Ethnomathematics at the Margin of Europe – A Pagan Calendar

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ABSTRACT

In 930, at the close of the settlement period in Iceland, a week-based calendar was adopted. Observations of the solar cycle soon revealed errors of the calendar, which were cleverly amended. In the 12th century, the week-based calendar, called misseri calendar was adjusted to the Roman Calendar used by the Christian Church. It remained in common use for secular purposes until the 19th century, and detailed guides to it were written. Special occasions related to it are still celebrated.

Keywords
Ethnomathematics, week-based calendar, Roman calendar, Julian Calendar, Gregorian Calendar, *misseri*, finger-rhyme counting method.

INTRODUCTION

> The construction of calendars, i.e. the counting and recording of time, is an excellent example of ethnomathematics (D'Ambrosio, 2001, 12).

What is ethnomathematics? "The term was coined by Ubiratan D'Ambrosio to describe the mathematical practices of identifiable cultural groups ... in its broadest sense the "ethno" prefix can refer to any group – national societies, labor communities, religious traditions, professional classes, and so on. Mathematical practices include ... measurement in time and space ... and other cognitive and material activities which can be translated to formal mathematical representation." (ISGEm, website).

In this article, practices of an ethnic group, a subgroup of the Viking culture, and their translatation into mathematical representation will be explained. The Vikings established a society in Iceland, an island in the Mid-North Atlantic Ocean, around 900 AD. The settlers came from different parts of Norway, the British Isles and Ireland. The calendar they had in common included a seven-day week and an empirical lunar calendar (Richards, 1998, p. 204). They constructed a new system of recording time, a calendar later called *misseri* calendar. On the basis of the above quotations, the *misseri* calendar will be considered as an example of ethnomathematics:

A parliament, *Althingi*, for the inhabitants of the relatively large country, 100,000 square kilometres, was established in 930. Its meetings were held for two weeks every summer. The short summer in Iceland and its vulnerable nature demanded that the meetings take place after certain farming duties were done, and before others arose. A fairly accurate calendar was therefore needed for the gathering. The seven-day-week calendar was extended to measure the length of the year, as will be explained in this article.

The parliament agreed to accept the Christian faith in about 1000 AD. The Christian Church as an institution was established around 1100, and due to its influence the Icelanders became literate in the first quarter of the twelfth century. Once literate, the Icelanders began to write voluminously, initially to document the laws of the newly-founded Commonwealth (Kristjansson, 1980, p. 29). A thirteenth-century manuscript of the law code *Grágás* contains a concise description of a week-based calendar, created in

Iceland in the tenth century (*Grágás*, 2001; 1980–2000). The Church introduced the Roman system of Julian calendar in the twelfth century, with one extra day added to the 365 days every fourth year: the leap year.

During the Commonwealth period, Icelanders had a close connection to Norway and in 1262 they submitted to the Norwegian King. By the establishment of the Kalmar Union in 1397, Iceland followed Norway into the Danish realm, to stay there until 1944 when the Republic of Iceland was established (Thorsteinsson and Jonsson, 1991).

THE VIKING CALENDAR

The common calendar of the settlers included a seven-day week, the days being named after the Norse gods (Bjornsson, 1990, pp. 71–74; 1993, pp. 18–19, 665–660):

Sunnudagur, Sunday, the day of the sun.
Manadagur, Monday, the day of the moon.
Tysdagur, Tuesday for Tyr, the god of war.
Odinsdagur, Wednesday for Woden, the cunning god.
Thorsdagur, Thursday for Thor, the thunder god.
Frjadagur, Friday for Freyja/Frigg, the goddesses of love/marriage.
Laugardagur, Saturday, the day of bathing.

The pagan names have survived in English and other Nordic languages than Icelandic, where they were abandoned by the Icelandic Church in the twelfth century for *thridjudagur* (Third Day) for Tuesday, *miðvikudagur* (Mid-week Day) for Wednesday, *fimmtudagur* (Fifth Day) for Thursday and *föstudagur* (Fast Day) for Friday. *Sunnudagur*, *manadagur* (later *manudagur*) and *laugardagur* have remained intact to this day.

Probably some of the settlers counted the time according to the cycle of the moon, which is 29.52 days. In Iceland the nights are light from April until late August, so the moon is barely seen. Counting the lunar months in summer was therefore abandoned and counting the summer weeks was taken up instead. Moreover, difficult weather conditions may mean that the moon cannot be seen regularly in wintertime and in time winter months were standardized at 30 days each (Richards, 1998, p. 204).

When a yearly parliamentary gathering was agreed upon in AD 930, some way to count the time had to be accepted. An agreement was reached that the next meeting would take place after 52 weeks or twelve 30-day months plus four extra nights. The year was divided into two terms, *misseri*, and accordingly the calendar was called *misseri* calendar. The winter *misseri* was to last six months, the summer *misseri* six months, and the four extra nights were added at mid-summer, after the 13th week of the summer *misseri*. The parliamentary meeting was to be in the tenth week of summer (Benediktsson, 1968, pp. 9–11, 15; *Almanak fyrir Ísland* 2008).

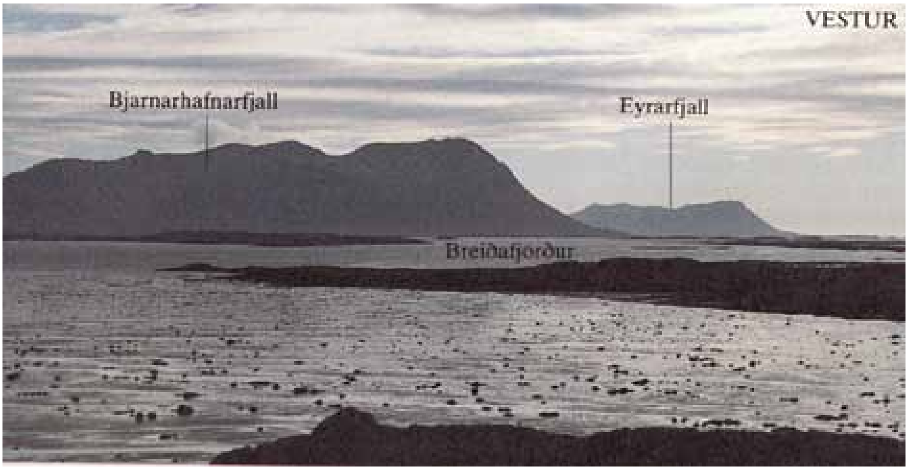
This system quickly revealed the need for a more reliable system of time-computing. By the 950s it had become clear that the summer 'moved back towards the spring', i.e. the summer according to this calendar began earlier and earlier vis-à-vis the natural summer. This was inconvenient, as the parliamentary gathering had to assemble after the completion of certain necessary farming tasks, and before others were due to begin. This is recorded in a brief history of Iceland, *Íslendingabók* (*The Book of Icelanders*, *Libellum Islandorum*), written by Ari the Learned in the period 1122–1133 that exists in manuscripts from seventeenth century (Benediktsson, 1968, pp. xvii–xvii, xliv–xlvii).

> This was when the wisest men of the country had counted in two *misseris* 364 days – that is 52 weeks, but twelve thirty-night months and four extra days – then they observed from the motion of the sun that the summer moved back towards the spring; but nobody could tell them that there is one day more in two semesters than can be measured by whole weeks, and that was the reason. But there was a man called Thorsteinn Surtur … when they came to the Althing then he sought the remedy … that every seventh summer a week should be added and try how that would work …[1] (Benediktsson, 1968, pp. 9–11).

Figure 1 below shows the view from Thorsteinn Surtur's farm where he may have studied the motion of the sun. Only at the summer solstice does the sun set on the right of Mt. Eyrarfjall (Vilhjalmsson, 1990, p. 21).

[1] All Icelandic texts have been translated by the author, KB.

Figure 1: The view from Thorsteinn Surtur's farm. Photographer: Gretar Eiriksson.



Thorsteinn Surtur thus realized the error around 955 AD by an observation of the location of the sunset, which in northern areas moves rapidly clockwise along the horizon before the summer solstice, and subsequently anti-clockwise. The extra week that Thorsteinn Surtur recommended every seventh year to be inserted at mid-summer is called *Sumarauki / Summer's Extra Week*, making the average year 365 days.

By 1000 AD parliament was meeting a week later than before, which indicates that the eleven missing leap years had also caused the start of 'summer' to move progressively earlier in the year, as explained above. In the *Book of Icelanders* it says: "Then it was spoken the previous summer by law, that men should arrive at *Althingi* when ten weeks of summer had passed, but until then it had been a week earlier." (Benediktsson, 1968, p. 15).

The reason why summer solstice may so easily be recognized in Iceland is that the track of the sun is flatter at northern latitudes than closer to the equator. Recalling that the declination of the axis of the earth is 23.5°, figure 2 is a simplified graph of the path of the sun, which shows how the sun moves rapidly along the horizon near the solstice.

The Mediterranean or 'Mid-Earth Sea' is known by that same name in Old Norse and in modern Icelandic: *Midjardarhaf*. The great city, Rome, was regarded as the middle of the earth. Rome is at 42°N. At the equinoxes the altitude of the sun there at noon is 90° – 42° = 48°. At the summer solstice the sun is 48° – 23.5° = 24.5° below the horizon at midnight and the night is completely dark. Thorsteinn Surtur lived at Thorsnes, near the modern town of Stykkishólmur, at 65°N (*Almanak fyrir Ísland*, 2008, p. 59). The altitude of the sun there at noon is 90° – 65° = 25° at the equinoxes. These computations are in agreement with the true altitude of the sun in Reykjavík at 64°N (*Almanak Háskóla Íslands*, website).

At summer solstice the sun is therefore only 25° – 23.5° = 1.5° below the horizon at its lowest position. Since the sun is so close to the horizon at that time, the night is bright enough for reading a book. The official calendar for Iceland does not record darkness in Reykjavík at 64° N from May 19 until July 23 (*Almanak fyrir Ísland*, 2008, pp. 22, 30).

As the path of the sun may be approximated fairly well by a graph of the cosine function, which becomes increasingly flatter when approximating its minimum value, one may understand the fast displacement of the sunset's position along the horizon in the period around the summer solstice (and the winter solstice as well). In figure 2 the altitude of the sun at 65°N and 42°N is approximated by

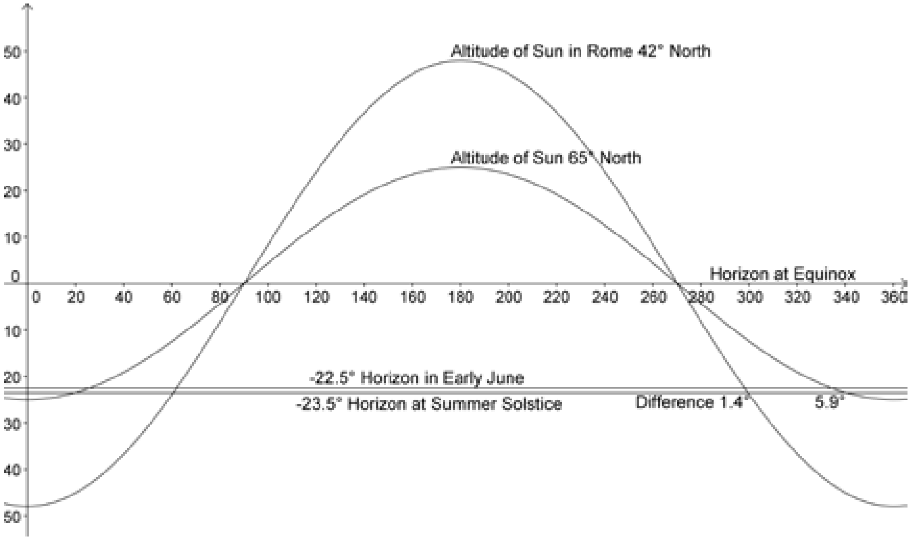
$f(x) = -(90-65)\cos(2\pi(x)/360)$ and $g(x) = -(90-42)\cos(2\pi(x)/365.22)$ respectively.

The scale on the horizontal axis, 0 – 360°, denotes the direction of the sun at the various times of the 24 hours' day, while the scale on the vertical axis denotes the altitude in degrees.

Figure 2: The altitude of the sun with respect to directions on the horizon.

The horizontal lines at -22.5 and -23.5 denote the horizon in early June and at summer solstice respectively. Their intersections to the two graphs denote the directions of sunrise and sunsets and their differences in Mid-Iceland and Rome at the indicated time of the year.

The time between the 22.5° and 23.5° lowering of the horizon is about 17 days. During that period the sunset moves about 1.4° at 42°N in Rome, while at 65°N in Thorsnes, it moves 5.9° along the horizon in the same number of days, or more than four times as far.



Figures 3 and 4 below show that this fact can also be easily realized in Reykjavík at 64°N. The pictures were taken at eight days' interval in mid-June 2008.

Figure 3: Sunset at 64°N on June 11, 2008 at 23:55
Figure 4: Sunset at 64° on June 19, 2008 at 24:04

According to the time difference of the two sunsets, 9 minutes, the sunset has moved 9/(60*24)*360 = 2¼° clockwise along the horizon in 8 days at 64°N.

ADVENT OF THE CHRISTIAN CHURCH AND ROMAN CALENDAR

After the establishment of the Christian Church as an institution in the twelfth century, the Roman Julian calendar was introduced as the calendar of the Church, with one extra day added to the 365 days every fourth year, in the leap year.

But the Julian calendar also contained errors. By adding a day to 365 days every fourth year, the average length of the year became 365.25 days, while in reality it is approximately 365.2422 days. The Julian calendar assumed the summer solstice to be on June 21, decided upon in Nicea AD 325, while in the twelfth century it fell on June 15, six days earlier, due to the addition of six too many leap-year days, which would have be skipped at years 500, 600, 700, 900, 1000 and 1100 according to the correction of the Gregorian calendar.

In the first half of the twelfth century Oddi Helgason, called Star-Oddi, a farm labourer, made observations of the annual motion of the sun, of which an account is found in the ancient treatise *Odda-tala/Oddi's Tale* (Beckman and Kålund, 1914–1916, pp. 48–53).

The treatise *Oddi's Tale* is preserved in several ancient manuscripts. In some of them it is a part of a chronological treatise, *Rím I* ('*rím*'/rhyme meaning calendar), while in the oldest manuscript, GKS 1812, 4to, written around 1192 (*A dictionary of old Norse prose*, 1989, p. 471), it is a separate treatise. *Oddi's Tale* comprises three sections, treating different aspects of the sun's motion. Firstly, Star-Oddi observed the summer solstice and the winter solstice to be a week earlier than the official date, i.e. on June 15 and December 15 instead of June 21 and December 21. Secondly, he explained the curve of the height of the sun during the year by counting the weekly increase in the first half of the year and decrease in the second half. As a measuring scale, he used the diameter of the sun, the sun rising a total of 91 diameters. The third part of *Oddi's Tale* concerns the time of dawn (Vilhjálmsson, 1991, pp. 27–34).

The Icelandic chronological treatise, *Rím II*, written in the late thirteenth century, says:

> Solstice in summer is four nights before the mass of John the Baptist ... It is so in the middle of the world. Some men say that it is close to a week earlier in Iceland (Beckman, and Kålund, 1914–1916, p. 121).

The error of the Julian calendar had thus been discovered in Iceland in the twelfth century. Better estimates of the year than that entailed by the Julian calendar had been made earlier, as listed in Table 1.

Table 1: Examples of early estimates of the length of the year (Richards, 1998: p. 33).

Researcher	*Location*	*Year*	*Length of the year*
?	Babylon	c. 700 BC	365.24579 days
Hippachus	Egypt	150 BC	365.2466 -
?	Mexico[2] (Mayan)	700 AD	365.2420 -
Da Yen	China	724 AD	365.2441 -
Al-Battani	Arabia	900 AD	365.24056 -
Al-Zarqali	Arabia	1270 AD	365.24225 -

The Icelandic week-based *misseri*-calendar was adjusted to the Julian calendar in the early twelfth century, in Oddi's time. By this adjustment the *Summer's Extra Week* was to be inserted every sixth year, or every fifth year if there were two leap years in between. The computations thus depended on the Julian calendar.

THE *MISSERI* CALENDAR

The pagan *misseri* calendar is adjusted to the Julian calendar but there is a basic difference between the calendars in determining dates. The year in the *misseri* calendar is counted in weeks. The years therefore have two different durations: 52 weeks with 364 days or 53 weeks with 371 days.

The *First Day of Summer* marks the beginning of the secular year. It was to fall on Thursday in the week April 9 to 15. In the late middle ages, April 9 was the beginning of the light-night period in Northern Iceland.

Thus the summer-*misseri* begins on a Thursday, and lasts 26 weeks and 2 days plus *Summer's Extra Week*. The winter-*misseri* begins on a Saturday in late October and is 25 weeks and 5 days. Dates are expressed in terms of days of a specified week of summer or winter. The following examples are taken from the 1920 national census of Iceland:

[2] So in the source.

> Sigurður Jónsson born on Sunday in twelfth week of winter 1859.
>
> Guðlaug Einarsdóttir born on the sixteenth Saturday of summer in 1850 (National Archives of Iceland, Statistics Iceland).

The Gregorian calendar was a reform to correct the discrepancies of the Julian calendar. By 1700, when the Gregorian calendar was adopted in the Danish Realm, eleven days were omitted, November 17–27 (Saemundsson, 1972, p. 131). The *First Day of Summer* was transferred to Thursday in the week April 19–25, and other dates, mid-summer, and beginning of winter were adjusted accordingly (Bjornsson, 1993: p. 16).

The *Misseri* calendar is basically a week-based calendar. However, there are also twelve thirty-day months and the four extra nights as quoted in the *Book of Icelanders*. The three last winter months have definite names that have remained unchanged through the centuries: *Thorri*, *Goa* and *Einmanudur*. *Thorri* and *Goa* were also names of pagan gods. *Einmanudur* means One-Month or Lone-Month. The name is believed to derive from the fact that when the month commences there is one month left until summer begins (Thorkelsson, 1928). The beginning of *Thorri* marks mid-winter and has been an occasion for mid-winter festivities.

Thorri. (masculine) begins on Friday in the 13th week of winter (in late January); this was Husbands' Day.

Goa. (feminine) begins on Sunday in the 18th week of winter (in late February); this was Wives' or Women's Day.

Einmanudur. (masculine) begins on Tuesday in the 22nd week of winter (late March); this was the Young Men's Day.

Harpa. (feminine), the first month of summer, begins on Thursday in April 19–25, First Day of Summer; this was the Young Girls' Day. (Björnsson, 1993, pp. 766–783).

The *First Day of Summer* has been a public holiday in Iceland for centuries. Youth and child-care organisations organize festivities in cooperation with local authorities. Furthermore, international Mother's and Father's Days are not much celebrated in Iceland: rather the first days of *Thorri* and *Goa* (Bjornsson, 1993, p. 31, 44–45, 766, 778, 780).

DACTYLISMUS ECCLESIASTICUS OR FINGER-RHYME

Great many calendars were preserved in manuscripts from the twelfth to the eighteenth centuries. The two first printed calendars in Icelandic were published at Hólar, one of the two episcopal sees, the latter *Calendarium: íslenzkt rím* (1597). It was a perpetual Roman calendar, but also explaining the *misseri* calendar. The information contained in the calendars was often partly built into verses and rhymes (Björnsson, 1990, pp. 68–69, 91–98).

Bishop Jon Arnason published in 1739 a detailed guide, *Dactylismus Ecclesiasticus or Finger-Rhyme (eður Fingra-Rím)*, to computing the calendar according to the new Gregorian calendar style, both by mathematical formulas and by counting on fingers. The title, *Dactylismus*, is drawn from the Latin word *dactylus* which again is drawn from the Greek word *dactylos*, meaning finger.

The *Dactylismus* was reprinted in 1838. On its front page it says that it is completely similar to the 1739 edition. A photographic facsimile of the 1838 printing was published in 1946 as a rare and appreciated book of earlier age. The facsimile has been used as a source to this article.

In his foreword, Bishop Arnason wrote:

> It is distressing to know that the art of finger-rhyme is mostly extinct in this country, which however was in my young days properly applied and used; many unlearned men and women could in a moment compute on their fingers both the dates of new moons and festivals ... (Arnason, 1739, 1838, p. 11).

The introduction of the Gregorian calendar in 1700 may have adversely affected the knowledge of the common people in this respect, but Bishop Arnason was hopeful that the lack of knowledge and skills concerning the calendar could be remedied by his work. The art had previously been practised with Latin rigmaroles so Arnason hoped that his *Dactylismus* in the vernacular would be a great support. His foreword concluded with a statement that he had composed the *Dactylismus* for the common people in the country. In this he differed from foreign authors he mentioned, who dedicated their works to the nobility, dukes and counts. The fact that there was no such class in Iceland meant that literature had to be aimed at least at the clergy and landowners and every common person, who could afford to own books.

The main bulk of the *Dactylismus Ecclesiasticus* is a guide to computing ecclesiastical moveable festivals, such as Easter, while a calendar of the 'farming-year', the *misseri* calendar, was attached as a second section. For both calendars, the Gregorian and the *misseri* calendar, so-called Sunday letters, or dominical letters, are important.

Each day of the year is assigned a letter, called *calendar letter*, A, B, C, D, E, F or G. Thus January 1 is assigned the calendar letter A, January 2 has B, January 3 has C, and so on. February 29 and March 1 have the same calendar letter. Each year is then assigned a letter, *dominical letter*, according to the calendar letter of the Sundays that year. As an example, the dominical letter of year 2010 is C as January 3 is the first Sunday, and so all Sundays in 2010 have calendar letter C. A regular 365-day year begins and ends on the same weekday, which entails that the dominical letters of succeeding years are displaced back one place (G for the next year after A). The rule is broken on leap years. As the leap-year day has the same calendar letter as the following day, the leap years need two dominical letters, one for January and February and another for the rest of the year.

Every fourth year is a leap year and the week counts 7 days. The lowest common multiple of 4 and 7 is 28, so that the sequence of dominical letters, called the Solar Cycle, was repeated every 28 years in the Julian calendar. Thus each year is assigned a number in the interval 1 to 28 in the Solar Cycle, beginning with 1 in the year 1600. Accordingly, years 1628, 1656, etc. were allocated the number 1 (Arnason, 1739, 1838, pp. 200–217).

The relation between the position of a year in the Solar Cycle and its dominical letter is found in Table 2. The years of *Summer's Extra Week* are marked by an asterisk by their dominical letter (Arnason, 1838, pp. 200–217).

Table 2: The Solar Cycle and the corresponding dominical letters. Years of *Summer's Extra Week* are marked by an asterisk.

Year of the cycle	*Dominical letter*	*Year of the cycle*	*Dominical letter*
1	B, A	15	E
2	G*	16	D
3	F	17	C, B
4	E	18	A
5	D, C	19	G*
6	B	20	F
7	A	21	E, D
8	G*	22	C
9	F, E	23	B
10	D	24	A*
11	C	25	G, F
12	B	26	E
13	A, G*	27	D
14	F	28	C

In the Gregorian calendar the leap years were skipped in years 1700, 1800 and 1900, so the Solar Cycles including these years were lengthened to 40 years. This is done by subtracting 12 from the number in the sequence at the turn of the century. For example the year 1699 is number 16 in the Solar Cycle, while year 1700 is number 5, year 1799 is number 20, but year 1800 number 9, and year 1899 is number 24 but year 1900 number 13. Year 1999 was number 28 and year 2000 number 1. The turn-of-the-century years, not divisible by 400, have only one dominical letter as they are not leap years, the latter letter of the two assigned to their number in their sequence (Arnason, 1739, 1838, pp. 200–217).

The *Summer's Extra Week* is inserted at mid-summer, beginning on Sunday after 13 weeks of summer. The years of *Summer's Extra Week* are those which begin on Monday, that is, when the dominical letter of the year is G, and those which begin on a Sunday, having dominical letter A, the year before a leap year. In that case, next year will begin with dominical letter G, only applying to the first two months of the year, and the remaining months of the year have dominical letter F.

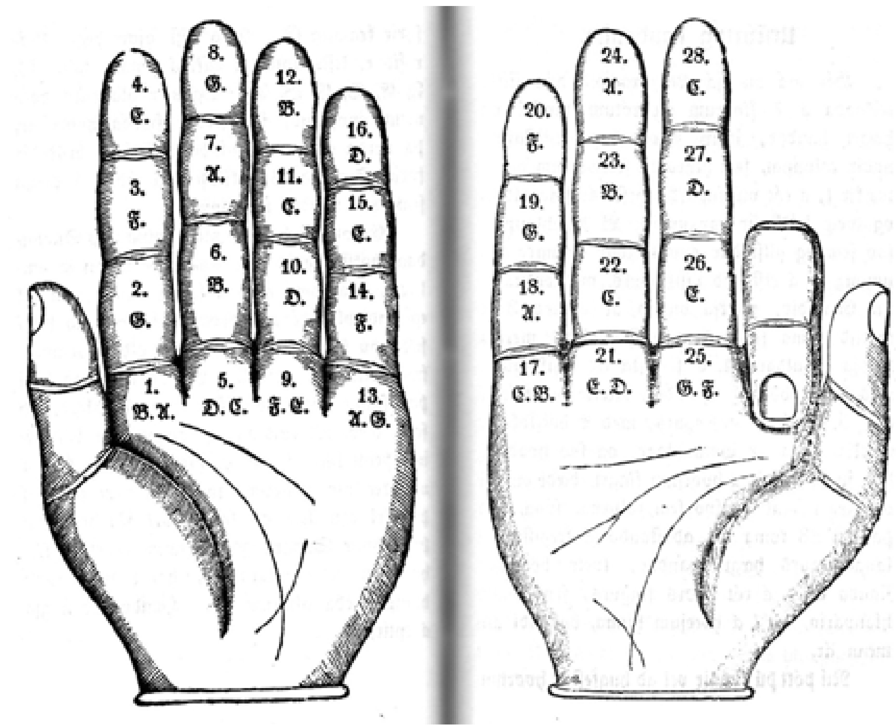
Dominical letter G brings July 22 on Sunday. This means that *Summer's Extra Week* always begins on Sunday July 22, except in the case of a leap year beginning with dominical letter G. In that case, *Summer's Extra Week* is inserted the year before, beginning on Sunday July 23 in a year with a dominical letter A.

Those *Summer's Extra Weeks* next before a leap year, beginning on a Sunday with dominical letter A, are called *Rímspillir* / Rhyme Spoilers in the *misseri* calendar. The Rhyme Spoiler moves all dates forward one day from *Summer's Extra Week* until leap-year day. This happens once in the 28-year Solar Cycle, in year 24, see table 2 above. In the Solar Cycles that contain the years 1700, 1800, 1900, 2100, and so on, when leap years are skipped, the Rhyme Spoiler year is number 36 of the cycle.

In his *Dactylismus*, Bishop Arnason explained how the dominical letters were remembered by their position on the fingers. Figure 4 shows how years number 1 to 28 in the Solar Cycle were assigned dominical letters in reverse alphabetical order.

The year 1600 was, as mentioned before, the first year in the Solar Cycle and had, as a leap year, two dominical letters, B and A. Thus for example, the year 1614 was number 15 in the Solar Cycle and had dominical letter E. The year 1623 was number 24 and had dominical letter A. It was a Rhyme-Spoiler year, as 1624 was a leap year.

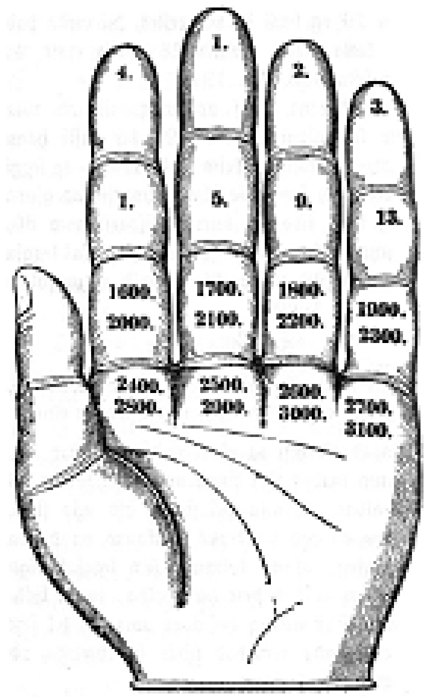
Figure 4: The numbers in the Solar Cycle and their dominical letters placed on palmes and fingers.



When finding a dominical letter for a year such as 1674, one could say that it is number 75 in the Solar Cycle, but it is far too high. The remainder when 75 is divided by 28 is 19 which becomes the number in the Solar Cycle so the dominical letter of year 1674 is G, according to Table 2 and the palms in Figure 4. The last year of the century, 1699, is number 100, with remainder 16 when divided by 28, so the dominical letter is D.

Due to dropping leap years at the turns of centuries, except when the year is divisible by 400, the larger Solar Cycle is 400 years. The *Dactylismus – Finger-Rhyme* helps to find the number of each year in the Solar Cycle, explained above. To correct the cycles due to missing leap years, the years at the turn of the centuries are assigned new numbers to be remembered on the left-hand palm and mid-bones of the fingers as shown in figure 5 below.

Figure 5: A memory scheme to find the number in the Solar Cycle at turns of centuries.



Years 1600, 2000, ... # 1
Years 1700, 2100, ... # 5
Years 1800, 2200, ... # 9
Years 1900, 2300, ... # 13

As an example, the years 1700, 2100, etc., instead of being assigned # 17, they go back to being #5, marked on the mid-bone of the middle finger. The length of the Solar Cycle is thus increased by 12 years to become 40 years. The years in-between the turns of the centuries are counted onwards in 28 year cycles explained earlier. The numbers 9 and 13 similarly correct the Solar Cycles due to missing leap years at the turns of centuries 1800, 2200, etc., and 1900, 2300, etc., respectively.

The numbers 4, 1, 2, 3 at the top bones of the four fingers denote the classes of the centuries within the 400-year cycle (Arnason, 1739, 1838, pp. 102–103).

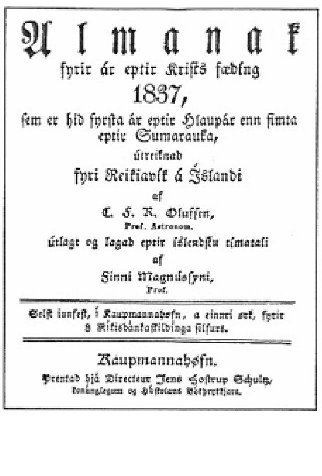
THE *MISSERI*-CALENDAR AND ALMANACS

Before Bishop Arnason's *Dactylismus*, Danish calendars were in use for a few centuries, but these did not meet the needs of Icelanders, most of whom were more familiar with the *misseri* calendar. The *Dactylismus* therefore must have been the main handbook for Icelanders during the 18th century.

Icelandic calendars of the years 1800 to 1836 exist in manuscripts, adjusted to the environment in Northern Iceland, but they were not continuous. In response to these, which were deemed to violate the University of Copenhagen's monopoly on publication of calendars, a calendar in Icelandic, the *Iceland Almanac*, was first published in 1837 by the University of Copenhagen. The calendars were computed by professors at the University of Copenhagen until 1923, and translated into Icelandic by prominent Icelandic scholars. They added the *misseri* calendar with all its features to the regular *Almanac*, which otherwise contained the ecclesiastical calendar of the Evangelical Lutheran Church in addition to local geographical information, such as time of sunrise and sunset in Reykjavík, the capital. (Sigurgeirsdóttir, 1969).

Figure 6 below shows the cover of the first issue of the *Iceland Almanac*. In translation it says:

> Almanac for year after Christ's birth 1837, which is the first year after leap year but the fifth after Summer´s Extra Week, calculated for Reykjavík on Iceland, by C. F. R. Olofsen, Prof. Astronom, translated and adjusted to the Icelandic calendar by Finnur Magnússon Prof. (Sigurgeirsdóttir, 1969).



Almanak
fyrir ár eptir Krists fæding
1837,
sem er hid fyrsta ár eptir Hlaupár enn fimta
eptir Sumarauka,
útreiknad
fyri Reikiavík á Íslandi
af
C. F. R. Olufsen,
Prof. Astronom.
útlagt og lagad eptir íslendsku tímatali
af
Finni Magnússyni,
Prof.

Selst innfest, í Kaupmannahöfn, á einni ark, fyrir 8 Ríkisbánkaskildinga silfurs.

Kaupmannahöfn.
Prentad hjá Directeur Jens Hostrup Schultz,
Konúnglegum og Háskólans Bókprentara.

Figure 6: The cover page of the first issue of the *Iceland Almanac*.

The publication of the *Almanac* was transferred to the University of Iceland in 1917, and from 1923 the computations have been made by Icelandic mathematicians or astronomers (Sigurgeirsdóttir, 1969).

WHY DID THE *MISSERI* CALENDAR SURVIVE IN A SOCIETY OF CHRISTIAN CULTURE?

The *misseri* calendar had been in use for two centuries before the introduction of the Roman calendar used by the Church. It was maintained as a secular calendar by parliament, which gathered in summer during the period 930-1800, and the calendar was registered in the law (*Grágás*, 2001; 1980–2000). It is rooted in the medieval literary heritage that was preserved and studied in Iceland through the centuries. Bishop Arnason respected it in his 1739 *Dactylismus*, as did the nineteenth century scholars at the establishment of the *Iceland Almanac* and its later calculators did so too.

Registrations of births and deaths, carried out by the Church, were only prescribed in Iceland from 1746. For this reason, the official Roman calendar was not in common use among the general public until after the 1739 publication of Bishop Arnason's *Dactylismus*. An inspection of the official census in 1920 reveals that a number of people, born before or around the 1860s, recorded their birthdates according to the *misseri* calendar, at a certain weekday in a certain week of summer or winter.

In northern latitudes like Iceland, the difference between darkness in winter and light in summer is extreme. Celebrating mid-winter *Thorri* and *First Day of Summer* and counting the weeks and months in between is a tribute to the light, and is intimately related to life in northern nature.

Figure 7: Celebration of First Day of Summer in 2008.

Photographer: Heida Helgadóttir.

CONCLUSION – THE *MISSERI* CALENDAR AS ETHNOMATHEMATICS

Iceland was settled while the northern people in Europe were still pagan. The settlers formed a new society on their own terms. A new system had to be made from scratch. The country lies on the margin of the North Pole area with continuous light in summer time and conversely long-lasting darkness during winter-time. The first Icelanders observed a new perspective on the heavenly bodies, the sun and the moon, the universal facts on which recording the time is based. Their intellectual instruments for recording time were deeply influenced by this environment. Their farming duties during the short summer also demanded more accurate dating than they had brought from their earlier domicile. They managed to establish a cleverly made dating system, adjusting a primitive week-based calendar to observations of nature phenomena, less visible in more southerly regions. The effort to adjust the length of the calendar year to observations of nature are examples of empirical adjustments of a mathematical model.

D'Ambrosio (2001) has explained the term *ethnomathematics* in the following way:

In the same culture, individuals provide the same explanations and use the same material and intellectual instruments in their everyday activities. The set of these instruments is manifested in the manners, modes, abilities, arts, techniques – in the *tics* of dealing with the environment, of understanding and explaining facts and phenomena, of teaching and sharing all this, which is the *mathema* of the group, of the community, of the *ethno*. That is, it is their *ethnomathematics* (D'Ambrosio, 2001, p. 24)

The Icelanders developed new instruments for dealing with their environment and explaining phenomena, different *tics*, from those living closer to the 'middle of the earth', the Mediterranian area, from where the Roman system of Julian and Gregorian calendars originated, adjusted to different modes of environment and different perspectives. The new environment and instruments created the settlers' own ways of sharing agreements on time-reckoning, manifested by the law, their own *mathema*.

Later adoptions of the Roman style did not overtake the domestic system of the week-based *misseri* calendar but served to refine it, to refine the *tics* and the *mathema* of the Icelandic community, the *ethno*. The Icelandic week-based *misseri* calendar may indeed be considered as an excellent example of the ethnomathematical concept.

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