DG 17: The changing nature and roles of mathematics textbooks: Form, use, access

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1. Introduction

The organisation of DG 17 was started by all members of the team. However, due to work commitments of some members of the team it was Berinderjeet Kaur, Birgit Pepin and Steve Rasmussen who eventually saw the DG complete its mission. Rheta Rubenstein, who attended all three sessions and meticulously took notes of the proceedings, has contributed significantly towards this report.

2. The proceedings

The primary aim of the DG was to steer discussion around the nature and roles of textbooks. Altogether nine papers were accepted. During the first session), three papers, namely Thompson and Senk (2008), Ding (2008), and Bae et al., (2008) provided the basis for the discussion that focussed on the role of textbooks and how authors represent mathematical knowledge. During the second session, six papers, namely Jun (2008), Heirdsfield et al. (2008), Ibarra et al. (2008), Shield and Dole (2008), McIntosh (2008), Pepin (2008) provided the basis for the discussion that focussed on teachers' use of textbooks, evaluation of textbooks/mathematical tasks in textbooks, and developing quality textbooks. The third session was used to consolidate the deliberations of the earlier two sessions.

The key issues that guided the deliberations and the many questions raised and discussed are summarised as follows.

2.1 What are the roles of textbooks?

Textbooks embody a teaching philosophy and pedagogy as well as the more obvious scope and sequence. Textbooks embody the ministry's view of what topics to teach, in what order, and how. Books may address needs of a diversity of learners. For example, in China, material for students at the extremes of accomplishment are provided at the end of chapters.

2.2 How do teachers use textbooks?

Teachers rely on textbooks for implementation of curriculum. Teachers use textbooks in differentiated ways, selecting different chapters, lessons, and exercises. Teachers use textbooks as dispensers of exercises, as a source of worked examples, and as way to have discourse. Teachers do not always understand an author's intent. At some times and in some places being 'a textbook teacher' was considered derogatory. At some times and in some places publishers tried to produce ‘teacher proof’ textbooks.

Often curricula are judged by student achievement, but teachers’ implementation of the curriculum may not have been studied. Teachers may omit a text topic because it is not on a test, but the topic is needed for the development of another idea later. Teachers may use an application to embody a mathematical topic, but the use of the application may or may not improve the understanding of the mathematical idea. It would be good to have research to know which topics are better taught or learned through an application. Teachers may prefer poor books that are simpler and more popular. You can’t compare books without addressing who the teachers are and how they operate. How books are used also depends on the length of teaching periods. In Argentina there are not enough books for all students. Teachers use books as a source for photocopying. Consequently, continuity and sequential development may be lost. In particular, justification for processes may be lost. Some papers discussed suggest that with respect to new materials, teachers experience a learning curve, understanding better the second and later years the intent, the mathematics, sequence, and the developmental approach.

2.3 How do authors represent mathematical knowledge?

Korean textbooks are characterised by systematic development, learning activities, connections, problem solving, calculations, and questions to the students. Korean authors include intermediary questions to engage students in understanding the key mathematical ideas. For example, in a primary textbook lesson on adding 9 + 4 using 10 as a benchmark, students are asked, “Why is 4 separated into 1 + 3?” How mathematics is represented may be the wrong question. A better question would be how do books develop mathematical thinking? How do they develop intuition, models, and formalisation? How mathematics is perceived differs in different countries. Key Curriculum books from the USA are mostly problems with a few worked examples. Chinese books provide questions to promote student representation and to build understanding. Diagrams and materials are used by authors to represent abstractions, but students may take them literally. Authors may propose tasks for students without providing the needed prerequisite definitions.

2.4 How are quality textbooks developed?

Quality texts should align with the frameworks or syllabi they represent. Chinese books use systematic variation (Gu et al. 2004, ICME-11 Chinese delegation, 2008). Key Curriculum Press books are produced through an iterative process of development, pilot, and revision.

2.5 By what criteria should textbooks be evaluated?

Criteria discussed centred around

- Content (mathematically correct, coherently developed, good topical examples)
- Approach (stimulating, creative, draws on mathematical history, driven by research on teaching and learning mathematics)

- Appropriateness (age-appropriate, appropriate number of exercises, aligned with testing calendar / school year calendar)

Synopsis of papers contributed to DG-17

(For the congress the papers were available at http://dg.icme11.org/tsg/show/18).

Bae et al. (2008) reported that elementary Korean mathematics textbooks, have been changed 8 times. They identified the philosophical or pedagogical shift for each change. The most recent curriculum emphasises learner's self-directed learning and differentiated learning, providing sections for the lower achievers and in-depth mathematics sections for higher achievers. The lesson format includes reality stage, model stage, agreement stage, and method stage. Samples in the paper illustrate the unit flow and lesson structure. One key element is the regular question to students, "Why do you think so?" This questioning promotes reflection and connections to justification. Authors note that some researchers (Grow-Maienza & Beal, 2004) showed that Korean elementary mathematics textbooks have strengths in their coherence and systematic approaches. Also, the authors suggest directions for the development of elementary mathematics textbooks.

Ding (2008), studying Chinese materials that are just beginning to incorporate applications, is concerned that the structure of the mathematics not be lost when applications are used, applications may detract from the time for the mathematics, and that applications need to be selected more carefully to ensure that the mathematics is primary. The paper includes examples of situations.

Heirdsfield et al. (2008) studied a series of primary books in Queensland, Australia that focussed on student thinking. As well as examining student materials, the study included interviewing authors, observing teachers using the materials, and examining the teacher sourcebooks. They found that "with teachers taking varied approaches to the use of textbooks, there is great potential for mismatch between the pedagogical intent of the textbook materials and the actual classroom practice that is enacted." In particular, they found marked differences in teachers with strong and weak implementations. The latter failed to listen to students, to pursue student thinking, and closed down rather than opened up lessons. Student misunderstandings are not recognized or are ignored. Their research suggests 3 levels of teacher use of curriculum materials:

- Are the students engaged and feeling good?
- Are the students learning? How are the students thinking mathematically?
- What am I learning about teaching and learning?

Consequently the authors suggest implications for texts:

First, they must 'capture' students, that is, they must support student engagement & self-efficacy. Second, they must provide ways for students to share their thinking and for teachers to see that learning has occurred, that is, they must support student learning. Third, "the mathematical knowledge and pedagogy exemplified in the text needs to align with appropriate mathematical knowledge and pedagogical practices", that is, they must support teacher knowledge and pedagogy.

Ibarra et al. (2008) analysed Argentinean middle school texts with respect to the construction of triangles. The analysis identifies tasks, techniques, technologies, and theory. The texts analysed seemed to lack continuity and depth. If teachers cannot bring more background to the lessons reasoning is lost. Books need an axiomatic framework. Applications are sometimes used. They need more coherent development aligned with the mathematics. Books lack attention to multiple solutions, showing only one solution.

Jun (2008) studied how Chinese mathematics schoolteachers design their teaching based on textbooks. She illustrates the teaching of the distance formula from a point to a straight line. She shows the teacher's active role in developing the new curriculum. Teachers' creative roles include seeking concise and new solutions, preparing a sequence of questions to guide the students' investigations, and posing a realistic related problem which creates an atmosphere of active learning. Some inappropriate changes made by schoolteachers in using textbooks are also indicated and discussed.

McIntosh (2008) studied a series of mathematics textbooks developed for Years 5 to 10 and piloted in schools across Australia. Pilot school teachers had been involved in professional development designed to enhance their skills and understandings. In her study, teachers found that their students had a deeper understanding of the material, but took longer to get through it. Feedback from teachers was taken into a review of the materials before a commercial version was produced and disseminated.

Pepin (2008) consolidated and synthesised much background related to textbooks, teaching for understanding, and tasks. Pepin concludes that while teachers may mediate connections for students while teaching, this cannot be relied upon to happen; textbooks must need to provide ways for students to build these links by

- emphasising relational rather than procedural or instrumental understanding;
- making connections with what students already know;
- making connections with the underlying concepts being learnt;
- making connections within mathematics and across other subjects;
- being embedded in contexts which help to make connections with 'real life';
- making high cognitive demand on pupils; and
- connecting different representations (analogies, worked examples).

Shield, M. and Dole, S. (2008) adapted the Project 2061 procedure (Kulm et al., 2000) to develop a set of criteria for evaluating middle-years mathematics textbooks. They tried to demonstrate the extent to which the material aligned with the syllabus and curriculum standards of the context in which they were used and the extent to which the materials recognised relevant teaching and learning research. They focused on the broad area of ratio, rate and proportion and the various ways these concepts appear in the mathematics curriculum. The authors evaluated the Rates and Ratios chapter of a Queensland middle grades textbook on each of the six principles (Equity, Curriculum, Teaching, Learning, Assessment, and Technology) and on each of the related curriculum content goals from the

Queensland syllabus. Current work involves training teachers to do comparable analyses.

Thompson and Senk (2008) shared the format for the University of Chicago School Mathematics Project secondary books. All units include skills, properties, uses, and representations (SPUR). All homework sets include covering the ideas, applications, review, and extensions (CARE). Research on teachers' usage showed that selections of lessons and of homework items vary considerably.

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