals of Basic Education revealed the presence of social, symbolic and cultural components linked to the objects of knowledge and their respective abilities, fruits of the questions about the development of basic mathematical competences necessary for the formation of the Brazilian students in the present times, as well as the presence of content selection criteria and curricular organization. The study indicates, with some limitations, a socio-cultural and postmodern view to the proposed curriculum proposal, relating the objects of knowledge, which can conceive the student the opportunity to perceive the meaning of the contents, to make social use them and to obtain a full access to the their citizenship.

In contrast, the lack of consideration of theoretical bases, advances in the area of Mathematics Education in the proposal and the centrality in the contents assume that the reform is tied to an "ideal" curriculum, according to studies by Kanbir (2016) in the United States, which may threaten Brazilian school autonomy. The construction of the curriculum from the prescriptions should not be understood as a product or static object in which there is a delimitation in what can be planned and implemented; he must present the real culture of society, seeking continuous reworkings that decide on what will be done in relation to teaching.

The proposal of the components of Bishop (1988) linked to the reflection and proposition of curriculum analysis criteria under the postmodern view of Doll Jr. (1993), extended by Silva (2009), helped in the comprehension and study related to curricular reform in Mathematics which emerges in Brazil, adding to the international literature on curriculum and Mathematics Education because they are the guide that refine the understanding of basic mathematical competences present in the prescribed curricula.

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THE ROLE OF INTERDISCIPLINARITY IN CURRICULAR REFORMS: THE CASE OF ANDORRA

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The presentation reflects about the theoretical foundation of an integrated curricula organized upon global situations from an interdisciplinary perspective with deep understanding of mathematical workshops for introducing mathematical objects and processes. It is explained the case of Andorran curriculum in which it was possible to find such an equilibrium. The empirical data shows the possibility to introduce these ideas in a curriculum for a country, not only as a school experience.

INTRODUCTION

Several studies in Mathematics Education describe the difficulties that students have in connecting their learning to their everyday life. In order to build these connections, it is suggested that environments should be created where students can learn competently without overlooking the contents meaning by using real problems (Clark & Lampert, 1986). Other authors focus on increasing the students' creative attitudes by promoting mathematical processes such as exploring, using different representations and using collaborative technological environments, or using activities as projects and placing problem solving activities in real world contexts (Verschaffel, Greer, & De Corte, 2000). Many projects have been introduced to make these connections possible as a curricular approach for many years (D'Ambrosio, 1976). An interdisciplinary approach is a key element for any successful educational enterprise that aims to prepare future generations for dealing with complexity and interconnectivity of our world (Sriraman & Frieman, 2009).

During the decade of the 90s some important European projects such as Wiskobas, focused on the importance of contextualizing processes to teach the abstract characteristics of Mathematics and so introduced the idea of Realistic Mathematics Education. Although interdisciplinary projects have been developed in many schools by using fields of experience (Boero, 1992), it seems that it is difficult for these proposals to occur in the curricula of countries that are above 20 million inhabitants. It has been said that training teachers in interdisciplinarity is difficult, because it means breaking with the tradition of specialized training. Nevertheless, some school systems—e.g. in Japan and Canada—have carved out more permanent spaces for interdisciplinary project work in the curriculum at different stages of the school curriculum (Howes, Kaneva, Swanson & Williams, 2013).

The twentieth century appears as the century of the democratization of the school not only in the United States (Kilpatrick, 2005) but also in many other countries. Problem solving has been the focus that has emerged as a key element in that framework (Ertmer, Schlosser, Clase & Adedokun, 2014). The low results that appear in the math tests in many countries, makes us lose sight of a curricular view on the scientific attitude only to return to focusing on the results. This results in the "return back" to proposals that, despite not focusing on the content, propose a look towards the definition of standards that must be fulfilled. The perspective of a world with hope but also uncertainty leads us to considering a school that must constantly change to respond to the demands of today's society. Thus, the STEM movement (Science, Technology, Engineering and Mathematics) seems to gain strength in the curricular developments of the second decade of the 21st century. In such integrated curricular experiences (Becker & Park, 2011), we will argue that a social, historical account is necessary, one that explains how disciplines have become both socially functional and yet also dysfunctional. Promoting STEM education, as an integrated curriculum is now a central aspect of educational policy in many countries worldwide (Riordain, Johnstone & Walshe, 2016); in order to prepare students for a more advanced scientific and technological society (Galev, 2015).

IINTERDISCIPLINARITY, COMPETENCIES AND SPECIALISED VIEW

Many curricular foundations serve when analyzing the role of Mathematics for helping the interpretation of science. We find first, the proposals based upon the analytical framework of deductive traditions; problem-solving approach and those facing complexity, inquiry and modelling approach by using projectual work. Such framework, recognize the need of preparing for productive knowledge, and those focusing on inter-related discipline connections. We should interpret the competencies approach as a pendulum movement against the "Back to basics" movement. The KOM Project introduced the notion of competency in a broader perspective as *having knowledge of, understanding; doing and using mathematics and having a well-founded opinion about it, in a variety of situations; contexts where mathematics plays or can play a role* (Niss, 2002: 182). One interesting idea of such approach is integration with the notion of reflective practice giving opportunities to promote a research attitude.

What is new in this competence perspective is the importance of semiotic analysis of mathematical practices. It supports the need for having a deep understanding about the role of interpretation and communication in all human and professional perspectives. The notion of competency relates to what neuroscience distinguishes by doing and learning mathematics, broadening the notion of learning when students use contextualized problems. When the teachers and researchers use all the human sciences interlock and can always to interpret one another: their frontiers become blurred, intermediary and composite disciplines multiply endlessly, and in the end, their proper object may even disappear altogether (Foucault, 1970: 357). Authors such as Peirce, considered as a precursor of contemporary interdisciplinarity define science as an interdisciplinary process in which communication —that is, love- produces new knowledge. The key to the advancement of knowledge and to the development of sciences is not revolution, but communication. Communication between the members of a science community is essential for scrutinizing the evidence and the results achieved in research (Nubiola, 2005).

"One of the most salient phenomena of the life of science is that of a student of one subject getting aid from students of other subjects" (Peirce quoted by Eisele, 1985). The key of interdisciplinary studies (according Peirce) is not the revolution, but sharing efforts assuming a singular mixture of continuity and falibilism. By far the most ordinary way in which one science extends a service to another is by furnishing it with a new fact, which the aided science treats as if it were a direct observation. (...) the science, which receives that fact, when it has performed its generalization of the fact, will return to the science, which furnished that fact an explanation of it. The rules, operative procedures in scientific investigations — material and associated discursive practices— are specific to the discipline, in particular in those situations where there is a recognition that they have to be appropriate to the object (Bourdieu, 1992). The regular framework of interdisciplinarity involves the work of specifying a *common* object-motive (product), which likely differs from object-motive$_1$ and object-motive$_2$ that characterize the respective mono-disciplinary efforts. Many of the recent experiences of an interdisciplinary and integrated unit, allowed connections between topics in mathematics, science and social studies. Interdisciplinarity usually creates a balance between disciplines as activities are shaped by the context while also respecting their individual curricula goals and objectives (Williams et al., 2016).

THE ANDORRA CASE AS AN INTEGRATED PROPOSAL

Andorra is a small country with three shared educational systems: the French, the Spanish and the Andorran one. Assuming a general educational perspective, as a result of the growing interest in developing a profile of increasingly competent citizens. This educational reform started in the 2012-2013 academic year and is reflected in a program called *Strategic Plan for Renewal and Improvement of the Andorran Educational System* (PERMSEA, 2016). The main issues of such a reform are: a) a plural use of four different languages and cultures (Catalan, Spanish French and English) in which students achieving competencies, to solve with efficacy complex situations. b) The development of specific and transversal competences that should allow students to be protagonists and regulators of their learning, and intervene in the different areas of life: personal, interpersonal, social and professional. c) A new assessment role of regulating learning experiences, by using a set of learning expectations. d) the teacher as a risponsable of a final level of decisions.

It is assumed that there is a need for developing a set of specific and transversal competencies students can be the actors and regulators of their own learning, no they can intertwine the various fields of their everyday life: personal, interpersonal, social and professional. Such competencies should promote the Andorran cultural identity, adaptability and autonomy, so there is a critical and creative citizenship spirit, with actors willing to cooperate with the population bringing peace and solidarity. This means a new curriculum is needed promoting an efficient and sustainable educational system and a new project for teacher training and selection.

It is not easy to think of a global and interdisciplinary curriculum design, but we know of many integrated experiences in a scientific arena (since Jacobs, 1989 until the ICMI 13 group study). In our case, we assume a position that there is a confluence between such proposals and a curriculum based on competencies. In this approach, the integrated school activities enrich the possibilities of connectivity of knowledge, facing complexity. In a way providing a "powerful idea", a cross-cutting idea, a perspective on perspective taking—that may be of great value (Ackerman 1989: 29). In the case of mathematics, it also allows to enter in a modelling perspective. In the Andorran curricular reform, it is assumed that the teachers are responsible for constructing integrated socio-cultural units and workshops to develop mathematical and transversal competencies. In the sequence construction proposal, the following design is followed: 1) Proposal: choose a single competence activity (situation-problem) to be carried out during the interdisciplinary project. 2) Rethink the situation-problem to extract the different transversal competences that will be worked on with its realization. 3) Think and plan the different phases (one by one) and the learning tasks in each one of them, to promote the chosen competence (global-situation) activity. 4) Contrast planning of the competence activity (situation-problem) with the scheme of the transversal competencies to see if we can introduce some other element to the competency activity (situation-problem) and favor the work of another transversal competency. In such approach, students then participated in decision-making processes based on conclusions drawn from the analysis of data. It is for this reason that the areas of training determine the orientation of the general competencies that configure the student's output profile and the various curricular elements. The training areas also help put students in a situation that needs to be identified, delimited, examined, debated, or solved. For this reason, they contribute to giving meaning and authenticity to the learning situations of the programming units, in order that these units are global and focus on lifelong learning. In addition, they can also orient the central activities promoted by the same school board. The areas of training around which the curricular framework of the compulsory education of the Andorran School is based are orientation and entrepreneurship; environment and consumption; mass media, health and welfare, and citizenship and coexistence.

There are competences or components of competencies that have obvious disciplinary support, that is, that its attribution is based on a particular discipline. There are other competencies or components of the competencies that are clearly achieved through interdisciplinary work and, therefore, are generated with the scientific support of various disciplines. Finally, there are other competencies which do not have epistemological support in any specific discipline and, therefore, are apparently more difficult to show from the disciplines themselves. That is, why we must consider the existence of a curricular field that integrates them. This concerns what is called the Transversal Area referring to the one that meets the competences of a metadisciplinary nature, whose learning objectives and resources are common to all areas (whether it is through personal intellectual tools or cooperative tools, such as learning management, regulation of emotions and conflict management, or participation in common projects).

The resources (mathematical objects or processes) considered, are grouped into five competencies. 1) Resolve everyday problems, based on the thinking skills and the processing of the information. To develop this competence, students must obtain, organize and interpret the information critically, comprehensively and creatively, to address difficulties of varying scope and relevance. 2) Prepare and communicate presentations and creations in different formats and media. Students should develop this competence, involving verbal communication, non-verbal and audiovisual language, to express themselves freely and creatively in different contexts and situations. 3) Plan and regulate the process of personal growth. Students should develop their competence based on the analysis and evaluation of characteristics, behavior, interests and their own productions (and those of others) in order to make appropriate choices and build self-concept. 4) Act skillfully in various social situations. Students should develop this competency by regulating the expression of their own emotions, and sharing and arguing ideas, feelings, thoughts and behaviors with empathy, assertiveness and respect. In order to analyze and manage interpersonal and social conflicts (in a way creative, flexible, tolerant and supportive), in order to assume social responsibility and moral autonomy and achieve healthy and rewarding social relationships.

The Andorran proposal, not only introduces STEM ideas, but includes the use of Languages, cultures and Social Sciencies as a global perspective.

SECONDARY SCHOOL MATHEMATICS SPECIFICITIES

Classroom work is proposed in various "spaces". Clearly interdisciplinary spaces are called ***global situations.*** In the preparatory phase, the use of mathematical representations is basic, as are arguments that justify the introduction of mathematical objects that are present in an interpretation of the global situation. In global situations, mathematical resources contribute to learning as they allow us to use numbers, functional relationships, geometric structures, as well as the reasoning for interpreting phenomena of such situations. In our perspective, global situations act as fields of experience (Boero, 1992) rather than simple contexts, because learners already have such experienced contexts and from which rich mathematical ideas can emerge.

The meanings of mathematical objects are justified, and then value is given to facts and concepts. In the resolution phase, the specific mathematical tools such as tables, schemes, relationships, etc. are constructed, which allows the organization of scientific information with specific procedures, such as calculus, the equation approach, the use of techniques of measure, approximation, etc. In the integration phase, the recognition of specific mathematical models is encouraged; the calculations are decontextualized and general rules are applied to other different situations, etc. It is the synthesis moment. In the specific ***disciplinary workshops*** associated with global situations, focused on the area of mathematics, resources are developed that should allow identification of specific mathematical elements, based on introductory situations in the mathematical field (numerical, geometric, statistical, etc.). The role of the teacher is to guide through the use of the resources proposing manipulations, discussions on processes, or resolution algorithms and techniques. In this space, they introduce and justify the elements of progressive mathematical abstraction, valuing processes resources such as: problem solving, generalization, planning, and hypothesis formulation. In the ***personal workspace***, the resources of mathematics that need more memorization, application and exercise tasks, are used beyond the activities carried out during the sequences of global situations and workshops. Some specific resources such as measurements and those related to control, analysis of changes would be developed throughout the different spaces (global situations, related workshops and specific workshops). A research workshop was proposed where the teacher helps to formulate mathematical questions of increasing quality for the development of mathematical processes. The fact that some of these processes have a global side as well makes some aspects common and transversal, such as resources that have to do with production of hypotheses, connections and information processing. It was used the name of resources, to escape the classical content proposals.

Interdisciplinary and disciplinary competencies and assessment criteria

We should consider the contribution of the area to interdisciplinary competencies (treatment of information and data, resolution of problems and constructions of hypothesis and communicative reasoning) and disciplinary competences to mathematise relate and model, and use problems, techniques and resources). It is decided to structure the disciplinary competencies from a perspective that we could call pedagogical / didactic, with three key competences, based upon the idea of *literacy*; *materacy* and *tecnoracy* (D'Ambrosio, 1990) stated as follows:

C1. Critically interpret real phenomena through objects and mathematical processes. This implies: a) Representing real phenomena from mathematical objects and processes; b) Associate mathematical meanings to various real phenomena; c) classify and define the mathematical elements constructed from real phenomena and d) formulate conjectures through mathematical language

C2. To analyze the change in real phenomena, establishing the corresponding mathematical models. This implies: a) Identifying the relevant variables in the observation of real phenomena; b) identify features that distinguish the exchange rate; c) carefully use algebraic language; d) perform predictions in random and random situations and reason the trials and solutions obtained.

C3. To solve complex mathematical situations from various mathematical techniques. This implies: a) Forming problems associated with situations in the real world solved with tools; b) Make good use of mathematical strategies, techniques and resources; c) interpret the validity range of statements associated with problem situations.

In order to recognize the regulation and assessment of competencies, some criteria are assumed. For the first competency, we consider: Relevance in the selection of mathematical models. Coherence in the construction of the meaning of a mathematical object; Effectiveness in the application of mathematical tools and processes; Accuracy in the use of concepts, languages and scientific symbols. For the second competency, we consider: Coherence in the realization of mathematical models, relationships and patterns of change. Relevance in the interpretation of change relations. Effectiveness in the use of properties and mathematical relationships and Accuracy in the analysis of variations, properties and mathematical patterns. For the third competency, we consider the following assessment criteria: Coherence in the development of processes and strategies involved in the construction of hypotheses and in the validation of the reasoning and conclusions. Relevance in the selection of the techniques, strategies and mathematical resources. Effectiveness in the application of mathematical techniques and adaptation when communicating the different processes.

THE TEACHER AS A CURRICULUM BUILDER

Working on global situations is not new to mathematical education. A global situation proposes an everyday fact that shoud be analyzed mathematically. The new curriculum proposes that teachers themselves find such global situations, and use workshops to introduce mathematical objects and processes. For example, for 12-13 years it was proposed an energetic study for a building. Not only does it offer a superficial realization, but also it starts with the intervention of an expert so that students can recognize an expert level. Thus, a Map of facilities is proposed as well as a Letter of demand to architectural firm on what is required in a reform of a house.

It starts with the idea of finding information on analyzing the conservation and dissipation of energy in energy transfers. A web space linked to the Languages is created. It is decided that the results should be communicated in the form of a mural. A second question is asked: how to improve the lighting of a house located in the dark area of the mountains. To think initially about the global situation-problem, questions are asked as the following: How to improve the thermal insulation of homes? How to improve the illumination of a house? How to distribute the heating in a house? How to isolate acoustically a house? From there, look at the different insulation and surface systems that housing needs, the materials with which it can be built, as well as the degree of sustainability they want to achieve. The problem of thermal insulation is then raised. The contrast of information leads to the relationship of the type of material with its thermal conductivity, and subsequent creation of a graph that relates the different types of materials and their thermal conductivity. The teacher also built an associated mathematical workshop to focus on basic proportional reasoning relating physical measurements. Other situations are focused on doing a radio program to introduce touristic ideas, in which the starting problem relates ski stations or medieval buildings, centering the interest upon economic contents.

CONSEQUENCES AND CONCLUSIONS

Curricular reforms that are based on jointly contemplating discipline and interdisciplinary work because they influence affective and professional development of mathematics teachers. The main idea is that it is not necessary a strong content knowledge in the particular disciplines, but it requires a great effort to hear students' voices. The need of all teachers collaborating to identify connections and activities for integrative proposals provides both the content and an important process for building 'situated' experiences. As an example of agreement with the curricular proposals, coordinator A mentioned that *the new program enhanced his teaching skills and provided him with practical opportunities to plan and develop interdisciplinary units—a skill that is necessary for successful teaching career. I now understand that Global Situations can develop students' mathematical competencies, even more than Problem Based Approach*.

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WHAT PRICE COHERENCE? CHALLENGES OF EMBEDDING A COHERENT CURRICULUM IN A MARKET-DRIVEN AND HIGH-STAKES ASSESSMENT REGIME

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Curriculum coherence is widely valued as underpinning the enactment of a curriculum consistent with intentions. Such coherence depends on good alignment between, among other things, the written curriculum, available resources, the range of assessments, and teachers' capacity, including their knowledge, skills and affect. We report on a group of longitudinal studies mapping the early enactment of an aspirational curriculum in England which initially appeared to be supported by a coherent system, and show how the maintenance of that coherence was threatened by competing teacher beliefs in a market-driven and high stakes accountability regime.

BACKGROUND

Mathematics curriculum intentions in England and elsewhere (and what we know)

Intended mathematics curricula (Mullis and Martin, 2015) are being reconceptualised globally in an effort to meet the perceived needs of students and of society in the twenty-first century. Priorities for that are contested (Gravemeijer et al, 2017), but in England have focused on an aspirational deep fluency, accompanied by a renewed focus on mathematical reasoning and problem solving, and effective communication of those. Curriculum change in England is driven by education ministers in consultation with subject and other experts, and the political timescale of that model means attempts at curriculum change must be accomplished in a short timescale. England's first national curriculum of 1989 and subsequent high profile 'National Numeracy Strategy' were followed by significant changes to relative profiles of content and process in the national curricula of 1999 and 2007, in response to both perceived shortcomings in enactment and to changing political, economic and philosophical perceptions, including international attainment comparison studies and an influential review of student and employer needs (ACME 2011a, b).

The approach adopted has been to publish a new curriculum 5-16 (DfE, 2014) which distinguishes three levels of demand at 14-16, tested at two overlapping tiers; mandate assessments at 16 ('GCSE') consistent with that, although offered in a market of three competing Awarding Organisations (AOs); and encourage but not mandate the production and use of curriculum-compliant resources. At age 11, national attainment tests are produced centrally. Additionally, ministers have funded a semi-autonomous organisation to support mathematics teacher development (see www.ncetm.org.uk). Changes at A-level, the post-16 calculus-based mathematics route, have followed, with similar aspirations and first large-scale assessment in Summer 2019. Since GCSE Mathematics is high-stakes for schools and teachers, as well as for individual students, ministers expect their adopted approach to result in a valid enactment of their intentions, despite similar attempts historically, and elsewhere, having proved intractably challenging at scale (e.g. Eurydice, 2011).

THE STUDIES

This paper draws on a set of longitudinal studies undertaken by a team of 9 researchers led by me, and focussing on the enactment of this new curriculum at a variety of student stages. The studies asked how teachers and students were experiencing the curriculum, particularly in relation to the renewed foci, and how the resources and assessments produced by a major publisher supported or otherwise impacted that experience and the student outcomes. They also asked in what ways teacher capacity affected the answers to those questions. The first two studies focused on the teacher and student experience and impact of resource schemes that included electronic teaching and learning packages as well as printed textbooks and workbooks etc, the GCSE study on the impact of summative assessments at 16 and the free surround to those, and the A-level study, still in progress, is focusing on both a similar range of publisher resources and A-level assessments. The scale and scope of the studies and the data collected are summarized in table 1.

Focus	**Study**	**Size**	**Data**
Primary: 5-11 (y1-6)	2 years Oct 2016-Sept 18 (y1-2, 5-6)	9 schools and mathematics coordinators, 18 classes and teachers	18 pre- and post-class assessment data. *Yearly:* 25 Autumn, 18 Spring, 25 Summer/Autumn teacher interview transcripts, 18 lesson observation notes, student focus group transcripts
Secondary: 11-16 (y7-11)	2 years Oct 2016-Sept18 (y7-8 or 8-9 and 10-11)	15+ schools and Heads of Mathematics (HoM), 32+ classes and teachers	32+ pre- and post-class progression data. *Yearly:* 35+ Autumn, 32+ Spring, 35+ Summer/Autumn teacher interview transcripts, 32+ lesson observations, 32+ student focus group transcripts, 32+ whole class surveys (>800 students)
GCSE Mathematics and progression: 15-16 (y11)	2+ years: Oct 2016-Nov 18	15+ schools and HoMs, 30+ GCSE classes and teachers, 16+ post-16 groups of students and teachers	*Yearly*: 30+ Autumn, 30+ Spring, 15+ Autumn GCSE teacher/HoM interview transcripts, 30+ student focus group transcripts, 30+ whole class surveys, 16+ post-16 focus group transcripts, 16+ post-16 teacher transcripts, 30+ class GCSE results.
A-level Mathematics/ Further Mathematics: 16-18 (y12-13)	3+ years: Sept 2017-Oct 2020	12+ schools and HoMs, 24+ A-level classes and teachers	*Yearly:* 24+ class progression data, 24+ Autumn, 24+ Spring, 24+ Summer/Autumn interview or survey transcripts, 24+ lesson observations, 24+ student focus group transcripts, 24+ whole class student surveys

Table 1: Curriculum 2014 impact studies

All samples achieved reasonable representativeness over a number of variables known to influence enactments; nevertheless, scale was such that generalizability of outcomes cannot be claimed with confidence. Tools were developed iteratively, and data were analysed using a grounded approach (Charmaz, 2014). Coding was validated by at least one other researcher, and final interpretations and reports offered to teacher participants for validation.

THEORETICAL APPROACHES

Coherence across the education systems.

'Coherence' appears in the education literature in various guises, sometimes meaning little more than alignment of intentions of various learning-related actors of influence (Schmidt and Prawat, 2006). Better-defined constructs are developed in e.g. Newmann et al (2001), who both argue for and demonstrate the importance of 'instructional program coherence', within classes and longitudinally, for supporting improvement in student learning outcomes. Here, I use Schmidt and Prawat's (2006) definition of curriculum coherence as alignment of all elements of a curriculum system (intended curriculum and related documentation, assessments and accountability systems, teacher knowledge and skills, related resources, support of a range of informed or powerful stakeholders, high proportion of intended curriculum actually studied,...) together with underlying age- and stage-appropriate sequencing and progression. Without that, curriculum enactment undergoes 'iterative refraction' (Spillane, 2004) both horizontally and vertically.

The reported studies are predicated on a claim that significant efforts to achieve curriculum coherence were attempted by central government, supported in these cases by a large publisher and examining body. Importantly, the 5-16 curriculum for mathematics (DfE 2014), and the subsequent A-level Mathematics criteria, were widely perceived to be consistent with recommendations in ACME (2011a,b) and enjoyed a high level of support among thse involved in mathematics education, if not in every last detail. They were, though, perceived to be highly challenging for both teachers and students (Golding and Grima, 2018), particularly in relation to mathematical problem solving and reasoning, with considerable demands being made on both depth of subject knowledge (SK) and on subject pedagogical knowledge (SPK), a contruct developed for mathematics by Ball, Thames and Phelps (2008). In England, even highly mathematically-qualified and experienced teachers have often had fairly limited experience of developing the key renewed-focus mathematical processes of reasonign and problem solving in the classroom when using previous curricula, and that has often been attributed to summative assessments that demanded little in these strands (e.g. Golding 2017b).

Teacher change for valid enactment

Clarke and Hollingsworth (2002) model teacher development as having an initial stimulus and then moving amongst different domains: 'the personal domain (teacher knowledge, beliefs and attitudes), the domain of practice (all professional actions, together with the professional context), the domain of consequence (perceived salient outcomes), and the external domain (sources of information, stimulus or support)' (p 949). Change within one of these domains can impact change in another by enaction, and by reflection. Thus, teachers learn and change through their professional activity, and within institutional systems that include choices of resources, teacher interactions, and approaches to external assessments. In the case of this curriculum change, the external domain includes curriculum documents, available resources, and external examinations; examination results are among the 'salient outcomes' in a high stakes assessment regime.

In England, most primary (5-11) teachers teach across the curriculum and few are specialists in mathematics. Secondary teachers are more likely to have studied a mathematics-intense discipline at university, but many have not, and those are typically allocated to younger students, or those who are less confident mathematically. Teaching preparation courses are typically of a year's duration, and many have very limited subject-specific input. The intended curriculum therefore makes considerable demands on teacher development. Golding (2017a) conceptualises the teacher-level characteristics needed as 'teacher capacity for change', and that capacity sits within Clarke and Hollingsworth's personal domain, incorporating a range of knowledge, skills and affect. Fan, Liu and Miao (2013) show how textbooks can both support and limit teacher development for a new curriculum, and similarly Madaus and Russell (2011) address and evidence the impacts, positive and negative, high stakes summative assessments can have on classroom practices adopted.

FINDINGS AND DISCUSSION

I address four aspects of curriculum coherence: the communication of curriculum intentions, especially with respect to renewed curriculum foci of mathematical reasoning and problem solving, the impact and use of resources, development of teacher SK and SPK for the new curriculum, and the role of emergent sample assessments and examinations. In each case I draw from a subset of the studies, but giving also a longitudinal lens on that area that describes how that area developed over the early years of enactment.

Communication of curriculum intentions

Almost all teachers across studies used proxies for curriculum documents: in primary and secondary schools those were often textbooks or (scheme-provided or school-developed) 'schemes of work'. Importantly, the publisher concerned used a team of internationally-respected mathematics education experts to support the production of both their resources and their initial examinations, so as to translate the stated curriculum intentions, themselves the product of comparative studies of international apparently successful practice, into high quality evidence-based artefacts. Teachers of students aged 14+ often used emergent sample or early live examination papers to triangulate textbook interpretations of curriculum, with official examination questions always 'trumping' those in textbooks or other resources: 'The textbook stresses mental methods as a first recourse, which is what I value, but the tests don't seem to want much of that, so we've changed our approach there' *(Year 6 teacher, Spring 2018).* Swan (2014) argues that greater exemplification ('beautiful examples', op cit p 628) in curriculum specifications would pre-empt reductive interpretations in either resources or assessments – at classroom level or in production. Within such curriculum 'translations', a majority of teachers across the studies began with fairly conservative use of reasoning and problem-solving opportunities provided in resources, or the more challenging aspects of fluency ('mastery') that underpin the intended functioning with problem solving and reasoning, and often limited such experiences to their higher attainers. Probing these enactments in interviews, it became apparent there was also a variety of grasp of teacher interpretations of these fundamental processes.

However, as sample and early external summative assessments emerged, teachers began to talk about the challenges of teaching the weakest students entering a paper for reasoning and problem-solving questions: 'we've got to get them all so they can solve problems, which is a challenge, though the problem solving examples in the books really help, and the hints they give' *(year 8 teacher, Spring 2018)*, or about perceived misalignments of textbooks with later papers: 'what we're seeing now, in these latest papers, is easier: we don't need to go as far as the textbooks do' *(Head of Maths, Spring 2018).* At A-level, though, whereas teachers still talked about the very real challenges of changing their teaching, and building up their professional knowledge, to accommodate genuine problem solving and reasoning, even early interviews, which followed on from first enactment of the new GCSE, commented on the learning already achieved by both themselves and their students in relation to these areas: 'Actually, even a year ago, I was thinking I can't do this, but it's amazing what the students have brought from the new GCSE, and for me too, that experience of teaching the new GCSE is underpinning my confidence to go for this' *(experienced year 12 teacher, Autumn 2017*). Coherence across year groups clearly bears fruit. These (largely specialist and mathematically well-qualified) A-level teachers unanimously valued the new expectations, while often simultaneously worrying about whether their weakest A-level students would be able to access papers: 'I don't know how I'm going to get them all to where they need to be, I simply don't know how to do it, but I do know that how I used to teach won't achieve that' *(year 12 teacher).*

Use and impact of resources

All participants in the first two studies, and many in the last two, used resource schemes produced by a leading publisher. All classes in the last two studies sat summative examinations from this publisher, one of three major GCSE and A-level AOs in England; many of those also used the publisher's resources ('the integrated offer'). Examination class teachers and students valued the coherence offered by adopting such an integrated offer, which eclipsed written curriculum specifications for almost all participants, though as perceived divergences among materials appeared, it was without exception the examination materials that prevailed.

In terms of pedagogy, use of the resources for most classes, and throughout the studies, was conservative, with more novel or creative features of printed and e-resources poorly harnessed. Limited use of e-resources was particularly conspicuous, with many teachers making no use of interactive features at all, for example (Evers et al, 2018). Teachers explained this with reference to the considerable demands of beginning to teach a new curriculum, feeling first, that there was insufficient time to explore new resources more fully, and second, that external examinations were such high stakes that experimenting with new approaches was in general too risky if it could be avoided. The range of participant teachers, even those with outstanding subject knowledge, talked about feeling insecure about teaching new topics, or preparing students for very different assessments. Such responses point to the considerable cost (in time, energy, money – and sometimes student learning) of coming to enact a new curriculum, especially one with associated high stakes assessment, even if an improved quality of education eventually emerges.

For non-specialist teachers at all levels, though, published resources supported confidence with planning for a new curriculum, and to a large extent, confidence with delivering that. In general, teacher positive affect (confidence, self-efficacy, enjoyment, motivation, willingness to take risks, persistence, etc) appeared to breed student positive affect (Barrow et al, 2018). Such confidence was not always well-placed: there was a range of quality of learning achieved in the lessons observed, with the strongest lessons, well-aligned with curriculum intentions and building deeply effective mathematical functioning in students, observed when teachers appeared to have robust capacity for change (Golding, 2017), underpinned by strong knowledge of the whole curriculum system, of appropriate mathematics and its pedagogy, and of their students. Such teachers, more than most, appeared more able to tolerate perturbances in emergent assessments or hiccoughs in student attainment; they supported the development of reasoning and problem solving through allowing student time to mull, to explore, to make mistakes, to harness multiple representations and resources, and supported them in learning to talk about their thinking and listen to others'. Such pedagogical expertise takes time to develop, and necessarily draws on deep subject knowledge.

Teacher development for the new curriculum

Teachers using such resources can access both included teacher e-development units and additionally paid-for face to face support, but very few participants had taken advantage of that – or indeed, acknowledged their existence. Yet by international standards, these were not in general teachers with strong subject-specific knowledge or skills. Resource planning affordances offer good subject development support, e.g. in identifying prior knowledge needed, likely misconceptions emerging, and suggesting ways to address those, but many teachers did not recognize a need to use that support (Evers et al, 2018). They often worked with school colleagues to support their development of teaching for the new curriculum, though in some cases this was the poorly-resourced leading the poorly-resourced.

This was true even of changes to the A-level curriculum, where, for example, the introduction of work with (analysis/interpretation of) a 'large data set' – that is, one not susceptible to analysis via the use of a hand-held calculator – was a source of considerable concern to teachers, who typically had little experience themselves with authentic use of such data sets, often lacked the IT skills they wanted students to learn, and of course needed to develop the pedagogical skills appropriate to such mathematics (Golding et al, 2018a). Different AOs have operationalised the requirement in different ways, sometimes drawing on the use of 'pre-release' material published in advance of final written summative examinations. It has to be remembered that for AOs also, significant curriculum changes can also bring challenges of demand for new types of assessment, as for the large data set – but still within a system that is high stakes for students, for teachers - and for AOs, representing large entry, high status examinations.

Development for such changes as the introduction of the large data set brings with it, then, a need for new teacher knowledge and skills. Resource writers, assessors and those providing a variety of support for teachers, whether as examination preparation support provided by the AO, resource-linked development, or otherwise channeled, all need to develop new skills and capacities. Without incentives for systemic investment in high quality professional development, it would appear that the validity of curriculum enactment for such an aspirational curriculum will inevitably be limited.

Role of emerging examination papers and sample assessments

Especially important in a high stakes assessment system is the quality of the assessment system and its alignment with curriculum intentions: I have described how influential emerging assessment materials were for teachers at all stages of the mathematics education system. As well as harnessing high quality external expertise to operationalise the curriculum via resources or initial examinations, the publisher also developed extensive 'free surround' support for teachers preparing students for mathematics GCSE or A-levels, and that was very much valued by teachers. Additionally, the national assessment body, Ofqual, invested, and continues to invest, in significant work, e.g. specifying the nature of mathematics problem solving in ways operationalizable by AOs, and analysing the nature of student difficulties in tackling such problems (Ofqual, 2017): central commitment to curriculum coherence remains high. The earliest examinations studied here, supported by external expertise, also showed high fidelity to curriculum intentions, but with resulting high demand on students and teachers (Golding and Grima, 2019).

However, the examination system in England at 16 and 18 is not only high-stakes but market-driven, and initial experiences of GCSE live papers suggested teachers perceived one AO to be offering papers much more accessible than others'. Ofqual responded with in-depth reviews and renewed criteria, but the result for the publisher studied, was the emergence of papers of lesser demand, but also arguably less coherent with curriculum intentions. Over time, that cycle has repeated, with a consequent growing gap perceived between the publisher's GCSE textbooks, for example, and their GCSE papers, as evidenced above.

Some participant schools have changed the AO used, for example for weaker students, since they perceive their papers to be more accessible, or more rewarding in terms of outcome grades: 'So now we use (AO x) for our two weakest sets: 'I like working with (AO y), I think they're mathematically better and they give teachers fantastic support, but at the end of the day these are more accessible papers and our kids feel they have done a better job. It doesn't make them better mathematicians, but it gets them better grades.' *(Head of Maths, Spring 2018).*

What we see, then, is a system in which, four years into a rolling introduction of a broadly-espoused new curriculum, considerable efforts by both central authorities and publishers to support curriculum coherence are being undermined by both the very high stakes nature of related assessments and a market-driven system of GCSE and A-level assessments which is leading to a downward pressure on aspirations. Teachers are challenged to respond ethically in such a situation: these studies offer evidence that many teachers, while limited in their subject-specific expertise, have been working hard to enact curriculum intentions while the system remains coherent, but they experience hierarchical and not always consistent beliefs relating to tensions between fundamental principles of mathematics education and the importance of external assessment outcomes. Any systemic change of the scale intended is expensive in many ways: if there is to be a net gain for student learning, it is important that ways are found to address such challenges to the coherence achieved.

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MATHEMATICS BASIC ACTIVITY EXPERIENCE: A NEW ASPECT OF CHINESE MATHEMATICS CURRICULUM

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We consider the shift from the double-base foundation of the Chinese mathematics curriculum to the new formulation of a four-base foundation. In particular we examine one of the key elements in the new Chinese curriculum: ***mathematics basic activity experience****. We discuss the meaning of mathematics basic activity experience in the context of Chinese mathematics curriculum documents and the treatment of basic mathematics and mathematical reasoning globally. We also consider the research basis for enacting this curriculum reform in China, including the results of a study that examined evidence regarding the mathematics basic activity experience of a sample of Chinese students in grades 7-9.*

BACKGROUND

In discussions of education across the globe, it has long been the case that mathematics is the school subject most often associated with the mastery of so-called *basics*. Yet, what has changed from time to time is the specification of what is included as basic mathematics. In the United States, for example, the popular conception of basics in mathematics during the 1970s was arithmetic and algebra facts and skills, but in the 1980s there was a press to include problem solving as a basic (NCTM, 1980). For the 1990s reasoning, communication, and connections were also treated as basic components of a high quality mathematics education (NCTM, 1989). Thus, though the importance of mastering the basics has remained a bedrock principle of mathematics education over time, there have been changes regarding how to define the basics.

In this paper, we examine a development regarding the nature of basics in mathematics education that has occurred recently in China. Because Chinese students, along with many of their peers in other East Asian countries, are viewed as being particularly adept in their mastery of mathematical basics, we think a shift in the Chinese conceptualization of mathematical basics is a strategic site for inquiry.

In China, the education system includes a compulsory education phase (Grades 1-9) and a high school education phase (Grades 10-12). The mathematics curriculum standards corresponding to these two stages respectively are the *Chinese Compulsory Education Mathematics Curriculum Standards* (CCEMCS, Ministry of Education of the People's Republic of China, 2011) and the *Chinese High School Mathematics Curriculum Standards* (CHSMCS, Ministry of Education of the People's Republic of China, 2017). One of the major changes found in these two new mathematics curriculum standards, in relation to prior curriculum documents, is the stipulation of the so-called "four-base" foundational requirements.

The “four-base” requirements are composed of mathematics basic knowledge, mathematics basic skills, mathematics basic ideas, and mathematics basic activity experience (CCEMCS, 2011, p.8; CHSMCS, 2017, p.8). Previously, Chinese mathematics curriculum documents referred only to a "double-base;" namely, mathematics basic knowledge and mathematics basic skills. The meaning of these terms in the Chinese curriculum documents is similar to the notion of basic mathematics knowledge and skills found globally. That is, mathematics basic knowledge refers to concepts, rules, formulas, axioms and theorems and mathematics basic skills refers to calculations, reasoning, processing data, drawing figures, and following procedures (Ministry of Education of the People’s Republic of China, 2000, p.2).

In this paper we examine mathematics basic activity experience -- one key element added in the shift from the traditional double-base to the new four-base structure. We explain what it is and why it was added to the curriculum in China. We also consider the research basis for enacting this curriculum reform, including evidence regarding students’ mathematics basic activity experience of a sample of Chinese students in grades 7-9.

MATHEMATICS BASIC ACTIVITY EXPERIENCE: WHAT?

Mathematics basic activity experience refers to aspects of mathematics that students are more likely to learn well experientially rather than through direct instruction. For example, though one might be able to learn through didactic instruction the rules of deductive logic or even ideas about heuristic problem-solving processes, it is likely that one would also need a rich experience base as well in order to develop proficiency in using this knowledge to solve novel problems or generate new mathematical ideas. Another example is, though young children might be able to learn through didactic instruction the truth “one plus one equals two”, it is likely that they would also need a rich experience base as well–one apple plus one apple equals two apples, one pear plus one pear equals two pears, one apple plus one pear equals two fruits, …one plus one equals two.

There are two main forms of mathematics basic activity experience: practical experience in mathematics and thinking experience in mathematics. In this paper we focus on the thinking experience component of basic mathematical activity experience. Our understanding of this new aspect of the Chinese mathematics curriculum is that it is the student’s way of mathematics thinking accumulated from experiencing and understanding the processes of mathematics inductive reasoning and mathematics deductive reasoning. A typical form of the inductive experience would be making an observation, posing a conjecture from one or more special cases, and then producing a general mathematical expression, rule or conclusion to express the generalization. Subsequently, students could use deductive reasoning approaches to verify or prove the conclusions or generalizations. This process is illustrated on the left side of Figure 1 (adapted from Guo & Shi, 2012-a).

The central column of Figure 1 captures a variety of mathematical processes used at various steps in the mathematical thinking activity sequence. The initial stage of observation entails noticing common characteristics and relationships among mathematical objects. As the eminent mathematician George Pólya (1954) observed long ago, inductive conjecture often involves analogy. Mathematical expressions involve the use of mathematical words and symbols. Verification or proof mainly depends on deductive reasoning, which may include deductive proof or negation by counterexample.

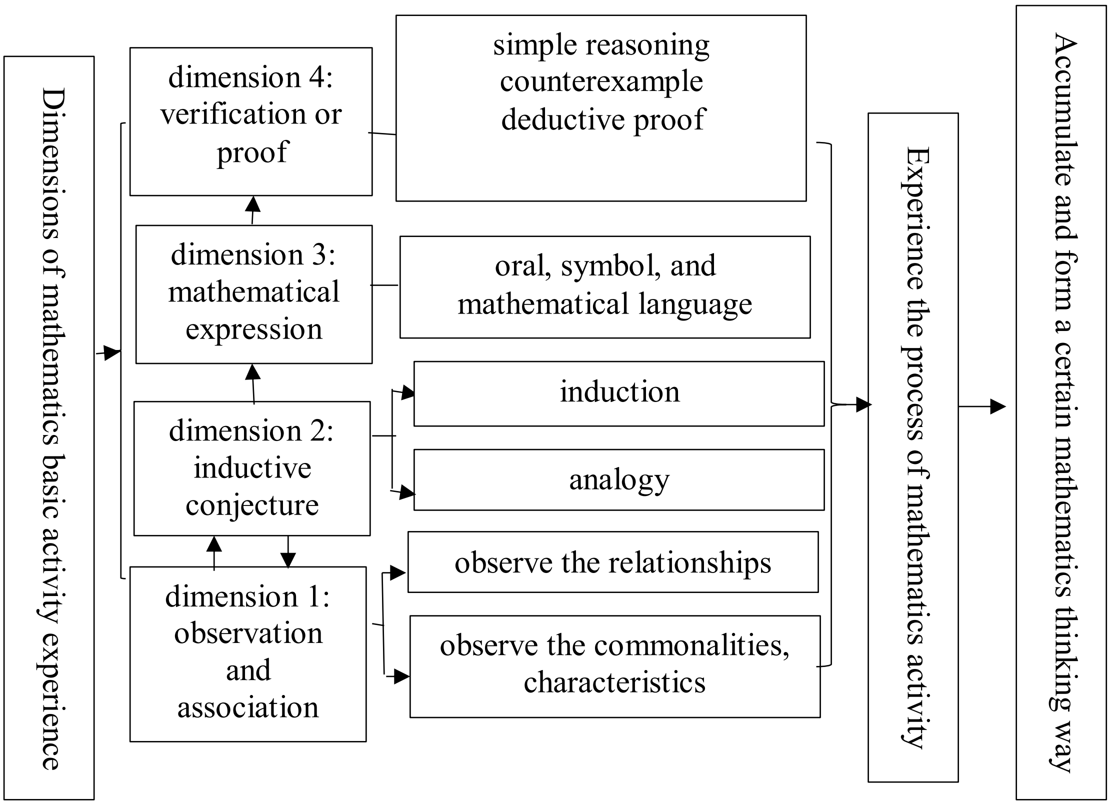

Figure 1: The dimensions of mathematics basic activity experience

MATHEMATICS BASIC ACTIVITY EXPERIENCE: WHY?

As noted above, there is a long tradition in the Chinese mathematics curriculum of emphasizing basic knowledge and skills. Attention has long been given to developing students' proficiency in numerical and algebraic computation, spatial reasoning, and logical reasoning. Not surprisingly Chinese students typically excel in performance on tasks involving these skills – often far surpassing the performance of students from most other non-Asian countries on such tasks (e.g., Cai, 1995, 2000; Fan & Zhu, 2004). On the other hand, Chinese students have performed less proficiently on tasks that involve nonroutine problem solving (e.g., Cai, 1995, 2000; Cai & Cifarelli, 2004). In response to these findings, along with indications of increased attention to mathematical processes associated with problem solving, invention and creativity in the curriculum documents produced in other countries such as the United States and Japan, thought leaders in the Chinese mathematics education community began to argue for some modifications to the Chinese mathematics curriculum to address these perceived shortcomings. Often such recommendations made explicit reference to problem solving, invention or creativity. For example, "We put too much emphasis on math drills, and neglect having students understand the nature and the significance of mathematics knowledge. Also, the students' personal experience in forming mathematical knowledge was not adequate." (Ma, Wang, Zhang, Liu & Guo, 2017, pp110-111).

The question of whether mathematics is invented or discovered has historically been debated since at least the days of Plato. Rather than choosing one view over the other, many believe that mathematics includes both inventions and discoveries. The abstraction of numbers and operations from experience with quantities in the real world; and abstractions about points, lines, surfaces and their relationships arising from graphical experience in the real world have yielded important inventions in mathematics. However, mathematics does not arise only from abstracting things that are found in real life. Some mathematical ideas are derived within the world of mathematical abstraction itself, such as the concepts of real numbers, the concept of high-dimensional spaces, operations of quaternions, and many more examples. Whether one focuses on mathematical invention or discovery, advances can often be seen to be based on an interplay between inductive reasoning and deductive reasoning. If we wish to cultivate in students an orientation toward mathematical discovery and mathematical invention, it is important for students to learn and have experience with both kinds of inference. Though deduction has long been valued in the mathematics curriculum, the process of inductive reasoning is also critically important as part of students' accumulated experience that forms foundation for their future mathematical invention and discovery. According to Guo and Shi (2012-b) this perspective is a key portion of the rationale for putting foreword a curriculum objective regarding mathematics basic activity experience in China.

MATHEMATICS BASIC ACTIVITY EXPERIENCE: HOW?

When new elements are proposed in curriculum reforms, the success of the reform depends on their being both a *compelling need* for the changes and a *readiness of the education system* to implement or enact the new approaches. Regarding students' mathematics basic activity experience in the Chinese context we draw on some data from a fairly recent study (Guo & Shi, 2013) to illuminate some aspects of the situation. In particular, we think the data provide an indicator of both the extent to which mathematics basic activity experience (especially with respect to mathematical thinking) is already accounted for in Chinese mathematics instruction and the likely challenges that might lie ahead as the curriculum transition from double-base to four-base moves forward.

Research questions

The research undertaken by Guo & Shi (2013) illuminates the following three questions: 1) How proficient are students with the kinds of mathematical thinking that are expected to be enhanced by students' mathematics basic activity experience?, 2) What are the relative strengths and weaknesses of students with regard to the dimensions of mathematics basic activity experience (see Figure 1)?, and 3) What are the different levels of students' mathematics basic activity experience?

Research sample and research method

Guo and Shi (2013) surveyed students from seven Chinese middle schools -- four in Beijing, two in Henan Province, and one in Guangdong province. In each school, two classes at grades 7 and 8 were surveyed, and at five schools two classes at grade 9 were also surveyed. A total of 1295 students were in the sample (435 grade 7 students; 515 in grade 8, and 345 in grade 9).

The survey was comprised of six mathematical problems, each of which had several sub-problems. According to the study authors, the problems were drawn from a variety of sources and were intended to assess students' proficiency in generating a general rule or conclusion through a process that starts from a specific and simple problem. Space limitations prevent us from reproducing the problems here, but each problem allowed students to demonstrate proficiency along all four dimensions of mathematics basic activity experience (see Figure 1). For example, one problem began with a sub-problem related to the number of segments created by 1, 2, 3 … n points on a line; progressed to consider the number of parts obtained from dividing a plane by 1, 2, 3…n lines; and finally asking about dividing space by 1, 2, 3, … planes; the work was organized in a summary table and further generalizations were invited in the final sub-problem (see Guo (2013), pp.246-248) for more about this problem).

Each student's response was scored using a grading scheme that yielded a maximum score of 100 for a correct response to all 6 problems and the 45 embedded sub-problems. The authors reported using the techniques of analytic hierarchy process (AHP) and cluster analysis (Liu & Huang, 2009; Gao, 2009) to analyze the data.

Selected findings

Reliability and validity. The authors reported acceptable levels of reliability and validity for the survey uses in this study. In particular, Cronbach's alpha coefficient, a measure of internal consistency reliability, was 0.74; and the criterion-related (mean area diversity) validity coefficient was 0.71. These reliability and validity analyses support validity arguments based on survey findings.

Overall classroom group performance. There was considerable variation in performance of classes both within and between schools. In all schools in the sample, grade 8 classes always outperformed grade 7 classes, but sometimes the difference was quite small. In the Beijing schools, grade 8 classes had scores that were 6 to 14 points higher (median 8.5); whereas, in the Henan Province schools the grade 8 classes only scored about 1 point higher than the grade 7 classes. Between-school variation was particularly striking. Average scores for grade 7 students ranged from 21.0 to 60.5 with a median of 29.7. For grade 8 students the school averages ranged from 21.6 to 69.3 with a median of 43.1. Across the five schools with grade 9 samples, the average scores ranged from 31.6 to 54.8 with a median of 36.5.

Student performance on the four dimensions of mathematics basic activity experience. Guo and Shi (2013) associated the sub-problems of each problem with one of the four dimensions of mathematics basic activity experience (see Figure 1), and the researchers used scores on each sub-problem to derive a total for each student on each dimension for each problem and for the entire survey. The distribution of student performance on each dimension is shown in figure 2 for the entire sample without regard to grade level. For each dimension, the horizontal axis represents the frequency and the vertical axis represents the score. Various descriptive statistics are provided for the score distribution obtained for each dimension.

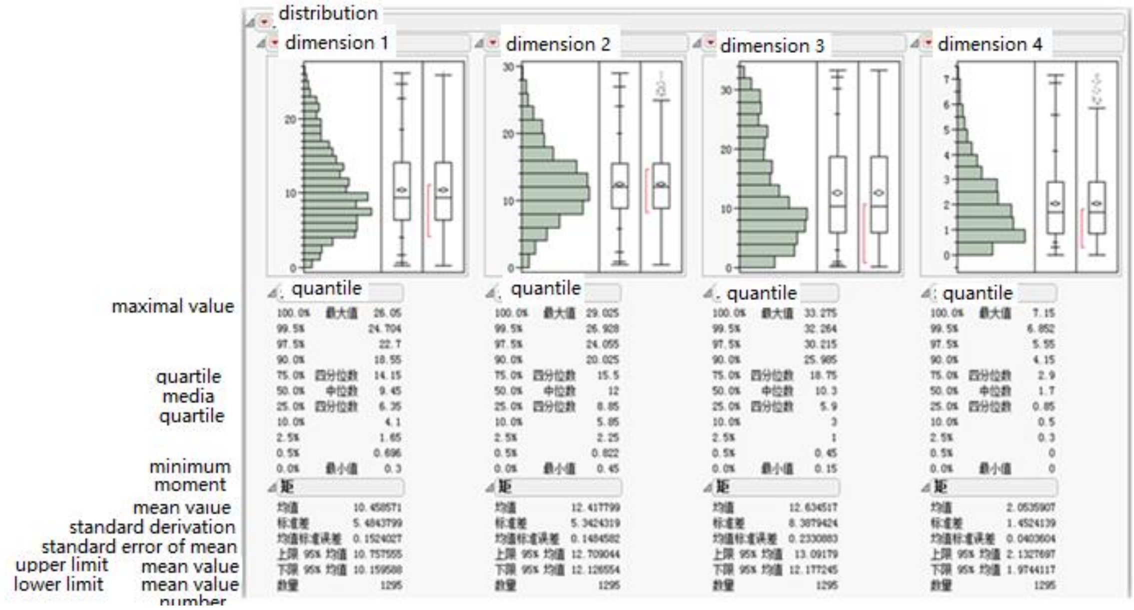

Figure 2: The distribution of students' scores in four dimensions'

Levels of student performance. Guo and Shi (2013) used cluster analysis techniques to detect different levels of student performance on the survey problems. Through their analysis they identified three distinct clusters of students. More than 80 percent of the surveyed students (1081 out of 1295) fell into the lowest performance category, which Guo and Shi judged to involve mostly imitation of taught procedures and very little mathematical reasoning. About 15 percent (195 of 1295) fell into a second category that they judged to involve evidence of some progress with mathematical reasoning. Only about 1 percent (19 of 1295) fell into the highest category that involved evidence of proficiency with mathematical reasoning.

Discussion of research findings

Viewing the findings of the Gou and Shi (2013) investigation in relation to the introduction of basic mathematics activity experience as a core goal of the Chinese curriculum, we think several observations are worth noting. All of these observations underscore a compelling need for the change.

First, the performance of classes of students in the schools included in the study was quite low. Though there was variation across schools and within school by grade level, there was no class that exhibited very strong performance on the survey. Because the survey problems were not familiar to the students and the students need to solve them based on their prior mathematics basic activity experience, the findings suggest that the situation on the students' mathematics activity experience is not optimistic and the current instructional practices and curriculum emphases in these Chinese classrooms and schools was not sufficient to support high levels of student proficiency on the types of reasoning required.

Second, considering the four dimensions of mathematics basic activity experience, the skewed performance distributions for three of the four dimensions indicated a preponderance of students at the lower end of the performance distribution. Moreover, the distributions also suggest very wide variation in student performance with respect to all aspects of the basic mathematical activity experience. Considering the survey students are only for about age 12-14 and their mathematics cognitive ability is still progressing, it is reasonable to get the skewed performance distributions for three of the four dimensions, however the result still reveals that some aspects are still missing or neglecting, such as dimension 1.We take this to suggest that the current instructional practices and curriculum emphases in these Chinese classrooms and schools have not been sufficient to support the majority of students to obtain the kinds of experience envisioned by the curricular reform.

Third, the research findings on levels of student performance underscore the fact that—under the conditions of the current instructional practices and curriculum emphases in these Chinese classrooms and schools—very few students have acquired high levels of proficiency in using mathematical reasoning to solve complex, novel mathematics problems that call for a combination of inductive and deductive mathematical reasoning. Thus, a curricular emphasis on mathematics basic activity experience seems timely and appropriate.

SUMMARY AND CONCLUSION

As we noted earlier in this paper the importance of mastering the basics has long been a bedrock principle of mathematics education across the globe; yet, there have been shifts from time to time regarding how to define the basics. We think that the notion of *mathematics basic activity experience* found in the recently promulgated Chinese national mathematics curriculum (CCEMCS and CHSMCS) is one of those shifts worthy of notice and examination.

The new emphasis on mathematics basic activity experience indicates that Chinese thought leaders regarding mathematics education have joined those in many other parts of the world in emphasizing the fundamental importance of helping students develop proficiency with mathematical processes and practices, such as reasoning and problem solving. Because this new emphasis will draw the attention of teachers and textbook authors to a heretofore underdeveloped aspect of the school mathematics curriculum in China, we expect that there will be many opportunities to learn from the instructional innovations introduced in the coming years in Chinese classrooms. Mathematics education scholars and practitioners across the globe should be able to learn from the Chinese experience in this endeavor because the curriculum goal represented in CCEMCS and CHSMCS as mathematics basic activity experience is widely shared in the global mathematics education community.

To the extent that the new focus on mathematics basic activity experience in CCEMCS and CHSMCS also supports broader efforts to cultivate students' creativity, additional benefits can accrue to the global mathematics education community. It can be argued that the by enhancing their proficiency along the four dimensions of mathematics basic activity experience represent key aspects of mathematical thinking processes and practices associated with the birth and development of mathematics itself. The findings of research undertaken in the service of implementing mathematics basic activity experience extensively in Chinese mathematics classrooms should provide guidance for teachers who wish to increase students' proficiency with mathematical reasoning and problem solving not only in China but also elsewhere in the world.

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1993 AND 2009/2011 SCHOOL MATHEMATICS CURRICULUM REFORMS IN MEXICO: COSMETIC CHANGES AND CHALLENGING RESULTS

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It is presented here a comparative study on the changes and coherence of the mathematics curriculum in Mexico, focusing on primary education, and the different curriculum reforms carried out in 1993 and 2009/2011. Three dimensions of the curriculum were considered for the study: the intended curriculum, the implemented and the attained. Following this frame, this study is carried out through a review, analysis and discussion of the official documents endorsed by the Ministry of Public Education in Mexico City. The results of PISA 2009 and 2015 are also considered, as well as several of the documents prepared by the National Institute for the Evaluation of Education in Mexico City (INEE). This paper is part of an ongoing research project that seeks to recover and categorize the information that teachers of primary education in Mexico have in practice on the understanding of concepts that are key to the development of mathematical thinking in this educational level.

THEORETICAL FRAMEWORK

There are several researchers (Remillard & Reinke, 2018, Van Zanten & Van Den Heuvel-Panhuizen, 2018, Hemmi et al., 2018, Lee et al., 2018) who recently document the characteristics of the curriculum reforms carried out in recent years in the USA, Holland, Finland, and Korea, through the study of the official documents in their countries of origin, such as curriculum documents issued by the state or government in turn. All these authors have agreed on conceptualize the curriculum on at least three levels, namely the intended curriculum, the implemented curriculum and the attained curriculum, as suggested by Travers in 1992 (see Preface, in Thompson et al., 2018). These authors (Thompson et al.) also report that already in 2014, Li & Lappan noted that the research around the mathematics curriculum was a relatively recent phenomenon.

According to Suurtamm et al. (2018, p.2. In Thompson et al., same year), the intended curriculum "focuses on what is expected to be attained through implementing the curriculum", as it exists on the printed page and is described in curriculum policies. The intended curriculum "includes the messages that are within the curriculum, ... such as suggested teaching practices" (Idem, pp. 2 & 3). Finally, the enacted curriculum "is viewed as the learning experiences jointly created by students and teachers and includes teachers' decisions ..." (Cal & Thompson, 2014. Cited by Suurtamm et al., P.3).

Similar to how these researchers have just been doing (Remillard & Reinke, 2018, Van Zanten & Van Den Heuvel-Panhuizen, 2018, Hemmi et al., 2018, Lee et al., 2018), in the comparative study we are presenting here, we have applied the methods and questions that these authors raised for the development of their research, and in particular to inquire about the characteristics of the curriculum proposals in their countries of origin (see Suurtamm et al., 2018). The same questions are then applied to the case of the study of curriculum reforms accomplished in Mexico: (i) How is the curriculum in Mexico organized? (ii) Who makes the curriculum in Mexico? (iii) What is the vision of mathematics and mathematics education that is portrayed in the Mexican curriculum, and what does this curriculum seem to value? (iv) What is the role in Mexico of evaluation in the curriculum intended and enacted? (v) What can be said about the implementation of the curriculum in Mexico? (vi) What is the role of textbooks and other resources in the Mexican enacted curriculum? (vii) What is the mathematical focus of the curriculum in Mexico?

It is interesting to note through the review of the work of Remillard & Renke (2018) that it is so important to have answers to the questions posed. For example, the answers given by Remillard & Renke make possible to understand the scope of public education policies in the United States. But curriculum studies could also indicate needs to move forward in the change of education policies that have been already taken but having strong arguments that justify required changes.

REVISION OF OFFICIAL DOCUMENTS, DISCUSSION AND DATA ANALYSIS

In next two tables, it is presented an overview of a summary analysis of the differences between the two 'moments' in reform, 1993 and 2009/2011.

Criteria	Curriculum	1993
	Elementary school	Middle school
Focus	**Problem solving** "Mathematics will be functional and flexible tools for the child, they allow him to recognize, draw and solve problematic situations confronted to him or her." (SEP, Plan and Programs of Study, Mexico, 1993, p. 49)	**Problem solving** "Students learn to use mathematics to solve problems with procedures and techniques that he or she learns in the school, with curiosity and imagination." (SEP, Plan and Programs of Study, Mexico, 1993, p.37)
Structure	Contents are articulated in 6 thematic axes: • Numbers, their relations and operations. • Measure • Geometry • Processes of change • Information treatment • Prediction and random Contents are grouped by axes and not necessary are linked between them.	There is a thematic organization in five areas: • Arithmetic • Algebra • Geometry (and trigonometry in third grade) • Information Treatment • Notions of probability Contents in each area are not necessarily linked between them.
General purpose	Students will be interested and find the meaning and functionality of mathematical knowledge, it should be valued by them, and they will make it a tool to help to recognize, to draw and solve problems presented in various contexts.	Developing students' operatory, communicative and discovery skills.
Evaluation	There is not a proposal	for assessment

Table 1. Table of summary analysis of the differences between 1993 & 2009/2011 curriculums/ Part 1

Criteria	Curriculum 2009/2011 (An integrated proposal for basic education)
	Elementary & Middle school
Focus	**Problem solving** [Teacher must] "take activities of study to the classroom that wake up the interest of the students and invite them to reflection, to find different ways to solve problems and to formulate arguments to validate the results." (SEP, Plan and Programs of Study, Mexico, 2009, p.80)
Structure	Contents in elementary and middle school are now organized in three thematic axes: • Numeric sense and algebraic thought • Form, space and measure • Information handling Contents are linked through the organization in thematic blocks including contents of the three axes. An important element that is introduced in the 2009/2011 curriculum is the denoted expected learning, which is described by 3 cycles of three years distributed from first to third grade and fourth to sixth grade in elementary school; and the three years of middle school (as is officially established in Mexico). Finally, at the beginning of each block, expected learnings are listed (knowledge and skills), which are supposed to be covered in each block.

General purpose	It is expected that students developpe the following mathematical competencies: • Solving problems autonomously • Communicating mathematical information • Validating procedures and results Efficiently handling of techniques
Evaluation	Here evaluation is addressed through three fundamental elements of the didactical process: teachers, activities of study and students. It is established that the first two can be evaluated through the register of brief judgements in the plans for the class, on activities pertinence and actions used to teach. Finally, students should be evaluated on their know-how and the application of the mathematical contents.

Table 2. Table of summary analysis of the differences between 1993 AND 2009/2011 curriculum/ Part 2

Seeing what was proposed about 1993 and 2009/2011 curriculum principles, it is noteworthy to mention that all the official discourse around the adjustments and changes involved in the curricular reform of 2011, were not really accompanied by significant changes in mathematical contents, nor of a new organization between them.

The Intended Curriculum in Mexico, focusing on the Case of Adding Fractions and in Curriculum reforms of 1993 and 2009/2011

For reasons of this paper's extension, only extracts of the collected information will be presented, the amount of data we have is very extended and here the available space does not allow to present all of them.

School grade	Curriculum of 1993	Curriculum of 2011
3th grade	- Formulation and resolution of problems involving the addition of simple fractions by manipulation of material	Solving simple problems of addition or subtraction of fractions, means, quarters and eighths (Theme: Additive problems, Content B-5). This does not have expected learning.
4th grade	- Formulation and resolution of problems involving addition and subtraction of fractions with equal denominators - Conventional algorithm of the addition and subtraction of fractions with the same denominator	Resolution with informal procedures, addition or subtraction of fractions with different denominator in simple cases (means, quarters, thirds, etc.) (Theme: Additive problems, Content B-3)

Table 3. Synthesis of topics and school grades in the curriculum of 1993 and 2011, in the case of learning the sum of fractions for 3rd and 4th grade of primary school

Considering the messages within the curriculum

The history of curriculum reforms in Mexico begins in 1992, when only primary education was compulsory (from 6 to 12 years of age). In that year (1992) the third constitutional article was modified and compulsory education was extended to 12 years, including pre-school education (from 4 to 6 years of age) and a first tranche of 3-year secondary education (from 12 to 15 years of age). In fact, in the last 25 years there have been 4 curriculums, where the most significant changes were made in 1993 and 2009. However, the latter had an adjustment in 2011. Since the 2009 curriculum was only valid for two years and that the one of 2011 has remained practically up to date, in this article only a comparison between the mathematics curricula of 1993 and 2011 will be carried out.

The 1993 curriculum was the product of the government program for educational modernization during the period of 1989-1994, which established as a priority the renewal of teaching contents and methods, the improvement of teacher training and the articulation of the educational levels that make up the basic education.

Purpose. It was to organize the teaching and learning of basic contents, to ensure that the children "acquire and develop the intellectual skills that allow them to learn permanently and independently, as well as to act effectively and with initiative in practical issues of everyday life ". (SEP, Plan and

Study Programs, Mexico, 1993, p.13). In the case of mathematics, the focus was on its application to reality with emphasis on the reflection on its meaning and in relation to fundamental knowledge. It is important to mention that the mathematics curriculum of 1993 was the first to postulate problem solving as the main focus for learning development.

Teaching approach. It was said constructivist, and it is to notice the included suggestion to follow the principles of didactical situations established by G. Brousseau, a French theoretician. In particular, it was expressed that "in the construction of mathematical knowledge, children can start from concrete experiences. Gradually, and as they go about doing abstractions they can do without physical objects. Dialogue, interaction and confrontation of points of view help to learn and build knowledge. Success in this discipline depends on the design of activities that promote the construction of concepts from concrete experiences, in interaction with others. In these activities, mathematics will be for the child functional and flexible tools that would allow him to solve the problematic situations that arise from ". (SEP, Plan and Programs of Study, Mexico, 1993, p. 49)

Textbooks. By official or governmental provision since 1957, at the beginning of each academic year, in primary education there is a free distribution of one set of official textbooks for each grade. According with the 1993 curriculum, the official mathematics textbook would be the axis that served the teacher to attend to his/her class, and in addition to textbooks, there were a series of activities in books cut-outs and a book for the teacher's guide explaining what to do and how to do it. In general, the structure of the textbook corresponded to the three moments proposed in Brousseau's theory of didactic situations.

Organization of content. Thematic was organized into six axes, namely: (1) Numbers, their relationships and their operations; (2) Measurement; (3) Geometry; (4) Change processes; (5) Treatment of information; (6) Prediction and chance

Teacher training. At that time, the Ministry of Public Education (SEP) offered training to in-service teachers to become familiar with the new approach.

For the curriculum reform of 2011, it is important to mention that since 2000, the elaboration of the mathematics curriculum in Mexico is carried out by an academic committee that is directly appointed by the Ministry of Education or educational authorities. It is assumed that the curriculum developed in this way must be passed by a period of auscultation among the educational communities directly involved in education development. After the period of auscultation, the new curriculum becomes the officially agreed by decree.

Organization of contents. They are now organized in three thematic axes: (a) Numeric sense and algebraic thinking; (b) Form, space and measure; (c) Treatment of information.

Teacher training. When the new mathematics curriculum was implemented (in 2011), there were not training for teachers, which did really make extremely difficult to implement the new proposal. In addition, new official textbooks or anterior classroom materials (as the book called "of math challenges") were not anymore functional. In summary, only best prepared teachers, used to develop their own materials for the class, could undertake the new approach confusion.

On the Enacted Curriculum in Mexico (Implemented and Attained)

It is important to remember that, in relation to the topics' list contained in a curriculum, in this paper the curriculum proposals are being presented only by short descriptions in the case of the sum of fractions, and for primary school.

School grade	1993	2011
3th grade	The work begins by fraction construction: halves, quarters, and eighths. The equivalence between fractions is worked on, and so on the resolution of equivalence problems, and order between fractions are solved. Beginning of problems of addition of the unit plus a half. Beginning of addition between halves, and quarters plus quarters	Identification of equivalent writings (additive, mixed) with fractions. Comparison of fractions in simple cases (with equal numerator or equal denominator) Beginning of resolution of problems of distribution with halves, quarters and eighths, problems of addition and subtraction with fractions with denominator 2^n. Comparison between fractions, equivalent fractions. Fractions as part of a turn. Graphic representation of fractions
4th grade	Continuation of sums of halves, quarters, and eighths using unit subdivisions, always with the use of paper cards, through solving problems that arise using paper strips of different lengths. Game of moving through halves, quarters, and eighths Introduction of halves, thirds, quarters, fifths, sixths, and eighths through partitions of areas Fractions' work with different numerator Sum of fractions through splitting the unit in 10 equal parts, locating each part in the real line, and each of the parts in 10 equal parts, also locating each part in the line, analyzing when two fractions are equivalent, and this will be the case when they are located at the same point. Order between fractions, which are obtained as the sum of other fractions Fractions with denominator 10, 100, 1000, etc. Fractions as turns	Resolution, with informal procedures, of addition and subtraction of fractions with different denominators in simple cases Resolution of problems where is important determine what fraction is a given part of a magnitude, resolution of problems that use addition and subtraction of fractions, using paper strips. Finding fractional numbers on the numerical line, adding fractions by concatenating them. Comparing fractions through problem solving

Table 4. Contents and activities suggested in official textbooks for sum of fractions in 3th and 4th school grades

For example, in Table 4 it can be read that in third grade and as the 1993 mathematics curriculum, it was suggested the approach to the construction and understanding of the fractions, halves, quarters and eighths by the manipulation of paper strips; something similar was suggested for the equivalence between them, and also for approaching the resolution of problems involving the simple addition of fractions (always with the same denominator). In summary, it is intended the child comprehension of the meaning of fraction's addition, but always through the manipulation of materials, and the contents proposal for fourth grade is very similar.

By other hand, in the 2011 curriculum and still in relation with adding fractions, the proposal is that students start to work with equivalence of fractions in third grade but considering always simple cases (see Table 4), and it is up to fourth grade that students begin to find, through the resolution of problems, what fraction is a given part of a magnitude; there are resolution of problems using addition and subtraction of fractions with informal procedures, including the use of strips of paper in simple cases, and placement of fractions on the number line. Finally, in fourth grade, the work with fractions continue to be on their placement in the number line, insisting on the equivalence between fractions and working on the sum of fractions with the same and different denominators through the usage of different concrete materials. In conclusion, and as in the 1993 curriculum, in the one of 2011 adding fractions is always studied through concrete and simple cases and by the manipulation of materials.

Finally, seeing the general comparison between the 1993 and the 2011 curriculum (in Table 1 and Table 2), and looking at the big changes announced in the official discourse, it is worth to say that they were never reflected in changes in the new official textbooks, or in the elaboration of new school materials that would facilitate the understanding or meaning of the new approach, and the curricular integration announced in the 2011 reform neither included substantive actions necessary for the academic update possibly required by the in-service teachers working in this educative levels.

The Attained Curriculum in Mexico

To discuss the attained curriculum, here have been considered the data and results published by the OECD. Especially important are those obtained by the application of the International Student Assessment Program (PISA) to the students living in countries affiliated to the OECD. PISA assessment measures mathematics student performance at the end of the first half of compulsory secondary education (i.e. at the age of 15 years). Its purpose is that OECD affiliated countries could use in a significant way the obtained data for decision making, law elaboration or application of public policies in education. Finally, serving as complementary PISA information, in this paper we have also considered the analysis of Mexican results in PISA that the INEE brings to the arena, INEE is precisely the institution that is responsible for coordinating the application of the PISA program in Mexico. INEE (see http://www.inee.edu.mx) documents are meant to expose or explain Mexican student results in PISA assessments.

At the time of establishing contrasts or similarities between the contents of the 1993 mathematics curriculum in Mexico and the one of 2011, it will be important to bear in mind that Mexican students participating in PISA 2003, PISA 2009 and PISA 2015, all of them began their primary education under the principles of the 1993 mathematics curriculum. But considering the students to whom the PISA 2015 was applied, although they were students whose primary education began in 2005 or 2006, that is, they completed the elementary school under the precepts of the 1993 curriculum reform. But 2015 PISA students' performance is extremely important in the analysis, because this generation of students had completed the first three years of compulsory secondary education already under the curriculum precepts of the reform of 2011.

Results of the performance of the 15-year-old Mexican participant students in the PISA 2009 and 2015

The PISA 2009 had 6 performance levels (from level 1 to level 6). In the global mathematics scale, level 6 is the highest and level 1 is the lowest. In order to establish an appreciation of the mathematical competences involved in this range it will be reviewed the definitions of the range extremities.

These definitions are found in the INEE document of 2010. It says that the students belonging to level 6: "know how to form concepts, generalize and use information based on research and modeling of complex problem situations. They can relate different sources of information and representations and translate them in a flexible way. They have an advanced mathematical thinking and reasoning. They can apply their understanding, as well as their mastery of symbolic mathematical operations and relationships, and develop new approaches and strategies to address new situations. They can formulate and communicate with precision their actions and reflections related to their findings, arguments and their adaptation to the original situations ". (INEE, 2010, p.101)

At the other end of the mathematical competence are the scopes of a student who belongs to level 1: "this student knows how to respond to questions related to familiar contexts in which all the relevant information is present and the questions are clearly defined. They are able to identify information and carry out routine procedures following direct instructions in explicit situations. They can perform obvious actions that are immediately deduced from the stimuli presented. " (INEE, 2009, p.102)

In addition, the INEE document (2010) indicates that "students whose performance falls below Level 1 are unable to succeed in the most basic tasks that PISA seeks to measure. This does not mean that they do not have any mathematical ability, but most of these students will probably have serious difficulties using mathematics as a tool to benefit from new educational and learning opportunities throughout their lives. " (INEE, 2010, p.102).

The results of the Mexican students in PISA 2009 and 2015 (INEE, 2010, pp. 101-102; INEE, 2016, pp. 62-68) are indicated in the following Table 5.

Percentage	Level 6	Level 5	Level 4	Level 3	Level 2	Level 1	Average score
PISA 2009	0	0.7	4.7	15.6	28.3	28.9	419
PISA 2015	0	0.3	3.2	12.9	26.9	31.1	408

Table 5. Comparing Mexican performance in PISA 2009 and 2015

It is to notice that, in PISA 2009, 21.8% of Mexican students do not reach level 1, and, in PISA 2015, the percentage of the same level is a little bit higher (25.6%). In other words, the percentage of Mexican students that in PISA 2009 are below level 2 (i.e., attaining the level 1 or zero) was 51%, and this percentage is 57% in PISA 2015, evidencing then an increment of Mexican students in the poor levels of performance. According to the INEE, students at levels 1 or cero are susceptible to experiment serious difficulties in using mathematics and benefiting from new educational opportunities throughout its life. Therefore, the challenges of an adequate educational attention to this population are huge, even more if it is also considered that approximately another fourth of the total Mexican population (33.3 million) are children under 15 years of age, a population in priority of attention (INEGI, http: //www.inegi.org.mx).

CONCLUSIONS

Pointing out possible connections between PISA results and curriculum reforms in Mexico, particularly those accomplished in 2009 or 2011 (see Table 1 to Table 5), it can be seen that almost all the PISA tests (for example, PISA 2003, 2009 and 2015) have been applied to young Mexicans whose primary education has been guided by the precepts of the curriculum reform of 1993. Since then, only those that participated at PISA 2015 were also educated in secondary school by the precepts of the curriculum of 2011, and it could be appreciated that in both Mexican mathematics curriculums, the one of 1993 and the one of 2009/2011 changes of approach to mathematical concepts were minimal. But it could be possible that drastic changes in the official discourse were causal for an increment in the percentage of Mexican students in the poor levels of performance in PISA 2015 (from 51 to 57%).

While the purpose of PISA has been to impact public education policies in the different participant countries, it appears that this has not been the case for Mexico. In practice there is no still until today registered progress on the promotion of the changes that public schools need, those that could show that the results of international evaluations, such as those of PISA, have been met by educational policies, or that these results had influenced decision making or the implementation of public policies that sought a sustained development of mathematics public education, especially for the most marginalized sectors of Mexican population.

Mexican government efforts made to advance in the development of the education of the population have only remained at a bureaucratic level, by the creation of specialized institutions devoted to the instrumentation or the coordination of the evaluation in Mexico, as for example the INEE itself (see http://www.inee.edu.mx), instead of having placed the emphasis on true teacher attaining or impacting in the strengthening of initial teacher education, or on teacher professional development for the promotion of mathematical thinking, as it is intended in the project initially mentioned in this paper.

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LEARNING EXPECTATIONS, DEVELOPMENT OF PROCESSES, AND ACTIVE CONTEXTUALIZATION IN COSTA RICA'S MATHEMATICS PROGRAM

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Educational changes should reflect the social, political, economic, and cultural changes of each community, but they should always seek development and improvement of the education system as a public service. Based on a characterization of the mathematics curriculum in effect Costa Rica, we analyze in greater detail the three key notions that structure the recent curriculum reform: general and specific abilities, processes, and one of the disciplinary core ideas, active contextualization. We reflect on the relevance of these notions and ultimately propose a series of considerations that arise from research currently being performed on the training of Costa Rican in-service teachers.

INTRODUCTION

An education system offers each individual "the best means to build his/her personality, develop his/her capabilities to the maximum, form his/her personal identity, and compose his/her own understanding of reality, integrating the cognitional, affective, and axiological dimensions" (Ministry of Education and Science, 2006, p. 17158). The social changes and needs that arise in developed countries thus usually have marked implications for these countries' education systems. It is in modern societies that the education system's main foundation is in the curriculum, which is structured as a proposal for educational planning and action (Rico & Lupiáñez, 2008).

The curriculum specifies a series of epistemological, pedagogical, and psychopedagogical principles that, as a whole, channel and define the general orientation of the corresponding educational system (Jonnaert, Barrette, Masciotra & Yaya, 2008). A great variety of knowledge is also considered and organized, and attention given to the complexity of teaching and learning processes in each discipline, among them, mathematics. The notion of curriculum is also important for the teacher's work. Niss (2006) characterizes a model of the competent teacher for teaching mathematics, stressing a curriculum-related facet that should form part of the teacher's knowledge and abilities: analyzing, evaluating, relating, and implementing training and curricular programs (p. 44). The implications of curricular directives and recommendations are also recognized in classroom activities: "What happens in the classroom is also, to a great extent, the result of factors, processes, and decisions that originate in other areas and levels, such as, for example, (...) the curriculum" (Coll & Sánchez, 2008, p. 21). When the curricular directives are isolated from other aspects of the educational reality, such as the teacher's work, problems and difficulties arise that work precisely to the detriment of this proposed curriculum (Harris & Burn, 2011).

The basis of the Mathematics Education reform in Costa Rica (Ministry of Public Education of Costa Rica –MEP-, 2012) is reorganization of the weight of the main dimensions and elements of the curriculum to give them greater cohesion and depth. As a whole, the changes represent an explicit bid to develop Costa Rican society in its full breadth and complexity.

A FUNCTIONAL FOCUS ON SCHOOL MATHEMATICS

Research that focuses on curricular comparisons shows evolutions in proposals and reforms, even in countries with a long tradition of prescriptive curricula (Bennet, 2005; Oberhuemer, 2005). Curriculum reforms generally seek to improve education (Calderhead, 2001), but it is interesting to analyze how these changes conceptualize some materials differently, expressing certain priorities and educational preferences for students.

Rico and Lupiáñez (2008, pp. 175-181) propose four curricular focuses that correspond to different orientations to how school mathematical knowledge is understood. First, the *instrumental* or *technological* focus stresses mastery and use of basic facts, skills, and concepts, considered as fundamental tools. Definitions should be ingrained in all students' memory, and operational routines executed with the greatest automatism possible. Second, in the *structural* or *technical* focus, knowledge consists of a structured system of rules and concepts, formalized and based on deduction. This focus is most clearly represented in the New Mathematics, but there are currently groups of teachers, networks in schools, and school materials commercialized by publishing companies whose practical supply maintains the influence of programs with a very formal and academic style. Within a *functional* focus, knowledge enables the modelling of real situations and is oriented to answering questions and solving problems in different contexts. The mathematical concepts and procedures have a purpose that is close to daily activities, serve to achieve something tangible, since mathematical notions are tools through which we act to answer questions, solve problems, and understand unknown phenomena in our environment. The notion of competence as a learning expectation fits the functional orientation of the curriculum. Finally, in the *integrated focus*, knowledge is an object of autonomous intellectual activity, creation, and interaction in a variety of situations and contexts. The integrated focus has attributes of both the structural and the functional models and is usually found in specific programs oriented, for example, to gifted students or to those with specific talent in mathematics.

We thus have different curriculum models, which present different options for educational plans according to the knowledge they stress, type of thinking they promote, weight of argumentation, and relationships of communication, complexity and diversity of capabilities considered in each case. That is, according to its ends.

From our perspective, the school mathematics reform in Costa Rica advocates and supports a functional focus of the mathematics curriculum:

> (the notion of) competence makes a contribution to the most general goals of Costa Rican education in development of the human personality, the participation of citizens with a sense of responsibility, understanding, and respect, which permits them to reconcile their interests with those of the community (a foundation of democratic life) and cultivate reflection that supports rational understanding of diverse cultural and social contexts, ideas and attainments that constitute human history. (MEP, 2012, p. 23)

The proposal for reform is far from a structuralist or formal view. Rather, it seeks to provide a comprehensive education of individuals so that they can use mathematics with rigor and good judgment to answer others and questions that they may encounter throughout their lives:

> (…) it could be assumed that the mathematical competence that it attempts to arouse in school education is mastery of mathematical structures or formalisms. It could also be assumed that one person is more competent mathematically than another if he/she knows a greater number of mathematical contents, which would within study programs give another specific perspective (for example, always seeking to introduce the greatest possible number of contents). (p. 23)

The functional focus of the mathematics curriculum advocates knowledge that focuses on development of one's own cognitive strategies, stressing the use of different forms of representation, argumentation abilities, and modelling techniques to pose and solve problems in context. In sum, its purpose is to develop schoolchildren's mathematical competence by improving their thinking and giving them certain autonomy.

This comprehensive education takes concrete form in expectations related to education, individual development, and personal autonomy, but also in expectations for communication and social and cultural interaction. Establishing different levels of expectations is a central element in the Mathematics Study Programs.

LEARNING EXPECTATIONS

Establishing learning expectations is related to establishing what the education community as a whole expects schoolchildren to develop as part of their compulsory education. In the Mathematics Study Programs, these expectations are proposed based on the notions of capability and ability. When capability is associated with specific short-term mathematical areas, we speak of specific abilities. When these are generalized to an educational cycle (also understood as from a mathematical point of view), general abilities arise (MEP, 2012). Capability and ability are thus related by both the specification of certain mathematics topics and an area of application.

In the case of mathematics, learning expectations express specific recognizable and desired uses of mathematical knowledge, which can be observed or inferred from the students' actions in response to tasks. Learning expectations in mathematics are based on demands for actions, contents, and tasks.

The depth and value of learning expected are based on the variety of connections, as well as the symbolic richness of mathematical knowledge mobilized and the difficulty of the problems tackled. As tasks and knowledge may have different levels, the students' actions can show different ranks of mastery and satisfy the higher cognitional abilities and capabilities articulated to different degrees. Mathematical learning is detected and confirmed by defining actions that make use of certain knowledge and respond to specific tasks. The programs have included different examples of tasks that demonstrate the activation of general and specific abilities. Their inclusion shows a high level of commitment to giving the programs coherence and depth from a pragmatic point of view.

MATHEMATICAL PROCESSES AND MATHEMATICAL COMPETENCE

The notion of *mathematical process* is also key in reform programs and in the subsequent evaluation models proposed (Ruiz, 2017). These processes, which do not depend on areas of mathematics, express ways of acting to solve and interpret problems. Fostering the putting into play of these areas leads to and structures the development of schoolchildren's mathematical competence. Mathematical competence is defined as the capability of the individual to formulate, employ, and interpret mathematics in a variety of contexts and the processes that make this notion operational, as the competence describes what individuals do to relate the context of a problem to mathematics in order to solve it. The processes selected in the Reform are: "Reasoning and argumentation," "Posing and solving problems," "Connecting," "Communicating," and "Representing."

Reasoning and argumentation involves posing and confirming hypotheses. It is also related to making inferences with inductive and deductive arguments and to the validation of statements and solutions found.

Posing and solving problems is mobilized when planning, applying, or evaluating strategies for solving problems in a variety of situations and contexts, as well as when evaluating the solution or solutions found. This process is also related to the ability to pose, formulate, and define different types of mathematics problems; in fact, this facet of invention has a proven educational interest, as it provides rich information on student learning.

The process of *connecting* is activated when different mathematical notions are related, when mathematics are related to other disciplines, and when their analytic and interpretive role is evaluated. Connecting also has to do with recognizing and applying mathematical concepts and procedures in nonmathematical contexts.

Communicating is a key facet of mathematical competence and is mobilized when problem situations are recognized and understood, when different mathematical ideas are read, decoded, and interpreted, and when results and procedures are summarized and presented.

Representing is a basic process in the learning of mathematics, and it is a key indicator of mathematical competence. It is activated when it is necessary to choose and use the form of representation best suited to the purpose, indicating potentials and limitations. Representing is also activated when the information provided by various representations is interpreted and described, and when forms of representation are related to or translated into each other.

These five processes can be activated to different degrees, also establishing important grounds for evaluation:

> When the degree of intervention of each process is specified, significant possibilities emerge that support a strategy to obtain progress in mathematics capabilities. In other words, we can adjust the degree of the processes in the mathematical tasks to feed into strengthening of mathematical competence. (Ruiz, 2017, p. 104)

ACTIVE CONTEXTUALIZATION OF TASKS

Another central notion in the mathematics study program is that of disciplinary core ideas (MEP, 2012):

> The core ideas here indicate priorities. They should therefore influence all elements of the curriculum. These priorities appear in the choice of topics, general instructions for management and method, and instructions and suggestions that accompany concepts and abilities when proposing the approach. Implementation of this curriculum proposal seeks to give special importance to each of these core ideas, although not all of them generate impact in the same way on each area or in each school year. (p. 35)

One of the five disciplinary core ideas considered is active contextualization, which focuses on the importance of proposing problems in real contexts. This focus can motivate the students to make connections and integrate various kinds of knowledge to propose creative and strategic solutions. These real contexts can have varying origins:

> (…) to awaken interest and participation, we propose using problems in real contexts that lead to the building or use of models. The idea is to design problems drawn from information in the press, school, community, class, or Internet. (p. 36)

Although these approaches are well grounded in research (Stillman, Kaiser, Blum & Brown, 2013), the difficulty of choosing or designing tasks framed in a real context has also been confirmed (García, Maaß & Wake, 2013). One way of collaborating on this difficulty is to provide guidelines for organizing fields of phenomena in which mathematics can play an interpretive, analytic, or model-building role. Although these guidelines do not appear explicitly in the basic document for the Costa Rican reform, the proposal for an evaluation framework (Ruiz, 2017) recommends using classification of situations proposed by the PISA framework (OECD, 2015) and that describe different environments in which mathematics problems are posed.

In addition to this organization of fields for the application of mathematics, it is also possible to characterize the relevance and authenticity of mathematical tasks, as a better qualitative step for addressing active contextualization. Maaß (2008) proposes a classification of tasks according to these two dimensions:

- Embedded word problem: The context is not important at all, it plays no real role in the solution and can be stripped away. Often the context is enormously simplified or it distorts reality.
- Reality related problem: The problem is related to reality but the context is simplified.
- Realistic context with a didactical relevant question: Within a realistic context a questions is asked which makes sense from a didactical point of view.
- Realistic context with an interesting question: The question asked is not authentic. However it is interesting because it gives a deeper insight into or a better understanding of this area.
- Realistic context with authentic question: Here we deal with questions which are important in a certain field and which are regarded as important by experts in this field. In the connection with the field of "everyday life" everyone may be regarded as an expert. (p. 2)

We are not arguing that tasks in relevant contexts with authentic questions have hegemony in the classroom. The definition of active contextualization in Costa Rica's Reform is consistent with the three most realistic levels presented, but the performance of more technical or algorithmic tasks can in any case be considered as contextualized within mathematics itself and should certainly also be present in school mathematics. To demonstrate the development of mathematical competence, schoolchildren should work on mathematics tasks that provide them with opportunities to reason, make arguments, and solve problems framed in real contexts:

> Presenting problems in real contexts motivates the student to make connections and integrate various kinds of knowledge to reach a solution in a creative and strategic way. For example, one student may tackle a problem algebraically, while another tackles it geometrically. In sum, students solve problems from real life that require the use of the accumulated skills, knowledge, and competences that they have acquired throughout their school life and through their experiences. (Caraballo, 2014, p. 49)

RESEARCH IN PROGRESS

Setting up the reform of the Mathematics Study Program in Costa Rica required the design and activation of various measures, one of which stresses promoting training of teachers (Ruiz & Barrantes, 2016) and regional advisors (Poveda & Morales, 2015), developing support materials (Ruiz, 2017), and disseminating directives, recommendations, and collaborations networks (like those found on www.reformamatematica.net), among others.

Research can also contribute to analysis of the design, putting into practice, and implications of this curriculum reform. In this case, we wish to synthesize some evidence from a study currently underway, even though very few results have been fully confirmed.

The study's general goal is to describe and analyze the nature and direction of the changes produced in the knowledge, capabilities, and attitudes of in-service mathematics teachers regarding their teaching practice in the context of the curriculum reform in effect in Costa Rica (Lupiáñez & Loria, in press). To achieve this goal, we apply qualitative research methods in a study whose purpose is descriptive, explanatory, and evaluative. We describe the performance of a group of teachers in a course-workshop that focused on the key notions of the reform and identify factors that contribute to explaining the changes that occurred in the teachers' knowledge, competences, and attitudes during this experience. Since we also analyzed the impact of this training on their classroom practice, we monitored four of the participating teachers, as well as another teacher who did not take the course-workshop, during five weeks of teaching.

The participating teachers showed considerable advance in conceptual clarification of various central notions of the reform. For example, as to their abilities, they are able to distinguish and characterize general and specific abilities, link different abilities to the handling of various mathematical concepts, and provide examples of tasks that could demonstrate the achievement of specific abilities. They also attribute a precise meaning to the notion of mathematical competence and relate it to the five processes.

As to the disciplinary core idea "active contextualization," although the teachers recognize the role and importance of its application, they have considerable difficulty proposing contextualized tasks; in the majority of their statements, context plays a very minimal role (embedded word problems or reality related problems). These difficulties were especially visible during the classes observed, where the teachers express regret and worry that they cannot find phenomena and fields of problems that enable them to propose relevant tasks and authentic questions. Although they do show solid knowledge of the differences between an instructional task and a evaluative task, they have some difficulty proposing tasks that evaluate some of the processes that compose mathematical competence.

In general terms, the teachers recognized that they developed professionally, expressing this in the final reflections on the course-workshop. They also demonstrated this development during the follow-up days in the schools, when they explained and justified the plans made and the class sessions taught.

It was precisely during the observation phase at the schools that we were able to confirm a difficulty inherent in the teachers who work in private schools. In these cases, the directives of internal organization and functioning require the teachers to make their lesson plans according to specific criteria, that do not necessarily agree with the priorities established in the Mathematics Study Programs. In some cases, new organizing notions appear that are not described conceptually but that must still be exemplified for each grade or level. In several of these cases, the teachers had to make a parallel lesson plan in which they detailed the ideas, sequences, examples, and statements of tasks that they really treat in the classroom.

FINAL REMARKS

The Mathematics Study Programs propose a complete compendium, over time and throughout the material, of sensible detailed explanation that gradually identifies important concepts and nuances, the terminology necessary to handle them, their origin and foundations, and recommendations for their subsequent development. But these plans also included sections dedicated to developing the praxis of this proposal with numerous examples that will be of great utility for the teachers. Reflective planning of an organized evaluation plan is crucial, since it will help in educational decision making by administrators, responsible policies, public opinion, teachers, parents, and the students themselves. It is very important, therefore that this curriculum model be based on the highest-priority needs of Costa Rica's sociocultural reality, as it must become a key reference to orient Costa Rica's education policy and give coherence to all of the renewal strategies, training, and assessment of schools and all members of the teaching staff who work in them.

A functional curriculum that proposes as educational priority the development of competent students represents in itself clear dedication, passionate challenge, and future commitment. But having legal documents establish

> mathematical competence from a functional focus is only the first step. Mathematical competence does not gain virtue merely by being included in the legal documents. Mathematical competence must be integrated into the entire curriculum framework, connected to the rest of its components, and made to act throughout the whole system. This network of relationships is established from a few coherent basic concepts with (…) a characterization of school mathematics through a functional focus of the curriculum (Rico & Lupiáñez, 2008, pp. 214-215)

The studies and research currently underway can play a very significant role in each component and agent of the system. The need to document and confirm initiatives, difficulties, advances, and concerns is now an absolute priority.

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DOMAIN-SPECIFIC FRAMEWORKS FOR CURRICULUM DEVELOPMENT OF EXPLORATIVE PROVING IN JUNIOR HIGH SCHOOL MATHEMATICS

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The present study conceptualizes explorative proving, and applies this idea to four domain-specific frameworks for the curriculum development of Algebra, Geometry, Function, and Data Handling. We highlight the significance and the way of mapping these frameworks to instructional units of "Course of Study" in Japan, and argue that the former have a better curriculum reform potential.

NECESSITY FOR CURRICULUM FRAMEWORKS OF EXPLORATIVE PROVING

The teaching and learning of proof is recognized internationally as a key component of mathematics curricula (Hanna & de Villiers, 2012). However, it remains the case that students at the junior high school level (and beyond) experience difficulties in learning proofs in mathematics (e.g. Harel & Sowder, 2007; Martinez and Pedemonte, 2014). To overcome their difficulties by not only reflecting on the nature of mathematics, but also cultivating generic competencies of authentic *explorative* thinking (Miyazaki & Fujita, 2015), we put forward a series of frameworks for the development of curriculum in explorative proving in lower secondary education (G7-9). We argue that they fare better than Japanese current "Course of Study" equivalents. In the first part of this paper, we provide a conceptualization of Explorative Proving in school mathematics. In the second part, we lay out the details about how we can implement this idea for curriculum frameworks in domains of junior high school mathematics such as Algebra, Geometry, Function, and Data Handling. In the last part, we compare our proposals with the existing "Course of Study" units, and argue that our proposals could offer a more effective way of teaching explorative proving.

EXPLORATIVE PROVING IN SCHOOL MATHEMATICS

Based on Fawcett (1938), Waerden (1967), and Lakatos (1976), we argue that proving activities in mathematics involve producing statements inductively, deductively, and analogically, planning and constructing proofs, looking back over proving processes and overcoming global/local counter-examples or errors, and utilizing already-proved statements in the context of working on further proofs (see Fig. 1) to reflect the nature of proving as an activity in mathematics (Freudenthal, 1971).

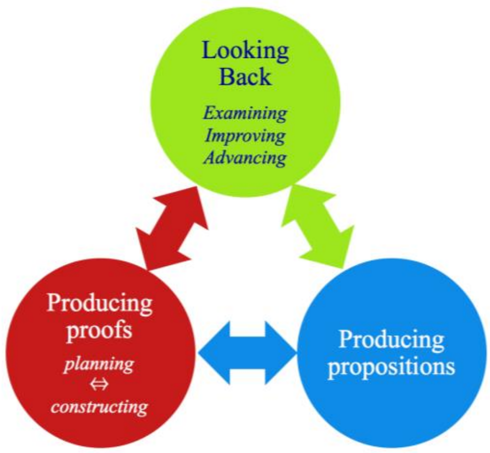

Fig. 1. Explorative proving

By considering insights from the above, we define explorative proving as having the following three components: producing propositions, producing proofs (planning and construction), and looking back (examining, improving, and advancing) (see Fig. 1) (Miyazaki & Fujita, 2015).

In developing domain-specific frameworks, producing proofs is focused on, and its elements (planning and construction; axes) and their interactions are represented a two-axes model in which each axis is divide into two or three parts according to the characteristics of domain, and intersections on the model means the level of learning of constructing and planning proofs. These levels should be subsequently transited, and learning how to construct proofs precedes learning how to plan them. For each stage, the component "Looking back (examining, improving, and advancing)" can be also expected and encouraged as explorative proving.

DOMAIN-SPECIFIC FRAMEWORK FOR CURRICULUM DEVELOPMENT

Offering a detailed proposal, this section shows how to theoretically implement the idea of explorative proving in curriculum frameworks in accordance with the characteristics of the domains of junior high school mathematics. We focus on producing proofs, namely planning and construction.

Geometry

Formal proofs in geometry in junior high school have a logical structure that connects premises and conclusions via deductive reasoning by adopting singular and universal propositions. By focusing on this characteristic, we propose the following learning levels planning a proof:

GP1: Clarifying what -and how- can be used to connect premises and conclusion.

GP2: Considering how to think backward from a conclusion, think forward from premises, and how to connect them.

In geometry, "Planning a proof" refers to the activity of seeking ways to connect premises and conclusions by deductive reasoning (Tsujiyama and Yui, 2018). This activity needs to expand the network of propositions that can be deduced from premises, and the other network of propositions that can be deduced from conclusions, and to seek the common propositions within the two networks. The first learning level (GP1) refers to differentiating the objects (what can be used to connect premises and conclusions) from the methods (how the objects can be used) that are necessary to plan a proof. GP1 also refers to the use of objects and methods in order to connect premises and conclusions. The advanced learning level (GP2) consists in thinking forward from premises to conclusions, thinking backward in the opposite direction, and making use of them in order to connect premises and conclusions.

When it comes to "Constructing a proof," the following learning levels can be considered:

GC1: Forming and expressing the deductive connection between premises and conclusions in a singular statement.

GC2: Forming and expressing the deductive connection between premises and conclusions while differentiating universal instantiation and hypothetical syllogisms from deductive reasoning.

"Constructing a proof" consists in finding the common propositions in two relational networks, and expressing the deductive connection between premises and conclusions, which are suggested by planning. Especially in a geometrical proof, premises and conclusions can be connected mainly by hypothetical syllogisms based on singular propositions. Considering a proof in more detail, each singular proposition would be deduced with a universal proposition (e.g., theorems). This deduction can be realized by universal instantiation. Finally, constructing a proof can be achieved by expressing the connection with language, diagram, etc. The first learning level (GC1) is dedicated to expressing the part of connection based on a hypothetical syllogism. The advanced learning level (GC2) is dedicated to differentiating universal instantiation and hypothetical syllogism from deductive reasoning, and to expressing singular propositions and universal propositions with a clear distinction.

The goal of our proposed curriculum is to help students achieve levels at GP2 and GC2 by the end of junior high school. However, it would be perhaps unrealistic to expect from students to construct proofs immediately at the highest stages. Therefore, it is necessary to set up several intermediate levels. By setting Stage O where there is no differentiation between planning a proof and constructing a proof, we can set five hypothetical stages if we combine two kinds of level related to planning and constructing. The transition from the lowest stage (Stage O) to the highest stage (GP2, GC2) needs to go through Stages GP1, GC1, in order to enhance planning and constructing a proof. Therefore, the transition can be divided into an earlier component [0 ⇒ (GP1, GC1)] and a later component [(GP1, GC1) ⇒ (GP2, GC2)]. Thus, we can establish the two transition processes as learning progressions (Empson, 2011)(see Fig. 2).

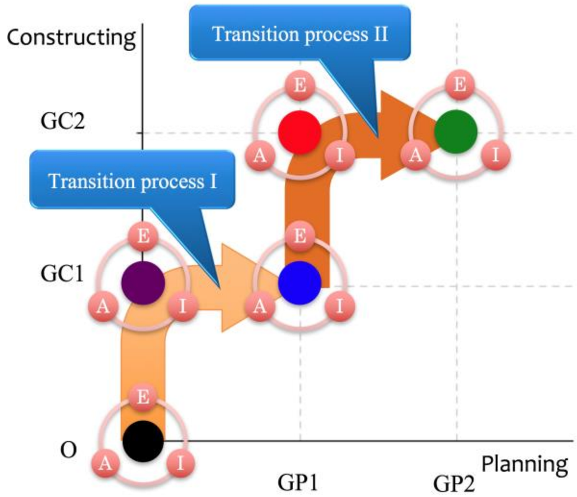

Fig. 2: Geometry-specific framework

Algebra

Miwa (1996) has illustrated three processes in the use of symbolic expressions: *Express*, *Transform*, and *Read* (Fig. 3). While planning a proof, one should consider how the premises and conclusions of a proposition can be symbolically expressed (*Express* and *Read* process, respectively), but also how to transform symbolic expressions representing premises into symbolic expressions representing conclusions (*Transform* process).

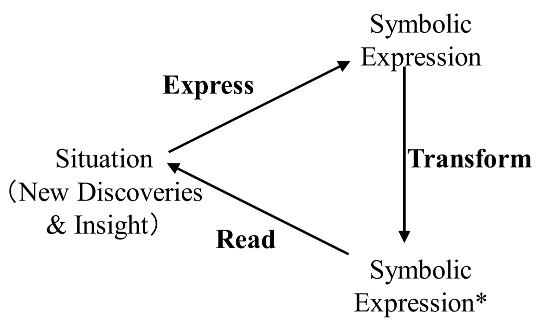

Fig. 3. Scheme of use of symbolic expressions

Taking the above into account, we propose the following learning levels for planning:

AP1: Clarifying how the premises of a proposition can be symbolically expressed by dividing a proposition into premises and conclusions.

AP2: Clarifying how the premises and conclusions of a proposition can be symbolically expressed by considering what the letters stand for.

AP3: Clarifying how the premises and conclusions of a proposition can be symbolically expressed and transformed to connect with each other by considering what the letters stand for.

To construct proofs in algebra, we propose the following learning levels:

AC1: Forming and expressing the deductive connection between premises and conclusions by using symbolic expressions and by explaining the relationship between premises, conclusions of a proposition, and symbolic expressions.

AC2: After proving the above and showing what the letters stand for, explaining the overall relationship between symbolic expressions and a proposition.

"Constructing a proof" consists in revealing the connection between premises and conclusions with the use of symbolic expressions. According to Miwa (1996), for students to understand the generality of proof in algebra, it is necessary to understand that literal symbols express arbitrary numbers. Therefore, we propose two levels in the *Express* process (Fig. 3). In the first level (AC1), students explain the relationship between the premises of a proposition and symbolic expressions. In the second (AC2), by being conscious of the generality of a proposition, they reveal what the letters stand for.

With regard to the *Read* process (Fig. 3), Miwa (1996) suggested that we need to read or interpret the result of transforming a symbolic expression ("Symbolic Expression*" in Fig. 3) in the context of the original situation in order to gain insight or uncover a new interpretation. Therefore, in constructing a proof, we propose two levels: In the first level (AC1), students explain the relationship between the result of transformations and the conclusions of a proposition. In the second (AC2), students explain the overall relationship between symbolic expressions and a proposition.

The goal of our curriculum is to help students achieve AP3 and AC2 levels by the end of junior high school. Three transition processes are assumed (see Fig. 4): In Transition process Ⅰ, students aim to clarify how the premises can be symbolically expressed by grasping the idea of a proof in algebra. In Transition process Ⅱ, students aim to think backward from the conclusions by being conscious of the generality of a proposition. In Transition process Ⅲ, students aim to consider how to transform symbolic expressions.

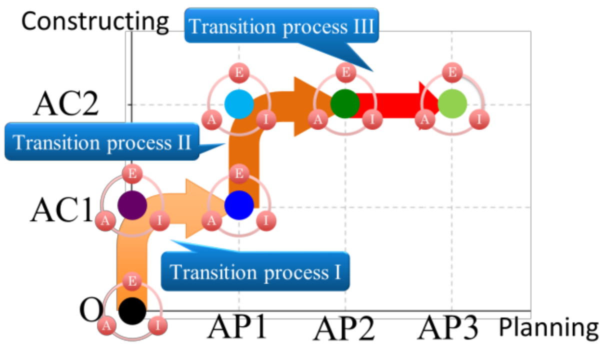

Fig. 4. Algebra-specific framework

Transition process Ⅰ reflects the transition from Stage O to (AP1, AC1) via AC1. This process passes through AC1, not AP1, because it is necessary to have a chance to construct a proof in order to learn how to "plan a proof" as with the domain of Geometry. Similarly, Transition process Ⅱ reflects the transition from Stage (AP1, AC1) to (AP2, AC2) via (AP1, AC2). Finally, Transition process Ⅲ is from (AP2, AC2) to (AP3, AC2) (see Fig. 4). Like geometry, the component "Looking back" plays important roles in each level. For each stage, the component "Looking back (**E**xamining, **I**mproving, and **A**dvancing, EIA)" can be expected and encouraged as explorative proving.

Function

In lower secondary education, one of the central aims of learning functions is to foster students' functional thinking, stressing the significance of its applicability. Hence, "producing propositions" in this process includes not only conjecturing about the properties of the functions, but also interpreting phenomena or predicting unknown situations by considering the relations among variables in the real world. We focus on the latter activity. In this process, *proving* refers to *justifying the prediction or interpretation produced by using functions*.

In the case of proving a prediction or interpretation (as well as in mathematical proofs), it is essential to inspect whether the inference process is appropriate. Such a process is similar to mathematical modeling, which is often grasped by following three sub-processes: *formulating* (*f*), *employing* (*e*) and *interpreting* (*i*). Therefore, we propose the following stages of learning:

*f*1: Showing what functions to be used in problem solving

*f*2: Showing mathematical evidence for the judgment about what function is to be used in problem solving

*f*3: Showing how the original real problem situation has been idealized or simplified

*e*1: Describing the entire process from the given conditions to mathematical conclusions

*e*2: Describing the entire process that leads to mathematical conclusions by means of specifying the mathematical model

*i*1: Taking into account the results of interpreting mathematical conclusions in the context of the original real problem situation

*i*2: Taking into account the results of interpreting mathematical conclusions that refer to limitations, and specifying their cause

The goal of our curriculum in the domain of function is to be able to construct the proof at stages *f3*, *e2* and *i2* (hereafter, we give a brief account in the form "proof at (*f3*; *e2*; *i2*)"). We set the following levels for proof construction:

FC1: Understanding the necessity of three sub-processes (*formulating*, *employing*, and *interpreting*) as a frame for proof, and constructing proofs at (*f*1; *e*1; *i*1).

FC2: Upgrading the description of proofs in terms of mathematical evidence, and constructing proofs at (*f*2; *e*2; *i*1).

FC3: Upgrading the description of proofs in terms of idealization and/or simplification, and constructing proofs at (*f*3; *e*2; *i*2).

In addition to being able to construct proofs autonomously, learning how to plan them is also required. Thus, we set the following levels for planning (corresponding to the levels of proof construction):

FP1: Investigating functions, and their usages to connect a premise and a conclusion from

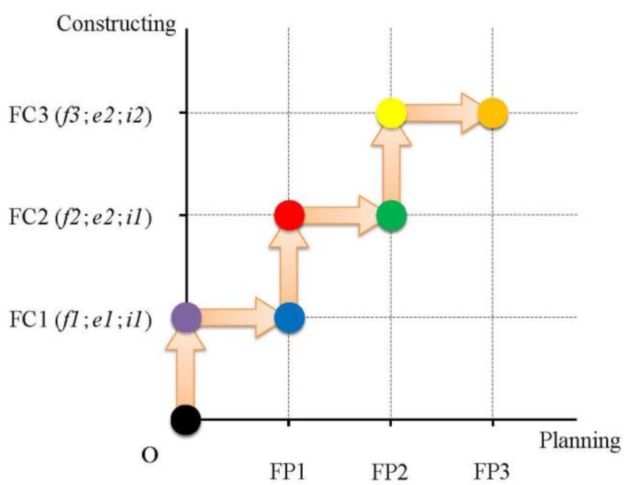

Fig. 5. Function-specific framework

the viewpoint of three sub-processes of mathematical modeling.

FP2: Investigating functions, and their usages to connect a premise and a conclusion from the viewpoint of criteria that justify three sub-processes of mathematical modeling.

FP3: Investigating functions, and their usages to connect a premise and a conclusion from the viewpoint of whole process of mathematical modeling.

By considering following points, we could postulate six transition processes of learning in the domain of Function (see Fig. 5).

Data Handling

While in the domains of algebra and geometry students are expected to prove the logical necessity of propositions, the domain of data handling deals with the plausibility of claims in real world contexts. Given this characteristic, we use the term *justification*, rather than *proof*. To characterize the meaning of justification, we employ Toulmin's (2003) layout of arguments. His model focuses on the soundness of practical arguments in everyday life situations, and thus it is relevant to the data handling domain. We use the simplified version of Toulmin's scheme where each inference contains three elements: the claim being argued, the datum used to justify the claim, and the warrant describing how the datum supports the claim. In our research, the datum means something obtained by statistically analyzing raw data (e.g., mean, median, and histogram), rather than the raw data themselves. *Planning a justification* refers to thinking about what datum is appropriate for supporting a claim, and *constructing a justification* refers to actually producing an argument that consists of the claim, datum, and warrant.

Task. The number set below shows how many books each of 20 grade 7 students in a secondary school read for one month. Can a student who read three books be regarded as reading relatively more books in this group? Explain your answer.

2	0	1	1	8	1	10
2	3	1	12	2	3	1
1	0	5	2	1	10 (books)	

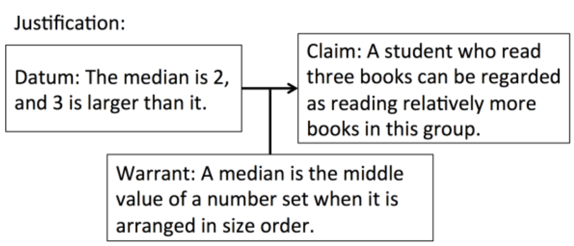

Fig. 6. Example of a task (left panel) and justification (right panel)

We briefly illustrate the above characterization with the task in Fig. 6 taken from a Japanese mathematics textbook for secondary school students. Before working on this task, students are expected to be familiar with several types of descriptive statistics such as mean, median, and mode. Thus, students could plan which average they should consider for answering the question posed in the task. Accordingly, students may construct a justification (as in Fig. 6, right panel).

Fig. 7 represents our framework for curriculum development in the domain of data handling. In this framework, we differentiate two levels of justification: DC1 involves constructing a justification based on a single datum, and DC2 involves constructing a justification based on multiple data (DP1 and DP2 involve planning the respective justifications). This is because, given that the data handling domain is related to uncertain empirical phenomena, a single datum sometimes may not be enough to justify a claim, and additional data may be necessary for strengthening the justification —for instance, in the task shown in Fig. 6, students may reinforce their justification by further taking a histogram into account.

In this framework, the transition from Stage O to Stage (DP1, DC1) is considered in the same way as the frameworks in other domains (e.g., geometry). With respect to the subsequent transition, the necessity for employing multiple data would arise in a situation where a single datum is not sufficient to represent the empirical phenomenon. In this case, on the one hand, the transition of planning would precede that of constructing since it is likely that students first plan what data they would add in order to strengthen their justification. On the other hand, it would not be reasonable to set up a stage exclusively concentrating on this kind of planning, namely Stage (DP2, DC1), because this stage means that students plan to employ multiple data but actually construct a justification using a single datum alone. Thus, we consider the direct transition from Stage (DP1, DC1) to Stage (DP2, DC2) as shown in Fig. 7. For each stage, the component "Looking back (examining, improving, and advancing)" can be also expected and encouraged as explorative proving.

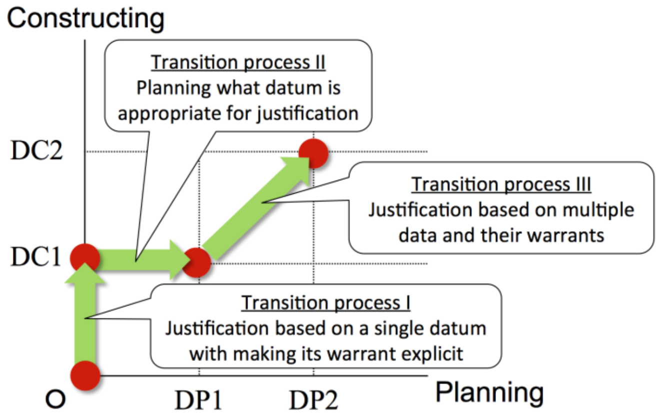

Fig. 7. Data Handling-specific framework

TOWARD DEVELOPING DOMAIN-SPECIFIC CURRICULUM BASED ON THESE FRAMEWORKS

In order to develop a domain-specific curriculum based on the previous frameworks, we examined the existing implemented curriculum "Course of Study" in Japan, and compared the implemented units with the transition processes that we proposed above. For example, in the case of geometry in Japanese junior high schools, our proposed framework can achieve to develop the desirable and realizable curriculum, echoing the idea of explorative proving, while "Course of Study" only requires realizing the idea, but does not propose the way to realize it as curriculum. Moreover, "Course of Study" requires that students learn various properties of plane and space figures mainly based on congruency and similarity, and also the meaning of proofs, and how to prove formally. Although "Course of Study" encourages the gradual introduction of formal proofs until the end of Grade 8, it does not offer a clear plan on how to gradually implement the learning processes of planning and constructing a proof. By combining local transitions of our frameworks with units in "Course of Study", the developed curriculum can propose teachers with a realizable plan on how to gradually implement the learning processes, and evaluate students' ability.

In order to show the advantages of our proposal, we show the table combining the implemented units in "Course of Study" with two transition processes in our theoretical framework (Table 1). Each unit has a plethora of contents. We have stipulated a correspondence between those and the local transitions. Taking this correspondence as a basis in an ongoing study, we have been implementing experimental lessons (Cobb et al., 2003), and investigate the feasibility of our provisional frameworks and curriculum by conducting a careful observation of students and their work (the so-called "method of lesson study"; Lewis, Perry and Murata, 2006).

Units in "Course of Study"	Local transitions
Properties of parallel lines and angles	(GP1, GC1)→(GP1, GC2)
Properties of angles of polygons	(GP1, GC2) + EIA
Meaning of congruent and conditions of congruent triangles	
Meaning of formal proofs and how to prove formally	(GP1, GC2)→(GP2, GC2)
Properties of triangles and quadrilaterals	(GP2, GC2) + EIA

Table 1: Correspondence of implemented units with local transitions in Grade 8 geometry

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RELATIONS BETWEEN MATHEMATICS AND COMPUTER SCIENCE IN THE FRENCH SECONDARY SCHOOL: A DEVELOPING CURRICULUM

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In the last decade, French curricula for secondary school have included contents in Computer Science inside Mathematics curricula. We study this particular situation by questioning the history of the introduction of Computer Science in school in France, by analyzing the links between Mathematics and Computer Science in the curricula (based on epistemological considerations), and by enlightening the issues of this curriculum under development.

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COMPUTER SCIENCE AND MATHEMATICS IN FRENCH SECONDARY SCHOOL

In the last decade, French secondary school has gradually incorporated in its curricula contents in computer science. Before that, there were already contents relative to computer tools (use of software) in the curricula (like spreadsheets in mathematics, or word processing program in other disciplines), but from 2009 contents concern computer science as a specific discipline and not only as a tool. Most of these contents in Computer Science have been integrated in the Mathematics curricula and addresses algorithmics and programming. Some advanced contents in Computer Science have also been introduced in optional courses in upper secondary school and are often taught by mathematics teachers; indeed in France, there are no positions of secondary teachers in computer science. This situation leads to tumultuous relations between two views about computer science education: the first view consists in defending the necessity of an autonomous scholar discipline, and the second one consists in underlying the links with Mathematics and the fact that Mathematics teachers could be trained for teaching of Computer Science.

This specific situation questions the relations between Computer Science and Mathematics, as disciplines, but also inside the French curricula of secondary school. This issues clearly regards STEM education, with a special focus on the relation between Mathematics and Computer Science among other scientific topics. In this paper, we address the following issues we have been working on since 2009 (Modeste & al., 2010, Modeste & Ouvrier-Buffet, 2011, Modeste, 2012, 2015, Modeste & Rafalska, 2017): What do Mathematics and Computer Science share as scientific disciplines and what kind of interactions between them can be developed in secondary school? How does the French curricula deal with this issue and in which direction are they developing?

In this paper, we first present briefly the theoretical framework that shapes our analysis. Then, we explore briefly the history of the introduction of computer science in school in France and we analyze the relations between Mathematics and Computer Science in the current curricula, based on epistemological considerations about the nature of these relations. Finally, we examine the institutional situation in France, through the processes that lead the development of these curricula and the actors who influence it. This will enable us to enlighten the issues of developing coherent curricula in mathematics and Computer Science.

A framework for analysing the curricula: ATD

In this paper, we will use some elements from the Anthropological Theory of Didactics (ATD) (Chevallard, 1992, Artigue & Winsløw, 2010, Bosch & Gascón, 2014). Indeed, ATD is relevant to analyze curriculum design and allows taking into account various levels of analysis such as society, pedagogy or discipline, as well as institutions involved in the process, which fits our purpose. We briefly describe the elements of ATD that are used below.

ATD considers unit blocks of activity called *praxeologies*. They permit to describe human activities and their organization in specific institutions. Here we are interested in Mathematics praxeologies and Computer Science praxeologies involved in secondary school curricula. Another concept linked with ATD is the *didactical transposition* (Chevallard, 1985, Chevallard & Bosch, 2014), which is the process through which knowledge (i.e. the praxeologies) is transposed from institutions where its is produced to institutions where it is taught. In the case of scientific knowledge (like Mathematics or Computer Science), the didactical transposition concerns the passage from scholarly knowledge to knowledge to be taught and then to the taught knowledge. This concept is closed to the notions of *intended* and *implemented curricula* but incorporates views on curricula as products and as processes. The didactical transposition of a piece of knowledge involves many actors and institutions, at a higher level than the one of the teachers, called *noosphere*. In the last part, we give example of it.

To understand how didactical transpositions occur or how curricula are designed and evolve, we will rely on the *levels of didactic codetermination* (Chevallard, 2002), which allow taking into account the context (in terms of conditions and constraints) in which praxeologies exist and develop. These levels, that interact with each other, are the following:

Civilization (9)↔ Society (8) ↔ School (7) ↔ Pedagogy (6)↔
Discipline (5) ↔ Domain (4) ↔ Sector (3) ↔ Theme (2) ↔ Subject (1)

In this article, we are interested mainly in *Society*, *School*, *Pedagogy* and *Disciplines* as levels that influence curricula, in particular on Mathematics and Computer Science and the way they interact.

Current situation in France: organization and design of the curricula

Currently, French secondary school is divided in two institutions. Lower secondary school (called *collège*) is mandatory and starts at grade 6 and finishes at grade 9, with a national assessment called *Brevet*. The next 3 years (grades 10 to 12) concern upper secondary school (called *lycée*). It has different streams (vocational, technological and general), and we will focus here on the general stream, which ends with a national assessment called *Baccalauréat*. The two national assessments, Brevet and Baccalauréat, have a strong influence on the way the intended curriculum is implemented by teachers.

Historically, France has a national curriculum elaborated by specialized committees. In mathematics, the curriculum is defined by different official documents. One is the *program* itself, describing contents and expectations about them, and the others are *accompanying resources* for helping teachers to implement the program, offering details about specific topics (propositions of learning activities and related explanations). Until 2016, the programs for secondary school were presented for each grade. Since 2016, in primary school and lower secondary school, the curriculum for each subject is derived from a *common core of knowledge, competencies and culture* and organized by 3-years cycles (lower secondary school is concerned by cycle 4 – grades 7 to 9 – and by the last year of cycle 3 – grade 6). These changes are clearly related to international movement towards enhancing competencies. For now, curriculum for upper secondary school has not changed but very recently the Ministry of Education has launched a wide project for changing the Baccalauréat and the whole structure of upper high school[1].

Historical overview on computer science in French Secondary School

This section is based on a previous collaborative work (Gueudet & al., 2017) in which details about French curriculum's structure and evolution are presented.

In the 1980s, after a few experiments, the teaching of computer science in upper secondary school was introduced in France for the first time with an optional teaching called "Informatique des Lycées" (Computer Science for upper secondary school). This teaching was done by specially trained teachers, essentially mathematics teachers, and was centered on algorithmics and programming; the Ministry of Education invested a lot of effort into teacher training for this option. At this time, international work is already developing about the interactions between Mathematics and Computer Science, as attested by the first ICMI Study: *The influence of Computers and Informatics on Mathematics and its Teaching* (Howson & Kahane, 1986). However in France there was no social consensus on the finality of this teaching (Baron & Bruillard, 2011), and Computer Science as a scholar discipline, disappeared in the 1990s, replaced in the curricula by teaching how to use computers as tools in every discipline.

In the 2000s, the CREM[2] (Kahane, 2002) recommended in its report to "introduce some Computer Science in the teaching of Mathematics and in teacher education" and defended the importance of interactions between Mathematics and Computer Science. The report addresses many arguments summarized as follows: Algorithmic thinking, implicit in the teaching of mathematics, could be developed and enlightened with the instruments of algorithmics; Programming promotes formalized reasoning; Questions about effectiveness of algorithms involve mathematics; Data processing and digital computations are common in other disciplines; and finally, Computer Science transforms Mathematics, bringing new points of view on objects, bringing new questions, creating new fields in Mathematics that are growing rapidly, and changing the mathematician's activity with new tools.

Just after this report was published, algorithmics was introduced in mathematics at grades 11 and 12, but only for literature series, and in optional mathematics courses in the last year of the economics series (in an introduction to graph theory) and the scientific series (in an introduction to number theory). Then, between 2009 and 2012 in new official programs, algorithmics was introduced as part of Mathematics into all series of the general stream (literature, economy, sciences) from grade 10 to 12 (French *lycée*).

1 A mission has also been given to the Fields medal awarded French mathematician Cédric villani on the teaching of mathematics at primary and secondary levels (http://www.education.gouv.fr/cid126423/21-mesures-pour-l-enseignement-des-mathematiques.html) but we will not analyze it here.

2 In 1999, the French education ministry appointed a commission, the CREM (National Commission for Reflection on the Teaching of Mathematics) for rethinking the teaching of mathematics for the new century headed by Jean-Pierre Kahane. Jean-Pierre Kahane is a famous mathematician, member of the Academy of sciences, and former president of ICMI (1983-1990). In 2002, the CREM published an important report on the teaching of mathematics.

Finally, in the 2010s, computer science as an autonomous discipline came back in the upper secondary school, together with algorithmics and programming as part of the contents in Mathematics. In 2012, a new optional teaching of computer science (called ISN, for *Informatique et Sciences du Numérique*) was proposed in grade 12 for students of the scientific series. These teachings are made by teachers from various scientific disciplines, including a lot of mathematics teachers. A similar option (*Computer Science and Numerical Creation*) has been created in grade 10 and tried in various schools since 2015. Since 2016, Computer Science is also taught in cycle 4 (grade 7 to 9), but divided between two disciplines: Mathematics and Technology.

Following this increasing need for teaching Computer Sciences in secondary school, a Computer Science option has been introduced since 2016 in the competitive exam for recruiting mathematics teachers for secondary school (CAPES), leading to the involvement of university scholars in Computer Science in the preparation of these.

We can already notice that the appearing of Computer Science in French curriculum is not a linear process, illustrating the tensions in the phenomena of didactical transposition, submitted to conditions and constraints that falls under different levels of codetemination: School, Pedagogy and Discipline, but also Society, inasmuch as the relationship with Computer Science, Mathematics and Sciences in the French society and French academic world clearly affects the curricula.

After this brief presentation of French curriculum structure and organization[3] and this short historical perspective, we examine relations between Mathematics and Computer Science in the current programs for grades 10 to 12 in the general stream and in the new programs for cycle 4 (grade 7 to 9) of lower secondary school.

ANALYZING SECONDARY SCHOOL CURRICULA

Epistemological considerations on the relation between Mathematics and Computer Science

In our previous work (Modeste, 2012, 2015), we have developed epistemological considerations – in the sense of Artigue (1990) – on the relations between Mathematics and Computer Science. We have identified four aspects that we summarized here.

A1. Foundations, logic and proof. Mathematics and Computer Science share common logic foundations (Computer Science is born, in part, from development of logic) which guide the role of language in both disciplines. Proof, as a mean of validation, plays an important role in Mathematics and Computer Science, and is an important component of their epistemologies.

A2. Continuity and interfaces. The frontier between Mathematics and Computer Science is impossible to draw, and a lot of objects and fields live at their interface like cryptography, games theory, graph theory, combinatorics, etc. Many new Mathematical fields developed in response to Computer Science needs and many algorithmic and computing questions arose from Mathematics.

A3. Computer assisted mathematics and experimental dimensions. Computer Science permits to foster the experimental dimension of mathematics, allowing systematic exploration of various cases,

3 For more details about the French Mathematics curriculum, see (Gueudet & al., 2017) or the ministry website: http://eduscol.education.fr/cid66998/eduscol-the-portal-for-education-players.html

or algorithmic solving of mathematical problems. Computer Science can bring complementary outlook on mathematical objects.

A4. Modeling, simulation and relation with other disciplines. Both disciplines offer means for formalization that permits to model and simulate (or compute) situations from other disciplines. Mathematical models can be computed and simulations often involve many mathematical models.

Based on these four aspects, we analyze below the curricula of French lower and upper secondary school in Mathematics and Computer Science. This provides an insight on the didactical transposition and informs us about the factors influencing the curricula and the levels of codetermination at which they intervene.

Computer Science and Mathematics in lower secondary school curriculum (French *collège*)

In lower secondary school, inside the Mathematics programs for cycle 4, one domain is *Algorithmics and Programming*. The main objective is developing pupils' skills on: "Write, elaborate, and execute a simple program". For this goal, the program enumerates various programming notions (loops, conditions) and programming paradigms (parallel, event-driven…) and suggests to use a programming language with specific properties, very close to the language *Scratch*[4], which is, in fact, used in the accompanying resource (and, thus, used by all the teachers). This follows an international trend for introducing Scratch and block programming in curricula, but the specificity, in France, is its inclusion inside the Mathematics class (Modeste & Rafalska, 2017). This leads to two different kinds of activity in class: on the one hand, algorithmics and programming are widely used to solve Mathematics problems, particularly in algebra and geometry (this is A3); on the other hand, they are also used in projects which can be very far from Mathematics. A4 could be developed at this occasion, but neither the program nor the accompanying resource develop this aspect. A1 is not present (proof and logic have a small place in this curriculum); it is the same for A2 (not any element of mathematics used in Computer Science has been introduced when Computer Science entered the Mathematics curriculum). The epistemological reference remains unclear about the status of these Computer Science contents.

Besides the teaching of Computer Science within Mathematics, contents related to networks and to programming the behavior of objects, machines or systems are taught within Technology. For now, very few interactions exist between teachers of the two subjects and few elements are given for that in the curricula and resources.

We can see various processes involved here, at different levels. At the civilization or society level, international trends seem to have a strong influence, but the transposition is influenced at many other levels like *School* (there are no Computer Science teachers in French schools); *Pedagogy* (the pedagogy of project, suggested in *algorithmics and programming,* is likely to collide with habits in Mathematics teaching in France, despite the efforts made since the 2000s to change them), *and Disciplines* (the potential links between the two disciplines are superficially taken into account). Another element affecting taught knowledge is the national exam at the end of lower secondary school (the Brevet). The contents in the final assessment strongly influence the type of tasks that are proposed by teachers and which domains, sectors and theme are emphasized. The recent

4 https://scratch.mit.edu/

introduction of algorithmics and programming tasks, oriented towards completing or interpreting programs or algorithms because of the paper-and-pencil nature of the assessment, is likely to impact teachers practices and hence the implemented curriculum.

Computer Science and Mathematics in upper secondary school curriculum (French *lycée*)

The current curriculum of Mathematics at upper high school is the one introduced between 2009 (grade 10) and 2012 (grade 12). Algorithmics has been introduced in Mathematics through these programs. In 2017, the Mathematics program for grade 10, and in particular the algorithmic part, has been adapted, as algorithmics is not a new subject anymore for pupils at this level, and in order to make of the transverse content of algorithmics an authentic chapter untitled "algorithmics and programming". The programming language Python has been introduced in replacement of Algobox, a language designed for French secondary school. This process will necessarily be followed by changes in grades 11 and 12 in the next years.

It is worthwhile to compare these two programs and the two corresponding accompanying resources, regarding the interaction of Mathematics contents with algorithmics. From 2009, algorithmics must be a tool for solving mathematics problems and contents (A3), but this role is often restrained to illustrate mathematical concepts and simulate random experiments (Modeste, 2012). There is also a collapsing between the notions of algorithm and program resulting in a collapse between input/output and typing/printing, and raising didactical issues (Modeste, 2012). In the 2017 program and accompanying resource, some of these issues have been settled, and a point of view on algorithms as functions is expected (in order to make this change effective, model subjects for the Baccalauréat have been published with an emphasis on the way algorithms must be structured). We interpret this change as a better consideration of some points of view of Computer Science in algorithmics, with a potential to develop also algorithmics for itself, but at the same time, the examples given in the new accompanying resource are strongly oriented toward application to calculus and numerical methods. This contradiction illustrates the tensions, at *school*, *pedagogy* and *discipline* levels, between teaching some elements of algorithmics for themselves and organizing the whole teaching of algorithmics around mathematics problems.

In 2009 as in 2017, A2 is not developed: although there is a strong potential to support interactions between mathematics or consistent algorithmic problem solving at secondary school (and namely the opportunity of being taught in the same class), very few contents shared by Mathematics and Computer Science are offered in the curriculum (for instance, arithmetic and combinatorics have remained negligible in the new curricula). For A1, although some contents of logic are present in upper high school Mathematics, there is no mention of their links with algorithmics. Finally, A4 is also very little present in these curricula.

Evolution of curricula: Actors, processes and debates. An insight of the noosphere

In the recent development of the curriculum of Mathematics and Computer Science, many actors have been involved or have influenced the situation. The academic societies in Computer Science have been, for a long time, lobbying for the introduction of Computer Science as a Scholar Discipline in France, associations like EPI[5] have been campaigning for introduction of Computer

5 Enseignement Public et Informatique : https://www.epi.asso.fr/, association founded in 1971.

Science in School and have been contributing to the refection on this for a very long time. In the 2010's, many opinion columns were published in French newspapers and the French Academy of Sciences (2013) published a report on the necessity to introduce Computer Science in school in France. Many of the actors of these institutions (mostly influential researchers) have also been invested in curriculum design (for ISN or the cycle 4 curriculum of Computer Science) but also in developing reflections on what should be and should contain a teaching of Computer Sciences, and in developing resources and textbooks.

The main debates about the future of teaching Computer Science are now about teachers and teachers' training, and the question of bringing together or not the different pieces of Computer Science present in the curriculum. The creation of a Computer Science option in the competitive exam for recruiting Mathematics teachers let think that we head towards a teaching of Computer Science by Mathematics teachers, or more likely Mathematics/Computer Science teachers. This debate is not closed at all and other hypotheses are also likely. In particular, some defend the close connection between Computer Science and Information Technologies (Tort & Bruillard, 2010) that would be likely to increase the distance between Mathematics and Computer Science. In any case, enhancing interaction between Mathematics and Computer Science in school will remain an important issue.

CONCLUSIONS AND PERSPECTIVES ON MATHEMATICS CURRICULUM REFORMS

Computer Science is still looking for its place in the curricula, and questions the territories of other scientific disciplines. As we have seen, the interactions with Mathematics are important in the scholarly knowledge. This leads to important curriculum design issues and questions the teachers' needs in order to handle the intended curricula. The perception of these changes in the curricula by Mathematics teachers could be enlightening for understanding their needs and representations of Computer Science, in order to understand the effects on the implemented curricula.

In the noosphere, many actors influence the didactical transposition of Computer Science which has a direct impact on Mathematics curriculum in the French educational context. In our view, an important issue is the place that a curriculum can let to the interactions between Mathematics and Computer Science (we have seen that these interactions are very limited at present). We have also seen that the French curriculum of Mathematics and Computer Science is influenced by two tendencies: international trends and French specificity.

We view the relation between Mathematics and Computer Science in French secondary curriculum as a prototypical example that illustrates the difficulties faced in STEM education to design curricula that take into account the disciplines themselves without neglecting their interactions (and reciprocally), and that address institutional constraints like the profile of the existing community of teachers.

To study the curriculum issues, we think that ATD and didactical transposition can be relevant.

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SCHOOL MATHEMATICS CURRICULUM REFORMS IN VIETNAM IN THE CURRENT PERIOD: CASE OF TEACHING MATHEMATICS MODELING

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Vietnam is currently in the process of fundamental reform of education. This article presents the current mathematics curriculum reforms that are designed to develop learner competencies. One of the issues is focused on interdisciplinary teaching and associated with solving problems in life. We will clarify the views of construction of the mathematics curriculum in Vietnam in the coming period as well as the challenges of implementing the reform program from the current situation of mathematics teaching.

1. DIRECTIONS OF MATHEMATICS CURRICULUM REFORMS

1.1. The overall program

The goal of education reform after 2015 is defined by Resolution 88/2014/QH13 of the National Assembly:

> To renovate general education curriculum and textbooks in order to make fundamental and comprehensive changes in the quality and efficiency of general education; […]; It contributes to transforming the education system into a comprehensive education in terms of quality and competence, harmony of the mind and body, and the best potential of each student.

From that innovation goal, one of the views of the program is:

> General education programs ensure the development of learners' quality and competence through educational contents with basic and practical knowledge and modernity; harmony of mind and body; focus on practice, apply knowledge to solve problems in learning and life; highly integrated in the lower classes, differentiation in the upper classes; through the methods and forms of educational organizations that promote the activeness and potential of each student, the methods of examination and assessment are in line with the educational objectives and educational methods to achieve these objectives.

The new curriculum is designed to create and develop students' core competences:

> a) Common competences for all subjects and educational activities that contribute to the formation and development: self-control and self-learning competence, communication and cooperation competence, problem solving and creativity competence.
>
> b) Professional competences are formed, developed primarily through a number of subjects and educational activities: language ability, computing power, natural and social learning competence, technology, computer skills, aesthetic competence, physical competence.
>
> In addition to the formation, development of core competencies, general education programs also contribute to discovering and fostering special abilities (talents) of students.

According to program planners, the major innovations in the new curriculum are:

- Access to competence development.

- Integrated education.
- Two periods of education curriculum:

+ Elementary education period from 1st to 9th grade: basic education, general education as the foundation for the disseminated education.

+ The career-oriented differential education from grade 10 to 12.

- One program with many textbooks.
- The decentralization of educational program implementation is based on the view that the school is a basic unit.
- Organize experiential activities.

1.2. Mathematics program

With the above educational orientation, the authors of the mathematics program determined that the mathematics program is designed to help students achieve the following major goals:

- Form and develop core competences (self-control and self-learning competence, communication and collaboration competence, problem solving and creativity competence) and mathematical competence (thinking and mathematical reasoning, mathematical modeling competence, mathematics problem solving ability, mathematical communication ability, ability to use tools for mathematics learning).

- Having basic and essential mathematics knowledge and skills; develop interdisciplinary problem-solving ability between mathematics and other subjects such as Physics, Chemistry, Biology, Geography, Informatics, Technology ...; Provide opportunities for students to experience, apply mathematics to real life.

- Form and develop common qualities (patriotism, compassion, hard work, honesty, responsibility) and the qualities that mathematics brings (discipline, persistence, initiative, flexibility, independent, creative, cooperative, adaptable to change and facing difficult challenges that arise in practice, habits and methods of self-study, excitement and confidence in mathematics).

- Have a general overview of mathematical disciplines for students to have career-oriented backgrounds, as well as have the minimum ability to self-study mathematics problems throughout life.

Some key points in the construction of the mathematics curriculum emphasize the application of mathematics in life and interdisciplinary with other subjects as mentioned below:

- Consciousness "math for each person", meaning everyone needs mathematics, but each person can learn mathematics in a way that suits his or her personal preferences and abilities. At the same time pay attention to the formation of good qualities such as diligence, hard work, persistence, careful in the work.

- The content of the Mathematics Program should be strengthened in practical application, linked to real life or other subjects, linked to the modern development trend of economy, science, social life and Global issues (such as climate change, sustainable development, financial education, etc.).

- Mathematics program is integrated around three circuits of knowledge: Number and Algebra; Geometry and Measurement; Statistics and Probability.

- Mathematical knowledge is exploited and used in other disciplines such as Physics, Chemistry, Biology, Geography, Informatics, Technology .... These subjects have contributed to strengthen the knowledge of mathematics, as well as contribute to training students the ability to apply mathematics in life.

2. MATHEMATICS MODELING IN TEACHING MATHEMATICS

From the point of view of constructing the mathematics curriculum above, we find that the application of mathematics to the problems of real life as well as other subjects is particularly emphasized. Indeed, in the drafting of the mathematics curriculum, after every subject of knowledge there is always the demande to practice the solving of real problems related that knowledge.

One of the core competences of mathematics is defined as the ability to model mathematics. The draft of the mathematics program states:

> Mathematical modeling capacity is demonstrated by the implementation of actions:
>
> - Use mathematics models (including formulas, equations, tables, graphs ...) to describe situations in real-world problems.
>
> - Solve mathematics problems in the established model.
>
> - Demonstrate and evaluate the solution in the real context and improve the model if the solution is not appropriate.

Accordingly, some of the highly usable content is taught in elementary school and extends through high school such as probability and statistics while in current program probability is only taught in grade 11 and statistics are present implicitly in class 5, which is officially taught in grades 7 and 10. In particular, in the current program, statistics is a content does not appear in the final exam questions as well as high school national examination. Therefore, most grade 10 teachers skip this section or teach it very sketchy.

Combined with the above objectives, the new mathematics program emphasizes experiential learning activities for students:

> The mathematics program at each level also provides adequate time to conduct mathematical experiential activities for students such as: conducting topics and projects in mathematics, especially topics and projects on the application of mathematics in practice; organizing mathematical games, math clubs, forums, seminars, math competitions; making reports (or journals) on mathematics; explore mathematics training and research facilities, communicate with mathematically gifted students and mathematicians ... These activities will help students to apply knowledge, skills, attitudes have been accumulated from mathematics education and personal experiences in the creative realities of life; develops students' ability to organize and manage activities, self-awareness and self-efficacy; helping students to identify their own strengths and weaknesses in order to orient and choose their careers; building some basic competencies for future employees and responsible citizens.

In particular, to prepare manpower resources for the Industrial Revolution 4.0 - the era of technologies such as virtual world, internet connection, 3D printing, artificial intelligence, the promotion of STEM education is taking place very vigorously in Vietnam. Since 2011, the Ministry of Education and Training has collaborated with the British Council to pilot a "STEM Lab in English" with four subjects: Robotics, Information Technology (IT), English and design, building intelligent devices at 14 schools in Hanoi, Ho Chi Minh City, Da Nang. Since 2012, many private education institutions have STEM education initiatives. In 2015, the Ministry of Science and Technology of Vietnam and the STEM Alliance organize the first STEM festival, followed by many similar events nationwide.

In the academic year 2015-2016, the Ministry of Education and Training has promoted the development of STEM education in secondary schools. Specifically, Official Letter No. 4325 / BGDDT-GDTrH dated 01/09/2016 on guiding the implementation of the task of secondary education in the school year 2016 - 2017 emphasized the renovation of teaching methods, in which pressing strong:

> Continuing to emphasize the integrated education of science - technology - engineering - mathematic (STEM) in the implementation of the curriculum in general education in related subjects. Piloting STEM education at selected schools.

3. ACTUAL SITUATION IN THE CURRENT MATHEMATIC CURRICULUM

In the current curriculum and textbook, mathematical modeling as well as interdisciplinary integration has not been emphasized. Specifically, teachers are trained in monographs, textbooks subjects are written independently of each other so there is no close link between subjects.

The study of current curriculum shows that the problem of applied mathematics in practice has also been addressed. For example, the high school matheamtics program emphasizes this view:

> The program is constructed and developed in the following way:
>
> [...]
>
> + Selecting basic, up-to-date, practical and systematic mathematics knowledge in the direction of streamlining, suitable with students' level of knowledge, demonstrating interdisciplinary and integrating educational contents, demonstrates the role of mathematics.
>
> + Strengthening the practice and application, teaching mathematics associated with the practice.
>
> [...] (From High School Math program, 2006)
>
> The first goal of the program is to achieve the meaning, application of mathematics knowledge to life, to serving other subjects.
>
> (From Algebra and Analysis program grade 11, 2006)

Thus, the practical application of mathematics and interdisciplinary in which mathematics plays a tool role is explicitly mentioned in the current program. However, reviewing textbooks shows that modeling teaching is not focused.

> The problem of modeling is not emphasized in textbooks and programs in Vietnam. We only find traces of modeling in the application of mathematical knowledge to some of the problems that arise from reality. In high school mathematics textbooks, these exercises are very rare and are often placed in the readings section or at the beginning of some chapters that lead to new knowledge.
>
> (Nguyen Thi (2011)

Thus, the textbook does not show the spirit of "strengthening practice and application, teaching mathematics associated with the practice" as the program mentioned.

We will consider a specific topic as the trigonometric function. This is a topic where mathematical modeling has a good environment for survival as it relates to many fields such as physics, astronomy, etc. Nguyen Thi Nga (2011) pointed out in vietnamese high school curriculum the presence of the contents related to trigonometric functions is described as follows:

Classe	Mathematics	Physics
10	Trigonometric values of any angle, trigonometric circle	Circular motions
11	Trigonometric function y = sin x, y = cos x, y = tan x, y = cotan x	
12		Harmonic oscillation, pendulum Sound, sinusoidal wave Alternating current

Table 1. Trigonometric functions in math and physics textbooks

Thus, in terms of program structure between Mathematics and Physics, we find a reasonable arrangement between the contents of the two disciplines. Specifically, circular motions are associated with the trigonometric circle is mentioned in grade 10. Next, the trigonometric function is studied in Mathematics in grade 11 and its applications in Physics like waves, sound, harmonic oscillation, ... present in grade 12.

Nguyen Thi Nga (2011) has shown in the topic of trigonometric functions the mathematics textbooks cover very few real-life problems. In addition, in these problems, the mathematics model is always given in the announcement. Thus, the work of the student is just to work with the mathematics model.

> Modeling teaching, especially the modeling of recurring cyclical phenomena narrows down in teaching using models. In particular, if the function belongs to the model, it will be presented in the assignment as soon as the actual introduction needs to be modeled.
>
> (Nguyen Thi (2011))

For example, consider the "real" problem after the trigonometric equation in the 11th grade textbook:

17. The number of hours of sunlight in the city A in north latitude 40 degrees during the day t is given by:

$$d(t) = 3 \sin\left[\frac{\pi}{182}(t-80)\right] + 12, t \in Z \text{ and } 0 < t \leq 365.$$

a) The city A has 12 hours of sunlight on which day of the year?

b) On which day of the year the city A has at least hours of sunlight?

c) On which day of the year the city A has the most hours of sunlight?

Figure 2. The "modeling" exercise in textbook Algebra and Analytic grade 11

The work of students in this situation is purely mathematics work (solving equations, selecting solutions). The astronomical phenomena of the variability of the period of day-to-day sunlight provide a reason for mathematics work in the harmonic oscillator model. This mathematics work in the best case allows to show how astronomical studies receive real results but does not allow entry into the phenomenological modeling itself.

For example, what is latitude data for? Is the latitude of the model to be built? How to construct the function d(t)?

Our research results raise the question of the ability to teach interdisciplinary physics - mathematics built around situations that contain a mathematics modeling of the phenomena studied in the subject. What mathematics and physics knowledges can be generated from a non-mathematics modeling process? Which praxéologie (math-physics mix) does the institution in high school need to build to teach mathematics modeling? What is the problem of teacher training, how should testing and evaluation be conducted to teach modeling that there is a real "living area" in mathematics instruction? These are questions that need to be addressed in order to promote the teaching of modeling and teaching by modeling in high school.

4. CHALLENGE IN IMPLEMENTING THE NEW PROGRAM

In order to implement the new general education curriculum, it is essential to change the contents of the curriculum, the teaching method and the method of examination and evaluation. In Vietnam, the assessment of students greatly influences the teaching process. Therefore, in order to teaching mathematics modeling and teaching by modeling can be "live", it is necessary to change the way of evaluation. Only when this content appears in the major exams such as final exam, high school national examination, they are taught thoroughly.

Moreover, how to assess students' competences is a matter of concern. Teachers should be equipped with assessment skills, trained, and guided in how to develop assessment scales to accurately assess student performance.

In addition, how to get interdisciplinary teaching situations, mathematical modeling teaching to form this competence for students is also a challenge. In order to be able to build such teaching situations, teachers need to have a broad understanding not only of mathematical knowledge but also of mathematical applications in practice and in other disciplines.

Our surveys show that mathematics teachers do not incorporate interdisciplinary knowledge (eg, physics) into their lesson because they do not well understand physics, they find that it is difficult to design teaching situations. In addition, mathematics and physics teachers in the high school rarely exchange on interdisciplinary subjects. They only talk about students (Nguyen Thi (2017)).

In order to promote interdisciplinary and modeling teaching in high school, we think it is necessary to promote some of the following solutions:

Training teacher about interdisciplinary and modeling teaching

It is necessary to organize teacher training for interdisciplinary and modeling teaching so that they can understand the theoretical as well as design interdisciplinary teaching situation. From there, they can apply to their teaching practice.

For pedagogical schools, these theoretical elements should be included in the teaching of the students and they practice them in the lesson of pedagogy. Specifically, the training program should be designed so that pedagogical students have enough basic knowledge of other subjects to be able to teach interdisciplinary subjects. From the subject of mathematics associated with other subjects, it is necessary to foster pedagogical student’s knowledge of these subject so that they have enough confidence in the design of the situation and organize teaching.

Set up interdisciplinary themes, teaching situations associated with modeling as a source for teachers to refer and use.

The development of interdisciplinary themes, teaching situations associated with modeling and teaching organization requires a lot of time and effort. In addition, it requires teachers to have a broad and deep understanding of the relevant knowledge. Thus, for teachers to teach interdisciplinary, modeling teaching, the development of interdisciplinary integration topics and interdisciplinary integration situations, modeling teaching situations for teachers to refer and use is really needed.

Organizing lesson study for mathematics teachers and other subjects teachers

To design and organize interdisciplinary teaching and modeling, the teachers (eg mathematics teachers) can integrate many different subjects into their own lesson. This requires teachers to understand the knowledge of those subjects (outside of mathematics) so that they can apply well in their lesson. In addition, many teachers in many subjects work together to develop and teach an interdisciplinary lesson on the same subject. This requires the teachers of different subjects to work together, discuss their subjects and their interdisciplinary subjects, and compose the topic to teach. The different subjects may be more closely linked and richer with this method of combining knowledge.

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MAKING INFORMED DECISIONS: TECAHERS' INTERACTING WITH CURRICULUM RESOURCES

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It is quite common for teachers to supplement the curriculum with web learning resources. To design curricular sequences skilfully, teachers need to be sensitive to aspects of curricular coherence, such as sequencing that avoids gaps in the mathematical progression, consistent and balanced handling of mathematical objects as well as coherence with national curricula. Currently there is a lack of tools to support such notions of coherence, both in-design and in-use. One of the key aspects that educators need are means to communicate and describe pedagogical moves and pedagogic materials in an accessible, understandable way. We suggest a set of methodologies, based on a pair of technological tools, for studying the voice and balance of a collection of learning resources. The first is a tagging tool that can be used to associate didactic metadata with individual learning resources. The second tool is a "dashboard" for visualizing and navigating a tagged collection or textbook, representing didactic aspects of the curriculum. Our findings suggest that tools should offer ways to perform large scale studies to provide research finding that will be informative to the variety of designers '/practitioners communities.

INTERACTING WITH CURRICULAR RESOURCES

Growing expectations that teachers integrate technology in their instruction, along with the emergence of "dynamic" e-textbooks (Pepin, Gueudet, Yerushalmy, Trouche, & Chazan, 2015), are placing teachers in the role of co-designers of the curriculum that they enact (Remillard, 2009). Technological advancements, mainly of Web2.0 participatory tools and norms that seem to be consistent with constructivist pedagogies, pose a challenge to the accepted function of the textbook. Teachers' interactions with textbooks are changing. Studies such as the TIMSS report (Hooper, Mullis, & Ma, 2015) show that though teachers in most countries still rely on a textbook for most of their curricular decisions, it is quite common for teachers to supplement the curriculum with learning resources available on the internet. In some contexts, teachers do not have a single textbook to rely on, and are responsible for the construction of teaching sequences. Teachers all over the world create and modify teaching materials, search through professional materials available on the Web, and share their repositories and ideas through on-line social networks. However, the web does not hold semantic information about content, which is necessary if one wishes to search for learning resources according to subtle epistemic or didactic features of activities. An important question to explore is whether teachers' tasks "to critically analyze curriculum materials, to examine the mathematical and pedagogical assumptions implicit in their design, and to consider how curriculum materials might be read, used, and adapted" (Remillard & Bryans, 2004, p. 386) will take on a different dimension in the new era of interactive textbooks and digital learning resources. Examples of teachers' participation in creating their own materials include the French mathematics teachers' online association, known as Sesamath, which is dedicated to the design and sharing of teaching resources (Gueudet & Trouche, 2012). Created over a decade ago, *Sesamath* has rapidly grown into an online community of practice. *BetterLesson* (http://betterlesson.com/home) is an open platform for constructing and sharing OER among the community of math teachers. Contributed ideas for lessons, reflections, interactions and tasks are mapped and can be sorted according to the topic defined by the Common Core Math State Standards and by the qualities of the contributing teacher. Another unique setting is the Integrated Mathematics Wiki-book Project (Even & Olsher, 2014) that invites teachers to collaborate in modifying the textbooks they use in their classes and to produce, as group products, wiki-based revised textbooks that are suitable for a broad student population. Among the changes the teachers suggested stands out a suggestion to incorporate organizing tools in the textbook to better communicate different ideas (e. g. what is the textbook core?) as well as making textbook contents more accessible (Olsher & Even, accepted). Even & Olsher's study (Even & Olsher, 2014) aimed at exploring how to expand the conventional relationships between teachers and curriculum developers, and expose the textbook developers to ideas about the textbook contents that originated from the teachers not just as evaluators, but also as professionals who can equally contribute to textbooks from their experience, their accountability to students learning, and their wisdom of practice. We acknowledge the fact that there might be conflicting aspects between the teachers experience and knowledge and the developers ideas and pedagogical stance, and invite an even-grounded discussion about these aspects, taking into account the fact that both sides could contribute. One of the key aspects that teachers and educators need in order to interact in a descriptive approach are means to communicate and describe pedagogical moves and pedagogic materials in an accessible, understandable way. We believe that solutions and ideas from the domain of data analytics should be explored in this effort (Cooper & Olsher, 2018), in ways that could provide novel methods for exploration of textbooks and other learning objects repositories.

INFORMED CURRICULUM DESIGN: TAGGING, BALANCE & SEQUENCE

Three key actions should guide any assemblage of curricular sequence, modification of an interactive textbook and actions relate to the use of a curricular sequence:1) Recognizing aspects of affordances of metadata categories that characterize the resources, Developing 2) an awareness of the balance among the learning objects and, 3) an awareness of the rationale of their sequencing. Useful organizing tools would help researcher study, as well as expose teachers to otherwise hidden aspects of the coherence of the textbook, and it would help them become involved, intelligent members in a participatory community focused on teaching with open, interactive, math textbooks. Understanding the scheme underlying coding of tasks, grasping and also describing the designed coherence in terms of the balance across variety of aspects that characterize the learning objects, and viewing the coherence of the sequence in use, is an essential but complicated mission that any teacher wishing to personalize her own resources would be confronted with. We suggest a set of methodologies, based on a pair of technological tools, for investigating the voice and balance of a collection of learning resources. The first is a tagging tool that can be used to associate didactic metadata with individual learning resources. The second tool is a "dashboard" for visualizing and navigating a tagged collection or textbook, providing an interactive visual representation of didactic aspects of the intended curriculum.

Tagging resources

The question that guided our design-based research was: What aspects of learning resources should be tagged to support teachers' needs as co-designers of the curriculum that they enact? We recognize that teachers' perspectives, influenced by instructional practices of supplementing conventional textbooks, may be different from the perspectives of researchers concerned with the coherence of a curriculum that is substantially co-designed by teachers. Our goal is not to reconcile these conflicting perspectives. Rather, we aim to develop tools that will support the emergence of new practices that draw productively on both perspectives. We describe design-based ideas related the development of a tool for communal tagging of learning resources – assigning didactic metadata to tasks – which can then be used by individuals to search for tasks in a didactically-informed manner. (Adopted from Cooper & Olsher 2018).

Task classification is an important but daunting part of any sequencing typology. Burkhardt & Swan (in Margolians 2013) offer a multidimensional framework for Balance-Based Task Design. The recent study of Huntley & Terrell (2014) further suggests that making the classification apparent to the user of the textbook is challenging. In the US, the CCSS Mathematics Curriculum Materials Analysis Project (http://www.mathedleadership.org/ccss/materials.html) was developed in order to "provide educators with objective measures and information to guide their selection of mathematics curriculum materials" (page 4). For the initial version of EDUMAP - our communal tagging tool- four main metadata categories were designed . *Curricular coverage*: We follow Schwartz et al. (1995) in our conceptualization of the mathematics curriculum: Categories of mathematical and general skills (e.g. modelling, manipulating, inferring), enacted in four main mathematical domains (number/quantity, shape/space, pattern/function, chance/data), involving a variety of operations on and with mathematical objects (e.g. numbers, functions, shapes). A balanced curriculum should cover all relevant combinations of skills, objects and operations. These categories support a modular approach to curricular design; tagging the mathematical nature of the task, and avoiding categories such as grade and difficulty levels, imply that tasks can be used in many different contexts. *Mathematical expressivity and curricular specificity* of a task (Sinclair & Jakiw 2005) refer to the richness of mathematical ideas, representations, and approaches on one hand, which often comes at the expense of the ease with which a task fits a specific curriculum on the other hand. *Representational modality* of mathematical objects: Yerushalmy (2005) has demonstrated the importance of linked multi-modal representations of mathematical objects (e.g. functions) in interactive learning resources. Each modality (verbal, numeric, symbolic, graphic) is tagged separately. *Resource usage*: We attend to two central aspects of enacting resources that are expected to be relevant for didactically-informed searching: curricular “role” (e.g. opening a topic, practice, homework, assessment, enrichment), which is relevant for sequencing tasks, and “class arrangement” (e.g. whole class, individual, pairs, groups). Another set of categories attend to teachers‘ enactment of curricular elements: *Curricular “role”* can be tagged as one of the following: opening a new topic, practice, homework, assessment, enrichment. *Class arrangement* can be tagged as one of the following: whole class, individual, pairs, groups. Taggers can specify which of three *types of technological aides* they would allow students to use: numeric manipulation (e.g. Excel), graphic representation (e.g. graphing calculator), symbolic manipulation (e.g. computer algebra system). And taggers can specify the *role of technology in explanations of students’ reasoning*, namely: explanations based only on technology are permitted, an explanation that relies NOT ONLY on technology is required, a non-technological explanation is required.

Practices of tagging and selecting learning resources

Though it was agreed that tagging, as a human endeavor, is inherently subjective, we considered some categories of metadata as more objective (e.g. mathematical topic, representations) than others. Objective categories were considered most reliable as a basis for searching, since the tagger and the searcher are likely to agree on how a task should be tagged. Nevertheless, often teachers recognized affordances in tagging contextual data, which is inherently subjective. We briefly summarize aspects highlighted in our design based research (Cooper & Olsher, 2018) 1) Emergent hybrid practice for searching and selecting tasks: Metadata was seen as a way to narrow down the search, eliminating irrelevant items, yet the final phase – selecting from among the search results – was based on personal preferences that cannot always be articulated in terms of didactic metadata. With this in mind, filtering on a range of grade levels, though inconsistent with the researchers' modular approach to learning resources, may nevertheless be an effective means of reducing the number of relevant resources that need to be reviewed. 2) Importance of instructional context for tagging and selecting tasks: While the researchers tended to discourage tagging contextual data, viewing it as inconsistent with a modular approach to curriculum design, teachers tended to embrace contextuality. 3) Sharing meanings within communities: Professional development may help teachers conform to the designers' curricular discourse our work suggests that tools should support the emergence of such communities. A more symmetrical approach, that does not privilege the designers' discourse, is to view tagging as a community endeavour. Real communities (teachers in a particular school) or virtual communities (teachers whose tagging one choose to follow) may develop shared meanings for keywords, and come to agree on tagging norms through joint work. 4) Tagging and the importance of "quality": Moving from "wisdom of trusted individuals" to "wisdom of crowds" requires careful deliberation on the representation of multiple-tagging for a single resource. Currently, teachers concerned with quality rely on wisdom of crowds ("likes", ranking), on the reputation of proven sources (developers, repositories), or on the recommendation of a trusted peer. Such a tool mediates – through particular categories of metadata – the action of an individual (e.g. a teacher) in her interaction with curricular material.

More generally, tagging resources before they are actually enacted can be seen as the first step in a teacher's interpretation of the designer's intentions. Analysis of teachers' tagging of curricular resources may provide a realistic view of how they are likely to enact intended curricula. Such an analysis could be automated to a great extent, and thus be feasible at scale.

Curricular balance

To design curricular sequences skilfully, teachers need to be sensitive to aspects of curricular coherence, such as mathematical correctness, epistemological stance to mathematical topics, sequencing that avoids gaps in the mathematical progression, consistent handling of mathematical objects, and consistency with national curricula (Gueudet, Pepin, & Trouche, 2013). Furthermore, this coherence-of-design needs to be maintained in the curriculum that is eventually enacted in classrooms (what Gueudet et al. call coherence-in-use, ibid.). As teachers, especially, have expressed a desire to personalize digital textbooks, we have identified a critical need for them to make informed decisions - decisions that not only reflect their own ideas and views, but also maintain the approach set by the national curriculum or the textbook's author. Currently there is a lack of tools to support such notions of coherence, both in-design and in-use. In response, we experiment and study a tool which provides a window into the underlying structure of the collection of resources. The tool's dashboard supports filtering of tagged resources according to values of metadata. By correlating various didactic aspects, it is possible to visually represent the balance of the intended curriculum represented in a textbook or any other sequence of learnign objects. Figure 1 is a screenshot of the balance representation tool, developed on an open platform for data analysis (Keshif LLC). Locking on a value of the metadata (in this case the value medium in the category duration) highlights the relative prominence of these tasks within categories of the collection.

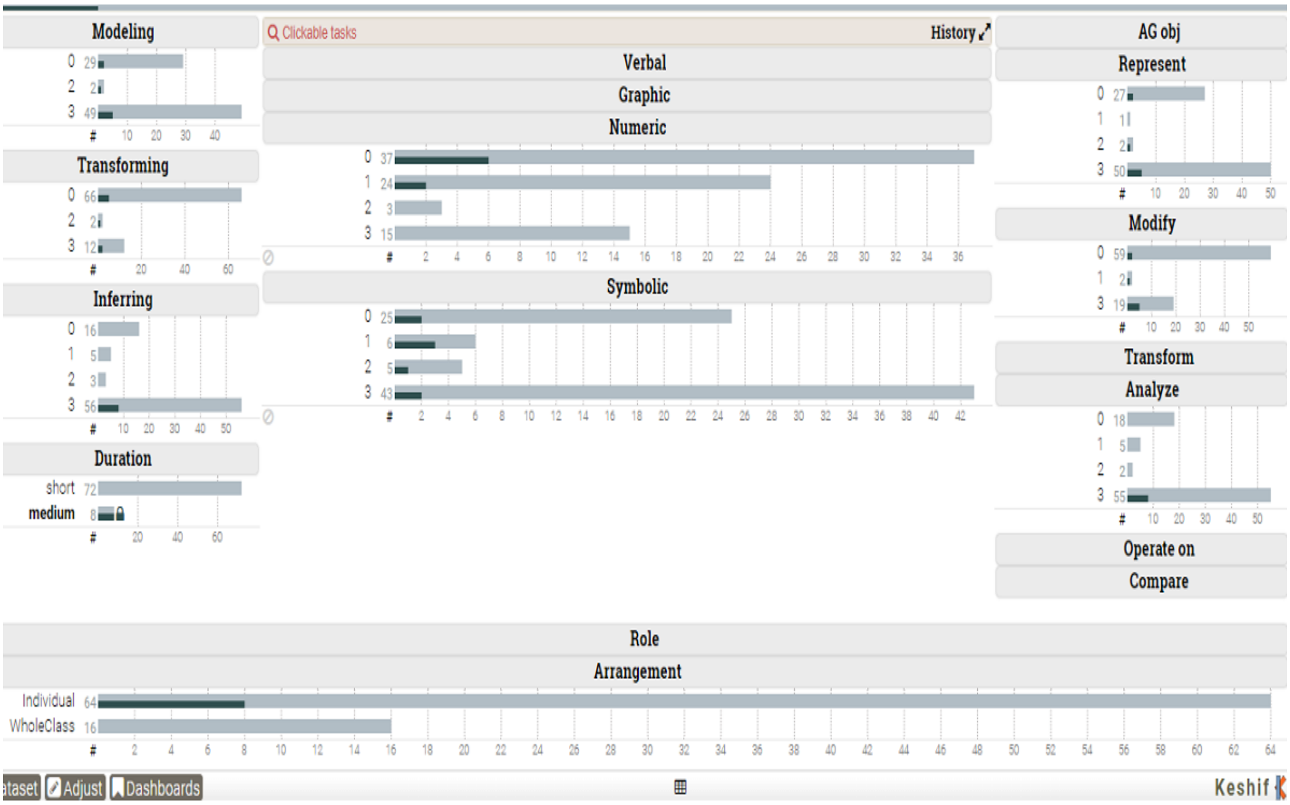

Figure 1. *Dashboard representing balance of the sequence.*

In our study, we have shown how frameworks of coherence (of-design and in-use) provide insights on the ways in which participants (practicing teachers and teachers-learners) made use of the tool. Our findings suggest that:

- The dashboard supports filtering with the goal of reducing a collection of tasks to a manageable size. This allowed participants to make use of new search schemes (filtering on metadata), while retaining the familiar and productive routine of reviewing a small number of tasks and choosing the most appropriate from among them. This also allowed them to apply idiosyncratic considerations along with the prescribed categories of metadata.
- The activity encouraged participants to make explicit the tacit considerations that they apply to curricular decisions.
- Comparing the search criteria across tasks invited reflection on subtle differences among tasks for opening a topic, for practice, and for assessment.

Moreover, recognizing that a tagged learning resource is not direct evidence of the nature of the resource, but rather of the tagger's interaction with it, we have investigated what we can learn about individual taggers through analysis of their tagging. Our analysis of four individual taggers, though relying on a limited number of tagged tasks, revealed some individual "profiles" of interaction with a textbook. It suggests that the categories of metadata sensitive enough to reveal patterns in the textbook that transcend idiosyncrasies of individual taggers, while at the same time highlighting characteristics of individual taggers. Following data analysis, we sought to explore the significance of the patterns that we had found in the structure of the textbook. Findings were presented to the author of the book, who was then interviewed in order to characterizing findings with respect to the intentions and constraints behind the design of the textbook. Such uses of the tagged metadata represented with the balance platform demonstrate the possible value to researchers, to Ministry officials – in the process of reviewing and authorizing textbooks – and to district and school professionals – in deciding which textbooks are a good fit for their local approach and teaching philosophy.

Curricular sequences

In curricular sequences we refer to how the mathematical content is organized for learning over time (along Remillard's definition 2018). Sequence may describe a planned sequence of lessons or a designed sequence of tasks in a book. It is an important mean of communicating the pedagogical approach. Figure 2 includes screenshots of the sequence representation tool, developed on the same open platform (Keshif LLC). The tool shows the progression over time of each chosen category of metadata.

Representation: Graphic

Representation: Numeric

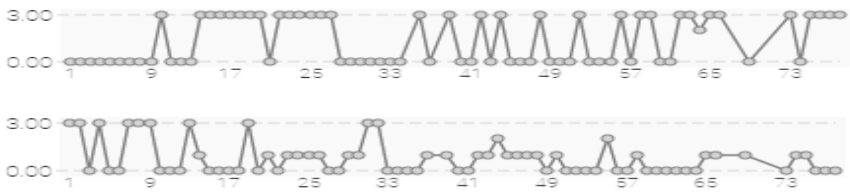

Figure 2. *Representing the sequence of tasks from 4 perspectives, corresponding to 4 categories of tagged metadata.*

In Figure 2, we see a seauence of over 70 tasks that were tagged and enacted by an Israeli school teacher (Cooper, Olsher & Yerushalmy, accepted). The sequence, describing tasks in the topic of Analytic geometry, could suggest that the first part of the sequence (namely the first 9 tasks) did not include any graphical representation, while using numerical representation quite often. As the teaching sequence progresses, there is more use of tasks with graphical representation compared to tasks with numerical representation. By using this method of representation on different characteristics over a sequnce of tasks from a textbook or enacted lessons there is a potential for understanding and representing various pedagogical and didactical desicions that are either concious or arbitrary, yet might influence the student's learnign experience.

DISCUSSION: MAKING INFORMED CURRICULAR DECISIONS

Teachers nowadays are involved in a range of interactions with curriculum materials and their role is shifting from enacting an intended curriculum designed by others to co-designing the intended curriculum. In this state of affairs, it is important that teachers be able to "perceive" the intended curriculum and the insights of the author and be able to communicate on and with the materials that they are co-designing. One of the key aspects that teachers and educators need in order to interact in a descriptive approach is means to communicate and describe pedagogical moves and pedagogic materials in an accessible, understandable way. In-practice researchers employ variety of useful methodologies for investigating the range of teachers' interactions with curricular material, but they are difficult to employ at scale in order to obtain a realistic picture of the ways in which curricula are interpreted and enacted. That is especially true in the era that technology introduces dramatic changes. We believe that solutions and ideas from the domain of data analytics should be explored in this effort. The categories of metadata provide a methodological framework for describing curricula. Tagging by a teacher can be seen as representation of a planned/enacted curriculum. Aspects of individual teachers' tagging, and in particular aspects that are invariant across a variety of instructional resources, provide a window on their pedagogical design capacity. Tagging by the author of a textbook or the designer of the resource represents the intended curriculum. The search for regularities in the tagged corpus that we studied could, in principle, have been carried out automatically. Yet, we feel that the visual and interactive representation in the Keshif dashboard is significant for at least two important reasons. Firstly, our findings suggest that while the more direct consequences from the analysis of the metadata could probably be found in the teacher-guide other insights were logical consequences of the author's principle, not explicitly stated and had revealed blind spots or oversights in the author's work. It is an example of how our methodology provides means and access for researchers and practitioners: For the author to explicate tacit consequences, for the teachers to gain curricular insights and for policy makers' decisions. Secondly, we acknowledge that traditional professional development may help teachers conform to the designers' curricular discourse. However, we argue that a more symmetrical approach that might better serve the evolving state of design is to view tagging as a community endeavor. Real or virtual communities may develop shared meanings for keywords, and come to agree on tagging norms through joint work. This suggests that tools should support the emergence of such communities and that methodologies should offer ways to study and provide research finding that will be informative to the variety of designers'/practitioners communities.

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IDEAS TO WORK FOR THE CURRICULUM CHANGE IN SCHOOL MATHEMATICS

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We refer to a variety of ideas that should be linked and managed by a mathematics teacher, either active or in training, either a coordinator or an educative administrator, either a counselor or a researcher, to properly interpret the components and understand the conditions that occur in a process of change at the curriculum of school mathematics.

We introduce two different and complementary approaches to curricular reflection. The first focuses on certain basic notions and reveals structural aspects of a general concept of curriculum. The second identifies categories and didactic contents of school mathematics, as analytic tools required for designing a mathematics curriculum, describe its components, interpret their changes, and manage their development.

Keywords

Conceptual Analysis; Curricular Levels and Dimensions; Didactic Content; Didactical Analysis Categories

BASIC AND GENERAL NOTIONS

School mathematics: a broad notion that refers to mathematics considered as a teaching and learning subject that is transmitted through the education.

Educational system: technical concept, which includes a set of social institutions responsible for the education of children, adolescents, young people and professionals, to initiate them in their cultural heritage, insert them socially and promote their personal, civic and professional development.

Mathematics education: set of ideas, knowledge, processes and attitudes involved in the intentional selection of mathematical contents that will be developed by the educational system giving place to the construction, transmission and evaluation of the school mathematics. Teachers' activity and their professional formation processes are also critical and decisive part of mathematics education (Steiner, 1980; Howson, Keitel & Kilpatrick, 1983; Romberg, 1992).

GOALS OF THE SCHOOL MATHEMATICS

Education is an intentional activity, guided by general goals. Studying the purposes of mathematics education is asking the question why is mathematics part of the compulsory education? It consists in looking for arguments and finding explanations about the fact that they are part of a shared cultural legacy, a social and educational heritage that citizens around the world receive from their ancestors. These questions and answers about the purposes and aims are fundamental for education. It is common to identify four issues, which classify the arguments about the goals of mathematics and that establish several dimensions to their study: Formative, Cultural, Political, and Social (D'Ambrosio, 1978; Niss, 1996).

Formative purposes

Educational teachers and experts aim to develop students' mathematical thinking, by means of determining facts, establishing relationships and deducing consequences, to enhance both reasoning and the capacity for symbolic action.

The formative or cognitive aims suggest the design of expectations and development of purposes as following: promote the capacity for expressing, elaborating and appreciating patterns and regularities, and their combination to achieve efficiency and coherence; reinforce the explanatory value of the mathematics that makes the world intelligible and endow it with meaning; participate in the construction of student's own knowledge (Freudenthal, 1981; Schoenfeld, 2011).

Cultural purposes

The goal here is to emphasize the academic function of each school system and highlight the transmission of a complex cultural heritage, essential in every society. The mathematics that forms part of the core curriculum is based on the values of the culture and society in which they are instituted. In the compulsory educational system of any country it is essential to teach mathematics, since mathematical knowledge is an important part of the culture of every society.

Most of school mathematics content expresses clear control mechanisms for behavioral management, as they address plans, formulas, rules, strategies, procedures and instructions. Mathematical objectives guide human knowledge to communicate, interpret, predict and conjecture. These objectives lead human thinking and attitudes: they provide patterns of rationality and well-founded criteria for individual behavior, help in decisions making and support our moral values (Weil, 1949; Whitehead, 1957).

Public purposes and policies

These goals encourage ethical-founded working rules such as cooperation, exercise of criticism, collaborative discussion, defense of one's own ideas, and joint decision-making. They develop scientific work by searching, identifying and solving problems and mobilize students' communication skills, stimulate justification for intellectual effort and a job well done.

The dissemination of democratic values and social integration, the exercise of criticism and the effort for communicative action are the key in planning of school mathematics. A critical vision of mathematics education considers different perspectives on mathematical knowledge (D'Ambrosio, 1978; Howson & Wilson, 1986; Niss, 1996).

Social purposes

Social purposes emphasize that mathematical knowledge is shaped and socially constituted, highlight its common and shared characteristics among people, and emphasize that it takes place through communication relationships. Experts state that mathematics has a public dimension and, therefore, it is essential to learn mathematics. They recognize mathematical representations as social constructions. They emphasize the conjecture of social construction that places knowledge, cognition and representation in the fields of their production, distribution and use. They argue that science is socially oriented and human groups support its objectives (Restivo, 1992; Niss, 1996).

Three areas show mathematics as a socially determined tool: Professional practice, Mathematical contexts, and Everyday habits through mathematics.

NOTION OF CURRICULUM

Curriculum is the usual term to express any activity to plan and implement an educational training program. A curriculum specifies a series of ideological, pedagogical and psycho-pedagogical principles that channel an orientation of either the educational system or the institution that applies it. The curriculum is situated between the declaration of general principles and their practical translation, between what has been prescribed and what really happens in the classroom (Stenhouse, 1984).

Questions to which a mathematics curriculum responds

A mathematics curriculum proposes concrete answers to some key questions, necessary to design and develop any training plan for whatever group of students. In our criteria (Rico, 1998), the central issues to which a math curriculum must respond are: What are you learning for? What content? How and when is the teaching carried out? What results show the achievement of learning?

In response to the previous questions, each curricular proposal offers answers in matters of:

1. The Interpretation of learning

2. Forms of content and knowledge understanding

3. Teaching planning and Implementation

4. Evaluating the utility and mastery of learning achieved

Mathematics curriculum seen as a multi- dimensional structure

These four questions are considered substantive and each one serves as the basis for one of the fundamental variables of a curriculum. The questions generate dimensions to organize thinking about the components of the curriculum. (See figure 1)

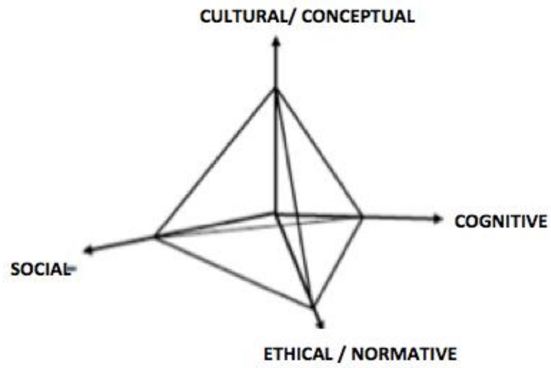

Figure 1: Dimensions of the mathematics curriculum structure

The dimensions of the mathematics curriculum that support our approach are:

- Cognitive dimension
- Cultural / conceptual dimension
- Ethical / normative dimension
- Social dimension

Levels of the mathematics curriculum structure

We will see how these dimensions provide a framework for analyzing a mathematics curriculum, based on a fact that each dimension admits different levels of reflection. These levels are presented when working the curriculum from a given priority or perspective. Dimensions and levels jointly establish a framework for analyzing and studying didactical contents of a curriculum (Steiner, 1980; Howson, Keitel & Kilpatrick, 1983; Romberg, 1992).

In this framework, the *action in the classroom* is a basic level, considered when the curriculum is assumed as a work plan for the teacher. Norms and rules that regulate the curricular description show this work plan that is expressed through specific objectives, its concrete mathematical content, specific methodology and some selected evaluation tools and criteria.

1st Dimension	**2nd Dimension**	**3rd Dimension**	**4th Dimension**
Cognitive	Cultural/ Conceptual	Ethical / Normative	Social
	First level: Action in the classroom		
Objectives	Contents	Methodology	Evaluation
	Second level: School system planning instrument		
Pupils	Knowledge	Teacher	School
	Third level: Academic Disciplines		
Learning theories	Mathematics, Epistemology, History	Pedagogy	Sociology
	Fourth level: Teleological or general goals		
Training and development goals	Cultural, conceptual and formal goals	Ethical and political goals	Social and utilitarian goals

Table 1: Dimensions and levels framework for the study of mathematics curriculum

As a second level, we contemplate the curriculum as a *planning instrument for the school system*. This is responsibility of the Educational Administration: courses are organized based on the proposed goals for pupils, knowledge is systematized through disciplines and subjects, teacher implements teaching tasks, and school socializes and promotes interactions among children.

Curriculum is also devised from another theoretical and investigative level, that of disciplinary and erudite reflection, in which different academic disciplines approach and study its theoretical foundations and its technical implementation. The Faculties of Education, Teacher Training Centers, and Departments of Mathematics Education, among others, are institutions that approach the study of the mathematics curriculum from a highly specialized disciplinary level. This level is established by the *academic disciplines,* each linked to one of the dimensions.

The fourth is the *teleological level*, based on general types of goals used to build table 1.

Each level of reflection characterizes the curriculum through didactic contents coming from each of the considered dimensions. Curricular questions are relevant at each level of reflection: What training? What is this training for? How it can be achieved? What training was has been accomplished?

The curricular changes will be determined by the emphasis given to certain purposes over others, their development will be conditioned by the priorities chosen in each case. Academics and politicians responsible for the new orientations will highlight those components that establish crucial changes. Counselors and administrators will give concrete form in their levels of responsibility to the respective curricular components. Finally, teachers will be responsible for the implementation and success of the changes at the classroom level (Howson, Keitel & Kilpatrick, 1983).

DIDACTICAL CONTENTS FOR CURRICULUM ANALYSIS

From principles to action is a well-known motto that has been recovered as a proposal for curriculum development, which experts in mathematics education have been raising since the beginning of this century. This orientation needs to renew the curricular concepts by reflecting on contents of the didactic of mathematics. It is necessary to show the suitability of these contents for the teaching and learning of school mathematics, as well as for the teachers' training plans. In summary, we intend to deepen, synthesize, order and deploy current curricular studies.

Well-established results from didactical researches, using methods of conceptual and content analyses, have been followed to identify, interpret, implement and evaluate these proposals, as they appear summarized in table 2 (Scriven, 1988; Cohen, Manion & Morrison, 2011).

We claim that didactic contents are located through a structure of curricular categories, which are explained and clarified by its components. Through the synthesis of the elements resulting from the conceptual and content analysis, a specific system of didactic contents for school mathematics emerges. Those contents describe each dimension and validate expert' knowledge (Steiner, 1969).

Organizers as categories for the mathematics curriculum

What are the organizers of the curriculum? To detect the presence of the dimensions of the curriculum in the normative texts, in the manuals and in the teaching practices, to analyze its usefulness and its functions, we identify some classification categories that we call curricular organizers. Those categories identify didactic contents through the information provided that, in each case, can be described by its components. These didactical analyses have a long tradition and are useful for the experts when selecting curriculum information, breaking it down, structuring it, and, when appropriate, using it in teaching (Steiner, 1969; Rico, 2013).

What are the organizers of the curriculum useful for? The organizers provide a stable classification system to identify elements that delimit didactic contents, according to dimensions. Therefore, the organizers deepen teachers' knowledge. Notions and basic elements are structured by specific components in each organizer, selected or elaborated from theoretical and empirical studies.

DIDACTICAL ANALYSIS CATEGORIES			
Dimensions			
Cognitive Content Analysis	**Conceptual Content Analysis**	**Instruction Content Analysis**	**Evaluative Content Analysis**
Study object			
Intentionality and learning conditions of school mathematics	Meaning of school mathematics contents	Planning and implementation of mathematics teaching	Evaluation and decision making from learning achievements
Organizers or categories used to perform the content analysis of the curricular dimensions			
1. Learning expectations	1. Conceptual structure	1. Tasks and sequences	1. Modalities and design
2. Limitations	2. Representation systems	2. Classroom work organization	2. Intervention and decision making
3. Opportunities to learn	3. Senses and modes of use	3. Materials and resources	3. Quality indicators
Components of the organizers to analyze didactically school mathematics			
1. Objectives, competencies	1. Formal and Cognitive functionality, emotional, moral and ethical attitudes	1. Task variables and its functions	1. Functions, regulations and moments
2. Errors, difficulties, blockages	2. Symbolic, graphic, numerical representations	2. Complexity, creativity and organization	2. Criteria, instruments and performance
3. Conditions, demands, challenges	3. Terms, contexts, phenomena, modes of use	3. Characteristics, types and uses	3. Strategic and comparative studies
Synthesis of main contents			
Tasks learning structure and coherence	Priority of meanings for teaching and learning	Teaching organization through units	Quality of the achieved learning
Mathematics didactic content obtained as synthesis of elements encompassed by didactical analysis			

Table 2: System of components, categories and contents for the didactic analysis of the mathematics school curriculum

Organizers and components provide a system to analyze the didactic contents and synthesize their elements in school mathematics.

How do the organizers work for a mathematical educator? In their practice the teacher, the designer or the expert need to perform a content analysis according to the different dimensions and their organizers. This analysis will be carried out with the help of texts. In this way, useful and necessary information will be obtained to design and plan the didactic materials to be for implemented in school mathematics. Each of the four curricular dimensions requires a specific type of categories, components and contents for the didactic analysis of the school curriculum (Rico, 2016).

Didactic analysis of school mathematical content

We consider the didactic analysis of mathematical content a complex method to deepen, structure and clarify said curricular content, with a view to its programming and implementation. This proposal arises from the four partial forms of curricular analysis: cognitive, conceptual, instructional and evaluative. Table 2 shows a summary of the didactic analysis structure.

The curricular organizers help the mathematics educator to select information needed to plan and structure teaching units and implement them in the training programs. Through the curricular organizers the teacher performs a didactic analysis of any school mathematics subject as presented in the texts. As consequence, a series of notions and proposals emerge, based on different modalities of content analysis and their subsequent syntheses, from the system of categories.

IDEAS FOR DISCUSSION

The processes of change and curricular development impose on its participants a commitment to the personal and social welfare of the citizens. These processes cannot be simple autonomous theoretical constructions designed to satisfy either vanity or ambition of the leaders. Researchers are important but they do not play an exclusive role, without the support of teachers and managers. How can a more complete and responsible participation of all the agents involved be guaranteed?

Curricular changes for deepening the didactic contents must be based on solid theoretical and empirical foundations. They also require intellectual and moral capabilities of experts and teachers, as well as of those managing curricular changes. Innovation in school mathematics, the training of qualified teachers and the development of research based on school practices, are the three main functions of the mathematics curriculum. Mutual support and coordination of these three functions is necessary in any process of change. How can we deepen and improve relationships and compromises between these curricular functions?

The mathematics teacher is a professional who assumes the responsibility of educating young people in a democratic and advanced society by teaching school mathematics. The mathematics curriculum is an instrument for this work. How is it possible to technically strengthen the responsibility and moral commitment of mathematics educators with all citizens?

The concept of curriculum is presented as a structure, as an organized system of ideas; it helps to raise questions and to obtain answers for the central questions about teaching and learning and attends to the functions of an educational system. A curriculum plans, implements and evaluates educational proposals in a social environment established for an institution. The curriculum helps in the reflection and work of the school math teacher.

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SECONDARY MATHEMATICS FOR THE SOCIAL SCIENCES

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The origins, rationale and development of the course 'Mathematics Applied to the Social Sciences' (MACS), created in 2001, is described and some ideas around the teaching of mathematics in secondary schools for students other than the future scientists and engineers are discussed.

There are two recurring debates about the mathematics curriculum in secondary schools, especially in countries like Portugal where compulsory education goes till the 12th grade. First, should all students study mathematics (not necessarily the same) or should the curriculum leave a part of the students "happy" without the mathematics "torture"? Second, should all students study the same "classic" mathematics, around ideas from differential and integral calculus with some Geometry and some Statistics?

When the 2001 revision (in great part in application today) of the Portuguese Secondary School curriculum was made (involving the 10th, 11th and 12th grades) different kinds of courses were introduced for the different tracks (but not for all of them) that traditionally existed. **Mathematics A** is for the Science and Technology track and for the Economics track and is a compulsory course. **Mathematics B** is for the Arts track and is an optional course. **Mathematics Applied to the Social Sciences (MACS)** is for the Social Sciences track and is an optional course. The Languages track was left without mathematics or science. Later the last two tracks were merged and the **MACS** course remained optional for the new merged track. The technological or professional tracks could have **Mathematics B**, **Mathematics for the Arts** or **Modules of Mathematics** (3 to 10 to be chosen from 16 different modules, depending on the professions).

THE ORIGINS OF THE MACS COURSE

The 1990 revision of the Portuguese Secondary School curriculum included for the first time a division of the Mathematics course into two different ones. **Mathematics** for some students and **Quantitative Methods** for others, including the students of the Arts track. This created quite a controversy at the time (Vieira & Abrantes, 1994) because the syllabus of **Quantitative Methods** included Logic, Real Numbers, Functions and no Geometry or Applications (the course was offered only for the 10th grade students, so it was only one year long). Students that had this course normally did not like mathematics or were weak at it (but they had to "suffer" only 1 more year). This situation motivated a number of projects that tried to present alternatives, namely for Arts students, that included some Geometry (Ponte et al, 1998) and proposals that included more modern topics like Graph Theory and Dynamical Systems (Carvalho e Silva, 1995). João Pedro Ponte, a leading researcher in Mathematics Education in Portugal and a former member of the Board of ERME (European Society for Research in Mathematics Education), questioned whether a single Mathematics course for all students, centered on Pure Mathematics, that was necessary to enter

Higher Education was or not one of the responsible for the very high retention rate. But there were no changes to the official curriculum till 2001 (Ponte 1998).

In 2001 there was a revision of the structure of secondary education, that defined seven different tracks with specific clear goals, but with the same structure: in each track there are 3 courses that are considered as foundational for that track and so are compulsory, there are several optional courses and a number of courses with an interdisciplinary flavor like "*Project Area*" (to develop project work) and ICT (Information and Communication Technology). This revision was implemented only in 2004, with some changes, but the general structure remained the same.

That's when it was proposed to have an optional mathematics course offered for the "*Social Sciences*" track. There were discussions about possibilities of having more courses (like "*Mathematics in History and Philosophy*" for the Languages track) for other tracks but finally MACS was chosen, only for the "*Social Sciences*" track, but not for other tracks (so, some tracks ended their mathematical studies at the 9th grade level).

THE MACS COURSE

When, in 2001, there was a possibility to introduce a new Mathematics course for the "*Social Sciences*" track, for the 10th and 11th grade students, there were some discussions of what could be offered. The model chosen was inspired in the course "For All Practical Purposes" (COMAP, 2000) developed by COMAP because it "uses both contemporary and classic examples to help students appreciate the use of math in their everyday lives". As a consequence, a set of independent chapters, each one with some specific purpose, was chosen for this syllabus, that included 2 years of study, with 4.5 hours of classes per week (normally 3 classes of 90 minutes each). The topics chosen were:

10th grade
- Decision Methods: Election Methods, Apportionment, Fair Division
- Mathematical Models: Financial models, Population models
- Statistics (regression)

11th grade
- Graph models
- Probability models
- Statistics (inference)

The stated goal of this course is for the students to have "*significant mathematical experiences that allow them to appreciate adequately the importance of the mathematical approaches in their future activities*" (Carvalho e Silva, 2003). This means that the main goal is not to master specific mathematical concepts, but it is really to give students a new perspective on the real world with mathematics, and to change the students view of the importance that mathematical tools will have in their future life. In this course it is expected that the students study simple real situations in a form as complete as possible, and "*develop the skills to formulate and solve mathematically problems and develop the skill to communicate mathematical ideas (students should be able to write and read texts with mathematical content describing concrete situations)*" (Carvalho e Silva et al., 2001).

This was a huge challenge for the Portuguese educational system because most of these topics had never been covered before, and most teachers did not even study Graph Theory at University. Election Methods, Apportionment and Fair Division were of course completely new to everybody. The reception was good from the part of the Portuguese Math Teacher Association APM, as it considered that "*the methodologies and activities suggested in the MACS program promote the development of the skills of social intervention, of citizenship and others*" (APM, 2007). The reception from the scientific society SPM was rather negative because they considered the syllabus did not have enough mathematical content.

These new topics had in part been proposed previously. Back in 1942 the mathematician and educator Bento de Jesus Caraça complained with the topics of the secondary school syllabus that had nothing to do with "contemporaneous life" and where practical applications were absent (Carvalho e Silva, 2014). Also, in 1994, the mathematics educator Paulo Abrantes wondered when topics like Graphs and Matrices would be introduced in our syllabuses, because they represent very different forms of mathematics reasoning (Abrantes, 1994).

THE CHOICE OF TOPICS

Being an optional course for secondary school students, the choice of topics is not constrained by further studies in Higher Education. The topics were chosen so that they could be used with secondary school students that normally are not a priority for mathematics studies, in order for them to encounter "*significant mathematical experiences*". It is hoped that, although teachers may find difficulties implementing this program, they will achieve some satisfaction when they see that "*students become aware how Mathematics is an important tool for their life*" (Carvalho e Silva et al., 2001).

Decision Methods were chosen because we live in a society where everybody is called to make decisions (for example in elections) and all need to be aware that mathematics gives some tools to choose an adequate method to arrive at a final decision.

The mathematical models are always incomplete but they can be useful to explain growth in a biological or economics situation, giving some information about when a population may become extinct. Graph models are useful to study in a more complete way systems of distribution or collection.

Probability and Statistics are so important in our times that they deserve to be discussed with some detail, and so these areas play an important part in this syllabus, including a new topic in any Portuguese Secondary School syllabus, the Statistical Inference, to show how scientific conclusions can be inferred from sets of data.

Other topics were proposed like game theory and cryptography, but no more topics were chosen so that teachers and students would have time to explore the syllabus, namely exploring concrete situations, look for data, develop some projects, explore the History of Mathematics (like the Konigsberg Bridges problem) and use relevant technology (graphing calculators being compulsory).

THE NATIONAL EXAMINATION

A very controversial matter about the MACS course is the existence of a national examination that counts 30% for the final grade of the student in the course. The present regulations state that students need to do a total of 4 national examinations in order to be granted the Secondary School diploma.

APM points out that the existence of the national examination is not compatible with the assessment suggested in the official syllabus (APM, 2007). The association complains that teachers lose their freedom and try to "prepare" students for the examination and this somehow does not allow the innovation aspects of this program to pass fully into practice. In fact, the official syllabus of MACS gives a great freedom to the teacher to use a number of assessment instruments and recommends to not give priority to timed tests. Group work and individual work is recommended, assuming different forms: essays, personal notes, reports, presentations, debates. With a national examination, teachers complain they would need a more detailed specification of what is covered by the examination, but if the questions remain very open it is not feasible to give these kinds of very precise details. The first year the examination was administered the authors of the program prepared 3 model examinations to guide teachers (it was chosen not to produce only 1 in order to give a more open view of possible questions and not introduce unintended limitations on the format of questions).

Today the national exam of MACS consists of several rather mostly open but simple questions, where some careful interpretation or model construction/analysis is required. We give two examples from the 2017 examination. The first involves an exponential model that needs to be compared with a previously studied logarithmic model, using a graphing calculator.

5.2. A empresa *ComPromo* disponibilizou uma bilheteira *online*, na qual também é possível comprar bilhetes para o parque de diversões. As duas bilheteiras entraram em funcionamento no mesmo instante.

Admita que o número total de bilhetes vendidos pela bilheteira disponibilizada pela *ComPromo*, ao fim de t dias após a sua abertura, é bem aproximado pelo modelo seguinte, com arredondamento às unidades.

$$c(t) = 35e^{0,14t}, \text{ com } 0 < t < 30$$

Ao fim de quantos dias, após a abertura das duas bilheteiras, o número total de bilhetes vendidos na bilheteira *online* do parque foi, pela primeira vez, inferior ao número total de bilhetes vendidos na bilheteira disponibilizada pela *ComPromo*?

Para responder a esta questão, recorra às capacidades gráficas da sua calculadora e apresente:

– o(s) gráfico(s) visualizado(s) que lhe permite(m) resolver o problema;

– as coordenadas do(s) ponto(s) relevante(s) arredondadas às décimas.

The second involves applying a given voting method to a concrete situation.

Tabela 2

Preferências \ N.º de votos	602	309	727
1.ª	C	A	D
2.ª	B	B	B
3.ª	A	C	C
4.ª	D	D	A

Concluída a votação, o apuramento da ementa vencedora é feito através do método a seguir descrito.

- Seleciona-se um par de ementas e atribui-se o número de votos registados em cada coluna à ementa mais bem posicionada, de entre as duas selecionadas.
- Comparam-se os votos obtidos por essas duas ementas. A ementa com o maior número de votos é a vencedora do par escolhido.
- Repetem-se os procedimentos anteriores até uma das ementas ter vencido em todas as comparações com as restantes. Essa ementa é a vencedora.

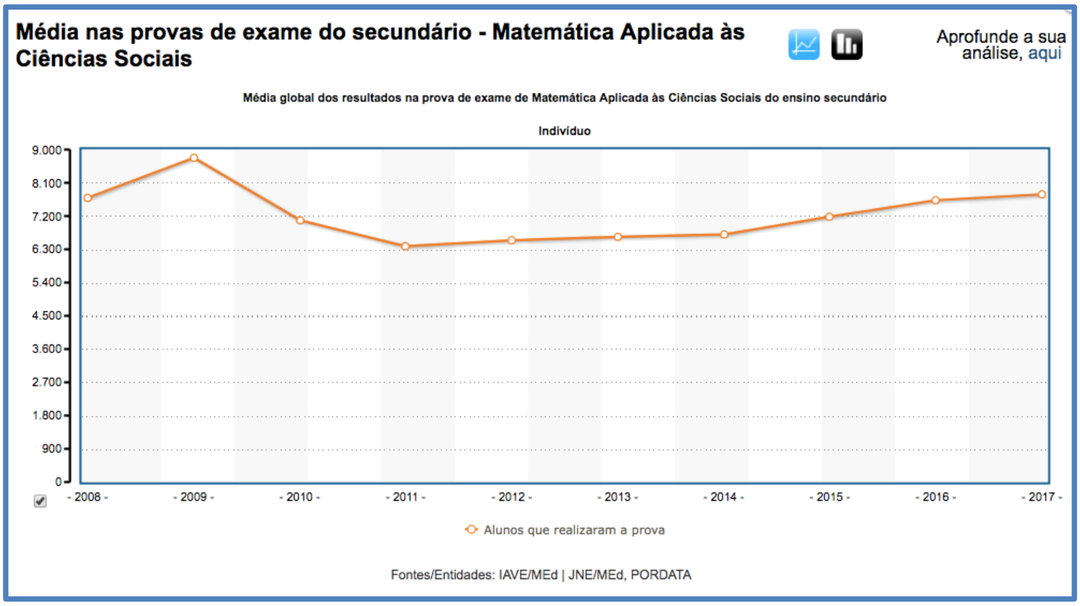
The marks students usually get on this national examination are similar to the results of other courses.

IS THE MACS COURSE A SUCCESS?

Being an optional course for secondary school students, and as mathematics is not very popular in Portugal, one would think that this course does not attract many students.

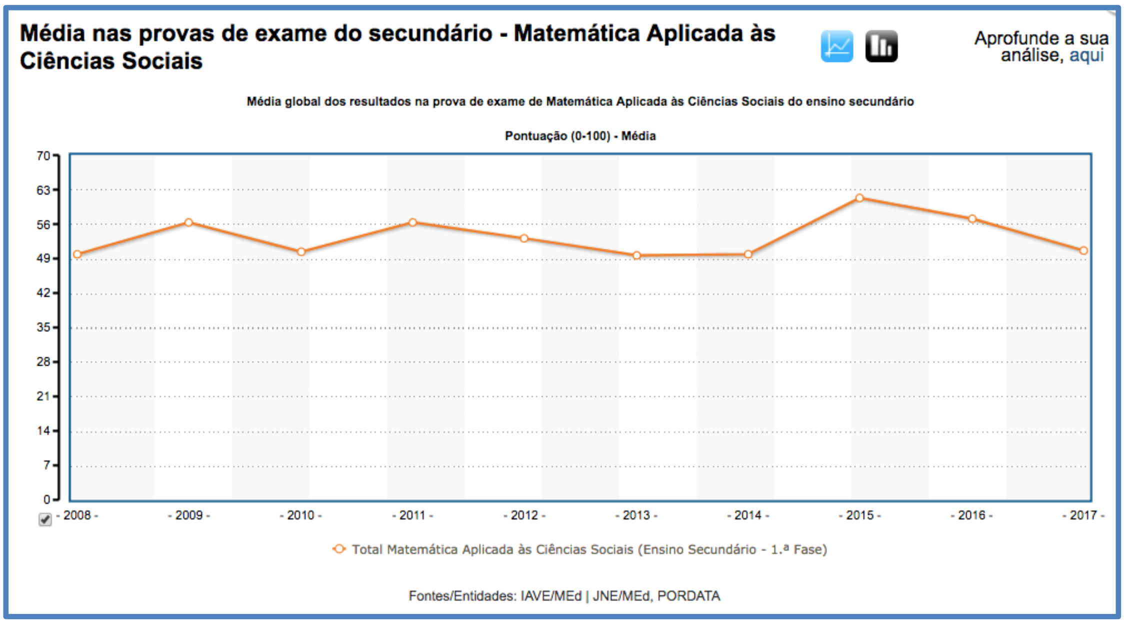
As this course is accepted by very few Higher Education degrees, students that take this course can easily opt not to take the National examination.

The number of students that take this examination is in fact very high. The total number of students taking exams is around 50 000, and some 30 000 take the main **Mathematics A** examination.



The number of students taking the **MACS** exam is around 8 000, which means around 25% of the students taking **Mathematics A**, higher than most people would expect.

The score obtained in the national **MACS** examination is normally higher than 50%, a little above the mean results of **Mathematics A**.



There was some controversy on the existence of this examination, as it apparently contradicted the stated goals of the course, but the results show that it did not seem to be a deterrent for a big number of students.

One clear topic winner of MACS among students was election theory. This topic clearly resonated in the student's experiences and there are numerous small projects connected to its use in very concrete situations, be it the parliamentary elections or the school's student union elections.

There are some research studies that test different parts of the MACS syllabus. We will mention three of them. The first, done with the students of the 11th grade (Gonçalves & Viseu, 2013), concluded that students managed to study graphs in a problem solving environment and evolved in the knowledge of this area using "real world" problems. The second, done with students of the 10th grade in the topic of Population Models (Gonçalves, 2014) concludes that students can do small investigations, work in groups with technology, communicate with others and understand how mathematics is used in the real world. The third was done again with students of the 11th grade but in the topic of probability models (Raposo, Nascimento, Costa & Gea, 2017); it uses technology to help students overcome difficulties with conditional probability. In all studies it is clear the role of "real world" problems.

WHY THE MACS COURSE SURVIVES TILL TODAY

As **Quantitative Methods** failed as a discipline, there was skepticism that an "alternative" to mainstream mathematics could be feasible. No tradition in Portugal, no teachers prepared, no publications, no textbooks. But the fact is that a carefully designed plan allowed today's situation where thousands of students opt for this course and do a no high-stakes national examination on it. There are no calls to end it, and we can think that this course might be offered to other kinds of students with the same success.

In 2001 there was a Consulting Committee at the General Directorate for Secondary Education that advised the Ministry on measures to be taken to improve the teaching of Mathematics at the Secondary School level. Under the guidance of this Committee, a set of Secondary School teachers was specially prepared in some working weeks, so that they would be prepared to make preparation of other teachers. Most of these teacher specialists in MACS produced professional development sessions for other teachers and so a way of preparing teachers to teach MACS was in fact functioning.

Written teaching materials were produced by the 3 authors of the MACS program. Several publications were made by several teachers, authorized translations of COMAP publications were edited by the Ministry of Education (including Election Theory, Apportionment and Graph Models), and new textbooks were produced (not without some trouble, one of the textbooks being taken out of the market due to serious errors in the statistics part). Areas like Election Theory and Graph Models sparked a lot of interest and we now have quite a few publications. As stated by UNESCO "*There cannot be any quality mathematics education for all unless quality resources are produced for pupils and for teachers*" (UNESCO, 2012).

Several universities, namely the University of Coimbra, included in their Master degree topics to prepare future secondary school teachers like election theory, apportionment and graph theory.

The Teacher Association APM, in its 2007 report said that "APM participated actively with proposals, teacher preparation, discussions, preparation of materials, etc." and "the process (…) has been exemplary". The authors of the MACS course had a permanent "contact with teachers in the field, asked for contributions from all the teachers, mathematicians and other specialists, integrated in a very satisfactory manner the several suggestions sent to them, and the authors also organized meetings to discuss the work being done in a very open way" (APM, 2007).

With all this national movement and the positive reaction of students, we can say that there were conditions for the course to contribute positively to the success of Social Sciences students in Mathematics.

CONCLUSIONS

After 15 years there is no thorough evaluation of how the course is run in practice in the schools, or which is the real impact on the further education or activities of the students that studied "**Mathematics Applied to the Social Sciences**". In Portugal there is no institution in charge of this type of work and evaluations are done on a case by case basis. All Secondary Schools need to do self-evaluations but normally just compare internal statistics to national ones to see where they are in the national scene. In the reports consulted there was no special mention to the MACS course and so we have the impression that the MACS course entered the normal Portuguese routine in Secondary School.

Now in Portugal compulsory education goes till the end of Secondary School, the 12^{th} year. I hope we will evolve to some significant mathematics studies being offered to all kinds students at the secondary level, and not only to some students on a partially optional level, in order to guarantee a quality mathematics education for all, following some ideas expressed in the UNESCO document (UNESCO, 2012).

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A TWIN CURRICULUM SINCE CONTEMPORARY MATHEMATICS MAY BLOCK THE ROAD TO ITS EDUCATIONAL GOAL, MASTERY OF MANY

Allan Tarp

The MATHeCADEMY.net

Mathematics education research still leaves many issues unsolved after half a century. Since it refers primarily to local theory, we may ask if grand theory may be helpful. Here philosophy suggests respecting and developing the epistemological mastery of Many children bring to school instead of forcing ontological university mathematics upon them. And sociology warns against the goal displacement created by seeing contemporary institutionalized mathematics as the goal needing eight competences to be learned, instead of aiming at its outside root, mastery of Many, needing only two competences, to count and to unite, described and implemented through a guiding twin curriculum.

POOR PISA PERFORMANCE DESPITE FIFTY YEARS OF RESEARCH

Being highly useful to the outside world, mathematics is a core part of institutionalized education. Consequently, research in math education has grown as witnessed by the International Congress on Mathematics Education taking place each 4 years since 1969. However, despite increased research and funding, the former model country Sweden has seen its PISA result decrease from 2003 to significantly below the OECD average in 2012, causing OECD (2015) to write the report 'Improving Schools in Sweden'. Likewise, math dislike seems to be widespread in high performing countries also. With mathematics and education as social institutions, grand theory may explain this 'irrelevance paradox', the apparent negative correlation between research and performance.

GRAND THEORY

Ancient Greece saw two forms of knowledge, 'sophy'. To the sophists, knowing nature from choice would prevent patronization by choice presented as nature. To the philosophers, choice was an illusion since the physical is but examples of metaphysical forms only visible to the philosophers educated at Plato's Academy. Christianity eagerly took over metaphysical patronage and changed the academies into monasteries. The sophist skepticism was revived by Brahe and Newton, insisting that knowledge about nature comes from laboratory observations, not from library books (Russell, 1945).

Newton's discovery of a non-metaphysical changing will lead to the Enlightenment period: When falling bodies follow their own will, humans can do likewise and replace patronage with democracy. Two republics arose, in the United States and in France. The US still has its first Republic, France its fifth, since its German-speaking neighbors tried to overthrow the French Republic again and again.

In North America, the sophist warning against hidden patronization lives on in American pragmatism and symbolic interactionism; and in Grounded Theory, the method of natural research resonating with Piaget's principles of natural learning. In France, skepticism towards our four fundamental institutions, words and sentences and cures and schools, is formulated in the poststructural thinking of Derrida, Lyotard, Foucault and Bourdieu warning against institutionalized categories, correctness, diagnosed cures, and education; all may hide patronizing choices presented as nature (Lyotard, 1984).

Within philosophy itself, the Enlightenment created existentialism (Marino, 2004) described by Sartre as holding that ‘existence precedes essence’, exemplified by the Heidegger-warning: In a sentence, trust the subject, it exists; doubt the predicate, it is essence coming from a verdict or gossip.

The Enlightenment also gave birth to sociology. Here Weber was the first to theorize the increasing goal-oriented rationalization that dis-enchants the world and creates an iron cage if carried to wide. Mills (1959) sees imagination as the core of sociology. Bauman (1990) agrees by saying that sociological thinking “renders flexible again the world hitherto oppressive in its apparent fixity; it shows it as a world which could be different from what it is now” (p. 16). But he also formulates a warning (p. 84): “The ideal model of action subjected to rationality as the supreme criterion contains an inherent danger of another deviation from that purpose - the danger of so-called goal displacement. (..) The survival of the organization, however useless it may have become in the light of its original end, becomes the purpose in its own right”. Which may lead to ‘the banality of evil’ (Arendt, 1963).

As to what we say about the world, Foucault (1995) focuses on discourses about humans that, if labeled scientific, establish a ‘truth regime’. In the first part of his work, he shows how a discourse disciplines itself by only accepting comments to already accepted comments. In the second part he shows how a discourse disciplines also its subject by locking humans up in a predicate prison of abnormalities from which they can only escape by accepting the diagnose and cure offered by the ‘pastoral power’ of the truth regime. Foucault thus sees a school as a ‘pris-pital’ mixing the power techniques of a prison and a hospital: the ‘pati-mates’ must return to their cell daily and accept the diagnose ‘un-educated’ to be cured by, of course, education as defined by the ruling truth regime.

Mathematics, stable until the arrival of SET

In ancient Greece, the Pythagoreans chose the word mathematics, meaning knowledge in Greek, as a common label for their four knowledge areas: geometry, arithmetic, music and astronomy (Freudenthal, 1973), seen by the Greeks as knowledge about Many in space, Many by itself, Many in time, and Many in space and time. Together they formed the ‘quadrivium’ recommended by Plato as a general curriculum together with ‘trivium’ consisting of grammar, logic and rhetoric.

With astronomy and music as independent areas, mathematics became a common label for the two remaining activities, geometry and algebra, both rooted in the physical fact Many through their original meanings, ‘to measure earth’ in Greek and ‘to reunite’ in Arabic. And in Europe, Germanic countries taught ‘reckoning’ in primary school and ‘arithmetic’ and ‘geometry’ in the lower secondary school until about 50 years ago when they all were replaced by the ‘New Mathematics’.

Here a wish for exactness and unity created a SET-derived ‘meta-matics’ as a collection of ‘well-proven’ statements about ‘well-defined’ concepts, defined top-down as examples from abstractions instead of bottom-up as abstractions from examples. But Russell showed that the self-referential liar paradox ‘this sentence is false’, being false if true and true if false, reappears in the set of sets not belonging to itself, where a set belongs only if it does not: If $M = \{A \mid A \notin A\}$ then $M \in M \Leftrightarrow M \notin M$. The Zermelo-Fraenkel set-theory avoids self-reference by not distinguishing between sets and elements, thus becoming meaningless by not separating abstract concepts from concrete examples.

SET thus transformed classical grounded ‘many-matics’ into today’s self-referring ‘meta-matism’, a mixture of meta-matics and ‘mathe-matism’ true inside but seldom outside a classroom where adding numbers without units as ‘1 + 2 IS 3’ meets counter-examples as e.g. 1week + 2days is 9days.

Proportionality illustrates the variety of mastery of Many and of quantitative competence

Proportionality is rooted in questions as “2kg costs 5$, what does 7kg cost; and what does 12$ buy?”

Europe used the ‘Regula de Tri’ (rule of three) until around 1900: arrange the four numbers with alternating units and the unknown at last. Now, from behind, first multiply, the divide. So first we ask, Q1: ‘2kg cost 5$, 7kg cost ?$’ to get to the answer (7*5/2)$ = 17.5$. Then we ask, Q2: ‘5$ buys 2kg, 12$ buys ?kg’ to get to the answer (12*2)/5$ = 4.8kg.

Then, two new methods appeared, ‘find the unit’, and cross multiplication in an equation expressing like proportions or ratios:

Q1: 1kg costs 5/2$, so 7kg cost 7*(5/2) = 17.5$. Q2: 1$ buys 2/5kg, so 12$ buys 12*(2/5) = 4.8kg.
Q1: $2/5 = 7/x$, so $2*x = 7*5$, $x = (7*5)/2 = 17.5$. Q2: $2/5 = x/12$, so $5*x = 12*2$, $x = (12*2)/5 = 4.8$.

SET chose modeling with linear functions to show the relevance of abstract algebra’s group theory: Let us define a linear function $f(x) = c*x$ from the set of kg-numbers to the set of $-numbers, having as domain $DM = \{x\in R \mid x>0\}$. Knowing that $f(2) = 5$, we set up the equation $f(2) = c*2 = 5$ to be solved by multiplying with the inverse element to 2 on both sides and applying the associative law: $c*2 = 5$, $(c*2)*½ = 5*½$, $c*(2*½) = 5/2$, $c*1 = 5/2$, $c = 5/2$. With $f(x) = 5/2*x$, the inverse function is $f\text{-}1(x) = 2/5*x$. So with 7kg, $f(7) = 5/2*7 = 17.5$$; an with 12$, $f\text{-}1(12) = 2/5*12 = 4.8$kg.

In the future, we simply ‘re-count’ in the ‘per-number’ 2kg/5$ coming from ‘double-counting’ the total *T*. Q1: *T* = 7kg = (7/2)*2kg = (7/2)*5$ = 17.5$; Q2: *T* = 12$ = (12/5)*5$ = (12/5)*2kg = 4.8kg.

Grand theory looks at mathematics education

Philosophically, we can ask if Many should be seen ontologically, what it is in itself; or epistemologically, how we perceive and verbalize it. University mathematics holds that Many should be treated as cardinality that is linear by its ability to always absorb one more. However, in human number-language, Many is a union of blocks coming from counting singles, bundles, bundles of bundles etc., T = 345 = 3*BB+4*B+5*1, resonating with what children bring to school, e.g. T = 2 5s.

Likewise, we can ask: in a sentence what is more important, that subject or what we say about it? University mathematics holds that both are important if well-defined and well-proven; and both should be mediated according to Vygotskian psychology. Existentialism holds that existence precedes essence, and Heidegger even warns against predicates as possible gossip. Consequently, learning should come from openly meeting the subject, Many, according to Piagetian psychology.

Sociologically, a Weberian viewpoint would ask if SET is a rationalization of Many gone too far leaving Many dis-enchanted and the learners in an iron cage. A Baumanian viewpoint would suggest that, by monopolizing the road to mastery of Many, contemporary university mathematics has created a goal displacement. Institutions are means, not goals. As an institution, mathematics is a means, so the word ‘mathematics’ must go from goal descriptions. Thus, to cure we must be sure the diagnose is not self-referring. Seeing education as a pris-pital, a Foucaultian viewpoint, would ask, first which structure to choose, European line-organization forcing a return to the same cell after each hour, day and month for several years; or the North American block-organization changing cell each hour, and changing the daily schedule twice a year? Next, as prisoners of a ‘the goal of math education is to learn math’ discourse and truth regime, how can we look for different means to the outside goal, mastery of Many, e.g. by examining and developing the existing mastery children bring to school?

Meeting Many, children bundle in block-numbers to count and share

How to master Many can be learned from preschool children. Asked “How old next time?”, a 3year old will say “Four” and show 4 fingers; but will react strongly to 4 fingers held together 2 by 2, ‘That is not four, that is two twos’, thus describing what exists, and with units: bundles of 2s, and 2 of them.

Children also use block-numbers when talking about Lego bricks as ‘2 3s’ or ‘3 4s’. When asked “How many 3s when united?” they typically say ‘5 3s and 3 extra’; and when asked “How many 4s?” they may say ‘5 4s less 2’; and, placing them next-to each other, they typically say ‘2 7s and 3 extra’.

Children have fun recounting 7 sticks in 2s in various ways, as 1 2s &5, 2 2s &3, 3 2s &1, 4 2s less 1, 1 4s &3, etc. And children don’t mind writing a total of 7 using ‘bundle-writing’ as $T = 7 = 1B5 = 2B3 = 3B1 = 4B\cancel{1}$; or even as $1BB3$ or $1BB1B1$. Also, children love to count in 3s, 4s, and in hands.

Sharing 9 cakes, 4 children take one by turn saying they take 1 of each 4. Taking away 4s roots division as counting in 4s; and with 1 left they often say “let’s count it as 4”. Thus 4 preschool children typically share by taking away 4s from 9, and by taking away 1 per 4, and by taking 1 of 4 parts. And they smile when seeing that entering ‘9/4’ allows a calculator to predict the sharing result as 2 1/4; and when seeing that entering ‘2*5/3’ will predict the result of sharing 2 5s between 3 children.

Children thus master sharing, taking parts and splitting into parts before division and counting- and splitting-fractions is taught; which they may like to learn before being forced to add without units.

So why not develop instead of rejecting the core mastery of Many that children bring to school?

A typical contemporary mathematics curriculum

Typically, the core of a curriculum is how to operate on specified and unspecified numbers. Digits are given directly as symbols without letting children discover them as icons with as many strokes or sticks as they represent. Numbers are given as digits respecting a place value system without letting children discover the thrill of bundling, counting both singles and bundles and bundles of bundles. Seldom 0 is included as 01 and 02 in the counting sequence to show the importance of bundling. Never children are told that eleven and twelve comes from the Vikings counting ‘(ten and) 1 left’, ‘(ten and) 2 left’. Never children are asked to use full number-language sentences, $T = 2$ 5s, including both a subject, a verb and a predicate with a unit. Never children are asked to describe numbers after ten as 1.4 tens with a decimal point and including the unit. Renaming 17 as 2.-3 tens and 24 as $1B14$ tens is not allowed. Adding without units always precedes both bundling iconized by division, stacking iconized by multiplication, and removing stacks to look for unbundled singles iconized by subtraction. In short, children never experience the enchantment of counting, recounting and double-counting Many before adding. So, to re-enchant Many will be an overall goal of a twin curriculum in mastery of Many through developing the children’s existing mastery and quantitative competence.

A QUESTION GUIDED COUNTING CURRICULUM

The question guided re-enchantment curriculum in counting could be named ‘Mastering Many by counting, recounting and double-counting’. The design is inspired by Tarp (2018). It accepts that while eight competencies might be needed to learn university mathematics (Niss, 2003), only two are needed to master Many (Tarp, 2002), counting and uniting, motivating a twin curriculum. The corresponding pre-service or in-service teacher education can be found at the MATHeCADEMY.net. Remedial curricula for classes stuck in contemporary mathematics can be found in Tarp (2017).

Q01, icon-making: “The digit 5 seems to be an icon with five sticks. Does this apply to all digits?” Here the learning opportunity is that we can change many ones to one icon with as many sticks or strokes as it represents if written in a less sloppy way. Follow-up activities could be rearranging four dolls as one 4-icon, five cars as one 5-icon, etc.; followed by rearranging sticks on a table or on a paper; and by using a folding ruler to construct the ten digits as icons.

Q02, counting sequences: “How to count fingers?” Here the learning opportunity is that five fingers can also be counted “01, 02, 03, 04, Hand” to include the bundle; and ten fingers as “01, 02, Hand less2, Hand-1, Hand, Hand&1, H&2, 2H-2, 2H-1, 2H”. Follow-up activities could be counting things.

Q03, icon-counting: “How to count fingers by bundling?” Here the learning opportunity is that five fingers can be bundle-counted in pairs or triplets allowing both an overload and an underload; and reported in a number-language sentence with subject, verb and predicate: *T* = 5 = 1Bundle3 2s = 2*B*1 2s = 3*B*-1 2s = 1*BB*1 2s, called an ‘inside bundle-number’ describing the ‘outside block-number’. A western abacus shows this in ‘outside geometry space-mode’ with the 2 2s on the second and third bar and 1 on the first bar; or in ‘inside algebra time-mode’ with 2 on the second bar and 1 on the first bar. Turning over a two- or three-dimensional block or splitting it in two shows its commutativity, associativity and distributivity: T = 2*3 = 3*2; T = 2*(3*4) = (2*3)*4; T = (2+3)*4 = 2*4 + 3*4.

Q04, calculator-prediction: “How can a calculator predict a counting result?” Here the learning opportunity is to see the division sign as an icon for a broom wiping away bundles: 5/2 means ‘from 5, wipe away bundles of 2s’. The calculator says ‘2.some’, thus predicting it can be done 2 times. Now the multiplication sign iconizes a lift stacking the bundles into a block. Finally, the subtraction sign iconizes the trace left when dragging away the block to look for unbundled singles. By showing ‘5-2*2 = 1’ the calculator indirectly predicts that a total of 5 can be recounted as 2B1 2s. An additional learning opportunity is to write and use the ‘recount-formula’ *T* = (*T*/*B*)**B* saying “From *T*, *T*/*B* times *B* can be taken away.” This proportionality formula occurs all over mathematics and science. Follow-up activities could be counting cents: 7 2s is how many fives and tens? 8 5s is how many tens?

Q05, unbundled as decimals, fractions or negative numbers: “Where to put the unbundled singles?” Here the learning opportunity is to see that with blocks, the unbundled occur in three ways. Next-to the block as a block of its own, written as *T* = 7 = 2.1 3s, where a decimal point separates the bundles from the singles. Or on-top as a part of the bundle, written as *T* = 7 = 2 1/3 3s = 3.-2 3s counting the singles in 3s, or counting what is needed for an extra bundle. Counting in tens, the outside block 4 tens & 7 can be described inside as T = 4.7 tens = 4 7/10 tens = 5.-3 tens, or 47 if leaving out the unit.

Q06, prime or foldable units: “Which blocks can be folded?” Here the learning opportunity is to examine the stability of a block. The block *T* = 2 4s = 2*4 has 4 as the unit. Turning over gives *T* = 4*2, now with 2 as the unit. Here 4 can be folded in another unit as 2 2s, whereas 2 cannot be folded (1 is not a real unit since a bundle of bundles stays as 1). Thus, we call 2 a ‘prime unit’ and 4 a ‘foldable unit’, 4 = 2 2s. So, a block of 3 2s cannot be folded, whereas a block of 3 4s can: *T* = 3 4s = 3 * (2*2) = (3*2) * 2. A number is called even if it can be written with 2 as the unit, else odd.

Q07, finding units: “What are possible units in *T* = 12?”. Here the learning opportunity is that units come from factorizing in prime units, *T* = 12 = 2*2*3. Follow-up activities could be other examples.

Q08, recounting in another unit: “How to change a unit?” Here the learning opportunity is to observe how the recount-formula changes the unit. Asking e.g. *T* = 3 4s = ? 5s, the recount-formula will say

T = 3 4s = (3*4/5) 5s. Entering 3*4/5, the answer '2.some' shows that a stack of 2 5s can be taken away. Entering 3*4 – 2*5, the answer '2' shows that 3 4s can be recounted in 5s as 2B2 5s or 2.2 5s.

Q09, recounting from tens to icons: "How to change unit from tens to icons?" Here the learning opportunity is that asking 'T = 2.4 tens = 24 = ? 8s' can be formulated as an equation using the letter u for the unknown number, u*8 = 24. This is easily solved by recounting 24 in 8s as 24 = (24/8)*8 so that the unknown number is u = 24/8 attained by moving 8 to the opposite side with the opposite sign. Follow-up activities could be other examples of recounting from tens to icons.

Q10, recounting from icons to tens: "How to change unit from icons to tens?" Here the learning opportunity is that if asking 'T = 3 7s = ? tens', the recount-formula cannot be used since the calculator has no ten-button. However, it is programmed to give the answer directly by using multiplication alone: T = 3 7s = 3*7 = 21 = 2.1 tens, only it leaves out the unit and misplaces the decimal point. An additional learning opportunity uses 'less-numbers', geometrically on an abacus, or algebraically with brackets: T = 3*7 = 3 * (ten less 3) = 3 * ten less 3*3 = 3ten less 9 = 3ten less (ten less1) = 2ten less less 1 = 2ten & 1 = 21. Follow-up activities could be other examples of recounting from icons to tens.

Q11, double-counting in two units: "How to double-count in two different units?" Here the learning opportunity is to observe how double-counting in two physical units creates 'per-numbers' as e.g. 2$ per 3kg, or 2$/3kg. To answer questions we just recount in the per-number: Asking '6$ = ?kg' we recount 6 in 2s: T = 6$ = (6/2)*2$ = (6/2)*3kg = 9kg. And vice versa, asking '?$ = 12kg', the answer is T = 12kg = (12/3)*3kg = (12/3)*2$ = 8$. Follow-up activities could be numerous other examples of double-counting in two different units since per-numbers and proportionality are core concepts.

Q12, double-counting in the same unit: "How to double-count in the same unit?" Here the learning opportunity is that when double-counted in the same unit, per-numbers take the form of fractions, 3$ per 5$ = 3/5; or percentages, 3 per hundred = 3/100 = 3%. Thus, to find a fraction or a percentage of a total, again we just recount in the per-number. Also, we observe that per-numbers and fractions are not numbers, but operators needing a number to become a number. Follow-up activities could be other examples of double-counting in the same unit since fractions and percentages are core concepts.

Q13, recounting the sides in a block. "How to recount the sides of a block halved by its diagonal?" Here, in a block with base b, height a, and diagonal c, mutual recounting creates the trigonometric per-numbers: $a = (a/c)*c = \sin A*c$; $b = (b/c)*c = \cos A*c$; $a = (a/b)*b = \tan A*b$. Thus, rotating a line can be described by a per-number a/b, or as tan$A$ per 1, allowing angles to be found from per-numbers. Follow-up activities could be other blocks e.g. from a folding ruler.

Q14, double-counting in STEM (Science, Technology, Engineering, Math) multiplication formulas with per-numbers coming from double-counting. Examples: kg = (kg/cubic-meter)*cubic-meter = density*cubic-meter; force = (force/square-meter) * square-meter = pressure * square-meter; meter = (meter/sec)*sec = velocity*sec; energy = (energy/sec)*sec = Watt*sec; energy = (energy/kg) * kg = heat * kg; gram = (gram/mole) * mole = molar mass * mole; Δ momentum = (Δ momentum/sec) * sec = force * sec; Δ energy = (Δ energy/ meter) * meter = force * meter = work; energy/sec = (energy/charge)*(charge/sec) or Watt = Volt*Amp; dollar = (dollar/hour)*hour = wage*hour.

Q15, navigating. "Avoid the rocks on a squared paper". Four rocks are placed on a squared paper. A journey begins in the midpoint. Two dices tell the horizontal and vertical change, where odd numbers are negative. How many throws before hitting a rock? Predict and measure the angles on the journey.

A QUESTION GUIDED UNITING CURRICULUM

The question guided re-enchantment curriculum in uniting could be named 'Mastering Many by uniting and splitting constant and changing unit-numbers and per-numbers'.

A general bundle-formula $T = a*x^2 + b*x + c$ is called a polynomial. It shows the four ways to unite: addition, multiplication, repeated multiplication or power, and block-addition or integration. The tradition teaches addition and multiplication together with their reverse operations subtraction and division in primary school; and power and integration together with their reverse operations factor-finding (root), factor-counting (logarithm) and per-number-finding (differentiation) in secondary school. The formula also includes the formulas for constant change: proportional, linear, exponential, power and accelerated. Including the units, we see there can be only four ways to unite numbers: addition and multiplication unite changing and constant unit-numbers, and integration and power unite changing and constant per-numbers. We might call this beautiful simplicity 'the algebra square'.

Q21, next-to addition: "With $T1$ = 2 3s and $T2$ = 4 5s, what is $T1+T2$ when added next-to as 8s?" Here the learning opportunity is that next-to addition geometrically means adding by areas, so multiplication precedes addition. Algebraically, the recount-formula predicts the result. Next-to addition is called integral calculus. Follow-up activities could be other examples of next-to addition.

Q22, reversed next-to addition: "If $T1$ = 2 3s and $T2$ add next-to as T = 4 7s, what is $T2$?" Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 3s, subtraction precedes division, which is natural as reversed integration, also called differential calculus. Follow-up activities could be other examples of reversed next-to addition.

Q23, on-top addition: "With $T1$ = 2 3s and $T2$ = 4 5s, what is $T1+T2$ when added on-top as 3s; and as 5s?" Here the learning opportunity is that on-top addition means changing units by using the recount-formula. Thus, on-top addition may apply proportionality; an overload is removed by recounting in the same unit. Follow-up activities could be other examples of on-top addition.

Q24, reversed on-top addition: "If $T1$ = 2 3s and $T2$ as some 5s add to T = 4 5s, what is $T2$?" Here the learning opportunity is that when finding the answer by removing the initial block and recounting the rest in 5s, subtraction precedes division, again is called differential calculus. An underload is removed by recounting. Follow-up activities could be other examples of reversed on-top addition.

Q25, adding tens: "With $T1 = 23$ and $T2 = 48$, what is $T1+T2$ when added as tens?" Again, recounting removes an overload: $T1+T2 = 23 + 48 = 2B3 + 4B8 = 6B11 = 7B1 = 71$; or $T = 236 + 487 = 2BB3B6 + 4BB8B7 = 6BB11B13 = 6BB12B3 = 7BB2B3 = 723$.

Q26, subtracting tens: "If $T1 = 23$ and $T2$ add to $T = 71$, what is $T2$?" Again, recounting removes an underload: $T2 = 71 – 23 = 7B1 – 2B3 = 5B\text{-}2 = 4B8 = 48$; or $T2 = 956 – 487 = 9BB5B6 – 4BB8B7 = 5BB\text{-}3B\text{-}1 = 4BB7B\text{-}1 = 4BB6B9 = 469$. Since T = 19 = 2.-1 tens, T2 = 19 -(-1) = 2.-1 tens take away -1 = 2 tens = 20 = 19+1, showing that -(-1) = +1.

Q27, multiplying tens: "What is 7 43s recounted in tens?" Here the learning opportunity is that also multiplication may create overloads: $T = 7*43 = 7*4B3 = 28B21 = 30B1 = 301$; or $27*43 = 2B7*4B3 =8BB+6B+28B+21 =8BB34B21 =8BB36B1 = 11BB6B1 = 1161$, solved geometrically in a 2x2 block.

Q28, dividing tens: "What is 348 recounted in 6s?" Here the learning opportunity is that recounting a total with overload often eases division: $T = 348 /6 = 3BB4B8 /6 = 34B8 /6 = 30B48 /6 = 5B8 = 58$.

Q29, adding per-numbers: “2kg of 3$/kg + 4kg of 5$/kg = 6kg of what?” Here the learning opportunity is that the unit-numbers 2 and 4 add directly whereas the per-numbers 3 and 5 add by areas since they must first transform into unit-number by multiplication, creating the areas. Here, the per-numbers are piecewise constant. Asking 2 seconds of 4m/s increasing constantly to 5m/s leads to finding the area in a ‘locally constant’ (continuous) situation defining constancy by epsilon and delta.

Q30, subtracting per-numbers: “2kg of 3$/kg + 4kg of what = 6kg of 5$/kg?” Here the learning opportunity is that unit-numbers 6 and 2 subtract directly whereas the per-numbers 5 and 3 subtract by areas since they must first transform into unit-number by multiplication, creating the areas. In a ‘locally constant’ situation, subtracting per-numbers is called differential calculus.

Q31, finding common units: “Only add with like units, so how to add T = 4ab^2 + 6abc?”. Here units come from factorizing: $T = 2*2*a*b*b + 2*3*a*b*c = 2*b*(2*a*b) + 3*c*(2*a*b) = 2b+3c\ 2ab$s.

CONCLUSION

A curriculum wants to develop brains, and colonizing wants to develop countries. De-colonizing accepts that maybe countries and brains can develop themselves if helped by options instead of directions from developed countries and brains. Some prefer a direction-giving multi-year macro-curriculum; others prefer option-giving half-year micro-curricula. Some prefer a curriculum to be a cure prescribing mathematics competencies and literacy; others prefer developing the existing quantitative competence and numeracy, defined by dictionaries as the ability to use numbers and operations in everyday life, thus silencing the word ‘mathematics’ to avoid a hidden continuing colonization. In the transition period between colonizing and decolonizing brains, grand theory has an advice to the ‘irrelevance paradox’ of mathematics education research: accept the brain’s own epistemology to avoid a goal displacement blocking the road to its educational goal, mastery of Many.

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SOME EFFECTS OF THE LACK OF COHERENCE BETWEEN THE NATIONAL HIGH SCHOOL EXAM AND CURRENT CALCULUS CURRICULUM IN HIGH SCHOOL OF VIET NAM

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In Viet Nam, the university entrance exam has a great influence on high school teaching all the more as the questions in the exam are not associated with the content of textbooks. In this paper, we summarize the curricular history and the the university entrance exams. We also show the differences caused by the change of entrance assessment in the 2016 – 2017 school year. Finally, we describe a major modification due to a new curriculum, which is expected to start to be implemented in 2019-2020.

THE EVOLUTION OF MATHEMATICS CURRICULUMS IN HIGH SCHOOL

After the country unification in 1975, Viet Nam underwent three great reforms in the mathematics curriculums. These reforms started with elementary school program and ended in the high school program respectively in 1990, 1998, and 2006.

For these changes, the author of the reforms was also the author of textbooks. Bessot and Comiti (2006) presented the first two reforms as follows:

> In Vietnam, the system of Education recently underwent the two following reforms: in 1990, Scientific, Techno-scientific and Literary final forms were created at the beginning of upper secondary school; vectorial method in geometry and introduction to computational science were introduced in form 10 and analytical geometry, basics of combinatorics, of integral calculus, and mathematical statistics and probability in form 12.
>
> In 1998, the distinctions between these three sections were abolished and a return was made to a final unique form before University. However this latest reform is partly formal, since competitive examinations giving access to the University are based upon the former distinctions; thus upper secondary schools distribute the students among pseudo-sections, with a common curriculum but applied at different levels.

In order to prepare for the most recent reform in 2006, the mathematics program was originally designed in two parts respectively for Natural Sciences and Social Sciences. However, this plan failed after commencing in the 2003-2004 school year because few students were choosing the Social Science orientation. This program has led to the publication of two sets of textbooks since 2006 including: standard and advanced textbooks. In fact, most teachers have used the standard textbooks.

THE EVOLUTION OF COLLEGE AND UNIVERSITY ENTRANCE EXAMS

In Viet Nam, the training time in colleges is 3 years long and the training time in universities ranges from 4 to 6 years. Vietnamese society attaches great importance to highly educated people. Therefore, going to a good university is the main goal of most students. University is more valued than colleges. So universities entrance exams are more difficult than the colleges entrance exams and have a great influence on the content and teaching methods in high school.

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From 1990 to 2001, the university and college exams were taking place after the high school graduation exam (around the end of May every year). Two university exams were organized between July 1 and July 20, and one college exam in August. Candidates could register for the three rounds to increase their chances to enter university or college. Universities created their tests themselves relying on the material provided by the Ministry of Education and Training. The test was complex and its content went beyond the content of textbooks. Content in the official textbook was only used for the high school graduation exam. This situation led many students to move to big cities to prepare for the exam which they wish to pass.

However, between 2002 to 2014, universities were no longer allowed to create their tests. The Ministry of Education and Training took responsibility for the university and college entrance exams, taking place one month after the high school graduation exam. During this period, a common exam took place on the same day for all universities, and the content of this exam gradually became more in line with the content of high school programs and textbooks.

Since 2015, high school graduation exam and university entrance exam have been merged into one single exam called the national high school exam. This exam and its content are organized by the Ministry of Education and Training. Its results are used both for high school graduation and admission to universities. The content is only based on standard high school textbooks. This leads teaching to only focus on the content of standard high school textbooks and on types of tasks which appear in the 180 minutes test.

From 2016 to 2017, the Ministry of Education and Training has changed the form of the mathematics exam, from essay to multiple choice questions. However, the curriculum and textbooks have not changed. The official and illustration exams at the end of June last year showed that traditional tasks had disappeared or had adjust into new forms. Along that, many new types of mathematical tasks have appeared creating more discontinuities than current program.

SOME DIFFERENCES BETWEEN THE NATIONAL HIGH SCHOOL EXAM AND THE CURRENT CURRICULUM

Change of typical tasks style in high school Calculus

"Study the variation and draw the graph of a function given by an algebraic expression", for example, "study the variation and draw the graph of the function $y = x^3 + 3x^2 - 4$, is a typical task in the Calculus program in high school. It is introduced as a lesson in the current textbook (taking 7 hours out of total 78 hours of Calculus in grade 12). The solving process of this traditional task in the Calculus textbook (standard textbooks, page 31) consists of the following steps: Find the domain of definition of the function (the set of all values of x makes y meaningful); Compute the derivative y'; Find the points at which the derivative y' is equal to zero or undefined; Determine the sign of derivative y' and infer from it the intervals where the function is increasing and decreasing; Find extreme values; Find limits at infinity, infinite limits and asymptotes (if they exist); Compile a variation chart from the results obtained; Based on the variation chart and the elements mentioned above, draw the graph. Note:

1. If a function is periodic with period T, then we only need to survey the variation and draw the graph over one period, then to translate the graph parallel to the Ox-axis.
2. We should calculate coordinates of some more points, especially, the intersection points of the graph and coordinates axes.
3. You should pay attention to the evenness and oddness of the function and the symmetry of the graph for accurate exact drawing.

This type of task always appears in the first question of a high school graduation exam and university entrance exams until the 2015-2016 school year. The technique which is used for solving this task is long and difficult. So with this type of tasks, the Ministry of Education and Training limits teaching only three types of functions: $y = ax^3 + bx^2 + cx + d$, $y = ax^4 + bx^2 + c$, and $y = \frac{ax+b}{cx+d}$ in high school. This limitation leads to the phenomenon that students memorize the graph shapes for each type of function and do not really need to understand the properties of the functions to draw their graphs. Many students are able to draw graphs correctly, despite that they make mistakes in the steps of studying of the function.

Since the 2016-2017 school year, as mentioned above, the mathematics test in national high school exams has been changed. There are now fifty questions to be solved in 90 minutes and for every question students are offered fours choices. The traditional type of tasks described above did not appear in the illustration and official exams. Instead of this, new types of tasks were introduced, such as the followings:

The first illustration test (October 2016)

Question 4: Let the function y = f (x) be defined and continuous on R and have the following variation chart:

x	$-\infty$		0		1		$+\infty$
y'		+	‖	-	0	+	
y	$-\infty$	↗	0	↘	-1	↗	$+\infty$

Figure 1: The variation chart for question 4

Which of the following statements is true:

A. The function has exactly one extreme.

B. The function has minimum value equal to 1.

C. The function has absolute maximum value equal to 0 and absolute minimum value equal to -1.

D. Function attains a maximum at x = 0 and attains a minimum at x = 1.

The third illustration test (May 2017)

Question 7: Let the function y = f (x) have the following variation chart:

x	$-\infty$		0		1		$+\infty$
y'		-	0	+	0	-	
y	$-\infty$	↘	4	↗	5	↘	$+\infty$

Figure 2: The variation chart for question 7

Which of the following statements is true:

A. $f_{max} = 5$.
B. $f_{min} = 0$.
C. Absolute minimum value equals 4.

D. Absolute maximum value equals 5.

This type of task, asking to read and interpret the variation chart poses many difficulties to both students and teachers because:

- in the current textbooks, there lacks tasks based on the definitions of local maxima and minima and reading of limit from a graph or a variation chart;

- furthermore there is no clear theory about how to read variation chart in current textbooks.

In an experiment with 134 high school students (on June 3rd, 2017) close to the beginning of the National High School exam in 2017 (between June 21st, 2017 and June 24th, 2017), we asked students to read the extremes, absolute maximum value and absolute minimum value of a function whose variation chart was as follows:

Let the function y = h (x) be defined and continuous on R and have the following variation chart:

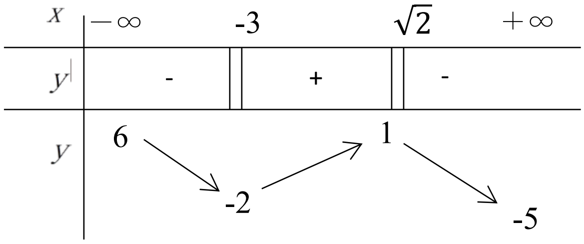

Figure 3: The variation chart for the experiment question

30% of students tested said that the function has no extreme because the derivative at -3 et $\sqrt{2}$ don't exist. 40% of students tested accepted that the limits of -5 and 6 are respectively absolute minimum value and absolute maximum value of function *h*.

According to our study, this typical task was still taught normally in the 2016-2017 school year. Limiting on the three types of functions that had been shown to contribute to explaining student failure when they confronted with the new task type - reads the variable chart. This does not exist independently contained in the current textbook.

Change in the role of graphical representations

In teaching Calculus, until before the 2016-2017 academic year, the teaching practice is exactly the same as that of Bessot and Comiti (ICMI 2006) describe:

> With such a context Calculus is greatly influenced by Algebra.[…]. In such a Calculus the graph plays a minor part in the study of elementary functions; it provides a synthesis of results obtained theoretically and helps to visualise the properties of the function studied.

In this context, Vietnamese students and teachers assume that the function is only determined by knowing its algebraic formula. However, some new types of questions have emerged along with the disappearance of the type of task: "Study the variation and draw the graph of the function" . This has changed the role of graphs from the 2016- 2017 school year. We give some examples below:

Question 14, code 102 (from the high school graduation exam in 2017)

The curve in the figure below is a graph of the function $y = ax^4 + bx^2 + c$ with a, b, and c are real numbers. Which of the following statements is true?

A. The equation y ' = 0 has three distinct real roots.

B. The equation y '= 0 has two distinct real roots.

C. The equation y '= 0 has no solution on the real number.

D. The equation y '= 0 has only one real root.

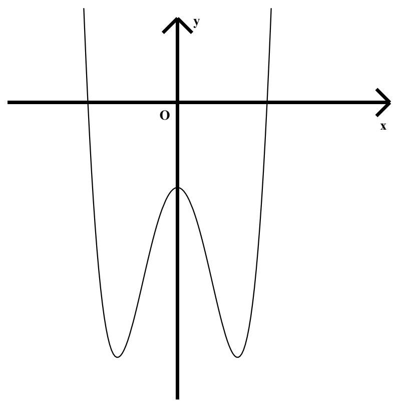

Figure 4: The graph of question 14

Question 24, code 103 (from the high school graduation exam in 2017)

The curve in the figure below is a graph of the function $y = \frac{ax+b}{cx+d}$ with a, b, c, and d are real numbers.

Which of the following statements is true:

A. $y' > 0, \forall x \neq 2$

B. $y' < 0, \forall x \neq 2$

C. $y' > 0, \forall x \neq 1$

D. $y' < 0, \forall x \neq 1$

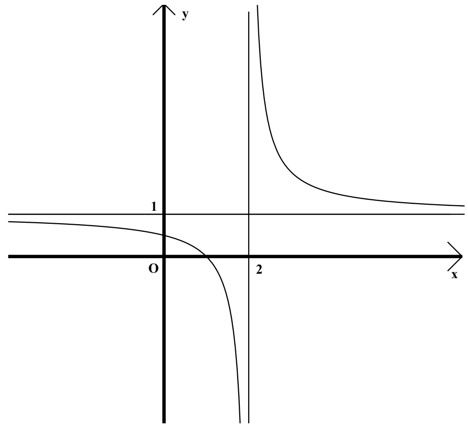

Figure 5: The graph of question 24

Students can read graphs to answer these questions quickly. However, as noted in the preceding paragraph, the use of graphs to answer the question about properties of function does not appear in the current textbooks. Our study shows that, in the 2017-2018 academic year, some teachers used summaries associated with each type of function to answer, others chose a specific algebraic formula representing functions having the same type of graph to survey the function and answer the questions.

Change in the types of tasks to reduce the use of hand held calculators

After the Ministry of Education and Training presented the first illustration exam on October 5th, 2016 with a multiple choice test, some types of tasks were changed. New techniques could emerge to solve the new tasks by using scientific calculators. In Viet Nam, students are allowed to bring handle calculators into the examination room. These calculators do not have the graphing capacities but they can approximate the solutions of an equation, integral values, etc.

For example, students can use handle calculators to solve the tasks that appear in the first illustration exam:

Question 12: Solve the equation $\log_4(x-1) = 3$

A. x = 63 **B.** x = 65 **C.** x = 80 **D.** x = 82

Question 25: Compute integral $\int_0^{\pi} \cos^3 x. \sin x dx$

A. $I = -\frac{\pi^4}{4}$ **B.** $I = -\pi^4$ **C.** $I = 0$ **D.** $I = -\frac{\pi}{4}$

If we ignore four results in A, B, C, and D then the above tasks belong to the main types of tasks in the relevant subjects such as exponential and logarithmic equations, definite integral in the current textbooks.

The wave of abuse calculator promotes the transformation of the tasks in textbooks so that it is difficult to use hand held calculators to solve them. As a result, there are so many new types of questions that teachers and students feel embarrassed by this sudden change. Here are some questions in the third illustration exam (issued May 14th, 2017):

Question 27: Given that $\int_0^1 \frac{dx}{e^x+1} = a + b\ln\frac{1+e}{2}$ with a, b rational numbers.

Compute $S = a^3 + b^3$

A. S = 2 **B.** S = - 2 **C.** S = 0 **D.** S = 1

Question 44: Let f(x) is continuous function on R and satisfy $f(x) + f(-x) = \sqrt{2 + 2\cos 2x}$, $\forall x \in R$. Compute $I = \int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} f(x)dx$

A. I = -6 **B.** I = 0 **C.** I = -2 **D.** I = 6

Question 45: How many m integers are there in the closed interval [-2017; 2017] so that the equation $\log(mx) = 2\log(x+1)$ has only one solution?

A. 2017 **B.** 4014 **C.** 2018 **D.** 4015

NEW PROGRAMS AND CHALLENGES

The school system of Viet Nam is now divided into three parts:

- Elementary school: 6 year old to 11 year old students, grade 1 to 5
- Lower secondary school: 11 year old to 15 year old students, grade 6 to 9
- Upper secondary school: 15 year old to 18 year old students, grade 10 to 12.

In January 2018, the Ministry of Education and Training of Vietnam issued a new high school curriculum. The draft mathematics curriculum was published in January 2018. With this new program, for the first time The National Assembly of the Socialist Republic of Vietnam approved a resolution saying that a program may have multiple sets of textbooks. This means that companies which do not belong to Viet Nam education publishing house can now write textbooks. The Ministry also announced the schedule for implementing the new program as follows:

The 2019-2020 school year will be implemented in grade 1;

The 2020-2021 school year: grade 2 and 6;

The 2021 – 2022 school year: grade 3, 7 and 10;

The 2022 – 2023 school year: grade 4, 8 and 11;

The 2023 – 2024 school year: grade 5, 9 and 12.

The biggest difference in the draft mathematics programs compared to previous programs is the appearance for the first time of the term "competence" in the program's aim: "Formation and development of mathematical competences, the most concentrated expression of calculation competence. Mathematical competences include the following core components: thinking skills and mathematical reasoning; mathematical modeling skills; problem-solving competence; mathematical communication skills; skills for using tools and means of mathematics, contributing to the formation and development of core competence. "(The draft mathematics programs, page 6).

The program describes the levels of attainment expected at the end of each education level (elementary, secondary and high school) for four competency groups: Thinking skills and mathematical reasoning; mathematical modeling skills; mathematical communication skills; skills for using tools and means of mathematics.

The biggest challenge is how to help teachers organize their teaching to reach the levels of attainment described, especially when these are more or less ambiguous. In addition, the question of assessing how these competencies will be implemented in practice in Vietnam with a very large class size (45 to 60 students per class) is open.

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PROFESSIONAL LEARNING COMMUNITIES AND CURRICULUM REFORMS

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Curriculum reforms should be accompanied by aligned teacher professional development to support teachers in working with curricula that they may not have experienced previously. In this paper, I review the work of a project in South Africa that set up professional learning communities among high school mathematics teachers in order to support curriculum and pedagogical reforms. I present three key findings of the project in relation to: shifts in teacher practice in the direction of their responsiveness to learners; conversations in communities in talking about responsiveness to learners; and teachers' accounts of why they stayed with or left the project. The analyses found shifts in practices for about half of the teachers, with nuances among teachers and communities. The content of the community conversations was related to the activities, and teachers spoke about both content and pedagogical content knowledge. A sense of professional agency characterized teachers who stayed with the project. The finding are explained and brought together through a discussion of professional agency and accountability in the South African context.

INTRODUCTION

It is generally accepted that curriculum reforms should be accompanied by aligned teacher professional development (TPD) to support teachers in working with curricula that they may not have experienced previously (Borko, Jacobs, Koellner, & Swackhamer, 2015; Zaccarelli, Schindler, Borko, & Osborne, 2018). This is particularly the case if the reforms encourage "ambitious instruction" (Lampert, Boerst, & Graziani, 2011), i.e. mathematics teaching that is responsive to learners' thinking and ideas, and develops conceptual connectedness among mathematical ideas.

It is also generally accepted that TPD programmes that support teachers with ambitious instruction should: be long-term and developmental, rather than fragmented once-off workshops; take the teaching-learning-mathematics relationship as an important object of study; support both a focus on and distantiation from practice, model and support reform practices; and build in support from colleagues and school management (Borko, 2004; Brodie & Shalem, 2011; Cohen & Ball, 2001; Katz, Earl, & Ben Jaafar, 2009; Llinares & Krainer, 2006). Professional learning communities are one possible model for TPD that enacts these principles.

PROFESSIONAL LEARNING COMMUNITIES

Professional Learning Communities (PLCs) can themselves be considered an ambitious reform in relation to the organizational structures and processes of schools and TPD. PLCs are groups of teachers who come together to engage in regular, systematic and sustained cycles of inquiry-based learning (Katz & Earl, 2010; Stoll & Louis, 2008). PLCs provide spaces where teachers can reflect and learn together, deliberately and systematically, to facilitate collective and sustainable shifts in their practice. PLCs aim to establish school cultures that are conducive to ongoing learning and development - of learners, teachers and schools (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006). These aspects of PLCs suggest that they can be useful mechanisms for TPD in relation to curriculum reforms, particularly when curriculum reforms are seen as requiring ongoing interpretation and re-interpretation by teachers in relation to their local contexts, rather than once-off, fragmented inputs by outsiders.

In this paper, I review the work of a project in South Africa that set up professional learning communities among high school mathematics teachers in order to support curriculum and pedagogical reforms. I describe briefly the design of the project, present the research questions that were investigated, discuss some of the findings and reflect on what the findings mean in relation to PLCs as a form of TPD in relation to curriculum and teacher development reforms.

THE DATA-INFORMED PRACTICE IMPROVEMENT PROJECT (DIPIP)

The immediate object of inquiry in the DIPIP project was learners' mathematical errors, particularly the reasoning underlying these errors. The assumption, based on the substantial errors and misconceptions research, is that systematic errors are built on partially valid mathematical reasoning and that making that reasoning explicit for teachers and learners can help them to value learners' current mathematical thinking and develop new ideas (Smith, DiSessa, & Roschelle, 1993). The focus on errors was a mechanism to access three important elements of teaching and learning mathematics: how learners' thinking makes sense to them and can be worked with, even (and especially) when partially correct; how teaching practice can shift to take account of learners' errors and thinking; and teachers' own knowledge, both content and pedagogical content knowledge. The project was therefore an ambitious TPD project focused on a key element of ambitious instruction – teachers' responsiveness to learner errors. The research aspect of the project investigated:

1. To what extent did teachers who participated in the DIPIP project shift their practices towards engaging with learner errors and learner thinking?
2. How did teachers engage with their own content and pedagogical content knowledge as they inquired into learners' errors in their communities?
3. What distinguished teachers who stayed with the project from those who left the project?

TPD Design

The PLCs were supported to participate in a sequence of developmental activities in which they analysed learners' errors in different teaching contexts. The activities were: test analysis; learner interviews; curriculum mapping; choosing "leverage" concepts; readings and discussion; planning lessons together; teaching the planned lessons; and videotaping and reflecting on the lessons together. Although the activities were set up before the project started, we did build in areas of choice and flexibility for PLCs. This TPD was therefore seen as somewhat adaptive (Koellner & Jacobs, 2015), since the model specified some key parameters but allowed for flexibility in relation to local contexts. A key area of flexibility built in from the start was for PLCs to choose the area of mathematics content to work on, based on their analyses of learner errors in their schools.

The tests that were analysed were international tests, national tests and teacher-set tests, depending on the needs and interest of the community. The test analysis provided an overview of strengths and weaknesses in learners' mathematical knowledge in a particular school or class. Based on the test analysis, teachers chose learners who had made interesting errors that they wanted to understand more deeply and interviewed these learners. They then took the results of these two analyses and mapped them against the curriculum, working out where the key concepts were taught and what curricular issues might have contributed to the errors. Based on these three activities, teachers chose a leverage concept, which is a concept that underlies many of the errors that learners made in a topic, for example: the equal sign and the differences between equations, expressions and formulae. Once a concept was chosen, the DIPIP facilitator found literature on that concept, including learner errors in the concept. The community read and discussed these papers and drew on these discussions to plan lessons together. The lessons aimed to surface learner errors in the topic and to find ways to engage them, rather than to avoid them. These lessons were taught and videotaped and the community then reflected on episodes in each teacher's lessons in order to understand their strengths and challenges in dealing with learner errors in class. In some years, communities changed the order of activities or emphasized some more than others.

DATA COLLECTION AND ANALYSIS

The TPD part of the project lasted from 2011-2014 and the research analysis and writing up continued afterwards. A number of postgraduate students investigated aspects of the above research questions and also developed their own question in relation to the data. Over the four years of the project 12 schools and 50 teachers participated with consistent participation for at least three years from 22 teachers in six schools in four communities - one community was made up of three neighbouring schools.

The main data sources for the above questions were the teachers' lessons, the community conversations and interviews with teachers. A total of 223 lessons from 19 teachers over four years, totaling more than 150 hours of lessons, were analysed using the Mathematics Quality of Instruction instrument (Hill et al., 2008). The MQI instrument has three levels, and we followed a methodology whereby we allocated numbers 1, 2 and 3 to each level, scored eight-minute episodes in each lesson, averaged across the episodes in each year for each teacher (Chauraya & Brodie, 2017; Koellner & Jacobs, 2015), calculated differences for each teacher across the years, and then calculated averages for each community. Since there were big differences across communities, we report on these separately. The requirements to reach level 3 were high, and only a few teachers accomplished this a few times, so we took a shift of 0,5 as a big shift in practice. Additional analyses, using the same instrument, were done by Molefe (2016) and Chauraya and Brodie (2017) over shorter periods of time and complemented with qualitative analyses.

The community conversations were video- or audiotape data from the four communities over two years. A total of 47 sessions involving 25 teachers and 55 hours of conversations were analysed, using a coding tool developed by the project to analyse the relationships between activity, content and depth (Chimhande & Brodie, 2016). In addition, Marchant (2016) analysed 17 conversations totalling about 20 hours of conversation and developed rubrics based on content knowledge and pedagogical content knowledge to analyse these conversations.

To understand why teachers chose to stay with or leave the project, we conducted eighteen in-depth interviews, with six teachers who stayed in the project for two years, seven who left within or at the end of the first year and the principal or deputy-principal in each school that the above teachers came from. We anlaysed these interviews using the features of PLCs from the literature: focus; long-term inquiry; collaboration/collectivity and leadership support. A notion of professional agency can account for the differences among the teachers across these features.

FINDINGS

Teacher Change

Community 1 made mid-level changes in two dimensions from 2011 to 2012: working with students and mathematics (0,31); and student participation in meaning-making and reasoning (0,17). Both shifts declined in 2013. The shifts could be accounted for by two of five teachers, both of whom made major changes to their teaching in 2012, and both of whom participated much less in the community in 2013, one because of illness. The other teachers' practices remained fairly stable over the four-year period. Community 2 made changes in the range 0,2 – 0,5 over each of two years 2012-2013 and 2013-2014 (they started the project in 2012) for three dimensions: mode of instruction, richness of mathematics and working with students and mathematics and in 2013-2014 in one additional dimension: student participation in meaning-making and reasoning. The changes are accounted for by six out of the nine teachers shifting in some or all of the dimensions, with three teachers not making major changes over the two years. In community 3, we saw changes in the range 0,5 to 1,5 in the first year in four dimensions[1] and somewhat lower changes, but still positive – around 0,2 in the second year. All of the five teachers made changes to their practice in the first year, with three continuing these changes and two declining in the second year. An analysis by year in the project over all communities shows changes of between 0,2 and 0,7 in the first year and these steadying off in the second. Chauraya's analysis (2017) of the pilot school in the project in 2010 showed substantial shifts for two out of four teachers in three dimensions with one teacher maintaining her shifts. So overall we see sustained shifts for about half of the teachers. It should be noted that very few projects have looked at teacher change in relation to PLCs (Vangrieken, Meredith, & Kyndt, 2017; Vescio, Ross, & Adams, 2008) and those that have, have shown modest shifts in practice, only some of which are sustained (Borko et al., 2015; Boston & Smith, 2011; Koellner & Jacobs, 2015).

Community conversations

The community conversations were analysed according to four content areas: learner, mathematics, practice, and learner thinking, where learner and learner thinking are distinguished by the extent to which the teachers considered the underlying reasoning for learners' errors. A description of learner errors counted as "learner" and an analysis of underlying reasoning counted as "learner thinking" (see Chimhande & Brodie, 2016 for more detail on these codes). Mathematics refers to teachers working on their own mathematical content knowledge, whereas the other three content areas relate to pedagogical content knowledge. The anlaysis was done for four key activities in the project: test analysis; reading and discussions; lesson planning; and lesson reflection.

A focus on learners, both understanding (42%) and thinking (37%), was most evident in the test analysis, because the test analysis distanced the focus from the teachers themselves and their lessons so that more focus was on the learners. A focus on mathematics was most evident in Lesson Planning (45%), because in these sessions, teachers often worked on the tasks and discussed them without

[1] The MQI instrument has five dimensions. I have excluded the dimension: errors and imprecision here as it requires a somewhat different analysis.

reference to learners, developing their own content knowledge. When planning their lessons, teachers spent 29% of their time on practice and much less time was spent on learner understanding (10%) and learner thinking (12%). A focus on practice was most evident in Lesson Reflection (56%). The lower percentages for learner thinking (13%) and learner understanding (19%) show that it was difficult to remove the focus from teachers themselves onto the learners when analyzing lessons. In lesson reflection sessions, teachers' conversations were mainly about how they dealt with learner responses in class and PLC members suggested ideas about how teachers could have dealt with learner responses differently, rather than focusing on the reasoning underlying the errors (Brodie, 2014). These findings suggest that the project made time for both content and pedagogical content conversations, and that a focus on pedagogical content knowledge did support teachers to work on their own content knowledge. Marchant (2016) found that about two-thirds of the conversations in one community in their third year were PCK conversations, and one-third were CK conversations. The PCK conversations occurred in test analysis and lesson reflection while the content conversations occurred more in the lesson planning thus confirming the above finding. Many PCK conversations triggered CK conversations (see also Brodie, 2014), showing that the TPD goal of supporting teachers to work on content knowledge, through pedagogical content knowledge, was achieved.

One main focus of the analysis was to see whether teachers spoke more about learner thinking over time. For the most part, they did not, with the percentage of time devoted to talk about learner thinking dropping from 24% to 13% from the first to the second year. There was an increase in talk about mathematics from 15% to 33% over the two years. Conversations about learner thinking declined substantially in Test Analysis and Lesson Planning, and the decreases were accompanied by an increase in talk about mathematics, particularly in Lesson Planning, from 24% to 71%. Talk about learner thinking increased in Lesson Reflection, as did talk about learners. This increase was accompanied by a decline in talk on practice, suggesting that teachers were beginning to shift their reflections from themselves to their learners when looking at their lessons, a finding that indicates some teacher learning in relation to the goals of the project.

Participation in the project

An analysis of the interviews with the participating and withdrawn teachers and their principals (or deputies) shows some interesting commonalities and differences. Almost all of the teachers indicated that they benefited from the collaboration in the communities and all of the teachers indicated that the biggest challenge to participation in the communities was time - with difficulties in finding time to meet during the school day, and other commitments, both professional and personal, taking precedence after school. A major difference between the two groups of teachers was that those who stayed with the project found the focus on learner thinking interesting and useful for their practice, while those who withdrew, saw the initial test analysis as "more marking" and thinking about learner errors as taking them backwards in the curriculum. The teachers who withdrew found that the time and energy required by the project created demands additional to, and outside of, their teaching work, while those who stayed, found that the project activities added value to their practice. While both sets of teachers made choices and exercised agency in relation to their participation in the project - to leave or stay - for some of those who stayed, their agency produced benefits in their teaching practice, as shown above, as well as in their conversations about teaching. We cannot know about those who left, as we did not continue to follow them.

DISCUSSION AND CONCLUSION

The findings show a number of generalities across the project as well as a number of nuances, as can be expected of a project of this size. In trying to understand changes across communities and teachers, as well as difference among them, it is useful to return to the notion of professional learning in relation to curriculum reform. Professional learning requires learning in relation to the knowledge base and practice, and the interaction between local and global concerns (Jackson & Temperley, 2008). It requires some forward thinking and choices among alternatives as to what learning might produce different results, and what is important for teachers, their schools and their learners. Wanting to learn more, based on one's own practice as well as external input, is an important element of agency for professional learning.

Professional agency is intimately linked to professional accountability, which means accountability to colleagues and clients in relation to the knowledge base, and working collectively to take the profession forward (Brodie & Shalem, 2011). While all professionals must work with some autonomy in making professional judgements, particularly about their own learning, they also must be responsive to the needs of the learners in their classrooms, their schools and society. This is particularly the case with curriculum reforms and ambitious teaching, which require ambitious professional development if they are to succeed. Our project suggests that many teachers who stayed with the programme were able to shift their teaching in the direction of responsiveness to learners.

Professional learning and professional agency require that teachers are positioned by society and schools as professionals. We have seen contradictory messages in this regard in relation to curriculum reforms in South Africa with regard to professionalism, with more recent curriculum reforms pulling back on professionalism and becoming more prescriptive. Yet PLCs are on the agenda for TPD reform (Department of Basic Education & Department of Higher Education and Training, 2011), suggesting mixed messages from different parts of our education system. The messages that teachers receive about PLCs, suggest variability in agency supported by communities in different districts. In some cases PLCS are strongly controlled while in others, they are supported to have some professional autonomy.

A key issue in PLCs is the extent to which school hierarchies are reproduced. Studies have shown that strong hierarchies work against effective learning in PLCs (Schechter, 2012; Wong, 2010) because the community is not seen as a collective. In this study, the two communities that shifted most in their practices were each located in one school, and in both cases the head of department, and in one the deputy principal, were active participants in the community. The conversations in community 1 - made up of three schools, and which did not shift strongly as a community - often resembled "teaching episodes", with one member taking on the task to "teach" the others, whom he perceived as less knowledgeable in content knowledge. In the two other communities, the heads of department, while taking some leadership and giving support, were just as interested in learning as the others, suggesting strong accountability to each other and to the PLC, as well as to learners who might benefit from the new knowledge generated in the PLC.

A key finding that needs explanation is the fact that teacher talk about learner thinking declined over time, and yet many teachers shifted in their practices in the direction of engaging more with learners and supporting learners to reason. Chauraya and Brodie (2017) have shown how the activities in one community can be linked to shifts in practice of the teachers and the analysis of community conversations discussed above suggests that the activities allowed for different foci across the four content areas. So the activities as a developmental sequence supported a focus on key areas of teachers' content knowledge, pedagogical content knowledge and a shift in their practices. We have argued that this shift was made possible by content knowledge being strongly related to pedagogical content knowledge through the design of the TPD programme (Brodie, Marchant, Molefe, & Chimhande, 2018; Brodie & Sanni, 2014).

In conclusion, I have argued that a model of extended inquiry in PLCs, with a developmental sequence of activities that focus teachers on different aspects of their knowledge and practice, can be a useful method of TPD for curricuum reforms that encourage responsiveness to learners. PLCs can be positioned differently by teachers and schools, and for them to work it is important for teachers to be positioned as professionals with strong notions of professional agency and accountability.

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PRESERVICE MATHEMATICS TEACHERS' PERCEIVED BELIEFS ABOUT MATHEMATICS TEACHING: A CASE STUDY IN NUMBER AND OPERATION IN SCHOOL MATHEMATICS COURSE

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The paper aimed to explore preservice mathematics teachers' perceived beliefs about Mathematics Teaching. Target group was twenty-seven 1st year preservice teachers who studied in Mathematics Education Program, Faculty of Education, Khon Kaen University. Data collected on Number and Operation in School Mathematics Course in the first semester, 2017. Data gathered from open-ended questions and self-reflection on every activity. Content analysis employed to analyze the data. The result revealed that the target group had perceived beliefs related to traditional mathematics teaching such as mathematics teaching focused on product and conveyed knowledge to students. On the other hand, perceived beliefs related to lesson study and Open Approach focused on process and students' ideas, problem situation based on real world and open-ended problem. Teaching style focused on posing problem situation, observe students' problem solving process or thinking process.

INTRODUCTION

School mathematics reforms are often conducted with changes in all difference aspects of the curriculum (ICMI Study 24, 2017). The currently movements to reform mathematics education need to rethink of mathematics teaching and teacher education program. These reform efforts tend to oriented toward increasing teacher education and support teachers who were educated under the traditional system of mathematics instruction to implement envisioned reforms to classroom (Simon, 2000). Many countries have reformed teacher education program implementing pedagogical content knowledge into curriculum such as Korean, Hong Kong, Singapore and Taiwan (Park, 2005). Inprasitha (2008) stated that teacher education program in Thailand has neglected significant research suggestions of teachers' knowledge. Thus, teacher education program should be reconsidered the structure of curriculum, which should emphasize on acceptable theoretical concepts in world mathematics education society and consistently adapt to meet requirement of the act of the basic education core curriculum. Both national and international teacher educations are undergoing closed scrutiny and over the last three decades there have been many moves to reform mathematics education (Clooney & Cunningham, 2017).

Mathematics Education Program, Faculty of Education, Khon Kaen University has improved and developed teacher education program in both undergraduate and graduate levels *(*Inprasitha, 2008; 2009*)*. These program challenged to design a new type of teacher education program *(*Inprasitha, 2012*)*, made a distinctive program by defining the major course of the specific course into three categories, that is; collegiate or advanced mathematics, school mathematics 24 credits such as Number and Operation in School Mathematics, Algebra in School Mathematics, and mathematical learning processes 21 credits such as Problem Solving in School Mathematics, Representation in School Mathematics, and related courses *(*Inprasitha, 2015*)*. There were some ideas that seemed impossible to implement in this new teacher education program at the beginning, such as we could not design courses related to mathematical learning processes separately from courses related to content. However, it was made workable by the implementation of two innovations, Lesson Study and Open Approach (Inprasitha, 2011; 2015).

The most important challenge in teaching and teacher education is in shifting views from traditional way to reform teaching and in the creation of communities of learning requiring the development of norms to guide program's and consistency (Tatto & Coupland, 2003). Richardson (2003) stated that the focus in the preservice teacher education literature has been on the very strong beliefs about teaching and learning that students brought into their programs with them based on 12 or more years as students in formal education. It could be claimed that teachers' beliefs influence their teaching practice (Thompson, 1992). However, change in teachers' beliefs might not lead to change in their practice. The most permanent change would be the result from professional development experiences that provide teachers with opportunities to coordinate incremental change in beliefs with corresponding change in practice (Philipp, 2007).

As mentioned above, this study aimed to explore preservice teachers' perceived beliefs about mathematics teaching during they participated in activities in a course of Number and Operations in school mathematics. The course was a part of school mathematics and relate to mathematical learning process such as Problem Solving in School Mathematics course.

METHODOLOGY

Participants of this studied were twenty-seven preservice teachers who studied in 1st year at Mathematics Education Program, Faculty of Education, Khon Kaen University. The data was collected during Number and Operation in School Mathematics Course in the 1st semester, 2017. As Inprasitha (2015) mentioned that the textbook and the pedagogical approach, are the most significant factors that affect the teaching of mathematics in Thailand. So, the course prepared for activated 1st year preservice teachers to recognize the difference between traditional and the new teaching style. This course focused on 1) analyzing mathematics textbook between Thai and Japanese textbook (see Fig.1), 2) watching and analyzing video of mathematics classroom innovation through lesson study. This mathematics classroom videos were a product of APEC Lesson Study Project (Isoda & Inprasitha, 2007) (see fig.2) and 3) observing mathematics classroom and reflection (see Fig. 3). Data collection consisted of open-ended questions and self-reflection on every activity.

Fig. 1: 1st years preservice teachers worked together to analyze Japanese mathematics textbook

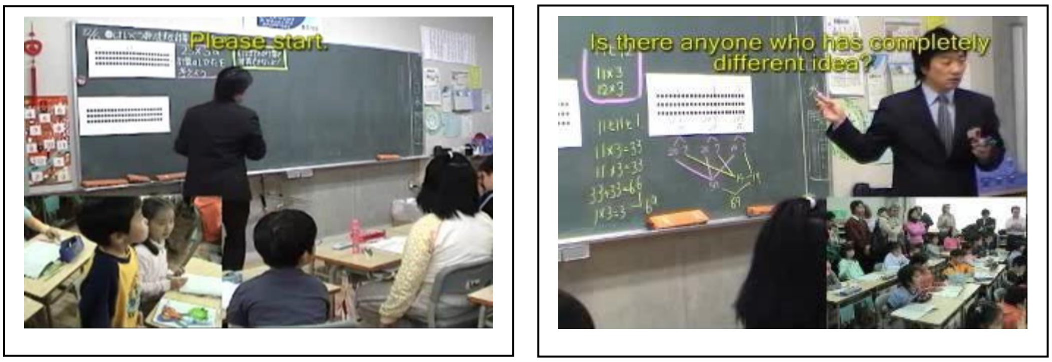

Fig.2**:** Watching and analyzing video of mathematics classroom innovation through lesson study

Fig. 3: Preservice teachers observed student intern mathematics class which taught by using lesson study and Open Approach. After the class, they attended reflection by lesson study team and experts.

Content analysis was used to analyze the data**.** Preservice teachers**'** perceived beliefs referred to those beliefs regarding mathematics teaching in which preservice teachers gradually become conscious of after they participated in number and operation in school mathematics course**.** Perceived beliefs about mathematics teaching was investigated into two categories; 1**)** traditional mathematics teaching and 2**)** mathematics teaching related to lesson Study and Open Approach**.**

RESULTS

Perceived beliefs related to traditional mathematics teaching

The results revealed that all preservice teachers perceived that traditional mathematics teaching focused on products such as right or wrong answer and teachers' roles focused on conveying knowledge to students (see Table1).

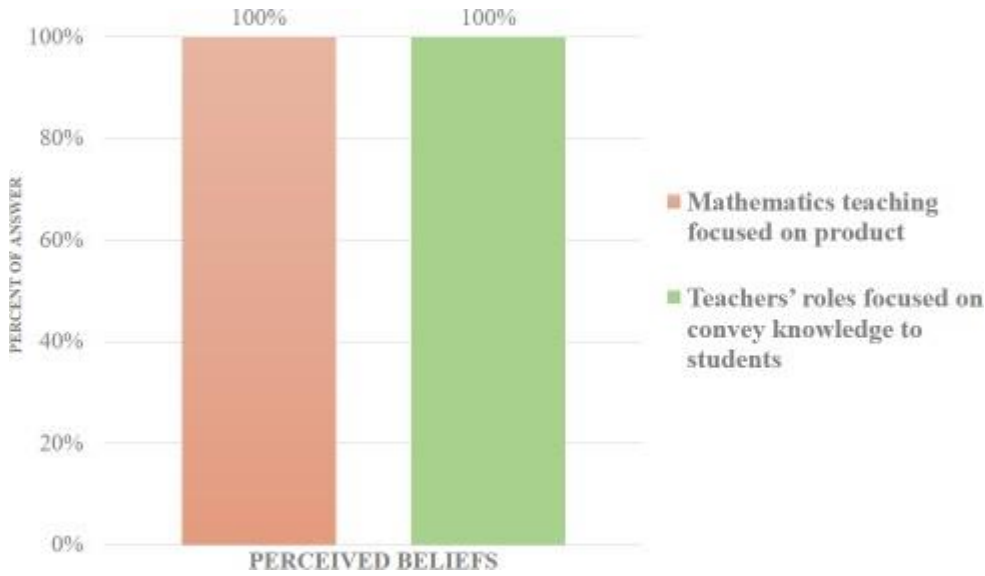

Table1: Perceived beliefs about traditional mathematics teaching

Some reflections were as below;

PT1: Mathematics teaching emphasized on teaching content. Students were taught to solve problem with fix solution or calculation. Teachers were not rather focusing on providing students to think by themselves.

PT2: Traditional mathematics Instruction concentrated on the answer which correct or incorrect.

PT3: In the past, teachers' role in mathematics teaching was to talk, describe and convey knowledges to students. They taught students to answer not to use thinking process.

Moreover, teachers' roles focused on conveying knowledge to students by lecturing or explaining such as some examples below;

PT5: Previously, I thought that mathematics teaching would have to provide definition, method and concept to students first to solve problems. This teaching style made them learn.

PT6: Teachers conveyed knowledges and thinking methods to students. Students memorized and use those methods to solve problems.

PT7: Mathematics teaching was focus on memorizing and doing problem. Teaching style based on teacher center by conveying, describing or transferring knowledge to students.

Perceived Beliefs about mathematics teaching related to lesson Study and Open Approach

The results revealed that preservice teachers' perceived beliefs could be categorized as the followings;

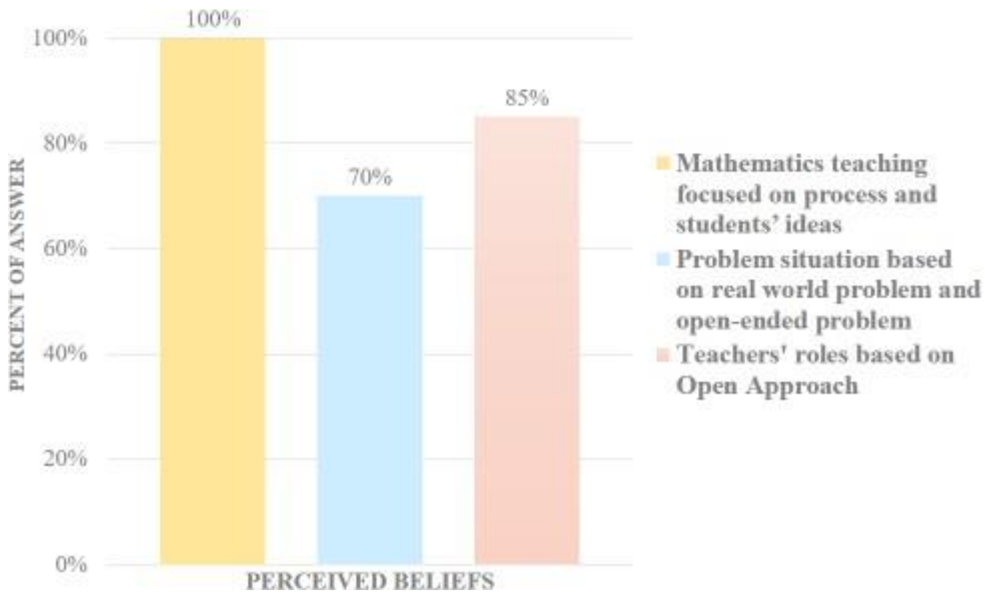

Table 2: Perceived beliefs related to mathematics teaching based on lesson study and Open Approach

1) Mathematics teaching focused on process and students' ideas. Some examples were as below;

PT2: Mathematics teaching emphasized on process and students' thinking methods.

Teachers created open-ended problem situation and provided a time for students to solve the problem by themselves. Teachers did not intervene students' ideas. So, students could create various ideas.

PT9: Mathematics teaching emphasized on thinking process. Students think by themselves then teacher collect their idea to summarize in whole class at the end of period.

2) Problem situation based on real world problem situation and open-ended problem such as below;

PT11: There are problem situations which related to students' daily life. Let students could think by themselves and do not resistant their thinking. Students enjoy to learn mathematics and they could express their ideas.

PT12: There are problem situations which have been created relate to students' daily life. These problem situations could encourage students' understanding.

3) Teachers' role were as followings;

PT13: Teacher posed problem situations for students to analyze and find the answer by using reasons to support their answers. Teachers listened to students' answers then shared to others at the board and asked others whether they agreed with this answer. Also teachers encouraged students to describe the reasons to classmates for understanding and believing in that. This teaching style indicated that students to solve problems by themselves and they could gain learning process.

PT14: Learning from Open Approach, Teachers should prepare content earlier and connect the content of each period. Moreover, they should have reflection from working with their teamwork. Teachers just post the problem situations and let students solve by themselves whereas teachers observe students' thinking and solving process. Finally, Teachers and students summarize contents on whole class to make them understand on the same way.

Concluding Remarks

Perceived beliefs that traditional mathematics teaching focused on product and conveyed knowledge to students reflected preservice teachers experiences during they studied mathematics in school for 12 years. It also reflected to the majority of in-service teachers taught by placing emphasis on lecturing or explaining laws and formulas and showing examples before assigning exercises for the students to do (Changsri, 2012; Inprasitha & Changsri, 2013). After they had experiences in analyzing Thai and Japanese mathematics textbooks, watching video of mathematics classroom innovation through lesson study and observing mathematics classroom using lesson study and Open Approach, they started to build new form of beliefs such as mathematics teaching focused on process and students' ideas, problem situation based on real world and open-ended problem. Teaching style focused on posing problem situation, observe students' problem solving process or thinking process. These results will be shared with teacher education program in the Capacity and Network Project (CANP) such as Lao PDR. and Cambodia.

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NEW MODEL OF TEACHER EDUCATION PROGRAM IN MATHEMATICS EDUCATION: THAILAND EXPERIENCE

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After the 1999 Educational Acts were enacted, Thailand adopted an educational reform movement. A national agenda in this educational reform was "to reform students' learning processes". In this regard, there has been attempting to initiate many types of teacher education programs among education faculty of most universities in responding to this demand. This paper reviews the traditional Thai approach to teaching mathematics, which is the consequence of the traditional teacher education and teacher training programs, and considers the effectiveness of a new model of teacher education to be implemented as part of the Mathematics Education Program. An exemplar illustrated students' activity during the Process of Problem solving Course.

INTRODUCTION

Teaching mathematics in Thailand for most of the teachers means preparing lesson plans by themselves, teaching those lesson plans in their closed classroom, checking the assigned homework, making some quizzes, and prescribing exercises. To teach, each teacher starts by explaining new content, giving some examples, then giving students some exercises, and assigning some homework, or demonstrating, questioning, describing and lecturing (Kaewdang, 2000; Khammani, 2005; Inprasitha, 2011) as shown in the figure 1. These kinds of activity have become a part of their own classroom culture and this is consistent with what Stigler and Hiebert (1999) mentioned, "Teaching is a cultural activity."

The teachers' roles in teaching, as mentioned above, are influenced from the teachers' understandings related to mathematical meaning as Dossey (1992) has mentioned that various comprehension for mathematical conceptual understanding are extremely important in development and successes in mathematics teaching and learning in school, and research understanding in school mathematics. An understanding that a nature of mathematical knowledge is outside the teachers and students (Plato, 1952 cited in Dossey, 1992) has made Thai mathematics teachers play roles of transmission their knowledge or contents to the students (Office of the Education Council, 2013). All new contents in mathematical textbooks, therefore, are things that the students have not known before. Beginner teachers' roles in classroom are trying to describe, lecture, ask short questions, etc. cooperated with using teaching materials as need, and the students just only receivers such knowledge, or a passive learner (Narot et al., 2000; Office of the Education Council, 2013). Relationships in the classroom are as Sekiguchi (1997) and Sierpinska (1998) has mentioned that the students' response is short and the teachers are evaluators of the student's response. Certainly, this kind of teaching activity cannot respond to the new demand of knowledge and skills for 21st century, which most countries around the globe are struggling for (Levy and Murnane, 2004).

NEW MOVEMENT IN TEACHING APPROACH

Before entering the 21st century, there have been many attempts to shift the paradigm for teaching, especially the way to teach mathematics from an emphasis on teacher-centered to students-centered approach (Calkins and Light, 2008). However, for many decades, the idea of student-centered has been taken for granted and seem not to be clarified when implementing the real classroom. In mathematics education, the development of mathematics teaching approach has been centering on the reconciling among these issues: new aspect of mathematics (Polya, 1954; Becker and Shimada, 1997), students' individual differences (Graff and Byrne, 2002), and problem solving as a teaching approach (Polya, 1954; Nohda, 1991; Becker and Shimada, 1997; NCTM 1980; Singapore MOE, 1990; Korea, 1997; Finland, 2004).

For example, in Japan there has been an endeavor to teach by emphasizing on mathematical thinking (Becker and Shimada, 1997) as appearing in the course of study since 1957 (Ueda, 2013). In Japan there is an emphasis on classroom teaching practice in cooperation with a focus on students' mathematical thinking. This approach is not without its challenges, not only because of the difficult mathematical content, but also due to individual differences. Therefore, this teaching approach has shown to be problematic to implement in classrooms around the globe (Isoda and Nakamura, 2010). During the 1970s, Japan developed a new teaching approach with the emphasis on students' mathematical thinking (Isoda, 2010). By changing the focus from that of the correct answer, or closed problem, to that of teaching to assess the students' mathematical higher order thinking (Shimizu, 1999). An interesting point of this change in the Japanese history of mathematics education is that the teachers consider the assessment first. The problem, that remains, is how to overcome the individual differences especially thinking differences between students (Takahashi, 2006; Mizoguchi, 2008; Miyauchi, 2010).

The new teaching approach has revealed an important step in the teaching process. It was found that by using an open-ended problem at the start of the lesson, teachers were able to focus the student's thinking process on the task at hand. An important point of this teaching approach is all students have their own problems, or the problems are not a given, and this can make the students solve the problems by themselves and drive, or foster, the students to think by themselves (Brown and Walter, 2005).

From these ideas, Inprasitha (2003) has been proposing a paradigm change in the Thai teaching approach from that which was mentioned in the early part of this paper to be an Open Approach incorporating in Lesson Study (Inprasitha, 2011).

INITIATIVE TEACHER EDUCATION PROGRAM IN MATHEMATICS EDUCATION

Since the new national agenda, *"Reforming Learning Process"* of the 1999 Educational Act, was declared a decade ago, the mathematics teacher education programs of most universities in Thailand has not been able to respond to this demand. Moreover, school teachers in our country lack both the professional tools to use in their daily teaching practices, and the professional learning community to participate in, that would enable them to continue their professional development. Certainly, this is a consequence of our traditional teacher education program. Regarding this point, there are many crucial aspects of the educational reform movement as in many countries. Among other things, professional development of teachers is a central issue. Teachers need to learn how to capture students` learning processes and to examine their own practice, etc. However, we lack clarity about how to best design initiatives that involve the examination of practice (*cf.* Ball, 1996; Lampert, 1999; Shulman, 1992; Fernandez et al., 2003).

Most teacher education programs in Thailand simply consist of three components: General Education courses, Specific courses and Selective courses, without describing the theoretical foundation of these elements components. During the last decade (2004-2013), the Faculty of Education, Khon Kaen University was challenged to design a new type of teacher education program. Based on the idea of Pedagogical Content Knowledge (Shulman, 1986; Park, 2005; Inprasitha, 2012), we made a distinctive program by defining the major course of specific course into three categories, that is, collegiate or advanced mathematics, school mathematics, and mathematical learning processes related courses (see Table 1). The idea for school mathematics is based on what Klein (2004a, 2004b; NCTM 1989, 2000) mentioned about elementary mathematics, that is, "Elementary mathematics has to be seen from an advanced standpoint" and the idea for mathematics learning processes related course come from what stated in NCTM standards (1989, 2000). Moreover, the program is intentionally planned to construct based on the idea of educational values and educational theoretical frameworks: reflective thinking (Dewey, 1933) and community of practice as a learning community (Lave and Wenger, 1991).

Component of the Program	**Credit***
Collegiate Mathematics Courses	36
School Mathematics Courses	24
Mathematical Learning Process Courses	21
Professional related Courses	54
General Education	30
Selective Courses	6
Total	**171**

Table 1**:** Mathematics Education Program of Faculty of Education in Khon Kaen University **(**2013**)**

Based on the perspective on values education, four cored values have been selected: valuing product/ process- oriented work, rather than only product- oriented work, attitudes towards collaboratively working, open- minded attitudes, and public concerned attitudes. The idea was practically implemented in the simplest fashion by allowing students the time to reflect upon whatever activity they had done. The central issues are on *"reflection"*, rather than what they had done. They were trained since their first year through a variety of activities such as Children's day (Y1), Math Camp and Sports Day (Y2), Study Tour (Y3), School Visit (Y4), and Internship (Y5) during which time the four cored values have been nurtured. The idea of community of practice brought into the program is the focus on individual participation in every activity (see figure 1).

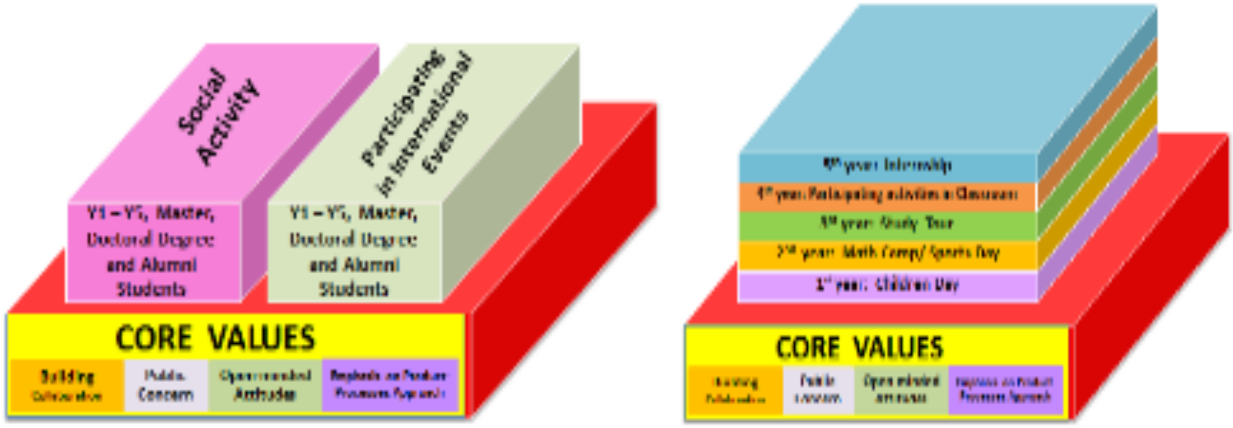

Figure 1. An Example of How to develop Core Value through 5 Year Teacher Preparation Program in Mathematics Education (Inprasitha, 2006)

There were some ideas that seemed impossible to implement in this new teacher education program at the beginning, such as we could not design courses related to mathematical learning processes separately from courses related to content. However, it was made workable by the implementation of two innovations, Lesson Study and Open Approach (Inprasitha, 2011) (see Figure 2.). Another seemingly impossible idea was to link the prospective teacher education program with the in-service teacher program. Building up this idea through the implementation of using lesson study as a professional learning community, experienced school teachers can work collaboratively with student interns and both groups have formed habits of '*teacher learning together*' and formed their long-term professional learning community.

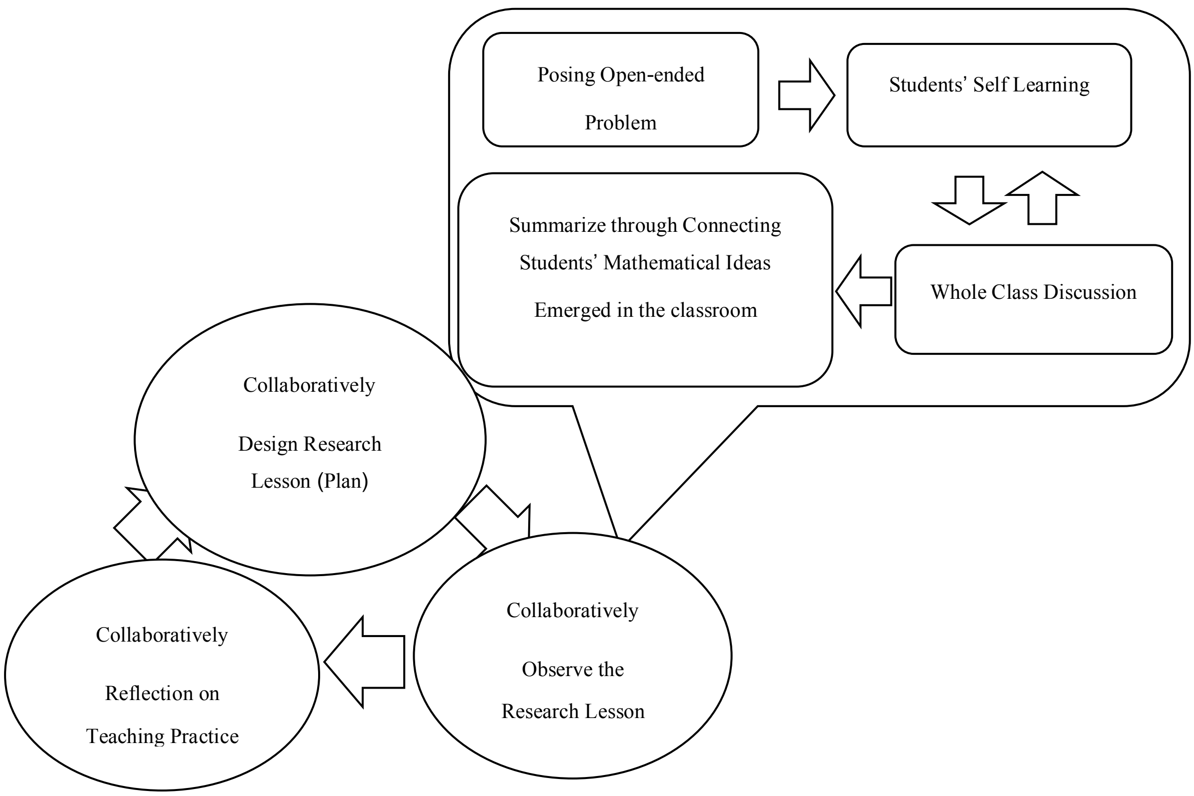

Figure 2. Lesson Study incorporating Open Approach (Inprasitha, 2011)

Exemplar of Activities in Process of Problem Solving in School Mathematics Course

This exemplar illustrated students' activity during the *Process of Problem Solving Course*. Students were divided into groups and worked to solve a problem by themselves. Each group included students who act as problem solvers and observers.

A learning unit was designed within the *"cylinder problem"* (Tsubota, 2005). This problem was a typical open-ended problem originated in Japan. A teacher and teacher assistants collaboratively planed this problem together. During the class all of them observed students' ideas. After the class, they had a reflection about the problems, students' ideas and the way to improve this problem. Four steps of Open Approach is used as a teaching approach as the Figure 3.

Cylinder Problem:

Session 1: From a given A4 paper and a model of cylinder with two lids, Imagine and sketching the cylinder to be cut and uncovering in the geometrical plane (not more than 5 minutes)

Session 2: 1) From another 2 A4 colored papers, sketching, designing an uncovered cylinder (cutting using scissors by one cut), then make a cylinder with fitted lids.

2) Paste the trace of cylinder cut on another A4 colored showing the trace to be cut.

1) Posing Open ended Problem

2) Students' Self Learning

4) Summarization

3) Whole Class Discussion and Comparison

Figure 3. Four steps of Open Approach used as a teaching approach
in *Process of Problem Solving Classroom*

CONCLUDING REMARK

This valued-driven teacher education program has been built upon major core concepts: problem solving as a driving force for mathematical thinking, teachers learning together, and reflection using two innovations; Open Approach as a teaching approach and Lesson Study as a way to improve teaching. When implementing the program in actual classroom, "*critical reflection*" has been highlighted throughout the program. For student teachers, first-hand experience learning through this teacher education program will form their attitudes toward learning when they teach in their own classroom. Moreover, student teachers will also learn the way to implement innovations in the classroom. With these innovations bringing into schools, they will become partners of school teachers and take part on academic leadership in schools. After a decade of implementation, the author become realized that beginning teachers is an important period and very crucial to smoothly land into professional teaching career.

Acknowledgements

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PRIMARY SCHOOL TEACHERS' KNOWLEDGE ON MATHEMATICAL TASK DESIGN THROUGH LESSON STUDY AND OPEN APPROACH

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The research aimed to investigated teachers' knowledge on mathematical task design through Lesson Study an Open Approach. The target group was teachers from Attached Primary school of Pakse Teacher Training College in Lao PDR. This target group had different two groups and experienced in employing innovation through Lesson Study and Open Approach. Data collected in the second semester of the 2016-2017 academic year. Research Methodology base on Lesson Study an Open Approach. The data were from video recording and taking field notes during the process of Lesson Study. The data include interviewing teacher and students' worksheet. Data analysis base on the three-step flow of lesson (Inprasitha, 2016). The result revealed that the targets group teacher did not participant in employing innovation through Lesson Study and Open Approach followed four phases. They lack of and need to improve on mathematical content knowledge and creating knowledge on teaching materials (Semi-concrete Aids).

Key words: Mathematical task, task design, teacher knowledge, Lesson study, Open Approach.

Introduction

Lee, Lee & Park (2013) mentioned teacher's knowledge was a potential factor because it was used to analyzed and adapted task design implementing in the classroom. Knowledge created concept development in specific content scope. As a result, Teachers' knowledge modification and task design issue was meaningful for students. It was necessary to improve in teacher education.
The teachers, researchers and mathematical communities taught mathematics were interested in task design because it was important in research perspective and mathematics education practice (Job and Schneider, 2013; Wason and Ohtani, 2015). The mathematical task was important for learning and teaching and it supported mathematical learning environment (Simon and Tzur, 2004; Clarke and Roche, 2010). Task design was a tool shaped thinking development, mathematical reasoning and encouraged students to have higher learning result (Stein et al., 1996; Shimizu et al., 2010; Henhaffer, 2014). According to Fujii (2013) pointed out that the task design through Lesson Study related to activities, namely students' problematic solution expectation when they wrote the lesson plan, classroom observation and the effectiveness evaluation of students' problem solution.

Lao PDR had practical training about Open Approach and Lesson Study for Mathematics and science teachers in 2002 (Hoshino's Project) (Inprasitha, 2007). Later, The Lesson Study was introduced in Lao PDR, 2004, supported by JICA (Japan International Cooperation Agency). This educational innovation employed to develop learning and teaching in Mathematics and Science to increased knowledge of collegial teaching (Saito, 2007). In the contrast, teachers were unable to get in depth understanding of teaching methodology and classroom management and teaching as student centered (Ministry of Education and Sports, 2014). It supported educational policy and encouraged teaching as student centered through Lesson Study in Teacher Institutes (TTCs) (Ministry of Education and Sports established the teacher department, 2014). Then the Lesson Study was studies in classroom. For example, Linphitham (2009) did studies about Teacher's belief in Mathematics classroom through Open Approach in Lao PDR, Xayyavong (2016) did studies Teachers' role in Mathematics Classroom at Primary Comprehensive Demonstration School, National University of Laos. Phailath et al. (2017) did two studies the effectiveness of using Lesson study to build the pedagogical knowledge and improve the teaching in the mathematics lesson and Professional development of mathematics teacher through lesson study. Thephavongsa (2018) did studies Enhancing the Teaching Skills of the Multi-Grade Teachers through Lesson Study And, Shingphachanh (2018) did studies Teachers' understanding and concerns about the practices of lesson study in suburb schools in Laos.

Lesson Study supported teacher community and improved knowledge linked to other knowledge such as content knowledge, pedagogical content knowledge, knowledge of students. This knowledge could be improved and employed in the classroom (Murata, 2011). This study focus on mathematical task design knowledge. it meant knowledge application to discuss mathematical content to coherence with students' thinking process. It is designed relying on framework of mathematical activity from the real world to the mathematical world accordingly which comprised of 3 steps based on Flow of Lesson: 1. Representations of Real world 2. Semi Concrete Aids 3. Representations of Mathematical World (Inprasitha, 2016). Knowledge includes mathematical content knowledge and knowledge of mathematical student. The content knowledge of mathematical task design meant Knowledge application to discuss content. Mathematical content can be applied as a tool supporting students' problem-solving (how to learn). And, Knowledge about students in mathematical task design meant teachers' ability in problem-solving anticipation of students, learning difficulty anticipation, and emotional understanding in learning Mathematics and interests of students toward mathematical content (Inprasitha, 2016).

According to the importance and issue above, result of Cannon (2008) found that students teachers were lack of specific content knowledge on task design as well as Lao primary school teachers lack knowledge and skill for organizing the teaching and learning activities effectively. They do not have deep understanding of how to make or create a lesson plan which can help them to accomplish their learning purpose (Ministry of Education and Sports, 2014; Thephavongsa, 2018). And, there were not researches were done about teacher knowledge on mathematical task design in Lao PDR. Therefore, the researcher was interested to investigated case of primary school teachers' knowledge on mathematical task design.

The idea of Lesson Study and Open Approach

Lesson study played important role in teacher profession and education system development in Japan. It supported concreted learning and teaching and a new perspective for teachers. It assisted the teacher in lesson planning and develop teacher profession. (Takahashi et al., 2006) and Isoda et al. (2007) stated Lesson Study was not only training for teachers but also supporting teaching, and work collaboratively with teachers in making lesson plan (Baba, 2007; Inprasitha, 2014). Lesson Study was introduced and employed in schools in Thailand in 2002. It composed of planning, observation, and reflection of teachers when they finished their class. They assisted each other to figure out problems of mathematics activities through Open Approach (Inprasitha, 2011). The important heart of Open Approach focused on the different individual ability of students. It fostered and extended students' concept and part of Open Approach comprised of Situation Problem and Tasks (Inprasitha, 2014). This paper focus on Lesson Study meant teachers or researchers' document study and collaboration for planning for task design and then employed it with students in the classroom. Teachers observed students' behaviors on task during class and have a reflection about mathematical task design after class. (Inprasitha, 2011). And, Open Approach meant the type of teaching approach which employed open ended problem in creating the problem situation. The mathematical task had to encourage student's thinking development; they could choose the answers correctly to answer questions in learning Mathematics Based on Inprasitha (2011), the answer methods comprised of 4 concepts 1. Posing mathematical task, 2. Students solve the mathematical task by themselves, 3. Discussion and comparison in the classroom. 4. Mathematical concept conclusion in the classroom.

The idea of Flow of Lesson

Inprasitha (2016) state that the Flow of Lesson is conceptual of mathematical activity designing. The activities emphasized on mathematical employment real life into mathematical world. The real world is meaningful for learning. It delivers real world to semi concrete aids. Teachers need to interpret task or problem situation to raise understanding of Flow of Lesson. It based on Flow of Lesson comprised of 3 steps

1. Representations of Real world : mentioned to real world outside the classroom and meaningful for learning of student. It is transformed its features into real classroom. For example, picture of children playing in the field.
2. Semi Concrete Aids : Types of aids connected to the real world and mathematization move to Mathematical World. For example, picture block, pattern box, diagram
3. Representations of Mathematical World : Represents to abstract world leads to task or problem situation stimulates thinking process to deal with problem solving by using semi concrete aids to representations of Mathematical World. For example Symbol sentence and formula.

Material and Method

Target group: Five teachers who taught at Attachment Elementary School, Pakse Teacher Training College, Lao PDR. This target group had different two groups and experienced in employing innovation through Lesson Study and Open Approach. It included four phases as follows.

Phase I: The all target group participated in educational innovative through Lesson Study and Open Approach training. The training content was about mathematical textbooks emphasized problem-solving under the lecturing by Assoc Prof Dr. Maitree Inprasitha and professors from Center Research for Mathematics Education from August 28, 2016, to August 29, 2016, at Pakse Teacher Traininng College, Pakse District, Champasack Province.

Phase II: The two targets group participated in the field trip of classroom innovation in schools employed innovation under mathematical higher order thinking project from Center Research for Mathematics Education, Faculty of Education, Khon khaen University on November 7, 2016, to November 9, 2016. The project had run more than 10 years.

Phase III: The all targets group got involved in real practice in three months with two master degree students from Mathematics Education, Faculty of Education, Khon khaen University. The first period was to create context from February to March 2017. The second period was to collect data in employing innovation in the classroom in April 2017

Phase IV: The all target group studied and trained about educational innovation employment in the classroom. They learned analytic perspective and received the recommendation from Mathematical education experts, Khon Kaen University. They followed Master degree students' research progression when they employed and implemented this education innovation – Lesson study and Open Approach, at Attachment primary school, Pakse Teacher Training College on April 28, 2017.

Data collection: The procedure was in the second semester, started from February to April 2017. It detailed as follows.

The created context of the target group, teachers, and researcher assistant collaboration. The process conducted through Lesson study and Open Approach and then conducted data collection with prepared instruments by Video recording, camera recording students' worksheet, teachers' interview form and portfolio used for field note during the process of Lesson Study.

Data analysis base on Flow of Lesson comprised of 3 steps 1. (Representations of Real world) 2. (Semi Concrete Aids) 3. (Representations of Mathematical World) (Inprasitha, 2016).

Results

Specimen: the result of data analysis of teachers' task design and learning plan in activity "Plus 2". The naughty monkey

Problem Situation: "There were five monkeys were eating fruits and then there were six monkeys running to join the fruits.
How many moneys were there? "
Instruction: Students write sentence symbol and find the answers

Fig. 1: Problem Situation

Step 1

Teachers' knowledge on Representations of Real world

Mathematical content knowledge

Protocol in planning

1 T 3: Students made understanding of adding number to be ten
2 T 2: Students had to make understanding of adding the number to ten and then students
3 divided number. Students knew it was eleven and then told them to divide it.

The protocol of T3 and T2 showed content analysis about ten divisions. Teachers knew how to deduct the number and plus it. They wanted students knowing how to plus number. This meant teachers had mathematical content knowledge in Math solution.

Students' mathematical knowledge

Protocol in planning

1 T 1: Student would be heard addition because they used to learn before. There were six sum of
2 moneys running to five monkeys, they might solve the problem.
3 T 2: They could not write the answer and it would be in speech. Some students thought there
4 were two monkeys on the tree and left on the land and five monkeys were in speech.

The protocol of T1 shown teacher understood students' words for comparing "running to five monkeys." This meant number addition was familiar with students. T2 teachers assumed students' problem solving based on the picture of the monkey, students could explain about the number of monkeys when it added the number. The teacher tried to interpret students' thinking with number addition when another group of money running to join one.

Protocol in reflection

The protocol of item 7 showed the mathematical education perspective of experts in reflection. It reflected monkeys eating fruits was a real world for students. The monkey's picture was no problem, students could explain the picture. This meant students understood real world and picture. Item 5 reflected situation which encouraged students knowing monkeys and its real world, group separation eating fruit and another group running to join one. It meant teacher had knowledge of Students on represented of the real world.

Step 2

Teachers' knowledge of semi Concrete Aids

Mathematical content knowledge

Protocol in reflection

The reflection protocol in item 5: The experts' perspective of mathematics education, Faculty of Education, Khon Kaen University, reflected students added 4 + 5 or 9 + 6 in step and step in semi concrete aids. They connected concrete aids in vertical to help the student write sentence symbol. It reflected teacher were lack of mathematical content knowledge on represented semi concrete aids as student' worksheet shown when used task during class.

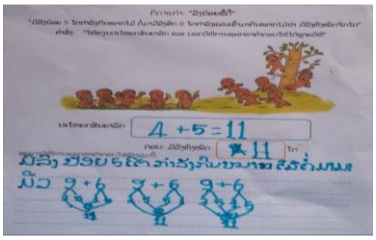

Fig. 2: The picture shown mathematical problem solution

Students' mathematical knowledge

Protocol in reflection

The protocol in the reflection of item 5: In the perspective of experts in Mathematics education, Faculty of Education, Khon Kaen University, reflected teachers' expectation deal with problem-solving how teachers eliminated problems when it occurred without expectation. It was difficult for teachers if they did not know to overcome the problems step by step. In addition, teachers did not considered which teaching materials to be used and when it was used in the classroom. It should be used to be matched with the learning environment and students as well. The reflection showed that teacher did not reach students in teaching and they did not enough Students' mathematical knowledge to task design on represented semi concrete aids.

Step 3

The knowledge on representations of Mathematical world

The mathematical content knowledge

Protocol in planning

T 2: The picture showed there were six sum of moneys on the tree and how added number to be ten. It was 5+5 and 1 equal 11

T 1: The first number was 5 and 6

The protocol of item 16 shown teachers analyzed content from monkey picture with number 5 and 6 and item 15 shown teacher analyzed content about how to add the number to be 10, with 5+5 and 1. According to number addition above, it is shown that teacher had mathematical content knowledge. They analyzed with Algebra incorporating the knowledge of task design on representations of Mathematical world

Students' mathematical knowledge

The field note of research assistant in planning

How does teacher assumes the students' concept in mathematical problem-solving in task design?

Students answered 5+6=11
Students enabled to divide number and added it

According to field note, it showed teacher could assume the students' concept in mathematical problem solving with questions created by teachers. Student enabled to answer 5+6=11 and they enabled to divide number and added it.

The information of teacher interview

Interviewer: The questions were created by teachers and it employed in the classroom. How did you think it enabled to solve mathematical questions and how was it difficult? How did teacher assume students' question-solving?

T1: Students enabled to write sentence simple as 5+6=11 or 6+5+11
T2: Students enabled to divide number and added it

According to teachers' interview form shown teacher had the ability to assume students' question-solving. Students enabled to write sentence symbol of Mathematics as 5+6=11 or 6+5=11. Students enabled to divide number and added it

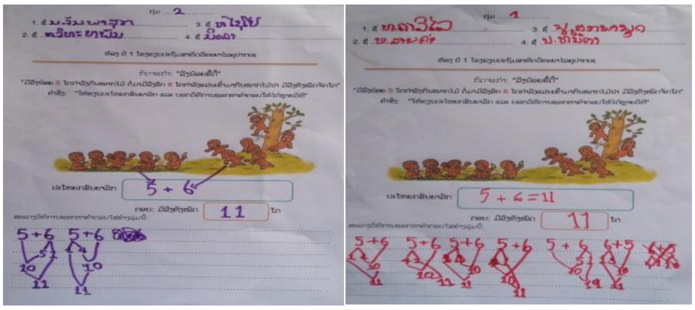

Fig. 3: The picture showed the result of mathematical question problems.

The picture illustrated the number division derived from sentence symbol writing of Mathematics as 5+6=11 followed expecting of teacher. It showed teacher had knowledge of Students in mathematical task design on representations of Mathematical world

Discussion and conclusion

This case of study to investigated primary school teachers' knowledge on mathematical task design found that

Targets group did participant in employing innovation through Lesson Study and Open Approach followed four phases. This target group teacher had mathematical content knowledge and knowledge of mathematical student followed base on Flow of Lesson of 3 steps because of they though attended in educational innovative through Lesson Study and Open Approach training from particular expert. They al so had opportunity in the field trip of classroom innovation in original schools employed innovation under mathematical higher order thinking project from Center Research for Mathematics Education, Faculty of Education, Khon khaen University. And, included they did collaborative in three months with two master degree students from Mathematics Education, Faculty of Education, Khon khaen University. Such that teachers' knowledge on mathematical task design come from Lesson Study enhance exchange and improve mathematical content knowledge and knowledge of mathematical student. Yoshida and Jackson (2011) claimed that Lesson Study promotes collaboration in writing lesson plan among teachers. They can assess students' thinking and use the results to develop teaching plans that is an effective technique to support teaching. Phailath et al. (2017) studies in Laos. They confirmed that the procedure of the Lesson Study can have an impact on the teachers and promoted student learning outcomes.

Targets group did not the participant in employing innovation through Lesson Study and Open Approach followed four phases. This target group teacher lack and need to improve on mathematical content knowledge and creating knowledge on teaching materials (Semi-concrete Aids) because they were lack of experience using the textbook on problem-solving. also, they did not familiarly Lesson Study and Open Approach before. Because it was the frits time into adapt at Attached Primary school of Pakse Teacher Training College in Lao PDR. Regarding Cannon (2008) found students teacher were lack of specific content knowledge in mathematical task design and employed the content in teaching. Furthermore, teacher's planning in teaching are unable to anticipating students' difficulties in learning Mathematics and the picture blocks used in teaching, the teacher cannot find guideline or preparing semi-concrete aids. These aids are able to extend and respond thinking process, students can use them to support learning and correctly mathematical problem-solving. As a result, thinking process of students align with goal setting. Lee, Lee & Park (2013) state that. It was challenging task for teachers in new task design because it required knowledge and experiences.

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A REFORMED MATHEMATICS CURRICULUM FOR LOW-TRACK STUDENTS IN ISRAEL: WHAT LESSONS CAN BE LEARNED?

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This paper reports on a reformed mathematics curriculum for low-track students in Israel, that began during the 1990's with the design of new learning materials, and continued with an innovative model of dissemination which is still ongoing. The rationale and pedagogical principles of this reformed curriculum are described, as well as considerations that led to the design of the dissemination model in the form of a personalized support system for teachers. The question of how the degree of success of this reform may be defined and assessed is discussed.

INTRODUCTION

This paper unfolds the story of a reformed curriculum of a special kind, and the challenging process entailed in its realization. I begin with explaining the rationale for this curriculum, specifying its scope and target population, and providing some details on the design phase. Then, I portray the difficulties involved in encouraging teachers to use the reformed curriculum, and discuss the ensuing definition of a supportive dissemination model. I describe the personalized professional development system that was designed according to this definition. Finally, I analyze this reform in an attempt to pinpoint characteristics by which its degree of success may be assessed, in light of both existing literature and local lessons learnt along the way.

Rationale and context

The reformed mathematics curriculum reported herein was designed for low-track students in Israel, aiming to enable more students to succeed in their mathematical studies. It began during the 1990's with an extensive phase of research-based design of new learning materials, and continued with an innovative model of dissemination still implemented in some of Israel's high schools today.

In most countries, failure in national exams at the end of high school is a serious impediment to enrolment in higher education and lucrative employment. In Israel, a prerequisite to entering universities, colleges, and many jobs is the eligibility for a Matriculation Certificate (MC), which involves a series of final exams in several obligatory subjects, one of which is mathematics. The percentage of students eligible for the MC within each cohort is around 50% (Svirsky, Connor-Atias & Dagan-Buzaglo, 2016), a statistic which is a constant concern for the Israeli Ministry of Education. In 1987, the Ministry created a special unit (the "Shahar Unit", in Hebrew) aimed at advancing students in grades 10-12 who are at risk of failing the Matriculation exams. One of the top priorities of the Shahar Unit is the study of mathematics; data shows that failing to pass the mathematics Matriculation exam is the single most common barrier preventing students from acquiring the MC (Israeli Ministry of Education, 2011; Shye, Olizky & Ben Shitrit, 2005).

The Israeli mathematics Matriculation exam may be taken in one of three levels: high, intermediate or low, and high school students are streamed to three tracks accordingly. Approximately 60% of the students study in the low-level mathematics track, however among those entitled to the MC, we find only about 20% who graduated this track. These data indicate that many Israeli students can be defined as "at-risk mathematics students", i.e., students' whose likelihood to fail the low-level final exam in mathematics, and thus become disqualified for the MC, is alarmingly high. The fact that nearly 50% of the low-level mathematics track students study in Shahar classes, suggests that there is a strong link between being at-risk in mathematics and being at-risk in general. This is coherent with the known correlation between low achievement or failure in mathematics, and issues such as social disadvantage and socio-economical inequalities (e.g., Balfanz, Mac Iver & Byrnes, 2006; Secada 1992). Added to this picture is the reality of Israel as a relatively young state merging numerous sub-cultures of immigrants and minorities, thus language and its relation to mathematics learning, as discussed in ICMI Study 21 (Barwell et al., 2016), needs to be considered. For example, the percentage of students of Ethiopian origin (immigrants arriving to Israel in the past 3 decades, who speak Amharic at home) studying in Shahar classes, is disproportional to their share in the general population, and their success in mathematics and eligibility for the MC is much below average (Koch-Davidovich, 2011). Consequently, the aim of advancing at-risk mathematics students should be perceived within a wider context of equity and opportunities for a better life.

A special case of a reformed curriculum

The reformed mathematics curriculum for low-track students in Israel is a special case of reform which is perhaps less explored, therefore a definition of what is meant by "a reformed curriculum" in this context is necessary. Arcavi (2000) defines a mathematics curriculum as follows:

> Curriculum: a conglomerate of materials of different kinds (textbooks, teacher guides, problem books, activity kits, games, material displays, computer software, teacher resource files, etc.) designed to teach and learn mathematics, which share an implicit or explicit rationale on both (a) the goals and (b) the nature of teaching and learning and of mathematical activity. (Arcavi, 2000, p. 156).

Following this definition, a "reformed curriculum" relates here to such a conglomerate of materials that introduces a radical change in the approach to learning, rather than a change in the topics of the existing syllabus. In other words, the topics included in the low-level mathematics track for the MC were determined by the Ministry of Education, however the underlying intention of the reformed curriculum (described below) was to make these topics accessible for mathematics students at risk. Such a reform is challenging since it not only includes design, but also needs to consider how to disseminate the curriculum among teachers, who are not obligated to adopt the suggested reform.

THE '3U' CURRICULUM AND THE 'SHLAV' MODEL OF DISSEMINATION

In many low-track mathematics classrooms we have visited across Israel, the atmosphere reflected low expectations on the part of both teachers and students, and a strong impression of "pseudo teaching and learning" (Karsenty & Arcavi, 2003). This is in line with well-documented low-track realities in other countries (e.g., Kajander, Zuke & Walton, 2008; McFeetors & Mason, 2005; Zevenbergen, 2003). However, we found that students' expectation to fail in mathematics had little to do with lack of cognitive capabilities, but rather with affective, social or behavioral problems. Many of them possessed normative sense-making skills, despite their deficiency in basic knowledge (Karsenty, Arcavi & Hadas, 2007). The target population for the reformed curriculum was defined accordingly as students whose cognitive abilities should have enabled them to pass the low-level mathematics Matriculation exam, however this prospect was jeopardized by various circumstances.

The 3U learning materials

From 1991, the Science Teaching Department of the Weizmann Institute of Science has developed learning materials for low-track mathematics students in grades 10-12, in line with the syllabus of the low-level Matriculation exam. The exam is divided into three separate tests named Units 1-3; hence the name of the curriculum was '3 Units', or in short '3U'. The syllabus included linear and quadratic equations; analytic geometry; trigonometry; arithmetic and geometric series; real-life graphs; real-life problems; statistics and probability; basic linear programming and basic calculus.

The materials were designed in an extensive research-based process (Arcavi, 2000), centering the approach of "learning by doing", as opposed to learning by receiving or by following rules. The endeavor was to enhance sound understanding through long-term student exposure to meaningful activities. The pedagogical principles guiding the design were to (a) engage students' common sense and real-life experiences; (b) base learning on visual and qualitative reasoning; (c) integrate multiple representations as different ways to envision the same ideas; (d) minimize technical manipulations and heavy notations; and (e) link students' thinking to their acting (for instance, by using various physical devices). The tasks encouraged estimated results, visual and numerical approximations, and the use of informal reasoning rather than symbolic algorithms and formulas. For example, students learn about the concept of slope by handling movable lines (drawn on a transparency) on a Cartesian grid, in order to get a visual sense of the "steepness" of different lines. The measure of "steepness" is done through reading the rise and the run from the graph. Gradually, students drop the use of the movable lines and turn to sketches of graphs in order to calculate slopes using the same definition of 'rise (or drop) over run' (the sign of the slope is separately determined by looking at the direction of the line). The formula $m=(y_2-y_1)/(x_2-x_1)$ which has little meaning for many at-risk students is presented, if at all, as the last strategy of this topic. We found that students can successfully solve Matriculation problems about equations of lines using visual reasoning, while they err considerably when attempting to use the formula for the same problem (Karsenty et al., 2007). Similarly, the midpoint of a segment AB is found by dividing the "step", created by the points A and B (i.e., the legs of a right-angled triangle where AB is the hypotenuse), into two equal steps, instead of using the formula $(x_{mid},y_{mid})=((x_A+x_B)/2,(y_A+y_B)/2)$. This visual strategy is particularly useful when the midpoint is given and one end needs to be found. Figure 1 presents an example of a Matriculation question that entails such reversed thinking.

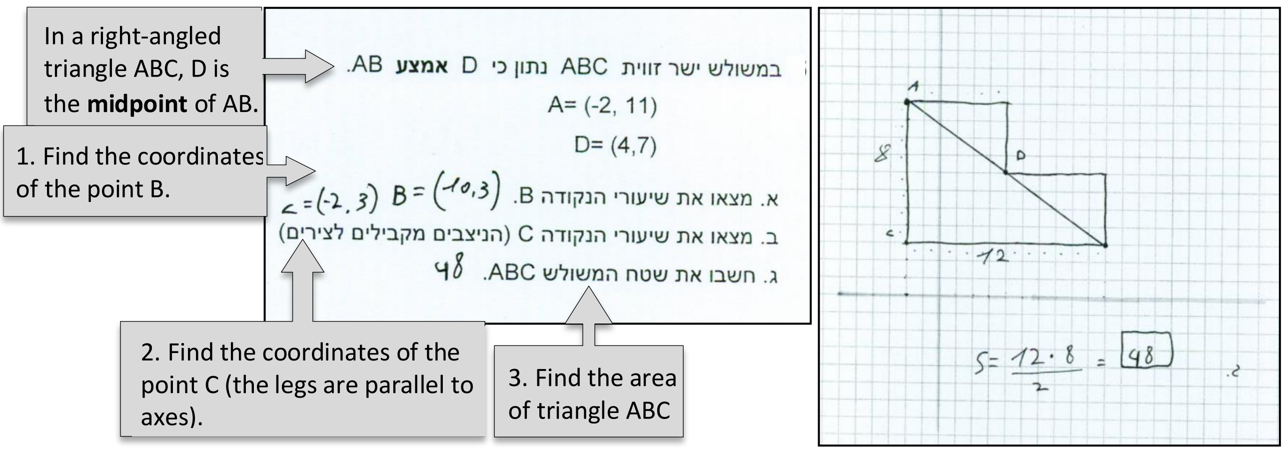

Figure 1. A Matriculation question in Analytic Geometry and an example of a student's answer

Rather than solving the equations $(-2+x_B)/2=4, (11+y_B)/2=7$, which require technical manipulations that are often insurmountable for many low-track students, a typical answer of students instructed within the 3U curriculum is based on visual means, i.e., sketching, counting, constructing the "mid step" and creating a congruent step (see a student's answer in the right-hand side of Figure 1). Note that this student did not use any words, nor algebraic notations; nevertheless, the answer clearly reflects a meaningful understanding of the concepts involved. This reflects the idea of deemphasizing formal solutions in favor of commonsensical and perceptual ways, which was a central design principle throughout all topics in the reformed curriculum.

In general, the design process of the 3U curriculum followed the principles of "didactical engineering", and consisted of an *in-vitro* creation of activities and tasks followed by *in-vivo* (and *in-situ*) classroom trials. The development team sought to study, within a realistic and authentic scenario, how the design of materials may match students' characteristics and at the same time strengthen their confidence in their abilities to do meaningful mathematics and succeed in exams. Therefore, members of the team have taught, throughout the school years of 1993-1995, three experimental classes of students initially tracked to non-matriculation bound classes. At the end of 1995, 54 students in the experimental classes took the Matriculation mathematics exam, and 49 have passed it. During these three years, a pilot edition of 3U books was published and circulated. Revised materials were re-tried by members of the team in other pilot sites (for further details on the 3U design process, see Arcavi, 2000). The teacher guides for each of the topics included the rationale of the curriculum in general and of the topic design in particular, solutions to problems, didactical suggestions, and samples of examinations. Throughout the guides, reports from students' data were included in order to sensitize teachers towards students' ways of thinking, help them prepare for possible student answers, and indirectly reassure them that these materials were developed "on" and "for" real students in the target population.

Disseminating the 3U curriculum: Pitfalls along the way and the emergence of a new model

Once the design phase was completed, the next step was to introduce the 3u curriculum to teachers. For that purpose, annual and summer in-service courses were offered to high school teachers of low-track students, during the years 1996-2005. The courses were conducted as workshops: Teachers were introduced to the materials through active work on selected activities, a discussion of their rationale, and some examples of students' work. At some point in this decade of intensive efforts, the developing team came to realize that there was a fundamental problem in the dissemination of the 3U reformed curriculum: Although hundreds of teachers have attended these courses, without external obligation[1], and while the feedback collected indicated that most teachers were positively impressed by the new curriculum, still relatively few of them were willing to actually implement it in their classrooms. The reasons for this reluctance fell into three main categories: (1) Claims about students ("my students are different", "they won't understand that", "they'd better do techniques"); (2) Claims about time ("this is great but I can't afford to devote time for deep understanding", "it takes too much time at the expense of massive exercising which is vital"); and (3) Claims about effort, i.e., teaching with these materials is demanding and requires careful preparation that necessities a lot of extra work. We realized that disseminating the reformed

[1] In Israel, teachers who take in-service courses obtain credit points for promotion, thus there is an external motivation for participation. However, the choice of which courses to take remains in teachers' autonomy.

curriculum through in-service courses was insufficient. Although the extensive phase of in-service courses was necessary for establishing initial links between design and practice, there was a missing ingredient. Balfanz et al. (2006), relating to high-poverty schools, argue that "any attempt to improve mathematics achievement at the schools would need curricular, professional development, and teacher support elements" (p. 36). It became clear that the curricular and the "formal" professional development elements were taken care of, yet the third element, i.e., teacher support, remained undeveloped. This was the starting point for SHLAV[2].

The SHLAV model: Supporting teachers in implementing a reformed curriculum

Our first step, back in 2004, was to explore the meaning of "support". What do teachers need when attempting to shift towards a reform-based practice, in the specific context of teaching mathematics to at-risk adolescents? As stated above, teachers' reservations referred to students' capabilities, time constrains, and extra effort. Thus, "support" had to relate to these three concerns. In addition, the literature provided us with further insights on what support may include. For example, Knapp, Shields & Turnbull (1995) discuss the supportive conditions needed for teachers in high-poverty classrooms, who face the challenge of maintaining a meaning-oriented instruction. They argue that a delicate balance must be attained between professional support, teachers' autonomy and external pressure for change. The perspective they offered was that teachers need not abandon their past practices but rather expand their repertoires to enhance academic challenge. We found this perspective to be both realistic and valuable. McLaughlin & Mitra (2001) emphasize the importance of feedback and encouragement provided to teachers by colleagues from their community of practice. Chazan (1996) pointed to the need to acknowledge the genuine difficulties faced by low-track mathematics teachers, struggling to change their practices. We learned that it is also important to acknowledge that mathematics teachers, assigned to teach at-risk students, are often afraid of failure. A reaction of a teacher after being told she was to teach in a low-track class, illustrates this:

> I was scared. Both of the character of students, who were known for their undisciplined behavior, and of the fact that in such a class I won't be able to demonstrate my skills and then my name will be connected to low rate of success in the Matriculation exam. […] The year before I prepared a class [for the Matriculation exam] and all of them passed successfully. Now, I knew, things will be different. I will not be able to repeat last year's success." (Bilia, 2003, p. 4, translated from Hebrew).

Our evolving interpretation of what a supportive model means was thus shaped to include the following components (Karsenty, 2012): (a) *On-site and ongoing advice on how to use the reformed curriculum with at-risk students*. Advice should be provided by a professional who is familiar with the local context in which the teacher works, with all its complexity and genuine difficulties, in a collaborative process based on experiences of both the teacher and the advisor; (b) *Authentic feedback on actual implementation of the reformed curriculum in the classroom.* Feedback must be given on a regular basis, to form a continual and meaningful process; and (c) *Opportunities to raise and discuss practical constraints and problems (including affective issues) connected with teaching at-risk students*. Teachers may want to share doubts and frustrations, as well as successful moments. They may also seek instruction regarding prioritizing activities and preparing for lessons ahead.

The SHLAV model of dissemination, applied by the Davidson Institute of Science Education (within Weizmann Institute) is therefore based on a professional and personalized support for

[2] The name SHLAV is an acronym of the Hebrew words for "Improving Mathematics Learning".

teachers in implementing the reformed curriculum. This support is given by a specially-trained counselor who functions inside the school (Karsenty, 2009). Each specialized counselor is assigned a secondary school, with a low proportion of students who pass the mathematics Matriculation exam, where s/he works for a full day every week. Main activities carried out as part of the counseling are: (1) group counseling: meetings with the mathematics teachers of low-track classes, where difficulties of students are discussed, 3U learning materials are studied and adapted, ideas are exchanged regarding affective and social concerns, and issues of assessment are considered. These meetings form a professional forum on low-track teaching, yet they also serve as a "support group" where doubts, frustrations, successful moments etc. can be shared. (2) one-on-one counseling: the counselor observes lessons, then meets with the teacher for a personal session, to discuss the lesson, decide together on future courses of action and confer on students requiring special assistance.

ASSESSING THE 3U REFORMED CURRICULUM: WHAT CONSTITUTES SUCCESS?

How can we assess the degree of success of a reformed mathematics curriculum for low-track students? What kinds of data are relevant for such assessment? These questions were (and to some degree still are) of concern to the development team, and to me particularly, as the founder of SHLAV and head of this program for 8 years. The quantitative criterion which is probably the first that comes to mind, and is the easiest to report, is the extent of participation. In the pilot years of SHLAV (2004-2006), 2 schools have participated in the program (6 teachers, 100 students). In the first year of post-pilot implementation (2007-8), there were 7 schools, 11 teachers and 195 students. Since then the numbers have increased at a moderate but steady rate. The most recent data collected shows participation of 32 schools, 191 teachers and 4750 students during the 2016-7 school year. However, I suggest that these numbers are limited in what they reveal about the success or failure of the curriculum; teaching with 3U and participation in SHLAV are not obligatory, i.e., schools can choose whether or not to use the reformed curriculum in their low-track classes, and moreover, they need to pay for the specialized counselor. Thus, the criterion of "how many" is linked to issues such as marketing and budget. Notwithstanding its importance, if this is the central criterion for success, then one might conclude that the 3U reformed curriculum failed, since it is implemented in only a small portion of Israeli high schools. Yet, there are other criteria that may be useful in assessing the degree of a curriculum's success, namely: what did students gain and what did teachers gain, in those classrooms that did endorse the 3U curriculum through participation in SHLAV? In an attempt to answer these questions, I draw on an assessment study of SHLAV conducted between 2007-2011 (Karsenty, 2009; 2012), within which students' grades in the Matriculation mathematics exam were collected and analyzed, and selected answers of students to exam items were investigated. In addition, various types of teacher data were collected: teachers filled an expectations questionnaire as they joined, and a summative questionnaire at the end of each year of participation. Interviews were conducted each year with a sample of teachers, and counselors' summary reports were examined. Main findings are summarized below.

Students' gains: On average, about 90% of SHLAV students took the mathematics Matriculation exam, and between 80%-93% passed it. In most years, the average grade of SHLAV students in the low-level Matriculation exam was 2-7 points higher than the national average in this exam.

Teachers' gains: The vast majority of teachers were highly positive about using the 3U curriculum as well as about the intensive counseling. In many cases, after a short period of adjustment, teachers and counselors began to form relationships somewhat similar to co-teaching in the classroom. Although teachers were solely responsible for presenting students with new ideas and materials, the classroom periods of seatwork were cooperatively directed by both teacher and counselor. Counseling sessions were more similar in nature to tutorials, where the counselor introduces ideas and strategies for discussion and responds to difficulties elicited by the teacher. These interactions were regarded by most teachers as very helpful. We found that new teachers tended to emphasize, in their reflections, the contribution of the counselor (e.g., "she took me by the hand through this unknown territory"; "it feels like you have something solid to lean on when you are not sure what to do"), whereas more experienced teachers tended to emphasize the use of the 3U materials (e.g., "It's not about giving the rule anymore […] you do everything in order to bypass the difficulties, bypass the formulas"; "The approach matches the level of students, you can't just teach them a formula and continue. […] I was really amazed by the materials, and now I teach exactly as suggested in them" (all citations are from Karsenty, 2012, p. 98).

Assessing the 3U reformed curriculum and the SHLAV dissemination model through the lens of students' and teachers' gains thus enables us to identify successful features of this special case of reform. Moreover, a deeper look can unpack two central ideas that, in retrospect, may have shaped the process that this reform had undergone, and contributed to its impact:

1. Contextual and tailored solutions. As Krainer (2014) puts it, when examining the notion of *Reflective Rationality*, "Complex practical problems require particular solutions. These solutions can only be developed inside the context in which the problem arises and in which the practitioner is a crucial and determining element" (p. 53). As described above, all along the process of the 3U design and its dissemination through SHLAV, the endeavor was to tailor the curriculum, based on research and on authentic experiences, to the specific context of low-track realities, and furthermore to adapt it to local practices and constraints, through counseling that reaches out to schools.

2. Respectful support. To quote Krainer (2014) again, "research and policy often seem to focus primarily on teachers' weaknesses […] Less attention is paid to the efficacy of the support system for schools […]. Such reactions indirectly blame teachers and - at the same time - they are unsatisfactory starting points for reform initiatives" (p. 52). The personalized support model of SHLAV breaks this asymmetrical role of designers-teachers, in that (a) the teachers' voice is salient in their professional development; (b) they are not a target of assessment whatsoever; and (c) the support given to them, within their "home court", is respectful and sensitive to their experiences.

To conclude, the 3U reformed mathematics curriculum can be seen as a case of reform that offers a new approach to an existing syllabus, for a distinct population of learners. While having no say regarding what topics to teach, thus avoiding conflicts attached to other kinds of reforms, such a reform carries its own unique challenges, with the main one being the dissemination among teachers whose experiences and beliefs might make it difficult for them to adopt the curriculum. The SHLAV model is an example of how this challenge may be overcome, through a sustained, mindful and professional assistance provided to teachers through a personalized system of support.

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RE-IMAGINING PRIMARY MATHEMATICS: RESOURCES SUPPORTING NATIONAL REFORM EFFORTS IN MEXICO AND ENGLAND

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In this paper we compare and contrast reform initiatives currently taking place in Mexico and England, with a focus on grades 1 and 2. These reforms embody a re-imagining of current curricula, through new resources. We analyse, from an enactivist perspective, the way in which these resources mark a break from previous practice in each country, in how the role of the teacher is imagined. In Mexico, resources within country-wide textbooks now include significant material aimed at teachers. In the UK, resources created by the National Centre for Excellence in the Teaching of Mathematics aim to promote a new focus on conceptual development. We contrast these new resources with their previous equivalents. We suggest the approaches warrant close attention since the new emphasis, in both countries, on teacher knowing represents a significant departure and one in which teachers are envisaged as innovators, just as much as the curriculum designers.

CURRICULUM REFORM IN MEXICO AND ENGLAND

The focus of this paper is on the implementation of reform and, in particular, an analysis of the design of resources (in the form of textbooks, classroom resources and teacher guidance). We have come to write this paper together through our recognizing, despite differences in contexts, remarkably similar kinds of reform taking place in Mexico and England, at primary school grades. In both countries reform is taking place through the introduction, nationwide, of new resources in the form of materials and guidance for teaching mathematics. These new resources represent significant breaks from previous traditions in each country and, in some sense, are a response to perceived failings of current teaching. In Mexico, PISA results show Mathematics performance improved slightly between 2003 and 2015, however a high proportion of students are low achievers (57% in 2015); in England, no recent reform has impacted on the 25% of 15 years-olds who do not reach PISA baseline standards with all the knock-on effects this has for life chances and social mobility (OECD, 2016).

The new materials and guidance being introduced in each country are innovative and worthy of study for what we can learn about ways of implementing reform. We will analyse: what are the new resources and their roles, in Mexico and England (including how they differ from previous resources)? and, what are their similarities and differences? At this point in time, in both countries, the materials have been created but are yet to be implemented at scale. Our analysis points to the ways in which the intended curriculum (in each country) is being elaborated differently to the past, with a re-imagining of the relationship with the teachers who implement it (Mullis and Martin, 2015).

Research in Mexico shows that, when facing reforms (including new curricula, past versions of country-wide textbooks and large-scale introduction of digital programmes) many teachers adapt new resources to previously used strategies: “teachers’ strategies tend to find a balance between their own practices and some elements of the new curriculum” (Block et al. 2007, p. 756, authors’ translation).

Some changes in teaching have been documented (Ávila, 2004; Trigueros, Lozano and Sandoval, 2014) but we know little about the conditions under which changes occur or the role of resources.

Similarly, research in England suggests that teachers integrate policy changes into existing practices in complex ways depending on prior experience and beliefs about self, teaching and learners (Boylan, et al. 2016). New thinking is needed to engage in systemic change (Rowson and Corner, 2015) and we will analyse the extent to which the roles being imagined for resources from Mexico and England represent such new thinking. We will first offer background details to each reform effort and then propose the methodology through which to analyse the new resources.

BACKGROUND TO MEXICAN CURRICULUM REFORM

In 2017 the current Mexican curriculum reform was presented through a new "Educational Model" which emphasizes quality in education for all students. There are three main components of the curriculum: academic background, personal and social development and curricular autonomy. The academic content in the new curriculum is organised around key learning outcomes, which are quite general (for example "reads, writes and orders natural numbers up to 100") (SEP, 2017, p. 317) and are to be attained by the end of each school year. The new model focuses on the transformation of teaching practices in schools, and the role of the teacher is seen as central.

In addition to the new educational model and the National Curriculum, the Secretary of Public Education (SEP) is developing different materials, including nation-wide textbooks and accompanying teaching guides for each subject, which each year are distributed freely to approximately 14,000,000 students and 570,000 teachers in primary education (http://www.snie.sep.gob.mx/descargas/estadistica/SEN_estadistica_historica_nacional.pdf). These textbooks are meant to provide "a common ground for education in the country [...], and are conceived as instruments which facilitate diverse and pertinent educational practices" (SEP, 2017, p. 126).

In the current reform, the importance of the teachers' guide is highlighted. They are considered to be tools through which teachers can enrich their knowledge and transform the proposals made in the textbooks, to be sensitive to their students and their context: "The teachers' guide will promote informed decision making, teachers' autonomy and reflection on pedagogical practice" (p. 126).

So far, grade 1 and grade 2 textbooks and teachers' guides have been produced for primary school, with Lozano and Sandoval (authors) coordinating the process and developing the materials together with others. The new materials will be used in the school year 2018-2019.

BACKGROUND TO ENGLISH MATHEMATICS TEACHING REFORM

There is an explicit government agenda at the moment in England to alter the practice of mathematics teaching, drawing on practices from East Asia, particularly Shanghai. There has been funding for a Shanghai Teacher Exchange programme (Chinese teachers brought to England and teaching in schools over a two-week period, visited each day by other UK teachers) and money devolved to regional "hubs" to promote what is being labelled a "mastery" approach to teaching mathematics in schools. The introduction of a new vision for mathematics teaching is taking place without a change in curriculum (the current curriculum having been introduced in 2014). There is inevitably a wide range of interpretations of the term "mastery" but the official government body, tasked with promoting and developing the new approach is the National Centre for Excellence in the Teaching of

Mathematics (NCETM). The NCETM write that, in a mastery approach: "Pupils are taught through whole-class interactive teaching, where the focus is on all pupils working together on the same lesson content at the same time" (NCETM, 2016). In contrast, an organizing principle in relation to typical primary school teaching prior to these reforms would have been that of "personalization", a concept at the centre of a previous reform of mathematics teaching in England (DfE, 2011, p.26).

Coles (author) has been "educational consultant" on a programme to develop mastery professional development materials which have so far been completed for grades 1 and 2. These materials are designed for teachers, although incorporating projectable resources that can be used directly in the classroom. Materials are freely available on the NCETM website (https://www.ncetm.org.uk) and grade 1 and 2 resources for "Addition and Subtraction" (making up over half the curriculum time of those grades) will be available for teaching in September 2018.

AN ENACTIVIST METHODOLOGY

Our methodological approach, which has inspired both our roles within the reforms described below and our comparison of them in this paper, is enactivist (Maturana and Varela, 1987; Reid and Mgombelo, 2015). A central idea, within enactivist research, is that multiple perspectives interact, not to lead to individual 'findings' but to new possibilities for action for participants involved in the research. From our perspective teaching and learning mathematics are conceived as a process of "expanding the space of the possible" (Davis, 2004, p.184) for learners. We also see our work in this paper as a form of learning and the spaces we will explore are the curriculum reforms in our own and each other's countries. Enactivist thinking orients us towards a focus on relationships of change and the synergies created through the interaction of the different contexts. To enrich our understanding, we will be seeking multiple views of our initiatives (Reid, 1996), making use of our different country perspectives, the different distinctions we inevitably make, and engaging in the systematic search for patterns across the innovations and new resources (Coles, 2015).

From the enactivist perspective, as humans, we are embedded in histories of interactions with all that is around us; we shape and are shaped by all the world in a process of co-emergence. In other words, what counts for us as meaningful arises through our histories of interaction in the world rather than from some objective features of the world. In this sense, no object or concept ever reaches a final form or has completeness. Resources linked to curricular reforms may occasion new kinds of actions, that students and teachers undertake, but they cannot determine those actions, which will emerge uniquely in each setting and cultural context in which the resources are used.

As a first step towards investigating and comparing the process of reform in Mexico and England we explore differences and similarities regarding those aspects which, for each set of resources, constitute departures from previous reforms and this is the focus of the next two sections.

COMPARING OLD AND NEW TEXT BOOK RESOURCES IN MEXICO

In Mexico, through the new nation-wide textbooks and teachers' guides, new ways of working are being introduced, with a stronger emphasis on guidance given to teachers. A different organisation is being used, and different kinds of activities are proposed, including many that go beyond the textbooks themselves and that involve exploration and collaboration. Specific strategies and representations for calculation and for analysing shapes are explicitly introduced, constituting a departure from previous approaches. Also, through a stronger emphasis on geometry and measurement (e.g., comparing object lengths and then measuring lengths), a previous imbalance that existed between those mathematical areas and the area of number is addressed.

Organisation of textbooks and conceptual understanding

The books are organised in groups of chapters which are meant to contribute, in a particular way, to the learning of a certain mathematical concept or idea. These groups of chapters are, in turn, linked together throughout the textbook, creating pathways in which there is an intention for deepening the work with those concepts and ideas, in cycles in which concepts are revisited from different angles. All this is made explicit in the teachers' guide. This organisation constitutes a difference from previous approaches in which pathways were left implicit.

Type of guidance given to teachers

The previous versions of nation-wide textbooks gave general recommendations for each area of mathematics (number, geometry and measurement, data handling), and they included brief suggestions specific to the chapters. In our new materials, specific guidance is provided for each chapter in the books, and particular attention is given to the following aspects: intentions related to conceptual learning; questions that can be asked to promote reflection; common mistakes and misconceptions; possible strategies for problem solving; strategies for differentiation; manipulatives or models that can be used.

Aspects that go beyond the didactics of mathematical concepts

Pedagogical aspects which are relevant for the teaching and learning of mathematics, but which are not necessarily included in the didactics of specific concepts such as number or proportionality, are included in the teachers' guide. For example, there is guidance on: ideas, beliefs and attitudes around the learning of mathematics; classroom culture and organisation, including different ways of working; doing mathematics; making mistakes in mathematics; teachers as learners.

Types of tasks: exploration, multiple answers and using particular strategies

Compared to previous approaches, there is a wider variety of types of tasks being used. On the one hand, there are open problems in which there is room for exploration and in which students are invited to investigate what happens in different situations to make observations and register and analyse what they notice. On the other hand, there are activities in which students are asked to use specific strategies and key representations are also included.

An emphasis is made on the use of problems with multiple answers, and guidance for teachers include invitations to work with those different answers, determining whether all possible answers have been found and exploring different ways of obtaining the answers. In previous textbook versions a few problems with multiple answers were included, but often the format in which the questions were posed (the space given for registering the answers) promoted finding one answer only.

Strategies for differentiation

Strategies for differentiation are for the first time included both in the textbooks and in the teaching guides. There are several ways in which differentiation is addressed: for each chapter, there are suggestions for modifications in case students are struggling and also suggestions for extending the task; in the textbook, there are further questions for students to answer after they solved the problem;

there are general suggestions given to teachers such as different groups working on different versions of the problems and even different problems. Suggestions for teachers centre around conceptual learning and are linked to planning and assessing.

Moments of reflection

Moments for reflection during lessons are suggested in different ways. For each chapter in the textbook, there are one or two questions or phrases which are meant to trigger discussion and reflection at the end of a lesson. In addition, suggestions are given in the teaching guides with questions that invite reflection throughout the process.

Collaboration

Different ways of collaboration are promoted by the activities. In some chapters, the participation of the whole class is needed. There are games that are to be played in groups and where different roles are assigned to the participants. Groups receive different versions of the problem (with different numbers, for example), and comparison is done at the end. Individual activities are also included.

COMPARING OLD AND NEW TEACHING MATERIALS IN ENGLAND

The current National Curriculum in England specifies learning outcomes for each year of study but is deliberately neutral about how these might be achieved. The new guidance materials and resources being produced by the NCETM offer an ordering of content within distinct "spines". The first spine to be developed encompasses "Addition and Subtraction" and includes what is also written in the National Curriculum under "Place Value". There is already an innovation inherent in the organization of materials, in the significance of place value is being downplayed (via its incorporation within "Addition and Subtraction") compared to previous versions of the curriculum, something called for in the work on Sinclair and Coles (e.g., Coles and Sinclair, 2017). Each section of work contains an 'Overview of learning', containing innovative features, compared to previous resources available for primary school teachers, which are drawn out below. There are also details of 'steps in learning' with further notes for teachers and resources (PowerPoint presentations) that contain some key images, representations and questions which can be used in the classroom.

Mathematical themes

There are themes such as equivalence, which are introduced at the very start of grade 1, and which are worked on and developed in a systematic manner throughout the primary school years. The current National Curriculum in England has themes running across the 5-16 age range of: fluency, reasoning and problem-solving. These themes are in essence about pedagogy and what is an appropriate balance of task and classroom organisation in order to develop confidence mathematicians. The new themes (such as equivalence) are more directly about the mathematics and supporting students to appreciate and use mathematical structures. There are closely linked pedagogical themes also, such as the use of 'variation' in presenting concepts, meaning for instance that non-examples are offered to students as well as examples.

Consistent representations

Students will be introduced to the number line and two different models for conceptualizing part-whole relationships (including a 'bar model', in which numbers are represented as the lengths of rectangle, so two 'parts' of a 'whole' would be two rectangles which, end to end, make the same length as the rectangle representing the 'whole'). Again, having core representations of additive structure that are introduced early in primary schooling and used consistently and repeatedly is an innovation from current practice. Rather than conceive of a progression from, e.g., concrete to pictorial to abstract representations of mathematical concepts, then emphasis is on teachers and students working on relationships and connections between representations, where as far as possible, the concrete, pictorial and symbolic (abstract) are all around from the start.

Emphasis on language

An emphasis on precise language, alongside the use of core representations, can be seen in the guidance for teachers, for example, with suggestions for how to encourage students to use full sentences in explaining their thinking. There is the suggestion of a practice of oral chanting and recitation and the use of 'stem sentences' that students will be offered and expected to use (e.g., a 'first, then, now' structure for classes of situations such as: 'first [there were four ducks on a pond], then [two ducks flew away], now [there are two ducks left]'). One innovative element of the curriculum organisation is the prompt to use a 'dual naming' of two-digit numbers. In English there are irregularities of naming, particularly in the range 11-19. Students will use an alternative naming, for example, "one ten five" for 15, alongside the standard "fifteen". Work on these numbers will also be delayed until after work on the more regular (in terms of naming) 20-99.

Number as object, number as length

A final distinction from previous practice to be highlighted, is being explicit about how number is being conceptualised and ensuring that there is a balance of cardinal and ordinal or measure-based approaches. Current practice in England would have worked on number in an almost exclusively cardinal manner in grades 1 and 2 (Coles and Sinclair, 2017). Students' very first introduction to number work (within the "addition and subtraction" spine) will be in the context of measures (drawing inspiration from the work of, for example, Dougherty, 2008).

SIMILARITIES, DIFFERENCES AND IMPLICATIONS

Comparing innovations, given our enactivist commitments, we have looked for patterns (Coles, 2015) to allow us to see more in each other's contexts. We have considered the guidance and our own distillations above (created independently) as data for comparison and, in the first instance, we have looked for common words. From these common words, we have distilled a range of meta-themes.

Explicit/implicit: In both countries there are changes regarding what is left for teachers to devise. There is more explicit guidance, both conceptually and pedagogically, than any previous curriculum reform. A new organisation of material is a common feature and with the aim of greater conceptual coherence (England) or conceptual understanding (Mexico). A similarity here is the making explicit of pathways through content that have been previously left for teachers to devise.

Innovative approaches to the teaching of concepts: The learning of number will now include core representations that will be used recurrently alongside themes across years in both countries, again, this is a break from current practice. In both countries there is an increased emphasis on measure. In Mexico, the introduction of work on measure and geometry comes through a recognition of a previous pre-dominance of number. In England, the emphasis on measure is within number work and speaks to a re-balancing of cardinal and ordinal approaches to number, to give each one equal weight.

Pedagogical aspects: The reforms are explicit about some elements of pedagogy, for example increased use of precise language in England and new classroom organizations in Mexico, including moments of collaboration and reflection. Linked to reference to concepts (above) is a new emphasis on themes across years (e.g., finding patterns in Mexico and equivalence in England). A difference is that the Mexican reform is working towards more classroom differentiation of tasks, whereas in England there is more emphasis on students working and progressing together.

Teachers' autonomy: More guidance is given to teachers than in the past, but at the same time more decisions are expected from teachers. In Mexico, this is seen in several ways: by offering a wider variety in the types of activities proposed, in the pedagogical suggestions and ways of working, in the strategies for differentiation and also in sections about planning and evaluating and a section in the teaching guide in which teaching is seen as a process of learning. In England, the materials are written for teachers with the explicit message that guidance is not taking the place of lesson planning but rather informing lesson preparation through a focus on the concepts being taught.

IMPLICATIONS

We observe new thinking (Rowson and Corner, 2015) embedded in the materials and guidance (above) in the cases of both Mexico and England. In each country, while there is in some sense an increase in prescription for teachers (for example in the use of particular representations), there is also a far greater emphasis than before on articulating, to teachers, the thinking behind the materials, both at the level of intentions beyond the mathematics (e.g., mathematical themes, or ways of working) and offering frameworks (for teachers and students) for making sense of what is being offered (e.g., 'number as object' versus 'number as length'). In neither case is there a sense of wanting teachers to 'implement', in a mechanistic manner, visions of a mathematics classroom held by policy makers or researchers. Whereas work on task design has often focused on the question of the 'gap' between the intentions of designers and the realization of tasks by teachers (Johnson, Coles and Clarke, 2017), in the materials from Mexico and England there is, instead, a sense of an 'offer' for teachers to use and adapt, as appropriate in their contexts. There are choices for teachers to make and, in neither country, are the classroom resources such that they comprise fully fledged lesson plans. In other words, it is the *expectation* behind their design that teachers using the materials will adapt. Therefore, instead of a concern with the fidelity of teachers' implementation of interventions or resources, that has characterised some previous reforms, these materials have the aim of supporting teachers to make informed (and inevitably differing) curriculum decisions, but decisions that can now be based on insights derived from research and practice, made explicit within the materials. There is no 'gap' between intended curriculum design and its implementation because, in keeping with our enactivist perspective, teachers are re-imagined as curriculum innovators and designers themselves.

We will, of course, be investigating the implementation of materials in both countries. We will be following in detail the extent to which the use of new material and guidance does in fact occasion classroom change in both teaching and learning. Our analysis, above, points to some particularly important lines of enquiry that will be significant internationally: is there, in actual fact, evidence of teachers innovating from the materials given? What kinds of professional development are required, alongside the materials and guidance, to support teachers in enacting new practices? Is there evidence that a focus on common representations and mathematical themes does indeed support student learning? Given the concern, internationally, to improve the provision of mathematics education in schools, the approaches being undertaken in Mexico and England warrant close attention. The ambition of both reforms is a transformation of teaching and learning, but one in which mathematics teachers and curriculum designers are re-imagined as partners in innovation.

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ATTAINED CURRICULUM AND EXTERNAL ASSESSMENT IN ITALY: HOW TO REFLECT ON THEM?

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In this paper we argue how the Italian standardized assessment tests can be used by teachers to perceive the intended curriculum. These tests aim at evaluating students' learning, but they can also become a means for teachers and students to deal with tasks that are constructed according to the goals explicitly stated in the National Guidelines. Through the analysis of an example, we highlight how merged qualitative and quantitative analysis of tasks, selected from Italian standardized assessment tests, may lead to reflections on the learning achieved by students (attained curriculum), and on the validity of educational choices in the implemented curriculum.

ASSESSMENT IN THE PROCESS FROM INTENDED TO ATTAINED CURRICULUM

IEA TIMSS and OECD PISA results have influenced different national curricula around the world. Osta (2014) stresses that, as a reaction to the results of international assessments in mathematics and science, many countries are tending towards more standardization and centralization in their mathematics curricular procedures. The PISA framework in particular has motivated an increasing trend toward the design of mathematics curricula in accordance with a set of mathematical competencies. Moreover, in Europe, documents from various political institutions (i.e. the European Parliament and Council recommendations) have highlighted the importance of fostering the design of educational activities and assessment of students' learning by focusing on competence development. In Italy, the National Guidelines (NG) for the first cycle of education (from grade 1 to grade 8: Primary and Middle School) indicate the learning goals to be achieved in terms of competence development at the end of grades 3, 5 and 8 (MIUR, 2012). Taking into account these national standards, each teacher can build his/her own path by choosing contents and methodologies, but it is not always easy for the teachers to fulfil NG expectations with adequate teaching and learning activities.

In Italy, the Ministry of Education (MIUR) entrusts the National Institute for the Evaluation of the Educational System of Education and Training (INVALSI, http://www.invalsi.it/invalsi/istituto.php?page=chisiamo) to assess the levels of learning of primary and secondary school students in Mathematics, Italian and English. The standardized assessment procedures are carried out in a form of census (while the processing of statistical data and the measurement of students' learning levels are carried out through sampling): each school receives its own data and each teacher can analyze the data of his/her students. The national assessment data provides a whole range of information that can be useful for different purposes. In this paper we look in detail at how Italian standardized assessment tests can disseminate the intentions and goals of NG and how teachers can reflect both on the tasks proposed in these tests and statistical data. We show how the Italian national tests and collected data can assist in passing on some important educational messages related to NG and thus be used to study the link between what are usually labeled as *intended curriculum* and *attained curriculum* (Clarke et al., 1996). From this perspective, the tests can constitute an object of reflection by teachers who subsequently have the task of developing the curriculum.

This paper presents the analysis of a task from the Italian standardized assessment tests which refer to the attained curriculum for Italian first cycle of education. In line with van den Akker's classification (2003), we focus on the *perceived curriculum* by means of reflections that link what is written in the documents relating to the curriculum (*formal/written curriculum*) with the tools used to evaluate what has been actually learned by the students (attained curriculum).

NATIONAL GUIDELINES AND STANDARDIZED TESTS

Italian National Guidelines for the first cycle of education

In the last ten years, new NG for the school curriculum in Italy have been proposed by the Ministry of Education (Ministero dell'Istruzione, Università e Ricerca, MIUR). For Primary and Middle school, the NG were first published in 2007 with the latest version arriving in 2012 (MIUR, 2012). Moreover, in 2018 a document "Indicazioni Nazionali e Nuovi Scenari" (National Guidelines and New Scenarios, MIUR, 2018) discussed the NG by underlining some key aspects to be developed in first cycle education; as regards mathematics, these included education on argumentation by means of group discussions and interactions between peers and experts (for example, in laboratory activities). Italian legislation does not lay down a strict curriculum, but it indicates the goals for competence development at the end of grades 3, 5 and 8 and then leaves it open to the schools to choose which path to achieve these targets. Therefore, teachers can implement curriculum according to the context in which it operates, interacting with the students, their experiences and their needs. This freedom enables teachers to make different educational choices. This means that there may often be inconsistency between the curriculum actually developed by schools (implemented curriculum) and the NG. In this context, the national assessment tests become a means for schools to tackle the expectations set out in the NG.

Italian national standardized tests (INVALSI tests)

INVALSI (www.invalsi.it) is a research institute with the status of legal entity governed by public law. The Institute carries out periodic and systematic checks on students' knowledge and skills, and on the overall quality of the educational offering of schools and vocational training institutes, also with a view to lifelong learning; in particular, it runs the National Evaluation System (SNV). Since the 2007-08 school year, the Italian Ministry of the Public education has annually established the standardized assessment of the Italian educational system, and commissions the INVALSI to carry out surveys nationwide to all students in the second and fifth classes of Primary School (grades 2 and 5), the third class of Middle School (grade 8), and High School (grade 10 and, from 2019, grade 13). The INVALSI tests were created for system evaluation and this is their primary purpose. The statistical representative sample comprises approximately 30,000 students (with tests administered under controlled conditions). Moreover, the tests are administered at census level and students' results are provided to each School Institution. The statistical area of INVALSI has been made available to schools via a web portal (http://invalsi-dati.cineca.it/) built ad hoc, which each school can access. The data returned by the INVALSI concern various aspects: the overall performance of the students' levels of learning compared with the average across Italy, the geographical area and region to which they belong; the performance of individual classes in the tests as a whole and the performance of the single class and individual student for each individual test. The INVALSI tests are designed by expert teachers, educational and disciplinary researchers, statisticians and experts of the school system. Before being administered in all classes of pertinent school grades, the INVALSI tests are pre-tested on a sample of schools: these pre-tests, or "field trials", are tools used to verify the relevant psychometric aspects. INVALSI mathematics tests are part of the external assessment of Italian students' learning outcomes according to the expectations stated in the NG. Therefore, they verify the attained curriculum, by measuring the learnings of the students. The challenge facing the group of teachers (in service teachers who work in primary and secondary schools) and researchers who produce the INVALSI tests is to try to combine, as far as possible, the needs of a summative system evaluation (the terms evaluation and assessment are distinct as stated in Niss (1992)) with the requests and perspectives given by current paradigms on the teaching-learning of mathematics and by the NG. For this reason, in the INVALSI tests, as we will see later, there are both multiple-choice tasks and open-ended tasks with a request for argumentation or of showing the work.

There is unanimous agreement that, for assessment of competencies, different types of interpretative and evaluative tools are necessary. Standardized tests, like any summative tests, can only provide certain information (which must be integrated into the competencies assessment process that is carried out at school). This is one of the reasons why, in the past, (even recent past), INVALSI tests have not always been accepted by teachers and, more generally, by the "school system": when teachers think about "assessment", they refer to the students' assessment, that is to say the long and complex process involving all the teaching activity and which is, in many ways, an integral part of it. The INVALSI tests have however become a source of reflection and provide examples of tasks in line with the intended curriculum: in fact, although these tests have, as already underlined, the purpose of evaluating students' learning, they can also become a means for teachers and students to deal with tasks that are constructed in accordance with the goals and objectives stated in the NG. INVALSI mathematics tests have their own framework (https://invalsi-areaprove.cineca.it/docs/file/QdR_2017_def.pdf) which is aligned (in the sense of Schmidt et al., 2005) with, and explicitly refers to, the NG (Arzarello, Garuti & Ricci, 2015). This framework is designed to help all those involved in the school system (teachers, managers, families) to interpret the results obtained by individual schools or by the individual classes in the INVALSI tests.

The benchmark proposed by the statistical analysis and carried out by INVALSI can constitute a term of comparison among different schools or among different classes in the same school. It is clear that the data useful to politicians or the Ministry of Education is different from the information needed by a teacher (Garuti & Martignone, 2015). In terms of curriculum implementation, it can be interesting for teachers to compare the statistical data about one's own classes or educational institution with the overall results of the tests, interpreted in the light of the specific context in which one's own school operates. This could be useful for teachers in order to identify the strengths and weaknesses of the learning path actually followed in class, and the educational choices made. It can support reflections on the learning achieved by the students, and on the validity of educational choices in the implemented curriculum.

INVALSI TESTS AS OBJECTS OF REFLECTION IN PROFESSIONAL TEACHER EDUCATION AND DEVELOPMENT

Background

Since the middle of the last century the topic of educational measurement has been the focus of debate and research. Assessment activities are varied and they can have different purposes. For example, classroom assessment is often formative and aims at supporting students' learning and informing teachers' instructional decisions. Classroom assessment can be distinguished from external assessment which often involves large-scale standardized tests and is most often used for summative or evaluative purposes (Goos, 2014). The use of standardized tests to assess students' learning and the general considerations derived thereby are often criticized. For example, Osta (2014) says: "the use of standardized tests was also contested, as these only provide scores which don't uncover the real learning problems, and which focus on recalling information and computation skills rather than mathematical thinking" (p.421). We have to stress that, even though a standardized test cannot propose tasks related to certain types of problems that could be very important in assessing mathematical competencies (e.g. the production of conjecture or the management of long and complex problem solving), it is possible to build tests which do not only offer questions requiring the recall of notions or the implementation of procedures.

In this paper, we show how INVALSI tests can contribute to the process of curriculum implementation as a tool for teachers to perceive the intended curriculum. In the work with teachers, we use analysis carried out in some studies on INVALSI tests (Branchetti et al., 2015; Ferretti & Gambini, 2018; Lemmo et al., 2015; Martignone, 2017). In these studies our group carries out a qualitative and quantitative analysis of INVALSI tests. We develop a vertical analysis, following a specific educational perspective also aligned with the ideals of the intended curriculum: in fact the NG are constructed and written according to the assumption that student learning unfolds over time, in line with a spiral model (https://eric.ed.gov/?id=ED538282). Vertical coherence of the goals at the end of the different grades becomes fundamental at this point. The qualitative analysis of INVALSI tests and results can provide interesting information about how effectively this perspective is attained. In order to do this, our research group first analyzed problems selected within the INVALSI tests, following which we shared the data and results of our study with teachers (during teacher education programs). The analysis of the tasks selected from the INVALSI tests fostered discussion among teachers and researchers about the task features and possible student answers (Martignone, 2017). This also becomes a means by which teachers may reflect on the relationship among intended, implemented and attained curriculum. Moreover, the qualitative analysis of INVALSI questions can be integrated with analysis of statistical data collected by INVALSI (Ferretti, Giberti & Lemmo, 2018).

An example of task analysis

As mentioned in the previous paragraph, in our research on INVALSI tests we analyzed tasks related to topics that are suitable for a vertical analysis work: for example, tasks concerning number line, geometrical transformations, decompositions of figures, fractions, etc. In this paper we focus on a probability task (which in the NG and INVALSI tests is part of items classified as "Data and Uncertainty") analyzed during some teacher education programs for primary and secondary school teachers. The importance of probability within the teaching of mathematics is already known, as well as its epistemological peculiarities and the difficulties related to its teaching and learning (Batanero, 2014). In Italy, the learning goals regarding probability at the end of Primary and Middle school are clearly stated in the NG: "In concrete situations, considering a couple of events, to guess and reason which is the most probable, giving an initial quantification in the simplest cases, or recognize if they are equally probable events" (grade 5); "In simple random situations, identify the elementary events, assign them a probability, calculate the probability of some event, breaking it down into disjointed elementary events" (grade 8) (translation by the authors). Even though this topic is present in the NG goals, it is rarely developed in depth in the curriculum actually implemented by the teachers. This fact can be attributed to various factors, some of which are specific to the Italian school tradition: in some university courses, it is generally taught at a formal, advanced level, while in secondary school, it is taught as a set of rules to solve standard exercises. In both cases, no room is left for discussion about the cultural importance of probabilistic thinking (Boero & Guala, 2008). Moreover, teachers need to be trained to teach probability (Batanero et al., 2004). To improve courses at national level for teachers of this subject, the IV Summer School (born from a collaboration between the Italian Mathematical Union (UMI), the Italian Commission for the Teaching of Mathematics (CIIM) and the Italian Association of Researchers in Mathematics Education (AIRDM)) presented probability and statistics as a key topic. The document that presents this summer school states: "on the one hand there is a widespread lack of adequate "probabilistic thinking" in a vast strata of our society, and an equally serious and related difficulty in teaching and understanding its elements in our school" (translation by the authors from http://www.umi-ciim.it/attivita-della-ciim/scuole-estive/4a-scuola-estiva-2017/). We took part in this summer school by presenting a workshop that proposed activities similar to those we have been preparing for teachers for several years now. The task that we will show in this paragraph (Fig. 2) is one of the examples used in that workshop. The tasks selected by INVALSI tests should be suitable for a vertical discussion on learning objectives in different school grades, and have the potential to reveal typical errors related to specific contents and requests. In particular, here we focus on probability tasks. Probabilistic reasoning seems to be scarcely developed without instruction (Fischbein, 1975). Literature in mathematics education offers many examples of persistent difficulties studied in different school grades. For the selection of the task analyzed below, we refer to the study by Fischbein and Schnarch (1997). These researchers analyzed some examples of problems faced by students in different school grades: grades 5, 7, 9 and 11, and CS (college students/future teachers). The goal was to generate assumptions about how specific misconceptions related to a series of probability problems can decrease, stabilize or increase in educational degrees. One example of a problem that shows a misconception linked to simple and compound events, which seems to persist throughout the different grades, is the following.

Suppose one rolls two dice simultaneously. Which of the following has a greater chance of happening?

- Getting the pair 5-6
- Getting the pair 6-6
- Both have the same chance
- Other answers

Figure 1: example of "simple and compound events" task (Fischbein & Schnarch, 1997)

In the INVALSI tests we find some typologies of tasks similar to those analyzed by Fischbein and Schnarch; these tasks were selected to be analyzed and discussed with the teachers. Below, we present an example related to that of Figure 1.

To choose who should wash the dishes after lunch, Marco, Lorenzo and Lidia decide to flip a 1 euro coin like the one you see in the picture:

Heads Tails

They establish that:

- if two tails come up, Marco will wash the dishes
- if two heads come up, Livia will wash the dishes
- if one heads and one tails come up, Lorenzo will wash the dishes.

Do you think all three will have the same probability of washing dishes?

□ Yes
□ No

Justify your answer.

Figure 2: task D11, grade 8, 2011 mathematics INVALSI test (translation by the authors)

This task asks students to identify which of the three compound events is the most probable and to justify this choice. We therefore note that in the INVALSI tests, argumentative skills are assessed (within the limits of a standardized test) as claimed in NG. According to the correction grid published by INVALSI, an answer is accepted as correct if the student explicitly states that the probability that "heads and tails" face up (or vice versa), is different from the probability that two heads or two tails face up. This correction grid lists different types of acceptable responses, which highlights the fact that different styles and resolution processes are considered acceptable in reaching the correct answer. This task is linked with the aforementioned grade 8 goal of the NG. In the first part of the item students simply have to identify whether the events have the same probability; already in this case, the percentage of correct answers is low, about 33.3%. In the second part, the percentage of right answer drops even further (correct answers 16.6%) because it asks students to justify the choice made. The data collected by INVALSI from the Italian national sample are consistent with those of the research by Fischbein and Schnarch in which the percentages of answers concerning the main misconception "have the same probability" are between 70% and 80%. According to Fischbein and Schnarch (1997), this misconception, linked to the identification of probabilities of compound events, may remain stable over time. In addition, we can see that the goal of the NG referring to the previously presented task is closely linked vertically with the NG goal for grade 5. It is therefore a learning goal that grade 8 students should have already achieved in Primary school. This task is an example of a problem that can be tackled even by grade 5 students (as well as high school students as stated by Fischbein and Schnarch (1997)). The INVALSI tests can thus become a bank of items for the teacher, in which there are tasks that assess some aspects of the competencies indicated in the NG, also for topic rarely developed in the implemented curriculum, as probability. Teachers can analyze responses, imagining the different solving processes and possible difficulties of students. These reflections can then be useful for planning educational activities consistent with the requests of the NG: in particular teachers can set classroom tasks, designed for formative assessment (Looney, 2011), comparing the different resolution strategies (correct and not) and discussing the results at different school grades.

CONCLUSION

In this paper we show that the qualitative and quantitative analysis of INVALSI tests can become a means for Italian teachers to perceive and reflect on the NG requirements, and on how these are evaluated through standardized tests. Even though the INVALSI tests are designed by Italian teachers and researchers to contribute to a system evaluation, we argue how an analysis of these test tasks can also be a tool to modify the system itself and carry messages related to the implementation of NG: e.g. by leading the attention on topic rarely developed in the implemented curriculum. As underlined by Santos and Cai (2016), assessment and curriculum are strictly tied. Italian standardized assessment can be both a tool for policy makers for the acquisition of comparative information on students' learning, and a vehicle to reflect with teachers on the goals for competencies development as stated in the NG. This is possible because the INVALSI framework is aligned with NG. Moreover, for teachers the INVALSI tasks can become tools for assessment, not only for evaluation: in fact, in the analysis shown, the focus is on the understanding of where, how and why students have difficulties. This can lead teachers to a formative use of standardized evaluation. This issue must still be investigated more deeply: we are carrying out a research that aims to analyze if, and how, Mathematical Knowledge for Teaching (MKT, Ball et al., 2008) changes during and after the activities carried out with teachers. In particular, it is possible to investigate how knowledge of the contents and curriculum varies. For this purpose, we collected materials from the analysis carried out by teachers during the educational programs and we are currently collecting and analyzing materials from long-term educational activities carried out in the classroom by teachers after the programs. Our research perspectives turn to the study of the teachers' role, not only as curriculum users, but also as curriculum developers and interpreters (Linares et al., 2004).

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FROM MATHEMATICS ENRICHMENT TO STEM: A SUCCESSFUL IMPLEMENTATION OF A SCHOOL-BASED CURRICULUM

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Following the globalization trend, Hong Kong government has promoted STEM education since 2015 and schools were encouraged to develop their school-based STEM curriculum. However, curriculum reforms do not always report success for the factors purporting the successful cases might be complex and much depended on the individual school and teachers. The authors report a successful school-based experience of reform initiatives via the case-study approach. Findings show that the school-based curriculum has gone through several milestone in journey to success namely, (1) A progressive model for launching of a mathematics enrichment program for the gifted since 2011; (2) a problem-solving approach for developing the curriculum lessons; (3) an evaluation with the evidence of students' performances; (4) Integrating with the new initiative of STEM education in the school-based curriculum. Finally, the authors argue that a key factor for successful implementation is to make classroom implementation as integral part of research.

Key words: curriculum reforms, gifted education, mathematics enrichment, STEM

1. INTRODUCTION

In line with the worldwide growing awareness of the importance to the education of Science Technology Engineering and Mathematics (STEM), the Hong Kong Government in the 2015 and 2016 Policy Addresses pledged to "renew and enrich the curricula and learning activities of Science, Technology and Mathematics, enhance the training of teachers, step up efforts to promote STEM education and encourage students to pursue the study of STEM–related subjects." According to the Report on Promotion of STEM Education issued in December 2016 by Hong Kong Education Bureau, the government proposed a range of strategies including, renewing curricula, learning activities for students, learning and teaching resources, and professional development of schools and teachers to promote STEM education by unleashing potentials in students' innovation. Schools were encouraged to develop their school-based STEM curriculum. However, curriculum reforms do not always report success for the factors purporting the successful cases might be complex and much depended on the individual school and teachers. In this paper, via the report of the experience of a successful case-study school, the authors attempt to provide some answers to the following question for the research theme (C) of "Implementation of reformed mathematics curricula within and across different contexts and traditions"

> C1: What processes, models, or best/common practices can be identified from the experiences in the implementation of new or reformed school mathematics curricula via a school-based experience?

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2. RESEARCH METHODS

The case-study approach was applied.

The school: The school was a very popular primary school in the neighbourhood, managed by a registered non-profit-making organization with a mission to promote the development of quality education in Hong Kong through the setting up quality schools. The school implemented ability-grouping policy with a selection strategy based on the results of the formative and summative assessment, the teachers' observation, supplemented with a measurement table based created according to the guidelines of the "Education Bureau Web-based Learning Courses (2013)". The school policy also facilitated a good teaching environment from the teachers' perspective, such as, appropriate grouping of the students, appointment of the teachers with relevant expertise for the program, the teacher would teach more than one class for the same year level to enable the teachers had the opportunity in a relative short time to improve and modify their own lessons based on self-reflection and catering for diversity of students.

The key-teacher: Leo, the co-author, is the key teacher participated in the project, so also playing the role of a participant researcher. Leo was the key teacher of the enrichment program (also co-author of the paper), is an enthusiastic mathematics teacher with a passion for learning, has a Master of Education degree specialized in mathematics education, is also a core-member of the Gifted Education Network (Mathematics) in the Education Bureau and an awardee of the Certificate of Teaching Excellence Award 2016. Leo designed, taught the enrichment program, during the implementation he developed and applied the problem-solving approach (Polya, 1954) in designing the lessons in combination with other popular learning theories such as, the Van Hiele Model (1999) and inquiry-based learning in the design of the curriculum and lessons.

Access of the data: For the research purpose of the study, a letter was written to the school principal for the access the school data of the implementation of the enrichment program and the STEM program, which includes: minutes of school meetings, PowerPoint presentations of different occasions, dissemination seminar, lesson materials such as lesson plans, worksheets, lesson video clips and student work; presentation of the internal school survey; the key teacher's personal audio record of PowerPoint presentation.

3. RESULTS: THE IMPLEMENTATION

3.1 A progressive model for launching the mathematics enrichment program for the gifted

Why was there a need for an enrichment program? In view of the results of TIMSS, Hong Kong students comparing with students in other parts of the world, despite of their good performance in mathematics, their confidence in mathematics and engagement in mathematics learning were relatively lower than other Asian regions. Also, the teachers in the school observed that the regular curriculum did not serve the purpose of exerting the potential of high ability students. Hence, the overall aims for the enrichment program were to raise their learning interest and confidence in mathematics, via enrichment of their learning of mathematics to enhance they thinking skills and problem solving skills. The school launched the enrichment program for the gifted students in the school with a progressive model (figure 1). It kicked off with 30 primary 5 students in 2011/12, extended progressively to include three levels (primary 4, 5 and 6) progressively in 2012/13, then integrated with STEM reform initiatives in 2015.

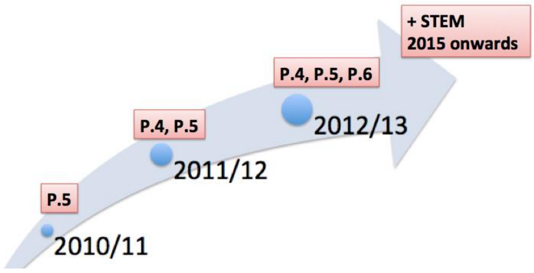


Figure 1: A progressive model for developing the 3-year Enrichment program

Selection criteria

The selection criteria for the high ability group took into consideration of the students' higher-order thinking ability (such as understanding and applying new mathematics symbols, motivation for learning difficult and extra mathematics knowledge, commitment and creativity in mathematics). Applying the core idea of the Three Ring Conception of Giftedness (Renzulli, 1986), the high (25%), middle (65%) and low (10%) groups were differentiated based on the students' performances in primary 3. The definition of "gifted " students in the school was different from the traditional definition that suggested IQ over 130 and less than 5% of students can be consider as gifted. Applying the Three Ring Conception of Giftedness, the gifted in the school included 25% of the students. The students began the program in primary 4 and the students' performance would be reviewed once before the promotion to the next year level and most of them would in the high ability group for three years. The class size was about 28 to 30 students (i.e., the normal class size in the school).

The program

The specific objectives for the school implementation of the enrichment program was: (i) to provide sufficient opportunities for students to develop their communication, analysis and problem solving skills; (ii) to let students fully utilize their normal lesson time. The reformed school-based mathematics gifted curriculum was a three-year enrichment program for the gifted students of primary 4 to primary 6, which encompassed the strategies of compacting, accelerating, deepening and widening:

1. Compacting: The teaching time of the regular curriculum in high ability group is reduced by approximately 25%-30% when compared with the other four groups in the same form.
2. Accelerating: Accelerating refers to enhancement that helps the students learn extra topics in higher level of the curriculum. For example, P.4 high ability group will learn extra topics such as addition and subtraction of decimals and fraction with different denominators, bar chart and rotational symmetry; P.5 and P.6 high ability group will learn the extra topics such as division of decimals and percentages.
3. Deepening: Problems stimulating exploration and thinking will be used, e.g, the problem will require the students solve a problem with minimum information, to look for alternative ways to find the solutions, to define and to design questions by themselves.

4. Widening: Problems with realistic context are used, such as, statistical analysis of BMI data of the class, the principles and operations of binary system after completing topic of large numbers in denary system, understand the principle of SOMA cube and create their own SOMA cube.

3.2 A problem-solving approach for designing the lessons

In this section, we used the lesson to show a possible way to create thinking space for the students in the enrichment program. The topic "the net of a cube" was taught in primary 4 in the mathematics curriculum in Hong Kong and the teaching in most schools aimed to teach the students how to draw a net of a cube and some teachers might feel frustrated when the students still got poor results after spending a lot of time on the topic, sometimes even ended up with rote memorization of the facts.

In the enrichment program, an alternative strategy based on Polya's problem solving model (Polya, 2014) was used in designing the lesson for the same topic, the lesson objective was revised to finding out the number of possible nets and a systematic way to draw all the possible nets of a cube. Under the teacher's guidance, the students would go through the 4 stages in Polya's model:

1. Understanding the problem: What does it mean by different nets? Are reversed figures different? Are rotated figures different?

2. Making a plan: Finding the major categories: 4 in a row, 3 in a row, 2 in a row, …, etc.

3. Carrying out the plan: After the categorization in the planning, the drawing and counting become systematic and simple.

4. Looking back and Extension: (a) For the above 3 steps, the students may have to look back and evaluate what they are doing at any stages? (b) How far can they go? For an extension to the SOMA CUBE problem (see figure 2), each group of the students was asked to draw the net of a block themselves. They could choose a block to draw the net according to their ability After gathering all the nets from all groups, they were further asked to discover the relationship among the blocks and made a simple conclusion. The purpose of such arrangement was the creation of thinking space, and the opportunity for explaining and communicating their ideas further. After the scaffolding discussion, the students were able to draw a more complicated net. Teacher printed out their own nets so that they could make their own toy. The students all found that the lesson was a rewarding experience.

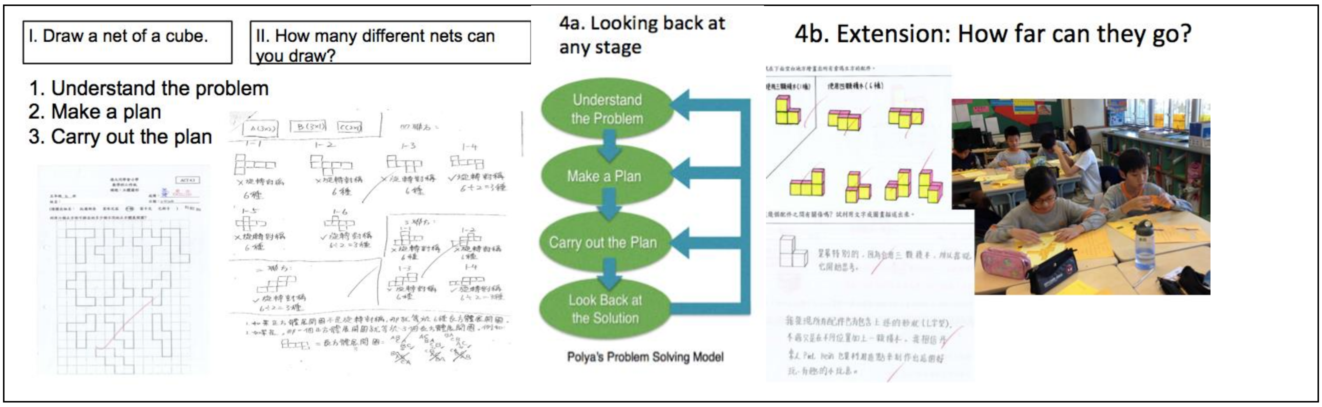


Figure 2: Applying Polya's problem solving model to the lesson of "The nets of a cube".

3.3 An evaluation with evidence of students' performances

Both quantitative and qualitative methods were used for evaluation of the program.

(A) Evaluation of each topic/lesson via the problem-solving approach

The topic of "The relationship between different types of quadrilaterals" was another well-known difficult topic. The teacher applied the problem solving approach in the process of designing the lesson.

1. Understand the problem: The problem is to find a relative method to categorize the quadrilaterals to design the lesson. In the analysis of the topic, the teacher applied the perspectives of the Van Hiele model (1999). It was noticed that the approach suggested in the textbooks put focus the first and second levels of the Van Hiele model, the visual level and the descriptive level.

2. Make a plan: To alter the teaching strategy, the enrichment program put emphasis on the informal deduction level, the third level in the Van Hiele model. The lesson design is to help student first getting familiar with the categorization system by the analysis the properties of trapezium, parallelogram, rectangle, rhombus and squares, equal sides; observing the various properties such as parallel sides, right angles; then they can decide to how to use the properties to categorize the quadrilaterals, also the flowchart online will be used as a tool.

3. Carry out the plan: Implement the lesson in which let the students build their own categorization system, discuss in groups the categories, criteria, methods and draw their own conclusion.

4. Evaluation: Comparing the student work produced in the enrichment lesson with lessons for the students with similar ability in the past. The results show: (1) Most students could create their own system of different complexity; (2) overall, the topic used 2 lessons less than the old approach, the spare lessons were used for building other skills such as paper folding and diagram drawing to enhance the students' spatial sense. Hence, in conclusion, it was decided to keep the newly designed lesson.

(B) Evaluating the students' academic performance before and after the enrichment program

A total of 90 students, 15 students from high ability group and 15 students from the middle ability groups in each of primary 4, 5 and 6 classes participated in the evaluation of the impact of the enrichment program (Sung, 2017). According to the report by Sung (2017), the implementation of the enrichment program for the enrichment program carefully evaluated for the following hypotheses:

H1: The program helped the students in the high ability group get higher academic achievement.

H2: The program developed the students in the high ability group high order thinking ability.

H3: The self-esteem of the students in the high ability group was raised.

H4: The test anxiety of the students in the high ability group was higher.

The test of H1 was based on the students' assessment results of the regular curriculum for P.4, P.5, and P.6 in academic year 2013-2014 Term 3 and 2014-2015 Term 3 respectively. The assessment results included the formative and summative assessments. The test for H2 was based on the formative assessment of non-conventional questions. The tests for H3, H4 were based on a questionnaire designed to collect the information of non-cognitive measurement for attitudes, belief and values, supplemented with student interviews.

Hypothesis	P.4 High ability group	P.5 High ability group	P.6 High ability group
Academic achievement	Slightly negative	-	Positive
High order thinking	-	Positive	Very positive
Self-esteem	-	-	Positive
Test anxiety	Positive	-	-

Table 1: The summary of the tests for the hypotheses

The results (Table 1) showed that P.6 students benefited most in the program and that the enrichment program could be a double-edge blade for some P.4 students. Possible reasons were that the student might take time to get used to the program and the students were likely to benefit more after participated in the program for a longer period. In addition, the school graduates sent back in retrospect very positive feedback about how they were benefitted in the enrichment program.

3.4 Integrating and implementing STEM component in the school-based curriculum

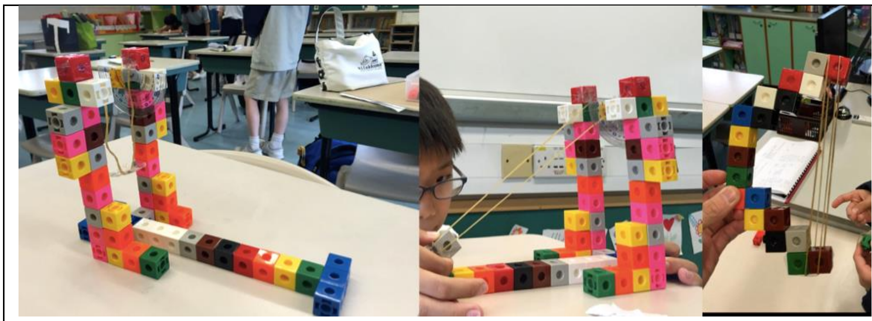
According to the teacher's summary of the enrichment program, the major features were:

- Compact and fast learning pace
- Enrichment topics for a board knowledge bases and interest
- Extension of existing topics for developing advanced thinking skill
- Introducing realistic contexts

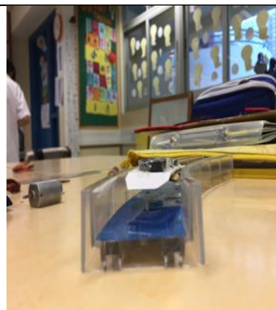
From the teacher's perspective, these features were compatible to the rationales for STEM education for the purpose of developing students' 21st Century skills, hence, the school decided to integrated STEM as an extension for the gifted curriculum. For the design of a STEM program, the teachers adapted the Maker concept with a belief that the nurturing of Makers should be that of helping students become good problem solvers, thus going beyond the experiences of assembling of a toy from parts. To achieve this aim, the school and develop a school-based strategy for the STEM program with four features:

1. Creation of "Thinking Space": Some aspects promoting the students' thinking opportunity include: The students' ideas are valued and they are encouraged to find out more possible solutions, alternative solutions and "proof" or justification for what they have to learn.
2. Creation of Hands-on Experience: The traditional mathematics curriculum provides little opportunity for hands-on experiences; hence, the students' ideas often become empty wishes. In this model, the students are encouraged to put their ideas into hands-on experience.
3. Provision of Trial-and-error experience: In the encouragement of experimenting their ideas, students also develop a deep learning for what are fair experiments and systematic trials.
4. Applying Polya's problem-solving model: For a guidance for systematic trials, Polya's problem solving model is applied to reflect upon on trials for identifying the crux of the problems and means for solving the problem.

In designing the STEM lessons, the gender issue was taken into account. According to the teacher, "Though some girls may not like mathematics, while choosing the topics carefully, the girls also develops in an interest." Some examples of the enrichment topics were modified to STEM topics, such as, building a shooting platform for paper cannon balls, building a maglev toy car, making a musical instrument, and handling Big Data (e.g., BMI of the class). (Figure 3)

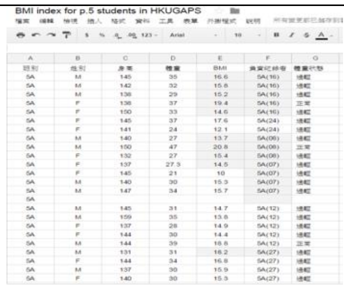


(1) Building a shooting platform for paper cannon balls



(2) Building a maglev toy car

(3) Making a musical instrument



(4) Handling Big Data: The students' BMI

Figure 3: Some examples of the STEM topics

5. FACTORS LEADING TO SUCCESS

Very often there is a gap between intended and implemented curriculum for the attention will shift to the monitoring of outcomes as soon as an innovation is planned (Fullan and Pomfret, 1977). Success implementation often depends on both contextual factors and personal factors. In an Asian context the limitation is often amplified for the competitiveness of the education system and the limitation of time and resources. For offsetting the limitation, the top-down policy plays an important role; the driving force for school-based changes is inevitably the alignment with the direction of curriculum reforms advocated by the Education Bureau. However, the success of the school-based curriculum is by no means by chance, it has gone through several milestones of development, namely, (1) a progressive model for developing the reformed curriculum; (2) a problem-solving approach for designing the lessons; (3) evaluation with the evidence of students' performances; (4) applying the evidenced-based approach in integrating and implementing STEM component in the school-based curriculum. Finally, in this report of the successful implementation of a school-based reform, there is a strong element of design experimental approach (Cobb et al., 2003). Leo, the key teacher, in the school-based reform designed the lessons with a genuine consideration of the curriculum goals and careful application of relevant learning theory in his design of lesson catering for the students' capacity. Furthermore, the plan was implemented and the lessons were scrutinized and evaluated with a deep reflection of the students' achievement in both cognitive and affective domains. Leo's approach of design research to some extent entails the features what Gu, Huang and Gu (2017) refer to as "a Chinese version of 'design-experiment'". The basic pedagogical approach in Leo's design is the inquiry-based learning but at the same time a primary goal of his design is to create an environment for his students to learn an effective way to master the key mathematics concepts via the inquiry. This is well demonstrated in his lessons for the topics of "the net of a cube" and "the relationship between different quadrilaterals." In addition, Leo's practice is a reminiscent of what is referred to as a successful research approach for realistic mathematics education that encompasses the features of "research-based" and "creating learning opportunities linking with well-specified learning goals" (Gravemeijer, Bruin-Muurling, Kraemer, and van Stiphout (2016). To conclude, the case study has exhibited a key for successful implementing a reformed curriculum:

> "A promising way of the closing the gap between research and practice is for researchers to develop and test sequences of learning opportunities, at a grain size useful to teachers, that help students move toward well-defined goals" Cai et al., (2017, p.342)

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IMPLEMENTING AN INTERNATIONAL REFORM INTO NATIONAL SCHOOL SYSTEM: THE STORY OF THE NEW MATH REFORM IN LUXEMBOURG

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This paper studies the implementation of the New Math reform during the 1960s and 1970s in Luxembourg. The New Math was an international math reform, which was disseminated in the Western part of the cold war with the help of the OEEC/OECD. The paper tells the story of driving forces and barriers, which shaped the implementation process of this reform in the small country of Luxembourg, with its specific geographical, cultural and lingual characteristics. This study shows how this international reform was adapted to be accorded with the existing culture, but also how the current culture and customs were changed and cultured by the new imported reform. This paper aims to bring an example from the past, which shows how a school mathematics reform with its international supports is implemented in a new context is adapted and adapts.

INTRODUCTION

This paper studies the process of implementation of the New Math reform and the translation of its original paradigm in a new context, Luxembourg. The paper proceeds in two main sections. The first section briefly looks at the creation of the supporting and background ideas and paradigms of the New Math reform. This section reviews how the idea of teaching modern mathematics was shaped, as well as how it gained the necessary power to get into curricula in the United States and travel to Europe and become dispersed. The second section investigates the process of the implementation of this reform into Luxembourgian school system during the 1960s and 1970s. The paper aims to study how the background idea and reasoning of the New Math reform through the journey of the reform from the United States to Europe and Luxembourg was changed and adapted to be applicable in the educational context of Luxembourg. The three main questions of the research are:

1. How was the New Math reform received and advanced in Luxembourg?
2. What did Luxembourgers expect from their school mathematics?

Furthermore, the paper shows that although the New Math, as an international educational idea, was not implemented as expected, it created a discourse, which led to the modernisation of school system and math education in Luxembourg.

THE NEW MATH REFORM AND THE CIRCULATION OF ITS IDEA

After the launch of the first Sputnik Satellite by the Union of Soviet Socialist Republics (USSR), the United States started a series of educational reforms. One of the most significant of these reforms was the reform of mathematics education, known as the New Math, which received the best funding from National Science Foundation (NSF) in the United States. The main characteristics of the New Math were to deemphasise "the rote calculation while infamously introducing sets and other new concepts, the designers of the new math attempted to fundamentally reform the way Americans thought about mathematics" (Phillips, 2014, p. 1). By this characteristic, the New Math claimed that it could prepare the future citizens for an unknown technological future world. However, a more scrutinised study of the story of the New Math shows that its philosophy rooted more profoundly in the history of math education. The New Math was created by a group of mathematicians and math educators at the University of Chicago who started working on a new plan for school mathematics in 1955. Their work was inspired by modern mathematics, which was introduced by a French group of mathematicians, named Nicolas Bourbaki.

Nicolas Bourbaki was the pseudonym of a group of French mathematicians, who started, since 1935, collecting the mathematical works of the 19th century and after with the intention of writing the 'Ultimate Mathematical Textbook' (Hacking, 2011). Bourbaki organised the new theories and approaches in a series of lectures entitled "Elements de mathématique" or "Elements of Mathematic" and called the mathematical concepts and theories used in these books "modern mathematic" (in French: 'mathématique modèrne). Bourbaki did not officially create a school curriculum, but their works and idea inspired the creation of the New Math school reform. However, when OEEC organised the Royaumont seminar in Paris in 1959 to introduce the New Math to European countries, some members of Bourbaki supported the act. For instance, Dieudonné (a member of Bourbaki) gave a controversial speech, wherein he articulated his famous slogan "Euclid must go" (OECD, 1961). This speech, which meant to accentuate the importance of the modernisation of mathematics education, was found too extreme by the audiences and created controversial discourse around the approach of the New Math reform.

The aim and promise of the New Math reform were to make the school mathematics focus on a deeper understanding of mathematical concepts, rather than just giving students some calculating skills, to improve critical thinking and learning ability. It aimed to offer a kind of mathematics, which can be useful for mathematical tasks, but also helpful in the understanding of science and other life problems, which the future would bring (Gardner, 1961). These aims and reasonings behind school mathematics were not new. Since the beginning of the 19th century when Arithmetic, as the third R of being literate, entered the school curriculum, questioning the purpose of math education was present in the discourses related to school math. The time was concurrent with industrial development and the need for more mathematical skills for industry. However, reviewing the relevant literature reveals another reason, which played a crucial role in justifying the necessity of math in the school curriculum (e.g., Alfonsi, 2012; D'Ambrosio, Dauben & Parshall, 2014). According to this literature, a significant reason was about the improvement of thinking ability in moral and philosophical education. Thus, two main ideas shaped the discourse around school mathematics since the beginning of the inclusion of mathematics as a school subject: math as a tool in everyday life and work (the paradigm of *Utilized math*) or math as a sort of mental workout (the paradigm of *Intellectual math*) (Nadimi, 2017). Each of these purposes can shape the approach of math education, including what to teach and how to teach it.

Before the new math, most attempts in the education of mathematics oscillated between the two aforementioned paradigms and in most cases, the intellectual math education had a disadvantage due to the complexity of its approach and unclear result (Nadimi, 2017). The New Math for the first time had both paradigms in its promised goals and purpose (Nadimi, 2017). The discussion around the purpose of the New Math reform shows that it strongly leaned toward the Intellectual mathematics.

However, due to the aim of at least some politicians and military people who expected the development in military and space science, was closer to the paradigm of *Utilized math*. After one year of practice in the United States, the New Math with all its background paradigms and discussions travelled to Europe with the help of the OEEC (Organization for European Economic Co-operation).

Royaumont Seminar

The OEEC organized a conference with the official title of "New thinking in school mathematics" and known as "Royaumont conference." It was held in L'abbaye de Royaumont in Paris, between 23rd of November and 4th of December 1959 (OECD, 1961). The conference was chaired by Marshall Stone, an American mathematician of the University of Chicago who was known for his efforts towards the internationalization of American mathematics research (Parshall, 2009). Each of the OEEC member states was supposed to send three delegates who should be outstanding mathematicians or mathematics teachers in their country. It was recommended that these delegates be among the "key people in their countries" because the organizers believed that "only" in this way "such a meeting could be successful" (OECD, 1961, p. 12). Before the conference, in March 1959, the OEEC sent a survey questionnaire to its member countries asking about the current conditions of their mathematics education and their suggestions for reform (OECD, 1961), which were then published in the proceedings of the Royaumont Conference (OECD, 1961). The Royaumont seminar, in general, had two major features to potentially impact the New Math movement in Europe and Luxembourg. First, it brought the discourse of the New Math movement up from national levels to an international level. It was a "rond point" at which delegates of different countries met and shared opinions on upgrading and updating school mathematics. These discussions helped to shed light on various aspects of the idea of school-math modernization that each representative might initially have had in mind. The second effect of the Royaumont seminar was that the conference created a precise subject of communication for the OECD to ask the member countries about it. Rather than generally asking about mathematics education, the OECD could ask about the implementation of the New Math reform (MEN0280, 1961 - 1973). This aim was one of the main aims of the conference as mentioned in the book of proceedings frosm the conference. The Royaumont seminar was the official establishment of ideas of the New Math. The ideas, meetings and discussions about a new way of mathematics education had already begun before this Conference, but the conference made it public.

Luxembourg also had two delegates in this seminar. These delegates were Lucien Kieffer and Marcel Michels, who were among influential figures of school mathematics in the country. The next section explores how Luxembourg received and developed the New Math reform for its school system.

THE NEW MATH REFORM IN LUXEMBOURG

In 1958, the Ministry of National Education of Luxembourg introduced a draft of a school reform plan, which was a broad and ambitious plan to restructure the whole school system from primary to secondary school. The goal was to create a harmonic school system which links all the levels of schooling with each other as well as linking to the life of citizens (Frieden, 1958). A revision of the family of mathematics education (arithmetic, geometry, etc.) was also a topic of discussion in this plan. The need for a math reform was discussed in various discourses among politicians, or public media showed that Luxembourgers were not satisfied by their math education. The discontent toward the school mathematics of the country was vastly reflected in different public press and teacher journals (e.g., d'Lëtzebuerger Land, 1959; Bour, 1959). The complaints were either about the unsuccessful experience of Luxembourgian students at the universities of neighbouring countries; or in general, it was argued that the school mathematics education was not useful enough for citizens in the different paths they took after finishing school. The above-mentioned reform was not successful in achieving its ambitious goal of linking different school subjects and different level of schooling. Primary and secondary schools implemented education reforms separately, and at different times. The same story happened for the reforms of different school subject including mathematics. The importance of this plan was that it coincided with the beginning of the New Math reform. Thus, it seems that the country was mentally prepared for a rigorous reform such as the New Math.

Articles published in the teacher journals also show that the idea of a reform plan like the New Math was already circulating before the Royaumont seminar, probably from a previous CIEAEM conference in Europe (e.g., Kieffer, 1959). Most of these articles sought to link math education to different aspects of the citizen's life and not just industry or economy and they believed including modern mathematics could help to achieve this goal. In general, reviewing the discussions before the Royaumont seminar seemed the country was prepared to implement the New Math reform when the reform movement started in Europe. However, in practice, Luxembourg was one of the last European countries that introduced a New Math style curriculum into its school system. The New Math officially entered the secondary school in 1968 while primary school has to wait until 1980 to formally see the New Math in the textbooks. After the Royaumont seminar, even those authors who supported a math education based on modern mathematics started to doubt about it (e.g. Kieffer, 196o). One reason might have been a result of discussions and debates during the ten days of the Royaumont seminar. The other important reason was related to the structure and the nature of the school system and the goal of the school system in Luxembourg, which is discussed next.

Cooperation with Centre Belge de Pédagogie de la Mathématique (CBPM)

Historically, the Luxembourgian and Belgian school systems were collaborating with each other, specifically in teacher training (Elz, 2009). Since 1961, the Belgian Centre of Mathematics Pedagogy (French abbreviation: CBPM) started preparing math curricula and math teacher training in Arlon, a Belgian city near the border of Luxembourg (Vanpaemel,, De Bock, & Verschaffel, 2012). Some teachers from Luxembourg were also collaborating with this centre to plan curricula and to prepare the necessary textbooks. Preparing teachers for this new reform started in 1962 at the CBPM. Almost half of the Luxembourgian teachers attended the teacher preparation courses organised in Arlon (Klopp, 1989). However, the two countries did not engage with the result of this cooperation similarly. Belgian schools started the New Math reform since the early 1960s, while, as already mentioned, Luxembourg introduced the New Math much later. The reason argued was the incompatibility of the Belgian reform for the Luxembourgian school system, as also summarized in a report to the ministry of education:

- The linguistic situation of the country necessitated to add more language classes in the secondary school compared to Belgium, which left fewer hours for mathematics.
- The passage exam between lower classes and higher classes of secondary school urged teachers to work on the necessary topics for passing the exam. Thus, they did not have time to teach the new concepts.

- The creation of pilot classes was difficult due to not having appropriate textbooks (MEN1158, 1967).

It is important to mention that Luxembourg has three official languages (Luxembourgish, French and German), which are also taught in schools, and thus a significant portion of weekly school hours is devoted to language education. Luxembourgish is a Germanic language, which is practised at home. In primary school, all subjects are to be taught in German, but students also learn French as a language, and it is the dominant language of the secondary school. Most subjects in secondary school, such as math and sciences, are taught in French, and students may learn English as an additional Foreign language. Thus, this language policy may restrict other school subjects in both quantitative and qualitative ways. A generous portion of school hours is devoted to the language education and the teaching language changes from primary to the secondary school. This language policy has been in effect since the school law of 1908, except during the World War II when the country was occupied by Germany. For all these reasons, the New Math reform could not proceed the same as it progressed in Belgium and had to take a different path.

First step: the secondary school

In 1968, a structural reform of the secondary school facilitated the introduction of the modern mathematics in the curricula. Before this reform, there were three tracks of education for the secondary level: classic education for boys, modern education for boys and education for 'young girls.' Classic sections offered three hours of mathematics for all the classes, except the section B, which provided one extra lesson to prepare students for foreign universities (MEN1145, 1947-1952). The reform of 1968 increased the weekly lessons of mathematics for some classes to 4 hours, which was not still sufficient, but it was an improvement. Before this reform, the number of hours for mathematics for boys and girls was similar, but the contents were different (MEN1145, 1947-1952). Mathematics for the young girls was lighter and rather just an introductory (Klopp, 1989) or mainly related to household work (Schreiber, 2014). Thus, the reform of 1968 had a capital favour for mathematics education of the country by stopping the differentiation between boys and girls regarding mathematics education (Klopp, 1989, MEN 1135, 1970). Before the reform of 1968, it was not accepted that girls could approach mathematics studies. According to Klopp (1989), some of the girls who had enough "obstinacy" to follow mathematics study at the university, had to confront various social problems in their environment (p. 255). This reform also eliminated the differentiation in math education between the classic and the modern sections. The whole reform, and not only its mathematics part, pointed to give more weight to scientific and technical study (MEN1145, 1947-52; Mémorial A n°23, 1968). As a result of such views, the somewhat formative mathematics of the classic sections (boys and girls) and the purely applied mathematics of the modern sections were combined with each other for all the sections. The reform practically gave a prominent place to mathematics in all of the sections from the science department to the economic and law (Klopp, 1989). Mathematics was no longer only a tool for the industry or some abstract concepts for a classic education, but it was considered to be relevant and applied in a broader perspective.

Second step: The reform process for primary school

Since 1965, primary school teachers were taught the notions of modern mathematics such as 'Set theory,' 'group theory' and statistics, and the possibility of introducing these concepts into the curricula of primary school was examined (MEN1158, 1967). Furthermore, a group of the teachers was also collaborating with CBPM to investigate the potential of modern mathematics in the school curricula. Some booklets (sometimes handwritten) were available for teachers to teach these new concepts (e.g., Dieschbourg, 1966). After the reform of 1968, some New Math was taught during the two last years of primary school, to prepare students for the secondary level (UNESCO, 1969). This period of piloting was dependent on the decision of teachers of each class, which could be not taken seriously due to other things that teachers had on their agenda such as teaching the official textbooks.

For the school year of 1970-1971, a pilot class in the primary school of Steinsel, a municipality in the centre of the country of Luxembourg. The school in Steinsel was chosen as a pilot school to examine the New Math program provided by the CBPM. During the 1970s, gradually more schools joined the program (Dieschbourg, 1971). In Luxembourg, all primary schools of the country use an official textbook for each subject provided by the ministry of education. During the 1970s, the official books of the Luxembourgian primary school had no trace of modern mathematics. After the school reform of 1979, new textbooks were introduced in 1981 in Luxembourg that included for the first time modern mathematics such as sets, groups and equality/inequality.

Language was one of the issues that challenged the process of the reform for primary school even more than with the case of secondary school. For the teaching of the New Math at the secondary school, the French textbooks from Bréard collection were used. However, despite the collaboration with CBPM, the Belgian textbooks could not be used in Luxembourg because the teaching language of primary education in Luxembourg is German. Another issue that was mentioned as an obstacle to the way of adapting the reform was the lack of "Luxembourgian" textbooks, which made even the realisation of the pilot classes challenging (MEN1158, 1967). The meaning of "Luxembourgian textbooks" or "manuels Luxembourgeois" was not only related to language. Having national textbooks in accordance to the need and the culture of Luxembourg was an important issue in the Luxembourgian school system for the primary school (Nadimi, 2017). Mathematics textbooks were changed once during the 1970s. Wherease these did not introduce modern mathematics, they did integrate more examples from the life of a Luxembourger (Nadimi, 2017). It was not until the 1980s that modern mathematics was presented for primary school, and it is evident that in general, the reform for primary school moved more cautiously than that for secondary school.

The role of OECD and international forces

Finally, at least briefly, it is important to mention the role of international support in the process of implementing the New Math reform in Luxembourg. As mentioned, in the first section of the paper, OECD after the Royaumont seminar was encouraging countries to implement the reform. During the era of the New Math reform, this institution frequently asked Luxembourg about the status of mathematics education and the new reform (Nadimi, 2017). An additional way that the OECD and specifically the ICMI encouraged the modernisation of school mathematics in Europe and Luxembourg was through organising conferences. Luxembourg hosted three ICMI conferences, and the representatives of the country participated in several OECD and ICMI conferences related to school mathematics.

CONCLUDING POINTS

This paper showed an example of an adaptation of an imported reform plan in a new context. The New Math reform needed to be adapted to the existing culture of the school system of this country and its conditions, goals and values. Two central values that we mentioned in this paper was the importance of multilingualism and having national textbooks for primary school. The New Math reform did not develop in Luxembourg in the same ways as other countries, or in the way that some teachers or mathematicians expected. Borrowing words from Foucault (2000), the result did not coincide the goal, but the reform raised discussions about school mathematics in Luxembourg. A previous study shows that the reasoning behind changes and revisions was developed together with the development of national identity (Nadimi, 2017). What it meant to ‘be a Luxembourger’ shaped the values and the goals of schooling. The reform movement could not increase the weekly hours of school mathematics much, but it made stakeholders, teachers, and people rethink what matters in the education of young Luxembourgers from the point of view of stakeholders. The average weekly hours of mathematics in most divisions of secondary education is still 4 hours in Luxembourg (MEN, 2016). Thus, no other further reform could change the quantity of the mathematics education in this country. However, the process of implementing the reform reconstructed the culture of math education in this country and modernised it. The New Math help to unify math education for girls and boys. Furthermore, it revised and modernised the background idea of how mathematics can be useful. The two paradigms which shaped the reasoning of mathematics education in other parts of the world were also present in Luxembourg. One representation of this duality was in the secondary school mathematics program, which was divided into two sections of Classic and Modern. The mathematics program of the Classic school was rather theoretical and thus closer to the paradigm of the *Intellectual math*. The mathematics curricula of the Modern section were more pragmatic and thus closer to the paradigm of *Utilised math*. The structural reform of the secondary school in Luxembourg, which came together with the New Math reform, also unified these two sections. As a result, the school math of Luxembourg also moved toward a kind of education that tries to consider both pragmatic closer needs and intellectual future needs.

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PREPARING PRESERVICE TEACHERS TO TEACH MATHEMATICAL LITERACY: A REFORM IN A TEACHER EDUCATION PROGRAM

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To meet the demand of incoming school curriculum reform focusing on competency-based learning in Vietnam, this paper reports on an innovation project on developing secondary mathematics preservice teachers' (PSTs) mathematical literacy and preparing them to teach mathematics contextually. We developed curriculum and studied the effectiveness of the implementation to a secondary mathematics PST education program that integrates mathematical literacy (ML) in methods courses. The courses offer the PSTs opportunities to experience ML as active learners and prepare them to teach ML. The preliminary results on a project-based modeling task show that the PSTs begin to develop an understanding of ML when they take real-life considerations into account in solving the authentic problem. Discussion about the tension between simplifying models and reflecting the real problem, and directions for future study are suggested.

INTRODUCTION

This paper reports on an innovation project on developing secondary mathematics preservice teachers' (PSTs) mathematical literacy (ML) to prepare them to teach mathematics contextually. This initiative is to address the incoming school reformed curriculum, which will be implemented in 2019. The curriculum focuses on competency-based learning (Vietnam Ministry of Education, 2018) and emphasizes the close relationship between mathematics and the real world. Teachers serve as an agent to make changes in their classrooms, and we argue that PSTs should be equipped with the skills to teach mathematics contextually. Therefore, our ongoing project strives to (a) investigate the process to prepare PSTs to teach mathematics contextually that meets the demand of the reformed school curriculum and (b) document the influences, successes, or failures of the implementation on PSTs' knowledge and practice. In this paper, we will address how the PSTs' mathematical literacy understanding is evident through their work on a project-based task in this innovative program.

SCHOOL REFORM – COMPETENCE-BASED CURRICULUM

The current national school mathematics curriculum in Vietnam, started in 2002, rarely highlights the relationship between mathematics and the real world and does not mention mathematical modeling or mathematical literacy explicitly. To meet the demands of the changing society, a reform in curriculum and textbooks following a competency-based learning will begin to be implemented in 2019 (Vietnam Department of Education, 2018). In this reformed curriculum, mathematical modeling is featured as one of the five competencies that involve communication, mathematizing, reasoning and argument, solving problems, and using mathematical tools. The curriculum indicates that it is necessary for students to use mathematics in everyday life. The curriculum underscores real-world contexts in each lesson, and students are required to apply mathematical knowledge they learned to solve real problems as well as to understand the meaning of mathematical knowledge in the real world. The reformed curriculum will provide opportunities for students to experience and apply mathematics to real-life situations and build the connection between mathematics and reality. However, it is challenges for teachers (including PSTs) to teach mathematics in the way the curriculum intended, as they rarely have such experience as a learner and are not trained to teach mathematics contextually. Therefore, we revamp the mathematics teacher education curriculum, implementing the innovation, and research its effectiveness.

CONCEPTUAL FRAMEWORKS FOR REFORMING A PST EDUCATION PROGRAM

Mathematical literacy

ML is an individual's ability to understand and use mathematics in a variety of contexts, including everyday life, professional, and scientific settings. Mathematics serves as a tool to describe, explain, and predict phenomena (The Organization for Economic Cooperation and Development [OECD], 2013). In turn, individuals appreciate the role mathematics plays in the world and how it prepares them to be constructive citizens and make well-founded judgments and decision. Additionally, OECD (2013) utilizes the mathematical modeling cycle (cf. Kaiser & Stender, 2013) to describe students' actions when facing challenges that require them to apply mathematics, including individuals' capacity to formulate, employ, and interpret mathematics in a variety of contexts.

Several researchers and mathematics educators highlight the difference between mathematics and ML and argue that some people who are good at mathematics are not necessarily good at ML (e.g., Steen, 2001). The focus on developing ML might be different from developing mathematical understanding. For example, whereas the aim of developing school mathematical understanding is to help students climb the ladder of abstract structure, ML is anchored in data that are derived from the empirical world. In addition, school mathematics tends to develop school-based knowledge, but ML involves mathematics acting in the world (Steen, 2001). It is important to note that ML used in this context is not limited to understanding and applying arithmetic but the abilities to use different mathematical knowledge, which might include advanced mathematics. Moreover, ML includes not only the skills and knowledge but also the beliefs, dispositions, and habits of mind people need to engage effectively in quantitative situations in life and work (International Life Skills Survey, 2000). The ML concept serves as a foundation to help PSTs make the connection between mathematics and real life.

Knowledge for teaching mathematics

Teacher knowledge is an important predictor of student achievement because a mathematics teacher's decision-making in class is a function, in part, of her/his knowledge (Schoenfeld, 2010). Educational researchers have conceptualized knowledge for teaching to include subject matter knowledge and pedagogical content knowledge (Shulman, 1987). In particular, pedagogical content knowledge (PCK) refers to:

> The most powerful analogies, illustrations, examples, explanations, and demonstrations—[. . .] the most useful ways of representing and formulating the subject that make it comprehensible to others.... Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them. . . (Shulman, 1987, p. 7)

This conceptualization of teacher mathematical knowledge informs us to provide the PSTs with opportunities to learn (OTL) ML and to teach ML to their future students. Therefore, we create opportunities for the PSTs to experience ML as active learners and engage in developing their knowledge/skills to teach ML through teaching tasks such as analyzing curriculum, selecting, adapting tasks, and using appropriate approaches to teach.

RESEARCH DESIGN

Research and Data Collection

This ongoing two-year project ran with a cohort of 120 PSTs. The cohort started participating in the project in September 2017, at the beginning of their third year in the program. We will continue following them to their last school placement. We adopted a design-based research (Cobb et al., 2003) that involves continuing data collection and data analysis, and curriculum development and implementation. First, we identified gaps related to ML in the current mathematics methods courses in the program. The current program indicates limited opportunities for PSTs to experience ML as learners and to develop PCK for teaching ML. In mathematics methods courses, the opportunity to learn in ML is limited (1.2 % of total training time) to an introduction to mathematical modeling and the Program for International Student Assessment (PISA). In 2017, we collected empirical data on the PSTs' opportunities to learn ML and their beliefs about mathematics and mathematics teaching and learning. The initial data analysis sheds light on our curriculum development, focusing on PSTs' mathematics methods courses and PSTs' field experience (school placement), and how to change PSTs' disposition/attitudes towards the subject.

In addition to measuring OTL and beliefs, we captured PSTs' modeling competencies as a proxy of their ML by using both a multiple-choice test and open-ended word problems. We adopted a research-based multiple-choice test (Haines et al., 2002) to measure the PSTs' understanding of ML when they started the methods courses in the program. This tool was developed measuring aspects of modeling competencies, which was administered to the PSTs individually within 60 minutes. We also got the PSTs to work in pairs for 120 minutes on open-ended tasks focusing on four content areas: shapes, quantity, data and chance, and change. The data provided us with information about the PSTs' current content knowledge related to ML and the weakness and strength the PSTs have prior to the methods courses. All the data were used to incorporate the opportunities to learn ML.

We have conducted interviews (task-based and stimulated recall). Other data sources include notes from classroom observations, PSTs' reflection on their placement related to ML, and their lesson plans. In 2019, the project will finish with a post measure of PSTs' ML OTL, beliefs, and modeling competencies. Post interviews will be conducted on some participants (cases). Table 1 summarizes the timeline of data collection.

Curriculum Development and Implementation

Opportunities to learn and teach ML were incorporated into four methods courses: Mathematics Teaching Methods and Assessment of Mathematics Learning (Semester 1 of 2017-18), Mathematics Curriculum Development and New Trends in Mathematics Teaching and Learning (Semester 2 of 2017-18). Additionally, PSTs were asked to reflect on ML when they were at their school placements. The first placement was mainly focusing on observing real classrooms and planning mathematics lessons but not implementing the lessons. In this instance, the PSTs were asked to reflect on how the observed lessons offer ML OTL and nominate their one best lesson plan that incorporates ML. In the second placement when PSTs will plan and implement their lessons in real classes, they will be asked to report on how they incorporate ML into the classes as well as reflect on the challenges/success they have when teaching ML.

Content	**Pre**	**Curriculum Implementation and Data collection**	**Post**
	• OTL ML measures (Individual) • Beliefs about mathematics and mathematics teaching and learning (Individual) • Multiple choice modeling test – Open-ended modeling tasks (Pairs) • Stimulated recall interviews about their OTL and beliefs • Stimulated recall interviews on modeling competencies	• Curriculum Implementation: Mathematics methods courses School Placement • Artefacts collections – Student works and presentations on ML tasks • Classroom observations	• OTL ML measures (Individual) • Beliefs about mathematics and mathematics teaching and learning (Individual) • Multiple choice modeling test – Open-ended modeling tasks (Pairs) • Special-cased interviews on their experience of the program
Timeline	09/2017	09/2017-05/2019	05/2019

Table 1: Project timeline and data collection

First, we exposed PSTs to tasks that offer rich opportunities to engage in ML as active learners. ML tasks have been integrated into mathematics methods courses, which range from standard applications to true (authentic) modeling problems (Tran & Dougherty, 2014). These tasks were adapted from research (e.g., PISA) to fit in the context of Vietnam. Some tasks were created based on the project team's experience with the training program and understanding of the local context, such as designing birthday cake boxes and designing the Hue University of Education parking lots for staff and students. We scaffold PSTs' experience with ML tasks by introducing them with increasing levels of authenticity tasks (Palm, 2009; Tran et al., 2016) that were solved within different time periods, such as several tasks in one session (Semester 1 of 2017-18 academic year), one task in a session (Semester 1 of 2017-18), and project-based tasks that last for several weeks (Semester 2 of 2017-18 school year). These tasks necessitate the use of realistic considerations, not merely mathematical tools. We aimed to help the PSTs to experience revising model and validating process as they went through the modeling cycle.

Second, we aimed to prepare PSTs with PCK to teach ML. In their third year of the program, PSTs were introduced to the modeling cycle (OECD, 2013) to inform phases that students generally go through when solving modeling problems and to reflect on the process of solving ML tasks. In Semester 2 of 2017-18, the PSTs were exposed to knowledge about ML and how to incorporate ML into the current curriculum. PSTs analyzed current school curricula to investigate how ML was introduced in the documents and contrasted with the reformed curriculum. They also explored how ML was emphasized in curricula from other countries. PSTs were asked to plan a lesson that integrates ML into the content specified in the curriculum. In Semester 1 of their fourth year, PSTs will be asked to analyze tasks based on the modeling cycle and the level of authenticity and then adapt them to incorporate into real lessons. In addition, they will analyze student works on modelling tasks and how to evaluate them as an assessment practice.

Project-based Task

In Semester 2 of 2017-18, PSTs were asked to work on the following task: "Currently, in our university campus, there are five parking regions that are close together which look quite messy. Can you design a parking lot for the university to solve the current issue so that it looks neat?" This task was similar to tasks found in literature, yet the uniqueness is that vehicles parked in this task include cars, bikes, electric bikes, and motorbikes, not just cars or motorbikes. PSTs were asked to work on this project for four weeks in groups of 4-5 and report to the class at Week 4. Students presented weekly on their progress of the project to get feedback/questions from peers (not in their groups) and the lecturers to improve their reports. They submitted their written report and gave a presentation to the class. The data for this task included their written report and their presentations.

Data Analysis

To evaluate the initial success of the implementation, we focused on preliminary results regarding different ways the PSTs approached an authentic modeling task of designing a parking lot for the university. A total of 15 written reports were collected. Especially, we looked for (a) evidence PSTs took realistic concerns into account (data, information, technical considerations) when designing the university parking lot and (b) their experience in different phases of modeling cycle when working on the task. We identified how the PSTs transferred from real life to mathematical problems and what variables they took into account to formulate mathematical models. We examined how they solved the problems and interpreted them back to the real-life issues.

PRELIMINARY RESULTS

A preliminary analysis shows that the PSTs formulated two mathematical problems or a combination of them: (a) design parking lots based on the information about the number of vehicles, and (b) find the cost to build the parking lots. The analysis reveals that the PSTs used a combination of arithmetic and proportions as main tools on this task. Some used sampling and data collection techniques to estimate the number of vehicles and used direct measurement and area formulas. Some built regression models to predict the cost.

In this paper, we report two samples from PSTs. The samples were chosen to (a) highlight the PSTs' considerations of real-life issues and the collecting of empirical data (measurement of the parking lot, surveying numbers of each of the vehicles) and (b) represent different mathematical tools the PSTs used to solve relevant mathematics problems formulated from real-world problems (e.g., arithmetic, advanced mathematical tools such as linear programming).

Surveying The Number of Vehicles and Designing Parking Lots

Group 1 specified real-life problems to address the issues messiness of the parking in Hue University of Education, when more people use vehicles to come to university as a result of advancing standards of life. They evaluated the quality of Hue University of Education's current parking and provided a plan for building the new facility with given funding. When attacking this problem, they found information about the number of vehicles present daily at the university and areas available for parking, which were the two sub-mathematical/statistical problems formulated from the real issue.

They searched the university website (http://www.dhsphue.edu.vn) for information about the number of staff members and students. However, the data might not reflect exactly the number of vehicles, which was the main variable to consider when solving the problem (validating). The group then surveyed the number of vehicles of each type on four random weekdays. When collecting the data from four days, on average, they found the percentages of vehicles in each of the parking lots: H (50%), DEG (40%) and GV (10%). In addition, they estimated the number of vehicles for each type as: motorbikes (60%), electric-bikes (15-17%), regular bikes (20-22%), and cars (2-3%). They estimated 1500 vehicles in the student parking lot and 180 in the staff parking lot. Therefore, they decided to design two parking spaces, one for students and one for staff. The student parking lot could hold 900 motorbikes, 250 electric-bicycles, and 350 bicycles. The staff parking lot could hold 135 motorbikes and 45 cars. During this process, the students used sampling and surveying as mathematical/statistical tools to collect data. Moreover, they utilized knowledge about proportion and percentages to decide the capacity for each type of vehicle in each parking lot. On the second sub-problem, the PSTs measured the sizes of current parking lots by applying their knowledge about the area. They drew a floor plan with specified dimensions for the parking lots.

The PSTs then searched for dimensions of each vehicle type to decide the appropriate space for them using a rectangular model and compared the area of the models to those of the real parking lots proportionally. They found that one-story parking lots would not be sufficient to meet the demand of the space for all vehicles; therefore, they needed to look for an alternative design. The PSTs investigated parking lots in other universities and those of a supermarket in the city to look for parking designs and how to operate the parking. As they found that no students travel to the university by cars, they decided to create one parking lot for staff and two for students. After collecting all relevant data, they designed a two-story parking that reserved one story for 45 cars and the other for motorbikes. Particularly, with their survey of car dimensions and spaces between two cars (length: 5,5 m, width: 2,3-4 m and the gap: 4-6 m), they figured out that the area of the parking should be about 36*30 (m^2).

For the students' parking lots, this group revamped the model of current parking lots by including specified dimensions for each row, taking into account the dimension of bikes, motorbikes, and electric bikes with the length of 2m and width of 0.8m. The distance between two consecutive rows is 1.8m. Therefore, they used a 2-meter square for each vehicle in these parking lots. They worked out the number of vehicles for each of the parking lots in the university and checked if the lots meet the demands of student vehicles from their survey (Figure 1). After finishing these sub-problems, they determined the cost to build such parking lots. They then submitted their findings and presented their plans to the class.

Focusing on predicting the cost of building parking lots on the number of vehicles

Group 2 surveyed the number of vehicles on three random days and found 1600 vehicles per day (motorbikes, electric-bikes, regular bikes) for both staff and student. This group measured the sizes of parking lots and calculated areas. They also decided that one of the parking lots needed to be two-story. They decided to build three parking lots: one two-story and two one-story that would connect to the three entrances into the University: G32, G34, G36. Additionally, they formulated a mathematical problem to predict the cost of the parking when knowing the number of vehicles.

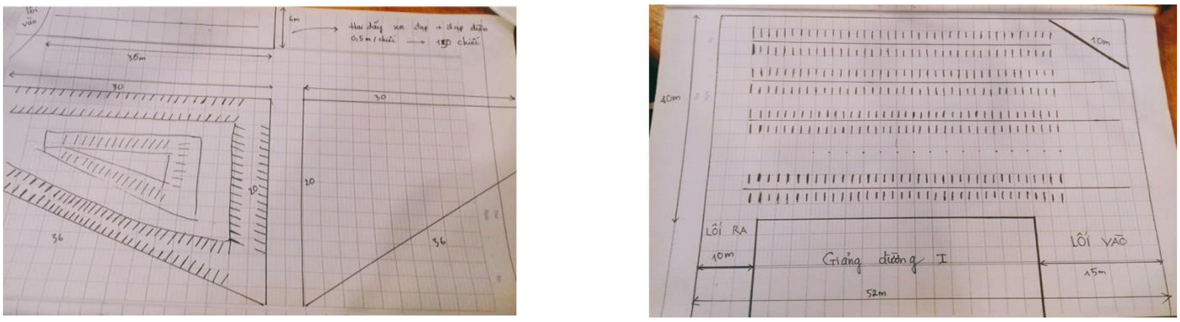

Figure 1: Layout of vehicles into roles in two parking lots

Based on the information about the cost of materials and relevant equipment needed to operate the parking lots (e.g., camera) and the cost to demolish the current parking lots in the University, they recorded the data on a table. The data were based on the following variables: the money to demolish the current parking lot, how much of the old infrastructure could be reused, the area of the parking lots, the number of stories, and the number of vehicles in each of the parking lots. They then graphed the data in a coordinate plane including one axis for the number of vehicles and the other the cost (in Vietnamese dong). They created a power function as an approximation for the collected data to come up with a model. The coefficients were an estimation based on the data, without checking if the models were good for prediction, or a regression model to minimize the total sum of square deviations (Figure 2). The two models follow:

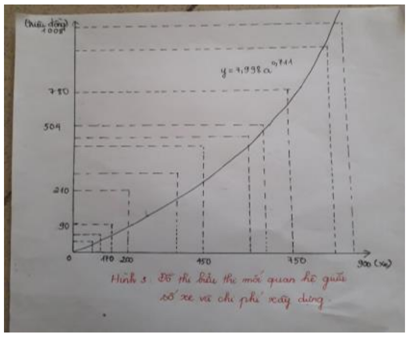

Figure 2: Power functions to predict the cost for one parking

Two-story parking lot at G34 for staff:

C = $\boldsymbol{C_{32}} + \boldsymbol{C_{34}} + \boldsymbol{C_{36}} = 0,\mathbf{918}.\boldsymbol{x_{32}}^{\mathbf{0,95}} + 2,05.\ \boldsymbol{x_{34}}^{\mathbf{0,95}} + 1,22.\ \boldsymbol{x_{36}}^{\mathbf{0,83}}$ (*x*.. is the number of vehicles in the parking .., and C.. the cost to build parking lot ..)

Two-story parking lot at G32 for students and keep the staff G34 parking lot as is:

Model 2: C = $\boldsymbol{C_{32}} + \boldsymbol{C_{36}} = 1,28.\ \boldsymbol{x_{32}}^{\mathbf{0,98}} + 1,22.\ \boldsymbol{x_{36}}^{\mathbf{0,83}}$

DISCUSSION

This ongoing project is in the process of implementing the innovative curriculum focusing on the developing PCK for the PSTs to teach ML. We have not collected the post data to investigate the effectiveness of the program. However, at this stage, the data show that the PSTs started to experience mathematics in a different way--not merely considering real-world contexts as a cover, which is easily stripped out to reveal the mathematics. Additionally, the PSTs experienced the uncertainty when using mathematics to solve problems they encounter in their lives. However, opportunities to discuss the difference between their estimations of vehicles were not taken, which could be powerful for validation. When predicting the cost to build parking lots, the PSTs need to balance how much they simplify the model so that they can formulate a problem that is solvable versus how to develop a model that is sophisticated enough to capture the real world, yet challenging to solve with their current mathematical knowledge. The PSTs were not familiar with regression models in prediction and unsure how to evaluate the goodness of their model; such findings calls for possibly having collaborations between mathematics educators and mathematicians who are responsible for training the students. A question emerged is what the program would look like if the mathematicians take an ML perspective when teaching their courses: how could PSTs' mathematical knowledge be strengthened?

Within the scope of this project, we investigated how PSTs' knowledge changed during the implementation of the innovation. However, it is challenging to discuss the impact of the implementation on their teaching practices because the PSTs have not had opportunities to teach in a real classroom. Future studies could build on this project and follow the PSTs in their second placement (teaching) and their first years of teaching to see how the training changes their practices.

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STATISTICS AND PROBABILITY IN THE SPANISH BACCALAUREATE: INTENDED CURRICULUM AND IMPLEMENTATION IN TEXTBOOKS

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Last curricular reform in Spain was introduced in 2015 in upper High School. The official curriculum issued by ministerial decree follows trends on statistical literacy and acknowledges some claims from scientific societies. Nevertheless, the intended curriculum still keeps some contradictory ideas about the use of ICTs for handling statistical data. Additionally, the representation of the curriculum in textbooks distorts some of these ideas, since it keeps many inertias inherited from previous curricula and from traditional uses of Statistics at school level. In this work, we analyze this curricular reform, and the way it has been implemented in textbooks, pointing out strengths and weaknesses.

INTRODUCTION

Statistics and Probability (SP) at school level have been a topic of interest, focusing the 18th ICMI Study on the challenges for teaching and teacher education derived from SP (Batanero, Burrill, & Reading, 2011). The latter was a frontrunner study and had impact not only on educational research, but also on subsequent curricular reforms, as we are trying to demonstrate in this work. Worldwide, the notion of statistical literacy (Gal, 2002) has been dispersed within the educational community, due to the novelty of this approach to reflect about what and how SP can be taught and learned in order to empower critical thinking among future citizens.

Despite the situation of SP curriculum has been studied previously in some countries (Campos, Cazorla, & Kataoka, 2011; Froelich, 2011; Newton, Dietiker, & Horvath, 2011; Wessels, 2011), most of this work focuses on intended curriculum. On the contrary, our approach is based on a double perspective. On the one hand, we analyze the intended curriculum in the last years of Spanish secondary school, as it is issued in the official documents from the Government. On the second hand, we study the implementation of the curriculum by examining how it is represented in the textbooks.

This paper is organized as follows: we first outline an overview of the current SP curriculum, analyzing the learning standards. Then, we collect and discuss the results of a study about the curricular implementation on textbooks. Finally, we provide some conclusions.

STATISTICS AND PROBABILITY IN THE SPANISH CURRICULUM

Structure of study cycles and previous curriculum

Since 1990, secondary education is divided into a first four-year-long compulsory cycle (called compulsory secondary education) from 11 to 16 years old, and a second two-year-long non-compulsory cycle (called baccalaureate) from 16 to 18 years old. We constraint our study to the latter cycle.

Since 2000-2001, mathematics courses are divided into two different modalities: one for science and technology (called Mathematics I and II, respectively for each year of baccalaureate) and another one

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for social sciences (called Applied Mathematics for Social Sciences I and II, respectively). In the previous curricular organization, SP were present only in the first year of science baccalaureate (that is, in the course Mathematics I). In social science modality SP were deeply developed. Rather than discussing curricular evolution, we prefer to focus on the new curriculum, and to underline differences with the old one.

Current curricular contents in SP

The current official curriculum in Spain was issued in 2015, and implemented in 2015-2016. National Government sets at least 55% of it, and then it can be completed by regional governments. Whereas differences among regions are considerable in other subjects, this does not happen with mathematics. Therefore, we focus on the national curriculum. The curriculum is structured into contents, assessment criteria, and learning standards. Figure 1 shows the SP contents within the four subjects.

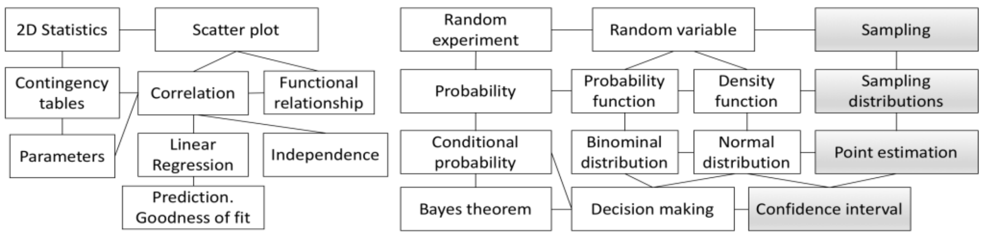

Figure 1: Curricular units within SP block for Mathematics I and II and Applied Mathematics to Social Sciences I and II subjects (shadowed units are exclusive for Applied Mathematics).

Mathematics curriculum is divided into five blocks in the science baccalaureate and into four blocks in the social science (same as in science, except geometry). The first block is always cross-wide and named “Processes, methods, and attitudes in mathematics”, and the last one is “Statistics and Probability”. For analyzing the contents, we are focusing on the SP block.

Mathematics I curriculum in SP consists of two-dimensional descriptive statistics, including regression and correlation. Mathematics II is dedicated to the study of probability and probability distributions. This is a novelty since previously SP were absent in the contents of Mathematics II. This inclusion responded to a claim made by scientific societies (Angulo, Ugarte, & Gordaliza, 2014) in a double way. On one hand, exit examinations for entering the university are based on the curriculum of the second year of baccalaureate. Thus, the omission of SP contents in Mathematics II caused that many teachers also avoided teaching SP in Mathematics I. On the other hand, almost all sciences and health and social sciences university degrees include at least one SP course. Thus, the absence of SP caused a lack in the background of future bachelor students.

Regarding Applied Mathematics to Social Sciences I and II, the curriculum of this course is different from that of science in the orientation more than in the contents: it is intended to be less concerned about formalism and more focused on applying mathematics into social sciences contexts, so that the students acknowledge mathematics as a useful instrumental tool (see Rodríguez-Muñiz, Díaz, Mier, & Alonso, 2016). In the first year the curriculum consists of two-dimensional statistics (similarly to Mathematics I), probability and distributions, whereas the second year it covers probability (axioms and Bayes’ rule) and statistical inference (exclusive content for Applied Mathematics to Social Sciences). Basically, with respect to the previous curriculum, the main change is the removal of hypothesis testing in the second year, which had been pointed out as a content over-passing the High School level (Batanero, 2000).

Curricular learning standards

At this point the reader maybe wonders: what is exactly new in this curriculum? Further than changes in contents, we can find the intended novelties in the learning standards, which were firstly issued in the 2015 reform. They can be found in MECD (2015). It is necessary to recall that, apart from SP block, the official curriculum includes a cross-wise block "Processes, methods, and attitudes in mathematics" that should permeate the rest of content blocks. Let us now discuss the most significant standards in terms of the curricular reform. We start with the standards from the cross-wise block, which are similar for Mathematics and Applied Mathematics (in the following, some codes are duplicate because we keep the original numerical codes, restarting numbers for each block):

> 2.3. Performing estimations and elaborating conjectures about the results of problems, appreciating utility and efficacy.
> 4.3. Using adequate technological tools for the type of problem, situation to be solved or property or theorem to be demonstrated, both in the finding of results and in the efficacy improvement to communicate mathematical ideas.
> 6.2. Finding connections between real context and mathematical world […] and among mathematical contexts […].
> 8.2. Establishing connections between real world and mathematical world problems: identifying underlying mathematical problem, and necessary mathematical knowledge.
> 8.4. Interpreting mathematical solution of a problem within the real context.
> 8.5. Making simulations and predictions to assess adaptation and limitations of models, suggesting measures for improving efficacy.
> 13.1. Selecting adequate technological tools and using them to perform numerical, algebraic or statistical calculations when their difficulty prevents from or advise against handmade calculations.

We find several standards, as 4.3 and 13.1, which are clearly aligned with the notion of mathematical literacy in PISA theoretical framework (OECD, 2017). They point out the need of integrating technology in the mathematical processes not only in the calculations but also in the interpretation of results and in the demonstration of mathematical properties. Another group of standards as 2.3, 6.2, 8.2, 8.4, and 8.5, underline the importance of context-based problem solving and the use, estimations, simulations, and conjectures in the solving processes. Despite not explicitly mentioned, ideas on statistical literacy and statistical reasoning are also underlying these standards, since understanding of graphics and performing estimations are core skills.

However, more properly, inspiration on statistical literacy can be found in the SP block learning standards for Mathematics I:

> 1.5. Adequate using of technology for statistically organizing and analyzing data, calculating parameters and generating statistical graphics.
> 2.1. Distinguishing between functional and statistical dependence and estimating whether two variables are statistically dependent or not by means of the scatter plot representation.
> 3.1. Describing situations related to Statistics by using an adequate language.

And for Mathematics II:

> 2.3. Calculating probabilities associated to a binomial distribution by using the probability distribution, the distribution table, calculator, spreadsheet or other technological tool.
> 2.4. Calculating probabilities of events associated to phenomena modeled by a normal distribution by using the distribution table, calculator, spreadsheet or other technological tool.
> 2.5. Calculating probabilities of events associated to phenomena that can be modeled by a binomial distribution by using its normal approximation, appreciating if required conditions hold.

We find not only technology-based standards but that technology plays the main role in the statistical process, as in 1.5. Standard 2.1 remarks variability, which is one of the components of the statistical reasoning (Wild & Pfannkuch, 1999). And the specific language of statistics is present in 3.1. Additionally, and after acknowledging this step forward statistical literacy and the use of technology, some standards seem to be too outdated. For instance, 2.3 and 2.4 incorporate ICTs to the calculation procedures but, in a clear contradiction with the spirit of the rest of the curriculum, still include tables. Final remark is related to the explicit inclusion of the normal approximation in 2.5. In this case, we have to underline that nowadays there are other interesting computational tools that allow calculating the exact binomial probability by using ICTs (calculator, spreadsheet, applets), and do not require the conceptual development of the notion of convergence, that can exceed the understanding of students at this level.

When looking into the SP curricular block in Applied Mathematics to Social Sciences I and II, we also find few differences between Mathematics and Applied Mathematics: all the aforementioned standards for science also appear here. This coincidence is partially contradictory with the aim expressed in the curriculum, emphasizing applicability and instrumentality in Applied Mathematics. Nevertheless, some standards are exclusive for Applied Mathematics to Social Sciences I, as:

> 3.3. Constructing the density function of a continuous variable associated to a simple phenomenon and calculating their parameters and some associated probability,

which clearly overpasses the level for the first year, since integration appears in the second year, and this fact obviously hinders dealing with density functions. Other standards are exclusive for Applied Mathematics to Social Sciences II, as:

> 1.4. Solving situations involving decision making under uncertainty by using the probabilities of the different alternatives.
> 2.1. Assessing representativeness of a sample depending on the selection procedure.
> 2.6. Connecting error and confidence in a confidence interval with the sample size, calculating each one of them given the other two, and applying this in real situations.
> 3.2. Identifying and analyzing components in the fact sheet of a simple statistical study.
> 3.3. Analyzing critically and reasonably statistical information in mass media and other spheres of daily life.

Standard 1.4 is fully aligned with the notion of statistical competence, putting into action probability as a tool for dealing with uncertainty (Rossman, 2008). We want to highlight standard 2.1 because it makes an invitation to discuss about different methods for random and non-random sampling, and their consequences, and this is connected to the highest levels of statistical literacy described in Watson (2006), as well as 3.2 and 3.3., that are directly related to critical thinking and reasoning within real life contexts. Moreover, standard 2.6 is appropriate for dealing with confidence intervals, and, at this educational level, this is more relevant than calculating the interval itself.

IMPLEMENTED CURRICULUM IN TEXTBOOKS

Goals

Research on textbooks is an established line within mathematics education community, and particularly in the case of SP and Spanish textbooks (see Rodríguez-Muñiz & Díaz, 2018, for a recent review). We omit here details about the importance that textbooks still have in the implementation of the curriculum and their influence on teachers' practices, but there is no doubt about the main role that textbooks have in the teaching/learning process, despite ICTs and teachers' own artifacts and materials. After analyzing the new curricular guidelines our interest lies now on checking how this is implemented in textbooks. More precisely, our goal with this study is to check if the new curricular alignments appear in the textbooks and, if so, how they are implemented.

Sample and methodology

For this study we have examined a sample consisting of five full series of textbooks of main Spanish editorials for the four courses considered (Mathematics I and II, and Applied Mathematics to Social Sciences I and II). Sampling was non-random but attending to circulation of the editorials within the school context. We have followed a theoretical framework based on Sierra, González & López (1999) (we avoid details here, see more in Díaz, 2017) which briefly consists of a triple analysis: conceptual, phenomenological, and didactical-cognitive. We organize our findings into three fields.

Findings in regression and correlation

We verified that, apart from one brief comment in one textbook, regression regards quantitative variables. There are no examples about qualitative ones. Obviously, well-known formulae are for using with quantitative, but examples or ideas about qualitative could be addressed, for avoiding misconceptions about regression as a procedure only for quantitative variables. All textbooks give explanations about how to calculate regression lines Y on X and X on Y, which is mathematically correct, but they assume that both variables can be independent, which is not true, in general, and it is statistically meaningful. We found scarce discussion about non-positive correlation. Further than one graphic showing how a negative correlated scatter plot looks alike, all the examples and exercises in the textbooks show positively correlated data. Additionally, it is very poor the representation of non-linear regression, which is not a curricular standard, but, again, not mentioning the existence of non-linear relationships among variables can lead to a misconception. Short notes or comments based on graphics about non-linear regression could be easily included.

References or comments to variability barely exist. Almost all books just explain formulae about linear regression (mostly without proofs) and then show the expression(s) of the line(s) and make predictions. But there is no discussion about the fact that different samples would produce different regression lines. Thus, regression tends to be taught in the textbooks as a deterministic procedure, far from the variation that should be inherent to statistics (see Lavalle, Micheli, & Rubio, 2006). Not only in the case of regression (we will underline it also in the next two topics) but especially when dealing with two-dimensional data, ICT tools are barely present in the textbooks. Hand-made calculations prevail over any other type. Specific statistical software is omitted, but also website applets that could help to ease the procedure, just two margin notes about the use of calculators. It seems obvious that, under this approach, regression becomes much more a calculation problem.

Regarding phenomenology in exercises and problems solved and proposed in the textbooks, we found (and it is also extensive to the next two topics) that more than 95% of them are algorithmic exercises (Butts, 1980), just a few are application exercises. Real and open-ended problems do not appear on the textbooks. Moreover, many exercises are still decontextualized, only posed within a mathematical context. This contradicts the aforementioned curricular standards asking students to make decisions, to use daily-life contexts, and to perform estimations and elaborate conjectures, which is impossible when solving decontextualized closed exercises and not proper problems.

Findings in probability

Classical Laplace's or geometric definition based on counting is the core of curricular standards in probability, and, therefore, it always organizes probability on textbooks. Additionally, some of the textbooks also make a frequential approach to probability. In two of the books it is explicitly explained that frequential probability can be used in cases in which Laplace is not applicable, but the rest of them just show how the frequencies tend to converge to the theoretical value of the probability. There is no place on textbooks for the subjective approach to probability. Moreover, books provide a mathematical definition of probability (either with the axiomatic definition or with the classical one) but none of them provide a conceptual definition of probability, explaining what this value means. At this point, we think that the approach to the meaning of probability can be attained by using a subjective definition (see Blanco, Díaz, García, Ramos, & Rodríguez-Muñiz, 2016), showing how it can be used in experiments in which nor the classical neither the frequential definition can be applied.

We already pointed out one contradiction in the curricular standards, asking for using density functions of continuous random variables when integration has not been yet studied. Textbooks try to sort this contradiction in very different ways. Some of them define the integral and notice the students that this is a concept that will be studied in the next year. Some others avoid the problem by using polygonal density functions and calculating the area with geometric formulae, but this method still does not solve how to proceed with curves. There are no comments on learning about distribution, how to know when some data could be normally distributed or not (Bakker & Gravemeijer, 2004). We explained above how curricular standards about the use of probability tables for binomial and normal distributions are obsolete. Nevertheless, the textbooks follow these standards and all of them allocate several pages to the normal distribution table but few and very marginal comments about calculators, spreadsheets or statistical software. Therefore, the curricular standard is not fulfilled.

Phenomenology is more assorted than in regression, with many of the exercises based on typical problems of urns, balls, games, raffles, etc., and anthropometrics for contextualizing normal distribution. However, there is an evident lack of problems regarding the curricular standard on decision making. Students are not asked to use probability as a tool for making decisions or justifying choices under uncertainty, which is clearly opposite to the spirit in the intended curriculum.

Findings in statistical inference

Textbooks contain interesting ideas and explanations about sampling and its consequences on inferences based on the representativeness of the sample. But only few ideas about non-random sampling can be found. Since statistical inference is the newest topic in this curriculum, the textbooks are much less standardized as in the previous two topics. Hence, we can find developments varying from not defining point estimation to defining properties as sufficiency or efficiency of estimators, which is clearly excessive for this educational level. Nevertheless, all the books follow a very classical approach, instead of taking advantage of technology for using new ideas and approaches (Ridgway, Nicholson, & McCusker, 2006).

Regarding confidence intervals it is clear that both the notion and the procedure can be too difficult for such a dense curriculum (Cumming, Williams, & Fidler, 2004). What textbooks include is basically a procedure without reasoning, or with a succinct explanation, to build the confidence interval in the different cases proposed by the curriculum, but without conceptual discussion. There are some books including also a brief explanation of the relationship among confidence level, sample size and estimation error, but this relationship is poorly used in the problems. Therefore, this interesting standard is scarcely implemented. A brief comment must be introduced about the Central Limit Theorem which is presented in most textbooks almost as question of faith. Assuming the conceptual difficulty of this part of the curriculum, in our opinion its implementation would be significantly improved by using simulation or experimentation to link distributions with inference as Rossman (2008) suggests.

Finally, we want to underline, again, the lack of decision problems regarding statistical inference. Decision making is reduced to check if a given value belongs or not to the confidence interval. The context, which is so important in statistics, becomes merely irrelevant, since most of exercises are algorithmic and context-based interpretation is omitted.

CONCLUSIONS

Results show that, although not explicitly formulated, we can confirm the influence of research on statistical literacy, reasoning and thinking on the intended curriculum of Spanish baccalaureate in SP. This research also proves how textbooks keep some inertias, inherited from previous curricula and from traditional uses of statistics, with many algorithmic procedures, leaving few spaces for critical thinking, decision making, and analyzing solutions within a context, and a deep lack of technologies, which are some of the new learning standards introduced in the curriculum. Additionally, the official intended curriculum still has some outdated standards regarding the use of tables, and an excessive emphasis on mathematization of statistical procedures.

The progress of the intended curriculum is very positive, especially by giving up too much formalism and trying to underline the importance of real data (Wild & Pfannkuch, 1999), context (Dierdorp, Bakker, Eijkelhof, & van Maanen, 2011), technology (Biehler, Ben-Zvi, Bakker, & Makar, 2012) and critical thinking (Batanero, Burrill & Reading, 2011). Nevertheless, the implementation of the SP curriculum in textbooks still needs to be improved in the way pointed by the official curriculum. First of all, technology must be much more present, not only as an instrumental tool but also for structuring contents. Secondly, some procedures regarding statistical inference could be lightened of formalism and heading for informal inference procedures (Makar, Bakker, Ben-Zvi, 2011). Third remark addresses to the need of introducing critical statistical thinking by means of context-based problems, real data, and open-ended situations in which students have to make decisions under uncertainty. Finally, a more general reflection, involving all the community of researchers in mathematics education: we need to try not only producing research results but also transferring them. Authors who write textbooks are not robots that copy and paste, but usually teachers, and it is our duty to let them know our research results for improving these resources.

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BUILDING PEDAGOGICAL DESIGN CAPACITY: EDUCATIVE CURRICULUM MATERIALS FOR COHERENCE AND SUSTAINABILITY

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All too often, resources developed to support reform-oriented school mathematics curricula gain short-term acceptance, but soon become little more than a set of good ideas or tasks to be used on an ad hoc basis if there is time. As such they fail to fulfil their goal of promoting sustainable reform in school mathematics. We use the key ideas of teachers' pedagogical design capacity and educative curriculum materials to describe how an Australian curriculum and human resources project is working towards achieving coherence, sustainability and scale. We argue that rather than seeing the human and material resources as separate, we ought to consider them as intricately intertwined elements of any curriculum resources development project.

The question of how effective various curricula are in helping students to learn mathematics oversimplifies the situation by ignoring the subtleties and complexities of curriculum enactment in the school context (Schoenfeld, 2006). These complexities include internal factors, such as the interaction between the curriculum and entrenched teacher beliefs, and external factors such as the impact of assessment regimes that may not assess the full range of goals of the curriculum. "Indeed, one can imagine curricular materials that, when used in the way intended by the designers, result in significant increases in student performance, but, when used by teachers not invested or trained in the curriculum, result in significant decreases in student performance" (Schoenfeld, 2006, p. 17).

Brown (2011) likens teachers' interpretations of the official curriculum to two different renditions of a jazz song. Each performer interprets the composer's score in their own way, and although the two renditions may sound different, the song is still recognizable. In some cases, the same song may be performed quite differently by a performer on different occasions, mirroring the activity of teachers contextualizing their use of curriculum materials for different groups of students. Brown argues that teaching is itself a design practice and that resource materials should therefore be *educative* rather than transmissive, supporting teachers to make decisions cognizant of the underlying philosophy and pedagogy of the materials. Educative curriculum material aims to speak *to* teachers about the ideas underlying the tasks, rather than *through* teachers en route to students (Davis & Krajcik, 2005).

In this paper we describe how we have sought to develop educative curriculum materials in an Australian national project, *reSolve: Mathematics by Inquiry*[1]. However, as Davis and Krajcik (2005) point out, used alone, even educative curriculum materials may serve as simply one more perturbation to the status quo. Hence, a critical aspect of the reSolve project is its dissemination via a cohort of

[1] Acknowledgement: reSolve: Mathematics by Inquiry is an initiative of and funded by the Australian Government Department of Education and Training.

300 Champions who are charged with working with colleagues to enact the project's philosophy through professional learning and use of the reSolve resources. We do not see the Champions as separate to the curriculum resources; rather, they are an integral part of the coherence of the reSolve curriculum project.

In this paper we first discuss the strengths and limitations of the base curriculum document, the Australian Curriculum: Mathematics (Australian Curriculum and Assessment Reporting Authority [ACARA], 2018). We then introduce the rationale for, and structure of, the reSolve: Mathematics by Inquiry project. We focus particularly on the reSolve Protocol, the vision of mathematics teaching and learning that holds the project together. We then explore the educative curriculum materials that are being developed in reSolve and the role of Champions in building pedagogical design capacity. We argue that the physical and human resources are intricately intertwined, and that they should be considered together rather than as separate elements of a curriculum resources project.

THE AUSTRALIAN CURRICULUM: MATHEMATICS

Davis and Krajcik (2005) emphasize that effective curriculum resources must be built on a base curriculum that is of high quality both in terms of content and pedagogy. The Australian Curriculum: Mathematics (ACM) (ACARA, 2018) provides the base curriculum upon which the reSolve curriculum resources are built.

The ACM was built from a shape paper (ACARA, 2009) that prioritizes a number of important design features. These features include the need to provide opportunity to learn and engagement for all students, connections to other learning areas, clarity in documentation that emphasizes the big ideas of mathematics, and the need for a contemporary orientation. While not taking a rigid stance on pedagogy and assessment, the shape paper (ACARA, 2009, p. 14) takes the view that:

- Depth is preferable to breadth;
- Challenging problems should be the basis of pedagogical strategies;
- Sets of ideas with key goals are preferable to disconnected experiences;
- Teachers make informed choices when they are aware of big ideas;
- Digital technologies can enhance the relevance of the curriculum; and
- The use of engaging experiences that allow for differentiation increases inclusivity.

This orientation is reflected in the aims of the ACM, which state that students should be confident and creative users of mathematics in their personal and work lives, should develop the proficiencies of fluency, understanding, problem solving and reasoning, should recognize connections between mathematical ideas and between mathematics and other disciplines, and should appreciate mathematics as an enjoyable field of study in its own right. The ACM has descriptions of essential content and desired proficiencies at each year level, and includes achievement standards with accompanying work samples and illustrations of practice.

Every curriculum is, of course, open to critique. Atweh, Miller and Thornton (2012) discuss the ACM in terms of its internal and external coherence. By *internal* coherence they mean the extent to which the content and articulation of the curriculum enact the aims and rationale, and by *external* coherence the extent to which the curriculum contributes to broader goals of education as described in national policy documents.

To investigate the curriculum's internal coherence Atweh et. al. (2012) examine the content elaborations for one year level of the curriculum (Year 8). Of the 43 content elaborations, they identify only 12% as relating to problem solving and a mere 7% as relating to reasoning. They raise the concern that "the curriculum document itself may not inspire teachers to appreciate the importance of these proficiencies and to think of valuable and exciting ways in which they can be used or developed in the classroom" (p. 9). As discussed below, similar concerns were raised in the two papers reviewing the current state of mathematics education in Australia that provided the background for the reSolve: Mathematics by Inquiry project.

THE RESOLVE: MATHEMATICS BY INQUIRY PROJECT

reSolve: Mathematics by Inquiry is an Australian Government Department of Education and Training funded project that aims to "transform the teaching and learning of mathematics in Australian schools". The project seeks to enact the goals of the ACM and to raise student engagement and achievement in mathematics from Kindergarten to Year 10.

In early 2015 the Australian Government commissioned the Australian Academy of Science and the Australian Association of Mathematics Teachers to develop position papers (Australian Academy of Science, 2015; Australian Association of Teachers of Mathematics, 2015) discussing current issues and gaps in Australian mathematics education and making recommendations for specific projects that might address these issues. The observations raised in these papers focus particularly on the extent to which the goals of the ACM are enacted in Australian schools. They include:

- A mismatch between contemporary advice on mathematics pedagogy and actual classroom practice;
- Students' limited exposure to pedagogies that promote 21st century learning;
- The lack of attention to reasoning and problem solving in current assessment regimes;
- The need for targeted and sustained professional learning;
- The plethora of resources available, many of which pay only scant attention to anything more than a narrow view of procedural fluency, from which teachers tend to "cherry-pick";
- The need to increase the intellectual rigor and complexity of problems typically tackled by students in school mathematics, and
- The need to support out-of-field teachers.

A strong recommendation was therefore that the proposed mathematics by inquiry project should focus strongly on the big ideas of mathematics and the proficiencies of the Australian Curriculum: Mathematics. Related to this there were strong recommendations for a concerted focus on building teacher capacity, through a coordinated and sustained focus on teacher learning of contemporary pedagogical approaches that would promote a spirit of inquiry, including the use of appropriate digital technologies. Above all, the recommendation was "that the (Australian) curriculum be taught with integrity" (Australian Association of Teachers of Mathematics, 2015, p. 24).

A project request for tender was subsequently issued, for which the Australian Academy of Science and Australian Association of Mathematics Teachers submitted a successful collaborative submission. The organizations were awarded the project, commencing at the start of November 2015. The project was subsequently branded reSolve: Mathematics by Inquiry, *reSolve* being an intentionally ambiguous word suggesting that multiple solution strategies are a key part of mathematical thinking and that showing resolve is a key attribute of successful mathematicians.

From the outset the project was designed for coherence, scale and sustainability through the development of an underpinning philosophical framework, the reSolve Protocol, and through the parallel activities of resource development and the recruitment and training of at least 240 Champions across Australia.

Notwithstanding the concerns about the ACM expressed above, the reSolve project took a “glass half full” view, engaging with the curriculum’s rationale and aims, and particularly with the importance of the proficiencies describing reasoning and problem solving. We took an expansive rather than limited view of the ACM content descriptions, attempting to add depth and make connections that were not necessarily obvious from their descriptions or elaborations.

THE RESOLVE PROTOCOL: PROVIDING INTELLECTUAL COHERENCE

The first product of the project was the reSolve: Mathematics by Inquiry Protocol, articulating a vision of mathematics, task and classroom culture that the project team believed would provide a sound base on which to build the physical and human resources of the project. The Protocol is based loosely on the Teaching for Robust Understanding (TRU Math) dimensions (Schoenfeld, & the Teaching for Robust Understanding Project, 2016). The three key elements in the Protocol are:

- reSolve mathematics is purposeful
- reSolve tasks are inclusive and challenging
- reSolve classrooms have a knowledge-building culture

By *mathematics that is purposeful* we wish to challenge perceptions that mathematics is a body of disconnected facts or procedures described in a curriculum document. By *tasks that are inclusive and challenging* we wish to challenge perceptions that mathematics is for the few and assert that it ought to be both challenging and accessible for all students. By *classrooms with a knowledge-building culture* we wish to challenge a view that mathematics is best learnt through demonstration, reproduction and repetition. The project team believes that these three key aspects of mathematics, tasks and classroom culture taken together can create an inquiry orientation to mathematics that is relevant and empowering, and that leads to strong cognitive and affective outcomes. The Protocol infuses all of the curriculum resources and is further intended to become a language through which teachers conceptualize and discuss mathematics curriculum and its implementation. It is thus central both to the coherence of the curriculum resource and to their enactment in schools.

EDUCATIVE CURRICULUM MATERIAL AND THE RESOLVE MATERIAL RESOURCES

A critical element of the early success of reSolve has been the coherence across the suite of resources being developed. The project has produced resources for professional learning and for classroom teaching. The resources are intended to be *educative* in that they enable teachers to understand the intent of the designer, and to engage with the resources in ways that are powerful for both teacher and students.

Remillard (2016) argues that the relationship between the teacher and the curriculum (or resource) is central to how it is enacted in the classroom. She identifies four ways in which a teacher might read the text of a resource: *following* or *subverting* the text, *drawing* on the text, *interpreting* the text, and *participating* with the text. Of these, participating with the text is the most powerful, in that teachers use the text to actively construct learning experiences for their students that faithfully enact the underpinning philosophy and aims of the text. The reSolve resources therefore aim to position teachers as participants rather than followers.

The professional learning modules elaborate elements of the Protocol to inform robust teacher learning. Each module focuses on a theoretical rationale and some practical strategies to enable teachers to better address an identified aspect of the Protocol, using the reSolve classroom resources as exemplars and discussion starters. The modules are thus a critical component of teachers participating with the resources as they make clear the pedagogical and mathematical intent and invite teachers to make the Protocol part of their decision-making in the classroom.

The classroom resources highlight elements of the Protocol through carefully constructed tasks accompanied by detailed documentation that makes clear how each element of the Protocol has been incorporated into their design. They attempt to progressively build understanding through a focus on big ideas and include prompts to enhance challenge and access as well as suggestions for consolidating learning. They do not attempt to address every content description in the Australian Curriculum, but rather serve as exemplars to engage teachers with the intent of the Protocol.

The documentation of the resources is designed to promote in teachers modes of engagement such as reading for big ideas and considering the variety of students' potential responses (Remillard, 2016). The materials value teachers' practical knowledge (Chapman, 2004), imagining an ideal reader as one who engages with the resources not as scripts or add-ons, but as integral to their project of ensuring a rich mathematics education for each of their students. The materials have a strong narrative structure, describing the designers' intent and imagining potential classroom interactions.

In keeping with Davis and Krajcik's (2005) guidelines and design heuristics for educative curriculum materials, the reSolve resources thus:

- Help teachers anticipate what learners may think about or do;
- Support teachers in their own learning of the subject matter;
- Describe ways in which the separate resources relate to each other and build progressive understanding;
- Make visible the designers' pedagogical decisions; and
- Promote teachers' pedagogical design capacity.

We maintain that of these guidelines the last is the most crucial. As has been pointed out by many researchers (e.g. Obara & Sloan, 2009; Polly, 2017; Schoenfeld, 2006) the potential impact of any suite of resources is dependent upon how teachers mobilize them in the classroom setting. We therefore see the bridge between the physical resources and their enactment in the classroom as integral to the design of the project.

PEDAGOGICAL DESIGN CAPACITY AND THE RESOLVE CHAMPIONS

Brown (2011) argues that, regardless of the intent of teachers, teaching by design is not so much a conscious choice as an inevitable reality. Every teacher makes choices about which materials to use and how and when to use them, noticing different affordances or constraints dependent on their experiences, intentions, beliefs and abilities. Brown introduces the concept of teachers' pedagogical design capacity (PDC) to describe the skill by which the various pieces of curriculum resources are brought into play. It includes the capacity to perceive affordances, to make decisions and follow through on plans, and to weave together the various pieces into a classroom setting. It is about creating "deliberate, productive designs that help accomplish instructional goals" (p. 29).

Enhancing teachers' PDC is therefore critical if a curriculum project is to realize sustainable outcomes for teachers and, ultimately, for students. The reSolve project seeks to achieve this through the Champions element of the program. The Champions are more than advocates for reSolve; they are more than conduits for the reSolve resources or deliverers of professional learning. Rather, they are catalysts for developing PDC among the wider teaching community.

Each Champion was recruited via an expression of interest process. No rewards were offered; in fact, participation is entirely voluntary and in teachers' own time including vacations. No criteria were set other than a commitment and passion to become a Champion; a deliberate choice was made to recruit Champions from as wide a range of backgrounds and experiences as possible. These recruitment decisions were crucial in guaranteeing that the intent of the Champions would match the intent of the reSolve project.

During the 12-month development program each Champion was required to participate in webinars, join an online platform where he or she undertook and discussed the reSolve professional learning modules, and trialed reSolve resources with colleagues. The key element, however, was participation in two face-to-face workshops. The first workshop, held in vacation time in October 2017, served to introduce Champions to the project, to each other and to their role within the project. The second extended over two days, with an optional additional day focused on theory and research, during the April 2018 vacation. This workshop introduced the idea of professional learning communities and positioned Champions as leaders of such communities in their contexts.

The six key elements of professional learning communities (DuFour & Eaker, 2009) with which the Champions engaged are:

- Shared mission, vision, values and goals;
- Collaborative teams, focused on learning;
- Collective inquiry;
- Action orientation and experimentation;
- Commitment to continuous improvement; and
- Results orientation.

The Champions engaged in activities that drew on their practical knowledge to synthesize overarching ideas about working in communities of inquiry and that provided models for how they might work with colleagues. The goal of the Champions program is to build a human resource committed to inquiry for both student and teacher learning. Their role with colleagues extends beyond transmission or advocacy for the reSolve resources towards, through the community of inquiry, building teachers' pedagogical design capacity.

DISCUSSION AND CONCLUSION

There is ample international evidence that policy or curriculum documents alone seldom lead to teacher change. How teachers interpret and implement reform curricula is dependent on factors including their efficacy beliefs (Charalambous & Philippou, 2010), their knowledge of mathematics (März & Kelchtermans, 2013), their beliefs about mathematics (Manouchehri & Goodman, 1998), and school leadership (Braun, Maguire & Ball, 2010). In most cases in which large scale reform is desired, the recommendation is for extended and focused professional learning. However, we take a different tack.

Sustainable and scalable teacher change requires both educative curriculum materials and teachers committed to developing their pedagogical design capacity. We have sought to achieve this through Champions whom we consider to be *part* of the curriculum resources, rather than separate from them. From the outset Champions have been involved in the development of educative curriculum materials through writing workshops and have built pedagogical design capacity through extended use with colleagues. We stress that while the Champions program has been a significant professional learning experience for those teachers involved, that is not its primary intent. It was established first and foremost as the human aspect of the curriculum resources, integrally intertwined with the physical aspect of the resources.

During the April 2018 workshop Champions were asked to complete the simile: "Doing a reSolve task is like…because…" The comparisons below are indicative of those written, and show how the Champions have engaged with a spirit of inquiry, seeing the resources not as a recipe to follow but as a text with which they can participate in a process of creating deliberate, productive designs to achieve instructional goals.

"Using a reSolve task is like organizing a dinner for guests because you are catering to various needs, you need to know each individual person, and bring everyone together and keep them entertained."

"Using a reSolve task is like listening to a song – it is enjoyable to do and when you listen carefully to the words there is a much deeper meaning."

"Using a reSolve task is like riding a bike without training wheels because when you start you have to trust yourself to balance otherwise you will put your foot down and stop and miss out on the ride."

Unlike reform efforts that have focused on written resources accompanied by professional learning programs that seek to help teachers implement the resources, in the reSolve project we see the Champions as *part* of the curriculum resources, not as separate from them. From the outset the reSolve project saw the human element as integrally intertwined with the curriculum resources—the physical materials do not stand apart from the Champions, nor the Champions from the materials. Together the physical and human resources of reSolve constitute educative curriculum materials that build teachers' pedagogical design capacity. Considering the Champions as part of the resources elevates them to a status as developers and designers, rather than as merely implementers.

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IMPORTED REFORMS: THE CASE OF ALGERIA

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This paper discusses the globalization of educational systems by presenting three different reforms, in Algeria, at different school levels, which brought more dysfunctions than positive outcomes. According to an ancient tradition in post-colonial countries, and also in developing countries, educational systems and reforms are imported from western countries, mostly motivated by political reasons. We argue that, a study of the local context, adaptation of any imported change, along with complete explanation and understanding of the objectives of any reform are necessary, otherwise it would be a complete failure.

INTRODUCTION

When we talk about reforms, changes are mentioned in one or multiple levels of an educational system (pedagogy, contents, assessment) for different reasons: to an adaptation to a new situation, to suggest solutions for some problems, to consider new data regarding contents or methods, and even to eliminate some elements like some contents, or leave some traditional adopted methods in teaching or assessing students. Sometimes, a reform is inevitable; it is the result of social and cultural mutations, but might also be part of political reforms. 'As students' learning of school mathematics is the fundamental training necessary for the development and up-keep of a system's economic strength, cross-system studies in school mathematics have been valued for informing educators about the effectiveness of their own practices and for suggesting possible alternatives for improvements (e.g., Postlethwaite, 1988; Robitaille & Travers, 1992)' (in F. K. S. Leung and Y. Li, 2010, pp. 1). The intention behind every reform is undoubtedly good, it is meant to improve the whole educational system, particularly to enhance students' performances, to increase the quality of exchanges in the classrooms between students and teachers, to develop teacher education or any other related objectives. However, in practice, these goals are not always reached, like multiple reforms undertaken in many countries, show. In many cases, reforms not only failed to bring an improvement, and thus to solve some existing problems as expected by its innovators; but have worsened the situation by creating new unpredicted problems.

The standard vs disparity aspects of educational systems

People tend to think that schools should work according to standards and what seems to work in one place, in the world, has no reason to not work, in another place. So, often, educational systems have been aligned according to some presumed general and international standards. Research (Bishop, 1990) about ethno-mathematics showed that mathematics is not culture free knowledge, consequently, many factors that shape the teaching and learning of mathematics have a strong role that affect directly any change, if they are not taken into consideration. It is illusory to think that a reform would be set independently from various factors constructing and shaping actively any educational system, even within the same country, which are, but not limited to: language of instruction and mother language, culture, students with disabilities, students coming from minorities, students with low ability and achievement levels, teachers' preparation and in-service training, school resources, pedagogy and methodology, teachers' practices and beliefs, institutional traditions, underrepresented schools, poor and isolated regions and assessment forms and technics. A reform undertaken in an educational system where teachers are prepared and trained to some familiar technics has rare chance to work in another system where teachers have little training and are not familiar with this particular technic. Reforms that require the use of technology are not welcome in countries where these means are still considered as luxury. This suggests that it would be important to think how to reach the same objectives with a reform even if we use different settings regarding the educational systems specificities, in two different countries. In other words, if the differences are considered as force points in one country, it might suggest working on them for adapting a reform, that worked very well in another country, to hit the same objectives. The mastery of two or three languages could be used to improve students' learning if they learn mathematics in a second or third language, the choice of questions and problems, when they are chosen respecting the culture specificities might help better students understanding their tasks. In fact, in this more and more globalized world, we believe that collaboration among people of different backgrounds is much more important than competition (Bishop, 2006, in F. K. S. Leung and Y. Li, 2010). But in practice, reforms are more initiated to a general standardization, influenced by general opinion and politics. Even in the western countries, particularly in the USA 'typically, when state or district policymakers did provide direction, they limited it to bare listings of course requirements or behavioral objectives. Few systems prescribed topics within courses or curricula; guidelines about teaching pedagogy were even rarer (Cohen and Spillane, 1993 in Massell and all. 1997). It is frequent that teachers, educators and even students mention reforms that have been implemented and caused dramatic situations in educational systems; this raises the question: when does a reform work, or what are the criteria to judge about the success or failure of a reform? How do people know if any reform was successful or failed? It is about to understand how the journey of school reform is a story of constant adaptation that ultimately undermines the common criteria generally used to judge success and failure (Cuban, 1998, p.453). Knapp (1997) focuses on alignment between different elements:

- A major constraint on the quality of science or mathematics teaching lies in the lack of alignment among key elements of the system (Fuhrman, 1993; Hill, 1995).

- Better teaching of mathematics and science will result when all elements of the system that bear most directly on the classroom—especially those dealing with what is taught, how it is taught, how learning is assessed, how teachers are prepared and supported, and how they are held to account for student performance—are aligned with challenging standards, applicable to the full range of students, embedded in a coherent, compelling vision of reform that reflects professional consensus among scientists and educators (Smith & O'Day, 1991; Sykes & Plastrik, 1993).

- The lack of alignment is best addressed at its source—that is, at the level at which policies and structures guiding each systemic element are set.

Reforms' success is also relative, 'suppose, for example, that, as time passed, an innovation that was once labeled a failure overcame early obstacles in its implementation and desired results came later than expected. Or suppose that the designers of a school reform judged the results to be unsuccessful because teachers failed to adhere to the plan. Yet these very same teachers took the design, adapted portions of it to their classrooms, secured favorable results from their students, and privately called the reform a winner. How can a school reform be judged successful by one group and tossed out as a miserable defeat by another?' (Cuban, 1998, p. 456).

We will present three problematic situations caused by reforms that either are imported or inspired by foreign reforms in three different school levels in Algeria. One in the primary school level, where children of the first grade should all pass to grade two, the second one is in high school level, where the contents about mathematics logic have been eliminated from the curriculum; and the third is at the university level, it is about adopting a new mark for students, which presents a kind of formative assessment.

EVERY CHILD SHOULD PASS TO GRADE 2

A reform was designed in 2009 in Algerian primary schools, similar to a previous one in France according to which, every child should pass from grade one (6 years), to grade two (7 years). It was supported by beliefs about the ability of children to learn effectively that starts at age 7, inspired by countries whose educational systems are the best ones, like Scandinavian countries, where children go to school at age 7. This reform has been adopted after a recommendation given by the ministry of education asking teachers to not allow children, at grade one, to remake the school year, in case their final marks do not allow them to pass to grade two. The why and how of this reform have not been clarified, teachers have not been given any information about this change and what, making all children passing to grade two, could mean. This reform could be of high benefit if the goals behind were about making all children achieving all the goals of grade one, by retake exams or catch-up sessions, through the schooling year. Which means, no child would be left behind. However, even if this was clear, teaching methods and approaches regarding how to assess students to select those who need help and how to catch up with them were neither modified nor adjusted, moreover, more appropriate resources to support teachers and children were not provided.

Consequences on teachers' practices

The first problem was that teachers who were not informed and not instructed to this reform did not understand its objectives; moreover having no instruction on innovative and formative assessment, teachers could not guess how to make these children, who begin the school with many disparities as they come from different backgrounds, reach necessarily all the objectives of the first grade. However, contrary to the true meaning of this change, most teachers have understood that children should pass to grade two, independently from their level of performance and marks. They thought that exams might be useless, and consequently, teachers do not catch up for children who have showed a weakness from the beginning of the school year. Yet some teachers proposed to eliminate the exams. Certainly, at grade two, where remaking the year was allowed, more problems appeared. Most of children who pass to grade two have difficulties, because of lack of prerequisites; besides, classes are overwhelmed due to the fact that more than half of children of grade two do not pass to grade three, which makes the number of children rises every year.

How this reform could be successful

Teachers who are the principal actors should be informed about the intended goals of the reform but more importantly the motivation and the reasons behind adopting it, then their instruction to reach those goals should follow focusing on the different resources and methods to use along with the corresponding duration of time. In fact, it is very common that teachers complain, at every school level, about the low level of prerequisites of children when they go from one grade to the successive one. This problem could be solved first, by making all children mastering the goals, from the very start at the first grade, then keeping watching the prerequisites of children to be completely acquired before passing from a grade to another. This method implies a change of the actual assessment, it requires an examination of whether the taught concepts are mastered by students, and not about classifying students (at the end of the school year) and deciding that some of them will pass (even with weak prerequisites) and others will fail. In other words, shifting from summative assessment to formative assessment. This requires also a shift in teachers' beliefs about the school mission and their own mission. Moreover, teachers should know what to do with children who show a weak level of concepts mastery, and how to make the corresponding assessment to reveal either the children who really have still some problems (to be taken care of them) or that the concepts are really mastered completely by children. If this point is reached, it would lift the teaching and the learning of mathematics to a better level, not only all children would reach the goals; but mostly, teachers who would act as experts, would detect what works better and the different dysfunctions in the curriculum along with their different practices. And this will certainly strengthen their continuous training.

ELIMINATING ELEMENTS OF MATHEMATICAL LOGIC FROM HIGH SCHOOL CURRICULUM

Reforms and mathematics education research are not two distinct fields; they rather evolve simultaneously. The curious fact is that the influence has tended to be mutual, that is, not only has research shaped reform, but reform has also shaped research (Amit, Fried, 2008). Research about formal logic has been a subject of disagreement among researchers, teachers and educators in the last two decades; who are split into two opposite positions. Those who are in favor of the necessity of teaching formal logic, keeping the use of logical symbols like quantifiers, implication and equivalence arrows. And those who argue that this was the origin of some of students' difficulties, therefore, excluding these problematic symbols and replacing them with the verbal expressions they stand for ('implies' instead of ' P ') would be of huge benefit for enhancing students' understanding in mathematics. In 2010, a new Algerian reform at high school level; was undertaken by the ministry of education decision in favor of eliminating logical symbols. This reform has been preceded with a similar one in France in (1981-2000).

Recommendations have been given to teachers to not use mathematics symbols related to formal logic and new textbooks have been printed without them; but there was also a subsequent change, set theory lessons were also eliminated entirely, together with all form of modes of proof like proof by contradiction, proof by contrapositive and direct proof, except proof by induction. These lessons were present, in the past, at the first or the second year of high school (students of 16-17 years); depending the specialty whether it is science, mathematics, or other. This could not be complete without excluding the use of the semantic meaning of logical conjunction: and, or, negation, equivalence, necessary and sufficient condition, and implication in mathematical activities; in other words, there was no additional work for the mastery of propositional and formal logic by using natural language.

For example, when students work the equality (x-1)(x-2)=0 in the affirmative form, they know that it means x=1 or x=2, but when they deal with the negative form; $(x-1)(x-2) \neq 0$, they still put $x \neq 1$ or $x \neq 2$, without feeling the difference. Truth-value of propositions is not taught, students are not instructed to discuss when mathematical statements are true or false, and what can affect their truth or their falsity.

Teachers' reaction

During the implementation of this reform, teachers of high schools were against this new approach of teaching mathematics, they did not see its goals and defended the presence of this material as the basis for mathematics in high school. Even if their position has been manifested, by a written report to the different directions of education and having had the support of the inspectors, the reform has been maintained with no changes. Teachers had difficulties to shift to new teaching approaches, as they have taught and have always learnt symbolic based mathematics. But, the most important was that, in missing the goals of this approach, they were not able to drive students to reach the same ancient goals, mainly mastering the semantic of the words, which replaced the symbols (like 'imply') and introducing semantic-based logical reasoning in mathematics by the new method.

We think that the main goal behind this reform could be to help students understand better mathematics by focusing on the semantic rather than on symbols to overcome their difficulties originated in high symbolic mathematics, like mathematics inspired by the 'new math movement' in France. As Algerian curricula have always been inspired by French curricula, mathematics has always had a strong symbolic approach.

Consequences on university mathematics

The point we would like to discuss here is the consequences on the teaching of mathematics at the university level, whose curriculum and courses were still a sequence to the ancient curriculum, where considerations (concepts, exercises activities and teachers' practices) were set as if formal logic, proving and set theory were still among high school curriculum, more importantly supposing and using a symbolic approach as ever. Let us consider a delicate recurrent example: to prove $(x+y)^2 \geq 2xy$. To be inspired, students generally start from the statement they should prove (which is not wrong), but they should find other equivalent statements and not advance by only implications. This means, when we start from $(x+y)^2 \geq 2xy$, we get $(x+y)^2 - 2xy \geq 0$ and by calculating the square, we get $x^2 + y^2 \geq 0$ which is a true statement. Taking into consideration that a false statement could imply a true one, we cannot confirm that the starting statement is true only because the last one is true, that is why we should use the logical equivalences or trying to arrive to $(x+y)^2 \geq 2xy$ by starting from $x^2 + y^2 \geq 0$, and using only implications. These obvious details are clear to someone who assimilated propositional logic (by symbols or by words) at both semantic and syntactic levels. It is not the case for students who did have no instruction about that. And how would teachers grade such a question, in the exam, if a student gave all the steps separated with nothing (neither 'imply' nor 'equivalent') from $(x+y)^2 \geq 2xy$ to $x^2 + y^2 \geq 0$ and claimed that $(x+y)^2 \geq 2xy$ is true as $x^2 + y^2 \geq 0$ is true?

At the university level, synchronization did not follow, thus as university mathematics courses (for algebra and analysis) consider that propositional and formal logic, modes of proof and set theory are among students' prerequisites, students who did not get them could not assimilate their first year courses from the first moment. Consequently, many of them fail and those who pass to the second year encounter many problems and fail again or pass to the successive levels with many difficulties. It is even worse, because the time dedicated for mathematics courses, at the first year, have been shortened, by a later university reform, so when teachers want to recall some material to catch up with some concepts (related to what was eliminated at high school), lack of time does not allow it. The present problematic situation is that most university teachers, who are not updated, are not conscious about this situation, instead, they believe that students are not high achievers and/or are less serious. While students keep thinking that teachers are not competent and/or mathematics is much more difficult than what they thought and so they are far less talented to be up to it.

How this reform could be successful

Focusing on the semantic rules is necessary but could be done without cancelling the symbols ($\Longrightarrow, \Longleftrightarrow, \forall, \exists$). The problem was to express the semantic and the syntactic rules with effective examples, using actual definitions and mathematical statements. If we replace the symbols by words and maintain the same ineffective use of them, students would not progress better. According to Durand-Guerrier et al (2012, p. 372), 'human reasoning […] involves an ongoing interaction between syntax and the interpretive role played by semantics'. Moreover, working on natural language and mathematical language would help students; in fact the mathematical logic is made explicit when it is expressed with language. Students, often, fail to perform well with logic because they have not developed their natural language and have problems to shift to mathematical language (Boero, P., Douek, N., & Ferrari, P. L. 2008). Teachers should be instructed on how to teach mathematics in the absence of these 'tools', and how students would master mathematical concepts using semantic and syntactic rules of mathematical logic. Coordination with university mathematics should follow up, either to adapt the courses to students with such background or to prepare them at the first year to face formal mathematics in case no changes are made for university mathematics.

CONTINUOUS ASSESSMENT MARK

A university reform that has been applied since 2006 in Algerian universities has brought many changes, focused particularly on contents and time duration dedicated to courses. The new system is called LMD (licence (bachelor, in French), master, doctorate) for respectively 3+2+3 years of formation. The main motivation given by politicians was the equivalence of degrees with European ones, which would facilitate both scientific and economic exchanges; we will focus on the novelty related to students' assessment. In the past, a student passes to the next level, when the average of three written exams marks (/ 20) is no less than 10/20, through the academic year (three trimesters). With the new system, the academic year is split to two semesters with only one written exam for each, and an additional new mark (/20) called a continuous assessment; for each semester. It is the average of one or more short tests made by teachers of the exercises' sessions for a group of students (25-35 ones) for a period of thirty minutes counted for 15 points; the other five points are given according to the presence and the work of the student in the class (only at the exercises sessions). The motivation behind this mark, according to administrators, is about assessing students for their continuous work during the semester and not only by one exam at one single day. This could be fair, if teachers and students were conscious and were given the means to do so. The other novelty is that the weight of this mark is the same for the written exam. The only written exam by semester is the same for all students (of the same discipline), at the same period and with the same scoring scale. The 'final average mark' to decide whether a student had passed or not is by calculating the average of the written exam's mark (/20) and the continuous assessment mark (/20) ((wem+cam)/2). Moreover, a student can contest the continuous assessment mark, even if it is the result of his/her own efforts, and not the result of the written exam. The objectives of this mark are not clear for teachers who became suspicious and resist grading students for their presence and their participation to the discussions in the classroom, while their scientific level could be very low. Teachers argue that the university is for high and advanced level, students should upgrade to be up to it, and nothing should intervene to decide about their ability to pass, but their conceptual mastery.

Consequences on students' beliefs and learning

It is incredible how this mark had a tremendous change on students' beliefs and learning; first, most of students are not interested in being present in lectures, because they are not graded, contrary to exercises' sessions; undoubtedly, it created a negative effect on their learning, reduced only to exercises' sessions. Second, students have less interest to reach the mastery of their concepts and make little efforts to prepare the official written exams; moreover, as they can contest their mark, they always ask their teachers for more. Some of them dare asking their teachers to give them a big mark even if they got a small one (in the test), to compensate a bad mark gotten in the written exam. Finally, the act of succeeding has been transformed from reaching the objectives assured by concepts mastery to a continuous negotiation between students and teachers and even begging for rising the continuous assessment mark as high as possible.

How this reform could be successful

This issue is the tree that hides the forest, because the real problem that has existed for a long period in the Algerian university, like in many countries, is the fact that students do not succeed at the university as they do in the school. The university norms are all different, especially with mathematics. The high level of failure of students could not be improved by making a gift of marks for students. Mastery of concepts, formative assessment and exams that are within reach of students and in accordance with what teachers presented to students can be in balance for students' success. Moreover, university teachers should be more aware about teaching challenges: students prior knowledge and students' beliefs and expectations about teachers and learning; while actual teachers are more focused in research and thus less interested in pedagogy (Rach & Heinze, 2016, p. 1347).

CONCLUSION

We have given three examples of imported reforms of different nature at different levels, in Algeria, which all failed to bring any improvements; not only that, but created dysfunctions at many levels. All three reforms have been set by political decisions without highlighting their reasons and their objectives; teachers who were not informed neither prepared are either against or, not understanding how to apply the new changes and how to adjust the current practices and to implement the new interventions. Their true motivations are certainly good and might have given good results in other countries; they could be of high benefit in Algeria too, but only, if they have been brought for that goal. Unfortunately, they are practiced in order to exist, because they exist somewhere else, and not to bring any specific improvement, and so they are.

These reforms, proposing new and different views of teaching mathematics, could be also an opportunity to reflect on one's practices and beliefs and try to think about how to make new changes that would bring positives outcomes. The change is not perceived as a problem, as everyone acknowledges that everything changes around us, it is rather how to create ways of adaptation, which is a permanent challenge. It requires first a mindset change and then all the rest would follow up. In fact, we cannot solve our problems with the same thinking we used when we created them (Einstein), which means if reforms are imported like any other goods, they will certainly fail. People need first to analyze them, and plan carefully to their application, by providing the conditions and the resources to do it. We cannot buy shoes of a different size, but we can make the same kind of shoes for any size!

Our conclusion is that 'effective reform' required more than a list of objectives agreed upon by "experts" with political motivations' (Mercer, 1993, p. 14). Our examples show also the limitations of adopting a reform as frequent radical changes inspired by the creation of a new situation capable to be up to a supposed better foreign system. However, 'given the frequency of reforms in mathematics instruction, it is reasonable to ask why educators are not in the process of *refining* math education rather than reforming it' (Mercer, 1993, p. 16), which would consider the adaptation of the local situation to a process of continuous improvements. 'According to Hofmeister, effective reform occurred when validated, replicable instructional programs with well-defined target populations, definitive treatments, reliable instrumentation, and meaningful evaluation measures became the vehicles for change'. (Mercer, 1993, p. 14).

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A COMPARATIVE ANALYSIS OF NUMERACY AS A DRIVER FOR CURRICULUM REFORM IN AUSTRALIA AND IRELAND

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Numeracy has emerged as a driver for curriculum reform in many international contexts. This paper compares curriculum reform processes in two countries, Australia and Ireland, which have introduced policies for embedding numeracy across the school curriculum. Our analysis examines the rationale for including numeracy in the curriculum, how numeracy is represented in the curriculum, and who is deemed to be responsible for developing students' numeracy. The findings highlight the need for "joined up" policy instruments addressing curriculum, assessment, and teacher preparation, as well as the relationship of numeracy to mathematics.

In many countries the notion of mathematical literacy as a 21st century competency has emerged from either international studies, such as the OECD's Programme for International Student Assessment (PISA; OECD, 2016), or national curriculum policy development. In some English speaking countries, however, it is more common to speak of *numeracy* rather than mathematical literacy. This paper sketches a comparative analysis of the role of numeracy as a driver for curriculum reform in two such countries, Australia and Ireland. One important difference in the curriculum policy contexts in these countries is that Australia is a federation of States and Territories with an overarching Commonwealth (national) government, and it is the States that have constitutional responsibility for public education; whereas in Ireland education policy is formulated and implemented at the national level. Nevertheless, in recent years in Australia the national government has taken a more active role in education policy development, affecting curriculum reform, numeracy testing, and teacher preparation (Stephens, 2014).

The main question address in this paper is: How is responsibility for students' numeracy development constructed by curriculum policy? The paper contributes to Theme D by comparing curriculum reform processes in two countries to shed light on the interpretation and expression of numeracy and its relationship to mathematics.

CONCEPTUAL FRAMEWORK: THE OFFICIAL CURRICULUM

Remillard and Heck (2014) defined curriculum as "a *plan for the experiences* that learners will encounter, as well as the *actual experiences* they do encounter, that are designed to help them reach specified mathematics objectives" (p. 707, original emphasis). They presented a visual model of the curriculum policy, design, and enactment system that distinguishes between the official curriculum and the operational curriculum enacted in classrooms. Our focus in this paper is on the official curriculum, as specified by governing authorities, and on two of its three components proposed by Remillard and Heck: the curricular aims and objectives and the content of consequential assessments. The third component, the designated curriculum, refers to the instructional plans and materials specified by a ministry of education to offer guidance towards addressing the curriculum's goals. Remillard and Heck note that across educational systems there is variation in the form and specificity of the designated curriculum. Thus we do not consider the designated curriculum in our analysis because in neither country are such materials treated as part of the official curriculum.

Our comparative analysis is structured around three dimensions: (1) the rationale for including numeracy in the school curriculum, (2) how numeracy is represented in the curriculum, both explicitly through curricular aims and objectives, and implicitly through the content of consequential assessments, and (3) who is responsible for developing students' numeracy.

RATIONALE FOR NUMERACY

In Australia, the rationale for including numeracy in the curriculum has evolved over 30 years and three national Declarations on the goals of schooling agreed by the State, Territory, and Australian Ministers for Education. In 1989 the Hobart Declaration (Education Council, 2014b) proposed a framework of national collaboration between the States and Commonwealth with ten agreed goals for schooling, including development of skills of numeracy and other mathematical skills. Ten years later, in 1999, the Adelaide Declaration agreed on eight key learning areas for the school curriculum and additionally stated that "Students should have attained the skills of numeracy and English literacy, such that every student should be numerate, able to read, write, spell and communicate at an appropriate level" (Education Council, 2014a). Whereas the previous Declarations were non-binding agreements, in 2008 the Melbourne Declaration foreshadowed action in referring to developing a national curriculum and national assessment program for literacy and numeracy (MCEETYA, 2008), replacing existing State-based curricula and assessments. Having skills in numeracy was seen as essential for creating "successful learners, confident and creative individuals, and active and informed citizens" (p. 8). While in each of these policy documents there was an emphasis on education for social and economic well-being, and an implicit suggestion that numeracy served this goal, nowhere in the three Declarations was numeracy ever defined.

In Ireland the rationale for numeracy driving curriculum reform is a more recent phenomenon, in response to the results of the Third International Mathematics and Science Study (TIMSS; Beaton et al., 1996) and Ireland's substantial decline in PISA mathematical literacy performance in 2009 (Shiel et al., 2016). Performance on these international assessments, together with the national economic crisis of 2010, provided impetus for development of a national literacy and numeracy strategy (Department of Education and Skills, 2011) and subsequent interim review of the strategy (Department of Education and Skills, 2017). The government has agreed that all young people in Ireland should leave school with the appropriate numeracy and literacy skills to live and participate as informed citizens in society. In the strategy document, numeracy is defined as follows:

> Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems. (DES, 2011, p. 8)

REPRESENTATION OF NUMERACY IN THE OFFICIAL CURRICULUM

Numeracy can be represented in the official curriculum explicitly, through curricular aims and objectives, as well as implicitly, via the content of consequential assessments.

Representing numeracy through curricular aims and objectives

In Australia, the relationship between mathematics and numeracy has been explored and contested for many years. The *National Numeracy Review Report* (Council of Australian Governments, 2008), although mixing together research and recommendations regarding both mathematics and numeracy, seemed to set a clear direction for distinguishing between these in its first recommendation:

> That all systems and schools recognise that, while *mathematics* can be taught in the context of mathematics lessons, the development of *numeracy* requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. (p. 7, emphasis added)

The *Australian Curriculum: Mathematics* was developed between 2008 and 2012, and is structured around the three content strands of number and algebra, geometry and measurement, and statistics and probability, and the four proficiency strands of understanding, fluency, problem solving, and reasoning (ACARA, n.d. a). At the same time, the Australian Curriculum has progressively elaborated the notion of numeracy as a "general capability" alongside literacy, ICT capability, critical and creative thinking, personal and social capability, ethical understanding, and intercultural understanding. The set of general capabilities is thus a curricular representation of 21st century competencies (e.g., Ananiadou & Claro (2009). General capabilities are meant to be developed in all learning areas, and the curriculum offers advice within each learning area for developing numeracy based on the following general definition:

> In the Australian Curriculum, students become numerate as they develop the knowledge and skills to use mathematics confidently across other learning areas at school and in their lives more broadly. Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully. (ACARA, n.d. b)

The General Capabilities section of the Australian Curriculum contains a set of Key Ideas in numeracy organised into the following elements: Estimating and calculating with whole numbers; Recognising and using patterns and relationships; Using fractions, decimals, percentages, ratios and rates; Using spatial reasoning; Interpreting statistical information; Using measurement. These elements are further represented in a numeracy learning continuum with statements describing what students can typically do by the end of the various years of schooling. For example, within the element of "Estimating and calculating with whole numbers", by the end of Year 8 students can typically "compare, order and use positive and negative numbers to solve everyday problems". However, it is difficult to see how this set of objectives aligns with the curricular aim of helping students "to use mathematics confidently *in other learning areas* at school and *in their lives more broadly*" (emphasis added). Nowhere in the learning continuum are other learning areas mentioned, and references to situations outside school are scant and superficial (e.g., money, maps, timetables). The numeracy learning continuum could easily be used to support teachers in implementing the *Australian Curriculum: Mathematics* without the need to engage with other learning areas, or the world outside school, at all.

Missing from this representation of numeracy are two key elements that are necessary for fulfilling the goal of "active and informed citizenship" proposed by the *Melbourne Declaration* (MCEETYA, 2008): *context* and a *critical orientation*. These elements are captured by the model of numeracy developed by Goos and colleagues for use in Australian classrooms (e.g., Goos, Geiger, & Dole, 2014). This model proposes that numeracy development requires attention to cross-curricular and real-life *contexts*, the application of *mathematical knowledge*, the use of representational, physical and digital *tools*, positive *dispositions* towards the use of mathematics, and a *critical orientation* to the use of mathematics in order to make decisions and to support or challenge an argument.

In Ireland, the first step to improving students' numeracy was through the introduction of a revised mathematics curriculum known as "Project Maths", introduced in all post-primary schools from 2010-2012. The overall aim of the new curriculum was to ensure that students had a deep understanding of mathematics in real life contexts (DES, 2010). However, Jeffes et al. (2013) reported that while students' dispositions towards mathematics have improved since the introduction of "Project Maths", they lack understanding of where in their future careers they would use the mathematics they have learned at school.

In Ireland, as in Australia, there is a lack of clarity in curriculum policy about the distinction between numeracy and mathematics. While the Irish document is referred to as the national strategy to improve literacy and numeracy among children and young people, throughout the document there is frequent reference to mathematics rather than numeracy. For example, the argument that young Irish people need to develop better numeracy skills was supported by claims such as "Repeated assessments of *mathematics* at primary level have revealed weak performance in important areas of the *mathematics* curriculum such as problem solving and measures" and "The proportion of students who are studying *mathematics* at Higher Level in post-primary schools is disappointing" (Department of Education and Skills, 2011, p. 13, emphasis added).

Nevertheless, a revised curriculum framework for the lower secondary years (known in Ireland as the Junior Cycle) has introduced a set of Key Skills that could be interpreted as 21st century competencies: being literate, managing myself, staying well, being curious, managing information and thinking, being numerate, being creative, working with others, and communicating (Department of Education and Skills, 2015). Teachers are meant to embed these key skills in the learning outcomes of every subject, but there is not yet any explanation within newly developed subject specifications of how this can be done. Whereas the Australian curriculum offers some further elaboration in the form of a numeracy learning continuum that arguably conflates numeracy with mathematics, the Irish Junior Cycle curriculum framework acknowledges the mathematical foundations of numeracy but also includes positive dispositions and the use of digital technologies to develop numeracy skills and understandings. This reference to dispositions and tools suggests there is potential for representing and embedding numeracy in the Irish curriculum in ways that reflect the rich numeracy model developed by Goos et al. (2014).

Representing numeracy through the content of consequential assessments

After several years during which the Australian States implemented separate tests of literacy and numeracy for school students, in 2008 the National Assessment Program – Literacy and Numeracy (NAPLAN) was adopted and the program continues annually for all students in Years 3, 5, 7 and 9. The NAPLAN numeracy tests "assess the proficiency strands of understanding, fluency, problem-solving and reasoning across the three content strands of mathematics: number and algebra; measurement and geometry; and statistics and probability" (ACARA, 2017). From this definition it is clear that these tests draw from the *Australian Curriculum: Mathematics* and could be regarded as assessing mathematics rather than numeracy as it is defined in the official curriculum.

Much data are generated from NAPLAN testing. National reports are prepared, and school results are publicly available on the MySchool website (https://www.myschool.edu.au/). Schools receive information on the performance of each of their students on the tests, and parents also receive the results for their children. Way, Bobis, Lamb, and Higgins (2016), in a critical review of Australasian research on curriculum policy, pointed to concerns that have been raised on the impact of NAPLAN on students and teachers, including a narrowing of the curriculum and restriction of pedagogical approaches. Although there is evidence that some schools have made productive use of NAPLAN data in conjunction with school-based assessments to identify and address students' learning needs, the consensus was that this high-stakes testing regime negatively influences public understanding of numeracy and of the mathematics curriculum.

Ireland, unlike Australia, has not implemented a comprehensive national assessment program in numeracy. Information on numeracy achievement in Ireland is limited to periodic assessments of samples of students at primary level collected in the National Assessments of English Reading and Mathematics, as well as from participation in TIMSS and PISA. Of these assessments, only PISA could be regarded as targeting numeracy in the broadest sense conveyed by the national literacy and numeracy strategy. Although there is qualified support in this strategy document for better assessment data, from standardised testing of numeracy as well as formative classroom assessment, these aspirations are unlikely to be fulfilled in the immediate future. Thus in Ireland, numeracy is *under*-represented in the content of consequential assessments, while in Australia it could be argued that numeracy is *mis*represented in the high-stakes NAPLAN assessment.

RESPONSIBILITY FOR DEVELOPING NUMERACY

In Australia, there has been acknowledgement for many years that numeracy is an across the curriculum commitment (e.g., COAG, 2008), and inclusion of numeracy as a general capability in the curriculum endorses the expectation that all teachers will be responsible for developing their students' numeracy, no matter what subjects they teach. Yet, the processes of curriculum reform in Australia that led, for the first time, to a national curriculum have not been explicit in setting out how all teachers should achieve this goal. This is perhaps a consequence of the designated curriculum – instructional plans and materials offering guidance towards addressing curricular aims – being left to local entities such as districts, schools, educational consultants and researchers, and textbook publishers (Remillard & Heck, 2014). So while there have been successful research and development projects that have helped teachers to recognise the numeracy demands and opportunities of the subjects they teach (e.g., Goos et al., 2014; Thornton & Hogan, 2003), numeracy is still widely regarded as the responsibility of the mathematics teacher or department (Carter, Klenowski, & Chalmers, 2015).

The situation is similar in Ireland, where the national literacy and numeracy strategy emphasises that the teaching of numeracy is not only the responsibility of the mathematics teacher but instead should be a priority across all subjects. Some teachers have been designated as "numeracy link teachers" with responsibility for disseminating the teaching and learning goals for numeracy to the rest of the staff in their schools, to ensure that numeracy was taught across the curriculum. However, these numeracy link teachers tend to be specialist teachers of mathematics, thus reinforcing the message that numeracy is the responsibility of mathematics teachers.

Another way to investigate whose responsibility it is to develop students' numeracy is to examine teacher preparation policies and standards. The Australian Institute for Teaching and School Leadership (AITSL) is a Commonwealth Government agency that plays a key role in regulating the teaching profession. AITSL (2017) has developed a professional standards framework for teachers at all levels within the profession: graduate, proficient, highly accomplished, and leading teachers. The graduate standards related to teachers' numeracy capabilities include the following:

> **Standard 2.5: Literacy and numeracy strategies:** Know and understand literacy and numeracy teaching strategies and their application in teaching areas.

The standards framework has assumed particular significance for initial teacher education as a mandatory accreditation framework for university programs, so that all pre-service teachers are required to demonstrate competence in order to graduate. Given the blurring between numeracy and mathematics evident in the official curriculum, expressed via curricular aims and NAPLAN numeracy test content, there must be some doubt as to how effectively and authentically this standard can be implemented.

Similarly, in Ireland, in response to the national literacy and numeracy strategy, all initial teacher education programs are required to "address student teachers' literacy and numeracy and their competence in promoting and assessing literacy and numeracy as appropriate to their curricular/subject area(s)" (The Teaching Council, 2017, p. 14). The learning outcomes for graduates frame this requirement as demonstrating knowledge and understanding of "the role of language in teaching the curriculum/syllabus together with a particular focus on literacy and numeracy" (p. 26). In both countries, then, despite the existence of standards or learning outcomes that give responsibility for students' numeracy development to all teachers, not only teachers of mathematics, there is weak framing of what teachers must know and be able to do in order to achieve curricular goals for numeracy. It is not surprising, then, to observe a variety of approaches to developing knowledge of numeracy teaching strategies in initial teacher education programs in universities in both countries. In Ireland, for example, these approaches range from semester-long courses on literacy and numeracy to no specific courses at all and instead an effort to incorporate numeracy across the teacher education curriculum.

CONCLUSION AND IMPLICATIONS

The aim of this paper was to offer a brief comparative analysis of curriculum reform in relation to numeracy in Australia and Ireland. We focused on what Remillard and Heck (2014) refer to as the official curriculum, and asked how curriculum policy is implicated in the construction of responsibility for students' numeracy development. Our analysis attended to the interpretation and expression of numeracy and its relation to mathematics in the curriculum and related assessments, and the implications for teacher preparation.

We can view the design and implementation of numeracy curriculum reform in terms of the policy instruments that enable these processes. Australia has a potentially fragmented system with States and Territories holding constitutional responsibility for education and the Commonwealth government increasingly seeking to exercise control nationally. Despite the difficulties in coordinating State/Commonwealth agendas, a national approach has emerged comprising separate policy instruments that have become "joined up"– a national curriculum, national assessment program, and national professional standards for accrediting initial teacher education programs. However, the articulation between these instruments is still inadequate, leading to inconsistencies in distinguishing between numeracy and mathematics, misrepresentation of numeracy in high stakes assessments, and a lack of resources supporting the goal of every teacher being responsible for developing students' numeracy in the subject(s) they teach.

By contrast, in Ireland, as in many other countries, curriculum is developed and implemented at a national level, and so it might seem that conditions are more favourable for national reform promoting the embedding of numeracy across the whole school curriculum. However, although there is increasing awareness of the importance of numeracy for informed citizenship and personal development, Ireland lacks well-developed policy instruments that connect curriculum, assessment, and teacher preparation in ways that support "joined up" implementation and monitoring of reform. In such circumstances it may be desirable to place more emphasis on the designated curriculum (Remillard & Heck, 2014), and develop instructional plans and material to guide teachers in implementing curricular goals for embedding numeracy in all school subjects.

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PRACTICES IN TEACHING A REFORMED MATH CURRICULUM IN THE JEWISH AND BEDOUIN (ARAB) SECTORS OF ISRAEL

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The study compares the way the primary school mathematics curriculum was implemented in Israel's Jewish and Bedouin sectors in terms of differences and similarities in "scripts" in fourth- and fifth-grade math lessons based on a recent reform in elementary mathematics education .The new curriculum was a component of on-going reform in mathematics teaching in Israel. Twenty classes, equally divided between the sectors, are observed. Five categories of teaching practices in class are analyzed. The results point to broad similarities, probably tracing to shared training, curriculum, and materials, and to differences, such as stronger teacher responsibility for learning in the Bedouin sector and more independent thinking and conduct in the Jewish sector. These tendencies in both directions probably trace to stronger adherence to tradition in the Bedouin sector. The results emphasize, among other things, the importance of comparing and contrasting teaching practices within countries as well as among them.

INTRODUCTION

In 2006, Israel introduced a new mathematics curriculum for primary schools in all population sectors. Its main aspect is emphasis on classroom culture and on the teaching that gives special importance to mathematical insight and inquiry. It also aims to have students develop non-standard algorithms as a way to strengthen their conceptual grasp of numbers and operations in them and to develop the ability to make estimates and produce numerical insights. The authors of the curriculum also note the importance of emphasizing students' activity in a rich environment of models of mathematical concepts and objects and phenomena that substantiate the subject being studied (Mathematics Curriculum, 2005). As the curriculum was being implemented, teachers and counselors assimilated its principles and emphases in multiple in-service activities and intramural counseling for all teachers of math was offered on a larger scale. Now, a decade later, the sectors are far apart in achievements generally and in assignments that test mathematical insight and inquiry particularly. One hypothesis about the source of the disparity is a material inter-sectoral difference in practice in mathematics classes. Therefore, the purpose of this study is to try to characterize these practices in order to get to the root of one of the main reasons for this inequality.

Much recent research demonstrates the substantial influence of a society's overall culture on teaching and learning methods in its schools and even on student achievements (e.g., Presmeg, 2007). According to Gallimore (2006), a classroom, all things considered, is an additional type of cultural activity; furthermore, the entire classroom environment is a micro-community in which learning takes place (Tabach & Schwartz, 2018).

Research on mathematics education shows that the math-class culture figures importantly in students' learning (Miao et al., 2015; Clarke, et al., 2006). In addition, comparative cross-country studies on math teaching find that the characteristics of a "good math teacher" or a "good math lesson" are different, diverse, and built on a combination of approaches and not on one approach alone (Mesiti &

Clarke, 2010; Clarke, et al., 2006; Hiebert et al., 2003; Stigler & Hiebert, 1999). Furthermore, many studies indicate that teachers even within one culture/country differ in teaching styles. It is argued, for example, that differences in teaching mathematics are greater among teachers in Beersheva than among counterparts in Hong Kong and Shanghai (Fried & Amit, 2005). Nevertheless, they point to the need to identify teaching patterns within geographic boundaries in response to teacher training and professional development requisites by profiling and defining successful practices in a context of shared curricula and textbooks (Hugener et al., 2009).

The intended curriculum contains what students are expected to learn (Chen, Reys & Reys, 2009). Studies about the implementation of intended curricula show that teachers tend to interpret messages of reforms in terms of their beliefs in regard to both mathematics and teaching of mathematics, instead of expected changes in teaching methods and their implications for these beliefs (Charalambous & Philippou, 2010; Boesen et al., 2014). Many factors—social, political, and cultural, among others—affect the difficulty in making the requisite change. This influence also finds expression in daily lesson planning, teachers' shared views on various teaching issue, and the support that they want or receive from official elements from school administration to the ministry of education (Remillard & Heck, 2014). One of the most significant effects in applying reforms and new curricula is cultural. This influence may be manifested in amplification, improvement, adjustment, or revision of the "spirit of the reform," all depending on how strongly the cultural environment seeks to preserve or discontinue the tradition at hand (Dee & Palmer, 2017).

Israeli society is heterogeneous, composed of groups differentiated in terms of nationhood, religion, ethnicity, class, and politics. Below we relate to two: secular Jews and Bedouin in Israel's south. The Jewish population is a largely urban and reflects a European cultural outlook on scholastic excellence as the assurance of a better future. The situation in Bedouin society is more complex. When Israel was established, Bedouin society underwent processes of social and economic change due to its transition from a traditional agricultural semi-nomadic way of life to a modern one. Young Bedouin are able to attend Israeli and foreign universities and travel beyond their tribal confines. These changes have affected the characteristics of education in Bedouin society in the direction of Western-style schooling for the masses. However, the Jewish and Bedouin sectors are far apart in cultural and economic respects among others. One way of surmounting the disparities is by education and schooling. Given the importance of the sociocultural aspects of the learning process, researchers are eager to document and examine behavior patterns in class generally and in math classes particularly. Although teachers in the Jewish and the Bedouin sectors are trained in the same higher-education institutes, it stands to reason that differences exist in their classroom teaching practices, manifested in the organization of the mathematics class, the goals of the lessons, and teachers' activity in class.

Several approaches exist in researching the culture of math classes. In this study, we relate to the approach proposed by Stigler and Hiebert (1999): description of the typical "script" of a math class as is accepted by the culture in question. Stigler and Hiebert, pioneers in researching typical scripts of math classes in different countries, define the classroom script as a mental version of teaching patterns that individuals learned while in school (ibid.). As an example of a pattern typical of mathematics lessons in the United States, they cite the tradition of checking homework before moving on to the lesson proper. One of the most influential studies in this direction is the international TIMSS video study, part of the comprehensive Third International TIMSS Video Study.

In its course, dozens of lessons were filmed in each participating country and were analyzed to yield a "typical script" of math classes in each country. As one may see in these scripts, math lessons in the U.S. Germany follow a script of acquisition/application. In Japan, script-"problematizing" comes to the fore. The commonalities among math lessons in Japan, which never drew the attention of researchers there, found expression in the international study (Santagana & Stigler, 2000).

Accordingly, the main purpose of this study is to profile math teaching practices among two population groups ("sectors") in Israel: Bedouin and Jewish. To accomplish this, we focus on two aspects: general organization of math lessons in each sector and analysis of the impact of culture on math teachers' practice. This will shed light on similarities and dissimilarities between the script of a "Jewish" mathematics lesson and that of a "Bedouin" one; it may also determine whether an "Israeli script" for a mathematics lesson in the country's primary schools exists.

METHODOLOGY

The study was conducted in twenty fourth- and fifth-grade classes in southern Israel: ten in primary schools affiliated with the Jewish sector and ten in the Bedouin sector. In each class, one math lesson was observed, yielding twenty observations in classes taught by different teachers. Although the participants in the study are not a representative sample, the classes chosen for the observations represent the socioeconomic differences and disparities in achievements on national tests. Some classes are in urban schools and others in rural and, in the Bedouin sector, some represent schools in townships and others in dispersed localities. The observations in each sector were divided into eight of arithmetic classes and two of geometry classes. The lessons were painstakingly documented by one of the authors, including the notes that she took during the observation. Throughout the study, we use the term "class" or "lesson" to denote a time-limited teaching unit. A typical lesson in Israel is 45–50 minutes long. Some lessons were twice as long; they were not taken into account.

All teachers who took part in the study hold B.Ed. degrees in mathematics. They were told that the purpose of the study was to observe typical math classes, their comportment, and their components. Thus, they were asked not to make special preparations for the lesson and not to tip off the students. Interviews were used to obtain information about each teacher's demographic particulars.

The method used to code the observations is based on that developed in the 1999 TIMSS video study (Hiebert, 2003), adapted to this study to accommodate research constraints. To describe the practices of the teaching methods in class, five categories were chosen: (1) organization of learning in class (full class, students' independent work, working in pairs or groups, didactic games); (2) the purpose of full-class activity (review of content previously taught, imparting new material, practicing new material); (3) material treated in full-class activity (determining its nature and checking its correspondence to the curriculum); (4) representations used by the teacher in class (symbolic, numerical, verbal, tangible, schematic); (5) teacher's activity during students' independent work (circulating in the classroom, working with a group of students, going about h/her own business).

RESULTS

a. Organization of learning in class

The organization of learning in most classes observed included full-class activity and independent work. All lessons in both sectors invoked the full-class element—some once, others twice, and a few from beginning to end. In both sectors, all lessons began with full-class activity. In both sectors, two teachers continued to work in this manner until the lesson was over (20%). Neither teacher assigned her students any independent or group work. Another phenomenon identified is the insertion of a full-class activity at the end of the math lesson. Some 70% of classes in the Bedouin sector ended with a discussion in full-class format and only one lesson ended without some kind of concluding full-class activity. In the Jewish sector, only two lessons (20%) ended with a full-class activity and 80% concluded with students' independent work. In both sectors, independent work usually began after a full-class activity; its purpose, by and large, was to practice material taught under the teacher's facilitation. Again, in the Bedouin sector independent work was followed by a summary before the full class; in the summary, usually, the teacher discussed solutions to problems that the students had tackled on their own. Summarizing the findings concerning the organization of classwork, we note that the most frequent lesson structure in the Bedouin sector is composed of three elements: "full-class—>independent work—>full-class." In the Jewish sector, in contrast, the most common structure comprises two elements: "full-class—>independent work."

b. Goals of full-class activity

Teachers' work before the full class revolves around the goals of the lesson. More than half of all classes in the Bedouin sector began with a review of matters previously taught. In half of these classes, teachers pursued this goal throughout the full-class time; in the other half, they convened the full class in order to give exercises in new material. Much the same was found in the Jewish sector. Even in most classes where full-class activity served the purpose of imparting new material, teachers pursued an additional goal: practicing new material. An additional practice identified in the analysis of the classes is "imparting new material" as a full-class activity in itself, with no review of issues previously taught. This practice appeared in both sectors but was more common in the Bedouin sector. In addition, full-class activity of the "practicing new material" type was common; in both sectors, most full-class activities were devoted to this. It is also noteworthy that Jewish teachers prefer to organize a full-class activity more for the purpose of "practicing new material" than for the other two purposes. Such activities usually deal with solving a mathematical problem in class or holding a class discussion on ways to solve problems concerning specific mathematical material. Bedouin teachers clearly prefer two types of full-class activity: review of matters previously taught and practicing new material. Notably, in terms of the content of full-class activity, in Bedouin classrooms there is less discussion of a given issue and more brief lectures by the teacher, who calls on certain students and asks them whether they agree or disagree with what s/he said, with no attempt made to obtain from the students an explanation or a rationale.

c. Mathematical material addressed in full-class activity

The primary-level math curriculum comprises two main kinds of material: arithmetic operations and geometry. Teaching time in each class is divided between numbers and arithmetic operations (75% of lesson time) and geometry and measurement studies. Some 70% of matters dealt with by math teachers in the Bedouin sector concern "numbers and arithmetic operations" and 30% goes to geometry, as the curriculum requires. In the Jewish sector, in contrast, 90% of class time is devoted to numbers and arithmetic operations and only 10% accrues to geometry, meaning that the apportionment of time specified in the curriculum did not correspond to actual conduct in the classes observed. In addition, many lessons in both sectors concerned matters related to natural numbers; fewer dealt with fractions of various kinds. Notably, in no lessons did teachers in either sector deal with multiple mathematical topics. Instead, they addressed themselves to one specific matter and made no effort to link it to anything else.

d. Representations used by teachers in class

Teachers use various representations to present and describe mathematical concepts or ideas. Since they may enrich their students' conceptual toolkit by using multiple representations, the number of representations that they use for one concept in the course of a math lesson is important. The findings show that, in both sectors, two to four representations of one mathematical concept or idea were used in the course of the same lesson. The average number of representations used in a lesson, however, was 2.6 in the Bedouin sector and 3.2 in the Jewish sector. Furthermore, teachers used four types of representation in four lessons in the Jewish sector and in one lesson in the Bedouin sector. Verbal representations were used in all math classes in both sectors, usually in verbal conversation between teacher and students, in the text of a verbal problem from a textbook, or in a handout. In no case did a teacher write a way of thinking or conclusions on the blackboard.

The most common representation is the numerical one. It was evidenced in all arithmetic classes in both sectors but not in geometry lessons in either sector. The schematic / graphic representation makes it possible to substantiate mathematical ideas visually. Teachers in both sectors used this form of representation—those in the Jewish sector in 80% of lessons and those in the Bedouin sector in only 50%. A symbolic representation is one that helps students to develop algebraic ways of thinking. Notably, teachers in the Jewish sector use this representation twice as frequently as those in the Bedouin sector do. The use of this feature in fewer than 50% of classes in the Jewish sector, however, is also insufficient.

e. What teachers do while students are working on their own

Some 80% of math classes include a component of independent work, in which each student is to cope individually, or together with another student, with a task or a set of tasks of the teacher's choosing. The question here is what teachers in both sectors do while the students are working on its own. Three patterns were observed: (1) *teachers circulate in class,* answering students' questions or helping those who get "stuck"; (2) *teachers call up groups of students* to give additional content); (3) *teachers go about their own business* at their desks. The findings show that the teachers in both sectors comport themselves quite similarly while their students are working on their own: in 60% of cases in the Bedouin sector, they circulate among the students; in 30% of cases, they sit with a group of students (mainly with those were having difficulty), and in 10% of cases they go about their own business at their desk. In the Jewish sector, in 70% of lessons teachers circulate among their students and in 30% of cases they stick with a small group. The only difference is that in 10% of cases Bedouin teachers go about their own business, a behavior not observed anywhere in the Jewish sector. However, no material difference was found in teachers' activities in this part of the lesson.

Our observations gave us an opportunity to see how class and learning activity were organized in both sectors. By and large, teachers in both sectors attended the same training institutes, use the same math curriculum, and teach from the same textbooks. Nevertheless, we found differences in their classroom activities and teaching methods. From looking on obliquely during the observations, we spotted a subtle component that influenced practice in class, even though the material taught came from the same source: Probably due to cultural differences, math classes are more traditional in the Bedouin sector than in the Jewish sector. Thus, their lessons are based more on conveying knowledge, giving exercises, and retaining responsibility for imparting the requisite material. Teachers in the Bedouin sector are more inclined to impart material by themselves than are their Jewish counterparts and are less predisposed to instigate discussions with students.

f. Typical math class script in each sector

For Bedouin math teachers, the purpose of the lesson is to impart skills or procedures that are needed to solve various tasks and problems. By and large, they focus on one skill or procedure and try to make sure that all students master it. Neither creativity nor different problem-solving strategies is sought; preference is given to teaching a procedure that "works." The topics of math lessons in this sector fit the curriculum. Teachers organize their lessons on the basis of the conventional traditional structure: "full-class—>independent work—>full-class." In full-class activity, two main goals are usually pursued: reviewing previously taught material and introducing new material. There is little discussion of any topic; teachers prefer to give a brief lecture and then call on several students. These youngsters are asked to agree or disagree with what the teacher has said; no attempt to solicit explanations or rationales is made. Teachers use various representations to present new material but avoid tangible representation. When students work on their own, Bedouin teachers usually circulate among them to answer questions and help those who find the task difficult to complete.

In the Jewish sector, math teachers also consider it their main goal to impart mandatory skills/procedures. Some teachers, however, try to challenge students by asking them to describe the rationale behind a given procedure and explain why it "works." Most material dealt with in class corresponds to the curriculum. Teachers pattern their lessons on a non-traditional model of "full-class—>independent work," usually omitting a summarizing full-class activity, and typically gear full-class activity to imparting and practicing new material. Teachers in this sector use various representations but, like Bedouin teachers, generally avoid means of substantiation and other tangible representations. In the independent-work part of the lesson, they circulate to answer questions or help those who encounter difficulties as they work. They usually play a supervisory role in this kind of work, as evidenced by the absence of a final full-class activity to summarize the contents taught.

DISCUSSION

Comparison of the scripts yields several interesting conclusions. First, both scripts deal with math lessons of the "acquisition/application" type (Hiebert et al., 1999), i.e., students learn to solve specific kind of problems or assignments by following a path to the solution that the teacher proposes (Stigler et al., 1999). In the second part of the lesson, students are expected to "imitate" the teacher by solving problems that strongly resemble those presented to the full class. Work usually takes place in one of two organizational forms: full class or independent work. The purpose of the former is to demonstrate a way to solve the problem; that of the latter is to determine where students have difficulties or find things unclear and to try to resolve these matters individually.

The two sectors' math lesson scripts are quite similar in terms of structure, teacher's activity during the lesson, and the goals of the lesson in its various segments. Just the same, intersectoral differences were found. First, lessons in the Bedouin sector are more traditional in structure: it is *de rigueur* to end them with a summarizing full-class activity and a homework assignment. In the Jewish sector, lessons often end with independent work. The difference probably traces to the difficulty that Jewish teachers have in convening their students for an additional full-class activity. The second difference, manifested in the various parts of the lesson, flows from the teacher's attitude toward the question of "Who is responsible for learning?" Pursuant to direct and oblique observation observations, a subtle element was detected that affects classroom practice, even though the sources of the material taught are identical: the stronger adherence to tradition in the Bedouin sector than in the Jewish sector, most likely tracing to cultural differences. Thus, to impart compulsory material, Bedouin teachers prefer to give over the contents by themselves and are less inclined to instigate discussions among their students. This behavior of teachers seems to follow the Bedouin culture that has special cultural characteristics; it is highly authoritarian, traditional, patriarchal and authoritarian. In terms of this culture the teacher's role in full-class activity is to give students examples that they may use to acquire a given skill or procedure. The students' role here is to listen to the teacher and be attentive to each stage in order to imitate the teacher in the second part of the lesson. Additionally, students must answer the teacher's questions, if any, or ask questions of their own if they find the teacher's presentation hard to understand or unclear.

Teachers in the Jewish sector, in contrast, gear their teaching to discussion in a full-class forum. The observations also showed that teachers in both sectors insist that students master a specific set of procedures in class and learn know how to use them when necessary. This approach stands out more strongly in the Bedouin sector than in the Jewish sector. In the latter, teachers try to tailor their work to the requirements of the curriculum but do not succeed full.

In addition to the foregoing, the results support Santagata and Barbieri's (2005) claim that teaching practices are hard to change and that historical, economic, political, cultural and other factors have much influence. This argument seems correct not only in the case of different countries, as cross-national studies show, but also in the case of one country, as demonstrated above (Clarke et al., 2006).

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INFLUENCE OF TIMSS RESEARCH ON MATHEMATICS CURRICULUM IN SERBIA

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Since 1995, achievements in mathematics and science have been assessed worldwide every four years by TIMSS (Trends in International Mathematics and Science Study), whose outcomes have influenced the development and (re)design of mathematics and science education curricula in a number of countries. This contribution to ICMI Study 24, concerned with its Theme D "Globalisation and internationalisation, and their impacts on mathematics curriculum reforms", examines how TIMSS has influenced changes in the mathematics curriculum in Serbia. Firstly, we briefly present TIMSS results for Serbian students. Then, we deal with the influence of TIMSS research on Serbian mathematics curriculum regarding educational standards. The contribution ends with a critical examination of these standards and suggestions for their enhancement.

INTRODUCTION

Since 1995, TIMSS has provided data on fourth and eighth grade students' achievements in mathematics and science for more than fifty countries around the world, every four years (see https://timss.bc.edu/). Apart from such achievement data, TIMSS international databases contain the values of many contextual variables, used to explain differences in students' achievements within and among countries, resulting in a great number of secondary analyses. Some of these have been undertaken by the author of this contribution (e.g. Kadijevich 2008, 2013), who is, as National Research Coordinator for Serbia (TIMSS 2003) and translator of tests items (TIMSS 2003, 2011), well-acquainted with TIMSS research.

Many outcomes of primary and secondary TIMSS research have influenced the development and (re)design of mathematics and science education curricula in a number of countries. The first curricular changes, which started in the end of 1990s, were described in Robitaille, Beaton, and Plomp (2000). Recent changes in the 21st century were documented in three TIMSS Encyclopedias (Mullis et al., 2008; Mullis et al., 2012; Mullis, Martin, Goh, & Cotter, 2016). One of these changes may be, for example, the incorporation of TIMSS cognitive domains into mathematics and science curricula (e.g., Mohd Zain & Goloi, 2012, p. 583; Ndlovu & Mji, 2012).

Since the 2007 cycle, TIMSS research, for both grade 4 and 8, has considered three cognitive domains: Knowing, Applying, and Reasoning (Mullis et al., 2005). Stated briefly, the first domain refers to knowledge the student needs to know, the second focuses on the application of this knowledge in solving routine problems, whereas the third refers to his/her ability to deal with complex contexts, unfamiliar situations, and multi-step problems. In this document, each domain was described by an exemplified list of behaviors. These behaviors are listed in Table 1.

Cognitive domain	Behaviors
Knowing	Recall, recognize, compute, retrieve, measure, and classify/order
Applying	Select, represent, model, implement, and solve routine problems
Reasoning	Analyze, generalize, synthesize, justify, and solve non-routine problems

Table 1: TIMSS cognitive domains and their underlying behaviors

While almost the same behaviors were considered in the next, 2011 cycle (only generalize was replaced by generalize/specialize; Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009), the list of behaviors in TIMSS 2015 and 2019 was updated for Applying and Reasoning. The new lists were: Applying – Determine, represent/model, implement; Reasoning – Analyze, integrate/ synthesize, evaluate, draw conclusions, generalize, justify (Mullis & Martin, 2013, 2017).

Being concerned with its Theme D "Globalisation and internationalisation, and their impact on mathematics curriculum reforms" (IPC, 2017; p. 12), this contribution to ICMI Study 24 examines how the TIMSS results for Serbian students have influenced changes in mathematics curriculum. Firstly, we briefly present these results in the previous four assessment cycles. Then, we deal with this influence regarding educational standards. The contribution ends with a critical examination of these standards and suggestions for their elaboration.

TIMSS IN SERBIA

TIMSS results

Serbian students have participated in TIMSS since 2003. In 2003 and 2007, TIMSS tasks were solved by eights graders. Bearing in mind that Serbian students participated in PISA 2003 and 2006, authorities at the ministry of education decided that as PISA would continue to involve 15-year old students, future TIMSS assessments would involve younger students. Because of that, in the 2011 and 2015 TIMSS assessment cycles, the study involved fourth grade students. The same applies for TIMSS 2019 (in process). All TIMSS studies in Serbia have been carried out by the Institute for Educational Research.

The results for these four assessment cycles are summarized in Table 2 (low scores that called for improvements are underlined). The main results (477, 486, 516, and 518) were much better for fourth graders.[1] The relatively unsatisfactory results of eighth graders in 2003 and 2007 (below 500 points) called for changes of the Serbian mathematics curriculum for compulsory education (grades 1–8), and the changes made around 2010 probably contributed to good TIMSS results in 2011 and 2015. Note that although Serbia was not among the top performing countries at fourth grade, it was definitely among them when countries with low GDP were considered (e.g. TIMSS 2015 Grade 4 countries whose GDP per capita was less than 8,000 $ in 2014).

[1] Fourth graders were also better when the balance among the results for cognitive domains was examined. By applying a min/max measure of this balance (the minimum of the three scores divided by their maximum; Kadijevich, Zakelj, & Gutvajn, 2015), the balance was 0.943 (467/495) in 2003, 0.948 in 2007, 0.983 in 2011, and 0.985 in 2015.

Year (grade)	Average MA	Average MA by content domain	Average MA by cognitive domain
2003 (8)	477	Number / Algebra / Measurement / Geometry / Data 477 / 488 / 475 / 471 / 456	Knowing / Applying / Reasoning 495 / 467 / 468
2007 (8)	486	Number / Algebra / Geometry / Data and Chance 478 / 500 / 486 / 458	Knowing / Applying / Reasoning 500 / 478 / 474
2011 (4)	516	Number / Geometric shapes and measures / Data display 529 / 497 / 503	Knowing / Applying / Reasoning 520 / 511 / 514
2015 (4)	518	Number / Geometric shapes and measures / Data display 524 / 503 / 517	Knowing / Applying / Reasoning 513 / 521 / 517

Table 2. TIMSS mathematics achievement (MA) for Serbian students[2]

TIMSS influence

TIMSS results have influenced the educational system in Serbia in a number of ways (Gasic-Pavisic & Kartal, 2012, p. 796). For example, TIMSS data have been used in various analyses of the primary education system. Also, TIMSS methodology and some of its accomplished test items have been used in national testing. The most important influence of TIMSS in Serbia may be recognized in the development of educational standards for mathematics and science in fourth grade.

Educational standards for mathematics, as with other school subjects in the Serbian compulsory education, have been defined by using three achievement levels, namely: basic, intermediate, and advanced. Mathematics in grade 4 was divided into several areas (e.g., Natural numbers and operations with them), and for each area, there were requirements concerning knowledge and skills required for these achievement levels (NEC, 2011). These requirements were carefully formulated after several rounds of discussion and empirical validation, so that these levels would be, respectively, reached by at least 80%, 50%, and 25% of students. Having in mind that the four TIMSS international benchmarks were respectively reached by about 90%, 70%, 40%, and 10% of Serbian fourth grade students (the exact figures were: 2011 – 90%, 70%, 36%, 9%; 2015 – 91%, 72%, 37%, 10%; Mullis, Martin, Foy, & Arora, 2012; Mullis, Martin, Foy, & Hooper, 2016), we may establish a correspondence between the three achievement levels and the three TIMSS benchmarks. In doing so, the basic level would correspond to the low benchmark ("have some basic mathematical knowledge"), the intermediate level to the intermediate benchmark ("can apply basic mathematical knowledge in simple situations"), and the advanced level to the high benchmark ("can apply knowledge and understanding to solve problems"). As a result, the basic level would

[2] Sources: Mullis, Martin, Gonzalez, & Chrostowski (2004); Mullis et al. (2008); Mullis, Martin, Foy, & Arora (2012); Mullis, Martin, Foy, & Hooper (2016)

primarily call for the cognitive domain of Knowledge, the intermediate level for the domain of Applying, whereas the advanced level would do that for the domain of Reasoning (possibly with some behaviors of other cognitive domains activated to some extent). The content of Table 3, taken from a document regarding educational standards for mathematics in grade 4 (NEC, 2011), supports these domain calls.

The same domain calls apply for other content areas in grade 4 although cognitive behaviors are not that rich for some areas and some levels (e.g. Fractions – basic level). It can thus be said that TIMSS cognitive domains have been incorporated in educational standards. However, these domains are not mentioned in official documents describing the development of these standards. Instead, the application of Bloom's taxonomy is mentioned (Pejić, Kartal, & Stanojević, 2013). Because Bloom's six cognitive categories (knowledge, comprehension, application, analysis, synthesis, and evaluation; Bloom & Krathwohl, 1984) may be viewed as building blocks of TIMSS cognitive domains (Knowing – knowledge and comprehension; Applying – comprehension and application; Reasoning – analysis, synthesis, and evaluation; Gutvajn, Džinović & Pavlović, 2011), it can be said that, through Bloom's taxonomy, TIMSS cognitive domains have been implicitly incorporated in educational standards for mathematics in fourth grade.

Is there some empirical evidence to support this incorporation of TIMSS cognitive domains?

To improve mathematics education (following unsatisfactory achievements in TIMSS 2003 and 2007 among other reasons), a project was carried out in the end of the 2000s, concerned with the development of criterion tests for the end of the first cycle of compulsory education (IEQE, 2009). As a result, a set of one hundred TIMSS-like tasks was carefully developed to assess mathematical knowledge in the fourth grade, recorded on CD, and sent to all schools in Serbia in May 2009, coupled with detailed documentation including a computer program to enter and analyze achievement data. Schools were recommended to use this material to arrange assessments by the end of the 2008/2009 school year, and most schools did so, which contributed to teachers' and students' familiarity with TIMSS-like context and tasks. The one hundred mathematical tasks were developed for twenty-five learning outcomes (with four similar tasks per outcome), and respective Bloom's cognitive levels and achievement levels were assigned to each learning outcome (Stanojević, 2010). When, for the purpose of this ICMI Study contribution, these achievement levels were examined as Bloom-based TIMSS cognitive levels (i.e. Level 1 – Knowing: knowledge and comprehension; Level 2 – Applying: comprehension and application; Level 3 – Reasoning: analysis, synthesis, and evaluation), the assigned cognitive level was present at the assigned achievement level for twenty learning outcomes. (The fact that some of Bloom's cognitive levels appeared at levels lower or higher than expected (e.g. analysis at Level 2 for fractions, or application at Level 3 for measurement and measures), shows that the overlapping of achievement levels cannot be avoided.) Because seventeen of these twenty outcomes were later used as a foundation of educational standards for mathematics in the fourth grade, there is also empirical evidence of the incorporation of TIMSS cognitive domains in these standards. Note that such a contribution was particularly strong for five learning outcomes in the Measurements and measures area.

Achievement level	Requirement (cognitive behavior)
Basic	1. know how to read and write the number given; know how to compare numbers; know how to locate the number on a number line (recognize/order/measure)*
	2. calculate the value of a numerical expression with a maximum of two operations of addition and subtraction within 1,000 (calculate)
	3. multiply and divide without reminder (three-digit numbers with one-digit numbers) within 1,000 (calculate)
	4. know how to set up an expression with one arithmetic operation on the basis of text (represent)
	5. know how to solve simple equations within 1,000 (recall/compute)
Intermediate	1. know how to apply the properties of natural numbers (odd, even, largest, smallest, preceding number, following number) and understand decimal number system (select/implement)
	2. know how to round the number given to the nearest ten, hundred, and thousand (select/implement)
	3. add and subtract, calculate the value of an expression with at most two operations (calculate)
	4. know how to solve equations (select/implement)
Advanced	1. know how to apply the properties of natural numbers to solve problem tasks (synthesize)
	2. know the properties of addition and subtraction and can apply them (select/implement)
	3. can calculate the numeric value of an expression with several operations, respecting their order (synthesize)
	4. can solve complex problem tasks given in textual form (analyze/solve non-routine problems)
	5. can determine solutions of an inequality with one operation (analyze/ generalize)

* Requirements translated into English by the author of this contribution

Table 3. Requirements (cognitive behaviors) for knowledge and skills
by achievement level for area “Natural numbers and operations with them“ in grade 4

CLOSING REMARKS

National educational standards is a critical educational component (e.g. Klieme, 2004). Among other things, not only can they promote a better, more focused education nationwide, but also enable the assessment of its outcomes in a more objective way, finding directions for an elaboration of these standards, if needed.

Our analysis of educational standards for mathematics in fourth grade in Serbia showed that the three achievement levels (basic, intermediate, and advanced) mirror the three TIMSS cognitive levels (Knowing, Applying, Reasoning) to a satisfactory extent. We also realized that at each achievement level, some behavior(s) used at other level(s) may be activated to some extent, which evidences the overlapping nature of these standards. Although this nature cannot be avoided, it may be reduced.

To reduce the overlapping nature of achievement levels, we should focus on 4–5 dominant cognitive behaviors that characterize each level (standards), and primarily these behaviors should be activated through the application of "what to know and do" requirements (cf. Long, Dunne, & de Kock (2014), who proposed to combine levels of processing (as our standards) with dimensions of understanding (as our behaviors)). When the development of standards begins with these behaviors, we can better specify these requirements and develop (possibly also novel) tasks to assess their attainment. With these dominant behaviors in mind, we should also avoid having just two or three requirements for some content area(s) at particular achievement level(s) (e.g., Fractions – basic level), which would limit the diversity of assessment tasks applied. As educational standards need be continuously examined and improved (through professional/theoretical discussions and empirical validations), these suggestions may be used in an elaboration of these standards, especially the mathematics standards examined in this contribution.

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CURRICULUM REFORM OF JAPANESE HIGH SCHOOLS AND TEACHER EDUCATION BASED ON LITERACY

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We give a short historical survey of curriculum reforms in Japanese education mainly focused on mathematics in these thirty years. These reforms are driven by international studies, TIMSS and PISA, but the recognition of the problems and the orientation for improvement have been already proposed in Japan. The orientation would be described with the international notion of "literacy", deeply influenced by PISA, but with Japanese local character, which we describe more concretely in the description of our curriculum reforms of mathematics in upper high schools. The reform is still on the way and is related to wider reform of Japanese education and, in particular, teacher education which has a big problem now in Japan. Here we need to re-consider literacy in wider context.

1. INTRODUCTION

In Japan the school system adopts the national standard curriculum determined by the Central Education Council and the curriculum is revised in almost every ten years. This occurred recently in 1998, 2008, 2017[1]. Here there was a big change of the governmental education policy in 2002. For this change the result of TIMSS (The Third International Mathematics and Science Study[2]) by IEA had a big influence. This gave a big shock also to science research communities including mathematics and they became involved in school education and published a report of science literacy. In the course of the preparation of the revision 2008 the result of PISA (Programme for International Student Assessment) by OECD exposed Japanese weakness of literacy (in a narrow sense) and influenced the revision strongly. The importance of PISA lies in not only the result of assessment but also the publication of the framework of assessment with the name of "literacy". The term "literacy" has several meanings but is used widely, in particular, "mathematical literacy" (See [Jablonka 2003] for example). Japanese curriculum reform would be understood with this notion. Namely a fundamental principle of reform is to foster literacy (in wide sense). For the success of curriculum reform, that of teacher education is necessary, namely to foster teachers' literacy. This process is still ongoing. For that purpose we have to re-consider and deepen the notion of literacy and we give several points to be discussed.

[1] The years of the publishment of curricula. Note that one needs preparation for about 5 years before publication.

[2] Current name is "Trends in International …"

2. TIMSS SCHOCK AND DECLINE OF "ABILITIES"

2.1 TIMSS shock

In 1995 (partial) result of TIMSS (The Third International Mathematics and Study[3]) by IEA (International Association for the Evaluation of Educational Achievement) was reported[4], and that of student inquiry gave a big shock to not only educators but also science (including mathematics) researchers:

Mathematics	It is important in life	Japan	71%	Int. average	92%
	Hope to be involved in future profession	Japan	24%	Int. average	46%
Science	It is important in life	Japan	48%	Int. average	79%
	Hope to be involved in future profession	Japan	20%	Int. average	47%

Moreover the (informal) result after 4 years in TIMSS-R (TIMSS-Repeat, later called TIMSS 1999) was even worse:

Mathematics	It is important in life	Japan	62%
	Hope to be involved in future profession	Japan	18%
Science	It is important in life	Japan	39%
	Hope to be involved in future profession	Japan	19%

Concerning test score itself Japan was in the first class, but for these inquiry scores in the lowest class.

2.2 "Encouragement of Learning" by Ministry of Education

Until the revision in 1998 they were going to reduce learning contents and school hours. From the middle of 90's academic circles were strongly against this policy because of "decline of abilities" of university students. The result of TIMSS and TIMSS-R gave a motivation to change public opinion which caused the change of policy of education. The turning point is the appeal of "Encouragement of Learning" issued by the ministry of education in 2002 January and this change reflected the revision in 2008.

2.3 "Science-for-All-Japanese"

On the other hand science research circles themselves considered the result of TIMSS as the lack of basic understanding of science, or "science literacy", and organized the project "Science for All Japanese" in 2006-08 to publish final report (but written only in Japanese) [SfAJ 2008]. We mention here only a few points (see [Nagasaki 2010] more in detail):

- The framework is modeled on that of "Science for All Americans" [AAAS 1989].
- The report keeps global character and intend to serve as a revision of SfAA.
- The report describes science knowledge which Japanese adults in 2030 are hoped to acquire.

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[3] ditto

[4] The row data given here were not published in the final international report by IEA.

Since many members in this project got involved with the revision of national curriculum, this report had indirect influence on the curriculum reform issued in 2008[5].

3. REVISION OF STANDARD CURRICULUM IN 2008 AND PISA, TIMSS

3.1 PISA shock

As is well-known, OECD began several projects on education, among which the so-called PISA project (Programme for International Student Assessment) would be the most important.

In the first assessment in 2000 Japan was in the top class, but in the next one in 2003 Japanese score of reading literacy went down near to the world average. This was taken to be serious and measures to it became one of the important issues in the revision of the curriculum. This problem of language ability, in particular logical thinking, was already pointed out by mathematicians before 2000 in the discussion on "decline of abilities".

3.2 Mathematical Literacy in PISA and TIMSS Video-Study

PISA publishes the framework of the assessment every time beforehand and revise each literacy in every 9 years. Among others mathematical literacy has high quality, in particular, that of 2012 [OECD 2013]. It is made under the strong influence of Prof. Niss who himself took the lead in the curriculum improvement in Denmark [Niss et al 2011]. It has influenced Japan certainly, in a sense that PISA gave a justification to the orientation of Japanese reform.

On the other hand international study made clear positive side of Japanese education. In 1994-95 TIMSS made a video-lesson study of USA, Germany and Japan, and Japanese lesson is very highly estimated [Stigler et al 1999]. Traditionally Japanese elementary school education (in particular training system in schools) has high quality, which would explain Japanese high score of mathematics in international assessments. Therefore, Japanese curriculum is rather stable in elementary schools (and junior high schools)[6], and curriculum change is needed mainly in senior high schools. Here we should learn from elementary school education (see the discussion **5.4** below).

3.3 Revised Standard Curriculum Issued in 2008 and Literacy in PISA

We come back to describe important points of the revised national curriculum (mainly of mathematics) issued in 2008.

Because of the lack of time we restrict ourselves to discuss the following two problems to be solved which became clear in international assessments:

1) Decline of the ability of language (cf. **3.1**);
2) Passive attitude or loss of motivation for learning (cf. **2.1**).

These problems are general ones but mathematics will play an important role.

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[5] There was no "math war" in Japan.

[6] The revision 1998 destroyed this stability partially by the reduction of lesson time etc, which was cancelled in the revision 2008.

We consider the first problem. The final report to the revision of the national curriculum considers the improvement of the language ability as one of the most important issues of which all subjects should be aware. We emphasize here that the report has declared explicitly mathematics as a subject relating to language together with Japanese and English, namely mathematics is a language. This would be clearly true for Indo-European people or scientific people, but most Japanese don't think so. Hence mathematics is responsible for the improvement of language ability as a language subject. Mathematics education is, of course, aware of it and in PISA literacy several capabilities are considered to be of this kind.

For the second problem the curriculum introduced a notion of "mathematical activity" which is by definition "various work related to mathematics which students practice positively with sense of purpose"([MEXTJ 2011] p.17). As important "mathematical activities", they mention the following (*ibid*.):

(i) founding and developing mathematical theories based on acquired knowledge;

(ii) using mathematics in daily life and society;

(iii) communicating to each other with reason and logic in use of mathematical expressions.

This leads to "active learning" nowadays. In all kinds of school the statement of the "object of mathematics" begins with "Through mathematical activity, …" In senior high school they created a new subject "Application of Mathematics", "in order to develop the ability to think phenomena mathematically and to foster attitude to make use of mathematics (the so-called *mathematical literacy*) both of which are indispensable for knowledge-based society nowadays"([MEXTJ 2012] p.59). Here the word "mathematical literacy" was used in Japanese curriculum for the first time.

As the last topic we mention the introduction of statistics to the common compulsory subject "Mathematics I" in senior high school, hence all high school students (which implies 96% young people of that age) learn this topic, which is an epoch-making event in the history of mathematics education in Japan. The reason of introduction is of course the importance of the topic in modern democratic and data-based society nowadays. Even the most important concept of mathematical education in the 21st century would be "uncertainty" as the concept of "function" (and possibly "axiom") was in the 20th century (Mumford 1999). The lack of experience of teaching, however, gives big difficulty, but also the material itself has its own essential difficulty caused by "uncertainty" (see the discussion below **5.4**).

3.4 Revision of Curriculum Issued in 2017 and Literacy in PISA

The revision of curriculum issued in 2017 follows that in 2008 in principle. The Central Education Council has requested to write not only contents but competences to be learned more in detail in all subjects to implement "proactive, interactive, and deep learning". As a result the size of teachers' guide of mathematics for senior high school just published has almost doubled than the predecessor.

4. TEACHER EDUCATION AND LITERACY

4.1 Reform of teacher education system in Japan

For success of such important changes of national curriculum, high quality of teachers is indispensable, hence reform of teacher education is also necessary. In fact CEC published in 2015 a wide plan to reform the system of teacher education. Changes caused by it is very big and is still under way. Here, however, we restrict ourselves only one topic related to literacy. In the report CEC critisized strongly the gap between the education of “subject content” and that of “subject education”, and now the system has changed so that both are put into one “category” in the curriculum of teacher education.

4.2 Establishment of Japan Society of School Subject Content Education

In order to improve the education of “subject content” in teacher education in general the Japan Society of School Subject Content Education was established in 2014, apart from that of subject education[7]. The novelty of this society lies in the fact that it contains all subjects and intend to establish common or synthesized framework. We are carrying out a project study on it and we hope to publish a report and sample curricula of teacher education in the near future. Here the author is going to propose a frame work based on literacy in general context (cf. discussion below). After Shulman’s pedagogical content knowledge (Shulman 1986), many studies have been done, but we hope that our trial can give some contribution in this direction.

5. DISCUSSION :REVISION OF THE NOTION OF LITERACY

As indicated above, Japanese reform of education is still on the way and we are looking for further improvement. We discuss its possibility based on the notion of literacy compared with mathematical literacy in PISA.

5.1 Generalizaion of the Notion of Literacy

In order to improve the education of “subject content” in teacher education in general the Japan, we need to widen mathematical literacy to two directions, namely to other subjects and to teachers. We might define general notion of “literacy” as follows:

“Literacy of A for B” (in a wide sense) means: a fundamental and comprehensive knowledge of A which people belonging to B are preferable to have, where B depends on the context.

For example the final report of the project: “Science for All Japanese” (**2.1**) says:

“We propose a fundamental knowledge (literacy) of science and technology which all Japanese adults are preferable to have in 2030 to spend a rich life in spirit”.

5.2 Literacy in the Narrow Sense :Mathematics as a Language

Traditionally the word “literacy” was used as the ability to read and write. As stated in **3.3** mathematics is a language but very special language, purely logical descriptive language. The awareness of this fact is strongly necessary in Japan.

5.3 Literacy in Learning :Cycle of Problem Solving

In PISA a model of mathematical literacy is proposed and there the cycle of problem solving plays the most essential role after Freudenthal ([OECD 2013] p.26). This model is a framework for the assessment but our literacy framework is for learning, another cycle of systematization inside our “knowledge” is necessary. Such a model is proposed by Prof. Shimada ([Shimada 1977] p.15).

[7] Many people, including the author, are strongly afraid that this reform would weaken the subject capacity of teachers.

5.4 Learning from Advanced Point of View and from Elementary Point of View

It is well-known that math teachers should have more advanced knowledge on mathematics which they are teaching as Felix Klein has shown, but also one needs to know more elementary mathematics. This is clearly true for basic notions necessary to learn, but some concepts in elementary mathematics is extremely important. An example is the distinction of "number" and "quantity" due to Toyama : "Number" is an abstract concept in mathematics and "quantity" is the property of things in real world which can be expressed with number. This gives a mathematically precise definition of "data" : finite set of quantities of the same kind (dimension). This definition explains theoretically the essential difficulty of the education of statistics.

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IRAN SCHOOL MATHEMATICS CURRICULUM: PAST, PRESENT AND FUTURE

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In this paper six mathematics curriculum changes in Iran from 1900 until now will be reviewed. Meanwhile change forces, barriers and main features of each reform will be represented. Specifically, first five curriculum changes described briefly and sixth one elaborated with more details as contemporary school mathematics curriculum change. After that this recent curriculum reform will be analyzed up on application and modelling approach. Then effect of globalization and research finding in the field of mathematics and mathematics education on Iranian math curriculum will be discussed. Finally, paper enclosed by a remark about necessity of paying more attention to information and communication technology as part of globalization, in particular recall policy-maker to consider computational thinking as null curriculum.

INTRODUCTION

Iran has a centralized educational system, so the mathematics curriculum and textbooks designed in national level and distribute around the country by ministry of education. All schools, teachers and students have to use same mathematics textbook for teaching and learning mathematics. Also, main resource in all sorts of exams (like classroom assessment and national wide external exams) is the same math textbook which all students access to that. upon TIMSS math teachers' questionnaire, most of Iranian math teachers in grade 4 and 8 reveal that they use mathematics textbooks as main source for their teaching in classrooms (Mullis, et. al., 2008 and Mullis, Martin, Foy, & Arora, 2012). So, in centralized educational system, textbooks have important role. First and only Iran national curriculum prepared and published by 2010. Therefore, in this paper Iranian mathematics curriculum reforms traced through Iranian mathematics textbooks changes. Indeed, mathematics textbooks changes show reflection of new aims, scopes and direction of mathematics curriculum.

Iran mathematics curriculum had experienced six reforms from 1900 until now. Five of them happen before launching national curriculum in Iran and one of them happen after that and upon direction of national curriculum. Indeed, Iranian national curriculum prepared and announced by ministry of education in 2010. After launching Iranian national curriculum, recent reforms in all mathematics textbooks started and gradually all of math textbooks in primary and secondary level changed upon new direction In this paper, I call this recent reform (after 2009) as contemporary mathematics curriculum reform.

Although, main focus of this paper is contemporary reform in Iran mathematics curriculum and analysis of that upon application and modelling point of view; but because of having a better understanding and getting comprehensive picture about Iranian mathematics curriculum reform, past reforms from 1900 until 2009 will be shortly reviewed.

PAST MATHEMATICS CURRICULUM REFORMS

In this section all educational changes and mathematics curriculum reforms from 1900-2009, briefly reviewed and change forces and barriers of past reform will be discussed. Some of these reforms previously distinguished and explained in national journals and magazines by other Iranian scholars in Persian (Farsi) language which is official language in Iran (e.g. Jalili, 2010, and Rezaie, 2016).

First mathematics curriculum reform started after establishing new type of school in Iran look like European school style. In 1851, first Iranian school (namely 'Dar Ul-Funun') established in Tehran (Capital city of Iran from 1788). In this school, foreign teachers employed to teach modern knowledge to Iranian students. Gradually, this school published some textbooks in different subject. These textbooks continue to use until 1938. Ministry of education try to unified all textbooks (include mathematics textbooks) by 1938. So, second mathematics curriculum reform start from 1938 and continue until 1962 and in this period of time all mathematics textbooks orchestrated and solidarity in all over the country.

In 1962, third reform in mathematics curriculum started after White Revolution (Revolution of Shah and Peoples). This curriculum change as third reform stimulated with educational system changes by 1967 and educational system divided in three section: Primary (5 years), Intermediate (3 years), and Secondary (4 years). All textbooks include mathematics textbooks changed during this reform (Rostami, 1978).

Forth reform start from 1975 and continued until 1992. In this time "New Math" introduced to Iranian school mathematics curriculum. Trace of "New Math" could be seen in all over the mathematics textbooks at that time from primary to secondary level. At that time, students were divided into two type of school (theoretical and vocation) after grade 8. In theoretical school, students were divided again into three different groups (Mathematics and Physics; Experimental Science; Human Science). As results of integrating "New Math" into the math textbooks, mathematics became meaningless for students. So, only small numbers of students (about nine percent) choose math and physics for their future study. Therefore, in future Iranian community encounter with shortage of candidate in math & science related job. As a reaction to this social phenomenon, High Commission of Fundamental Changes in Educational System decides to launch new reform.

Fifth reform start from 1992 until 2009, as a reaction to new math, and this curriculum reform influenced just secondary level (grade 9-12). This reform also stimulated with educational system changes. Educational system divided in four sections: Primary (5 years), Intermediate (3 years), Secondary (3 years), and Pre-University (1 years). As another change in this reform, school year which start from September to June divided in two parts (as semester) instead of three parts (as before). There is special attention to vocational education in secondary level during this reform. In this reform, all mathematics textbooks in grade 9-12 were changed. During this reform research finding in mathematics education domain was be used widely and in some of mathematics textbooks in this time, there are some features of constructivism and problem solving which were draw upon new finding in the field of mathematics education. Mathematics changes also considered and "discrete mathematics" added to grade 12 as a separate mathematics textbook for students in mathematics and physics branch.

CONTEMPORARY MATHEMATICS CURRICULUM CHANGES

Iran national curriculum project started from 2006 in High Commission of Fundamental Changes in Educational System (under High Commission of Cultural Revolution) and first edition of this document published in 2009. After approval of this national curriculum at 2010, Sixth and recent mathematics curriculum reforms started and continue until now. In this national curriculum document (2010) there are eleven learning domain which Iranian students have to study during their formal education. Mathematics is one of these learning domains which defined as a science of pattern, asymmetry and art, and finally precise language. In Iranian national curriculum document consider several roles for Necessity and function of mathematics as below:

- Understanding laws of the nature (anticipating and controlling different natural situation);
- Solving real world problems;
- Developing method of thinking in other natural and human science;
- Enhancing rational reasoning.

There are four content standard (Number and Operation; Algebra and Symbolic Representation; Geometry and Measurement; Data and Statistics and Probability), and seven process standard (Problem Solving; Modelling Real Data; Reasoning; Visual Thinking; Creative Thinking; Connection; Communication) in Iranian national curriculum document.

In this document there is emphasis to express role of Iranian mathematician in developing of mathematics in Golden Islamic Age (Europe Dark Age). In some of mathematics textbooks which published after approval of Iran national curriculum, there are some descriptions about works of Iranian mathematician and scientist in Golden Islamic Age which have major impact on Muslim culture and civilization. In Iran national curriculum, also, there is emphasis on use of technology (such as calculators and computers) in mathematics. But, not seen in new mathematics textbooks!

After enactment of Iran National Curriculum, contemporary mathematics curriculum reform started in Iran and two mathematics textbooks (one from primary level and another from secondary level) were changed in each school year. Now, almost all of school textbooks were changed or modified upon national curriculum.

Sixth curriculum reform also stimulated with educational system changes. Educational system divided in four sections: Junior Primary (3 years), Senior Primary (3 years); Junior Secondary (3 years), and Senior Secondary (3 years).

ANALYSIS OF CONTEMPORARY MATHEMATICS CURRICULUM CHANGES

It seems to new contemporary mathematics curriculum changes have small influences in the process of teaching and learning mathematics in Iran. Several studies show that there is still plenty of work to do. As an example, TIMSS 2015 reveal that Iranian students' performance in mathematics in both grade 4 and 8 increased but this result show that Iranian students' performance in still below the international average and still not good (Mullis, Martin, Foy, & Hooper, 2016). There is public opinion between Iranian people about Iranian students' performance in mathematics. Most Iranian think Iranian students had or must had good performance in mathematics because of Iranian students good record in mathematics Olympiad contest which usually end up in top 10. But, when first results of TIMSS published around 2000, Iranian were shocked and after that poor performance of Iranian students' became one of the researchers and policy-makers concern.

Gholam-Azad (2015) after analysis of some of new mathematics textbooks which published during recent contemporary curriculum reform mentioned some of challenges of new textbooks. In my opinion, one of the important of these challenges is instrumental understanding of recent research findings instead of rational and deep understanding of them. Gholam-Azad (2015) said instrumental (superficial) understanding of recent research findings cause to reverse outcome. To clarify of superficial understanding of research findings, I will focus on application and modeling (which is one of the mathematics processes in Iranian National Curriculum) and continue my discussion through systematic literature review of research studies which related to mathematics textbooks and investigation of teacher education program.

Modelling in Mathematics Textbooks

Modelling approach means a process which starts with a problem situated in the real world. The modelling process continues with formulating the real world problem in mathematical terms. When this process is complete, the mathematical problem can be solved by the application of mathematical concepts and solution processes. Finally, the mathematical solution must be interpreted to provide an answer to the real world problem, and checked for its adequacy in answering the original question. A new cycle of formulation may then begin for improving the model. In figure 1 a simple diagram of modelling cycle presented.

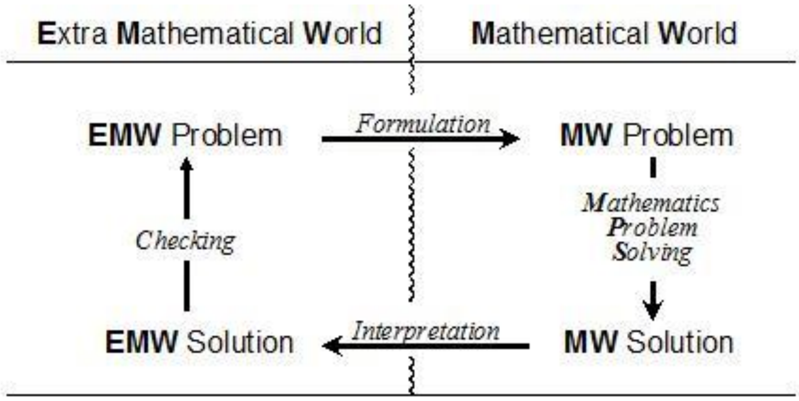

Figure 1: A model of the modelling cycle (Verschaffel, 2002)

There are some other related expressions which close to modelling but they are different. For example Standard Application problems refer to a real world problem, which the mathematical model of solution already exists in hands of students (Niss, Blum, Galbraith, 2007). Another proximity term is Word problem or Story Problems which are some sort of mathematical problems that covered in language form. Although Standard Application problems and Word Problem have some overlap with modelling but they are different.

In all of new mathematics textbooks which wrote after reenactment of national curriculum, authors of textbooks mentioned their loyalty to national curriculum in preface of each textbooks. So there are some parts in these mathematics textbooks related to application of mathematics in real world, bust almost all of them are superficial usage of mathematics in real world, not real modelling. Study of Rafiepour, Stacey & Gooya (2012) show that modeling activities are absent in new mathematics textbook in grade 9, while there are some standard application tasks in these mathematics textbooks. Research studies show that in new Iranian mathematics textbooks series, at primary and secondary level, there isn't actual modelling activity (e.g. Rafiepour, 2012; Rafiepour, 2014; Khani, Rafiepour, 2015).

Modelling in Teacher Education Program

Consequently of centralized educational system, there is a centralized system for mathematics teachers' education in Iran. Recently, new program designed for pre-service mathematics teacher education program which have mission for teacher training for all of country (ministry of science, technology and innovation, 2015). In this new program, there isn't any course about modeling in teacher education program, but there is some small emphasis on application in other courses such as problem solving. Therefore there is small support for mathematics teachers in pre-service program in Iran to implement modeling approach in their future teaching.

Now, what happened in in-service program for mathematics teachers in Iran? After textbooks changes some short time courses defined for mathematics teachers for familiarity with new changes of textbooks by ministry of education. In small number of these courses, application and modeling introduced to Iranian mathematics teachers. There is another chance for supporting Iranian math teachers regard to using modeling and application in their classroom. In master program of mathematics education, most of students are teachers of mathematics. They usually are teaching mathematics simultaneously with study in master program. So, we can consider this master program as an in-service teacher training program. In curriculum of master degree in mathematics education, there is an optional course about modeling and application. But, still not enough for supporting teachers for implementing modelling approach into their actual classroom setting.

GLOBALIZATION

In the Iranian national curriculum document (2010) explicitly does not mention globalization and its position in the national curriculum. Just a definition for globalization is expressed in appendix of national curriculum document. However, investigation of Iranian mathematics textbooks show that recent research finding in the fields of mathematics and mathematics education were be used for shaping new changes in math textbooks during the time. For example during 'Dar Ul-Funun' period of time (1851-1938) European teachers jointly with Iranian partner try to develop new textbooks for updating Iranian students with new knowledge. As another example, during Forth reform (1975-1992), "New Math" introduced to Iranian school mathematics curriculum. In more recent reforms in Iranian mathematics school curriculum, writers of textbooks try to use recent finding of mathematics education. For example in fifth Iranian mathematics textbooks reform (1992-2009), in some of math textbooks, there is emphasis on constructivism and problem solving. In contemporary and sixth Iranian mathematics textbooks reform, writers continue to emphasis on problem solving and cover some application of math through the math textbooks in all grads.

Another context for effect of international experiences on changing Iranian mathematics curriculum related to TIMSS data. Iranian students participated in TIMSS study from 1995 until now, in different grades. Iranian students' performance in mathematics was not good and in all TIMSS study (1995, 1999, 2003, 2007, 2011, and 2015) was below the international average (TIMSS, 2016). Although, in grade 4, Iranian students' performance increased from 387 in TIMSS1995 to 431 in TIMSS 2015 and in grade 8, Iranian students' performance increased from 418 in TIMSS1995 to 436 in TIMSS 2015; but this situation still is not desirable. Educational policy-maker frequently and during several conferences ask researchers and curriculum developers to reform Iranian school mathematics curriculum toward enhancing Iranian students' performance in TIMSS study. In contemporary school mathematics textbooks reform, writers try to direct change in such a way to focus on problem solving in all mathematics textbooks, to response educational policy-makers concern regard to Iranian students' performance in TIMSS.

Results of another large scale international assessment, namely the OECD PISA, influence mathematics curriculum in many countries (De Lange, 2017). Iran mathematics curriculum also was not exception. Although Iran didn't participate in PISA until now, but PISA had also some implicit effect on Iranian school mathematics (Stacey, et al. 2015). PISA introduced to Iranian community through mathematics educators research, at first. After that, several teachers of mathematics which start their master degree, research on mathematical modelling and applications, which is one of the focal points of PISA. Results of these researches published in national and international conferences and journals. Through this sharing other mathematics teachers familiar with PISA and in that way PISA affect practice of teaching and learning mathematics in Iran. As mentioned in before sections, there some sort of application of mathematics in all new version of mathematics textbooks to response passion of community.

FINAL REMARK

Review of mathematics curriculum reforms in Iran show that every 15-20 years, Iranian mathematics textbooks were changed. In all of these reforms, there is some sort of barriers which abort progressive. It seems one of the important barrier is related to teacher education program. For example, Gooya (2007) research shows that traditional mathematics teachers didn't believe to constructivism point of view and they are opposite to geometry curriculum change in direction to constructivism. In contemporary curriculum change, lack of adequate knowledge of teachers of mathematics caused to disappointed results (e.g. in TIMSS 2015 Iranian students' performance in mathematics were below the international average). Indeed, teachers of mathematics hadn't access to good resources to improve their knowledge and skills in direction with new educational reforms. They didn't received suitable content and pedagogical knowledge during their pre-service and in-service program regard to new curriculum changes. In such situation, teachers stand alone with their problems and they didn't receive suitable and adequate support. It is more and more important to support math teachers for future mathematics curriculum reform, especially in 21st century which everything changes very soon and school mathematics curriculum must reflect these new changes in new reforms. Teachers are most important and smallest loop in curriculum chain. If teachers properly supported through pre-service and in-service program, then we can wait and see good results after any educational reform.

Review of the history of education reforms in Iran reveals that there were several different reasons in different educational changes such as:

- Varying goals, perspectives and educational expectations upon social changes;
- Assessment of implemented curriculum;
- New research finding in the field of mathematics and mathematics education;
- Wide spread usage and pervasiveness of technology such as computer, Internet, smart boards, calculators and so on.

This latter reason which is contemporary concern of almost all educational system around the world has been neglected and should be given more attention in Iran educational system. Human being society become more and more technology based in 21 century and all people of this society have to have appropriated knowledge to perform their work in better manner in such society which heavily integrated with technology. Gardner (2008) mentioned about future world with search engine, robots and other computer based instrument, which need new capacity that until now have been mere options. To meet the new world demanded capacity, One of these necessary knowledge which every students have to learn is CT which Wing (2006) suggested, "To reading, writing, and arithmetic, we should add CT to every child's analytical ability" (p. 33). CT will change paradigm of many business in the future and really change job environment.

Today many countries around the world address to computational thinking (CT) in their school curriculum as part of mathematics curriculum or separately (Bocconi, Chioccariello, Dettori, Ferrari, & Engelhardt, 2016). CT defined as abstraction, algorithmic thinking, decomposition and pattern recognition (Hoyles and Noss, 2015). I would like to conclude this paper with a recommendation for policy-makers in Iran toward consider CT in future curriculum reform. At this time, CT is a neglected in Iran national curriculum and can be consider as null curriculum as terms of Eisner (2004). Although different countries have different issues and problems to each other and curriculum developer must keep in mind local issues (like environmental challenges, natural disaster and son on) but they also have to prepare their citizen for 21st century requirements. In this regard CT is necessary for Iranian Students.

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THE MATHEMATICS CURRICULUM DESIGN FROM AN INTERNATIONAL PERSPECTIVE: METHODOLOGICAL ELEMENTS FOR A COMPARATIVE ANALYSIS

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This article submits methodological elements for analysis of curriculum design from an international perspective. It furthermore presents a brief summary of the results of their application in a comparative analysis of the Mexico, Chile, South Korea and England curricula. We focus on the curricular design (intended curriculum) looking for laying the groundwork for a larger study which should take into consideration critical aspects of curriculum implementation.

INTRODUCTION

In spite of the generalized recognition of the decisive role of curricula in teaching, the research literature of mathematics education is notoriously scarce on this topic (Kaiser & Sriraman, 2014). In recent years, researchers have paid more attention to curricula and a good measure of published papers report that regardless of the fact that education systems differ in many aspects, including socio-cultural context and motivation for curriculum changes, these changes are inevitably linked to common factors of the educational process in all systems (such as, curriculum development, education policy, the schooling concept, teacher training, student learning, among others) (Li & Lappan, 2014). The existence of such common factors allows for the study of mathematics curricula to be undertaken in two ways, throughout different educational systems using a common lens; or from within a single system from an international perspective. A significant amount of the research developed on both paths in the last two decades is compiled and summarized in the book entitled Mathematics Curriculum in School Education (Li & Lappan, 2014). One of the conclusions of this review on the state of the art is that a solid research foundation is paramount for guiding design and necessary curricular changes. In order to contribute to building research pieces in this sense, this article submits methodological elements for analysis of curriculum design from an international perspective. It furthermore presents a brief summary of the results of their application in a comparative analysis of the Mexico, Chile, South Korea and England curricula.

COMPARATIVE STUDY OF THE CURRICULUM DESIGN ACROSS DIFFERENT COUNTRIES

Purpose of the Study and Reference Framework for the Analysis

The study aims to evaluate the current mathematics curricular design for compulsory education in Mexico from an international perspective [1]. A methodological and reference framework was designed in keeping with the scope of the main purpose of the study. This framework comprises elements for the intrinsic analysis of the quality of curricular design, as well as evaluation of such design from an international perspective. The reference framework is made up of four components, namely: 1) International context of mathematics education; 2) Mathematics education research and the curriculum; 3) The role assigned to mathematics in the curriculum; and 4) Distinctive traits in the quality of curriculum design (figure 1).

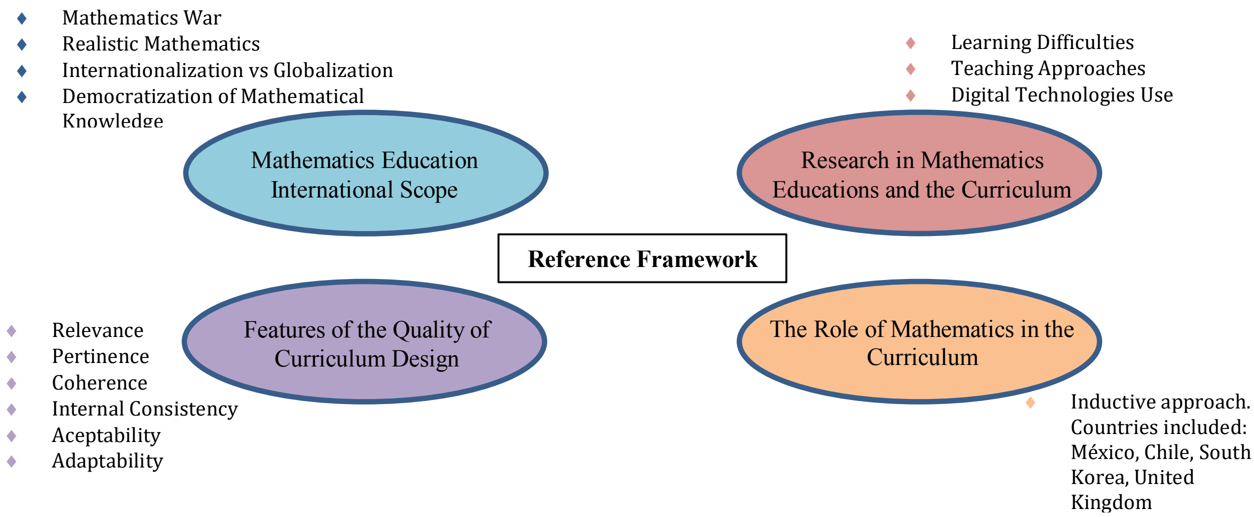

Figure 1: Reference Framework

We decided to focus on the curricular design (intended curriculum) because it gives us the basis for a larger study, which should take into consideration critical aspects of curriculum implementation such as the dynamic role of teachers' knowledge, beliefs and traditions, as well as the cultural and historical contexts of each country. However, we consider that a solid analysis of the design is a preliminary step for defining the comparison axes for such an implementation study.

Component 1. International Context of Mathematics Education

The specialized literature reports movements and events that have occurred in the field of mathematics education, and that have led to international tendencies on the conception of school mathematics. Some of these tendencies are shown in the choices that the different educational systems make of teaching contents and approaches, and they are the most important elements of this component. These events and currents are described as follows:

1) The debate known as 'the mathematics war', which juxtaposes the approach through the teaching of traditional contents and the guidelines of the reform started in the United States in the 90s, which suggest prioritizing conceptual teaching and problem solving, above algorithmic skills and symbol manipulation (Van de Walle, 2007). 2) The 'realistic mathematics' movement, which conceives school mathematics as an activity of carrying out mathematization processes, and argues that this activity should be connected to the reality of the students (Gravemeijer, K., 1990). 3) The counter position 'internationalization of mathematics education' (through education research, seeking to identify common issues) versus 'globalization of mathematics education' (triggered by applying international tests to student populations of well-differentiated socio-economic contexts) (Clarke, 2003). And 4) Focusing on the access to digital learning environments, we consider the 'democratization of mathematics knowledge through the use of technology environments, which promotes the idea that these environments open the possibility for groups of students of all ages to gain access to powerful mathematics ideas (Kaput, 1994).

Component 2. Mathematics Education Research in the Curriculum

For a good number of years, mathematics education research has focused on unraveling the difficulties intrinsic to the learning of notions that pertain to various mathematics areas, such as arithmetic, algebra, geometry, probability and differential and integral calculus. Dissemination of the results that report such difficulties led in some cases to important decisions being made regarding the curricula. For instance, recognition of difficulties related to the learning of fractions and their operations influenced the curricular decision to reduce the teaching of this content or to transfer it to higher grades (Streefland, 1991). Other examples are early introduction to algebra, to topics of mathematics of variation and to three-dimensional geometry, as well as inclusion in basic education of the study of regularities and generalization (Rojano, 2008). This component is about the influence of this sort of outcomes of mathematics education research in curriculum updates and reforms.

Component 3. The Role Assigned to Mathematics in the Curriculum

This component refers to the role (social, cognitive, cultural) assigned to teaching mathematics in the education systems of various regions which, in some cases is defined on the basis of answers to the following questions: Why teach mathematics? What about mathematics should be taught? How should mathematics be taught?

Component 4. Distinctive Traits in the Quality of Curriculum Design

The quality traits that are the elements of this component correspond to the quality criteria for curricular design that are taken into account by the National Institute for the Evaluation of Education in Mexico (INEE-DECME, 2014). These criteria specify that the curriculum must be relevant, pertinent, fair and flexible, and that it must ensure internal consistency both on each educational level and throughout compulsory education. In addition it must contain all the elements that allow readers to have an adequate understanding of the approaches that are presented within it. For the purposes of the study, a specific interpretation in the field of mathematics education of these quality criteria was undertaken.

Choice of Countries for the Study

The countries in the comparative study were selected according to several criteria. On the one hand, the possibility of making comparisons among different curriculum designs, that is to say that the designs make the essential elements that conform the curriculum explicit, in terms of mathematical content, didactic approach and the number of years of compulsory education. On the other hand, the authors sought countries where research elements in mathematics education were applied to development of their curriculum design. Lastly, another selection criterion was performance in international assessments, specifically PISA 2012 (OCDE, 2014), TIMSS 2011 and TERCE 2013 (OREAL-UNESCO, 2014). Even if there are numerous opinions pointing out that ranking and competitive testing is not necessarily a sound basis for assessing quality, we considered interesting contrasting the international examination scores of countries included in the study with the quality of their mathematics curriculum design.

In this way, the United Kingdom, Chile and South Korea were chosen for the comparison. In addition to meeting the criteria mentioned above, the Chilean curriculum was included for its social and cultural proximity to Mexico and its outstanding performance in international assessments vis-à-vis other countries of the region. In the case of the United Kingdom, for instance, the authors identified research carried out by mathematics education specialists that were specifically undertaken for analysis, follow up and improvement of curriculum design (Royal Society/JMC, 1997). Lastly, the South Korean program was included for several reasons, namely: because it offers the possibility being compared with the other designs; because it is from a country standing geographically on a different continent; because of its outstanding results in international assessments.

Delimitation of the Analysis Corpus

Given that this is an evaluation of design quality, the analysis corpus consists of the following sources: a) Official documentation detailing the current mathematics curriculum in Mexico and the other three countries, for the preschool, elementary, secondary and tertiary levels; b) Official (or authorized) supplementary documents (teaching guide, textbooks, others).

Method of Analysis

The method of analysis is linked both to the four reference framework components and to the quality criteria for mathematics curriculum design, as well as to the comparison axes (that are described below). The analysis was performed in the following two stages:

Intrinsic analysis. Operative description and application of the lines of intrinsic analysis of the Mexican curriculum: a) By school level, according to the criteria for design quality (relevance, pertinence, congruency, consistency, accessibility and flexibility) and the influence of education research. b) Longitudinal, throughout all four school levels, according to the criteria for design quality (relevance, pertinence, congruency, consistency, accessibility and flexibility) and the influence of education research.

Comparative Analysis. Operative description and application of the lines of comparative analysis: a) Transversal (Mexico and other three countries) by school level, according to the influence of education research. b) Transversal (Mexico and other three countries) and longitudinal (preschool to tertiary), according to international tendencies and the role assigned to mathematics in the curriculum.

The comparative analysis was performed for the purpose of enriching the intrinsic analysis of the first stage, for which three comparative axes were defined.

Axes for the comparative analysis

Axis 1. The sense of mathematics in the curriculum, defined pursuant to the specifications provided by the curriculum regarding the purpose of teaching mathematics, teaching contents and its organization, and the approach used in teaching mathematics.

Axis 2. Organization and teaching of the discipline contents, which is defined as a result of the inductive exercise on identifying comparison traits. This exercise was performed as per the programs of three pre-selected countries, in addition to Mexico, in terms of the following headings: presentation of the curriculum (synthetic vs. extensive presentation; flexible vs. prescriptive; static vs. dynamic), mathematics content (curricular map, structure, segmentation and organization of thematic contents), age range, duration of school cycle, teaching methodology, ICT use, and connection with other subject areas.

Axis 3. International tendencies and research in mathematics education, defined in terms of the following headings:

- Position
  - Teaching of traditional content – Priority afforded to teaching of concepts and problem solving.
  - Symbolic and abstract approach – Mathematics in context approach.
  - Global Approach – Approach that takes education research findings into account
- Explicit inclusion of elements in mathematics education research. For instance:
  - Learning difficulties in terms of the notion of number, proportional reasoning, geometric thinking, probabilistic reasoning, transitions from numerical thinking to symbolic algebraic thinking, and from algebraic thinking to mathematics of variation.
  - Early introduction is considered regarding mathematical concepts such as variation and algebraic notions.

Figure 2 shows our analytical framework for the comparative study, displaying the relationship between the four components of the reference framework and the analytical axes.

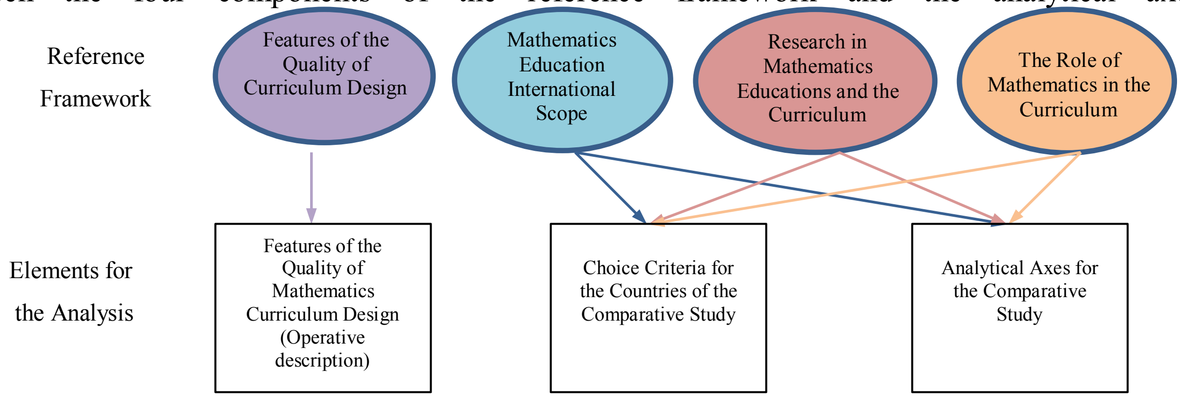

Figure 2: Analytical framework for the comparative analysis

SUMMARY OF RELEVANT FINDINGS OF THE COMPARATIVE ANALYSIS

Below is a summary of the main findings of the study, resorting to specific elements of the reference and comparison framework of the Mexican curriculum with the curriculum proposals of the other countries.

On the Relevance and Sense of Mathematics in the Curriculum

The four curricula reviewed in the study share common interests with respect to developing mathematics competency and the purposes of training individuals that are able to solve problems.

Important differences were found concerning how school mathematics is conceived in the four curricula. The South Korean curriculum stresses teaching and learning of mathematics rules, principles and concepts. The Chilean design stands out given that it emphasizes and makes explicit the instrumental nature of mathematics to access knowledge in other scientific areas. Whereas the UK curriculum highlights: the creative nature and cultural tradition of mathematics, the usefulness of its applications, and its use in every day life and work life. The Mexican design does not take an explicit position in this regard; such absence of positioning may be related to flaws of consistency in the elements that make up the Mexican curriculum, as is discussed below

On Curriculum Design Quality: its Communication and Contents

One of the aspects being assessed is the manner in which curriculum designs are presented, the amount of elements they are composed of and the level of detail with which they are described. The authors found that the Mexican and Chilean proposals are broken down in detail, meticulously specifying the content, learning and competencies that the students are to acquire in bimonthly periods. In contrast, the United Kingdom and South Korea specify compulsory content by grade or key stage. This characteristic might contribute to a flexible organization of the content throughout the school cycle. Likewise, the Mexican and Chilean proposals resort to a large amount of elements in order to present the objects of teaching: Thematic Content, Expected Learning, Abilities and Attitudes. Once again, in contrast, the proposals of the UK and South Korea have fewer elements, focusing on thematic content.

Now on the relevance of content, taking into account both the ages at which content is presented and the internal consistency of curricular designs, the authors found that throughout schooling (from elementary to tertiary education) the contents being taught coincide greatly, with some differences in terms of the times at which some of them are introduced. For instance, in Mexico probability topics are formally introduced in secondary school, while in South Korea they are studied as of elementary school. In the United Kingdom, South Korea and Chile, algebraic language is used in the sixth grade for solving equations, while in Mexico it is done in the first year of secondary school (seventh grade).

The programs of all four countries are positioned in the international tendency of prioritizing problem solving, while lessening the emphasis on teaching algorithmic and symbol manipulation skills. They nonetheless afford different roles to problem solving. For South Korea, problem solving enables development of increasingly complex solution strategies throughout each school year; the curriculum has specific indications on learning and the teaching methods for problem solving. In Chile, problem solving is a skill for which the recommendation is to favor a refining of various problem-solving strategies. For The United Kingdom, the approach also includes a focus on the ability to apply mathematics; specific suggestions are made in this design for teaching problem solving, for example, it is suggested to subdivide the problem in simple steps, fostering perseverance in the search of a solution, without rushing forward to new content. In Mexico, problem solving is both a competency to be acquired and a teaching methodology; however information is not provided as to how development of the competency is to be fostered.

Regarding the role of contexts in teaching mathematics, the Mexican curriculum emphasizes problem solving in daily life and in mathematical contexts. South Korea in contrast, has a curriculum that encourages development of arithmetic and symbol manipulation skills, as a complement to development of other skills, among which is the solving of context problems.

Globalization and Education Research in the Curriculum

The presence of “problem solving” and the emphasis on “teaching mathematics in context” places the programs of the four countries studied within the global tendency in line with the PISA international assessment program, which assesses competencies rather than knowledge.

On the influence of education research, notable in all four curricula is the lack of explicit references to research literature, as well as their inclusion of innovations in terms of content or teaching approaches (use of ICT, early algebra, for instance). However, all four programs showed signs of the influence of education research. For example, in the Mexican program there are several signs of the presence of research findings on proportional reasoning, broaching different aspects the likes of ratio, percentages, scale factor. The Chilean and United Kingdom programs incorporate pre-algebraic notions from elementary school, as they introduce variation between two quantities ‘x’ and ‘y’, which is represented in graphs and tables.

Assessment and ICT use

All four programs contain scarce presence of assessment expressions. In Mexico program assessment plays a prescriptive role at the preschool level and a marginal role at later stages. In contrast, in Chile the assessment heading is a basic component of the curricular document. In South Korea and the UK, presumably there are complementary curricular documents that provide assessment guidance. Except for the Chilean curriculum, the proposals analyzed make reference in general terms, occasionally marginally, to the suggested (not-mandatory) use of ICT. And this is in spite of the fact that education research has demonstrated the potential of digital environments for meaningful learning of mathematics.

FINAL REMARKS

The analytical framework developed from the four components has made it possible to evaluate the Mexican compulsory education curriculum from an international perspective. Results from the intrinsic and comparative analyses provide guidelines for future curriculum reforms, as well as for curriculum development and implementation. Based on the comparative analysis, it can be argued that the Mexican curriculum shares an important nucleus of mathematical content with the other countries, as well as traits of modernity, such as development of competencies, problem solving, mathematics in context and the influence of education research. In this sense, the Mexican program stands within predominant global and international tendencies. In particular terms, at the elementary school level one can observe a wide unfolding of didactic phenomenology of different arithmetic contents, which attests to the influence of international education research. However, the deficiencies found in the Mexican curricular design suggest the need for a redesign that addresses the following:

- The absence of a position regarding the conception of school mathematics: What is the purpose of teaching mathematics to every citizen?
- A redesign of contents and didactic approaches, based on foundations of the nature of school mathematics and that avoids following each and every one of the modern tendencies (trends), which leads to multiple ambiguities.
- The explicit consideration of findings of research in mathematics education. So as to create hypothetical learning trajectories for the different thematic lines of mathematics content, throughout the four school levels.
- The consideration of research findings and of experience in use of digital learning environments for inclusion of educational innovation.
- A complete redesign of the manner in which the curriculum is presented.

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Notes

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EMBEDDING ALGORITHMIC THINKING MORE CLEARLY IN THE MATHEMATICS CURRICULUM

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Mathematical reasoning takes many forms, such as algebraic, spatial and geometric, and statistical. Algorithmic thinking is one particular form of mathematical reasoning, emphasizing decomposition (breaking a complex problem down into component sub-problems and sub-tasks), pattern recognition, generalization and abstraction. With a growing global emphasis on using algorithmic thinking in coding and computing programs in schools, it is necessary and timely to examine whether algorithmic thinking should be included more explicitly in the teaching and learning of mathematics, and to what extent its inclusion can foster improved mathematical learning.

GLOBAL CONTEXT AND CHALLENGES

A capacity to solve unfamiliar problems and to reason mathematically is now an accepted goal of mathematics education in most countries where it is an expected outcome for all stages of schooling. Mathematical reasoning is defined broadly in the Australian Curriculum: Mathematics (ACARA, 2015) as a "capacity for logical thought and actions", mathematical reasoning shares common ground with problem solving, but it also relates to students' capacity to see beyond the particular to generalise and represent structural relationships. This ability is seen as a key not only to further study in mathematics but also to further studies in science, technology and engineering (Wai, Lubinski & Benbow, 2009). However, definitions of mathematical reasoning and mathematical literacy have and will continue to evolve as a result of these externalities. Nor can the purposes of school mathematics be isolated from these concurrent developments. This paper will focus on the challenges presented to school mathematics internationally by developments in coding and computing programming.

Challenges to the Mathematics Curriculum come from three directions. In the first place, significant curriculum developments are already under way to include programming and computing in the school curriculum starting in the elementary school. Secondly, there are concurrent developments in promoting coding among school students, sometimes outside formal school hours. And thirdly, in some countries there are emerging changes to the school mathematics curriculum in response to these developments.

The inclusion of programming and computing in schools has been discussed by Webb et al. (2016) who presented vignettes of five countries – the United Kingdom, New Zealand, Australia, Israel and Poland – where programming is part of the school curriculum. Documenting these developments in a comprehensive way is beyond the scope of this paper, but it is a task which the International Commission on Mathematical Instruction is urged to take on. Several instances are pertinent.

For example, in recent years, the United Kingdom (Department of Education, 2016) has introduced *Computing Programmes of Study* in Key Stages 1 to 4 to equip students to use computational thinking, including abstraction, logic, algorithms and data representation. France has a new national curriculum, *Algorithmique et programmation* (Ministere de l'Education Nationale, 2016) for all grade levels which includes algorithmic thinking and computing concepts. From 2020, the Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT, 2018) will make computer programming a mandatory subject for all primary school students. This will be expanded to include middle schools and high schools in 2021 and 2022 respectively (Bethune, 2016).

In the case of Japan, the press release announcing the formal inclusion of computer programming education into the school curriculum was made by on behalf of three Japanese government ministries, setting up a *Learning Consortium for the Future* (METI, 2017):

> The Ministry of Education, Culture, Sports, Science and Technology (MEXT), the Ministry of Internal Affairs and Communications (MIC), and the Ministry of Economy, Trade and Industry (METI) will jointly establish a *Learning Consortium for the Future* as a preparation for the implementation of computer programming education to foster programming-thinking skills and other theoretical approaches for school children, which will be introduced into schools under the next curriculum guidelines. The consortium will aim to disseminate and promote computer-programming education in schools through the development of diverse and sophisticated educational materials for computer programming, experienced-based computer-programming activities with the cooperation of companies, and other efforts in collaboration with school officials, companies or venture businesses related to the fields of education or IT, and industrial worlds.

In Australia, the national curriculum now includes *Digital Technologies* (ACARA, 2016) which all States and Territories implement. These national developments reflect an overall goal to improve the international competitiveness of each country's labor market, where those countries with the best-trained labor force can be expected to reap the rewards. Commenting on the inclusion of *Digital Technologies* in the Australian Curriculum, Clark (2016) places these developments in a wider frame:

> For many teachers, the content of this subject will present quite a challenge, as it requires them to teach algorithmic thinking from the Foundation year and to introduce coding from as early as Year 3. Previously, this has been the preserve of a small number of courses in the senior years of the curriculum, but there is a worldwide demand for greater coding skills as a part of core education; Australia is not alone in promoting this type of thinking as a part of the compulsory curriculum.

A second line of challenges come from concurrent developments, sometimes outside formal school hours, in government sponsored initiatives, such as in Malaysia with its *Coding@Schools* and a *National Code Challenge (NCC)*, in Taiwan with its *Hour of Code*, and in Singapore with extra-curricular programs to introduce young children to coding and computational thinking skills.

In both these cases, the focus may appear to be in teaching coding or programming using a student-friendly form such as *Scratch*. In this paper, coding is not the primary focus. Programming is really about solving problems and developing a logic-focused mindset. Coding should be understood as a formalized means of recording and executing algorithmic thinking. But there is an even stronger argument for thinking about what underpins coding or programming, as Clark (2016) emphasizes: there is little point in learning a programming language without a good understanding of the algorithmic thinking which sits behind any purposeful computer program.

A third and emerging result of the first two trends is to formally embed elements of Algorithmic Thinking in the school mathematics curriculum, such as in the State of Victoria, Australia (Victorian Curriculum: Mathematics, VCAA, 2017). These developments are not about teaching students how to execute standard algorithms for the four operations. The goal is to show students how to think logically as they solve mathematical problems, to think like a computer and deconstruct complex systems, breaking down a problem into smaller steps to develop (test/revise) a structured approach to solving a problem. Flexibility is a characteristic of this kind of algorithmic thinking which is the opposite of lucky guesses and/or trial-and-error strategies. Students are taught represent algorithms in everyday language, or as flow charts, or in a simple coding language. Algorithmic thinking should arise naturally in and support mathematics learning in the primary and junior secondary school years.

Faced with pressure from two directions, in both the formal school curriculum and in related extra-curricular forms discussed above, a clearer positioning of algorithmic thinking within the school mathematics curriculum is both timely and necessary. In Finland, coding and programming are now part of the curriculum, which students tackle from a young age. Finnish children are taught to think of coding and programming more as tools to be explored and utilized across multiple subjects. As Clark (2016) agues, teachers of mathematics should not think of algorithmic thinking as yet another thing which they have to teach, but rather as a pedagogical approach to problem solving in general, a skill which will be transferable across many disciplines.

In subsequent sections, it will be appropriate to give a more explicit definition of what is meant by algorithmic thinking. The next step will be to present a case for its more explicit inclusion in the school mathematics curriculum. To explain what this might mean for the teaching and learning of mathematics. Finally, I argue that ignoring these opportunities is likely to isolate school mathematics from what children are learning in other areas, and hold back needed research into the kind of teacher professional learning that will support these developments.

DEFINING ALGORITHMIC THINKING

In his classic article on Algorithmic thinking and mathematical thinking, Donald Knuth (1985) describes algorithms;

> as encompassing the whole range of concepts dealing with well-defined problems, including the structure of data that is being acted upon as well as the structure of the operations being performed; some other people think of algorithms merely as miscellaneous methods for the solution of problems, analogous to individual theorems in mathematics (p.170).

Twenty-one years later, Wing (2006) makes similar points, describing algorithmic thinking, or computational thinking, as it is sometimes called, as an approach to solving problems. It is the thinking behind coding. It is a way of thinking that helps students to look at a problem and to focus on the best ways to solve it. To do this, students must understand the various constraints in which the problem is embedded, and adopt flexible approaches to its solution.

The following section will examine in greater detail some specific sources of challenges to a contemporary mathematics curriculum.

INTERNATIONAL CURRICULUM DEVELOPMENTS IN CODING AND COMPUTING

Coding has been taught in Estonia since 2012. Australia, Belgium, England, Finland, France, Greece, Italy and Netherlands include coding in their national curricula. Luxembourg, New Zealand and Japan are in the process of introducing it. Across these countries there appear to be commonalities about what students are expected to be able to do using coding in the primary and junior secondary years – understanding what algorithms are, explaining how they work, writing algorithms using sequencing, selection, sorting and repetition, designing and applying algorithms to solve problems, debugging algorithms (detecting and correcting errors), and comparing and evaluating alternative algorithms designed to solve the same problem. Four countries – England, France, Japan and Australia – will be discussed further in this paper because their official documents are familiar to the author. A more comprehensive international treatment, while beyond the scope of this paper, is clearly needed.

The aims of courses in computing programming have moved beyond teaching students about information technology or teaching a programming language. These aims are exemplified by the *National Curriculum in England: Computing Programmes* (2016) under four headings:

The English national curriculum for computing aims to ensure that all pupils:

- can understand and apply the fundamental principles and concepts of computer science, including abstraction, logic, algorithms and data representation;
- can analyse problems in computational terms, and have repeated practical experience of writing computer programs in order to solve such problems;
- can evaluate and apply information technology, including new or unfamiliar technologies, analytically to solve problems;
- are responsible, competent, confident and creative users of information and communication technology.

The first three of these are directly relevant to this paper in illustrating how the intended curriculum is most likely to complement and impact upon what students are expected to learn in Mathematics. In Key Stages 1-4 (covering the school Years 1 to 11, Ages 5 to 16), the importance given to algorithms and algorithmic thinking is evident. In Key Stage 1 (Years 1 and 2), where pupils should be taught "to understand what algorithms are, how they are implemented as programs on digital devices, and that programs execute by following precise and unambiguous instructions". In Key Stage 2 (Years 3 to 6), pupils should be taught to "use logical reasoning to explain how simple algorithms work, and to detect and correct errors in algorithms...". In Key Stage 3 (Years 7 to 9), "understand several algorithms that reflect computational thinking [for example, ones for sorting and searching]; use logical reasoning to compare the utility alternative algorithms for the same problem. In Key Stage 4 (Years 10 and 11), "all pupils should be taught to develop and apply their analytic, problem solving, design, and computational thinking skills". As these quotations show, with their repeated emphasis on logical thinking, analytic, and problem-solving skills, there are deep links with mathematics, as well as science, technology and design. A key goal is having pupils becoming digitally literate – at a level suitable for the future workplace and as active participants in a digital world.

Regrettably, the underlying mathematical and logical skills that are needed to execute these operations are often left implicit. It is therefore important and helpful to note that four underpinning logical and mathematical skills are described explicitly in the French course *Algorithmique et Programmation* (Ministere de l'Education Nationale, 2016). These four underpinning skills are:

- *decomposition*: analyze a complicated problem, break it down into sub-problems and into sub-tasks;
- *pattern recognition*: recognize patterns, patterns, invariants, repetitions, highlight interactions;

- *generalization and abstraction*: to identify the logical sequences and to translate them into conditional instructions, to translate the recurring schemas into loops, to conceive methods related to objects that translate the expected behavior;
- *algorithm design*: write modular solutions to a given problem, re-use already programmed algorithms, program instructions triggered by events, design algorithms running in parallel.

Having stated these underpinning skills explicitly, it is impossible not to see the connections between the underpinning skills that are being developed through coding and two central goals of mathematical reasoning and problem solving in school Mathematics.

This year in Japan, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) released its *Elementary school programming education guide* (MEXT, 2018). Sponsored by three ministries, referred to above, this development is a key element of a *Learning Consortium for the Future.* The new curriculum guidelines are intended "to promote the smooth implementation of programming education in the elementary school curriculum, and to resolve any anxiety that teachers hold against programing education" (p. 10).

Although the formal term "algorithm" (アルゴリズム in Japanese) is used only once in the document (p.10), the term "programming thinking" (thinking in the way of programming) can be found many times in the document. Defined as the power to think logically, programming thinking is described the core of programming education at elementary school (p.11). Programming thinking is also connected to the ability to utilise information (p.14). Elsewhere, it is emphasised that programming thinking is not something "to be done" but rather has to be nurtured by programming efforts (p.13). Accordingly, students need to be trained in programming thinking through guidance in each subject (p.13), clearly implying that programming thinking is intended to be embedded throughout the curriculum. Through its repeated emphasis on programming thinking, the MEXT (2018) document seems to provide an emphatic endorsement of what has been discussed above as algorithmic thinking.

ALGORITHMIC THINKING IN THE AUSTRALIAN CURRICULUM

This section focusing on the primary school years and into the early secondary school will show how algorithmic thinking fits readily with many areas of mathematical content in the primary and junior secondary years. It will elaborate on two principles for a more explicit inclusion of algorithmic thinking. The first principle (*Principle 1*) is that the school Mathematics curriculum cannot be isolated from those logical and structural ways of thinking that students encounter in other areas of the curriculum. The second (*Principle 2*) is that explicit inclusion of these ways of thinking is justified whenever they can assist and advance the development of students' mathematical learning.

Algorithmic thinking is included in every stage of the Australian Curriculum: *Digital Technologies* (ACARA, 2016) as a key process and production skill. Starting in Foundation Year and up to Year 2, students "follow, describe and represent a sequence of steps and decision (algorithms) needed to solve simple problems". In Years 3 and 4, students "define simple problems and follow a sequence of steps and decisions (algorithms) needed to solve them. In Years 5 and 6, students "design, modify and follow simple algorithms represented diagrammatically and in English involving sequences of steps, branching and iteration (repetition)". In Years 7 and 8, students "design algorithms represented diagrammatically and English; and trace algorithms to predict output for a given input and to identify errors”. In Years 9 and 10, students are expected not only to design algorithms, but also to “validate algorithms and programs through tracing and test cases”.

At present, the State of Victoria is the only Australian State where algorithmic thinking is explicitly included in the school Mathematics curriculum. Clark (2016) notes that other Australian states are looking at how this implementation works in practice and how best to deliver algorithmic content.

The Victorian Curriculum: *Mathematics* (VCAA, 2017) gives specific attention to the development of algorithmic thinking from the first year at school. Algorithmic thinking is situated in the major Strand of *Number and Algebra*, in the sub-strand of *Patterns and Relations*. Table 1 sets out the content descriptors of algorithmic thinking from Foundation Year to Year 7. While descriptors from higher years are not included in this table, there are evident parallels with those descriptors relating to algorithmic thinking excerpted in the Australian Curriculum: *Digital Technologies* (ACARA, 2016). However, their inclusion in the Victorian Curriculum: *Mathematics* (VCAA, 2017) shows how and where algorithmic thinking is directly relevant to the teaching and learning of mathematics.

In the early primary years, child-friendly robotic devices such as *BeeBot* and *Sphero* introduce young students to systematic thinking to solve problems. These codable digital systems allow students to design and test solutions to simple problems using a short sequence of steps and decisions.

Table 1: Algorithms in the Victorian Curriculum: *Mathematics* – Foundation Year to Year 7

Level	Content Description	Level	Content Description
F	Follow a short sequence of instructions	4	Define a simple class of problems and solve them using an effective algorithm that involves a short sequence of steps and decisions
1	Recognize the importance of repetition in solving problems	5	Follow a mathematical algorithm involving branching and repetition (iteration)
2	Apply repetition in arithmetic operations, including multiplication as repeated addition and division as repeated subtraction	6	Design algorithms involving branching and iteration to solve specific classes of mathematical problems
3	Use a function machine and the inverse function machine as a model to apply mathematical rules to numbers or shapes	7	Design and implement mathematical algorithms using a simple general purpose programming language

In Years 2 and 3, students use algorithmic thinking to apply repetition in arithmetic operations, such as representing multiplication as repeated addition, and division as repeated subtraction.

In Years 3 and 4 students are introduced to simple function machines and the inverse machine as models to apply mathematical rules to numbers and shapes. As they progress, students can be introduced to combinations of more than one function machine. A function machine allows students to successively transform numbers from input to output, to identify rules, and understand how reverse (inverse) operations go from output back to input. Function machines also serve as important instruments to express and represent algorithmic thinking. Their use remains important in subsequent years. Algorithmic thinking is suitable for helping students to articulate rules for the ordering of multi-digit numbers from largest to smallest; and for ordering multi-place decimal numbers.

In Years 4 and 5 and in later years, students are introduced to simple classes of problems and learn to solve them using one or more effective algorithms that involve a sequence of steps and decisions. In applying algorithmic thinking to problem solving, students can be encouraged to find that problems may be solved using more than one approach. In evaluating the merits of different approaches, it is essential that students learn to record their work through tables and charts which show how the steps and decisions and their associated operations have been applied. These representations allow students to move easily to construct spreadsheets and tables to explore specific values, to identify various constraints in which the problem is embedded, and to explore alternative approaches to its solution.

In the upper primary and junior secondary years, students learn to follow a mathematical algorithm involving branching and repetition (iteration); for example, applying a suitable branching algorithm and decisions points to classify various quadrilateral shapes, and solving more complex problems.

CONCLUSION

In the foreseeable future, we can expect continued momentum to consolidate the place of coding and computing programming in the schools. There are powerful economic and political drivers promoting these developments, and their potential to impact on school mathematics cannot be ignored. With no powerful forces policing the boundaries of school mathematics, those boundaries remain porous. As Principle 1 implies, as developments in computer programming, coding and algorithmic thinking become more widespread the more likely they are to impact on evolving definitions of mathematical literacy, problem solving and reasoning. In a globalized and evolving world, it becomes more and more difficult to isolate the purposes of school mathematics from these concurrent and powerfully driven developments in coding and computing programming.

More explicit attention to algorithmic thinking in school mathematics could help students to expand their problem-solving techniques, and to explain and justify their mathematical reasoning (Principle 2). Results from large scale international assessments such as TIMSS and PISA (Thomson, De Bortoli & Underwood, 2016; Thomson, Wernert, O'Grady & Rodrigues, 2016) have consistently shown that too many students in Years 4 to 9 have difficulty in solving unfamiliar problems and explaining and justifying mathematical reasoning. This may not be surprising given that school mathematical textbooks used at these levels often focus on relatively low-level repetitious exercises that are unlikely to be conducive to the development of deep understanding or mathematical reasoning.

Four themes for discussion

The following four themes appear ripe for discussion at ICMI-24. A first theme is to chart more clearly the range of interfaces between algorithmic thinking and school mathematics as presently constituted. A second theme is to illustrate where fruitful connections can be made, building on the potential of algorithmic thinking to enhance students' mathematical understanding and ongoing development. A third theme is to identify areas of mathematical content or approaches to content which might be expected to change or, as algorithmic thinking becomes more embedded in the school curriculum. These three are intended to open up discussion for the international community and to suggest lines of ongoing investigation. Consequently, a fourth theme is to identify where further research is needed into how a more explicit embedding of algorithmic thinking in the school mathematics curriculum might improve the teaching and learning of mathematics.

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CURRICULUM CONSTRUCTION IN FRANCE: SOME REMARKS

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In this paper, I give a witness view of the evolution of the construction of the curriculum (more precisely syllabus, "programme") in France in the last twenty years, its driving forces, and its shortcomings. Frequent changes without follow-up, and the short time given in the last years, because of the political calendar, to the realization of the work, explain a certain lack of consistency in mathematics and across disciplines, and the small weight given to research findings in mathematical education. A number of new ideas have nevertheless percolated but in a rather inefficient way. A notable evolution during this period is a self-organization of the mathematical education community in a wide sense, and, more generally, the emerging of an organized science education community, partly as a response to the problems in curriculum construction; one explicit goal of this self-organization was to give weight to this community in the discussion, and to prevent reforms from being completely top-down.

INTRODUCTION

I was involved for accidental reasons in the group of people writing the mathematics curriculum of French high schools in 2000-2002; since then, I have continued to follow syllabus construction from various viewpoints, as a member of the teaching commission of SMF (*Société Mathématique de France*) from 2000 to 2011, as the president of CFEM (*Commission Française de l'Enseignement Mathématique*, corresponding member of ICMI) from 2009 to 2012, as a member of an evaluation commission for the application of program reforms from 2011 to 2014, and as a participant in various initiatives.

Others have related the story of the famous *mathématiques modernes* reform (the French version of the new maths movement) and its aftermath; I will try in this short paper to describe the sequel: the quite complicated story of curriculum construction in France in the last 20 years, its main actors, and some lessons we can draw from it.

In the rest of the introduction, I give a quick sketch of the changing framework and the main actors; then, in a first part, I give a more detailed history of the program construction. In a second part I discuss the evaluation commissions, and in a third part the self-organization of the educational community. I then draw some conclusions; the main one is that the instability and the lack of follow-up and evaluation explain in part the dissatisfaction widely expressed about the programs.

As I will show below, the construction of curricula is a very political subject, not in its content (ministers, with some exceptions, do not have ideas about trigonometry or algebra), but in its organization, which changes frequently. Since some programs (e.g. history or economy) can occasionally be the occasion of political fights, the first actor, political power, wants to keep control at all times on the construction of programs, with restrained communication with the rest of society and notably with the research community. In the last years, an accelerated political calendar has made long-time work and communication between disciplines rather difficult.

A second actor is the administration of the Ministry of Education, which tries to keep the monopoly it used to have on program construction, and to avoid the intrusion of outsiders, be they high school teachers, mathematicians or didacticians. This central administration can align or oppose political power, depending on the situation; but their joint influence dwarfs any other.

Since political parties differ on education, political changes have resulted in a discontinuous method of construction of school programs. Before 1990, the administration was in charge of the programs; between 1990 and 2005, a special Council, the CNP (*Conseil National des programmes*), was in charge of the curriculum; it was suppressed in 2005, and the administration drafted programs between 2005 and 2012, before a new Council, the CSP (*Conseil Supérieur des Programmes*) was again created in 2013 by law as an independent body to supervise program construction. One might argue that an essential element of successful reform is long-term consistency; this is made difficult by such an unstable structure.

Another important element, universally recognized, is an independent evaluation, but recent experiences prove that this evaluation is very difficult because most of the actors of the reforms are still in power positions, and independence does not fare well if it is not in line with the policy of the moment. An example is the follow-up commission (*Commission de Suivi*) created in December 2011, with an engagement letter restricting it to aspects of the reform implementation; it handed back a report in 2013 [6], after the change of political power in the 2012 elections. The report was published, ignored, and the commission did not meet again.

There is, however, another side to this history; faced with these constant top-down changes, the third actor, the educational community, has markedly organized itself in the last 20 years, first at the level of mathematics, and then at the level of sciences (mainly chemistry, computer science, mathematics, and physics).

There was first the CREM, *Commission de Réflexion sur l'Enseignement des Mathématiques*, or *Commission Kahane* from the name of its promoter and first president, Jean-Pierre Kahane. It produced influential reports on various aspects of mathematical teaching, which were published in book form [2] and are accessible online at [3]. In the following years, the CFEM (French member of ICMI) served as a meeting point of the various stakeholder in mathematical teaching, from primary school to mathematical research and research in didactics, even including parts of the administration (*Inspection Générale*); regular reunions have helped to prevent the type of "math wars" that have occurred at other moments. This change was noticed by the government, and, under the name *Stratégie mathématique*, there were in the last years several meetings between the ministry, the administration, and the mathematical community.

More recently, there have been regular meetings between mathematicians, computer scientists, and physicists. For example, a tentative program of mathematics for computers was written by a group of computer scientists and mathematicians; the resulting text showed that the gap between both viewpoints was much narrower than most people believe.

I have here only written about the program elaboration; there were evolutions of program content, with an increased emphasis on statistics, and the apparition of discrete mathematics, algorithmic and computer science. But the method of production of the programs hindered consistency and made the training of teachers in these new domains difficult.

PROGRAM CONSTRUCTION

Before 1990, the programs were the prerogative of a part of the central administration of the Ministry of Education, the *Inspection Générale*. An independent Council, the CNP (*Conseil National des Programmes*), was founded in 1990 (see [4] for more details); its role was to select groups for each discipline (the GTD, *Groupe technique disciplinaire*) to write the program. A first reform was done in 1992-1995 (since upper high school is a 3-year cycle, it takes 3 years for a reform to be fully applied).

Programs of 2000: The CNP

A new reform was decided in 1999. I was asked in 2000 to participate in the GTD in answer to a letter I wrote to the group leader, and which was very critical of a first draft. It turned out to be a rather long time project (3 years), which was completed in 2002. This calendar allowed thorough work and a number of innovations. Among others, for the first time, the programs of the various series (in the French high school system, the general route, which accounts for about 50% of students, is divided into 3 series, Economy, Literature, and Science) were each devised independently, and not as a subset of the program of the science series; in particular, the program of the economy and social sciences series contained a discrete mathematics chapter.

A very notable innovation, made possible by this long-time frame, was a common text in the programs of Biology, Physics, and Mathematics about radioactivity and the exponential function which allowed a real connection between disciplines and was quite appreciated.

The group proposed to the Ministry a follow-up commission, to allow a quick reaction to the problems which could surface during the application of the program; this was refused by the administration, possibly because it implied sharing some authority.

Programs of 2010

After a change of government in 2002, the CNP was left idle. In 2003, the administration canceled and replaced the program for the literary series. The CNP was formally disbanded by law in 2005. In the new organization, the writing of programs was left to the *Inspection Générale,* in a very informal way.

A new reform was decided in 2008, and a committee of 4 people was drafted to write a new program. The reform was however canceled by the government at the end of 2008, the committee disbanded, and the draft put away. A more modest reform was decided the following year; new programs were rapidly written, partly on the basis of the draft, and applied in 2010.

These programs were generally criticized by the mathematical education community, see [7], for their lack of general perspective, the disconnection with other disciplines, notably physics, and the insufficient preparation they gave for the entry to university.

Programs of 2016: The CSP

After the presidential election of 2012, a new law established the CSP (*Conseil Supérieur des programmes*), a slightly different version of CNP; it was given in December 2013 the task of writing new programs for primary and lower secondary schools (the first 9 years of compulsory education), with a deadline in April 2014, see [5]. This proved too short; there was a general consultation in fall

2014, a first version presented in April 2015, an intensive work of consultation with the mathematical community (teacher association, learned societies, didacticians) during the summer, and a final project published in November 2015, to be applied in fall 2016. It should be noted that, except for the last 3 levels, didacticians were this time closely associated with the construction of the syllabus.

This calendar also proved too short to allow consistency between disciplines; for example, the new program of computer science is shared between mathematics and technology, but no meeting took place between the corresponding program groups.

The application of this new curriculum made unavoidable a change in upper high school at short notice, but due to the political changes, no instruction was given to this effect to the CSP.

A new high school reform was acted at the beginning of 2018, following the work of the commission Mathiot in fall 2017 [8]; the minister sent on February 28, 2018, an engagement letter [10] asking for a report on April 15, 2018, on the principle of new programs, with a complete document expected in fall 2018 for application in 2019.

As it can be seen, the time frames have been accelerated in 20 years, from 2 years to 6 months for the realization of programs. This seems to reflect an acceleration of political time, with continuous information and social networks. It results in a concentration on immediate results, with the famous "EDL-TTU" (*Eléments De Language - Très très Urgent*), sound bites for interviews summarizing complicated questions in a few seconds; this makes difficult the discussion of long-term questions.

It is worth to note that all members of the CSP (except the president and the secretary general), as well as those of the GEPP, are working overtime, keeping a full time position elsewhere (or retired).

Content changes in the last 20 years

Some long-term trends in content can be noted in all these program changes.

The most notable are, on one side, a large decrease in the importance of algebra, geometry, and of the technical side of analysis, with much less insistence on proofs; and on the other side, a growing insistence on probability and statistics - which constitutes now more than a quarter of teaching time at high school level - the introduction of a program of discrete mathematics (graph theory) in some sections, and more recently the introduction of algorithmic and computer science.

Because the French tradition imposes a complete separation between the writing of a program and its implementation, in-service and, to a lesser degree, initial training of teachers has lagged behind; this has, in particular, led to difficulties in the teaching of statistics and probability. Teachers who lack formation in this domain tend to transform it into a series of rote manipulations, which do not prepare the students for university or professional life.

The problem is even more serious for computer science ; while everybody recognizes this as one of the crucial problems, all administrations have kicked the can down the road, refusing to create a new group of computer science teachers, but also refusing to allocate the resources needed for a real in-service training of mathematics teachers in computer science.

THE QUESTION OF EVALUATION

The problems of the French curriculum have been recognized in the last years (in particular because of PISA and TIMSS studies). But, while there is a mounting pressure for the evaluation of public policies, this turns out to be difficult in practice: these policies have been conceived and implemented by a stable group of civil servants and political personalities, who fear the political damage that could result from a critical commission. It suffices here to remark that Luc Ferry, initially a university professor in philosophy, became president of the CNP for 8 years before becoming the first minister of education of Chirac; Xavier Darcos, initially a literature teacher, later university professor, was the dean of *inspection générale* from 1995 to 1998 before becoming minister of schools under Chirac and the first minister of education of Sarkozy ; Jean-Michel Blanquer, a university professor in law, was rector of two main academies, and then director of the central administration of public schools before becoming the present minister of education of Macron: these people (and several others) have been at the top of the system for 20 years. This strong link between the administration and the political power makes the intervention of other stakeholders difficult: since its creation in 2013, two presidents of the CSP have resigned because of disagreement with the minister.

There is, of course, a strong tradition of internal evaluation of the system : this is (as the name itself suggests) one of the main goals of *inspection générale*, which regularly write very interesting reports, and one branch of the minister, the DEPP (Direction de l'Evaluation, de la Prospective et de la Performance) keeps very accurate and reliable data on the educational system, which have become more accessible in the last years. But this remains internal, and under the authority of the administration; independent and public evaluation is still a problem.

The Follow-up Commission (Commission de Suivi)

This was evidenced by the evaluation commission installed in 2011 by the ministry to answer a growing discontent on the implementation of the 2010 reform. This commission answered a demand from several associations, and its members represented a wide part of the education community. However, the engagement letter severely restricted its mission to the conditions of the implementation, and not to the content of the reform. The commission worked for 2 years and produced a report [6] in January 2014. This report was published after a change in the political power; since the commission was seen as a creation of the previous administration, its report was not even acknowledged by the minister, and the commission never met again.

Recent developments: Mission Mathiot, Mission Torossian-Villani, and reform.

There have been two evaluation commissions in fall 2017; the *mission Mathiot* who dealt with the general structure of high school, and the *mission Torossian-Villani* on the teaching of mathematics. Both commissions audited a large number of persons and delivered interesting reports.

The mission Mathiot was largely aligned with the intentions of the minister, and a number of its recommendations served as a basis for the reform which was proposed soon after the commission made its report public; its report can be found in [8]

The mission Torossian-Villani, whose report (see [9]) was widely praised, also made a number of recommendations, but at the moment, it is unclear which ones of them will be followed; some, as the proposition of mathematics laboratory (a very old proposition, already made by Borel a century ago), are very interesting and could change the teaching conditions for active teachers.

SELF ORGANIZATION OF THE MATHEMATICAL EDUCATION COMMUNITY AND BEYOND

It has been clear for some time now that the top-down method of program construction, which is a consequence of the need to control, strongly restricts any real participation of the mathematical education community, and any long-term perspective.

In reaction, this community has sought for a long time to organize itself, independently of the administration. There are the trade unions, and a teacher's association (the APMEP). More specific to France, there are the 28 IREMs (*Institut de Recherche sur l'Enseignement des Mathématiques*), created in the 70s after the "modern math" reform, which are situated inside math departments of universities and allow collaboration of secondary and higher education teachers.

This self-organization has taken new steps in the last 20 years, in an effort to articulate a common message and make itself heard.

The CREM

An important step was the CREM (*Commission de réflexion sur l'enseignement des mathématiques*), which resulted of the acceptance, in April 1999, by the minister of education of a proposition of 4 associations of mathematicians and teachers of mathematics, see [1]. While it was formally a commission created by the ministry to work with the CNP, it was largely organized by the community itself. In its 4 years of existence, it published several reports, who were edited as a book [2] and can be found online at [3]; the reports on calculus, on geometry, on computer science, and on teacher training, among others, still make worthwhile reading.

Recent developments in the mathematical community

The last 10 years, in particular because of problems with teacher training, have seen an increase in the organization of the mathematical education community at large. The CFEM (*Commission Française pour l'Enseignement des Mathématiques*), which is the French correspondent of ICMI, was instrumental here since it is a place where all associations and organizations interested in mathematical teaching meet regularly (see on the website http://www.cfem.asso.fr/ the list of the 12 organisms that take part in CFEM today). All these associations meet regularly and are often able to take a position quickly on new developments.

The ministry seems to have taken note of this new situation; after the shock created by PISA and TIMSS results, a new so-called *stratégie mathématique* was decided by the minister, and associations were regularly invited for follow-up meetings with the administration. This created opportunities to discuss frankly a number of problems about the curriculum.

The Torossian-Villani commission was another occasion for the mathematical community to be heard; several propositions of its report emerged from the consensus created by this common work.

Recent developments across disciplinary boundaries

It has been clear for a long time that fights between disciplines and lobbying strategy at the expense of others was not a way to create an efficient curriculum, and there have been several tentatives of informal federations of associations; a collective named *Action Sciences*, involving 14 associations in Biology, Chemistry, Mathematics, Physics, and Technology was active between 2003 and 2010. It organized a conference on the future of science teaching on April 5, 2008, with a number of contributions on scientific teaching at large.

Although it made a number of public interventions and press releases, it seems in retrospect quite clear that its main interest was opening new lines of communication between disciplines.

More recently, members of various associations came to the conclusion that the present disjunction between teaching of different disciplines (among them, computer science, mathematics, and physics) was harmful to high school education, and that the method of curriculum construction, with its time constraints, did not permit to change this state of affair. They decided to form a group, supported by their respective associations, to list some main goals of a general scientific education, and to coordinate curricula on different subjects.

As of August 2018, they have written principles for a general basic curriculum for all high school students, and are working on the principles of an advanced science curriculum; the idea is not to write a detailed content, but some basic and limited principles that such a curriculum should satisfy. It should, however, be noted that the principle of the new reform is to force each student to choose 2 disciplines in the last year; hence it would be impossible to choose, for example, mathematics, physics, and biology, which might make the harmonization of content between discipline meaningless. The influence this group will have on curriculum writing remains to be seen. Another interesting advance was the redaction by the computer sciences and mathematics research societies of a mathematical program oriented toward the needs of computer science, see [11].

SOME CONCLUSIONS

As I have tried to show in this short historical panorama, the curriculum construction in France is subject to turbulent conditions. This has consequences which might partly explain the poor efficiency of the system in the last years; these changing conditions imply a disconnection between disciplines, making interdisciplinary cooperation more difficult (at a time when interdisciplinary teaching is strongly promoted with various innovations and a reduction of hours attributed to fundamental teaching). They also seem to prevent a number of teachers from appropriating the curriculum, seen as imposed from the top, without consideration for the teacher, and in many cases, without sufficient formation.

The problem of initial education and in-service formation of teachers has been recognized for a long time, and several reports have insisted on this question, which has also been raised at all meetings between the administration and the various elements of the community. There have been many grassroots efforts to build in-service formation, in particular in the domain of probability and statistics, and more recently in all the aspects of computer science (the IREMs have been very active in this domain, proposing a number of innovative formations); but the lack of funding and of persistence has severely restricted the impact of these formations, which should be part of a long-term effort.

It would be desirable to create a more stable process, involving teachers and researchers, to test a curriculum before generalizing it (as was done for a reform of lower high school in 1985), and to disconnect this process from political alternations; but this does not seem to be realizable in the near future.

An alternative possibility would be a collaborative work of the education community to propose elements and principles for a science curriculum, which could be useful for building sound programs in the short time left by the present method; this is what the interdisciplinary group cited above is trying to initiate.

Since an independent self-evaluation of the educational system seems to meet serious obstacles at this moment, this evaluation should be initiated by the education community itself, and there have been some tentative in this direction.

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CURRICULUM DOCUMENTATION AND THE DEVELOPMENT OF EFFECTIVE SEQUENCES OF LEARNING EXPERIENCES

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In this article we focus on ways that the documented curriculum can inform the construction and implementation of planned sequences of experiences to support mathematics learning. We report on the early stages of a research project which is examining ways that thoughtfully created, cumulative, challenging and connected experiences can both initiate and consolidate mathematics learning. It is intended that through an iterative cycle of design-test-redesign-retest we will ultimately transform the documented curriculum into a set of refined and empirically developed sequences of learning experiences that are accessible by a diverse range of students.

THE RATIONALE FOR LEARNING SEQUENCES

The focus of this article is on ways that the documented curriculum might be transformed into planned sequences of learning that can inform teaching programs. The article reports on the early stages of a research project which is examining ways that thoughtfully created, cumulative, challenging and connected experiences can both initiate and consolidate mathematics learning.

The prompt for the project was earlier research by Sullivan, Borcek, Walker, and Rennie (2016), which found that cognitive activation is more likely when learning experiences are structured in particular ways, including:

- the ways tasks are posed in the introductory phase;
- actions taken to differentiate the task for students who might require additional support and those who finish quickly; and
- ways that the student activity on the task is reviewed emphasizing students reporting on their explorations and fostering classroom dialogue between students.

Sullivan et al. hypothesised that learning would be further enhanced if purposeful follow up experiences were posed to consolidate the learning. The nature and effectiveness of those follow up learning experiences is the focus of this new project. This process for consolidating learning is connected to considering sequences or trajectories of learning over a longer time frame than the single task and single lesson.

The notion of sequences of learning is informed by Variation Theory which was described by Kullberg, Runesson, and Mårtensson (2013) as follows:

> In order to understand or see a phenomenon or a situation in a particular way one must discern all the critical aspects of the object in question simultaneously. Since an aspect is noticeable only if it varies against a back-ground in invariance (emphasis in original), the experience of variation is a necessary condition for learning something in a specific way. (p. 611)

Similarly, Sinitsky and Ilany (2016) argued that considering both change and invariance illustrates not only the nature of the mathematics but also the process of constructing concepts. In the application of Variation Theory to the creation of sequences intended to consolidate learning prompted by an initial task, the intent is that some elements of the original experience remain invariant, and other aspects vary so that learners can focus on the concepts and not be misled by over-generalisation from a solution to a single example.

The hypothesised advantages of learning sequences are as follows:

Sequences can help students see the 'bigger picture'. One of the disadvantages of conventional approaches to mathematics and numeracy is that mathematics can seem to be broken into sets of micro skills rather than contributing to a coherent whole. Sequences may help students see connections by making the big ideas and progression of learning more obvious to the student.

Concepts are learned as much by what they are not, as from what they are (such as, for example, the attribute of length is different from volume). Carefully varied tasks within sequences can emphasise what the central ideas are (and what they are not) so allowing students to discern the essence of concepts.

Sequences of challenging tasks can prompt "light bulb" moments. But there are no light bulbs if students are told what to do. Students can benefit from working on tasks that are challenging, and progressively see meaning by experiencing connected tasks with success developing progressively especially where the insights or "aha" moments are the result of their own thinking.

Sequences can reduce the sense of risk experienced by some students. Many teachers report that some students do not embrace challenges possibly fearing the risk of failure. One of the goals of the sequences is for students to see that, even if they cannot do the current task, there is a similar task coming and they can learn how to do subsequent tasks by engagement in the current task, even if not successful yet.

The focus of this article is on ways that the documented curriculum can inform the construction and implementation of learning sequences.

Some characteristics of sequences

Part of the research focusses on validating the structure and principles that inform the design of the sequences. At this stage, our focus is on the first three years of formal school. The sequences are proposed to:

- represent one to two weeks of classroom mathematics lessons;
- facilitate movement from concrete to pictorial to symbolic/mental images;
- be challenging for students in that they will not initially know how to solve the problems;
- not only address important mathematics concepts and language as identified in the curriculum but also reflect the ways that young learners approach that mathematics;
- be applicable for Years F to 2 (although teachers of Foundation classes might spend more time on the initial suggestions and teachers of Year 2 class would extend the latter suggestions);

- allow students time to make choices on the type of answer and/or approaches to solution;
- be explicitly differentiable through a "low floor high ceiling" nature or enabling and extending prompts; and
- be structured similarly, especially identifying relevant curriculum focus and learning goals, presenting sequenced task suggestions, and assessment rubrics specific to the sequence.

In our project, the iterative cycle of design-test-redesign-retest will ultimately lead to a set of refined and empirically developed sequences of learning. While the sequences are intended for early years students (F to 2), the approach is applicable at all levels.

A SEQUENCE IN LENGTH

To illustrate the ways that the curriculum might inform a sequence, the following uses the example of a sequence of learning experiences focusing on the learning of length concepts. The section first presents a discussion of some earlier research on the development of length concepts then examines how length concepts are presented in the Australian Curriculum: Mathematics. The section also presents examples from a draft sequence.

A perspective on the learning of length

In describing the development of length concepts in the junior and middle primary levels, McDonough and Sullivan (2011, p. 34) wrote:

> Teachers of children in the first year of school can reasonably aim that nearly all students are able to compare the length of two objects, to order a third object even if not necessarily directly comparing it to the others, and begin to move towards quantifying lengths. It is relevant to note that … structured activities that provide experiences in (iteration) of length learning are important. For example, asking students to compare the lengths of two objects that cannot be placed next to each other.
>
> Teachers of children in the second year of school could emphasise activities that facilitate the movement of all students toward using informal units iteratively to quantify lengths, both using a single unit repeatedly and using multiple versions of the one unit. It is worth noting that approximately two thirds of the students at this level are either at or will become ready for using standard units during the year.
>
> Teachers of children in the third year of school should expect most children to be moving towards using standard units such as cm. Again it is noted that many students are ready for more sophisticated tasks involving measuring length.

The stages described above give a strong indication of the ways that these concepts develop. In summary, it seems that direct and indirect comparisons are followed by experiences in which a unit is used iteratively, which then lead into opportunities to use formal measurement units.

It is worth noting that McDonough and Sullivan found from an analysis of large data sets that students' responses to measurement items and their responses to number items were quite different and that facility with one did not imply facility with the other. This finding emphasizes another aspect of the sequences proposed in our current study in that they are designed for mixed achievement, whole class teaching.

Length in the Australian Curriculum

Of course, it is not expected that teachers will read research articles on the many topics that make up the mathematics curriculum. Therefore, the curriculum needs to communicate key ideas hopefully clearly and succinctly. The references to Length in the Australian Curriculum are as follows. Note that Foundation students are commonly aged 5.

Foundation Year

Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language (ACMMG006)

Year 1

Measure and compare the lengths and capacities of pairs of objects using uniform informal units (ACMMG019)

Year 2

Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units (ACMMG037)

Year 3

Measure, order and compare objects using familiar metric units of length, mass and capacity (ACMMG061)

Year 4

Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084)

Compare objects using familiar metric units of area and volume (ACMMG290)

Year 5

Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108)

Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109)

In other words, the same developmental sequence of key length concepts identified by McDonough and Sullivan (2011) are evident and appear in the Australian Curriculum. The descriptions of the content are succinct but the key terms – direct comparison, indirect comparisons, uniform informal units, familiar metric units, etc – are prominent.

A notional sequence

The following are the headings of five suggestions in a sequence that is intended to guide teachers in the first years of school. The sequence extract gives the title of each suggestion along with a possible learning focus to indicate the intention of matching experiences.

Suggestion 1: Direct comparisons

Learning focus: It is possible to compare lengths by putting one object against another

Suggestion 2: Indirect comparisons with lines

Learning focus: It is possible to compare one object against another using a third object

Suggestion 3: Indirect comparisons with different shapes

Learning focus: It is possible to compare one dimension against another using a third object

Suggestion 4: Using informal units iteratively

Learning focus: It is possible to compare lengths by using an object over and over again

Suggestion 5: Using informal units to compare different objects

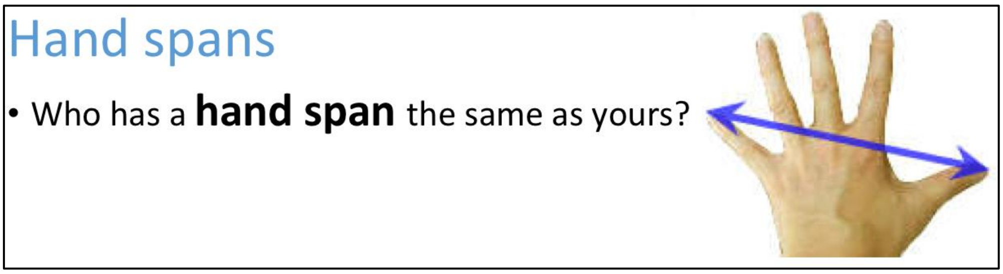
Learning focus: When comparing different informal units, the unit in each case must be constant

All of the experiences accompanying the respective suggestions are challenging for students. This allows Foundation students to engage with the tasks but also means that Year 2 students are required to think even in the initial suggestion. Our notion of 'challenging' experiences incorporates characteristics that require students to make connections between different aspect of mathematics, to devise solution strategies for themselves, to explore more than one solution pathway and to explain their strategies and justify their thinking.

We are currently working with early years teachers to test and refine the individual experiences, the sequences in which they are presented and accompanying support documentation.

A suggested sequence

The details of three connected learning experiences from the suggested length sequence intended for Foundation students moving from *Suggestion 1: Direct Comparison* to *Suggestion 2: Indirect comparisons with lines* are presented in this section. This sequence is representative of the type of connected experiences and accompanying support information provided to teachers. The first suggested experience is as follows:



A suggested connected experience following this, which still involves direct comparison, is:

> Find something that is longer than your hand span, but shorter than your foot.

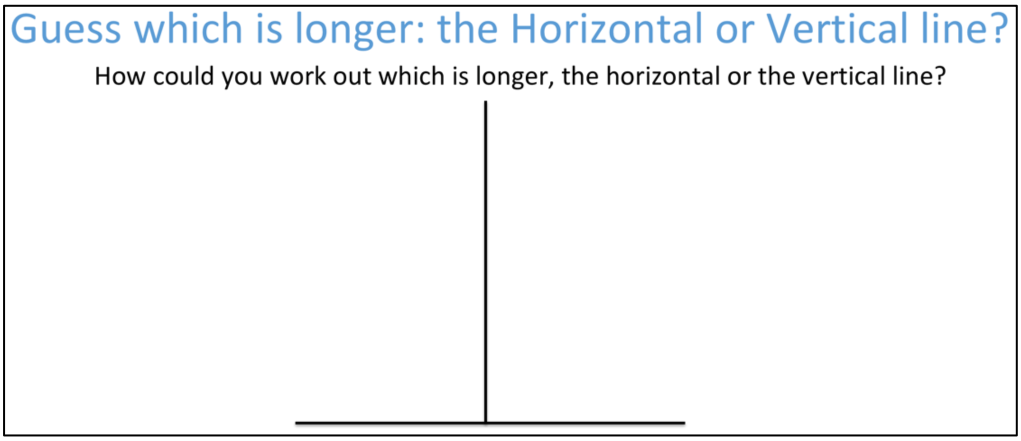
In the accompanying documentation for teachers, it is emphasized that key terms such as long, longer, longest, short, shorter, shortest, height, width are appropriate for use with students. A rationale for the suggested sequence and pedagogical considerations are also provided to teachers, including:

> The intention is to develop an intuitive sense of length so prompt students to estimate before measuring;
>
> The intention is to consolidate the concept before moving to formal units;
>
> All tasks allow students opportunity to explain their thinking; and
>
> The tasks are only illustrative – choose your own contexts.

Key ideas for measuring length described in the Australian Curriculum and by McDonough and Sullivan (2011) are evident in the sequence of experiences appropriate for early years students. The initial suggested experience introduces students to the concept of length using a familiar experience involving their own hands. The key ideas of 'longer', 'shorter' or the 'same' length are openly explored by direct comparison of just two objects. Students determine their own approach for finding someone with the same hand span as their own.

The connected consolidation task is intended to extend the learning by involving a third object ("your foot") and has an additional element that explicitly requires students to simultaneously consider "longer than" and "short than". This task is considered challenging because it allows students opportunities to devise their own strategies for directly comparing three objects that are not easily placed side-by-side to enable direct comparison. While multiple solutions are available, the range of possibilities is also constrained.

A third connected experience involving *Suggestion 2: Indirect comparisons with lines* is:



For indirect comparison experiences, teachers are advised to consider:

A third tool (string, streamer) is needed to compare the lengths; and

Some tasks involving the ordering of more than one length.

While this learning experience has a single correct answer, the intention is that the students will devise their own indirect comparison strategies involving a third object or tool. The task is considered challenging because multiple strategies are possible, the answer is not obvious, and students are required to explain their reasoning for their response. In terms of developing key concepts of length described by McDonough and Sullivan (2011), the task could involve unit iteration (e.g., measuring the length of each line with a paper clip rather than a single streamer), the length of two lines are compared when they cannot be placed next to each other and, therefore, requiring indirect comparison using a third object as a tool.

TEACHERS' RESPONSES TO SEQUENCES

The following data were from early years teachers responding to an online survey during a professional learning day to introduce them to the notion of sequences as a guide to planning. The teachers were asked to rate each statement on a 5-point Likert scale (strongly disagree to strongly agree). Table 1 presents the responses of 96 teachers.

As can be seen, around half of the teachers strongly agreed with the statements, with most of the rest agreeing. Most importantly, the majority of teachers indicate that they will implement the sequence as it was presented to them and feel confident teaching the challenging tasks to their students. Given that the length sequence is long and the time for discussion was brief, and that teachers came from schools of very diverse socio-economic backgrounds this is strong endorsement that the sequence suggestions would be welcomed by many teachers.

Statement	Strongly disagree (1)	2	3	4	Strongly agree (5)
The way the sequence builds is clear to me	0	0	7	41	52
The sequence is easy to follow and logical	0	0	6	40	54
I plan to use this sequence more or less as it is written	0	0	11	43	46
The tasks look interesting	0	1	6	25	67
I feel confident teaching this sequence of tasks	0	0	2	45	47

Table 1: Responses (%) of early years teachers to prompts about the length sequence (n = 96)

We also asked teachers to indicate which year level the sequence would be suited for. Table 2 presents their responses. Note that teachers could indicate more than one level.

Year level	Number of teachers (n = 96)
Foundation	69
1	67
2	73
3	42
4	32

Table 2: Numbers of teachers indicating particular levels at which the sequence is applicable

The majority of the teachers saw the suggestions as suitable for students aged 5 years old, there was also one third who felt the suggestions were applicable to students in their fifth year of school. Given that the experiences were explicitly designed with a "low floor high ceiling" nature in mind, it is affirming that the suggested sequences will be suitable for mixed achievement whole class teaching.

The teachers are currently teaching the sequence to their students and will complete a similar survey after they have taught the entire sequence.

CURRICULUM-INFORMED EMPIRICALLY DEVELOPED LEARNING SEQUENCES

In this article, we have presented an approach to curriculum development that is informed by the Australian Curriculum: Mathematics. The documented curriculum influences student learning through the sequences of experiences teachers plan and implement as part of their instructional program. It therefore makes sense that the potential of such sequences to improve student learning are empirically tested.

Guided by Variation Theory our research aim is to develop cumulative, challenging and connected sequences of experiences that can both initiate and consolidate students' learning of mathematics. Thoughtfully designed sequences of learning experiences are more likely to assist students see connections between mathematics concepts by making the big ideas and progression of learning more obvious. Carefully varied tasks within sequences can emphasise the big ideas to students, allowing them opportunities to distil the essence of concepts over time.

It is important to note that the suggested sequences do not disempower teachers from making professional judgements regarding appropriate experiences for their students. Teachers must still structure their lessons, orchestrate whole class strategy discussions, make 'in-the-moment' decisions about providing appropriate enabling and extending prompts, and assess student thinking to determine 'where to next'. Each suggested sequence of learning matches the development of concepts reflected in the Australian Curriculum, so teachers are free to adapt the experiences to reflect their contexts.

The sequences of learning experiences we are examining hold great promise for building student understanding of important mathematics concepts. It is the aim of our research to explore this potential.

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THE MATHEMATICAL DIMENSION OF THE CURRICULUM REFORM IN PERU

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The present study, in the first place, reveals important characteristics of the National Curriculum for Basic Education development process, in force since June 2016, which includes the learning standards for mathematics. Subsequently, a comparative table presenting the differences between the curricular designs is elaborated, in order to carry out an analysis and understanding the changes. Finally, this paper develops the National Quality Assurance System for Accreditation, Evaluation and Certification proposal for validating the learning standards for mathematics using authentic tasks. In conclusion, it can be said that National Curriculum for Basic Education of 2016 represents progress in the Educational System, but it is necessary to make changes in the proposal.

THE CURRICULUM REFORM IN PERU

The Peruvian Educational System is regulated by the General Education Act No. 28044 of 2003. The Basic Education is the first stage of the Educational System in which there are three modalities: Special Basic Education (SBE), Regular Basic Education (RBE) and Alternative Basic Education (ABE). RBE is formed by three levels, seven cycles and thirteen degrees (Table 1).

EDUCACIÓN BÁSICA REGULAR													
NIVELES	Inicial		Primaria						Secundaria				
CICLOS	I	II	III		IV		V		VI		VII		
GRADOS	años 0-2	años 3-5	1º	2º	3º	4º	5º	6º	1º	2º	3º	4º	5º

Table 1. Levels, Cycles and Degrees of RBE (Ministerio de Educación, 2017)

Basic Education and the National Curriculum

General Education Act No. 28044 states that the National Curriculum for Basic Education (NCBE) is a basic document of the National Pedagogical Policy. The NCBE "is the result of a series of national public hearings that took place between 2010 and 2016 and involved different stakeholders, such as civil society, teachers, education specialists and national and international experts in curriculum" (Opertti, Kang & Magni, 2018, p. 41). In the NCBE, the learning standards (LS) "are the references of how far or close a student is in relation to the achievement of competencies for a given grade", and "are argued to be the references to evaluate learning both at classroom and system level" (Opertti et al., 2018, p. 38). The NCBE (Ministerio de Educación, 2017) is in force in Peru since June 2016 and is the result of a process began in the mid-1990s. But the Curriculum Reform cannot be understood in isolation, it has to be seen in the context of global educational policies. In that sense, at the end of the last century similar reform movements were promoted in many countries around the world by international financial organizations, in compliance with international agreements on quality and equality in education like Jomtien (1990), Dakar and the

Millennium Summit (2000) (Rosales, 2010). Some researchers point out that reforms fostering neoliberal social project, promoting the cult of individuality and competition, changing relationship between education and citizenship and the problem of democratic participation of the majority, by the competent knowledge and concertation (Soares cited by Rosales, 2010).

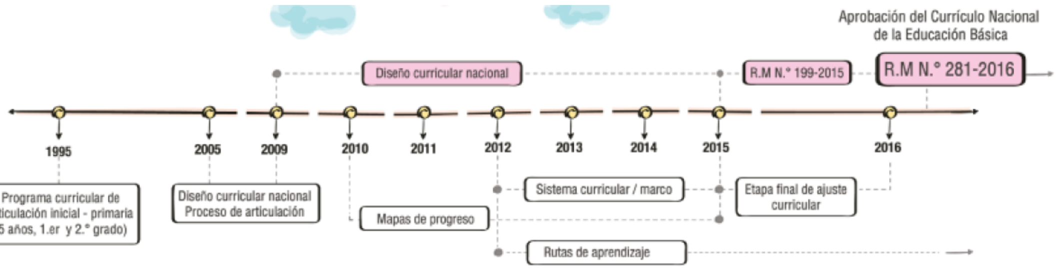


Figure 1. NCBE development process (Ministerio de Educación de Perú, 2018)

In the Curriculum Reform, the NCBE development process was progressive (Figure 1). The following are key moments of development process:

- 1995, Initial Education with Primary Education Articulation Program (five years, 1st and 2nd Grade). 1997, Basic Curricular structure of Primary Education, first cycle (lst and 2nd grade). 1998, second cycle (3rd and 4th grade). 1999, third cycle (5th and 6th grade).
- 2005, the "National Curricular Design for Regular Basic Education. Articulation Process" was published (Ministerio de Educación, 2005). Initial Education, Primary Education (PE) and Secondary Education (SE). Beginning of the Articulation Process.
- 2006, the National Education Council published National Education Project to 2021, which established as one of its national policy priority the preparation of learning standards.
- 2009, the National Curriculum Design for RBE (NCD) (Ministerio de Educación, 2009) was published. All cycles, levels and grades have the same organization model. End of Articulation Process.
- 2010, the National Quality Assurance System for Accreditation, Evaluation and Certification – SINEACE – initiated the learning standards (LS) development process, under the modality of progress maps (PM). The process ended on December of 2015 (SINEACE, 2015). They used authentic tasks in the validation process.
- 2016, the NCBE (Ministerio de Educación, 2017) was published as the basis for elaboration of curriculum program and tools of SBE, RBE and ABE.

ANALYSIS OF THE NCB DEVELOPMENT PROCESS

In the process, three different curricula have been proposed in 2005, 2009 and 2016. Table 2 shows most important characteristics of the three curricular designs: competence definition, formulation of mathematical competences, capability definition, elements of curricular design, content categories and mathematical capabilities. The constructivism and problem solving are disciplinary and pedagogical approaches of the NCD of 2009 (UNESCO, 2013). The curriculum of the Peruvian Educational System has stopped being focused on objectives learning. The NCBE adopts a constructivist educational paradigm and pedagogical approach of learning by competencies (Tapia & Cueto, 2017).

	NCD of REB. Articulation 2005	NCD 2009	NCBE 2016
Modalities	**RBE**	**RBE**	**RBE-ABE-SBE**
Competence definition	Competencies are learning achievements obtained when certain capabilities, knowledge, abilities, skills and attitudes are put into play when an activity or task is carried out.	Competencies are achieved throughout a continual process by developing duly articulated capacities, knowledge, attitudes and values. All of them favor the student know-how.	Faculty having a person to combine a set of capabilities to achieve a specific purpose in a given situation, acting pertinently and with ethical sense.
Formulation of mathematical competencies	Each level has a different organization. In Primary Education to write a competence begin with: "The student solves problems ...". Each competence requires contents, processes and attitudes. In Secondary Education competencies are not formulated.	Competencies are formulated in each cycle. To write a competence begins with: "The student solves problems...". Competencies mention contents, processes and attitudes.	Four competences: The student solves problems of: 1) quantity 2) regularity, equivalence and change 3) shape, movement and localization 4) data management and uncertainty
Capability definition	Capabilities are learning achievements. Set of mental capabilities and motor skills that are evaluated with observable behavioral indicators.	There is no definition of capability.	Capabilities are resources in order to act competently. Resources are knowledge, skills and attitudes that students use to act in a particular situation.
Elements of curricular design	Learning achievements: Competencies Capabilities Knowledge Attitudes	Competencies Capabilities Knowledge Attitudes	Curricular approach by competencies Competencies Capabilities Learning standards
Content categories	Components: Number, relationships and functions. Geometry and measurement. Statistic and probability.	Organizers: (SE) Numbers, relationships and functions Geometry and measurement Statistic and probability	Content categories are not indicated. The contents are writing within the competencies and are specified in the levels of development.
Mathematical capabilities	Capabilities: Reasoning and demonstration Mathematical communication Problem solving	Transversal processes: Reasoning and demonstration Mathematical communication Problem solving	The student: -translates -communicates -uses strategies -argues affirmations -represents data -sustains conclusions -models

Table 2. Comparative table on curricular designs (2005 – 2016). Source: Own elaboration.

Analyzing the comparative table it can be seen that competence definition in the three curricular designs does not have significant differences. In the first, the competence is considered as learning, in the second, it is student know-how and, in the third, it is a faculty. The three definitions of competence take into account achievements, capabilities and attitudes. In the three curricular designs, mathematical competences begin with: "The student solves problems of ...". Subsequently, specific mathematical content is indicated. To discuss about the formulation of mathematical competencies proposed in NCBE 2016, is it necessary to remember what problem solving means to renowned authors.

George Polya, a hungarian mathematician, described methods to solve problems. He defines problem solving as finding "a way where no way is known, off-hand… out of a difficulty…around an obstacle" (Polya, 1949/1980, p. 1, quoted by Laterell, n.d.). Similarly, according to the National Council of Teachers of Mathematics (NCTM, 2000, p. 52) "problem solving means engaging in a task for which the solution method is not known in advance". On the other hand, the Programme for International Student Assessment (PISA) 2012 (OECD, 2014, p. 30) defines problem-solving competence as: "…an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious." Finally, in the NCBE 2016 (Ministerio de Educación de Perú, 2017, p. 36) with respect to competence, it is said:

> Being competent means understanding the situation that must be faced and evaluating the possibilities to solve it. This relates to the ability to identify and implement the knowledge and skills that one possesses or that are available in the environment, analyzing the combinations most pertinent to the situation and the purpose, and then making decisions; and execute or put into action the selected combination.

Considering the above mentioned definitions, in problem solving the way is not known and method is not obvious. As the aforementioned paragraph says, in NCBE 2016 competence is related to the ability to identify the knowledge and skills, making decisions and put into action the selected combination. It means that if the student has the ability to identified knowledge and skills, it is because they are not known. However, mathematical competencies described in the NCBE 2016 (Table 2) indicate a content category in each one of them, addressing the solution path. Besides, if the problems are real and complex, knowledge of different areas may be needed.

The NCBE of 2016 (Ministerio de Educación de Perú, 2017, p. 143) explains the competence 'The student solves problems of quantity' as follows: "It consists of the student solving problems or raising new problems that require him to build and understand the notions of quantity, number, number systems, their operations and properties." It can be seen clearly, in this paragraph, that the notion of problem solving is being handled from a traditional perspective. In traditional Math-Method of instruction, the teacher presents a mathematical concept, reviews the procedures required to find the solution, and then the students practice these procedures solving additional problems (Chapko & Buchko 2004, p. 9, quoted by Ferguson 2010). It restricts the path that the student can follow to solve the problem, generally oriented to use what the teacher taught, when it is known that there is more than one way to get the correct answer.

Therefore, it can be affirmed that the formulation of mathematical competences of NCBE 2016 is not coherent with the competence definition (Table 2) and the aforementioned paragraph in the page four. A competence does not consist of using certain specific contents and skills to solve a specific type of problem. In the Peruvians curricular designs, the influence of the PISA Program is notorious. The PISA 2015 fundamental mathematical capabilities have been a reference to the construction of the NCBE 2016 mathematical capabilities. PISA 2015 points out that in mathematical literacy are present the mathematical processes (formulate, use, interpret, evaluate) and the underlying mathematical capabilities. However, those mathematical processes have not been considered in the NCBE 2016.

PISA 2015 (OECD, 2017) defines mathematical literacy as follows:

> Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens.

Regarding the challenges, one problem in the Peruvian curricular design is that the formulation of mathematical competencies gives a lot of importance to contents. Implicitly, there are lags of an academicist curricular approach (Roman & Diez 2003, quoted by Lamas, Manrique & Revilla 2014) in the NCBE 2016, which are very difficult to overcome. The PISA 2015 definition of mathematical literacy addresses deeper aspects, contents are on another dimension and are transversely crossed by the mathematical processes. Paradoxically, the NCBE does not indicate contents. The NCBE considers the contents within formulation of competencies and the development levels pointed out in the LS.

Other authors (Tapia & Cueto, 2017) propose that the achievements of the NCBE are summarized in: a) it gives a greater emphasis on learning, it does not focus on prescribing what should be taught, but describes what students should learn; b) it improves competence definition and what it means to work for competencies in classroom; c) it emphasizes that learning is progressive and continuous.

AUTHENTIC TASKS TO VALIDATE LEARNING STANDARDS

Since 2010, SINEACE was preparing the Mathematical LS, in the PM modality, in order to create a progressive and continuous learning that would be incorporated into the NCBE. The process was advised by international experts and the experience developed in many countries was investigated. When the PMs were ended, it was necessary to initiate a validation process. With that purpose, the instruments were developed to show that students achieved the learning expectations described in each performance level of mathematical PMs. The instruments contain 28 authentic tasks (7 for each PM), which were validated by specialists in area and applied in a pilot phase. Subsequently, tasks were readjusted to finally be applied, in its final version, to students of different cycles of RBE.

In the validation process, the instruments were developed and applied to 2776 students of all grades of BRE. Information collected in fieldwork was recorded by written and audiovisual means. To investigate what students have learned in each mathematical competence, the answers were evaluated using analytical rubrics prepared by a team of specialists, who analyzed and discussed level of performance achieved by each student. Finally, they were selected examples of student responses that would be published as evidence of what a student can achieve at each level of the PM of mathematics.

In December 2015, SINEACE concluded the PMs, in the first half of 2016 it is delivered to the ministry and published. However, the proposal presented by SINEACE is not considered in its entirety by authorities of Ministry of Education, since other competencies and LS were officially presented.

Authentic tasks are similar situations to out-of-school life and have an explicit purpose for students (Atorresi & Ravela, 2009), to ask them to elaborate a specific product. They have been designed considering following elements: context, slogan, complexity and product (Isidro, Ordoñez & Paz, 2017). Authentic tasks allowed students to put into play various knowledge and capabilities to solve a given task, obtaining responses from students of different performance levels of mathematical competences.

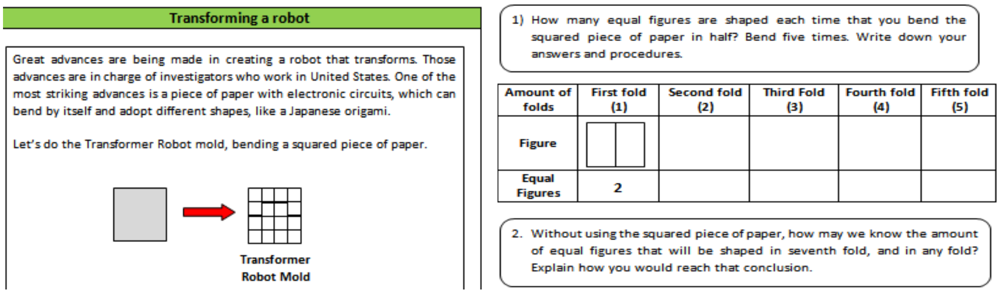
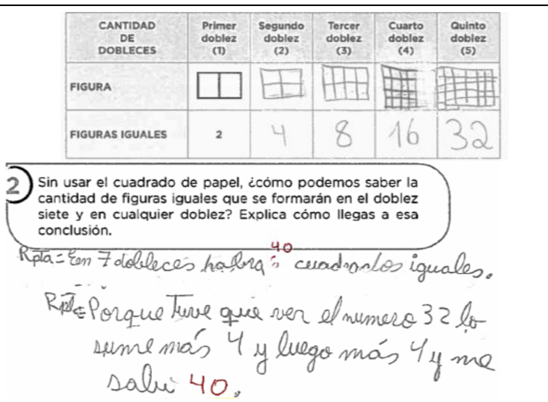
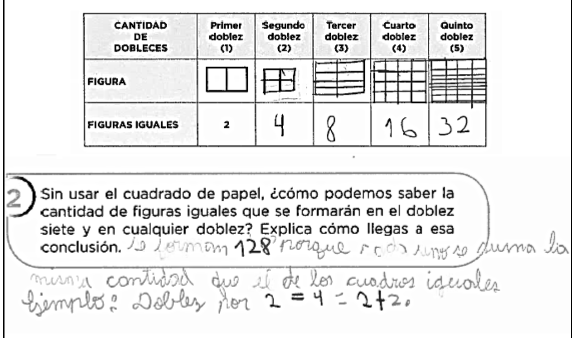
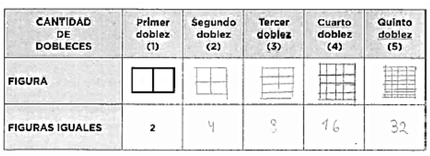


Figure 2: Sample of regularity situation task - Cycle IV. Source: (Isidro et al., 2016)

Level	Evidence	Description
1		The student writes the answer in the table, but fails to associate the number pattern with the operation of multiplying by 2, instead he/she states that it must be added 4. He/she produces a wrong answer because uses an additive and not multiplicative pattern. He/she explains his/her answer using some mathematical terms. He has learnings below IV cycle.
2		The student correctly registers the data in the table, but identifies an additive pattern. He/she argues that term of sequence is formed by adding the same amount and tests it for one of the terms of the sequence. Uses imprecisely mathematical language. Student shows some learnings IV cycle, but has not consolidated others yet.
3		The student relates the data registered in the table and expresses this relationship with a multiplicative pattern "x 2". To determine the seventh term, he/she doubles

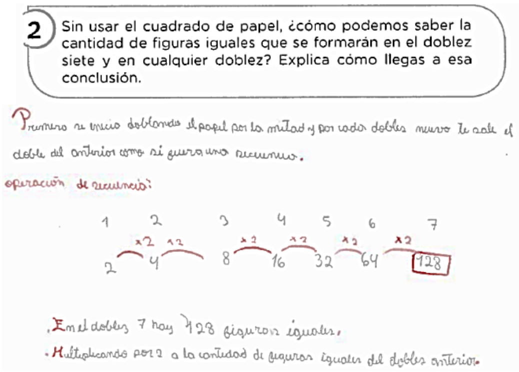
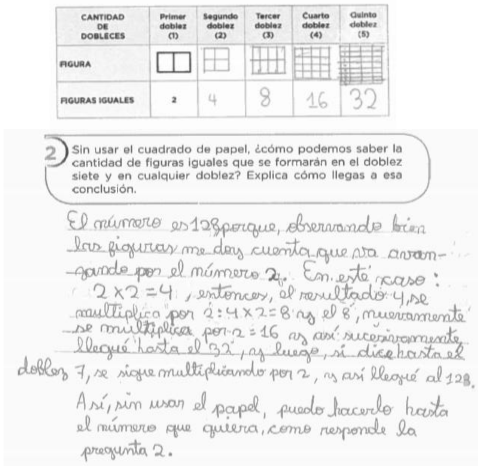
	 	the previous term. Explains that each term of the sequence is formed by multiplying the amount of the same figures 2 fold above, and justifies validating the pattern meets the first seven terms of sequence. He/she uses properly the terms, symbols and mathematical operations to solve the situation. Therefore, the student shows learning of IV cycle.
4	 	The student completes the table data and relates it to a multiplicative pattern. He/she describes in detail the procedure used justifying it using special cases, and is able to generalize the procedure to determine any term of the sequence. Therefore, the student shows learnings that exceed expected for the cycle IV.

Table 3. Examples of performance levels in a regularity situation task. Source: (Isidro et al., 2016)

In this case, to evaluate/validate description of IV cycle (9 years of age) of competence *Act and think mathematically in situations of regularity, equivalence and change*, it was proposed to observe student capability to translate data and conditions of situation posed to multiplicative patterns from use of simple tables and graphs and justify it using some mathematical terms. For this, task *Transforming a robot* was designed and applied (Isidro et al., 2016) (Figure 2). After application, answers were obtained from students corresponding to four levels of performance, as can be seen in Table 3. In this way, authentic tasks applied have served to evaluate degree of development shown by students with respect to mathematical competences defined for RBE.

In response to these authentic tasks, students constructed substantial answers that revealed their understanding of multiplication concepts and skills to apply, analyze, synthesize and evaluate those concepts. This analysis revealed that some students understood multiplication concepts because they were required to constructed new mathematical knowledge and not just to select a response for the task. The validation of learning standards was determined when some students used the tasks to demonstrate specific multiplication skills by applying them to solve the problems posed.

CONCLUSIONS

Throughout present study it has been possible to appreciate strong influence that the PISA Program has on educational policy of Peru. Since 90s a Curricular Reform has been developed and financed by international financial organizations, which have managed to spread continuous and progressive character of learning through LS in educational community. For this purpose, a valuable fieldwork was carried out in the classroom in which levels of student performance were observed and evaluated, which is expected to be used by government authorities. There is still little clarity in formulation of mathematical competences and in role of contents in NCBE. This study aims at contributing to debate over the subject.

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CURRICULAR MODIFICATIONS REGARDING MATHEMATICS TEACHING OVER THE LAST DECADES IN CHILE AND THEIR INFLUENCE ON TEACHING PRACTICES: TOWARDS AN UNDERSTANDING OF CONDITIONS FOR THE DEVELOPMENT OF GOOD PRACTICES.

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In this article, a description and an analysis of the type of difficulties and tensions experienced by teachers in the field of Mathematics are presented, regarding the last two curricular reforms in Chile, focusing on the implementation of two teaching strategies in the country developed within each of these reforms: "The Literacy and Mathematics Strategy (LEM, for its acronym in Spanish)" and "The Singapore Method". The purpose of this study is to understand the conditions and restrictions that make it possible for teachers to develop appropriate and relevant teaching practices according to officially established criteria, as well as those conditions that hinder them. Framed within the Epistemological Approach in Didactics of Mathematics and, specifically, in the Anthropological Theory of the Didactic, we identified didactic phenomena linked to each reform process and its implementation in the classroom in the different levels of didactic co-determination, and a series of reflections that arise from the questioning about the didactic economy is presented in the conclusion.

INTRODUCTION

Over the last decades in Chile, important modifications have been made to the curricula of all the learning areas, particularly mathematics. Of course, these modifications have been oriented by the demands of the 21st century society, which is highly technological, changeable, and increasingly empowered by its rights and that demands not only higher quality in the teaching processes but also greater equality in access to knowledge; an inclusive teaching that integrates all students in meaningful learning processes.

In this sense, there have been two major reforms in the last 20 years in Chile, one that took place between 2004 and 2012, and the other that was created in 2012 and it is currently under development. Both reforms respond to specific needs of the era in which they were proposed, which are stressed by the demands for change established at international level.

The purpose of this article is to analyze the conditions and restrictions that each of these reforms have presented to teachers, to illustrate the tensions that have led them to move from one reform to the other one, and the possible advances that have been achieved with these changes. Another purpose is to provide reflections on the didactic economy associated with initiatives like these, that is to say, to discuss the benefit achieved by the Educational System as a whole with all these efforts, in relation to the human, financial and social cost that it has meant to carry them out.

The strategy used to address this issue is to examine the introduction of two teaching strategies implemented as models quite generally in the country during both periods. On the one hand, "The Literacy and Mathematics Strategy (LEM, for its acronym in Spanish)", on the other hand, "The Singapore Method", respectively.

In order to carry out this study, we considered the Epistemological Approach in Mathematics Didactics, and we used the tools that the Anthropological Theory of the Didactic (ATD) by Yves Chevallard (2009) and Theory of Didactical Situations (TDS) by Guy Brousseau (1997) provide for that purpose.

CHARACTERISTICS OF THE LAST TWO CURRICULAR REFORMS IN CHILE IN THE FIELD OF MATHEMATICS EDUCATION AND TENSIONS EXPERIENCED BY TEACHERS REGARDING THE DEMANDS MADE

The first Reform presented in this section is the one that took place between 2004 and 2011. The previous curriculum was organized in terms of Mandatory Minimum Contents and Fundamental Objectives. These objectives were defined more in terms of teaching than in terms of learning learning, and delimited, through brief examples, suggestions of what the teacher could do for the students to achieve these objectives. There was a high valuation of the mathematical contents over the mathematical activity to be carried out with said contents with very little reference to the development of skills. Mathematics appeared as important in itself; so there was a clear interest in learning their properties and characteristics; a type of cultural knowledge, generalized and universal.

The new reform of 2004 drastically changed the focus, centering on the students and their learning process. The first modification was to replace the Fundamental Objectives with Expected Learnings. The center of the teacher's task was to generate relevant activities for the students, which gave meaning to the mathematic content that they had to learn, making explicit the evaluation indicators to validate the progress and the students' achievements regarding those previously named Learnings.

The mathematical contents remained more or less the same as in the previous period; however, there was more emphasis on the importance of context in order to provide meaning to the same contents. The Study Programs were structured in four annual thematic units for each level of education with mathematical contents of the different thematic axes of the numbers program; operations, geometry, algebra and functions. The study of the measurement of magnitudes and statistics were not thematic axes in themselves, but they were considered within one of the previously mentioned thematic axes.

Teachers who had to face this challenge, focused more on the new pedagogical issues, than on the mathematical activity itself. In this way, the teaching decisions were mainly related to the type of material to be used (concrete, work guides, etc.); type of organization for work (group, collective or individual); and on the type of questions to ask for all students to participate. In this context, the National Literacy Strategy appeared.

TEACHING FOCUSED ON THE LEARNER AND THE MATHEMATICAL ACTIVITY: THE CASE OF THE LITERACY AND MATHEMATICS STRATEGY, LEM

In order to support Primary Education teachers in the implementation of the new curriculum, the Ministry of Education of Chile along with the Felix Klein Center, of the Universidad de Santiago de Chile, the design and implementation of the LEM strategy was carried out. The strategy included the development of four Didactic Units (DU) (downloaded http: //lem.uct.cl/? Page_id = 27) by level, to be disseminated massively among public school teachers. Each unit organized a study process of six lessons around a central mathematical theme included in the Official Program. In this way, LEM did not have the requirement to cover the entire school curriculum. The aim was to provide the teachers with four "illustrative examples" on how to bring the new curricular demands to the classroom and, based on those experiences, transfer the didactic principles of the DUs to the teaching of the other contents of the Program.

The DU were designed for students to experience a set of Study and Research Activities (Chevallard, 2001) involving the mathematical knowledge through fundamental situations (Brousseau, 1997) that lead to a construction of mathematical knowledge along with the teacher. Each DU, in its beginning, stated a problematic question that, through the exploration carried out by the students accompanied along with an adequate management of the teacher, would make the corresponding mathematical knowledge emerge. Then, the students, individually and/or collectively, would work the knowledge in other situations and under different conditions, until they achieve an adequate mastery of it. The teacher with his students would institutionalize and evaluate the mathematical knowledge. Each DU included a detailed and well-argued description of its Didactic Strategy. These foundations were described in a general way and by lesson, incorporating what was called the "Lesson Plan". Thus, the Didactic Units raised certain notions that were introduced in the country, existing until today: the "Lesson Stages". Using the notion of didactic moments (Chevallard, 2002), the lessons were structured in three phases: warm-up, development and closure. In the warm-up phase, relatively new problematic issues were often raised for the students, which they had to try to solve on their own without the teacher's help. As a result of this study, it was expected that concepts and/or mathematical procedures that were discussed would be raised. In this way, during the warm-up, the exploratory moment was essentially experienced. During the Development phase, activities were carried out to consolidate the procedures that arose during warm-up, to question them and to understand how and why they work, and the predominant moments are the moment of work of technique along with the technological-theoretical moment. Finally, during closure, a series of questions were suggested that allowed evaluating and systematizing what was learned, so that the moments of evaluation and systematization were experienced.

During those years, Centro Felix Klein was in charge of offering Chilean teachers who worked at public schools organizations that embodied the principles of the new curricular reform, through the aspirations of the ATD and TDS, and at the same time be feasible. After more than 6 years of implementation of this Strategy, we were able to obtain valuable conclusions. The majority of teachers, when managing the teaching of the four DUs, altered their habitual practices, trying to adhere to the "didactic strategy" and the lesson plans, with more or less success. However, once the implementation of the LEM strategy was completed, the teachers returned to a classic teaching style, far from the didactic principles they had put into practice during the implementation of the DUs. Regarding the challenge the teachers faced to extend the didactic principles for teaching the other subjects of the curriculum, they chose to desist from those principles. As time went by, even when managing DU teaching, the teaching practices ended up becoming distorted and moving away from the original ones, and finally diluting.

This left us two important conclusions: The first is related to the fundamental need to previously know the implicit didactic theory of teachers (assumptions, operating principles, conceptions about mathematical contents, etc.) as well as their own limitations, and from there propose Didactic Organizations that are relevant and viable. Otherwise, we face the risk of proposing organizations that are very distant from the usual practices, which require teaching tools that teachers do not have and, for this reason, they end up rejecting them. The second conclusion was to recognize the importance of the spontaneous didactic praxeologies (Chevallard, 2002; Barbe et al, 2005) that teachers of the same school create, there is a common didactic-technological-theoretical discourse on the mathematics studied in the school and its teaching. Faced with the existence of different theoretical elements from different approaches and currents of didactic research, it is very difficult to promote a reflective discussion that promotes the change of practices. In the absence of a common reference, teachers often act with implicit technologies, from perspectives sometimes contradictory to each other.

In the case of the LEM strategy, although we proposed a didactic theory and technology consistent with the new curricular demands, teachers of different ages and educational trajectories had elements of traditional didactic technologies that clashed with the new ones and, at the same time, were different from each other. Apparently it would be preferable to stabilize a common discourse among teachers, although less ambitious didactically, and, from there, to work and reflect with teachers about the changes required in their practices.

TEACHING FOCUSED ON THE DEVELOPMENT OF CERTAIN MATHEMATICAL SKILLS: THE CASE OF THE SINGAPORE METHODOLOGY – MATHEMATICS

From 2012, a new process of curricular reform in Chile has been in force up to the present, this time placing emphasis, as we have discussed above, on the development of mathematical thinking skills in students. They appeared explicitly in the new Curricular Bases as axis articulators of the curriculum, the notions of mathematical modeling and representation, new for the Chilean teaching staff; and the notions of argumentation, communication and problem solving, notions already known by teachers before this period. The teacher's task is focused on living learning experiences that initially require mathematical modeling, and are strongly linked to problems of daily life faced by students. The requirement proposed by the international community that works around mathematics teaching appears, especially from PISA and TIMMS, that mathematical learning must be useful for the lives of students (OCDE, 2003). Therefore, in any activity proposed to students, there must be a problem arising from a clear context in which mathematical knowledge appears as a solution tool, a set of mathematical skills put into play to study and solve the problem, and certain conditions of specific realization of the activity. At the Curriculum level, stressed by the international community, data and chance appear as a new axis to address the problems that require statistical analysis and/or probabilistic use. Likewise, the study of the measurement of magnitudes is raised as an axis that begins in the first years of childhood education and ends towards the final stage of Primary Education. To give space to the new thematic axes, the number of concepts and mathematical topics of the programs decreased, and three strong ideas inspired by the curriculum of Singapore (Syllabus, 2006) appeared: the use of concrete material in support of teaching, the different types of representations, concrete, pictorial and abstract for the study of problems, and the spiral curriculum. The units articulate different mathematical topics that were previously covered separately. The concepts of additive problems appeared in opposition to addition and subtraction problems, as well as the field of multiplicative problems in opposition to multiplication and division problems that were in the previous curriculum. Thus, subtracts is studied strongly linked to sum, and division strongly linked to multiplication. At the same time, the study of algebra begins early, as a modeling language for arithmetic operations through equations, where there is an unknown value.

Within the framework of these new curricular demands and challenges, and taking into account the difficulties that teachers had faced in the implementation of the previous reform, the Ministry of Education decided to promote the use of school texts inspired by the Singapore Method (SM). A pilot program was developed in three hundred schools in the country in which a series of Singaporean textbooks adapted to the Chilean curriculum was provided. In general, the SM is based on an articulated organization of five fundamental components of the learning process for problem solving. These components are: concepts, processes, abilities attitudes, and metacognition. This methodology includes textbooks for each of the Primary Education courses, which are responsible for teaching all the contents of the official program of each course.

Likewise, it incorporates a guide for the teacher that explains, in a very detailed way, how to implement the textbook in the classroom, and a set of concrete materials to support learning. The proposal contained in the chapters of each textbook follows a common structure: it begins by setting a contextualized problem out that involves the mathematical content to be learned, and subsequently several procedures are presented for its solution.

Then a study is carried out that ends by characterizing and specifying the procedures and the mathematical content in question, and exercises and problems similar to those solved for the work of the students are proposed. At the end of each chapter, there is a mathematical challenge that leaves room for students' free exploration, but that is not part of what they should learn. The mathematical problems included in the proposal are, in general, much more diverse than what Chilean teachers commonly do in their lessons. The sequences of problems and exercises are well articulated and organized in order of increasing complexity. However, the student's activity is still very pre-established and directed by the teacher or the textbook.

The exploration of problems is strongly led by the teacher, so it does not incorporate classroom management that encourages the participation of students in the construction of mathematical knowledge. The mathematical questions that remain in charge of the students are generally simple, since the complexity is usually answered by the teacher. Ultimately, the role of the student is apparently active, but on mathematical issues very well delimited and previously simplified by the teacher. Regarding the sequences of types of activities, they tend to present a fairly careful articulation to solve the same problem; several techniques and certain associated technological discourse are presented. However, the didactic strategy remains classical, in the sense that knowledge is already built, giving few opportunities for real participation in the construction process for students.

DISCUSSION

In this section, we discuss and contrast the main advances achieved by both strategies and their difficulties. To contrast these two elements, we identify certain common aspects of the didactic organizations that teachers develop in each of the strategies, using quality and equity criteria (Gellert, et al, 2013) and levels of didactic co-determination (Florensa, et al, 2017), to then contrast those aspects. In the case of the LEM Strategy, the didactic organizations proposed to the teaching staff were very innovative for the time; they were based on ambitious and revolutionary didactic principles that were at the base of the curricular reform of that period and that required a significant mathematical construction activity by students, as we described in section 2. In turn, the DUs had didactic orientations for classroom management that were strongly supported but scarcely patented. Moreover, they never intended to guide teachers to teach all the mathematical contents of the curriculum, but the expectation was to provide the teachers with some examples that illustrate how to teach under this new paradigm, and be used as a reference to organize the teaching of the rest of the topics of the programs. This resulted in teachers gradually abandoning the aspirations of the LEM and returning to traditional classes based on the exposure of knowledge and the reproduction of techniques by the students. The LEM strategy didactic technology was very distant from the one that the teachers had at that time, and the curricular exigencies overflowed them. Building educational organizations for the teaching of other mathematical contents of the curriculum following the principles of LEM strategy was frankly complex to them. This required, among other things, teachers to have some didactic tools for the reconstruction of problematic learning-generating issues that they did not have. Finally, the educational system as a whole could experience ways to manage the teaching of mathematics in a different and more coherent way with the new paradigms. However, teachers ended up aborting this experience by slightly modifying their practices, and students continued to be content replicators. In the case of the Singapore Method, the main focus was on the development of certain mathematical skills, emphasizing the learning of techniques, focusing the study on certain essential mathematical contents and trying to promote the appropriation of argumentative discourses through reflection on them. Thus, cross-cutting aspects of the mathematical learning process, such as metacognition, were highlighted. Problem solving became the center of mathematical activity, but with few spaces for exploration by students. Although there is richness in the mathematical problems proposed in the SM, once they are presented, the texts quickly raise strategies for their resolution, leaving little space for students to participate in its elaboration. Mathematical questions appear simplified in terms of the work that students must do since the complexity is addressed by the teacher, and the didactic organizations proposed to the teachers are clearly defined and previously stated. This way of proceeding allows stereotyping the didactic organizations simplifying the work of the teaching staff. There is ambiguity in what students learn: they understand the importance of solving problems, their mathematical modeling and the existence of various forms of resolution. However, the teacher is the one who in charge of modeling the problem mathematically and proposing different representations and forms of resolution. Students also understand the importance of communicating and justifying mathematical procedures; however, given their limited participation in the search and construction of such procedures, the real need to clarify key mathematical issues is superficial.

Through a follow-up to several schools in Chile that have used the method for more than 6 years in all the classes of Primary Education (Espinoza 2016), significant improvements were observed in the learning achievements of their students in comparison with to students from other similar schools who did not use the method. Likewise, there were achievements in the teaching practices, although not in a generalized way, mainly related to processes of reflection and critical analysis on teaching, managing to go beyond pedagogical issues and to tackle problems of didactic nature related to mathematical activity that students develop. The teachers, after making several implementations of the method, managed to appropriate the didactic technology present in the teacher's textbooks and began to wonder about technological aspects, specifically, didactic and mathematical. This questioning allowed the teachers to reflect on their own practices and be aware of the need to adapt them to the SM.

Regarding students who used the SM between 1st grade and 6th grade of primary school, they had the opportunity to solve a greater diversity of mathematical problems, along with a diversity of strategies for their resolution. In turn, these students expressed the need of having argumentative discourse that justifies the resolution reasons in mathematical study.

Questions about their own study looking for arguments for the understanding of the mathematical concepts were raised. The implementation of the SM demonstrated samples of a greater degree of appropriation of the didactic strategy by the teachers than the LEM strategy. However, it is important to point out that the teachers of those schools where the LEM strategy lasted for a period of five years did manage to be empowered by the didactic technology and techniques made explicit in the DUs.

CONCLUSION

After these two experiences in Chile during the last two curricular reforms for a considerable implementation period, it is possible to draw some conclusions.

The first conclusion is related to the importance of math teachers of the same school having a common didactic technology about teaching and learning mathematics. When there is a shared discourse on teaching, a common reference agreed by consensus and established in the school, teachers can discuss with each other the appropriateness of teaching with such activities and problems, or with such support devices for learning, or with such management within the classroom, regardless of the level of specific schooling. If the criteria are clear, it is less difficult to discuss their adequacy and relevance. Otherwise, in the absence of a common discourse, the problems of teaching that can be discussed by the teachers are general, of a pedagogical and contextual nature, since the specific didactic considerations are different between the different levels of schooling. Although this finding could be considered quite obvious, in Chile this understanding obtained from the evidence and the associated research processes is a great advance, since the country's public policy distributes textbooks, from different publishers, that rely on different didactic technologies, sometimes even contradictory.

The second conclusion drawn from the contrast of both experiences is that teachers accept with relative ease the incorporation of new praxeological elements to their teaching practices, as long their spontaneous praxeologies are not questioned. However, when the praxeological elements question their practices deeply, it is necessary to carry out a work of support and systematic reflection so teachers accept them. The main challenge that teachers faced with the LEM strategy implementation was to place students in the role of authors of the studied mathematical knowledge, a challenge not present in the SM implementation.

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REVIEW OF AGENTS AND PROCESSES OF CURRICULUM DESIGN, DEVELOPMENT, AND REFORMS IN SCHOOL MATHEMATICS IN COSTA RICA

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A synthesis of the curricular implementation processes and actions carried out since the approval in 2012 of new programs of study for primary and secondary Mathematics within the framework of mathematics reform in Costa Rica is presented. Here some of the contributing factors that have played an important role and that have sometimes generated complementary or synergistic effects are incorporated. In particular, the pragmatic perspective of a curriculum influenced by international ideas and constructions by the country's own researchers, research that has originated from the curriculum, decisive participation of multiple partners in the Reform of Mathematics Education project in Costa Rica, innovation in teacher professional development processes and the support of the private sector in this process are highlighted. Likewise, it is suggested that the strategies followed in a developing country like Costa Rica, provide lessons that can serve other countries in similar conditions.

INTRODUCTION

In the search to strengthen higher order cognitive abilities as part of the approach to the new challenges posed by a modern society, on May 21, 2012, the Higher Council of Education of Costa Rica (the constitutional body responsible for guiding and directing the different levels, cycles and modalities of the Costa Rican Educational System from a technical point of view) approved new mathematics study programs for all pre-university education in that country.

The previous curriculum had a strong behaviorist influence (programmed objectives) and was quite linear (evaluation associated with each disaggregated objective, one by one). In general, despite the language of "objectives", it was basically lists of contents, almost no interaction with the curricular foundations (their concepts and objects), absence of mathematical tasks with increasing levels of complexity, minimal problem solving and erroneously conceived, minimal participation of real contexts. There was no formal mathematical modeling and only minimal use of technologies.

The Curriculum, approved in 2012, is ordered based on mathematical contents, but the essentials are the abilities associated with the content, often in cycles of two to three years. The purpose of school mathematics program is general mathematical competence, which is interpreted as a capacity to understand and use mathematical objects in various contexts. That is why the central issue here is to promote the development of what was called transversal higher cognitive abilities: Reasoning and Argumentation, Posing and Solving Problems, Communicating, Connecting, Representing.

In terms of content, several changes were made: introduction of coordinate geometry and transformations (before there was only traditional, synthetic geometry), spatial visualization, statistics and probability in all 11 school grades, algebraic thinking beginning in the primary grades, more complete treatment of functions, including their use in analyzing algebraic relations, although trigonometric functions were eliminated (for diverse reasons). The approaches proposed are crucial: for example, in statistics, what matters is the analysis and interpretation of information, not the calculation of measures; and relations and functions are to be associated with modeling.

The curriculum proposes a model of four steps and two stages for classroom practice. The two stages are construction of learning, followed by mobilization and application of the constructed learning. The four steps for implementing the first stage are presentation of a problem, independent student work, collaborative phase for testing strategies, and the final closure. The problem is the beginning of the lesson. In the previous methodologies, instruction began with a presentation of the mathematical elements (for example, Pythagoras theorem), then examples, and later routine practice and maybe, but not most of the time, a problem was finally used. Now the flow in the classroom action has been reversed.

In summary, this Curriculum aims to bring a pragmatic perspective to mathematical knowledge and the development of higher abilities for understanding the society's realities. In addition, it formulates a new pedagogical strategy: "Problem solving with an emphasis on real contexts", which implies a substantial transformation of classroom practice. These particularities make necessary various actions that support this curricular reform.

Starting in 2013, a process of curricular implementation began. It has been characterized by changes that are deliberate and gradual. The national reality has been taken into account in light of a curriculum substantially different from those that have preceded it and with an international perspective that requires new educational scaffolding.

WHAT ARE THE PROCESSES DEPLOYED, IN THE CURRICULUM REFORM?

The Ministry of Public Education of Costa Rica (MEP) has concentrated its institutional efforts on the implementation of new Mathematics programs of study in 2012-2017. It is important to note that the Mathematics Education Reform Project in Costa Rica (of the Ministry of Public Education with the support of non-governmental entities) (PREMCR) has participated in a relevant way in the design and execution of strategies for the implementation of the curriculum. One of the first actions was the preparation of transition programs that sought a gradual introduction to the programs of study. By 2016 it can be said that the implementation process was complete up through regular high schools, but it was not until 2017 that the extra year of technical high schools was added. However, it should be pointed out that the reform requires many more stages for its definitive consolidation.

Costa Rica, as is the case in other developing countries, has major weaknesses that may hinder or slow down a curriculum implementation process: budgetary limitations, instability in the continuity of educational policies and reform processes, weak teacher preparation, lack of well-focused professional development processes, absence of monitoring mechanisms, among others. Taking into account all these weaknesses in the education system, the curriculum had to go beyond being a guiding document. It had to anticipate conditions and face resistance on multiple flanks.

This new curriculum is more than a content adjustment (readjustment, increase or decrease). It implies a different paradigm, and therefore its implementation has been a complex process that has required multiple tasks with different lines of attention. For example, the classroom implementation process brought challenges to the work of teachers. One of the challenges has been lesson planning, as indicated in the Fifth State of Education Report (2015):

> The preparation of the classroom action acquires a more relevant place with this curriculum than with the previous ones. A greater preparation in the various pedagogical and cognitive aspects present in the lesson is demanded of teachers: mastery of the curriculum and not just the mathematics content of the curriculum, mathematical preparation in the new topics and also in the corresponding approach to each one of them (p.156).

Traditionally, teacher planning revolved around mathematical content and meeting objectives. However, Ruiz (2015) points out that the vision of problem solving that is introduced in this curriculum is a strategy for organizing lessons. This puts an emphasis on a particular style of pedagogical mediation where teacher planning is a transcendental element; as well as being more demanding and involving more intellectual effort.

Being a substantial transformation, the role of the teacher in the implementation of this methodology is transcendental. However, one of the challenges in the realization of this curriculum is precisely the initial and ongoing professional development of in-service teachers (Gaete and Jiménez, 2011).

Trying to solve these deficiencies, a column of "Specific Suggestions" has been added to the Program of Studies document. This is a novel way of offering not only brief methodological suggestions associated with concepts and skills, but to specify what is desired for implementation in each case, visualizing the meaning of the proposed skills.

Although this column of suggestions can guide lessons in some ways, it was considered that this was not enough to fill so many training gaps. This is why, given the depth of the differences between this curriculum and those that preceded it, PREMCR designed a large number of curricular support documents and made the decision to unleash a large-scale teacher professional development process through different strategies and modalities.

From 2011 to 2017, blended courses (face-to-face sessions and online independent work) were carried out for primary and secondary school teachers separately. It is a novel strategy that involves two types of sessions: one to work with teacher leaders and another to train large populations of teachers. Local officials and teacher leaders with the administrative support of the central offices of the Ministry, were responsible for offering in the different educational regions the same course they received in the first phase. The documentation, the self-assessment practices, the exams and all the resources were essentially the same in the two phases. This process helped to guarantee significant academic quality in each course in both phases. The details of this project and its actions can be seen in Ruiz (2013), however it is important to note that through this strategy it was possible to serve almost all teachers of secondary mathematics (2500), and 50 to 60% of primary teachers (20,000).

The needs of the mathematical reform implied a need to serve the educational community in an even broader way. There was a desire to reach populations that had not received the blended courses, to serve those who had not successfully completed those blended courses, as well as those who wanted to complement their preparation. Here the blended strategy was no longer enough. During 2014 and 2015, completely virtual courses were offered as MOOCs (Massive Open Online Courses).

In 2016, also using MOOCs, support was provided to high school students who had to take exit tests that are also used for entrance to higher education. This was the first time that the new curriculum had been used as a reference for the design of the tests even though the students taking the tests had just begun to use the new curriculum three years earlier and most of those years it had been through transition programs.

The experience of the years 2014 to 2016 led to a new innovation: the Mini MOOC. These are courses with the same virtues as MOOCs, but are focused on specific, compact, short and self-sufficient topics. Mini MOOCs can be completed in less than 15 hours. The Mini MOOCs are grouped into collections. The perspective that has been taken is to create spaces that respond more to individual (personalized) needs. Between 2017 and 2018, more than 50 of these mini courses have been designed and executed. This modality has been applied for both teachers and high school students.

This strategy is innovative for the country. Ruiz (2013, 2017) points out that MOOCs and Mini MOOCs drastically modified what had been usual in the professional development processes that were taking place in Costa Rica and opened up new horizons that use communication technologies intelligently. At the same time, in an intrinsic way, the experience in the use of this type of platform not only brings teachers closer to the use of Information and Communication Technologies (ICT) but also modifies their professional profiles, promoting a modern vision of the educator.

A relevant aspect is that this experience of curricular implementation has served as a model for other educational reforms that are being carried out in the country. That is why it is considered that in the context of a developing country like Costa Rica, the actions and strategies followed in this process of consolidating a curriculum, which still has not concluded, provide lessons that can be useful to other countries with similar conditions.

In summary, it should be emphasized that historically these actions have not been usual in Costa Rica when there has been a change in curricula. This reform has been possible thanks to the active and transcendent participation of PREMCR in leading various implementation strategies.

ROLES OF TEACHERS, TEACHER EDUCATORS, RESEARCHERS AND MATHEMATICIANS IN THE CURRICULUM REFORM?

Costa Rica's Mathematics Curriculum (MEP, 2012) was written by a team of university researchers and independent experts external to the MEP. Angel Ruiz, with the Minister's support, formed the writing committee for the new curriculum with five researchers in mathematics education from public universities with whom he had worked in some cases for more than 20 years. Although the initial education of the committee members was in mathematics, over time they had specialized in such areas as history and philosophy of mathematics, use of technology, statistics and mathematics education. This group was joined by six primary and secondary teachers, four were released from their school assignments by the MEP to work comprehensively in the preparation of the curriculum.

Not including MEP officials on the writing team did generate friction and, in some cases, they provided little support to the reform process. However, despite all these negative reactions, there has been continuity during two government administrations. This situation is documented in Ruiz (2013).

One aspect that was decisive in the reform was that the same team that wrote the curriculum also assumed a decisive role in the implementation of the reform. This was possible through PREMCR. Both in the design and in the implementation, in addition to the researchers from the public universities, the project included in-service teachers primary and secondary teachers and specialists in communication technologies (Ruiz, 2013, 2015, 2017). Teachers have been provided by the MEP, and specialists and researchers have been hired with private financial support.

Given that this mathematics reform touches several components of the national education system, the team that has guided the reform can be considered the factor that has made this process more visible than would the implementation of a textbook series. One dimension of this group is its important connections in the international mathematical education community (ICMI, CIAEM, NCTM, among others).

Another transcendental aspect was that it was based on the premise that a reform requires a network of leaders willing to promote it throughout the country and serve as a reference in their educational institutions. This strategy not only brings forth outstanding teachers who underpin the implementation of the curriculum in different educational regions of the country, but in this process an important new actor has emerged: the Regional Mathematics Advisor.

Poveda and Morales (2015) point out that the reform in Mathematics Education has brought changes in the role of the Regional Mathematics Advisor. Ruiz (2015) suggests that the Regional Advisers have become true leaders and reference points in their regions. They have become decisive in the implementation of the reform.

As mentioned above, one of the weaknesses for the curricular implementation process is the initial training of teachers, and the absence, until recently, of continuous professional development processes (Alfaro, Alpízar, Morales, Ramírez & Salas, 2013).

In recent years, the public universities in one way or another have made changes to articulate their teacher preparation programs with the reform curriculum and the work of classroom teachers. It would be expected that in the following years these institutions will provide teachers with appropriate skills that will be prepared to consolidate the curricular implementation. Ruiz (2015) points out that universities have a great responsibility with respect to the success of the curricular implementation; as they will nurture teachers who must be prepared to face the challenges posed by this reform.

PUBLIC ENGAGEMENT AND THE MEDIA IN THE CURRICULUM REFORM?

The processes of curricular implementation in a developing country are complicated because the social, political and educational contexts impose conditions and limitations. A condition in Costa Rica is that the Minister of Education is appointed by the incoming government every four years, and this often affects the continuity of projects and reform actions. Much of the success of these processes depend on historical and political conjunctures. This is how the second administration of Minister Leonardo Garnier Rímolo (2010-2014) established a commitment, not only to the design of the Mathematics curriculum, but also to its implementation.

In the process of implementing this reform, a strategic alliance between the public and private sectors emerged. PREMCR was born as a joint project between the Ministry of Public Education (MEP) and the Costa Rica United States Foundation for Cooperation (CRUSA). Between 2012 and 2013 the

Project had the commitment and support of both entities. Even so, the reform has always had obstacles and threats. This is because the new curriculum and its implementation processes have meant a significant change in tasks and a much greater preparation demand, not only for teachers, but also for MEP officials in general. The challenges have been perceived differently by the various actors.

Uncertainty has been present at different times. In the last six months of the Garnier administration much of the support and resources that the MEP had provided, and that were needed by PREMCR, were weakened. Another moment was in 2014, because Costa Rica had a change of government and a new political party emerged in power. This was perceived as a new threat to the curricular reform. However, the new Minister, Sonia Marta Mora (2014-2018), assumed the reform as a priority of her administration, and therefore the chances of success increased considerably.

Here we must highlight that this educational reform has received the support of non-governmental organizations. In the 2012-2015 period, the reform project received decisive financial support from CRUSA that was executed through the Omar Dengo Foundation. In the period 2016-2017, the Business Association for Development (AED) supported the reform, and CRUSA has maintained its contribution. Many of the resources that have been developed, and a large number of the actions carried out, were possible thanks to this national support. This expresses that the mathematical reform has been seen until now as a country-wide project where public and private sectors have converged.

Although Costa Rica is a small country geographically, it presents a great diversity of conditions and particularities. The progress in implementation of the reform has differed in the various regional education directorates (Costa Rica is organized into 27 educational regions). In some regions there has been greater intensity than in others, and there are differences at the school and classroom levels as well. While the reform continues to advance in the national consciousness, full implementation will be a complex and long-term process that invokes multiple dimensions of national life (including some outside of education). New actions should use and enhance the high-quality resources that were already generated in the 2012-2017 period, as well as improve them with what has been learned. Providing continuity to the efforts is undoubtedly the first priority.

The year 2018 brought another change of government and uncertainty regarding the direction of educational reform arose again. Past political support has facilitated the very solid steps that have been taken. However, it is not possible to assure, a priori, that new governments will provide the necessary continuity. Nor is it certain that the progress already made is sufficient to assure that the country will not regress in the absence of political support.

RESEARCH INFORMING THE CURRICULUM DESIGN AND DEVELOPMENT?

Costa Rica has been able to count on researchers from the public universities who for many years have identified findings in national and international Mathematics Education, and who have contributed their work in this project of change and curricular implementation. In particular, there is a vision of Mathematics as an historical and cultural construction with a strong influence from the empirical, physical and social worlds, which support the design of this curriculum. This is a perspective based on the works of A. Ruiz (1987, 1990a, 1990b, 1992, 1995a, 1995b, 1995c, 2000, 2001, 2003).

This theoretical influence directly impacted the curriculum of Mathematics of Costa Rica and inscribes it within the latest trends in mathematics education: those that use competencies or skills as an essential factor for teaching and learning. However, although the program of studies emphasizes the strengthening of superior cognitive abilities, here a pragmatic vision of mathematical competence is presented from an original perspective: The curriculum is not organized by means of competences, but rather they are proposed as objectives to develop during pedagogical mediation. (MEP, 2012).

The curriculum integrates, in its foundations, theoretical elements of the international community, especially the NCTM and PISA, adjusted to the national reality under the influence of ideas contributed by Costa Rican researchers. Likewise, in the organization of the lessons, concepts or ideas raised in the theoretical frameworks of the French Mathematics Didactics and classroom action models of Japan are incorporated.

Here it should be noted that an intellectual construction of its own has been made, advancing in ideas about curricular design and its implementation, which can serve as a contribution to international research and experience. It must be emphasized that a model external to the country has not been adopted. There is in the foundation, in the programs of studies and in the curriculum in general, an autochthonous and functional use of the elements that are identified in the research and experience of International Mathematics Education adjusted to the conditions of a peripheral, developing country.

This richness from theory and this international perspective on the curriculum have encourages a great amount of research in recent years, including multiple undergraduate and postgraduate works on various subjects of mathematical education.

In addition, the mathematical reform in Costa Rica has been widely documented with several publications: Ruiz (2013, 2015), Ruiz and Barrantes (2016). New intellectual constructions have also been developed in the light of the curriculum (Ruiz, 2017).

The pragmatic nature of the curriculum has motivated in-service teachers to make public their successful classroom experiences. This also prompted some regional pedagogical consultancies in Mathematics to generate teaching materials. In conclusion, this curriculum has served as a pivot to promote educational research and development in Costa Rica and beyond.

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SHARED PERSPECTIVES ON RESEARCH IN CURRICULUM REFORM: DESIGNING THE CAMBRIDGE MATHEMATICS FRAMEWORK

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Different groups of stakeholders in curriculum development hold different perspectives on teaching, learning, and the domain of mathematics. The degree to which they coordinate or align their efforts based on these perspectives affects curriculum coherence, both in the design of the intended curriculum and how it is enacted in schools. Stakeholders must be able to share or at least understand each others' perspectives on the potential implications of research for curriculum design in order for research to have a coherent influence. We are designing the Cambridge Mathematics Framework to link research to mathematics learning in a form that can be mutually considered and applied to the processes of curriculum design and enactment by curriculum designers, resource designers, and teachers. We describe work in progress on the design of the Framework and the processes underway to incorporate feedback into the design and evaluate whether the Framework represents research in such a way that it is likely to be meaningful, useful and used.

INTRODUCTION

The decisions and actions of a diverse set of stakeholder groups shape the ways in which any given mathematics curriculum is *intended* to function by its designers, *enacted* in schools, and *received* by students (Stein, Remillard, & Smith, 2007). Coordination, or lack thereof, between these groups affects how coherently the domain of mathematics is presented in the intended curriculum, and how coherently the intended curriculum can be enacted. These each affect what mathematics students have the opportunity to learn (Schmidt, Wang, & McKnight, 2005). Research in mathematics education – including philosophy of mathematics learning, learning in particular subdomain areas, and pedagogy – has the potential to be applied in the design and enactment of curricula. However, different stakeholder groups (and different stakeholders within groups) are likely to be familiar with different subsets of existing relevant research, and when they look at the same research they don't always see it in the same way. This might limit the effectiveness of actions that any one stakeholder might take based on this research, if these actions are not coherently supported by the work all groups do to form the curriculum as a whole. In this paper we describe work in progress on the design and evaluation of the Cambridge Mathematics Framework, which we intend will help to coordinate perspectives on applying research in curriculum design for three umbrella categories of stakeholder roles: curriculum designers, resource developers, and teachers. Of the processes of design, development, and reform that drive curriculum change, our project is focused on contributing to curriculum design and development, but we work with the larger process of curriculum reform in mind.

Curriculum coherence and a shared perspective on existing research

'Coherence' is frequently called for across the curriculum design and mathematics education literature as a way to increase effectiveness of teaching and learning, by coordinating policies, resources, and actions. Discussions of coherence in curriculum reform often fall into two categories: cultural and cognitive, each with a distinct set of implications for the goals and design of the Framework.

A cultural lens focuses on curriculum coherence through coordination of diverse perspectives (Hall, Morley, & Chen, 2005; Robutti et al., 2016; Thurston, 1990) or standardisation towards one perspective (Pring, 2012; Schmidt et al., 2005). This affects how a curriculum is decided upon and enacted through the education system. These cultural approaches need not be mutually exclusive but can be invoked depending on the nature of a given curriculum change (Schmidt et al., 2005).

A cognitive lens places the focus on coherence in the learning process that a curriculum is intended to support (Cobb, 1988), or the nature of the domain of mathematics itself (Dewey, 1938; Schmidt et al., 2005; Thurston, 1990), abstracted from individual experiences. In each context, the scale at which coherence is discussed can range from single concepts and individual learners through to regions and entire jurisdictions. The implications of aiming to support coherence, both for equity and for the structure of the curriculum, may therefore be very different.

We apply these two perspectives on coherence to our design in different ways. From a cognitive perspective, we represent mathematics learning according to a particular set of considerations for *what* is described and *why*, and *how* it can be experienced by students through their actions. We share this representation with members of the communities of practice that generate, review, and improve the research we refer to in our research base. From a cultural perspective, we seek to support curriculum coherence by designing the Framework to present research in a form that is relevant to stakeholders when they are making decisions. In this way we hope the Framework will help to foster shared meanings and practices in communities with diverse perspectives. Shared knowledge representations have been shown to facilitate working between groups who have differences in their constraints and priorities (DiSalvo & DiSalvo, 2014; Lee, 2005; Robutti et al., 2016; Star & Griesemer, 1989). We can't control how widely across a system the Framework might be used, but we can seek to make it appropriate for use in coordinating curriculum approaches, materials, and actions.

Designing and evaluating the Cambridge Mathematics Framework

When released, the Framework will comprise (1) a database of mathematical ideas and experiences, defined, referenced, and exemplified as actions and informed by research synthesis and consultation, (2) an interface providing a set of tools for searching and visualising mathematical content and the research base, and (3) a guiding structure that determines what and how ideas are expressed in the database. Eventually we also plan to include connections to specific classroom activities, assessments and professional development resources.

Our design process, described in the methodology section, is guided by the following questions: how can the contents of a mathematics curriculum framework be expressed in a way that: (1) has core features that designers, teachers, and subject experts can interpret and assess relative to their context? (2) emphasises connections? (3) expresses and describes research influences in localised parts of the framework and across the structure of the framework?

Our evaluation of the design is likewise guided by the following questions: is this framework (1) as informed and meaningful as we can make it given the resources at our disposal? and (2) does it make reasonable use of existing research and feedback from collaboration and evaluation?

METHODOLOGY

We consider our approach to be research-informed design. It is a qualitative, interpretive process of expressing mathematics learning, combining theory and empirical research from a variety of sources with descriptions and experience from practice in a way that is explained and documented at a fine-grained level. We draw on models for design processes in education that have been developed and refined within design research methodology in education for over twenty years (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; McKenney & Reeves, 2012). Particular aspects of design research that make some of its methods appropriate for us include: linking specific design priorities and choices to theory; using initial design work to develop design principles that inform ongoing work; engaging in iterative cycles of design in which feedback on work in progress is incorporated into new design versions and practices; and participation in design by experts in multiple relevant communities (Barab & Squire, 2004; McKenney & Reeves, 2012; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). Although our goals differ in some important respects from the goals of design research, we aim to conduct and document our work in such a way that our resulting design might later be able to contribute to research. However, until the Framework is complete enough to be implemented, we rely on face validation with experts to evaluate the content and the structure of the Framework. We are aware that such validation may not lead to generalisable conclusions with respect to curriculum design.

Comparable approaches have been described by framework development projects in other contexts. In a retrospective review of the standards writing process for the NCTM *Principles and Standards of School Mathematics* framework, the writers noted that a set of theoretical perspectives emerged as important influences over time as they collected, analysed, and incorporated feedback on work in progress (Ferrini-Mundy & Martin, 2003). Currently, the UNESCO Institute for Statistics is developing the Reference List & Coding Scheme (RL&CS) framework, intended to provide rich qualitative support for mapping theory and curricula to assessment frameworks. This project's interpretive approach similarly required the designers to evaluate trustworthiness on the basis of practical value of the construct, as expressed through feedback on work in progress by expert members of the communities that would be making use of it (Cunningham, 2017).

Literature review and the research base

We want research to influence the design and contents of the Framework in a way that is meaningful and valid. As mathematics curriculum framework designers have noted in similar contexts, however, there is more to draw on in the literature for some areas of mathematics education than others, and it is also more feasible to employ review methods that identify relevant, essential areas and themes than to complete exhaustive systematic reviews of work in every subdomain (Cunningham, 2017; Ferrini-Mundy & Martin, 2003; Sfard, 2003; Thomas & Harden, 2008). This means that while a design can be grounded in research it cannot be *prescribed* by research, and we do not suggest that our design is the only way of interpreting the research. Rather, we want our design to draw on existing research in the context of our goals for the Framework as a whole. These include considerations that are as much about how mathematical ideas are represented, recognised, and put into action by different users as they are about the nature of concepts, skills, practices, etc., that have been identified and characterised in mathematics learning.

Our inclusion criteria for this process are broad and functional, with the understanding that use of sources will be subject to further review as part of our external expert review process (described below). We note and exclude any source from a particular review that we judge to be irrelevant or lacking reasonable support for its claims. To facilitate expert review and our goal of transparency, we record metadata for research sources to help characterise and communicate our influences. This includes: (1) the source's level of influence on a particular area of the Framework, (2) the search method that retrieved it, (3) publication context and intended audience, and (4) broad category of focus. When sources are used in writing Framework content, they are entered into the Framework database and linked to that content. This makes it possible for writers, reviewers, and users to summarise and examine the influences that have contributed to specific areas of the Framework.

Design and evaluation processes

We have developed a guiding structure for positioning ideas in the Framework that allows us to make them explicit, set scope and boundaries, and find patterns. In this way, it acts as an ontology (Schneider, Siller, & Fuchs, 2011), which Gruber (1993) defines as "the objects, concepts, and other entities that are presumed to exist in some area of interest and the relationships that hold among them." This ontology is not fixed but is something we are continuing to add to and refine. Like any model, our ontology highlights or includes some ideas at the expense of others – often by intent, but sometimes as an unintended consequence of another decision. This means that our Framework may alleviate some problems involving shared understanding while failing to address others, and it is essential that we evaluate our decisions and their implications so that we can both communicate them and identify important changes to make.

We treat designing the ontology and writing the contents of the Framework as intertwined processes. We laid the groundwork for the design with high-level review of theories and approaches and we continue with cycles of review, writing and refinement. Initially, we reviewed a variety of perspectives on classification schemes and 'big ideas' in mathematics education, as well as curriculum frameworks and content documents from a selection of jurisdictions. We used ideas from this process to create a tentative "top-down" way of dividing parts of the domain among members of the writing team. At the same time, we imagined what we might need from the construct from the bottom up and reviewed existing frameworks for conceptual understanding in mathematics (Freudenthal, 1983; Michener, 1978; Pirie & Kieren, 1994; Schoenfeld, 1992; Skemp, 1979; Tall, 1988, 1999; Usiskin, 2015; Vergnaud, 1996 among others) and for learning with understanding (Bransford et al., 2000; Hiebert & Carpenter, 1992; Kieran, Doorman, & Ohtani, 2015; Martin A. Simon, Nicora Placa, & Arnon Avitzur, 2016; Sfard, 2003; Simon & Tzur, 2004; Swan, 2014).

Currently we are in a cycle of writing, discussion, feedback, and refinement of the content and the construct. We have developed a set of tools for writing content into the structure of the Framework, searching and visualising content, and collecting reviews of content. Each writer works according to a cycle of (1) literature review, (2) generation of content, relationships, glossary definitions, research records and any other features called for in a particular area (discussed in more detail below), (3) internal discussion and review, (4) formal and informal external evaluation, and (5) refinement based on feedback.

Good feedback is necessary in order for this cycle to be effective. While informal review has been ongoing, we have enough work in place to begin more formal evaluation, which we divide into processes for two aspects: expert face validation of the structure of the Framework in general (ontology), and the representation of mathematical ideas in specific topic areas. To evaluate the Framework ontology, we are currently conducting a Delphi study with a panel of experts in mathematics curriculum research and curriculum design. Delphi is a structured group survey method for identifying areas of consensus and dispute among experts (Clayton, 1997). It is especially useful for ontology evaluation because it allows us to work with a range of international experts who could not otherwise be convened in the same place, and it helps to mitigate some forms of bias in face-to-face interactions between members of specialised communities. We expect to be able to report the results from this Delphi study in late 2018. When evaluating specific topic areas, external reviewers will be provided with access to the visualisation and search tools used by the writing team. We will then gather feedback through surveys and semi-structured interviews.

At the same time, we are working to characterise relevant existing ways of working among potential users of the Framework so that we can anticipate discrepancies between our initial design assumptions and what might be necessary in order for us to meet our goals for the design of the framework and user interfaces. In addition to user surveys and interviews, we are considering methods for evaluating representations and interfaces that we would use when we are closer to being ready to work directly with potential users of the Framework.

DESIGN OF THE FRAMEWORK IN PROGRESS

The Cambridge Mathematics Framework treats mathematics as a web of ideas with multiple levels of organisation. This web is built as a network in a graph database, in which the mathematical ideas are expressed at nodes and relationships between ideas are expressed as edges. We have developed tools which allow us to search, filter, and visualize the ideas expressed in the Framework, and view different levels and types of information as connected layers (see Figure 1). Currently we are using these tools to design, author, and evaluate the Framework, and in the future they will also form the basis for a set of tools that others will use to interrogate the Framework.

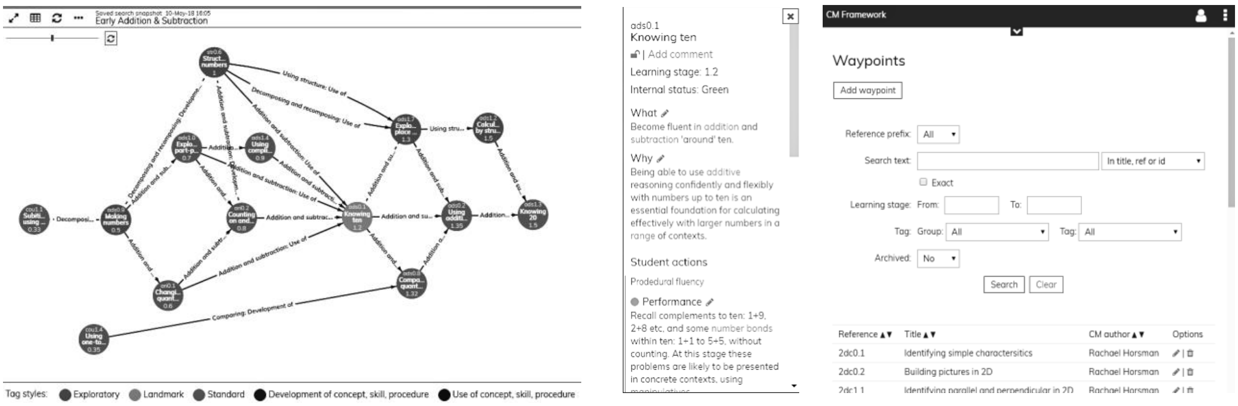

Figure 1: Design tools used to work with and visualise the contents of the Framework

The *mathematical ideas* layer is where we describe mathematical ideas and relationships. The nodes in this layer are *waypoints*, defined as ‘places where learners acquire knowledge, familiarity or expertise'. This definition is influenced by characterisation of learning sequences by Michener (Michener, 1978) and Swan (Swan, 2014, 2015). Each waypoint contains a summary of the mathematical idea (the 'what') and why it is included (the 'why'), and lists examples of 'student actions' that would give students the opportunity to experience the mathematics in meaningful ways. All waypoints in the Framework have the above characteristics, but there are also two special cases. *Exploratory waypoints* usually come at the beginning of a set of linked waypoints. At *landmark waypoints*, ideas are brought together such that the whole experience may seem greater than the sum of its parts. We refer to specific waypoints as *standard waypoints* if we need to distinguish them from exploratory or landmark waypoints. Relationships (edges) between waypoints are *themes*, named according to the concept/skill/procedure we believe the relationship to represent (e.g. 3D Shapes, Inference, etc.). The connection between the waypoints is either described as the *development of* a concept/skill/procedure or as the *use of* a concept/skill/procedure.

The Framework is a construct built by individual authors, and so their decisions about themes and waypoints determine which mathematical ideas are expressed in the Framework and how. The tools we use allow us to connect mathematical ideas in multiple ways and to focus on different sets of ideas and connections at different times. Others might make different choices that could still be entirely reasonable representations of a set of mathematical ideas. This is why we write short white papers which we call *research summaries* to explain specific decisions about the creation and structuring of individual themes. The *research* layer contains these research summaries, along with research nodes and edges, all of which are linked to corresponding features in the Mathematical Ideas layer. We are also developing a Glossary layer, which contains glossary nodes in which key mathematical terms or phrases are defined. These are also linked to the appropriate features in the Mathematical Ideas layer. Ultimately, we expect to create additional layers with features which will contribute to task design, professional development and assessment uses of the Framework.

DISCUSSION AND NEXT STEPS

The influence that research can have on curriculum design and development depends in part on the meaning of that research in the work of various stakeholders in curriculum design, and the ways in which different stakeholder groups are able to coordinate their decision-making and actions (whether informed by research or otherwise). In order to evaluate whether the Cambridge Mathematics Framework shows promise in terms of making a positive contribution to this coordination, we will continue our current and planned evaluation efforts, expanding the process of face validation of the contents and the structure of the Framework beyond our core group of collaborators to a broader range of representatives of stakeholder groups. In addition, we are working with collaborators from curriculum design and resource design stakeholder categories on several small pilot projects in order to develop scenarios for use, and we hope to be able to disseminate the results of these in the coming year. In order for the Framework to help designers to have new insights and develop new solutions, so that they can put the raw material for reform into action, we also work according to our knowledge of the context for reform. We continue to deepen and inform our perspectives on the processes and agents of reform, the dynamics between different stakeholder roles, and issues of communication between immediate stakeholders and the public. While our project is focused on the influence of research in curriculum design and development, the other questions in Theme E are considerations which are equally essential to the eventual impact of the Framework in curriculum reform.

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CHALLENGES INTHE DEVELOPMENT OF REGIONAL MATHEMATICS CURRICULUM STANDARDS: THE CASE OF SOUTHEAST ASIA MINISTERS OF EDUCATION ORGANISATION (SEAMEO)

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This case study illustrated the roles of agents inthe development of the ASEAN regional curriculum standards, particularly the challenges and elaborations to consolidate different perspectives of diverse background in three phases: Firstly, the mathematics curriculum of ASEAN countries were compared and mapped to find the minimum essential contents. The comparison of topics and gradesshowedno intersecting common curriculum between the countries. Secondly, the union of the mappingsfor contents was benchmarked with curriculum standards of developed countries. However, this did not match well with the 21st-century curriculum reform issues. Thirdly, the 21st-century curriculum framework was established emphasising on values and thinking skillswith collaboration oflocal and global agents to finalize the curriculum standards.Comparison of curriculums with other countries isa necessary step toknow the current status of each country, even withmethodological limitations.The 21st-century mathematics curriculum can be realized with the perspective of the process of mathematisation to distinguish the conceptual differences.

INTRODUCTION

There are several efforts for curriculum integration to sharecurriculum standards for the establishment of quality education and securinghuman capitalmobility. Common Core States Standards in the USA is an effort from the state to federal level. Regionally, the Bologna process is established to strengthen the quality assurance of higher education in the European countries.Likewise, Asia Pacific Economic Cooperation (APEC, 2017) is also seeking about integrated efforts to be projected until 2030.In the case of the Association of Southeast Asian Nations (ASEAN) Community, Southeast Asia Ministers of Education Organization (SEAMEO) established Education Agenda #7 *"Adopting a 21st Century Curriculum*" up to 2035 to integrate regional curriculum standards. What are the necessary activities and challenges for curriculum integration?This paper illustrated the challengesin developing the Southeast Asia Basic Education Standards (SEA-BES) in Mathematics under this objective.

RESEARCH QUESTIONS AND METHODOLOGY

Based on the research objective and discussion document of theme E, the role of agents for designing and developing curriculum was chosen. This research illustratedthe role of agents and the challenges in designing and developing the SEA-BES in Mathematics.There are four questions to be answered in relation to the roles of the agents: Q1. How the agents set the format of the standard document?Q2. How was the content of teaching chosen?Q3. What are the principles applied in choosing the contents and writing the standards? Q4. What issues and challenges encountered among the agents were solved? Through answering those research questions, the four foci ofE1to E4 posed in theme E will be answered.

The curriculum development project of SEA-BES in mathematics up to 9th grade was initiated since 2014 and completed in 2017. The project was managed by SEAMEO RECSAM (Regional Education

Centre for Science and Mathematics) under the mandate of the SEAMEO Secretariat. The outcome of the project, which is the proposed curriculum standards, can serve as a platform for curriculum development and assessment of each member country and professional development of teachers imbued with ASEAN ideals in building the ASEAN Community.

Many agents participated and contributedin this project. The SEAMEOSecretariat (2 persons) provided the information regarding related issues of educations and directions were set as in the SEAMEO 7 priority areas. RECSAM director and the specialists (4 persons) were responsible to plan the activities and engaged in the integration of the comparedcurriculum amongASEAN countries.The results on the spreadsheet showed the map for comparison, and subsequently the writing of the standard documents. RECSAM consultant (1 person) suggested the curriculum reform movements of various countries, the formats of writing the standards,informed the content knowledge for teaching and discussed the aspects that werelacking. The RECSAM collaborators, who were leading teachers, teacher educators and professors (30 persons) in Malaysia were involved in the mapping for comparison, developing the initial draft of the standards and checking the proposed document. The curriculum specialistswere government officials in every SEAMEO countries (11 persons) provided the information of their curriculum, critiques of the draft with a comparison to their curriculum standards and provided suggestions for improvement. The international I-experts (8 persons), contributed ideas about on-going curriculum reforms, roles of technology in the reform, inquiry-based and critical thinking as the trend of teaching, and professional development. The Japanese J-experts (7 persons) explained the reform movements, roles of textbooks in Japan and teaching of proof. In the analysis of activitiesand the roles of the agents, only the underlined names were used in the writing. However, contributor's names'was quoted and acknowledgedon the website of SEA-BES.

RECSAM and the consultant wereauthors of this article. The data for discussionin this article were as the following: RECSAM and the consultant retained every edition of their working using MS-words, Excel and e-mails and official reports of the meeting could be seen on the web. These data sequenced by the timeline werethe data for analysis. Analysis of data was done by the following steps: Firstly, based on the timeline, the challenges faced by RECSAM, the consultant, and other agents were specified. Secondly, from the specified challenge, the three phases of the project were clarified throughthe contributions of collaborators, specialists, the consultant and experts. Personal information of the agents waswithheld in the writing. Based on this context,the three phases of the project in relation to the research questionsand the specified foci of E1 to E4 in the themeE were elaborated.

First phase: Comparison of curriculum standards and the mapping

RECSAM initiated the proposal to SEAMEO Secretariat for developing the SEA-BES curriculum standards in 2014 which was aligned to the SEAMEO 7 priority areas.With the consent, the SEA-BES was developed as a part of the 21st-century curriculum with minimum essential contents.Regional meetings were then held to carry out the comparison of curricula from the 11 SEAMEO member countriesand addressed the issues of minimum essentials. Due to the constraints that most curricula were not translated into English, only curriculum of six countries submitted in English were used for review and comparison based on the mathematics terminologies usedin one of the countries. Specialists fromall the 11 member countries were invited to provide commentaries during the second phase.

c	Year 1	Year 2
Domain		
1. NUMBER & OPERATION		
1.1 COUNT, READ AND WRITE, NUMBER TO 20.	Counting orally to 20 and back by reciting zero, one, two, three..	COUNTING, READING AND WRITING, NUMBER TO 1000.
Count Numbers	Giving a reasonable estimate of up to 20 object	Counting numbers in steps of 100 UP TO 1000
Read and write numbers	Reading and writing numbers from 0 to 20 in numerals	Counting in steps of 1 between 100s to 1000.
Counting by progression	Writing number symbols and number names up to 20	Count numbers in steps of 10s and 50s up to 1000.
Recognize zero	Recognising zero, its meaning, and its symbol	Representing numbers up to 1000 in different representation such as concrete, oval and symboloc
Compare and order numbers	Comparing and ordering numbers from 0 to 20.	Reading and writing numbers up to 1000 in numerals and in words
Math symbols	Using words such as more, less, greater, smaller, fewer, the same as, as many as.	Recognizing and representing place values of 3-digit numbers using models and expanded notation
Arrange numbers	Arranging a set of numbers within 20 according to size	Comparing and ordering numbers within 1000
Skip count	Skip counting and recognizing simple number patterns.	Skip counting and recognizing simple number patterns
Ordinal and Cardinal	Using ordinal numbers up to 10th (first, second, third, …, tenth) to describe the position of an object in a row of objects or the order of a set of events.	Using ordinal number up to 30th.

Figure 1. Mapping of Brunei Curriculum (a Part)

Topics	Year 1	Year 2
Domain 1:		
1. Numbers and Number Sense		
1.1 Number Notation	**Visualizing and representing numbers from 0 to 100 using a variety of materials.**	
1.2Count objects	**Counting the number of objects in a given set by ones and tens.**	
1.3Compose and Decompose Objects	**Composing and Decomposing a given number. e.g. 5 is 5 and 0, 4 and 1, 3 and 2, 2 and 3, 1 and 4, 0 and 5**	
1.4Representing objects by numerals		**visualizing and representing numbers from 0-1000 with emphasis on numbers 101 – 1 000 using a variety of materials.**
1.5Grouping Objects		**grouping objects in ones, tens, and hundreds.**
1.6Numbers Visualization		
1.7 Place value -whole number	**visualizing and giving the place value and value of a digit in one- and two-digit numbers.**	**giving the place value and finds the value of a digit in three-digit numbers.**
1.8 Cardinal and Ordinal Numbers: Visualization	**Recognizing Cardinal and ordinal Numbers from o to 100**	
1.9Cardinal and ordinal numbers	**identifying the 1st , 2nd, 3rd, up to 10th object in a given set from a given point of reference.**	

Figure 2. Mapping of Philippines Curriculum (a part)

Primary Mathematics							**Country: Malaysia**					
	pic Progressi						**Learning Objectives**					
Topics	1	2	3	4	5	6	BRUNEI	SINGAPORE	PHILIPPINES	MALAYSIA	CAMBODIA	THAILAND
Domain 1: NUMBERS AND OPERATIONS												
1. Read , write count	/						/	/	/	/	/	/
2. Skip count	/						/			/		/
3. Mathematics symbols	/						/	/	/	/		/
4. Arrange, compare numbers	/						/	/	/	/		/
5. Ordinal cardinal							/	/	/		/	

Figure 3: Mapping in the case of whole numbers for comparison of curriculums

Figure 1 and 2 are samples of the curriculum mapping done by the collaborators based on curriculum of the selected countries. Figure 3 shows an example of a consolidated results. The terminologies in the left column was chosen by RECSAM based on the Malaysian curriculum.The top row shows the countrieswith the intersections showing the grade. In this mapping, the consultant pointed out the weaknesses and inappropriateness of terminologies based on the standards sequence from one country such as Malaysia because other countries may practise differently. For example, cardinal and ordinal number is the basic knowledge of number concept. However, Malaysiadid not use these terminologies in their curriculum document. There wereindeed, several missing terminologies in the first RECSAM mapping. Some of thesewere related tothe cultural-language dimension: In the Malaysian curriculum standards, shape and figure were not distinguished in Malay. On the other hand, some standards in other countries distinguished shape and figure distinctly. Based on these points highlighted, RECSAM revised their screening by adding new terminologies, in the case of 'shape' which was used in other countries. However, the newly added terminologies, 'shape', emerged at the end of the first list of terminologies, whereas the cardinal and ordinal number in Figure 1, 2, and 3emerged at the end of line. The second map lost the linkage amongst terminologies. The first map was based on the terminologies of one country showingthe comparison between that one country and other countries. The second map shows the differences clearly but did not succeed to show it inthe map at a glance. For example, 'shape' is usually taught before 'figure'. However, 'shape' as a terminology appeared later.

The differences clarified by the second map enable theinclusionof terminologies into domains and overcome the differences of content teaching. For example, one country specified that 'money' was included in the domain of numbers while some other countries specified money asa measurement. In another case, some countries never teach geometry with proving until the 9th grade. Under the domain of geometry, angles were related to calculation of geometry, however it was a part of measurement in others. It was found that thereweremore orientation of calculation in the content of mathematics and on contrary, explanation and reasoning was not enhanced relatively. Such differences were clearly seen through the maps (It will be discussed in Second Phase).

Based on those maps, RECSAM attempted to show the curriculum standards with the minimum essential contents. Through the mapping, RECSAM expected that the intersections of all terminologies became the minimum essential contents. However, the results of the mapping still showed the difficulty inidentifying the minimum essentials. There were cases where the same terminologies were used, but the teaching grades differ and resulted in terminologies being not shareable. There was a country that initiated division from the first grade while another country initiated this from the fourth grade. This implied that all SEAMEO countries should initiate the teaching of division from the fourth gradeif the common curriculum standards were defined by the minimum essential contents sharable in ASEAN countries.Based on this particular example, RECSAM would considerinitiating division only at the fourth grade. The above discussion was made among RECSAM and the consultant. Due to thediscrepancy of grading, RECSAM finally decided to change the grading to three key stages: Key stage 1 (grade 1-3), Key stage 2 (grade 4-6) and Key stage 3 (grade 7-9).

Second phase: Benchmarking based on outcomes of the mapping

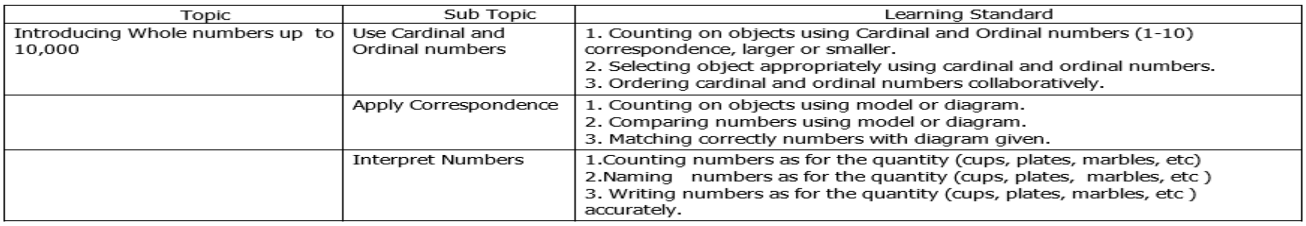
The Secretariat recommended RECSAM to produce the 21st-century curriculum. SEAMEO Priority No. 7 states that 'Pursuing a radical reform through systematic analysis of knowledge, skills, and values need to effectively respond to changing global contexts, particularly to the ever-increasing complexity of the Southeast Asian economic, socio-cultural, and political environment, developing teacher imbued with ASEAN ideals in building ASEAN community'

Topics DOMAIN 1 - NUMBERS AND OPERATIONS_ALGEBRA	subtopic	KS	Countr
1. Whole numbers up to 10000	Whole numbers and Operations		
	Cardinal and Ordinal numbers (1-1 correspondence, larger or smaller,	1	S
	Composition and Decomposition of Numbers)		
	Denominations of numbers as for the quantity (cups, plates, RM, Rp, TB, SD	2	M
	cents, Converts between single denominations and multiple denominations)		
	Understanding and using the Based-10 place value system with model (units	1	M
	cube, tenth bar, hundred square, thousand cube, 0-9 digits, places)		
	Various ways of counting (e.g., Count all, Count on, ascending, descending ..	1	B
	Various units for counting (e.g., Count by 10's, count by 100's, multiplication	2	B
	by 2, multiplications by 5, multiplication by 10's, 100's, 60mins, 20RM, 16oz..)		
	Integers on the Number Line	2	P
	Compare Integers to other Numbers	2	P
	Arrange Integers	2	P
	Compare and Order numbers 1 - 1000; 1000 - 100,000		
	Denominations of numbers as for the quantity (cups, plates, RM, Rp, TB, SD		
	Converts between single denomination and multiple denominations		

Figure 4: A sample of benchmark edited based on collaborator's report for the domain of Numbers and Operation, Algebra. S means Singapore, M means Malaysia, B means Brunei and P means Philippines, KS means number of key stage

RECSAM was seeking ways on how to utilize the maps to formulate curriculum standards that fits the ASEAN countries and challenged to meet the needs of statement No. 7. RECSAM began to draft the curriculum standards for every key stage in relation to the mapsof all the standards of thesixcountries and attempted to quote all countries standards to produce a meaningful documentas the bases for the benchmarking (Figure 4).

RECSAM set the same domains across all three key stages such as 'Numbers and Operations, Algebra'. The collaborators decided the topic names such as 'Numbers up to 10000' based on curriculum documents of the ASEAN rising countries or choosingmore advanced contents in other cases.. RECSAM and the collaborators used the whole map instead of the intersections of mapping for the selection of contents. During the criteria selection on this benchmarking, the consultant guided and provided important information such as meanings of competency (OECD, 2005), and several curriculum standards to RECSAM for clarifying the 21st-century curriculum and the current world curriculum reform movement, which also included Sustainable Development Goals by UNESCO and STEM. RECSAM also provided the curriculum standards of advance countries as reference to the collaborators. These included the NCTM standards (2000), Common Core State Standards (2010), Australia Curriculum Standards (2009) and Japan Curriculum Standards (2012) and the new English edition, curriculum standards of Malaysia and Cambodia in ASEAN countries. The contributions of the collaborators were found to be biasedto their own national curriculum in terms of content selection and description (Figure 4).This was overcomeby RECSAM in consolidating their contributions for benchmarking through discussion among RECSAM and the consultant with the perspectives of curriculum standards in advance countries.



Topic	Sub Topic	Learning Standard
Introducing Whole numbers up to 10,000	Use Cardinal and Ordinal numbers	1. Counting on objects using Cardinal and Ordinal numbers (1-10) correspondence, larger or smaller. 2. Selecting object appropriately using cardinal and ordinal numbers. 3. Ordering cardinal and ordinal numbers collaboratively.
	Apply Correspondence	1. Counting on objects using model or diagram. 2. Comparing numbers using model or diagram. 3. Matching correctly numbers with diagram given.
	Interpret Numbers	1.Counting numbers as for the quantity (cups, plates, marbles, etc) 2.Naming numbers as for the quantity (cups, plates, marbles, etc) 3. Writing numbers as for the quantity (cups, plates, marbles, etc) accurately.

Figure 5: A Partof summarisededition for regional meeting of curriculum specialists from ASEAN

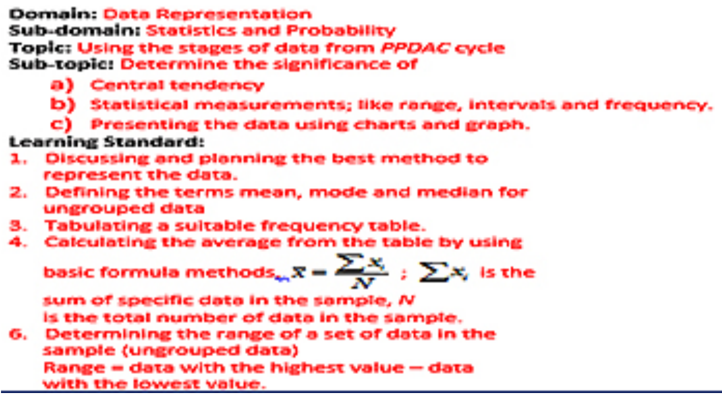
At the regional ASEAN curriculum specialist meeting, the document inFigure 5 was proposed and discussed about the appropriateness of the content mapping for every key stage based on the explanation of RECSAM. The curriculum specialists provided the feedback and shared the difficulties to set the regional standards. Additionally, the consultant also sharedthe fundamental concerns of a 21st-century curriculum for embedding the necessary competencies.The first concernwas'What wasto be benchmarked?'At that juncture, RECSAM proposed (Figure 5) to select the final expected achievement for the topic name up to the 3rd grade in Key Stage 1. If the topic name 'Whole Number up to 10000' is given in Key Stage 1, the competency such as 'Number up to 120' is embedded and need not be described. However, such a way of writing the benchmark wasnot suitable to developthe necessaryprocess skills. For example, counting objects, inthe early first grade, the first object for counting, direction for counting, and the last object for counting were necessary. It is not the counting by the base ten system such as 10 of 100, 10 of 1000 and so on. In that case as in Figure 5, most of the process skills for learning and doing could not be shownunder the limitation of writing "up to 10000": we never count by onesup to such large numbers.

The second problem is the choice of terminologies. For example, in Geometry: Shape, Figures, and Plane Figures are different terminologies for specific objects in the curriculum. The Plane Figures extend the sides, otherwise, there will be no discussion on the equality of area with the same height,and ex-circles can be constructed by extending the sides of a triangle. Plane Figures are the object of proof in geometry. Some countries never teach Plane Figures which deprivesthe learning opportunities for explanation (proving). Explanations with critique areusually done by examples and counterexamples.If Shape, Figure, and Plane Figuresare distinguished, in discussion for redefining the meaning of terminologies, critiques are possible to be initiated even from Key stage 1. Amending the sequence to develop thinking mathematically is possible by distinguishing the conceptual differences with various terminologies. The third problem is the usage of “domain”.Keeping the domain names across the key stages enhanced the compartments while “strands” enhancedthe connectivity of different concepts.It is not necessary to keep the same names for strandsand standards beyond the key stages because atevery stage, the concept of numbersis extended and reorganized in appropriate manners.

For solving those problems, specialists, RECSAM and the consultant neededto consolidate the description of the regional curriculum standards and a further (third) meeting was set.

Third phase: Formulating the framework and finalising the document

In the thirdmeeting, RECSAM and the consultant revised the draft of the standards based on the outcomes ofthe previous meeting (see, Figure 6). Before the third meeting, the draft document was sent to all specialists in every ASEAN country with questionnaires, such as:1.Meaningof the national curriculum in your country. There were diversities on the meaning of national standards,includingthe assessment standards, textbooks and every daylesson plans.The diversities itself had produceddifficultiesfor sharing; 2. Structure or format of the national curriculum document with terminology for formatting curricula such as standards, learning outcome, and teaching and learning activities. The format of the standards was the major problem for a common consensus; 3. Diagram for explaining aims of the national mathematics curriculum standards for 21st century; 4. Explaining some examples of their national standards from their aims and diagrams; 5. Commentary about the tentative format of the RECSAM standards (Figure 6); 6. Commentaries to the RECSAM standards for feedback, such as how far or close to your national curriculum, and what arethe challenges for youto overcomethediscrepancy.


Domain: Data Representation
Sub-domain: Statistics and Probability
Topic: Using the stages of data from *PPDAC* cycle
Sub-topic: Determine the significance of

a) Central tendency
b) Statistical measurements; like range, intervals and frequency.
c) Presenting the data using charts and graph.

Learning Standard:

1. Discussing and planning the best method to represent the data.
2. Defining the terms mean, mode and median for ungrouped data
3. Tabulating a suitable frequency table.
4. Calculating the average from the table by using basic formula methods, $\bar{x} = \frac{\sum x_i}{N}$; $\sum x_i$ is the sum of specific data in the sample, N is the total number of data in the sample.
6. Determining the range of a set of data in the sample (ungrouped data) Range = data with the highest value – data with the lowest value.

Figure 6:Sample format proposed from RECSAM for the third meeting

Third meeting was carriedout with additional presentations from I-experts and J-experts for consolidating the standards from the international perspectives. In relation to the first and third problems on phase two, I-experts provided information on the followingitems: Common Core States Standards in the USA, Exploration with Technology, Reform of Curriculum for Open-Ended Approach, Curriculum Development with Collaborative Enquiry, Verb- based Curriculum Numeracy, Critical Thinking, Curiosity, Possible Challenges in Thailand, and Professional Development. In relation to all three problems in the second phase, J-experts provided the information on the following items: Ongoing Reform of Junior High School, Proof and Proving for Developing Critical Thinking on Geometry on Joint Project with the UK. Those inputs of the experts were to clarify the aims, objectives, roles and contents of the standards. For example, some specialists who recognised geometry as measurement were also able to recognise the calculation of angles as a form of proof in geometry.

Atthe meeting, the specialists presented their answers for the questionnaires andgave theircommentaries.RECSAM and the specialists learned various frameworks from each other such as the diagrams in Figure 7. Against theRECSAM standards description,the specialists gave positive commentariesand provided the comparisons of their content standards highlighting thedifferences and theircapacities in handling challenges that emerged. Based on the discussion, RECSAM and specialists chose the three rectangular-framework as in Figure 8,consisting of components like Mathematical Values, Attitudes and Habits for Human Character; Mathematical Thinking and Process,and Contents.For embedding every countries' aims such as in Figure 7.For allowing the possible interpretation of sentence in the standards on the context of every country, the four hierarchies format instead of the five hierarchies in Figure 6 were shared and set the way of writing by using verbs and adjectivesto clarify the process-humanity strands in the content and showing the conceptual differences and connectivity between standards within the same strands and between different stages.

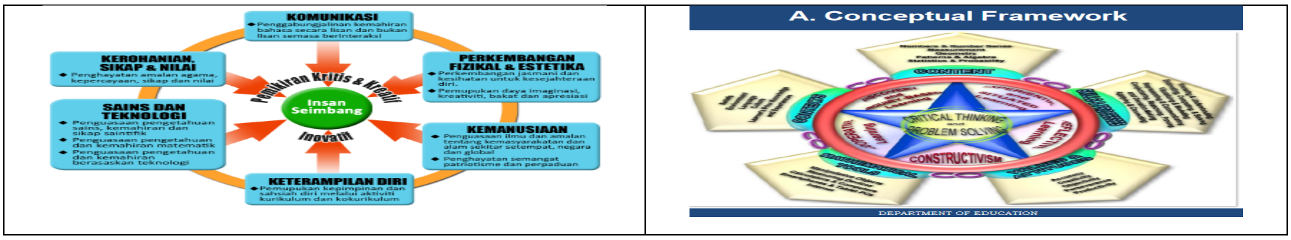


Figure 7: Malaysian Diagram (left) and Philippine Diagram (right)

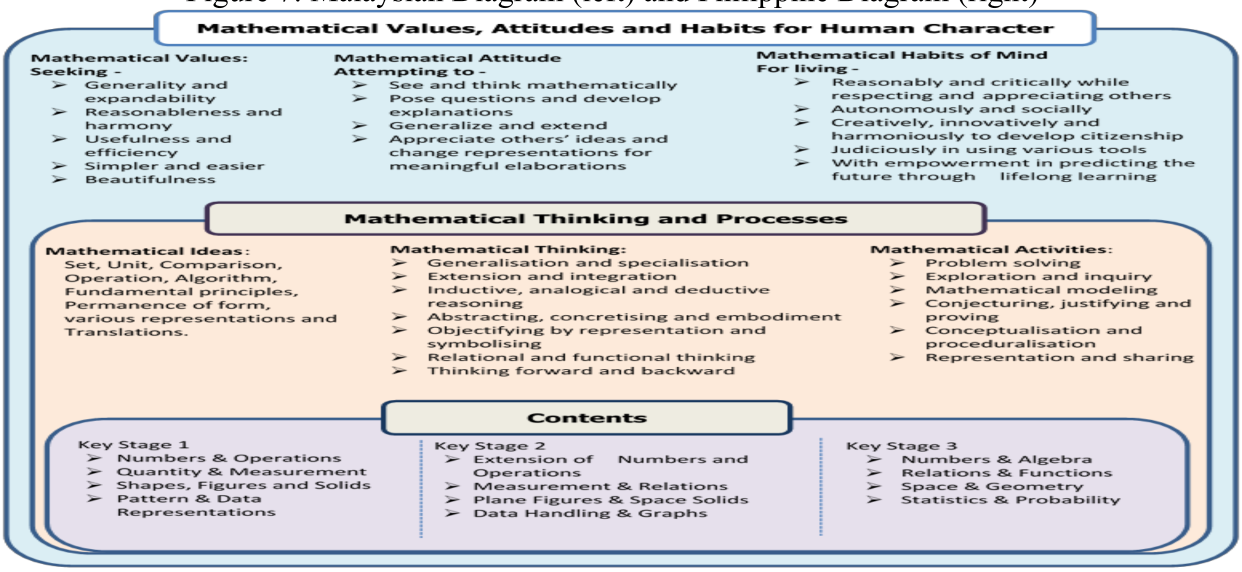


Figure 8: SEA-BES Common Core Regional Learning Standards in Mathematics Framework for the 21st Century

CONCLUSION

In the analysis of the roles of agents, this report concludes the following answers to the research questions. About Q1, RECSAM changed the bases for formatting the standard from the map based on terminologies under the minimum essential contents to describe the process and humanity for the 21st-century curriculum with contributionsfromother agents at every stage. About Q2, contents were initially chosen from the intersection of the maps, and later from the whole map for benchmarking, and further included the process-humanity for 21st-century curriculum in Figure 8. About Q3, the first hypothesis for writing the standards by mapping was based on the mathematics terminology which can be divided into the consistent domain names. The second hypothesis for writing standards focus on distinguished conceptual differenceswhich enabledthe process-humanity strand based on the mathematisation which was also symbolised by the different strand names in different key stages in Figure 8. About Q4, there were challenges and elaborations among the different agentsresulting frombias for their own national standards. The inputs from I-experts, J-experts and experts from non-ASEAN member countryfor the 21st century curriculum were able to set international perspectives and overcome biases.

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DIVERSITY IN CURRICULUM COMMITTEES: CHALLENGES AND OPPORTUNITIES FOR CROSS-COMMUNITY COLLABORATION

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Reform committees typically comprise members from diverse communities of practice. Research has shown that achieving productive cross-community collaboration in mathematics education is far from straightforward. The work of reform committees is typically confidential, yet circumstantial evidence suggests that cross-community interactions are less productive than they could be. Due to the crucial influence of such committees on mathematics education at the national level, we suggest that cross-community interactions in reform committees should be an explicit topic of research. We propose boundary-crossing as a framework, and apply it to analyze cases of collaboration, including a setting that simulates the work of committees – a mathematics education forum where mathematicians, educational researchers and teachers (including past and present members of reform committees) meet to discuss issues of common interest. We identify the crucial role played by brokers in facilitating cross-boundary learning. We propose that boundary-crossing should be an explicit aim for reform committees, and call on the community to intentionally study the role of broker in this context.

INTRODUCTION

Committees concerned with mathematics curriculum and reform in Israel, as in many other countries, are culturally diverse. They are usually chaired by a research mathematician, and comprise teachers, teacher educators, mathematics-education researchers and ministry officials, as well as mathematicians. As a result, committees can be said to have multi-faceted expertise in issues of mathematics education, providing valuable resources to draw on, and increasing the likelihood that outcomes will be aligned with the interests of all stakeholders in mathematics education. However, these communities bring diverse values and perspectives regarding the teaching and learning of mathematics, which may lead to conflicting opinions and tensions that could hinder reform processes. One way to address this challenge is to limit cross-community interaction by differentiating between epistemic, pedagogical and institutional issues, and designating less diverse sub-committees to address particular issues. In such a model, committees can be said to function as “the sum of their parts”, drawing on individual fields of expertise separately. We would challenge such an approach, claiming that issues of mathematics-education policy are rarely single-faceted. We would rather see committees functioning as more than the sum of their parts, where diverse communities learn from and with each other, and draw on multi-faceted expertise to develop novel ideas and insights. The internal workings of committees usually remain confidential, even after their work is done, yet based on circumstantial evidence and on informal discussions with past and present members of committees, there is reason to believe that cross-community interactions are less productive than they could be. Furthermore, our own research on interactions among stakeholders in mathematics education shows that, though possible, achieving productive collaboration is not straightforward. Yet it may have a crucial effect on educational policy at the national level.

The question that guides this article is: How can members from different communities in mathematics education collaborate productively in curriculum and reform committees? This study is part of a long-term research project that explores cross-community collaborations in mathematics education, the potential contribution of such collaborations, and what support is required in order to achieve this potential. In this paper, we re-visit several of our studies that examined cross-community collaboration in the context of teacher-education and re-examine the data and the findings from the theoretical perspective of boundary-crossing, to draw implications for the current research question.

WHAT DO WE KNOW ABOUT CROSS-COMMUNITY COLLABORATION IN THE CONTEXT OF REFORM COMMITTEES?

Mathematicians' involvement in mathematics education has a long history, starting with eminent figures such as Felix Klein, Hans Freudenthal and George Pólya. However, over the years mathematics education has established itself as a separate academic discipline, with its own distinct epistemology, philosophy and methodology, drawing on theories and paradigms from the social sciences and psychology. This shift has led to increased diversity in the community of mathematics education, to the point where the views on mathematics teaching and learning prevalent among various stakeholders are not just profoundly different but are essentially incommensurable (Sfard, 1998). This diversity has contributed to severe conflicts that have emerged over recent decades between mathematicians, teachers and mathematics educators, including the debate over educational reforms in the U.S. known as the Math Wars (Kilpatrick, 2001). In particular, disputes over the question of who is most competent to decide in matters of curriculum reforms in mathematics education threaten to destroy "the productive, reasonably cohesive community [that] has flourished in U.S. mathematics education [and] that has brought together research mathematicians, schoolteachers, and university mathematics educators to address issues of curriculum and instruction in school mathematics" (ibid., p. 104).

We do not know whether or how these disputes influence the work of committees in mathematics education. Research on the challenges and opportunities that cultural diversity brings to mathematics education is scarce, and in relation to the work of mathematics education reform committees is non-existent, as far as we can tell. Yet there is ample circumstantial evidence. Research that examined cross-community collaboration in mathematics education has suggested that while the mathematical knowledge of research mathematicians might be highly valuable for school mathematics education, it requires substantial transposition to become relevant (Bass, 2005). Our own research (e.g. Cooper & Karsenty, 2017; Cooper & Pinto, 2017, Accepted; Pinto & Cooper, 2017) has corroborated and refined this observation. We found that cross-community collaboration was achieved in a deep sense only when members from different communities made explicit and brought for discussion their (often tacit) goals, norms, values and perspectives. This finding echoes Sfard's call (1998) for an informed dialogue that will advance cross-community collaboration in mathematics education by clarifying what the words 'mathematics' and 'learning' mean in the different communities of practice. Our research has also indicated that capitalizing upon cultural diversity is far from straightforward. More often than not, cultural diversity hinders cross-community collaboration rather than enhancing it. Interviews that we have conducted with past and present members of mathematics curriculum and reform committees, including mathematicians, school-teachers, educational researchers and ministry officials, suggest that these committees rarely capitalize on the opportunities of their diversity.

THEORETICAL FRAMEWORK

From our perspective, for the work of mathematics education policy and reform committees to be considered productive, it should support committee members from different communities in learning from and with each other, thus gaining new curricular understanding and insight – both individually and collectively. We frame this criterion theoretically, as follows. We draw on Commognitive theory (Sfard, 2008) in viewing knowledge pertaining to mathematics curriculum as a form of *discourse*, i.e. the ways in which individuals or communities communicate, think and act with regard to the mathematics curriculum. This *curricular discourse* comprises mathematical, pedagogical and institutional sub-discourses. In commognitive theory, discourses are distinguished by 4 partially overlapping aspects: relevant keywords (e.g. understanding mathematics), commonly used visual mediators (e.g. multiplication tables), endorsable narratives (e.g. "by 4th grade all children should have memorized the multiplication table"), and repeating routines (e.g. procedure for long division). Communities of practice (e.g. research mathematicians, teachers, teacher educators, educational researchers) can be distinguished by these 4 aspects of their curricular discourse. For example, they may make use of different keywords, or of the same keywords with subtly different meaning, and they may disagree on which narratives about curriculum should be endorsed/rejected, or even on the grounds for endorsing/rejecting narratives (epistemic, didactic, institutional).

We view the work of committees comprising members from different professional communities as boundary-encounters, where the notion of *boundary* is defined as "sociocultural differences leading to discontinuity in action or interaction" (Akkerman & Bakker, 2011, p. 133). From our perspective, these sociocultural differences are taken as differences in curricular discourse, and discontinuities are also discursive in nature, such as *commognitive conflict* – communication across incommensurable discourses, which differ in their use of keywords, mediators or routines (Sfard, 2008). Akkerman and Bakker's (2011) review of research on boundary encounters has characterized processes of learning through *boundary-crossing*, which we take to mean transitions and interactions across different discourses. Boundary-crossing may require that individuals "enter onto territory in which [they] are unfamiliar and, to some significant extent therefore unqualified" (Suchman, 1994, p. 25) and "face the challenge of negotiating and combining ingredients from different contexts to achieve hybrid situations" (Engeström, Engeström, & Kärkkäinen, 1995, p. 319). Two learning mechanisms are pertinent to our research context. *Reflection*: explicating aspects of one's discourse with respect to the discourse of others, thus coming to learn something new about one's own perspective (a process coined *perspective making*), while possibly changing one's perspective in the process (*perspective taking*). *Hybridization* – the emergence of a new cultural form in which ingredients from different contexts are combined into something new and unfamiliar (Akkerman & Bakker, 2011, p. 148). This can take the shape of new tools or signs (Engeström et al., 1995) or of a new practice that stands in between established practices (Konkola, Tuomi-Gröhn, Lambert, & Ludvigsen, 2007).

Two elements have been found to be instrumental in boundary-crossing. *Boundary objects* and *brokers*. Boundary objects are artifacts that fulfil a bridging function between incommensurable discourses. For example, in (Cooper & Pinto, 2017) we demonstrated how the narrative "the square root of 18 is closer to 4 than to 5 because 18 is closer to 16 than it is to 25" can serve as a boundary object that can support boundary-crossing between mathematical and pedagogical discourses, inviting mathematicians and teachers to reflect on an explicate the norms of argumentation that are acceptable in their communities. Brokers are individuals who are conversant in the discourse of more than one community, and engage in – or facilitate – boundary-crossing events, for example by helping discussants articulate tacit norms and views. In these terms, we reiterate our research question as:

How can the work of policy and reform committees in mathematics education be organized to support and encourage boundary-crossing among representatives from different communities of practice?

METHODOLOGY

Circumstantial evidence suggests that the work of curriculum committees is not as rich in productive collaboration (i.e. boundary-crossing) as it could be. Since the work of reform and policy committees is confidential, we must look elsewhere to answer our research question on how productive collaboration can be encouraged and supported. Thus, we examine data collected in prior research that we have conducted on interactions among various communities that are stakeholders in mathematics education (for the most part research mathematicians and teachers) and reframe findings in the theoretical perspective described above in order to investigate the nature of productive learning that took place. On the basis of our analyses we suggest implications for the makeup and the functioning of policy and reform committees. In this paper, we discuss in detail two cases from separate studies. The first case, previously analyzed in (Cooper & Pinto, Accepted), is a 2-hour discussion that took place in the "Math-Ed Crossings club" (MEC) program at the Weizmann Institute of Science, a forum formed by the first author of this paper that brings together teachers, mathematicians and researchers of mathematics education approximately once every 2 months, to discuss issues pertaining to mathematics education that are of common concern and interest. Several members of the MEC forum hold prominent positions in present and past curricular committees, and we view it as a "lab" for investigating collaboration across communities. The second case, previously analyzed by Cooper and Karsenty (2016), is a lesson taught by a mathematician in a professional development course for primary school teachers. The meeting of two communities presented numerous opportunities for boundary-crossing, encouraging the mathematicians to investigate pedagogical aspects of elementary mathematics, while encouraging teachers to delve deeply into epistemic nuances of the content (see also Pinto & Cooper, 2017). We also draw on an exploration we conducted of mathematical argumentation from mathematical and pedagogical perspectives (Cooper & Pinto, 2017), in which we demonstrated how the interaction of these perspectives can lead to new insights on the mathematics at stake, on its learning, and on its teaching.

ANALYSIS

Case 1: Exploring the pedagogical affordances of enrichment material for school students

In this section we analyze the boundary crossing in a discussion that took place in the MEC forum, which focused on pedagogical affordances of enrichment material for secondary-school students designed by a mathematician. These data were previously analyzed in (Cooper & Pinto, Accepted).

The boundary object: Henri, a research mathematician who is deeply involved in mathematics education policy in Israel, developed a game as enrichment material for school students. The game, a computer-based version of Tic Tac Toe, draws on the popular card-game SET in that it models a geometry of an affine space over a finite field (see details in Cooper & Pinto, Accepted). At the time, Henri acknowledged that he had not yet given much thought to what children could gain from playing the game, but he was confident that the game could provide a worthwhile learning activity, inside or outside school. In particular, Henri maintained that even without mediation, playing the game could inspire students to rethink or even abstract and re-conceptualize the geometric concept of 'line'. Henri presented his game in the MEC forum, with the participation of 2 research mathematicians (including Henri), 3 experienced secondary mathematics teachers, and 3 researchers of mathematics education (including the authors). Thus, the game became a boundary object in a 2-hour discussion in which the 8 participants investigated, both pedagogically and mathematically, the relevance of the game for school mathematics. We followed up with semi-structured interviews with Henri and with Abby, an experienced advanced-track high school teacher who holds a Ph.D. in Mathematics Education and who is a member of a mathematics education reform committee.

Nature of the boundary: While both Henri and Abby were enthusiastic about extra-curricular enrichment material that engages students in mathematical inquiry, they had very different perspectives about the relevance of the game for school mathematics, the opportunities for learning mathematics that it affords, and the kind of mediation that would be needed for capitalizing upon these opportunities. Henri viewed mathematical activity as an iterative process of abstraction and concretization. Accordingly, Henri framed mathematics learning as developing competency in abstraction and concretization, which could be fostered through student engagement with concrete mathematical objects outside the curricular content, such as a line that consists of a finite number of points. In his design, Henri did not consider mediation, and he maintained that even if students are unaware that they are 'doing mathematics' while playing the game, over time the mathematics would 'percolate' at some intuitive level. Abby viewed school mathematics primarily as a problem-solving activity. She maintained that students should be presented with interesting and meaningful problems within the curricular content in order to encourage them to inquire and gain further insight into the content they are already familiar with. Abby was critical of the notion that students can construct new mathematical objects on their own, and maintained that the teacher has a crucial role in mediating new mathematical ideas if student inquiry is to be productive.

Nature of the learning through boundary crossing:

We recognize mechanisms of both reflection and hybridization in learning that took place through boundary-crossing. Henri, in response to Abby's criticism, took a clear stance on the affordance for students' learning through abstraction, while conceding that mediation might be required for significant learning of mathematical ideas. Abby eventually agreed that there is pedagogical merit in extending students' conception of line. However, she felt that Henri, from his university perspective, was underestimating the gap between what she called a concrete, continuous line and its extension to something discrete, which for students does not yet justify the name line. To bridge these perspectives, she suggested that playing the game on a grid larger than the one Henri designed might bring the "new" line object, which is discrete and cyclic, perceptually closer to the familiar continuous notion of line.

Case 2: Mathematicians teaching professional development courses for primary school teachers

In this section we analyze the boundary crossing that took place in a lesson taught by a mathematician in the context of a professional development course for primary school teachers (PD hereafter).

The boundary object: Cooper and Karsenty (2016) analyzed in depth one lesson on division with remainder (DwR). The impetus for this lesson was an inadequacy – in the eyes of the mathematician

– of the standard notation for DwR. Applying the transitivity property of equality to the expressions 17:5=3(2) and 11:3=3(2), or to 7:2=3(1) and 14:4=3(2), would respectively imply that 17:5=11:3 and 7:2≠14:4, which the mathematician considered "nonsense". To resolve this issue, the mathematician initially suggested that DwR should employ an alternate notation for equality, one that is not symmetric (e.g. "=>"). Yet, after discussions with the researcher, he agreed that an alternate notation for the *result* would be more appropriate. The proposed notation was: 17:5=3(2:5), where the result would be read as "3 with remainder 2, which we tried to divide by 5". Though the relevance of an alternate notation, in the eyes of the mathematician, was mathematical in nature (retaining the equivalence properties of equality), it was also found to be pedagogically relevant in paving the way to fractions, and as such served as a boundary object, inviting the mathematician to investigate pedagogical aspects of mathematical notation, and teachers to investigate epistemic aspects.

Nature of the boundary: The mathematician felt that unlike other mathematical objects learned in primary school, expressions like 3(2) – read "three remainder two" – lack a well-defined and inherent meaning as a quantity. Thus, from the mathematician's perspective, the use of '=' in expressions such as 17:5=3(2) is an "abuse of notation" (cf., Wu, 2011, p. 379). The teachers, in contrast, were not conflicted about this notation, accepting 3(2) as a quantity made up of two whole parts (quotient and remainder). Both communities were acutely aware of the mathematics on the horizon of DwR – fractions – yet this horizon served different roles. For the mathematician, it was primarily the criterion by which the equality 17:5=11:3 was judged to be nonsense (17/5≠11/3). The teachers, in contrast, had no problem with this equality, recognizing that "having the same DwR result" is an equivalence relationship on whole number division exercises, though not the same equivalence as "having the same rational result". Their attention was on the affordances of such a notation for a smooth transition to fractions in the 5th grade – 2 years hence. Thus, though they agreed that the new notation has merit (endorsing a new visual mediator), they disagreed regarding the (in)adequacy of the standard notation and the inherent (in)correctness of the narrative 17:5=11:3. These different notions of correctness extended to the endorsement of other mathematical narratives. For example, the mathematician and the teachers debated on whether 21:3=7(0) is a valid solution for the exercise __:3=7(__). Mathematically, 0 is a legitimate remainder, yet in some teachers' experience, this solution often indicates that students have not fully grasped the notion of remainder as "something left over".

Thus, the incompatibility of the parties' discourse regarding the mathematical correctness of the equality 17:5=11:3 was revealed as incommensurability of their discourse regarding the relationship between DwR and fractions. Their discourse was also to some extent incommensurable in their use of the keyword "correct", which they used with mathematical and pedagogical meaning respectively.

Nature of the learning through boundary crossing: Much of the learning that took place in this meeting came from interactions between mathematical and pedagogical discourse. The mathematician was persuaded by the pedagogical critique of the initial notation he suggested "17:5=>3(2)", and eventually endorsed a notation that stretches his notion of quantity, and which is a *hybridization* of the different discourses. One of the teachers recognized a specific pedagogical affordance of the proposed new notation; in the transition to decimal long division her students often neglect to divide the "remainder", and she felt that introducing the new notation in 3rd grade would alleviate this difficulty in 5th grade, signaling the remainder as something that is "pending division". We view this as a kind of hybridization, where a new notation draws meaning from two discourses.

DISCUSSION

We have described some encounters of boundary-crossing among stakeholders in mathematics education. We now discuss the relevance of our findings for the work of curriculum committees.

The discursive diversity in curricular committees poses both challenges and opportunities. We have distinguished between *incompatibilities* between discourses (i.e. endorsing conflicting narratives) and *incommensurabilities* (i.e. differing in their use of keywords, mediators or routines). Incompatibilities that are not incommensurable may be quite simple to resolve. For example, Abby's and Henri's disagreement on the extent to which Henri's game would motivate children could be resolved through empirical research, assuming that they ascribe similar meaning to the keyword *motivation*. In contrast, incompatibilities that are grounded in incommensurabilities, for example Abby and Henri's disagreement regarding the *concreteness* of affine lines, may be difficult to resolve, since the conflicting discourses may not share criteria for deciding which narratives to endorse and which to reject. However, we have shown that such situations can be valuable in their affordance for learning.

We have demonstrated two general mechanisms of learning through boundary-crossing. *Reflection* is a process by which communities can learn from each other by sharing perspectives - explicating their own perspectives (e.g. Henri on learning through abstraction) or re-aligning them (e.g. Abby on the value of extending the familiar notion of *line*). *Hybridization* is a process of transforming practice, drawing on two or more discourses to create something new (e.g. a new notation with complementary mathematical and pedagogical affordances). We have demonstrated how these processes occurred through boundary-crossing in professional development and in a culturally-diverse mathematics education forum, and propose that such processes can and should occur within curriculum committees. What might it take to support such processes? To answer this, we reflect on what supported boundary-crossing in our examples.

<u>Extrinsic motivation to resolve incompatibilities</u>: At a point early in the PD (Case 2), participating teachers began to complain that what they were learning was not relevant for their classroom teaching. This induced the mathematicians running the PD to explicate their own notion of relevance for teaching, and to adopt parts of the teachers' notion of relevance. Similar extrinsic motivation may exist in committees, for example, crucial decisions sometimes require a unanimous vote, creating an incentive to reach agreements. However, we have shown in the case of the PD (Pinto & Cooper, 2017) that the effect of such motivation was often limited, generating a rather ritualistic adoption of the teachers' perspective on the part of the instructors, and leaving both sides discontent. In other words, extrinsic motivation to resolve incompatibilities did not necessarily entail boundary-crossing. Yet in some cases the mathematicians did cross-boundaries, engaged more exploratively with the teachers' notion of relevance-for-teaching, and drew on it to articulate new learning goals. We have shown that in these cases they achieved a deeper relevance. Applying this insight to committees, we suspect that in itself, the requirement to vote unanimously will not instigate boundary crossing.

<u>Brokering</u>: Research often focuses on individuals who either engage in or facilitate events of boundary-crossing. Such individuals, called brokers, are typically members of more than one community, or are at least "conversant" in these communities' discourse. Much of the boundary-crossing that we have described was brokered by the authors, each of whom is a member of at least two communities: both are researchers in the field of mathematics education who hold graduate

degrees in mathematics and have some experience in mathematical research. Furthermore, the author who brokered in the case of the PD has experience as a school teacher. Having a “mathematician’s perspective” proved to be valuable when interpreting the teachers’ claim regarding the equivalence properties of equality in the context of DwR and for communicating it to mathematicians. A “teacher’s perspective” was important for appreciating that a new notation for remainder would encounter less objection from teachers than a new notation for equality. An “educational researcher’s perspective” was instrumental in teasing out incommensurable notions of the keyword “concrete” in Abby’s and Henri’s discourse. We would argue that educational researchers are natural candidates for brokering among these communities, since their profession requires them to take an explorative approach to the discourses of multiple communities, in an attempt to see the sense in them.

Currently, committees often do include individuals who are members in more than one community, yet we propose that this may not be enough. In our view, boundary-crossing should be an explicit aim for committees, and the role of brokers as facilitators of boundary-crossing in this context should be intentionally studied. Designating committee members as brokers may support committees in making the most of their diversity, thus functioning as much more than the sum of their parts.

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THE SIGNIFICANT NARRATION OF MATHEMATICS TEACHERS' PROFESSIONAL IDENTITIES IN AN IRISH MATHEMATICS EDUCATION POLICY DOCUMENT

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The aim of this paper is to examine how the professional identity of mathematics teachers is constructed in a mathematics curriculum reform policy document. The policy document is analyzed within a theoretical framework based on discourse theory and narrative identity. The paper investigates how policymakers may act as significant narrators through policy documents in an attempt to 'oughthor' – that is, to author what ought to be – the designated identities of mathematics teachers. The public narrative told about Irish secondary school mathematics teachers in this document describes two figured worlds. The paper argues that images of figured worlds presented through public narratives in policy documents serve to influence teachers' professional identities as part of the process to initiate and implement mathematics curriculum reform.

INTRODUCTION

The aim of this study is to examine the role of policy documents as part of the processes of curriculum design, development and reform in secondary school mathematics. This paper reports on the use of public narratives in an Irish mathematics education policy document which may (re)construct teacher identities and initiate curriculum reform. The paper does not report on the identity of secondary school mathematics teachers in Ireland: rather it illustrates how a public policy document about secondary school mathematics education can produce certain understandings and self-understandings of teacher identity. The policy document is analyzed using discourse analysis (Gee, 2011) combined with Sfard and Prusak's (2005) narrative approach to identity which focuses on discursive positioning and the work of Holland, Lachicotte, Skinner, and Cain (1998) on figured worlds. This study aims to build on similar work carried out by Søreide (2007) in Norway, Thomas (2005) in Australia and Morgan (2011) in England to further examine how policy documents can influence teacher identities by framing how teachers can perceive their experiences teaching, perform their role as a teacher, justify their practices, and position themselves as "good" teachers. This study addresses the following research questions:

- How do agents lead or dominate aspects of curriculum reform through policy documents?
- What processes are deployed in curriculum documents to aid in the development and implementation of mathematics curriculum reform?
- How is research used to inform or influence curriculum design and implementation processes in policy reform documents?

This paper addresses theme E: Agents and processes of curriculum design, development and reforms in school mathematics.

THE CONTEXT OF CURRICULUM REFORM

In Ireland since the turn of the 21st century, the mathematics education provided for students (5-13 years old) at primary level and for students (13-18 years old) at secondary level has been subjected to constant review which resulted in curriculum reform at both levels of schooling (Leavy, Hourigan, & Carroll, 2017). This paper focuses on the curriculum reform of secondary mathematics education in Ireland. At the end of secondary school, students complete the Leaving Certificate, a national terminal examination overseen by the Department of Education and Skills with the results used for entrance to tertiary education. The students have the option of completing the examination at higher-level, ordinary-level or foundation level. A passing grade in Leaving Certificate mathematics is a requirement for entry to any university program. In the early 2000s, there was considerable dissatisfaction with the mathematical skills and knowledge of secondary school students and the extent of examination focused teaching by secondary school mathematics teachers (Lyons, Lynch, Close, Sheerin, & Boland, 2003; State Examinations Commission, 2005). It was the surfacing of these issues that resulted in the National Council for Curriculum and Assessment (NCCA), the body responsible for the formulating and reforming curriculum and assessment policy in Ireland, publishing a review of secondary school mathematics education in 2005 (Oldham, 2006). This review entitled *Review of Mathematics in Post-Primary Education: a discussion paper* is the focus of the paper. Following the review, in 2008 the NCCA began the phased introduction of the revised 'Project Maths' syllabus. In 2017, for the first time, all students who undertook the mathematics Leaving Certificate examination had studied the Project Maths syllabus from their initial year in second level education (State Examinations Commission, 2015).

In many countries, there is evidence that mathematics curriculum policy documents can be significant indicators of the intent of policy makers to effect change. For example, in England, in 2014, a new mathematics curriculum for students (11-14 years old) at Key Stage 3 and students (14-16 years old) at Key Stage 4 was developed (Department for Education, 2014b). By September 2016, all schools under local authority in England had to teach the reformed programs of study (Department for Education, 2014a). The reform process in England began with the publication of the Teaching White Paper which was followed by a review of the existing curriculum by an expert panel, consultations and research into mathematics education in a number of countries which were regarded as high performers (Donnelly & Wiltshire, 2014). In Australia, in 2009, the Australian Curriculum, Assessment and Reporting Authority (ACARA) was established to develop a National Mathematics curriculum. As part of this process ACARA published papers documenting the development of the curriculum (Donnelly & Wiltshire, 2014). A later review of the new curriculum (Donnelly & Wiltshire, 2014), highlights that the phase for shaping the curriculum consisted of three steps. Firstly, a position paper was developed that identified and responded to key issues that needed to be resolved. The second step consisted of preparing initial advice papers and discussing ideas at national fora. Finally, the process was complete with the development and publication of draft and final shape papers. This paper examines a document produced as part of phase one for mathematics curriculum reform in Ireland to provide a new way of theorising the relationship between discourse, curriculum reform and reform of mathematics teachers' practices.

THE PUBLIC POLICY DOCUMENT

The document used for this research was selected because it preceded public consultation, a report on this consultation and a commissioned review of international literature on mathematics curriculum and assessment. This policy document was the first step in the process of shaping the reformed mathematics curriculum as it identified existing issues on the basis of empirical studies, international test scores and Leaving Certificate examination performance. It was a significant link in the genre chain[1] as it targeted the existing curriculum, teaching, learning and assessment practices with a view to changing the culture of mathematics education in Ireland (NCCA, 2012). The subsequent introduction of the Project Maths curriculum, provision of professional development for qualified mathematics teachers and out-of-field mathematics teachers[2] and the growing body of research on mathematics education in Ireland demonstrates the change to the mathematics education landscape that was partly instigated by this public policy document. The public narrative presented in this policy document is examined with regard to the identity it is trying to attribute to secondary school mathematics teachers (Gee, 2011).

THEORETICAL FRAMEWORK

A teacher's professional identity refers to the way teachers defines themselves to themselves and to others as they account for their practices and their positionings within official and unofficial discourses of teaching (Lasky, 2005; Morgan, 2011). It comprises self-understandings which are discursively and narratively constructed by the ways the teacher relates to the world and to other people, their decisions, their practices, the language they use and the narratives they tell and hear about themselves and others (Holland et al., 1998; Sfard & Prusak, 2005; Søreide, 2007). These self-understandings are the meanings one makes of oneself as one engages in the process of authoring the self (Holland et al., 1998). Gee (2011) uses the term Discourses to describe the characteristic ways of saying, doing, and being. These Discourses exist within figured worlds, which are historical phenomena that individuals are recruited into or enter. These figured worlds are socially organized and reproduced wherein participants' positions matter. They consist of processes and traditions that give form to the lives of their inhabitants and are populated by identifiable kinds of people (Holland et al., 1998). By enacting Discourses which are located within figured worlds, Gee (2011) contends that individuals project themselves in a bid to be recognized as a certain kind of person engaged in a certain kind of practice. As such, these Discourses which are practiced identities construct subject positions which are then attributed to participants (Morgan, 2011). In the context of teaching, the accessibility of the subject positions influences teachers' understanding of their job (Søreide, 2007). These subject positions, which prioritize a specific view of the world while simultaneously blocking other ways of understanding and experiencing the world, can be presented in public narratives (Søreide, 2007). These are narratives told by policymakers in documents about persons who are affiliated with large cultural and institutional formations (Somers & Gibson, 1994 cited in Søreide, 2007). Public policy school documents which author the official Discourse through public narratives about teachers have the ability to construct teacher professional identities (Gee, 2011; Morgan, 2011; Søreide, 2007). They provide persons with resources for identity construction as they author

[1] The genre chain refers to the strategy and associated documents that are part of a reform agenda (Taylor, 2004).

[2] An out-of-field teaching mathematics teacher refers to a teacher who is teaching mathematics without the requisite qualifications (Bosse and Törner, 2015).

utterances which communicate socially situated identities and socially situated practices (Gee, 2011; Søreide, 2007). Gee (2011) explains that the socially situated identities refer to the *who* while the socially situated practice refers to the *what*. The *who* and the *what* are inseparable as you are who you are in part due to *what* you are doing, and *what* you are doing is identified by *who* is doing it. Gee (2011) explains that utterances communicate an integrated "*who-doing-what*".

Sfard and Prusak (2005) operationalize a narrative discursive view of identity by equating identities with stories about persons. This identifying technique replaces utterances about *doing* and *actions* with reifying sentences about *being* and *having*. Sfard and Prusak (2005) purport that identities may be defined by narratives about individuals which are reifying, endorsable and significant. The reifying aspect refers to the repetition of actions and is demonstrated by words such as *be*, *have* and with adverbs such as *always* and *never*. A narrative is endorsable when the identity-builder claims that it reflects the state of affairs in the world. A narrative is significant when any alteration to the story would affect the storyteller's feelings about the identified person. Sfard and Prusak (2005) note that every identifying story can be represented by a triple which describes who is telling the story, whom the story is being told about and to whom the story is being told. This indicates that the story is a bid to be recognized as a kind of person and may be told differently or interpreted differently depending on who is telling the story and to whom it is being told.

In the policy document that is reviewed in this paper, the story being told about secondary school mathematics teachers is authored by the NCCA and being interpreted by researchers. This is referred to as a $_{\text{B}}{}^{\text{A}}{}_{\text{C}}$ triple $_{\text{NCCA}}{}^{\text{Secondary school mathematics teachers}}{}_{\text{Researchers}}$ – whereby the identity-builder, the NCCA, is a third party telling the story about secondary school mathematics teachers to a third party. Sfard and Prusak (2005) explain that the identifying narratives of a person can be divided into current identity (formerly referred to as 'actual identity') and designated identity. *Current identity* narratives consist of stories about the current state of affairs told using *present tense verbs* and *factual assertions*. *Designated identity* narratives consist of narratives which for some reason are expected to be the state of affairs in the future as they have the potential to become part of one's current identity. These narratives which guide action can be identified by the use of *future tense* and *modal verbs*. Designated identities are created from already existing narratives through a process of discursive diffusion. In this way, those to whom the stories are told about the identified person(s) and those who tell the story about the identified person(s) may be co-authors of the identified person(s)' designated identities. Sfard and Prusak (2005) explain that the significance of the storyteller to the identified person(s) determines whether the story told makes it into one's designated identity. The storytellers who are referred to as *significant narrators* have the most influential voices given their position of authority and power. These significant narrators transmit the cultural messages which have the greatest impact on one's actions. In mathematics education in Ireland, the NCCA are significant narrators of mathematics teachers' designated identities given their role in policymaking. Therefore, by analyzing the Discourses advocated for by the NCCA through public narratives contained in public policy documents, the identity resources that are produced as part of the curriculum reform process can be examined.

A DISCOURSE ANALYTIC APPROACH

The analytic approach undertaken for this research applied Sfard and Prusak's (2005) framework for 'Telling Identities' combined with the work of Gee (2011) on situated meanings and figured worlds as tools of inquiry. For phase one of the analysis, the text was analysed under the themes of assessment, pedagogy and resources. For phase two of the analysis, the discourse relating to these themes was analysed using Sfard and Prusak's (2005) operationalization of current identities and designated identities by examining the text for identifying adverbs and past tense, future tense and modal verbs. For phase three of the analysis, Gee's (2011) situated meaning tool was applied by examining what specific meanings teacher-readers may attribute to the words and phrases in the document that depict current and designated identities, given the context and how the context is construed by the NCCA. Gee's (2011) figured worlds tool was also applied to text pertaining to current and designated identities by examining what typical stories and figured worlds the words and phrases in the policy document are assuming and inviting the readers to assume about assessment, pedagogy and resources. These typical stories about these themes were analyzed based on the participants, activities, ways of interacting, forms of language, people, objects, environments, and values referenced in the text.

FINDINGS

Identifying Functions

The public narrative told in this policy document has identifying functions as it is *endorsable, reifying*, and *significant*. The public narrative is *endorsable* as the identity-builder, the NCCA, indicates that the policy document is an evaluation of the state of affairs in mathematics education in Ireland.

> The review is not simply an exercise in syllabus revision—although this may be an outcome of the review—but rather a more fundamental evaluation of the appropriateness of the mathematics that students engage with in school and its relevance to their needs. (NCCA, 2005, p. 3)

The public narrative is telling the typical stories of mathematics education at second level in Ireland and is *significant* as any change in this story would result in a change in the NCCA's views about secondary school mathematics teachers.

> A change of culture is required, together with a change in practice. (NCCA, 2005, p. 6)

The public narrative produced in this document provides a reification of secondary school mathematics teachers: The document explicitly refers to the *what*, but this also implies the *who* and contains words with reifying properties such as '*have*'.

> … the 'elitist' status that *Higher level mathematics* can sometimes *have* in schools among students and teachers. (NCCA, 2005, p. 10, emphasis added)

A Discourse surrounding higher level mathematics and higher-level mathematics teachers – that is, the *who-doing-what* – is presented here. The situated meaning of *elitist* implies higher level mathematics teachers may see themselves as superior to other teachers in the school. This text is an example of the reification of the identity of higher-level mathematics teachers in the document.

As Gee (2011) indicates the integrated nature of *who-doing-what*, then if the story of the socially situated practice changes then the socially situated identity changes also. This indicates that the policy document has the capacity to influence teacher identity construction as it is an identifying public narrative. The ways in which it may affect teacher identity are examined, based on the current and designated identities that are illustrated through the Discourses and respective subject positionings available within two figured worlds presented in the text – namely, the traditional mathematics classroom and the reform-oriented mathematics classroom.

Current Identity – The typical story told about secondary school mathematics teaching

This policy document presents images of the current state of affairs in secondary school mathematics teaching. Research findings are used to suggest that the majority of secondary school mathematics teachers teach in the same way.

> The findings of research (Lyons et al., 2003) into the teaching and learning of mathematics in second-level schools in Ireland suggest a high level of uniformity in terms of how mathematics lessons are organised and presented. (NCCA, 2005, p.17)

The policy document describes the currently enacted style of teaching with reference to international studies.

> Evidence from international studies suggests that Irish classrooms are largely 'traditional', involving teacher exposition and (probably, followed by) individual pupil work (Lapointe et al., 1992; Beaton et al., 1996). (NCCA, 2005, p. 17)

The use of the word *are* describes the current state of affairs. As Gee (2011) notes that as practice is in part recognized by who is doing it, then the practice of being largely traditional constructs the identity of secondary school mathematics teachers. The policy document describes the enacted Discourses within the figured world of the traditional mathematics classroom.

> As evidenced by inspection visits, teaching is highly dependent on the class textbook (which tends to reinforce the 'drill and practice' style). (NCCA, 2005, p. 21)

Ma and Singer-Gabella (2011) characterize the traditional figured world of mathematics teaching as being driven by didactic teaching that is highly ritualized with teachers presenting procedures followed by students practicing these procedures. In this figured world, students are the receivers of knowledge. This is the figured world presented in the document as being the current state of affairs. The policy document uses value-laden terms such as *suffered* and *helplessness* to depict the effects of this style of teaching on students. This approach positions students within the figured world as victims of poor teaching methodologies and mathematics teachers who teach in this way as inadequate.

> Students who have suffered from a 'tell and drill' or 'busywork' approach (bereft of meaning) may already have learnt this helplessness before they enter second level school. (NCCA, 2005, p. 18)

Designated Identity – The typical story to be told in future about secondary school mathematics teaching

The policy document contrasts the figured world of the traditional secondary school mathematics classroom with the figured world of a reform-oriented mathematics classroom.

> A teacher who believes that mathematics is a bag of useful but unconnected tricks is likely to emphasise different things than will a teacher who believes that mathematics is a body of knowledge as near to absolute truth as we can get, a web of beautiful relationships, or an activity involving the formulation and solution of problems. (NCCA, 2005, p. 18)

The comparison between *a bag of unconnected tricks* and *a web of beautiful relationships* demonstrates that a reform-oriented mathematics teacher is passionate about the subject. They don't carry the tricks of the subject with them in a bag: rather they are totally immersed in the beauty and value of the subject. The policy document employs future tense and modal verbs to author what ought to be, '*oughthor*', the designated identity of mathematics teachers.

> If a genuine re-appraisal of mathematics education is to lead to significant change, attention *must* be paid to the *need* for teachers to move away from the traditional approach, which may have been their own experience as students and/or which may have served them well as teachers up to now, and to embrace a new philosophy and associated methodology that *will* best serve future generations of students. (NCCA, 2005, p. 18, emphasis added)

The enacted Discourses of secondary school mathematics teachers within the reform-oriented classroom are described throughout the document. The document refers to the need for teachers to be aware of the links between mathematics and other subjects and within mathematics, and that students should acquire relational understanding and an appreciation of mathematics in their lives. Ma and Singer-Gabella (2011) describe that in the figured world of the reform-oriented mathematics classroom students are active learners making sense of knowledge and the relationships between mathematical ideas. The narrative plot told in this document can be described as students are suffering within the traditional mathematics classroom and that to rectify this, teachers need to move away from their comfort zone to enact a new Discourse by adopting alternative teaching philosophies and methodologies that foster a new reform-oriented figured world within their mathematics classrooms.

DISCUSSION AND CONCLUSION

The analysis of this mathematics curriculum policy document illustrates that agents such as policymakers can lead curriculum reform by acting as significant narrators of teachers' designated identities. In the policy document, this is achieved by the presentation of an identifying public narrative. Søreide (2007) states that public narratives are a powerful force as they circulate intentions, expectations and demands, which in turn produces certain Discourses, subject positionings and current and designated identities. The narrative plot in this document described the preferred identities of teachers teaching within a reform-oriented classroom. This plot in turn positions teachers who are not teaching in this way as inadequate. Curriculum documents either implicitly or explicitly advocate for particular approaches to teaching and learning to be employed in the classroom (Donnelly & Wiltshire, 2014). In the document analysed in this study, the NCCA employed evidence from research studies to construct images of the traditional figured world of mathematics teaching and the figured world of reform-oriented mathematics teaching. The policy document challenges the identities of secondary school mathematics teachers and argues that these teachers ought to enact different Discourses to become a different type of mathematics teacher. In doing so, the review serves to assist in initiating and implementing curriculum reform by *oughthoring* the designated identities of secondary school mathematics teachers. By analyzing mathematics education curriculum policy documents' public narratives through the lens of figured worlds and discursive narrative identities, it is possible to examine how reform is encouraged through the endorsement of a new discourse for teachers to account for their practice.

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HOW TEACHERS AND RESEARCHERS CAN COOPERATE TO (RE)DESIGN A CURRICULUM?

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This paper is grounded on a design-based research, in which a curriculum for the First and Second Grades in Arithmetics has been built. This curriculum, termed Arithmetic and Comprehension at Elementary School (ACE), has been elaborated within a team composed of teachers and researchers, and implemented in classrooms. We present a concrete example of the way teachers and researchers work together to improve some aspects of the ACE curriculum, by designing "Inquiry and Training Threads". This improvement work relies on the use a Picture-Text-Audio Hybrid System (PTAHS) that we briefly sketch.

INTRODUCTION

This paper is grounded on a design-based research, in which a curriculum for the First and Second Grades in Arithmetics has been built. This curriculum, termed *Arithmetic and Comprehension at Elementary School* (ACE), has been elaborated within a team composed of teachers and researchers, and implemented in more than 500 classrooms since 2012.

There are numerous interesting research works in Arithmetics at the beginning of the elementary school (for example Brousseau, 1997; Fuson 1990; Fyfe et al., 2014; Ma, 2010; Ding & Li, 2014; Bartolini-Bussi & Sun, 2018, etc.) but it is difficult to find curricula specially designed to provide a *practical synthesis* of these works, specially at First and Second Grade. ACE is an attempt to build such a curriculum, which is informed by research. In this paper, we address two questions of the Theme 3 of ICMI Study 24. We try to give the reader a general understanding of the ACE Research, from two points of view. The first one refers to the way research could "inform or influence curriculum design" (in question 3 of the theme 3); the second focuses on "What are the processes, and how are they deployed, in the development of and during a mathematics curriculum reform? What agents lead or dominate and what is their influence on the aspects of curriculum reforms?" (question 1 of the theme 3). To concretize that, we present an *in-situ* example of the way the ACE research team work together to improve some aspects of the ACE curriculum, by designing "Inquiry and Training Threads", that one may see as a way to work out the two questions above. This improvement work notably relies on the use a Picture-Text-Audio Hybrid System (PTAHS), an exploratory digital instrument that we briefly sketch. In the limited space of this paper, we will only present a few features of the ACE curriculum process and structure.

THE ACE CURRICULUM: PRINCIPLES AND RATIONALE

The ACE Curriculum: a brief description of the research design

The ACE project refers to the construction of the concept of number, which we are experimenting with 6 and 7 year-old-students (First and Second Grade). ACE gathers together five French research teams, involving teachers, mathematics psychologists, and mathematics didacticians, each of them having designed a part of the whole curriculum. In this paper, we focus only on the Brittany-Marseille team, which has in charge a domain called "Situations". The making of the curriculum is based on the implementation of a *cooperative engineering*, that is a specific form of design-based research (Cobb et al., 2003), which develops specific relationships between teachers and researchers, according to a symmetry principle (Sensevy et al., 2013). This principle relies on the idea that there is no *a priori* division of work between teachers and researchers, and that each member of the collective is able to reflect on all the problems encountered by the collective.

The first year (2011-2012) of the experiment consisted of designing a curriculum for the building of the concept of number, at first grade. This designing process was carried out in a specific way, in that the elaborated situations were first experimented in four classes, called Study Classes, and redesigned iteratively. *The Study Classes Teachers were members of the research team*, they completed a Thesis Master, and they should be considered as teachers-researchers. The curriculum was implemented at first grade in 60 experimental classes the second year of the experiment (2012-2013), in 120 experimental classes the third year of the experiment (2013-2014). Since 2014-2015, a second-grade curriculum was designed according to the same structure and process.

This research was a quasi-experimental design. In effect, each year, students' learning in the experimental classes has been compared with students' learning in control classes (pre-test/post-test assessment). It worths noticing the two main results that have been obtained. 1) For each year of the experimentation (2012-2013 and 2013-2104, first-grade; 2015-2016 and 2016-2017, second-grade) the experimental classes students outperformed the control classes students. 2) For each year of the experimentation, the gap between students from underserved communities (Priority Education Zones, in the French System) and students from middle class communities was largely widening along the school year in the control classes, and stay at the same level in the experimental classes. The ACE curriculum can be viewed as built in a specific kind of evidence-based research (Fisher et al., accepted).

The ACE Curriculum: principles and rationale from a mathematical viewpoint

From a conceptual viewpoint, the ACE curriculum is based on the following principles. **1)** Familiarizing the students with numbers and relations within numbers by focusing first on "small numbers" for a long amount of time (Ma, 2011). **2)** Giving a prominent importance to the study of the equivalence, in that students become able to think of the equality sign not as a hint to produce an operation, but as a relational sign (Brousseau, 1997, McNeil, 2014). **3)** Using first the arithmetic operations as means to explore numbers and build significant relations between them. For example, in the core situation of this curriculum, the students are guided to be able to refer to a number in an additive form (a sum) and to compare it, with other additive forms in particular, by using seminal conceptual strategies of relevant composition/decomposition ($3 + 4 = 3 + 3 + 1= 6+1$; $8 + 4 = 8 + 2 + 2 = 10 + 2$), decimal understanding ($24 = 20 + 4 = 10 + 10 + 4$). **4)** Using manipulatives and representations in a systematic way, by satisfying two criteria. The first one refers to the necessity to enable students to rely first on manipulative and concrete "objects", then to study iconic (analogical) representations of numbers (e.g. the "number line"), then to write down equations in "canonical" form. This process seems very close of the Chinese textbooks tradition (Ding & Li, 2014) and can be thought of as "concreteness fading" (Fyfe et al., 2014). The second criterion lies in a "translational principle". To understand various properties of numbers, students had to compare different representations of the same mathematical reality in order to become progressively able to recognize the differences and the similarities between these representations. **5)** The fifth principle of the ACE rationale holds as follows. To have the pupil acquainted with the historical-cultural sense of mathematics (Radford, 2014), and to apprehend their deep conceptual structure (Richland, Stiegler & Holyoak, 2012), students had *to write* mathematics, and to develop a first-hand relationship to mathematical writing. **6)** The last principle of the ACE rationale refers to the conception of mathematics this curriculum enacts. Mathematics should be worked out in an inquiry process, in that students become progressively able to build a first-hand relationship to mathematical inquiry.

The ACE Curriculum: principles and rationale from a cooperative viewpoint

The ACE curriculum has been designed through a specific form of design-based research, that we term *cooperative engineering* (Sensevy et al., 2013; Joffredo-Le Brun et al, 2018). It may be characterized through its twofold structure. One may describe the research design in two spheres. The *Sphere 1* is composed by the research team, which means Study Class Teachers, researchers, PHD Students, teachers trainers, and pedagogical advisors. The Experimental Classes Teachers represent the *Sphere 2*, especially among them 50 teachers in Provence and 50 teachers in Brittany, which are in ongoing contact with the research team (Sphere 1). To understand the research and design process, one has to take into account the impact of this twofold structure on the implementing of the ACE curriculum 1. The first year of the research project, the Sphere 1 designed a first curriculum, tested in the Study Classes, then refined to be proposed to the Experimental Group (The Sphere 2) the second year. 2. The second year of the research project, the Experimental Classes Teachers (Sphere 2) implement the curriculum 2 for the first time. The Study Class Teachers (Sphere 1) implement the curriculum in the same conditions as the experimental group. For them, it is thus the second year they carry out most of the main situations of this curriculum.

Before the implementation process, an initial training session enables the Experimental Classes Teachers (Sphere 2) to understand the main components of the curriculum 2. During the implementation process, the relationships between Sphere 1 and Sphere 2 are as follows. Firstly, we have developed a national website (http://blog.espe-bretagne.fr/ace/) and an internet forum in which every teacher may raise some issues encountered in her implementing the sequence. Some other teacher may answer these questions; the research team does it systematically. Secondly, there is a mailing list specific to the geographic zone (i.e. Provence or Brittany…), through which the teachers may notably share various kinds of reflections and resources. Thirdly, every six weeks Sphere 1 and Sphere 2 meet in a training session in which they work out together the implemented curriculum. Fourthly, a week-long training session, called “Assessment and Project” gathers together the two Spheres (the research team and the experimental teachers) at the end of each year of experiment. On this basis, one feature of this cooperative engineering consists in the continuous changes in the curriculum that the research team may propose to the experimental classes teachers (Joffredo-Le Brun et al, 2018). In effect, frequent meetings within the research team (Sphere 1) enable these Sphere 1 participants to rely on the study classes' implementation, and on the observation of some experimental classes, to design hypothesized curriculum improvements. These improvements are proposed as working hypotheses to the experimental classes teachers, on the research website or during the teachers training sessions. This type of structure enables the research team (Sphere 1) and the experimental classes teachers (Sphere 2) to enact together a specific kind of *cooperative inquiry*, in which every participant (member of the research team or experimental teacher) is considered capable of relevant proposals (symmetry principle). This is the first part of such a process (within Sphere 1) that we describe in the following section.

COOPERATION IN THE IMPROVING OF THE ACE CURRICULUM

In this section, we will focus on an example of cooperative work within the Research Team, at second grade. As we mentioned it above, the second grade ACE curriculum has been implemented within classrooms since 2015-2016, and it is now relatively stabilized. The main research team's work consists of analyzing the implementation of the curriculum in various classrooms, and specifically in the Study Classes, to propose some improvement. Some of these modifications slightly change the curriculum. Some others ask for a kind of redefinition of some curriculum's features. In what follows, we focus on one of these important changes, the implementation of what we called "Inquiry and Training threads".

Inquiry and Training Threads

We argue that the implementation of a curriculum should rely on a continual assessment of its mathematical efficacy. One way to insure this assessment is the statistical analysis of performances, as it has been performed, in ACE research, within the quasi-experimental paradigm that has been carried out. Another complementary way is to observe the implementation process directly and then try to propose local improvements of the curriculum. As an example of a melting of these two strategies, we will start from one exercise of the second-grade posttest. It included, at the end, a problem that has been presented during the test. The statement of the problem was as follows (in French): "At recess, Dimitri play marbles. At the beginning of the game, he has 37 marbles. At the end, he has 72 marbles. How much has he won?". Students were given 5 minutes to solve the problem, using a pencil and an empty 8.5 x 16.5 cm rectangular frame (included in the student's booklet) for research and computation. The results of this posttest problem were very interesting. The data show that the students in the experimental group (ACE) produced the correct response "35" slightly more in total than the students in the control group, 20.20% versus 18.23%, $\chi^2(1) = 2.90, p = .089$. However, *they produced considerably less the response "109"*, which results from the working out of the false operation, 5.08% versus 12.33%, χ^2 (1) = 80.45, $p < .001$. This kind of result was very encouraging, in that ACE students demonstrated a better understanding of the mathematical structure of the problem, but a bit disappointing too, given that the ACE students, even though they perform in a better way that the control group students, were not able, for 80% of them, to provide the good answer. An examination of the way ACE students who have found the good answer performed led the research team to study similar ways of solving the problem, by using a number line of the following type (figure 1, a ACE student's production):

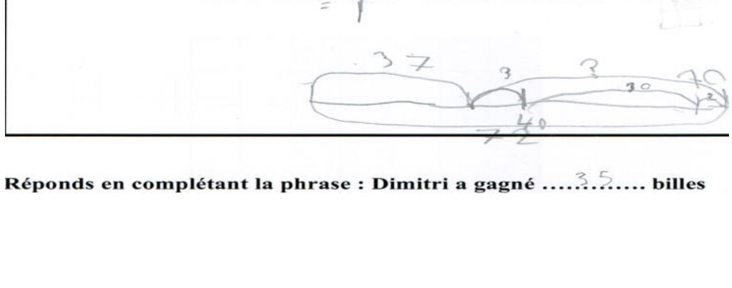

Figure 1: A student's way of solving the marbles problem by using the number line

In this production, the student used a representational system at the core of the ACE curriculum, the *number line.* He first drew a "bridge" which represented 37, one which represented 72, and one which represented the difference between 37 and 72, with an interrogation mark on the top. Then he proceeded on the number line. He added 3 to 37 to reach 40 (first bridge after 37), 30 to reach 70 (second bridge after 37), and 2 to reach 72 (third bridge after 37). After adding these three numbers, he wrote the "good answer" (Dimitri a gagné 35 billes/Dimitri gained 35 marbles) under the representational system. It is important to notice that this student's procedure was not at all an invention of this own. Such a use of this number line was a distinctive strategy of the ACE curriculum, and it can explain why the ACE group outperformed the control group in the understanding of the mathematical structure of the problem. But the test showed the research team that the calculation process was not mastered enough, while giving them an evidence, by considering students' productions as the one above, that using the number line as a topological device was very promising. This kind of result brought the research team to become aware of a fundamental fact. Students should carry out an inquiry process, a first-hand relationship to mathematical inquiry, that is at the core of the ACE curriculum. *But they should also train themselves a lot* to acquire a deep number sense and calculation skills. In this perspective, the research team (sphere 1) began to develop, in synergy with the experimental teachers (sphere 2), what has been called "Inquiry and Training Threads". An "Inquiry and Training Thread" is a mathematical practice in which students *gave themselves a small "exercise"*, by choosing the structure and the "numbers" of this exercise, work it out, then share their answer with other students, by discussing the various proposed exercises. The teacher plays a prominent role in this discussion, notably by helping students understand the mathematical techniques they used, and by institutionalizing some of these techniques to make them available for each of the students in the classroom for their future productions. To ensure the "training" part of an "Inquiry and Training Thread", this mathematical practice occurs several times a week in the classroom, often every day, on a short lapse of time (from 5 to 15 minutes), but on the long duration of the whole classroom year, as a work in progress. Students then experienced an enduring mathematical work, growing into culture of mathematical inquiry.

Sharing and Improving an Inquiry and Training Thread

The "Inquiry and Training Thread" practice was integrated to the ACE curriculum, firstly in Second Grade, since the end of 2016-2017. The first "Inquiry and Training Thread" implemented was termed "Exploring the number line". The students gave themselves a difference exercise (for example 64-37), then used the number line as roughly described in the example above to find the solution. This mathematical practice ("Exploring the number line") gradually enriched the ACE curriculum, along with other "Inquiry and Training Threads" (for example "Problem Posing"). Two fundamental questions were then raised: i) How to share deeply this kind of practice through the experimental group; ii) how to improve this kind of practice. To answer these questions, the Research Team designed digitalized films of practice, "Picture-Text-Audio Hybrid Systems". Such devices aimed to enable people a better knowing and understanding of the filmed practice (e.g "Exploring the number line" practice). We describe now more precisely an example of such a design (its first part).

SHARING AND IMPROVING AN INQUIRY AND TRAINING THREAD: AN EXAMPLE

This iterative process unfolded as follow. First, a text was elaborated about "Exploring the Number Line", which is available online in its consecutive versions, each of them modified trough the research meetings. Based on this text, PTAHS (Picture-Text-Audio Hybrid System) were designed in an iterative process. We focus now on our concrete example. *At step 1*, the research team simply decided to videotape an ordinary current practice of one of the Study Class teacher (Grade 2). *At Step 2*, the whole session (15 min) was filmed. It consisted to having two dyads of students working on an "Exploring the Number Line" task, then analyzing these productions with the whole classroom. We give below the final state of this work for the first dyad (figure 2), which shows how different semiotic systems may accompany the number line, in a kind of "translation" process between different representations.

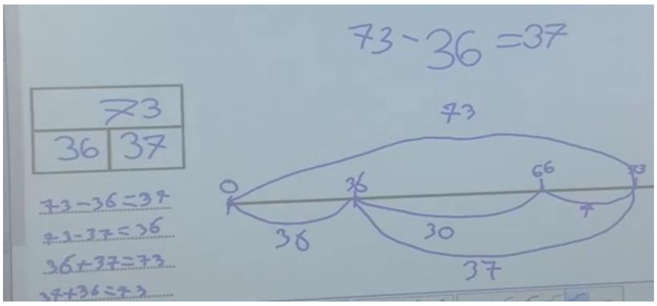

Figure 2: First dyad's production (Step 2

At Step 3, the discussion inside the research team brought it members to notice that "Exploring the number line" could be conceived of as a mechanical technique, in which approximation processes were discarded. The research team designed a new session, in which students should estimate the result by "computing on tenths" (for example 73 - 36 could be estimated as 7 tenths minus 3 tenths, so 4 tenths). *At step 4*, a new version of the "Exploring the number line" session line was so implemented. Students were asked to first estimate the result, by "computing on tenth". Students, who were trained in ACE to consider numbers through the decimal numeration structure, could estimate very well results of subtractions, but a very interesting problem occurred when the Study Class Teacher tried to write down students' reasoning. Consider the following photogram (figure 3) in the classroom, where the teacher writes down the used solving strategies under the students' dictation.

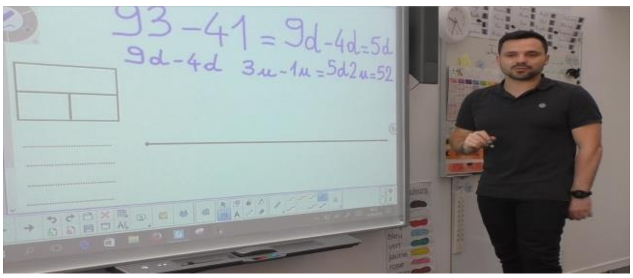

Figure 3: The writing down of the "computing on tenth" (Step 4)

At Step 5a, by studying the writing of the equality (figure 3), the Study Class Teacher said to the other members of the team that he was not satisfied by this writing, the affirmed equality 93 – 41 = 9 d – 4 d = being false. Another concern referred to the second line of this calculation. How to find a rigorous and simple way to show the “separate calculation” of tenths and units? A discussion followed within the research team, which lead to a new working hypothesis. The research team decided to test the use of the “approximately equal symbol” (≈) to solve the first problem. This symbol was introduced in the following session, as shown below (figure 4, step 5 b). While listening to the students’ dictation, the teacher proposes new ways of writing mathematics.

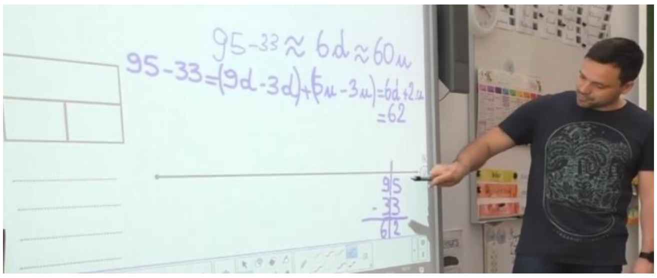

Figure 4: Using the “approximately equal” symbol (Step 5b)

It is interesting to notice that the Study Class Teacher found a way to solve the second teaching problem (showing the “separate calculation” of tenths and units) by using a parenthesis system. As a member of a research team, this teacher had participated in many discussions in which the use of parenthesis was proposed as a working hypothesis. Reminding these conversations, he used the parenthesis system above (figure 4) to show the separate calculation. A few days after, the students currently used these two semiotic systems in their “Exploring the Number Line” activity. Even though such semiotic system (“approximately equal symbol” and parenthesis system) are generally not used at primary school, they seem useful in that they provide students with a shared way of symbolizing their mathematical though. Finally, as a glimpse, we give a copy of the “homepage” of the PTAHS designed from this example (English translation and subtitling are in progress).

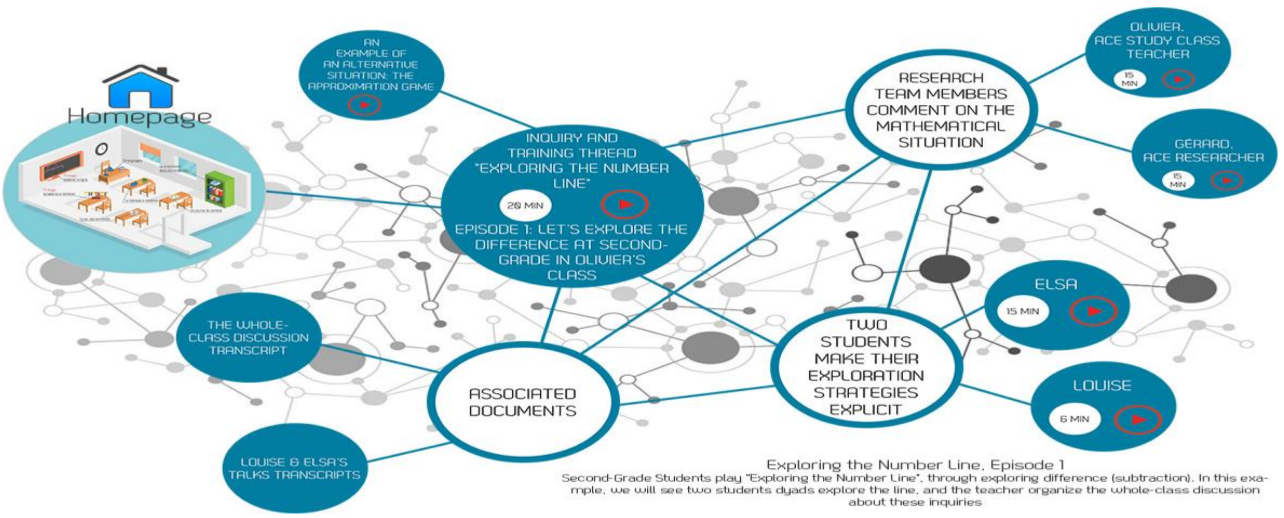

Figure 5: the designed PTAH “Homepage” (Link below)

The central disc above (figure 5) refers to annotated and commented films of the classroom practice about the thread (“Fil rouge Explorer la ligne”). Other discs refer to 1) associated documents (notably transcripts of the films dialogues), 2) comment on films by the teacher, researcher, and students, 3) an annotated film of the new classroom approximation mathematical practice. The user of the PTAHS thus may enter this network of practice to better understand the Inquiry and Training Thread, and to propose new improvements.

SUMMARY AND CONCLUSION

In this paper, we have shown first how considering the result of a test led the research team to transform the ACE curriculum (second-grade) by introducing "Inquiry and Training Threads", and, more precisely, the "Exploring the Number Line" thread. We have described how the Cooperative Engineering collective redesigned this thread by integrating to it firstly the estimation strategy, and secondly the semiotic systems which may help perform this strategy accurately. The whole improvement process is now available on line on the Picture-Text-Audio Hybrid System (PTAHS) (http://pukao.espe-bretagne.fr/public/tjnb/shtis_ace/reseau_analyse_approximation.html) whose we present the "homepage" above. A further step in this research will consist in transmitting the improved thread from the research team (Sphere 1) to the experimental ACE teachers (Sphere 2), as it is planned in the next ACE meetings.

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International Commission on Mathematical Instruction

ICMI Study 24

SCHOOL MATHEMATICS CURRICULUM REFORMS: CHALLENGES, CHANGES AND OPPORTUNITIES

DISCUSSION DOCUMENT

Prepared by the International Program Committee

December 2017

1. INTRODUCTION AND BACKGROUND

School mathematical reforms have taken place in many countries around the world in the recent past. Although contexts vary significantly much could be learnt from deeper and more substantial reflections and research about different aspects of these reforms.

Reforms have been large-scale involving an education system as a whole, at a national, state, district or regional level in which mathematical curricula, standards or frameworks have been developed and implemented. Changes have taken place at all levels of mathematics in the school educational system from pre-primary through senior secondary.

School mathematics reforms are often conducted with changes in all different aspects of the curriculum: mathematics content, pedagogy, teaching and learning resources (e.g. texts and technologies), and assessment and examinations.

It is possible to observe different influences on school mathematics curriculum reforms over time. During the mid-twentieth century school mathematics curriculum reforms were shaped by developments within the discipline of mathematics and by the ideas of some mathematicians. This is captured in an address by Dieudonné, one of the proponents of what was then termed the "New-Math" in 1959:

> "In the last fifty years, mathematicians have been led to introduce not only new concepts but a new language, a language which grew empirically from the needs of mathematical research and whose ability to express mathematical statements concisely and precisely has repeatedly been tested and has won universal approval.
>
> But until now the introduction of this terminology has been steadfastly resisted by secondary schools, which desperately cling to an obsolete and inadequate language. And so when a student enters the university, he will most probably never have heard such common mathematical words as set, mapping, group, vector space etc."

(Cited in Howson et al., 1981, p. 102)

The New-Math reform, took place in a particular historical context of the "cold war". It became a mathematical movement that spread to many countries around the world with different influences on national curricula and practical implementations in schools. The character of this reform and its challenges was a departing point for many developments and discussions in the teaching of mathematics. Since then, with the lessons from the New-Math reform movements, the field of mathematics education has progressed immensely.

Another major influence on school mathematics curriculum reforms in the second half of the twentieth century has been from outside mathematics, that is, developments in other disciplines, most notably, psychology. Studies and theories in behaviourism, the rise and development of cognitive science and constructivism, to name a few, have especially impacted pedagogical approaches advocated in mathematics curriculum reforms. Other trends in mathematics curriculum reforms included problem solving, and back to basics, (among others).

More recent influences on mathematics curriculum reforms, in this twenty first century, have come from other areas, such as large international studies, especially those focusing on student achievements. These studies have enabled comparisons of mathematics curricula (such as intended and attained curricula) across many countries and generated particular conceptions (such as mathematics literacy), which have found their way into mathematics curriculum reforms. Nowadays international comparative studies like the Trends in International Mathematics and Science Study (TIMSS) (Mullis et al, 2016) and the Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) (OECD, 2016), which attract a great deal of public attention and media focusing on student and teacher performance in mathematics education (and to which politicians and policy makers are especially responsive), are impacting and shaping school mathematics curriculum reforms as countries or regions both compete and share curriculum policies, materials and approaches. These studies have raised the stakes significantly, and arguably, entrenched a focus on student performance and better test scores as opposed to better student learning within mathematics curriculum reforms. There are a diversity of studies and findings from international experience and research that can and does influence the nature of curriculum changes, and the possibilities of educational reform and its implementation: - curricular design results; a revised role for components in the teaching of mathematics (e.g. mathematics content, pedagogy, and assessment); the role of technology; and new cognitive, sociocultural and sociopolitical perspectives.

In recent years the internationalisation and globalisation of the economy, universality of technological development and related needs for new skills and knowledge play the role of strong motivations for curriculum reforms that have brought calls for unified standards for mathematics in school. In the international debate, many scholars, teachers and policy makers now speak of the "*21st century competencies*" and consider important items like: "critical and inventive thinking; communication, collaboration and information skills; and civic literacy, global awareness and cross-cultural skills"[i]. In many countries, the so called "21st century competencies framework"[ii] is being worked on, in order to guide the development of the national curriculum and to design school-based programmes to nurture these competencies.

In relation to this, new mathematics curriculum discourses have emerged and taken hold. Notions of mathematical "competencies" and that of mathematical "literacy" are important examples that have been raised, from different perspectives around the world (Niss, 2015). In particular, from the approach of OECD's PISA, several notions (and their underpinning theoretical framework) have become very influential in many countries in the changes being made in local curricula and standards; for example, in Denmark, Germany, Japan, South Korea, Costa Rica, Spain, Norway, Mexico, Sweden. PISA stresses the role of mathematical literacy as a central goal in school mathematics education, because it improves the life chances of most students, and justifies why mathematics is essential to describe, explain and predict the world. According to the PISA 2015 Mathematics Framework,

> "Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens."
>
> (OECD, 2016, p. 65)

However, the word literacy itself is ambiguous with multiple meanings, and trying to translate it into different languages and cultures is a difficult, if not sometimes impossible task. In the literature, one finds different names and definitions; and many changes over the years showing the notion of literacy to be

> "a socially and culturally embedded practice, and …[its] conceptions … vary(ing) with respect to the culture and values of the stakeholders who promote it"
>
> (Niss & Jablonka, 2014, p.395).

The differences in approach are directly linked with the goals that are pursued in mathematics education in individual countries. Its inclusion in curriculum reforms identifies new demands about what citizens are obliged to know (or not allowed to ignore). Hence a careful analysis of this notion is required in order to focus its rationale in a curriculum.

Moreover, international studies that examine the successes and failures in achieving the promised aims from different reforms, across these countries, equally need to be undertaken. For instance, the aftermath of the New-Math reform alludes to the importance of reflecting on the requirements for a new curriculum, suitable to escape the causes of the complete, or partial rejection of this reform in so many countries.

The challenges of this particular reform and others that followed opens a discussion about different aspects of curriculum reforms, which go beyond content, such as:

- The existence of different epistemological and cultural positions concerning mathematics and its relevance in different societies;
- The distance of the proposed reform from the mathematical, educational and material conditions and possibilities in different countries, including teacher quality, their preparation, knowledge, beliefs and expectations;
- The relationships with the social, cultural and personal contributions brought by the students in the classroom, so relevant to avoid students' alienation from their social and cultural environment and to allow students to engage in learning in a productive way; and
- The influence of political and institutional scenarios within educational systems, that can promote, discourage or weaken curricular reforms.

A consequence of these reflections is that the communities of researchers, teachers and policy-makers need to become more aware that considerations of curriculum reforms from various perspectives and constructs (mathematical literacy or competencies, for instance) raises many issues, from a scientific, political and cultural point of view.

This ICMI study topic invokes not only questions about changes in curriculum design but – with force - questions about the implementation of these changes across an educational system. A curriculum reform will be influential or have impact in so far as it can be implemented and sustained. What has functioned (or not) at the time of implementing a curricular change? What are the limitations? How have resources (e.g. textbooks and technology) influenced the reforms and their enactment? How must large scale teacher preparation be conducted to achieve the reform goals? How do diverse social, economic, cultural and national contexts condition the nature and extent of curricular reforms; especially teacher expectation, attitudes and beliefs; and the social and cultural background of students? How are assessments of students' learning influential in curriculum reforms? An ICMI Study offers an opportunity to provide a synthesis and meta-analysis of different aspects of school mathematics reforms historically, geographically and globally.

There are many studies conducted in different parts of the world about these issues of mathematics curriculum reforms and their implementation with findings that can be systematized, compared and studied. The way curricula are elaborated, proposed, changed, and reorganized is, however, still a rather under-explored area in mathematics education. This ICMI study can allow a more informed and comprehensive analysis of the roles of different actors and of the many aspects influencing and shaping mathematics curriculum reforms that are or have taken place; and of the possibilities and means to tackle a curricular reform in the current scenario we live in and unfolding future developments.

2. AIMS AND RATIONALE

Nowadays, a central issue for many countries and education systems, and for many social and educational actors, is to carefully reconsider and revise the nature of school mathematics; to come to a more precise meaning of curriculum reform; and to scrutinise the diverse strategies for its implementation. There is a need to identify common ground, and to point out findings and good practices to enable effective, efficient and successful school mathematics reforms.

A study that sheds light on what "works" and what does not in school mathematics curriculum reforms and their implementation across diverse contexts would be of great value and use not only for policy makers, administrators, and researchers, to learn from each other's countries and regions, but also to practitioners and to educational communities as a whole. It is a key imperative for many countries to enhance the competencies of students who will become key players in changing societies, given the internationalisation and globalization of the economy, and rapid advancement of communication and other technologies. What is currently taught and learnt as school mathematics is challenged in this evolving context.

It is as crucial an issue for developing countries, as it is for developed ones, given by the global changes taking place in societies, as they confront different challenges of growing inequality, unemployment, poverty, mass migration, environmental disasters, and conflicts (to name but a few), and within which school mathematics reforms must take place. However, the processes of curriculum reform may differ in developed versus developing countries due to different protocols followed, different intentions and agendas as well as policy and political rhythms. Other comparative fault lines are, for example, East-West differences in mathematics curricula and reforms which have gained much interest, largely from the results of international studies.

A further rationale for this study is to stimulate further research and publications that explore mathematics curriculum reforms especially at a policy level and across multiple and diverse contexts. Some recent volumes such as by Li and Lappan (2014) point to the growing need for further work in this area and its potential for more research- and evidence-based policy generation as well as implementation models and frameworks.

An ICMI Study offers a unique opportunity to examine past and present mathematics curriculum reforms in different parts of the world, from a macro perspective and meta level and to investigate larger questions of who or what sectors of society drive and most influence curriculum reforms, what reforms precisely are taking place, how are these being implemented, and if they are deemed successful (or not), what count as success. Hence, this study has the potential to build understanding of the implications – current and future – of these larger questions for school mathematics, for different aspects of teaching and learning mathematics, and for its role in the broader society.

Clearly a wide range of specific questions may be raised with respect to this broad topic of school mathematics curriculum reforms. However, these may be engaged by clustering them within a selection of themes as set out in the next section.

3. THEMES AND QUESTIONS

The overarching question of this ICMI Study is to explore what school mathematics curriculum reforms have been or are taking place, especially at a meta, macro or system level; and to learn about the many different aspects of mathematics curriculum reforms from past experiences, to specify the current status and issues in reforms world-wide, and to identify possible directions for the future of school mathematics.

The following five themes are selected for the study to address the research questions.

A. Learning from the past: driving forces and barriers shaping mathematics curriculum reforms

B. Analysing school mathematics curriculum reforms for coherence and relevance
C. Implementation of reformed mathematics curricula within and across different contexts and traditions
D. Globalisation and internationalisation, and their impacts on mathematics curriculum reforms
E. Agents and processes of curriculum design, development, and reforms in school mathematics

Each of these selected themes is aligned with a group of specific questions to be addressed in the study.

In the following discussion, we need to note that key distinctions are needed in conceptualisations of curriculum in a study on curriculum reforms (e.g. Mullis & Martin, 2015).

- Intended curriculum, implemented curriculum, and attained curriculum
- Curriculum at the system level, classroom level, and student level
- Curriculum as a product and curriculum as a process

We focus on an intended curriculum and insofar as it is concerned with and takes account of the implemented and attained curriculum at the level of the classroom and student respectively, and on the level of educational systems, and on the dynamics of curriculum as a process, at the phase of educational reforms and in the context of societal needs expected of school education in different countries.

For each of the themes below, different curriculum components may be analyzed such as content, pedagogy, textbooks, technology, assessment, initial and continuing teacher professional development, curriculum development and design processes, and the role of agents. Contributions are invited to the separate themes and will be distinguished by the theme's specific foci and questions.

A. Learning from the past: driving forces and barriers shaping mathematics curriculum reforms

School mathematics curriculum reforms are contested spaces with many different vested interests because of the multiple goals and intentions they are expected to serve. Therefore, in any curriculum reform, there are both driving forces and barriers in shaping mathematics curricula. This first theme sets a general background and the context, and invites studies of school mathematics curriculum reforms in the past decades.

A1. What aspects of school mathematics curriculum reform carried out in the past decades are considered to be the most important (for example, in content, pedagogy, and in the underpinning theoretical approaches)? What potentially crucial aspects of mathematical curricula have not been considered, and even less, touched upon?

A 2. Which goals and values in school mathematics curriculum reforms, carried in the past decades, have been the most important (for example, in the selection and organisation of mathematics contents, or process aspects of mathematical activities)?

A3. How have the questions of content become linked to the notions of mathematical competencies, capabilities, and literacy; and how have these evolved to become a driving force in the curriculum development and reform initiatives?

A4. What has been the role and function of curriculum resources, materials, and technology, including digital curricula and textbooks in curriculum reforms and their implementation as driving forces or barriers?

B. Analysing school mathematics curriculum reforms for coherence and relevance

The role, content, and importance of mathematics as a school subject are examined in each educational system from time to time. All mathematics curricula set out the goals expected to be achieved in learning through the teaching of mathematics; and embed particular values, which may be explicit or implicit. Recent emphases on STEM (Science, Technology, Engineering, and Mathematics) education in many countries raises both the question of the place of mathematics among these subjects, and the discussion of introducing an integrated or interdisciplinary subject. Questions about the study of school mathematics curriculum reforms are raised in this context for their coherence and relevance.

B1. What is the extent of coherence within and among different aspects of reformed curricula such as values, goals, content, pedagogy, assessment, and resources? How are curriculum ideas organised and sequenced for internal coherence in a curriculum reform? What are the effects of a lack of coherence? For example, regarding relations between high-stakes examinations and curriculum reforms.

B2. How are mathematics content and pedagogical approaches in reforms determined for different groups of students (for e.g. in different curriculum levels or tracks) and by whom? How do curriculum reforms establish new structures in content, stakeholders (e.g. students and teachers), and school organisations; and what are their effects?

B3. What interrelation between mathematics and other disciplines, or movement toward integrated or interdisciplinary curricula, can be observed in mathematics curriculum reforms, given the current emphases on STEM education? What is the relationship between school mathematics and mathematics as a discipline in school mathematics curriculum reforms?

B4. What curriculum materials development and technology are or have been engaged, and what are their roles, goals, and underlying values in school mathematics curriculum reforms?

B5. What theories and methodologies are appropriate for studying phenomena related to mathematics curriculum reforms?

C. Implementation of reformed mathematics curricula within and across different contexts and traditions

The cultural, social, economic and political contexts and positions for the implementation of the school mathematics curriculum are important considerations. The processes of implementing new or reformed curricula may differ according to the cultural and historical contexts and traditions due to different protocols followed and the processes of political decision making.

C1. What processes, models, or best/common practices can be identified from the experiences in the implementation of new or reformed school mathematics curricula?

C2. What are examples of successful or unsuccessful reforms and what are the reasons for their success or failure? What criteria are used for assessing curriculum reforms and their degree of success or failure?

C3. How is the implementation of new or reformed curricula monitored, evaluated, and acted upon? What are models or mechanisms of continuous improvement in school mathematics curricula? How does the existence of such a mechanism affect the frequency, (dis)continuity, and perceived challenges and successes of curriculum reforms?

C4. What models or processes for professional teacher preparation and continuous development have been carried out in different countries in the implementation of new or reformed curricula; and what are their influences, effectiveness, successes or failures?

C5. What are the types of resources and what are their roles (e.g. textbooks, materials, technology) in the implementation of reformed curricula?

D. Globalisation and internationalisation, and their impacts on mathematics curriculum reforms

There are a number of factors that advance globalisation and internationalisation through rapid changes in the nature of communication and availability of information. This internationalisation and globalisation of life in the twenty first century seem to affect mathematics curriculum reforms. These influences appear to increasingly lead toward a "convergence" in school mathematics curriculum reforms. Commonalities and diversity may be observed through comparative studies.

D1. How have results of international experience and research in the teaching and learning of mathematics influenced curricula changes? To what extent can local curriculum reforms be examined against an emergent "international" mathematics curriculum?

D2. How have particular international studies become drivers for school mathematics curriculum reforms? What new discourses with dominant theoretical and conceptual underpinning have emerged; and how have these been taken up in curriculum reforms in different contexts? For example, how have the OECD's PISA notions of mathematical literacy and mathematical competencies been interpreted and expressed in curriculum reforms?

D3. How are mathematics curriculum reforms varied (or similar) in different social, cultural, economic and political contexts such as developing versus developed countries or East versus West? How do selected curriculum components such as content, pedagogy, materials technology and teacher preparation vary from one reform, tradition, country or context to another?

D4. How can comparative or meta analyses of curriculum reform processes and implementations shed light on what works or does not work in mathematics curriculum reforms in contemporary societies?

E. Agents and processes of curriculum design, development, and reforms in school mathematics

Curriculum reform processes are as much an educational matter as they are political; and nowadays involve a broad range of stakeholders with vested interests. Educational, social and political actors influence and shape curriculum reforms – from business, industry, media, teacher unions, and parents on the one hand; to those with different expertise such as curriculum policy makers, educators, mathematicians, researchers, on the other hand.

E1. What are the processes, and how are they deployed, in the development of and during a mathematics curriculum reform? What agents lead or dominate and what is their influence on the aspects of curriculum reforms?

E2. What different roles do mathematics teachers, teacher educators, (education) researchers and mathematicians play in curriculum reforms? What kind of influences do these role players have in mathematics curriculum reforms?

E3. How (if at all) is public engagement with the mathematics curriculum reforms organised and managed; and who takes or is given this responsibility? What is the role and influence of different media in curriculum reforms?

E4. To what extent does or could research inform or influence curriculum design and development processes in reforms?

4. THE STUDY CONFERENCE

ICMI Study 24 on school mathematics curriculum reforms is planned to provide a platform for teachers, teacher educators, researchers and policy makers around the world to share research, practices, projects and analyses. Although these reports will form part of the program, substantial time will also be allocated for collective work on significant problems in the topic, that will eventually form parts of a study volume. As in every ICMI Study, the ICMI Study 24 is built around an international Study Conference and directed towards the preparation of a published volume.

The Study Conference will be organized around working groups on the themes. These groups will meet in parallel during the conference. It is the work of these groups that is captured as chapters in the ICMI Study Volume.

Papers are invited in each theme to address the different questions. We encourage papers that are analytical rather than only descriptive. *It is expected that interconnections between themes will emerge and warrant attention therefore, papers may be re-allocated if necessary.*

4.1. Location and dates.

The Study Conference will take place in the Tsukuba International Congress Center, Tsukuba, Japan and will be hosted by University of Tsukuba. The conference will take place from 26 to 30 November, 2018, with an opening reception on the evening of Sunday, 25 November, 2018.

4.2. Participation

As is the usual practice for ICMI studies, participation in the Study Conference will be by invitation only for the main/corresponding authors of the submitted contributions, which are accepted. Proposed papers will be reviewed and a selection will be made according to the quality of the work, the potential to contribute to the advancement of the Study, with explicit links to the themes contained in this Discussion Document and the need to ensure diversity among the perspectives and representation. The number of invited participants will be limited to approximately 100 delegates.

Unfortunately, an invitation to participate in the conference does not imply financial support from the organizers, and participants should finance their own attendance at the conference. Funds are being sought to provide partial support to enable participants from non-affluent countries to attend the conference, but it is unlikely that more than a few such grants will be available.

4.3. ICMI Study 24 Products

4.3.1 The **first product** of ICMI Study 24 is an electronic volume of conference proceedings, to be made available first on the conference website and later in the ICMI website. It will contain all the accepted papers as reviewed papers in a conference proceeding with an ISBN number, which can be cited as a refereed publication, but are published online only.

4.3.2 The **second product** is the ICMI Study 24 volume. The volume will include the outcomes of the discussions at the conference on the themes in this Discussion Document, informed by the papers. It must be appreciated that there will be no guarantee that any of the papers accepted in the study conference proceedings will appear in the book.

The ICMI Study will be an edited volume published by Springer as part of the New ICMI Study Series. The editing process and content will be the subject of discussion among the International Programme Committee (IPC). It is expected that the organization of the volume will follow the organization and themes set out in this Discussion Document, although some changes might be introduced to incorporate the discussions raised during the conference. Hence the chapters in the volume collectively and consensually integrate the outcomes from the parallel working groups of the ICMI Study Conference.

A report on the study and its outcomes, if not the completed ICMI Study 24 volume, will be presented at the 14th International Congress on Mathematical Education (ICME 14), to be held in Shanghai, China, from 12 to 19 July, 2020.

5. CALL FOR CONTRIBUTIONS

The IPC for ICMI Study 24 invites submissions of contributions of several kinds which include: research papers related to school mathematics curriculum reform issues; theoretical, cultural, historical, and epistemological essays (with deep connection to curriculum reforms); discussion and position papers analysing curriculum policy and practice issues; synthesis and meta-analysis reports on empirical studies; reviews of curriculum reform efforts, especially at macro levels; and papers on comparative studies in curriculum reform initiatives.

Authors must select one theme from among the five described in this Discussion Document to which their paper must be submitted. Authors are expected to consider the questions listed below each theme in making their decision to submit papers.

To ensure a rich and varied discussion, participation from countries with different economic capacity, and different social, political and cultural heritage and practices is encouraged.

The IPC encourages people who are not familiar with such conferences to submit early and request assistance for finalizing their contribution (by 28 February, 2018 - this assistance concerns the choice of the paper topic, theme or structure, not the editing of English language). In this way, the IPC supports a tradition of helping newcomers to the international mathematics education community[1].

An invitation to the conference does not imply that a formal presentation of the submitted contribution will be made during the conference or that the paper will appear in the study volume published after the conference.

5.1 Submission

A template for the submission of papers is available on the ICMI Study 24 website (see below). Papers prepared in English (the language of the Study Conference) according to the template and a maximum of 8 pages must be submitted by the deadlines set out below.

5.2. Deadlines

30 April, 2018: Submissions must be made online no later than 30 April, 2018, but earlier if possible.

30 June, 2018: Papers will be reviewed, decisions made about invitations to the conference, and notification of these decisions will be sent to the corresponding/main author by the end of June.

Information about registration, visa application, costs, and details of accommodation may be found on the ICMI Study 24 website:

http://www.human.tsukuba.ac.jp/~icmi24/

6. MEMBERS OF THE INTERNATIONAL PROGRAM COMMITTEE (IPC)

IPC Co-Chairs:

Yoshinori Shimizu (Japan, yshimizu@human.tsukuba.ac.jp)

Renuka Vithal (South Africa, vithalukzn@gmail.com)

[1]Those who need assistance for finalizing their contribution must submit a tentative copy of their paper requesting assistance no later than 28 February, 2018. Their submissions will be examined immediately. An IPC member may be assigned to help with the final preparation of the paper. Then the final paper will undergo the standard review process.

IPC Members:

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7. REFERENCES

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[i] See for example the Singapore Ministry of Education website accessed 6 December 2017: https://www.moe.gov.sg/education/education-system/21st-century-competencies

[ii] See for example *Towards Defining 21st Century Competencies for Ontario, Canada* accessed 6 December 2017: http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf

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