

*Once upon a time... Historical vignettes from the Archives of ICMI:
The Aarhus ICMI-seminar on the teaching of geometry*

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A perpetual challenge, for any organization supported by an archive, is to ensure that as many documents as possible, pertaining to its mission, are collected and duly preserved for the benefit of posterity. Sometimes a longed-for “gold nugget” will be found after a patient and systematic search, and at other times it will simply be encountered serendipitously. I want in this vignette to report on a recent “success story” of the latter kind related to the ICMI Archive.

My former colleague Claude Gaulin (1938-2020), from the Faculty of Education of Université Laval, was a long-time devotee (and “Old Hand”—see [1]) of ICMI. A few months after a small-scale (and friendly) scientific gathering organized at Université Laval on the occasion of his 80th birthday [2], I had the good fortune of visiting Claude in his house, where he had invited me to have a look at the hundreds of books accumulated in his basement—he was aware, in relation to my duties as the ICMI Archive Curator, of my deep interest for ICMI-related documents. I had the strong feeling that he would have some “ICMI treasures”...



*Portrait of Claude Gaulin by Maria Verônica Azevedo (1985).
(Source: [2])*

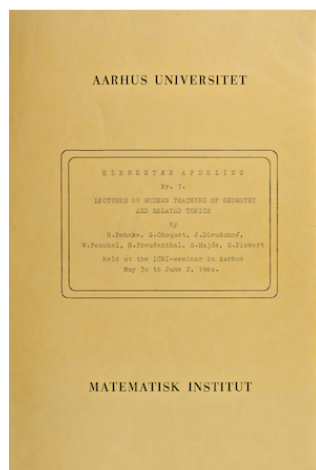
What a shock it was for me! I don’t think I had ever seen so many mathematics and math education books and journals belonging to a personal collection. Even books from his days as an undergraduate and graduate math student, as well as numerous books from the 1960s and 70s, in the early days of Claude’s migration to the world of *didactique des mathématiques*. Not to speak of the scores of boxes piled up on the floor which had been packed with personal notes and documents of all kinds at the time of his retirement, when he had to empty numerous filing cabinets at his university office.

Claude had generously allowed me to search at my will into his math education collection for documents of interest from an ICMI perspective. (We had quickly agreed that it was not part of my task to dig into his boxes of personal notes.) Over a period of a few weeks, a few hours each

time, I visited him on four occasions and came out with some fifty items now deposited in the ICMI Archive. A very rich crop indeed, from an ICMI Archive perspective. But for me, in addition, what beautiful moments I did spend with Claude! He was obviously pleased to witness my deep interest for these documents, and eager to add personal memories about people, places or events connected to almost each of them.

I must add that only a few months after my exploration into Claude's collection, a serious water infiltration occurred in the basement of his house, damaging a large quantity of books and other paper documents. And slightly later, Claude quite sadly began being significantly afflicted by illness.

The gold nugget that I wish to present in this vignette is the proceedings [3] of an ICMI sponsored seminar organized at the University of Aarhus on May 30 to June 2, 1960. In some way, a small gathering, with 30 participants from 10 European countries. But still a rich testimony both about trends of the time related to the “elementary teaching of geometry” [4, p. 140] and about foremost contributors to such reflections. It was from Mogens Niss, my predecessor as ICMI Secretary General, that I first learned of this event and of its importance in connection to the mathematics / mathematics education relationship in the New Math era. I was thus really excited when I discovered that document in the basement of Claude Gaulin's home. It is now available online via the Digital Library page of the ICMI website.



Proceedings of the ICMI-seminar (Aarhus, 1960)
(Source: IMU Archive)

In a memorandum note to the members of the Organizing Committee he was thereby appointing, ICMI President Marshall H. Stone crystallized the “purpose” of the Aarhus meeting as aiming “to advance the study of the three topics which are to be taken up” [5] during special meetings organized by ICMI and connected to the education section of the International Congress of Mathematicians (ICM), to be held in Stockholm in 1962 [6, p. xxxvi]. These three topics had been identified earlier by the 1959-1962 ICMI Executive Committee [7, p. 285] as:

- *Which subjects in modern mathematics and which applications of modern mathematics can find place in programmes of secondary school instruction?* (Rapporteur: John G. Kemeny)

- *Connections between arithmetic and algebra in the mathematical instruction of children up to the age of 15.* (Rapporteur: Stefan Straszewicz)
- *Education of the teachers for the various levels of mathematical instruction.* (Rapporteur: Kay Piene)

These themes were partly rooted in recommendations [8, pp. 218-219] from the previous ICMI EC, with Heinrich Behnke as President—and Vice-President on the new ICMI EC. (Reports on the first two topics have appeared in *L'Enseignement Mathématique* 10 (1964), pp. 152-176 and 271-293.)

The Organizing Committee of the Aarhus ICMI-seminar, chaired by Svend Bundgaard and composed, in addition to Behnke and the three rapporteurs mentioned above, of Ole Rindung and Willy Servais, promptly “decided to concentrate most of the lectures and discussions on Modern Teaching of Geometry in secondary schools with particular emphasis on ways of treatment opened up by developments lately, in particular by the algebraization of mathematics” [3, p. ii]. (It is also mentioned in the Preface of [3] that some of the sessions in Aarhus were devoted to the three topics of the 1962 ICM, but with no report on these therein.)

Seven mathematicians—who, in addition to a deep expertise in geometry, had connections (of various kinds) to mathematical education—presented the lectures published in the Aarhus Proceedings:

- Heinrich Behnke, *Felix Klein und die heutige Mathematik*;
- Günter Pickert, *Axiomatik im Geometrieunterricht*;
- Jean Dieudonné, *The introduction of angles in geometry*;
- Gustave Choquet, *Recherche d'une axiomatique commode pour le premier enseignement de la géométrie élémentaire*;
- Werner Fenchel, *On the relation between synthetic and analytic geometry in secondary schools*;
- György Hajós, *Über die Genauigkeit im Geometrieunterricht*;
- Hans Freudenthal, *Logik als Gegenstand und als Methode*.

In spite of dating back to more than six decades, these lectures are per se still worth being read.

I will concentrate here on three protagonists with an importance of their own who took part in this event: Gustave Choquet (1915-2006), Jean Dieudonné (1906-1992) and Hans Freudenthal (1905-1990).¹

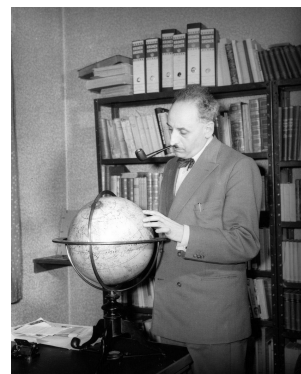
¹ **Photo credits.** Gustave Choquet: photo ID 667, MFO, Oberwolfach. Jean Dieudonné: photo ID 847, MFO, Oberwolfach. Hans Freudenthal: Hofland, L.H., fotograaf; Auteursrechthouder: Het Utrechts Archief, CC BY-SA 4.0, via Wikimedia Commons.



Gustave Choquet (1973)



Jean Dieudonné (1970)



Hans Freudenthal (1957)

Both Choquet and Dieudonné are recognized as outstanding and prolific mathematicians of their generation. Their connection to the ICMI-seminar was directly related to their long-time involvement in geometry, which was evidenced in particular when they each published a landmark book on this topic in 1964: Choquet's *L'enseignement de la géométrie* [9] and Dieudonné's *Algèbre linéaire et géométrie élémentaire* [10]. Freudenthal was then also seen as a respected research mathematician, in particular in relation to his results from the late 1930s on the homotopy groups of spheres and the so-called "Freudenthal suspension theorem". It may be noted that at the time of the Aarhus event, he was still active as a researcher in mathematics, as shown by his short communication related to Lie algebras presented at the 1962 ICM [11, p. 30]. But already at the time of the Aarhus seminar, Freudenthal had started his earnest and exceptional involvement in mathematical education. He had previously served as a rapporteur on the topic of geometry for one of the ICMI special meetings at the 1958 ICM in Edinburgh [12]. His deep and original reflections on the teaching and learning of mathematics, combined with his energetic character, would soon bring to play many pivotal roles in the field of didactics of mathematics, including as a member of the ICMI Executive Committee (1963-1966) and ICMI President (1967-1970), and as the initiator of both the journal *Educational Studies in Mathematics* in 1968 and the series of International Congresses on Mathematical Education (ICMEs) in 1969.

A specific interest of the Aarhus seminar stems from the (brief) reports, published in the Proceedings, on the discussions that followed the various lectures. Discussions which sometimes may have been quite animated, as can be seen from a comment by the discussion rapporteur, after Pickert's lecture on axiomatics in geometry, where it is noted that: "Apparently the heat of the discussion came from mutual misunderstandings as nobody wanted to teach the pupils to construct silly axiomatics systems and prove silly theorems" [3, p. 33]. Or in the summary of the discussion after Dieudonné's lecture on angles: "Here followed a rather heated discussion about the sum of the angles in a pentagon" [*ibid.*, p. 45].

From the discussion after the lecture by Pickert, one reads that Dieudonné observed: "Pickert made a too strong distinction between algebra and geometry. It would be better to use algebra and the properties of the real numbers from the very beginning of the teaching of geometry" [*ibid.*, p. 31]. In reaction to Freudenthal's remark: "The most important part of the pupils are those who are not going to study mathematics later on", Choquet replies: "We can't distinguish between two

kinds of mathematics. What is taught from the age of 15 should be mathematics (i.e. correct deductions etc.) and it isn't important whether it is called axiomatics or not" [*ibid.*, p. 32].

An interesting part of the debates at the Aarhus seminar is related to a certain tension between didactical considerations, including psychological or pedagogical ingredients, as opposed to an intrinsically mathematical standpoint. This is seen for instance in different interactions between Freudenthal and Dieudonné. On one occasion, after the lecture by the latter on angles, Freudenthal comments: "It is dangerous with too radical changes. We don't obtain anything by introducing too much too early without thinking about the psychological and pedagogical problems." To which Dieudonné's reaction is: "There are many psychological difficulties; but we don't get anywhere if we are too cautious. There must come a change" [*ibid.*, p. 46].

In the same vein, after Choquet's lecture proposing in a great many details "a convenient axiomatics for the first teaching of elementary geometry", Freudenthal notes that "we cannot teach the pupils everything. There are certain psychological and pedagogical principles which mustn't be violated", pointing in particular to Choquet's suggestion of introducing an order relation axiomatically, which Freudenthal considers as too difficult for the pupils. To which Dieudonné retorts: "The important thing is to teach the students some good mathematics. The psychological considerations are of secondary importance." In order to make Dieudonné's point clear, the rapporteur even adds his comment verbatim in French: "La psychologie, je m'en fiche."—that is, "Psychology, I don't give a damn." [*ibid.*, p. 104] After another lecture, to Freudenthal's comment: "We could teach everything, drive the children in any direction. But there exist other things at school. We must see the whole together", Dieudonné ripostes: "No. We are here to discuss mathematics and nothing else." [*ibid.*, p. 127] Those who have known Dieudonné and Freudenthal, and are familiar with their physical and moral statures, can easily imagine each of them making his case...

The Aarhus ICMI-Seminar was considered in many ways to be a real success. The final word possibly belongs to Heinrich Behnke, who in his capacity as Vice-President of ICMI presided over the meeting, at the request of Stone [5]. In a letter to IMU Secretary Beno Eckmann, only a few days after the symposium, Behnke wrote:

"Vielen Dank für Ihr Schreiben vom 4. Juni, das ich gerade bei meiner Rückkehr von dem ICMI-Seminar in Aarhus erhalte. Ich war von dieser Tagung sehr angetan, die vor allem in der scharfen und langen Diskussion zwischen Dieudonné und Freudenthal ihren Höhepunkt fand. Es hat sich wirklich gelohnt. Ein europäischer Vorschlag für den Unterricht auf den Oberklassen ist im Werden. (Wir haben sogar den Plan besprochen, ein deutsch-französisches Lehrbuch herauszugeben.) Selten habe ich ein Seminar mitgemacht, welches insgesamt so harmonisch verlief." [13]

English translation:

"Thank you for your letter of 4 June, which I just received on my return from the ICMI seminar in Aarhus. I was very impressed by this conference, the pinnacle of which was above all the lively and long discussion between Dieudonné and Freudenthal. It really was very worthwhile. A European proposal for teaching at the upper-secondary level is in progress. (We even discussed the plan to publish a German-French textbook.) Rarely have I participated in a seminar that went so smoothly overall."

NB: Further information on events surrounding the ICMI-seminar held in Aarhus can be found on the *History of ICMI* website [14], edited by Fulvia Furinghetti and Livia Giacardi. See especially the detailed analysis of the time spans 1955-1959 and 1960-1966, accessible under the *Timeline* browser tab.

Sources

- [1] Dionne, J.J., & Hodgson, B.R. (2020). Claude Gaulin (1938-2020)—In memoriam. *ICMI News* (July 2020) pp. 15-16. [www.mathunion.org/icmi/publications/icmi-newsletter/icmi-newsletter-archive]
- [2] Grossi, E.P. (Ed.). (2018). *Homenagem Claude Gaulin*. Porto Alegre (Brasil): GEEMPA.
- [3] Bundgaard, S. (1960). *Lectures on modern teaching of geometry and related topics*. (ICMI-Seminar, Elementær Afdeling Nr. 7), Aarhus (Denmark) : Aarhus Universitet.
[www.mathunion.org/fileadmin/ICMI/Digital%20Library/ICMISeminar_AarhusUniversitet_1960.pdf]
- [4] ICMI (1960). Compte rendu de la réunion de la C.I.E.M (Belgrade, 21 septembre 1960). *L'Enseignement Mathématique* 6, pp. 140-141. (Translated from the French)
- [5] Stone, M.H. (1960). Memorandum to H. Behnke, S. Bundgaard *et al.*, 1 March. *IMU Archive/ SF 1 / Ser 14: ICMI 1952-1966. [Box 14A]*
- [6] IMU (1963). Secretary's report. In: *Proceedings of the International Congress of Mathematicians 1962*, pp. xxxiv-xxxvii. Djursholm (Sweden): Institut Mittag-Leffler.
[www.mathunion.org/fileadmin/ICM/Proceedings/ICM1962.1/ICM1962.1.ocr.pdf]
- [7] ICMI (1959). Rapport de la réunion de la C.I.E.M (Paris, 7-8 décembre 1959). *L'Enseignement Mathématique* 5, pp. 284-287. (English translation from the 1962 ICM Programme [11], pp. 29, 38, 53)
- [8] Behnke, H., & Desforge, J. (1958). Lettre-circulaire du Bureau de la C.I.E.M. aux dirigeants des Sous-commissions nationales (28 juin 1958). *L'Enseignement Mathématique* 4, pp. 213-219.
- [9] Choquet, G. (1964). *L'enseignement de la géométrie*. Paris: Hermann.
- [10] Dieudonné, J. (1964). *Algèbre linéaire et géométrie élémentaire*. Paris : Hermann.
- [11] IMU (1962). *Programme of the International Congress of Mathematicians 1962*. Stockholm.
[www.mathunion.org/fileadmin/ICM/Proceedings/ICM1962.p/ICM1962.p.ocr.pdf]
- [12] Freudenthal, H. (1959). A comparative study of methods of initiation into geometry. *L'Enseignement Mathématique* 5, pp. 119-139.
- [13] Behnke, H. (1960). Letter to Beno Eckmann, IMU Secretary, 7 June. *IMU Archive/ SF 1 / Ser 14: ICMI 1952-1966. [Box 14A]*
- [14] Furinghetti, F., & Giacardi, L. (Eds.). *The first century of the International Commission on Mathematical Instruction (1908-2008)*. [www.icmihistory.unito.it]