# DG 9: Promoting Creativity for All Students in Mathematics Education


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## 1. Overview

Some mathematics educators tend to think that creativity in mathematics is only for a small elite of gifted students. In contrast, others hold the view that mathematical creativity is something that all students can develop if stimulated and assisted in the right kinds of learning environments. What do we actually mean by mathematical creativity? Is it true, or is it wishful thinking, that it can be promoted amongst all students at all educational levels? If it is true, how can it be promoted, and at what costs?

Mathematics educators do not agree on a common definition of mathematical creativity or whether all students can or should be creative. DG 9 explored these and other questions. What is mathematical creativity—a property of a person, a problem, a solution, a process, or a teaching technique? Which students can or should be creative? How does mathematical creativity relate to general concepts of mathematics, problem solving, problem posing, research, and creativity? Is an in-depth knowledge of mathematics a prerequisite for becoming creative? What might teachers do to foster (or inhibit) creativity? Will a focus on creativity distract from other critical areas of mathematics education? How might we recognise and assess mathematical creativity and use technology to promote rather than inhibit mathematical creativity?

Forty papers in DG 9 from 62 participants from 25 countries were published in a proceedings edited by Emiliya Velikova and Agnis Andžāns through the generous support of the University of Rousse, Bulgaria; the University of Latvia, Riga, Latvia; and the Faculty of Natural Sciences and Education of the University of Rousse, Bulgaria.

## 2. Aims and focus

The goal of DG 9 was to support productive discussions about important current problems, issues and challenges relevant to promoting creativity for all students.

The forty papers from the participants and the discussion surrounding them were divided into four sections:

Section 1: Goals and definitions
Section 2: The role of the teacher
Section 3: The use of problems and assessment
Section 4: Technology and the environment

Several papers addressed more than one aspect of creativity. All papers were designed to instigate discussion about goals, research and exemplary practices for promoting mathematical creativity for all students.

Creativity and innovation are often cited, along with problem solving, critical thinking, communication and cooperation, as critical skills, knowledge and expertise that students need to be successful in work and in life in this twenty-first century world. The need for creative, innovative individuals with a strong foundation in mathematics is overwhelming. As seen throughout the activities of DG 9, this topic is of international interest and concern. It is not enough to be proficient at computation or at memorising rote procedures to solve routine problems. These skills are important, but even more important are the abilities to recognise and define problems, generate multiple solutions or paths toward solution, reason, justify conclusions, and communicate results. These are not simply abilities that one is born with and they do not generally develop on their own. For students to become creative mathematicians, these talents must be cultivated and nurtured. DG 9 addressed a variety of issues surrounding this topic.

## 3. Goals and definitions

The papers in this section addressed the question of what it means to be mathematically creative. Aralas discussed the connections of creativity to mathematical imagination, noting that mathematical imagination can enhance pupils' creative work and appreciation of a diversity of mathematical structures. Meissner also included imagination as well as intuitive and unconscious components in his description of mathematical creativity. In addition, he compared and contrasted what it means to be mathematically creative and mathematically gifted. Choi and Do, Daniel, Lenart, and Meletea also discussed the connection of mathematical creativity to giftedness, and Tsvetkova applied creativity to team competitions in mathematics. Other views of creativity were presented by Kadijevich noting creativity is manifested in non-conventional problem solving, Hansen-Smith defining mathematics as an art that engages young students in playing with its functions and expanding its boundaries, and Mina characterising creativity as the ability of humans to establish new relationships and change reality. These papers set the stage for a discussion of questions including:

- What is mathematical creativity and which mathematics students can and should be creative?
- Is mathematical creativity a property of a person, a problem, a solution, a process or a teaching technique?
- Is mathematical creativity domain specific? What does it mean to think like a mathematician; is this creative by definition?
- How does mathematical creativity relate to general concepts of mathematics, mathematical problem solving, problem posing, research, and creativity? Can 'concepts' be creative, rather than simply 'problems' or 'solutions? What would this mean?
- Should mathematical creativity be something new to the world or can it be just new to the creator?
- Is it enough to have ideas that are novel and innovative or must creative mathematics be applied to mathematical problem solving?
- Can creativity be developed? Is it innate? Can it be taught?

## 4. The role of the teacher

Teachers play a critical role in the development of students' mathematical creativity. In some cases, education does more to destroy creativity than to enhance it, but the promotion of creativity and innovation are increasingly important to the future of the world.

Papers in this section addressed a variety of other techniques and strategies that teachers might use to nurture students' creativity. As Foong noted, teachers' conceptions of mathematical creativity have a strong influence on their teaching strategies. She found that the majority of preservice teachers saw creativity as an event linked to problem-solving where more experienced teachers noted the importance of a teacher's actions and not just the problems presented. Gal et al. and Karsenty and Friedlander discussed their work with preservice and experienced teachers and the techniques they use to prepare them to work with mathematically gifted and talented students. Millman and Jacobbe also looked at work with preservice teachers when they discussed mathematical habits of mind that need to be developed for creative problem-solving including exploring mathematical ideas, formulating questions, constructing examples and problem-solving approaches, generalising concepts and reflecting on answers. Ong also discussed habits of mind noting the importance of persistence and thinking interdependently. Building on the idea of thinking interdependently, Toncheva discussed the importance of students talking to each other and the teacher about their intuitive arguments. Beswick addressed this as well, stating that classrooms are complex systems where teachers should structure classrooms to maximise creativity of the class as a whole as well as of individuals. Alfonso and Martinez discussed the use of formative evaluation and learning strategies to move college students from the reproductive stage of solving routine problems through the practical level based on traditional mechanisms and theoretical understanding to understanding at the investigative level. Teoh suggested ways to create opportunities for developing creativity by helping teachers make connections that allow children to revel in the magic of mathematics. Abdounur took teachers out of the classroom into a museum where students can creatively explore relationships among mathematics and music as they make and test conjectures, solving and creating problems as they establish and express analogies between mathematics and music. The papers in this section raised a variety of questions for discussion related to the connections among teaching, learning and mathematical creativity, including the following:

- What is the role of the teacher and others in recognising and promoting mathematical creativity? What is the goal in doing this?
- How should we prepare teachers to foster mathematical creativity in all students?
- Is there a difference between the 'creative teacher' and the 'productive teacher'? Between the 'creative teacher' and the teacher of mathematical creativity?
- Must the teacher be mathematically creative to foster student creativity?
- Should mathematical concepts and skills be learned creatively or should they be memorised before students are encouraged to be creative?
- What methods of instruction might stimulate students to create new problems, solve problems uniquely, conduct research work in mathematics, etc.?
- Is it ever too late to search for creativity?

## 5. The use of problems and assessment

Rich tasks or problems are critical for encouraging mathematical creativity. The papers in this section presented a variety of tasks and problems that might be used to develop and evaluate mathematical creativity. Kim presented problems of rabbits and chickens, Pascal's Triangle, and the use of pattern blocks for introducing fractions, and discussed how these might be used to promote creative problem-solving. This idea of digging creatively into a simple problem was extended in Holton's paper with the six circle problem and in Chan's paper with the model-eliciting biggest box problem. In a similar manner, the Group MUSA.E1 used examples from Gaussian theory and Pythagorean theory in their discussion of the development of creativity. Bilchev presented a chain of steps for guiding students to pose problems and create a group of connected problems that lead to mathematical results that are new to the creator and even new to the world. He gave examples from his extensive work with the Rousse Mathematical Circles as well as his university work. Sedrakyan illustrated this idea of making connections and generalising from special cases to a wide range of related problems with examples of geometric inequalities. Cuador Gil extended the concept of creativity to the geosciences using geostatistics for engineering students. Sinitsky looked at essential features of activities for eliciting creativity and noted that any creative activity must take the student, the problem and the teacher all into consideration. Gogovska and Malcevski noted the importance of using counter-examples, Hall discussed using ill-defined problems, and Li emphasized open-mind questions, all to develop students' mathematical creativity. Swirski, Wood, Carmody and Godfrey discussed the use of creativity in the assessment of engineering students and Fong noted the importance and difficulty of incorporating more creative questions in assessments. Cibulis and Lace presented problems, games and toys used to recognise and assess mathematical creativity. Many of the papers addressed the idea of teaching less so students might learn more, giving students more time to delve deeply and creatively into topics of interest. The papers in this section introduced questions related to the use of problems in the development and assessment of mathematical creativity, including the following:

- How might mathematical problems be used to develop mathematical creativity? How might mathematical creativity be assessed? How do we evaluate our success in developing mathematical creativity in all students?
- What are examples of good investigations and problems that can be useful for promoting mathematical creativity?
- How might problems best be used to develop mathematical creativity?
- Can all mathematical problems be used to evoke a creative response? Should they be used in this way?
- How might assessment be used to promote rather than inhibit mathematical creativity so that all students might be creators and not simply consumers of knowledge?
- What are the effects of standardised or standards-based assessment on mathematical creativity?
- What criteria might be used to recognise, encourage, and assess creativity?

## 6. Technology and the environment

The environment plays a critical role in the need for, and nurturing of, mathematical creativity. No longer do individuals work in isolation. Technology gives students the opportunity to quickly access information, peers, and mentors around the world as well as actively investigate problems that would be nearly impossible without technology. Successful problem-solving in the twenty-first century will require that students are able to work efficiently and creatively with ill-defined problems, large amounts of information, calculators, computers, and others around the globe.

The papers in this section looked at the ways that technology and the environment affect students' mathematical creativity. Velikova described a successful teacher-training programme for preservice secondary teachers of mathematics and informatics where preservice teachers created a variety of multimedia applications, essays, and presentations to promote their own mathematical creativity, and that of the secondary students whom they will teach. Watanabe compared secondary students' problem solutions using paper and pencil to practice solving problems where the teachers have both the questions and the answers to creatively solving the same problems using graphing calculators or computers. One goal was to increase student interest in mathematics and to reduce their dislike for it. Bonka and Andzans also acknowledged the effect of interest and noted several tools for fostering mathematical creativity ranging from positive emotions and textbooks to correspondence courses, summer camps, and contests in newspapers and on the Internet. Johny also described the importance of improving attitudes and self-concepts while reducing mathematical anxiety. Using dynamic geometry software to develop mathematical creativity was discussed in articles by Flores Samaniego; Wurnig; and Kakihana, Fukuda, and Watanabe, while the use of computer algebra systems to stimulate mathematical creativity was investigated in papers by Windsteiger, Wurnig, and Siller. The papers in this section raised questions including the following:

- How do technology, other resources, and the environment affect the mathematical creativity of the student? Does the use of technology promote or inhibit students' mathematical creativity?
- What environment, technology and other resources best nurtures creativity? How does this environment differ from student to student, from grade to grade, from level to level, from subject area to subject area?
- How do these environments vary in different cultures? Does 'creativity' mean the same thing in different social contexts? How is the creative individual viewed in different cultures?