DG 13: Challenges and possibilities posed by different theoretical approaches in mathematics education research

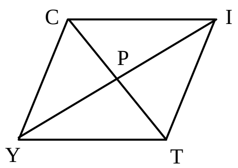
Team Chairs: Susanne Prediger, IEEM Dortmund, Germany, prediger@math.uni-dortmund.de
Luc Trouche, National Institute for Pedagogical Research, France, luc.trouche@inrp.fr
Team Members: Patricia Camarena Gallardo, Nat. Politechnical Institute, Mexico
Jana Trgalova, Nat. Institute for Pedagogical Research, France
Soledad Ulep, University of the Philippines National Institute for Science and Mathematics Education Development, Philippines

The large diversity of different perspectives and theoretical approaches in the mathematics education research community has often been described (for example, in Sriraman & English, 2010). It poses challenges for international communication, for the integration of empirical results into a bigger corpus of scientific knowledge and in the long run for a cumulative and joint progress in the research field.

This diversity is not only unavoidable due to very different traditions and focuses of research, but it can be considered as a rich resource that enables researchers to grasp the complexity of their research objects (Prediger, Arzarello, Bosch, & Lenfant, 2008). In order to approach this perspective on diversity as richness, this DG worked on practical examples (Section 1) and discussed different strategies for connecting theories (Section 2 and 3).

1. A first experience: Classroom data in different perspectives

The data of a first DG activity consisted of a transcription of students' group interaction in a Philippine mathematics classroom in Grade 8. English is the official language of teaching, while students mostly discuss in Filipino (code switching is not documented here due to space restrictions). The lesson was videotaped as a part of the Learners' Perspective Study, an international research on classroom practices (Clarke, Keitel, & Shimizu, 2006). The excerpt of the translated transcript shows how a group of eight students collaboratively worked on the following task.



CITY is a parallelogram.
Angle of C is 5x - 10, angle of T is 4x + 10.
Find the measure of each angle of ▱CITY.

23:37	Arn	What's that? Supplementary? Are these supplementary? Supplementary? *[Laine does not answer.]*
23:44	Arn	What?
23:45	Laine	What? Which? Which is your problem here?
23:48	Arn	I'm asking if these are supplementary.
23:50	Laine	Supplementary? They are equal because, aren't C and T opposite angles?
23:54	Arn	Yes.
23:55	Laine	Opposite angles are congruent, oh. Isn't it placed there that angle C is congruent equal to angle T? 5x minus 10 is equal to 4x plus 10. *[The teacher asks Laine to put their poster on the board.]*
24:09	Laine	That's it, then just find their values.
24:14	Arn	Okay. Why does it not have this? *[Arn points to his notebook.]*
24:16	Laine	What? It's there already, oh.
24:20	Arn	What I mean is, why is it like that?... Why?
24:22	Laine	There's no more like that because 5x minus 4x is already x. You don't need to get it.
24:28	Arn	Wait, wait.

24:29	Laine	4x. 5x minus 4x is equal to 1x. We divide don't we?
24:34	Arn	5x?
24:35	Laine	Minus 4x
24:36	Arn	Oh, 1x
24:37	Laine	So do you still need to divide 1x by 1? *[Arn nods.]*
24:41	Laine	Isn't that you don't need to? Isn't it that it's just the same as x?

The participants in the DG started to analyse the transcript from different theoretical perspectives, among them the following three.

The focus of a socio-cultural perspective (Clarke et al., 2006) is on the students' roles and opportunities for participation: While the activity was intended to involve all group members, Laine contributes all mathematical ideas. She leads the group and asks her group mates questions. Arn, whom Laine designates as the reporter for their group, keeps asking her clarifying questions, as he is anxious to carry out his role. The group interaction is reduced to these two students who interact in a more-knowledgeable and less-knowledgeable relationship.

The Local Theoretical Models perspective (Rojano, 2008) enables the researcher to understand the cognitive processes while taking into account the subject-specific obstacles and difficulties. For the selected transcript, this concerns the typical confusion of supplementary, complementary and consecutive angles on the geometric side. On the algebraic side, the main linguistic and cognitive obstacle concerns the fact that Laine could not accept that 1*x* is equal to x. The Local Theoretical Models perspective allows us to compare the students' work with the expert work, for example the length of students' method to determine the *x* value.

The Instrumental Approach perspective (Guin, Ruthven & Trouche 2005) focuses on the artefacts (material as well as symbolic) that are present in the environment: artefacts for remembering the needed knowledge (here Arn's notebook), for expressing and sharing ideas (here the board and the poster), for measuring, computing, reasoning (the drawing of the parallelogram; the algebraic language). An analysis of the interplay between these artefacts is crucial to understand what prior knowledge is mobilised, and what knowledge is constructed through activity. From this point of view, we miss in the available data the interplay between native and official language.

Already these short sketches of the three analyses make clear how different perspectives focus on different parts of the complex classroom reality.

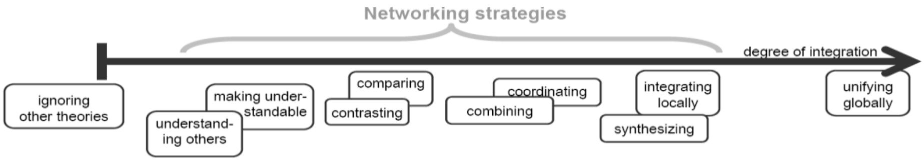
2. Strands and Issues for Dealing with Diverse Theories

The ICME discussion group focused not only on differences like in the first case study in Section 1, but also on possible strategies for connecting perspectives with the aim of better understanding the complex phenomena in mathematics teaching and learning. Selected case studies served as concrete starting points for discussing the following strands and issues:

- How are different perspectives and theoretical approaches reflected in concrete research practices (for example, practices of data gathering and data analysis)?
- How can the diversity of perspectives and theoretical approaches be used as a resource for a better understanding of complex phenomena?
- What strategies can be used to connect different perspectives and approaches, for enhancing a concrete research project or for developing the theories in the long run? How can different theoretical approaches learn from each other?

Ignoring other theories and unifying theories in a global way are extreme poles of a scale that allows us to distinguish between different intended degrees of integration of the theories. In between these poles are fruitful ways of networking theories like comparing, combining, synthesising. This scale was used to organize a landscape of strategies in Bikner-Ahsbahs & Prediger (2010).

Whereas the analysing activity presented in Section 1 served as a starting point for understanding other perspectives and for comparing them, the following section gives a short insight into a more elaborate case of comparing and coordinating theoretical perspectives.



3. Case study of networking: Cross-experimentation in TELMA-project

The European research project TELMA (Artigue et al., 2007) brings together six teams with a strong tradition in technology-enhanced learning of mathematics, to promote construction of a shared scientific vision. As the teams had to cope with the diversity of their theoretical frames, a better mutual understanding of these frames appeared as a necessity for an effective collaboration.

For this purpose, TELMA developed a specific research methodology based on a cross-experimentation approach: Each team experimented, in real class settings, with an ICT tool developed by another team. The development of local experiments was systematically explored with respect to the role of theories in the design of classroom activities. The notion of didactical functionality of an ICT tool (Cerulli et al., 2005) was introduced as the ICT tool's characteristics and modalities of use that may enhance teaching/learning processes, with respect to a specific educational goal. The introduced theoretical notion provided a common, theory-independent perspective on various ways of technology use.

Comparative analysis of local experiments pointed out the ways that theoretical frames affected these. Theories first impact the analysis of the tool used in the experiment and the didactical functionalities assigned to it by the experimenting team. They affect design choices by determining what needs to be planned in advance, and what roles will be assigned to teacher and students. Furthermore, theories impact the design by influencing researchers' visions of "distances": "distances" between representations of mathematical objects and actions in usual contexts and those provided by the tool, "distances" between educational cultures underpinning the design and practice.

By cross-experimentation, each team's experiment could be analysed with external eyes of researchers from other teams, who adopted their own theoretical lens. The cross-analysis allowed deeper insights in how different theories shape the design and implementation of an experiment, but also to understand better the theories themselves. In some cases, the analysis of an experiment from different theoretical perspectives contributed to make sense of unexpected events occurring during the implementation with which the designer team could hardly cope. In this way, theoretical perspectives could be locally coordinated. Thus, cross-experimentation has turned out to be an interesting method for comparing and possibly coordinating different theoretical frames without loosing the richness of the diversity of approaches.

4. Perspectives and concluding remarks

The shortly sketched two examples of comparing theories and other networking case studies considered in the DG (Gellert, 2008; Bergsten, 2008) gave rise for some general considerations.

There are different ways for crossing theoretical perspectives (Section 2), not a single royal one. Different strategies can be located on a scale of different degrees of integration, but the extreme poles appear impossible. It is impossible to lock oneself in a single theoretical framework, each science has to keep open to other sciences, each theoretical approach has to keep open to other ones. Mathematics education is a living scientific discipline, and an open mind attitude is a condition for designing common research projects, beyond various cultural traditions. On the other hand, it is impossible and not desirable to unify all existing frameworks in a single one. One grand unified theory could never grasp the complexity of the field, as each theory has its blind spots.

So the current situation with the co-existence of diverse theories will last, and it has to be considered as a chance for the researchers in the field. The short examples (in Section 1 and 3) showed that it is productive to compare different theoretical perspectives for a deeper analysis and a better understanding of complex situations of learning and teaching mathematics. This interest is particularly obvious in complex situations like those involving ICT (Lagrange et al., 2003).

Connecting or crossing theoretical approaches seems to be more useful if it happens at the beginning of a research; it could help, not only for analysing data, but also for conceiving a relevant methodology, and gathering data needed by each theoretical perspectives (as experienced in the first case study).

The connection of theories is rarely easy, as it necessitates an understanding of concepts which are not familiar for “foreigners” to a given framework. The TELMA project seems to have explored one fruitful way to these share resources.

As the necessity of connecting theories is crucial for the daily work of a researcher, the methodology of networking theories has to be further developed. For this purpose, the meta point of view on the interest—and difficulties—of this development must also be further elaborated. This programme has been discussed in Mexico, it is continued on other occasions like CERME (Prediger et al., 2008).

References

Artigue, M. Bottino, R.M., Cerulli, M., Georget, J.-P., Maffei, L., Maracci, M., Mariotti, M.A., Pedemonte, B., Robotti, E., & Trgalova, J. (2007). Technology Enhanced Learning in Mathematics: the cross-experimentation approach adopted by the TELMA Research Team. *La matematica e la sua didattica* 21(1), 67-74. (telearn.noe-kaleidoscope.org/warehouse/TELMA_UMI_for_telearn.pdf).

Bergsten, C. (2008). How do theories influence the research on teaching and learning limits of functions. *ZDM - The International Journal on Mathematics Education*, 189-199.

Bharath, S. & English, L. (eds.) (2010). *Theories of mathematics education: Seeking New Frontiers*. New York : Springer.

Bikner-Ahsbahs, A. & Prediger, S. (2010). Networking of theories – an approach for exploiting the diversity of theoretical approaches. In B. Sriraman & L. English (eds.): *Theories of mathematics education* (pp. 483-505), New York: Springer.

Cerulli, M., Pedemonte, B., & Robotti, E. (2005). An integrated perspective to approach technology in mathematics education. In: M. Bosch (Ed.). *Proceedings of CERME 4,* Sant Feliu de Guíxols, Spain 1389-1399.

Clarke, D, Keitel, C. & Shimizu, Y. (2006). *Mathematics classrooms in twelve countries: The insider's perspective.* Rotterdam: Sense Publishers.

Gellert, U. (2008). Validity and relevance: Comparing and combining two sociological perspectives on mathematics classroom practice. *ZDM - The International Journal on Mathematics Education*, 215-224.

Guin, D., Ruthven, K., & Trouche, L. (Eds.) (2008). *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument.* New York: Springer.

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and Mathematics Education: a Multidimensional Study of the Evolution of Research and Innovation. In: A. Bishop, et al. (Eds.). *Second International Handbook of Mathematics Education* (pp. 239-271). Dordrecht: Kluwer.

Prediger, S., Arzarello, F., Bosch, M., & Lenfant, A. (Eds.) (2008). Comparing, Combining, Coordinating - Networking strategies for connecting theoretical approaches. *ZDM - The International Journal on Mathematics Education, 40* (2), 163-327.

Rojano, T. (2004). *Local Theoretical Models in Algebra Learning.* In D. MacDougal (Ed.), *Proceedings of PME-NA* (pp. 37-56). Toronto: PME-NA.