Collaborative learning for mathematical level raising, what does it take?

Rijkje Dekker, University of Amsterdam, r.dekker@uva.nl

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SUMMARY

In this contribution I will give an overview of my work as researcher of collaborative mathematics learning during 20 years. I will focus on characteristics of learning materials, a helpful theoretical model, the role of the teacher, the size of small groups and new research lines.

1. INTRODUCTION

At ICME-6 in Budapest in 1988, I gave a presentation about the learning of mathematics in heterogeneous small groups. I was a PhD student and completely involved in classroom observations and the designing of good learning materials for small group learning (Dekker, 1987). Freudenthal, whose ideas about the heterogeneous learning group had influenced me, was in my audience, giving me support with his presence. Now, 20 years later, I have been involved in many research projects on collaborative learning of mathematics. We know a lot more about the process of interaction which stimulates mathematical level raising. We also know more about the characteristics of the learning materials. For level raising isolated problem solving activities are not sufficient, we need at least a series of problems, with special problems in it to provoke level differences between the students. We know more about the favorable size of small groups, the pros and cons of couples: easily accessible for research, but less rich for a critical discussion between students. And we start to know more about the role of the teacher. Which interventions stimulate the interaction and the process of level raising? Which interventions can be disturbing? Which sort of whole class discussions supports the learning in small groups? Some say that whole class discussions are crucial to establish good social and socio-mathematical norms and to consolidate level raising. Others think that they are mainly time-consuming and evoke all sorts of stereotypical behavior of the students, including off-task behavior. I will present some of our research findings over the last twenty years and I am sure we will have enough to discuss!

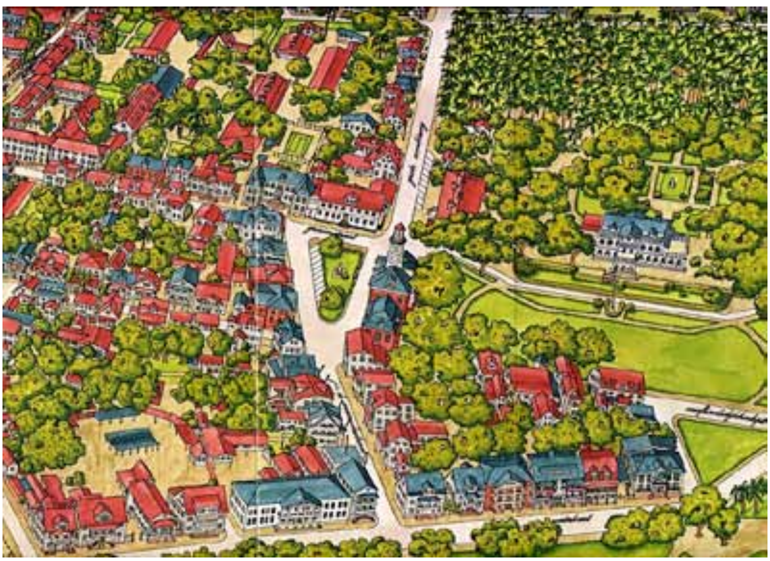
2. LEARNING MATERIALS

While finishing my PhD, one of my supervisors asked me to formulate characteristics of learning materials which evoke interaction and level differences between children, which I did in my thesis (Dekker, 1991). First, the problems are placed in a *realistic context* in order to appeal to the students and to make it possible for them to realize the situation. Second, there are problems in the learning materials which are *complex*, in order to stimulate interaction between the students. To solve these problems different abilities are needed, like finding relevant information in a text, measuring precisely, making calculations well.

They also have to take into account all sorts of different information, data from a text, a map, a table or from earlier solved problems. A third characteristic is that something has to be made, *constructed*, like a graph, a table, a model, a little story. That stimulates students to draw, write or make calculations. In that way they can see each other's work and the differences in it. An important characteristic of the learning materials is the aiming at *level raising*. At certain places in the learning materials there are problems which, when approached on a too low level, cannot be solved well. I will make the characteristics concrete by giving an example from the learning materials I have developed for my PhD research.

The learning materials for small, heterogeneous groups of students age 12, 13, consist of one map for each small group and a letter of a girl Merlien, living in Paramaribo, Surinam. Figure 1 shows a fragment of the map. Figure 2 shows a fragment of the letter.

Figure 1. Fragment of the map.



Fragment of the letter:

'It was raining too hard, so we waited for a moment. Fortunately it was cooling down a bit.
Suddenly the shower stopped, we walked on and soon the sun was burning again.
We walked slower and slower.
But when we strolled into the Palm Garden, it was pretty cool under the trees.'

The letter of Merlien is about a walk she makes with her friends from school till the Palmgarden (see the upper right corner on the map). She tells about differences in temperature because of the heath and a sudden tropical rain shower and about differences in their speed of walking, strolling by the heath, and running by the shower. In the first problems in the learning materials the small groups are asked to tell Merlien's story in graphs: a temperature/time graph about the differences in temperature during the walk, a speed/time graph about the differences in speed during the walk, and finally a distance/time graph about the growing of the walked distance during the walk.

The learning materials are clearly placed in a *realistic context*. Many children never have been in Surinam, but the map, the letter and the presence in many Dutch classes of children with parents from Surinam, make the situation very well realizable. The problems are also *complex*. In order to make the graphs, the map has to be studied, the letter as well, some measurements have to be made and all has to be combined. The graphs have to be *constructed*; decisions about the axes, about some numbers on the axes and about the global shape of the graphs have to be made. Van Hiele once explained that the making of the temperature/time graph and the speed/time graph are activities on the visual level. Changes in the temperature and in the speed are in direct contact with the changes in the graph: when the temperature or the speed goes up, the graph goes up as well and when the temperature or speed is constant, the graph is flat. Although making the graphs is not an easy thing, students don't have to know much about graphs to construct them well. However, the making of the distance/time graph is a different thing: when the speed is constant, the walked distance grows regularly, when the speed is zero, the walked distance remains constant. One really has to understand the construction of the graph, which means a jump to the descriptive level where not the objects themselves, but their properties are central (Van Hiele, 1986). So the learning materials aim at *level raising*.

Analysis of audiotapes of the small groups revealed that the construction of the distance/time graph leads to level differences in the answers of the students, which are intensively discussed. Students frequently explain their work and criticize each other's work. Level raising is already evident in some students.

3. A HELPFUL MODEL

During my PhD work I was puzzled by the question which elements in the interaction between students contribute to level raising. Freudenthal mentioned the role of explaining as a mean for reflection (Freudenthal, 1978). I thought about the role of critic. I made a model in which I described what I thought was crucial for level raising. After my PhD I became a researcher of mathematics education and I started to collaborate with Marianne Elshout-Mohr, a cognitive psychologist with whom I shared interest in learning processes. I showed her my model and we discussed it in detail. She was very interested, but also raised some sound critic. She convinced me that criticizing the work of someone else is not crucial for one's own level raising, but the justifying that it evokes, is. We reconstructed the model together and published it in Educational Studies of Mathematics (Dekker & Elshout-Mohr, 1998). The model is presented in Figure 2. For an extended explanation and theoretical justification of it, I refer to that publication. Here I will explain parts of it.

In the process model for interaction and mathematical level raising we divide *key activities*, *regulating activities* and *mental* activities. Key activities for a person A, who is working on a mathematical problem, are the main activities for A's level raising. They are:

A tells or *shows* her work
A *explains* her work
A *justifies* her work
A *reconstructs* her work

In a collaborative learning setting a person B can regulate the level raising of A by performing the regulating activities:

B *asks* A *to show* her work
B *asks* A *to explain* her work
B *criticizes* A's work

I will show her two parts of the model to give insight in the relation between the key, regulating and mental activities:

B asks A to explain her work (regulating)
A thinks about her work (mental)
A explains her work (key)
B criticizes A's work (regulating)
A thinks about B's critic (mental)
A justifies her work (key)

The main idea for level raising is that when A justifies her work and notices that her justification fails, she will criticize her own work and come to reconstruction of it. The reconstruction can reveal A's level raising.

A help for bringing the process model alive is to read only the middle column. That way one can imagine what kind of discussion between students can stimulate level raising.

Figure 2. Process model for interaction and mathematical level raising.

A and B are working on the same mathematical problem. Their work is different.

A is working		B is working
A asks B to show his work	*What are you doing?* *What have you got?*	*B asks A to show her work*
A becomes aware of her own work		B becomes aware of his own work
A shows her own work	**I am doing this…** **I have got this…**	**B shows his own work**
A becomes aware of B's work		B becomes aware of A's work
A asks B to explain his work	*Why are you doing that?* *How did you get that?*	*B asks A to explain her work*
A thinks about her own work		B thinks about his own work
A explains her own work	**I'm doing this, because…** **I've got this, because…**	**B explains his own work**
A thinks about B's work		B thinks about A's work
A criticises B's work	*But that's wrong, because…*	*B criticises A's work*
A thinks about B's criticism		B thinks about A's criticism
A justifies her own work	**I thought it was right, because…**	**B justifies his own work**
A thinks about her justification		B thinks about his justification
A criticises her own work	Oh no, it isn't right, because…	B criticises his own work
A reconstructs her own work	**I'll better do it like this…**	**B reconstructs his own work**

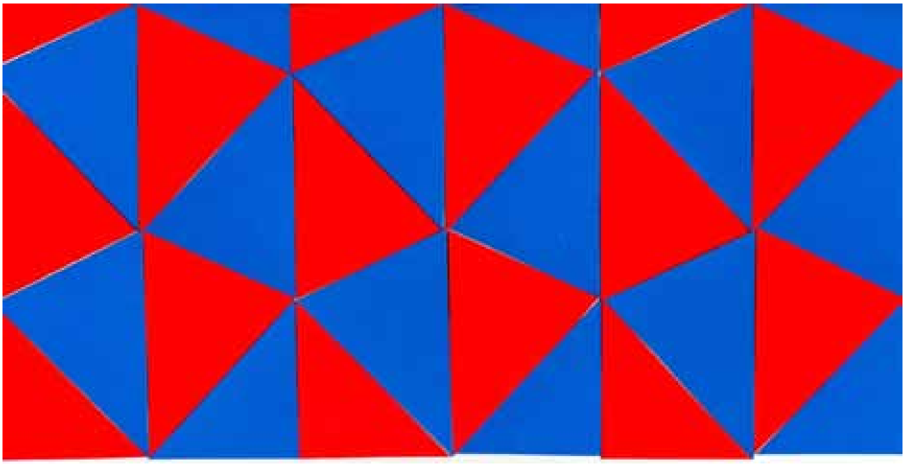
bold: key activities
standard: mental activities
italic: regulating activities

4. ROLE OF THE TEACHER

After reflecting on the findings of my PhD research and the development of the process model, Marianne Elshout-Mohr and I discussed the role of the teacher during collaborative mathematics learning. We argued that if we take our own model seriously, then a teacher who promotes the activities as described in the model is more effective in relation to level raising, than a teacher who gives 'normal' help. We assumed that in both cases help should be minimal, in order to stimulate independent learning of the small groups. To make a clear distinction of both roles, we wanted the process teacher not to give any product help and to make this clear to the students. The focus is to stimulate the students to perform key and regulating activities and the process teacher should make this clear to the students. The other teacher, we called the product teacher, as for content help to small groups the product of the small group is an important source of information for the teacher, should refrain himself from process help.

We prepared an experiment, this time with older students, age 16, 17, working in triples on learning materials about geometrical transformations (see Figure 3). Normally they follow a program on abstract mathematics.

Figure 3. Fragment of new learning materials about transformations.



The main finding of our experiment was that students with a process teacher reach more level raising than students with a product teacher. This was in line with our hypothesis, but as the quality of the help of the product teacher was very high and the help of the process teacher was almost absent, this was not what we expected during our experiment. We have described our findings, including more details about the learning materials and teacher interventions in Dekker and Elshout-Mohr (2004). Another finding from our experiment was that the discussion in triples is very intense. In the meantime my PhD student Monique Pijls also started research on the role of the teacher during collaborative mathematics learning. She developed learning materials on chances, partly on the computer. For that reason she worked with couples. Her students were younger, age 15, 16 and did a program on applied mathematics. She also worked with a process teacher and a product teacher. Her main finding was that students with a process teacher reach as much level raising as students with a product teacher. She also found that couples got stuck at level raising problems and giving process help without content help was very frustrating for the process teacher (Pijls, 2007; Pijls, Dekker & Van Hout Wolters, 2007a, 2007b).

5. SIZE OF THE SMALL GROUPS

In my PhD research I worked with groups of 4. It was very hard to listen and work out the audiotapes, but the mix of students and the level differences in their solutions led to rich discussions with a lot of showing and explaining.

In our research about teacher interventions we worked with triples. Also with triples the level differences led to rich discussions, but more than with the groups of 4 the discussions in triples were very intense. Monique Pijls worked with couples, in this case because of the computer. On the other hand, in much research on collaborative mathematics analyses of conversations between couples is dominant. Together with Terry Wood, Marianne Elshout-Mohr and I analyzed a protocol of a couple, age 8, working on a mathematical problem. We analysed the protocol from different perspectives and studied how the students regulated their own learning (Dekker, Elshout-Mohr & Wood, 2004, 2006). We felt that in a couple their can be an implicit division of roles, which can disturb the level raising process. That became more evident in the work with Konstantinos Tatsis. Tatsis analyzes collaborative mathematics learning from the perspective of the role theory of Goffman (Tatsis & Koleza, 2006). We combined our perspectives in an analysis of the protocols of couples, future primary school teachers, working on mathematical problems. We studied the influence of the different roles, students take in pairs, on the performing of the key and regulating activities. One of our findings is that a smooth collaboration can lead to shared knowledge building, but at the same time level raising is at risk, as during smooth collaboration there is less need for explaining and justifying, which are key activities for level raising. We continued our analysis on a protocol from the research of Pijls and also found a division of roles, which is in some parts counterproductive for level raising (Tatsis & Dekker, in press). It seems that working in a triple gives more chances for level raising. As a student, age 16, once said:

> "I prefer to work in a couple, because then you really have to build upon each others thoughts… …
> But in a group of three there is more knowledge."

Or is expressed in an old Chinese saying:

> 'Where three deliberate, wisdom arises.'

6. MORE RESEARCH

Monique Pijls and I reflected on our research projects and the role of the teacher. We were convinced that a process teacher gives chances for level raising, but that the role of a process teacher is not 'normal' for teachers. Teachers like to explain. That is crucial for them. So we started to think how we could persuade teachers to stimulate students to perform key and regulating activities. We were also curious if teacher maybe already do that in unexpected ways. So we observed 'normal' teaching in search of (chances for) key and regulating activities, discussed our observations with the teachers, deliberated how key and regulating activities could be stimulated more and observed more experimental lessons. It led to mixed results and feelings, as expressed very clearly by one teacher:

> "I like very much to explain. Now I had to say, ask your neighbor and then go away quickly, because otherwise they keep on asking me. I found that very hard!"

> "I saw students really working more intensely together and sometimes that worked very, very well. They really started to ask each other to explain and they have helped each other."

Monique Pijls and I described our findings in an article to be published (Pijls & Dekker, submitted). In the meantime Monique Pijls has started as a professional trainer of process help. I am trying to find new ways to implement process help in the daily practice of mathematics teachers. Sonia Palha, my new PhD student is developing switch problems to be used during the work with a chapter from a textbook that is very popular with teachers. The idea is, that during their normal teaching at certain moments, when the learning is hard, the teacher forms triples of students of mixed levels, give them switch problems to work on collaboratively in stead of working on problems in the book, and takes the role of a process teacher during the work on the switch problems. We use the word switch problems in a double meaning. The teacher switches role, from 'normal' to process teacher and the problems are to stimulate level raising, so to switch from one level to the other. The problem of making a distance time graph, presented in the beginning of my talk, is an example of such a switch problem. Sonia Palha will compare this working with the normal teaching of the chapter. Our hypothesis is that working with the chapter with switch problems, leads to more level raising than working with the chapter in the normal way. The first findings during try-outs are promising (Palha & Dekker, 2007). To be continued…

7. AN OVERVIEW

So, to sum up 20 years of research of the question 'Collaborative learning for mathematical level raising, what does it take?' We can say:

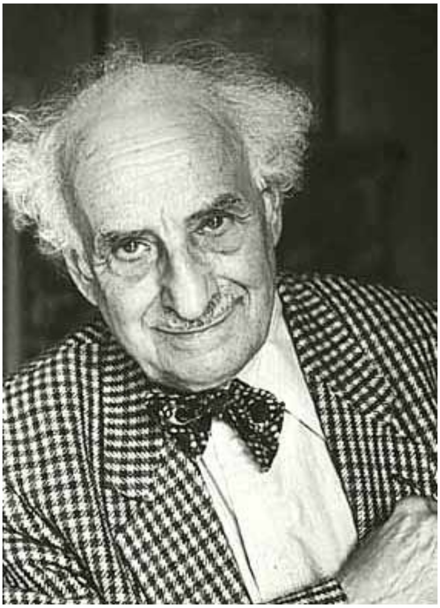
- Carefully designed learning materials with switch problems
- A teacher who stimulates students to perform key and regulating activities
- Small groups of 3
- More research!

Not a normal teacher

Ending my overview I come back to the person who once stimulated me to do research on collaborative mathematics learning. His genuine interest in my developing ideas and experiences with collaborative mathematics learning and his encouragement by saying 'go on', gave me the courage to continue my research.

And I did go on…
I still do.

Figure 4. Hans Freudenthal (1905–1990).



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