MATHEMATICS FOR THE 21-st CENTURY.

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1. The mathematical essence of nature.

The discoveries made by pre-Aristotle philosophers were to mark mathematics throughout the ages. The existence of exactly five regular polyhedra, named *Platonic solids*, the simple numeric relations in the harmonic scale, the Pythagorean relations of the triangle, the existence of infinitely many prime numbers, the axioms of elementary geometry, brought thinkers to express their awe for mathematics and its methods. From Plato's "God is a geometer" to Paul Dirac's view that "the fundamental laws of nature are expressed by mathematical theories of great power and beauty, we could say that God is a geometer of highest quality who used the most sophisticated mathematics in order to create the universe".

In his eloquent writings on the scientific method published around 1620, Galileo Galilei claimed that God's design for the universe "is written in the language of mathematics, and the characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to comprehend a single word of it." Those sentiments were controversial in an era when most learned men venerated Aristotle, who had described the world quite coherently using neither formulas nor equations. But from the perspective of the twenty-first century, the unique explanatory power of mathematics seems self-evident. We know that mathematics is essential to conceive the development of sciences as well as to the design of every device we use to augment our senses or control our environment, from the Internet to the refrigerators.

The force of Galileo's, and others' of his contemporaries, view of science was overwhelming. It implies, for instance, that scientific laws exist independently of human consciousness. Hence scientists are, like explorers of the shores of an unknown continent, gradually mapping the preexisting contours of nature through observation, experiment, and reflection. It is a perspective that inspires scientific research to this day: "Our universe is not just described by mathematics," for many scientists, "it is mathematics."

But, is this really so? There are those who argue that the mathematical structure of the universe is mere appearance, a human invention that results from our brain's natural tendency to impose order on the raw experience of our senses. This point of view was fed by mathematicians themselves in the 1800s, when Euclid's geometry, long thought to be the only way to describe the world, was found to be only one among many axiomatic systems that did the job. In this view, mathematics tells us more about the organising principles of our minds than the underlying structure of nature.

Euclid's *Elements* is probably the most-read book of all time apart from the Bible — an illustration of the surprising longevity of mathematics. In the *Elements* Euclid set out the rules of mathematical exposition as they are still followed today: start with a set of axioms, self-evident truths that no-one in their right mind would call into doubt, and then proceed to derive mathematical results from these axioms using only the rules of logic. During centuries, this was the essence of the methodology of scientific work, even Newton, when writing down his theory of gravitation followed this strict axiomatic approach.

One of Euclid's original axioms, however, did not appear that self-evident, and for centuries mathematicians had tried to prove that it followed directly from the others, so that it could in fact be left out. The nineteenth century crisis ensued because it became clear not only that the axiom was independent of the others, but also that without it, you could build other geometries that were also perfectly free of contradiction. Euclid's geometry, which had been assumed to be universally woven into the fabric of nature, was only one of several self-consistent alternatives.

This devastating realisation caused a shift towards a view of mathematics as a game whose rules could be changed arbitrarily to create another game. The rules that western mathematicians throughout the centuries had chosen to adopt are by no means the only ones available, they are simply the ones that chime best with our perception of the abstract aspects of the physical world. Mathematics, then, should be viewed not only as a human product which is separate from the physical world, but also as being devoid of content: a collection of abstract objects whose interactions are governed by whatever set of rules you choose. At the center of mathematics were not Euclid's geometric shapes or Pythagoras's numbers, but the rules imposed by logic.

At the beginning of the twentieth century efforts were made to unify all of mathematics into a single abstract and axiomatic system. But a second crisis was soon to follow, as logicians proved that any axiomatic system which generates even only the simplest of mathematical objects, the natural numbers and their arithmetic, will always contain a statement that no-one can ever prove to be true or false. In other words, there are things you can say of the natural numbers that are undoubtedly true, but which you will never be able to derive from the axioms. But proving things from first principles is of course what mathematics is all about. This view of mathematics as not only a human thought experiment, but an "imperfect" one at that, is a long way from the divine truth envisioned by the likes of Plato and Descartes.

Nevertheless, the crisis was about the founding philosophy of mathematics not about the importance and variety of applications of mathematics available to solve every sort of problem. Wigner's claim of the surprising efficiency of mathematics is not disputed. Many theories that were originally thought to be pure abstractions, later turned out to describe aspects of the natural world. For instance, group theory, devised in the 1800s as a description of the relations between various mathematical operations, provided the key for the understanding of subatomic structure a century later.

This approach describes the conceptual view of most working mathematicians through their own work. Mathematicians invent mathematical concepts, inspired by nature or otherwise, and they discover mathematical truths about those concepts, guided by the rules of axiomatic game.

Acknowledged as a powerful conceptual tool or dismissed as a capricious game, mathematics never lost its position as a problem solver. Applications of mathematics range from the very abstract to the very concrete solution of real life problems. Several questions emerge.

How important are these applications of mathematics? Surveyed in a superficial way, as the vast activity of mathematicians around the world, it is easy to come away with the impression that mathematics is not actually all that important. If you ask a mathematician to explain what he or she works on, you are usually told that it is not possible to do so in a short time. If you ask whether this mysteriously complicated work has practical applications, then there are various typical responses, none of them immediately impressive. One is the line taken by the famous Cambridge mathematician G. H. Hardy, who was perfectly content, indeed almost proud, that his chosen field, Number Theory, had no applications, either then or in the foreseeable future. For him, the main criterion of mathematical worth was beauty. At the other end of the spectrum there are mathematicians who work in topics where mathematics merges with other sciences. Then the criteria of worth are simplicity, accuracy of predictions, broadness of application and, yes, beauty. When asked what the chances of success of his new mathematical approach for solving a problem in quantum mechanics, the leading physicist Paul Dirac did not give it any chance since the proposed theory was not beautiful enough.

Do mathematicians really deal with reality? Contrary to the Platonic feelings of mathematicians and most users of mathematical tools, the answer is no.

Mathematicians inspect and manipulate, not natural phenomena but models of them. Models are intellectual constructions which try to express as many features of the phenomenon as possible, keeping a level of simplicity which allows mathematical manipulation of the information. This applies to all mathematics, even to the simplest mathematics.

Working on models brings the right amount of abstraction and generality which allows mathematical considerations to work. On the other hand, mathematicians, including those who work in useful-sounding branches of mathematics, do not work on problems with direct practical applications. They deal with models in the same way that pure mathematicians deal with the most abstract subjects.

2. The success of mathematical models.

For centuries, mathematics was used as the paradigm of an idealised concept of science. This idea of science was based on the following principles:

(a) The belief in a single, eternal, universal truth independent of the particularities of observers, history, or social conditions.
(b) The belief in objective, that is, observer-neutral, context-free knowledge.
(c) The belief in a stable, reliable, trans historical subject of knowledge, about which we can formulate true statements and construct objective knowledge.

These "pillars" of science were shaken and reinforced in critical moments of the history of science: first, Euclid showed that mathematical truth was eternal and universal, Newton show that the laws that govern heavens were the same as those describing the fall of an apple to earth. Mathematical intuition and the principles of science constructed so far were shaken by the discovery of alternate geometric models. Were these alternate models of real worlds? Then came quantum mechanics and relativity requiring acceptance of probabilistic models of atomic phenomena and a new role for the observer of phenomena. Both theories were built in the old axiomatic style but represented a shift from "understanding nature" to "modelling nature".

For us, a model is a simplified, often mathematical, representation or analogy of some aspect of reality, for the purposes of description and calculation. A mathematical model is usually a built-in element of a theoretical model. The physical theory interprets the mathematical model, including its assumptions and constraints. The mathematical model is needed to quantify the physical model, thus enabling the theoretical model to make precise predictions and applications.

Nevertheless, one is generally not satisfied with a model that merely saves the phenomena: it should preferably do so in terms of plausible physical principles. The plausibility of physical principles in turn depends very much on our worldview. One important function of mathematical models is to provide a precise mathematical connection between one's worldview and one's observations. The compatibility of the mathematical model and the physical concepts behind was so important for Newton that he postponed the publication of his *Principia Mathematica* for 20 years, until after the invention of integral calculus allowed him to show that the integrated attraction of a spherical body is the same as the attraction of a body whose mass is concentrated at the center of the sphere, a hypothesis used in the model.

One remarkable feature of mathematical models is their often astounding success. Frequently they work much better than might be expected. This is best illustrated in physics. It is remarkable that a wide range of physical phenomena can be modeled in terms of a very small number of physical principles. For example, general relativity can be used to describe the behaviour of objects ranging from billiard balls and bicycles to rockets and planets. Maxwell's equations allow us to describe all electro-magnetic interactions. Quantum mechanics provides the basis for chemistry. Physics has been a highly successful science primarily because the basic physical principles can be readily modeled by precise mathematical equations.

Why are mathematical models so successful? In 1960 Eugene Wigner, a Nobel-prize winner in physics, gave a famous lecture on "the unreasonable effectiveness of mathematics in the natural sciences". He concluded that the amazing applicability of mathematics to the physical world is a mysterious, undeserved and inexplicable gift.

But such a mathematical research strategy for making discoveries is essentially an anthropocentric (that is, man-centered) strategy. It presumes that humans have a special place in nature. This is because mathematics relies on human standards such as simplicity, elegance, beauty and convenience. Anthropocentrism is most blatant in those cases where even the notation of mathematics plays a role in scientific discovery.

Steiner gives the example of Paul Dirac's discovery of the positron. In 1930 Dirac applied quantum mechanics and special relativity to electrons. He ended up with a quadratic polynomial that had to be factored. When real and complex numbers did not work, Dirac introduced higher-dimensional number-like objects (4 dimensional matrices). This factoring yielded several extra solutions, in addition to the one corresponding to the electron. One of these solutions implied the existence of a particle identical to the electron but with a positive charge. Two years later, the existence of such particles--called positrons--was confirmed experimentally. Thus a mere mathematical trick, invented for computational convenience, resulted in a major physical discovery. Remarkably, the mathematical method Dirac applied (known as Clifford algebra) had been developed in the 1800's for entirely different, purely mathematical, purposes.

So the philosophical problem is not so much the applicability of mathematics to our descriptions of physical reality but, rather, the major role of human mathematics in the discovery of new phenomena. Of course, this would be justified by a realist view that mathematics exists objectively, in some ideal, non-physical realm, with both the physical world and our human minds somehow reflecting aspects of that mathematical realm. These features would explain the huge success of our mathematical models.

However, we must be careful not to over-rate mathematical models. For example, according to Roger Penrose, the concrete world of physical reality emerges mysteriously out of the ideal world of mathematics. Penrose views the mathematical world as the primary, real world; the other two worlds of our consciousness and physical things being mere shadows of it (1994). These views illustrate a peril facing scientists, particularly mathematical physicists. This danger is what Keith Ward calls "the fallacy of misplaced concreteness". One is so impressed by the beauty and predictive power of the model that one starts thinking this is reality. The fallacy is to mistake the abstract model for the concrete reality.

3. The quest for mathematics with a social face.

Counting and measuring, two of the most elementary mathematical processes are at the basis of every scientific development. Indeed, it is the notion of quantity and how to measure it that opens and establishes the first step of scientific methodology. But also for the individual and the community, counting and measuring yield the most primitive form of knowledge and control of material resources. In a few words, counting and measuring empower those who do it.

The 17th century enthusiasm for the sciences, building upon the underground age-old numerological mysticism of the hermetic and Kabbalah tradition, led to a frenzy of enthusiasm for a quantitative and mathematical study of social life among the scientists. Unfortunately, "hand in hand with this revolutionary ideal went a devout but misplaced notion that to measure and to understand were one and the same". In his classic *Gulliver's Travels*, Swift effectively lampooned the crazed scientists of Laputa and elsewhere who were putting into effect what would now be called a Baconian "research programme." Finally, in 1729, Swift followed up this satire with his famous *Modest*

Proposal, which was thought to be "the last word on political arithmetic as an instrument of social policy."

Disgracefully, political arithmetic reached new extremes during the 20th century and its applications to “social policies” range as the most vicious idea ever conceived. Indeed, during the 1930s and the years of World War II, the American-based multinational corporation International Business Machines (IBM) and its German and other European subsidiaries collaborated with the government of Adolf Hitler. IBM's technology helped facilitate Nazi genocide against the Jewish people through generation and tabulation of punch cards based upon national census data. In words of author of *IBM and the Holocaust*, Edwin Black:

> Mankind barely noticed when the concept of massively organized information quietly emerged to become a means of social control, a weapon of war, and a roadmap for group destruction.... Hitler and his hatred of the Jews was the ironic driving force behind this intellectual turning point. But his quest was greatly enhanced and energized by the ingenuity and craving for profit of a single American company and its legendary, autocratic chairman. That company was International Business Machines, and its chairman was Thomas J. Watson.

The 20th century was characterised all too often by critical situations involving politics, economics, and science. Just to mention a couple of landmarks, we recall the absurd support by Stalin to the anti-Darwinian views of the Lisenko school; the letter of Einstein to President Roosevelt suggesting the possibility of building the atomic bomb; the Manhattan project as the biggest concerted effort of scientists and politicians, an effort which produced not only the bomb but computers, radar and many civilian-friendly applications; as a byproduct, a new, higher, status for science which has had huge budgets in the developed countries ever since; the quest to conquer outer space and the paradigmatic image of setting foot on the moon. Mathematics and mathematicians played a major role in the exhaustive revision of the school curricula which led to the notion of *new math reform* and its disastrous consequences.

Tension in science during the 20th century is also illustrated by the changing role of philosophy of science. Until the mid 20th century, the philosophy of science had concentrated on the viability of scientific method and knowledge, proposing justifications for the truth of scientific theories and observations and attempting to discover at a philosophical level why science worked. Already Karl Popper had begun to attack this view. Popper denied outright that justification existed for such concepts as truth, probability or even belief in scientific theories, thereby laying fertile foundations for the growth of postmodernist attitudes.

During this time there had also been a number of less orthodox philosophers who believed that logical models of pure science did not apply to actual scientific practice. It was the publication of Thomas Kuhn's *The Structure of Scientific Revolutions* in 1962, however, which fully opened the study of science to new disciplines by suggesting that the evolution of science was in part sociologically determined and that it did not operate under the simple logical laws put forward by the logical positivist school of philosophy.

Kuhn described the development of scientific knowledge not as linear increase in truth and understanding, but as series of periodic revolutions which overturned the old scientific order and replaced it with new orders (that he called "paradigms"). Kuhn attributed much of this process to the interactions and strategies of the human participants in science rather than its own innate logical structure. Some interpreted Kuhn's ideas to mean that scientific theories were, either wholly or in part, social constructs, which many interpreted as diminishing the claim of science to representing objective reality. More radical philosophers, such as Paul Feyerabend, argued that scientific theories were themselves incoherent and that other forms of knowledge production (such as religion) served the material and spiritual needs of their practitioners with equal validity as did scientific explanations.

The attack upon the validity of science from the humanities and the social sciences worried many in the scientific community, especially when the language of social construction was appropriated by groups who claimed to proffer alternative scientific paradigms. Many scientists perceived that as attempted political control of science in society, for example, so-called ‘creation science,’ ‘intelligent design,’ and the continuing creation-evolution controversy.

Probably the best known event of these *science wars* is the Sokal affair. Physicist Alan Sokal had submitted an article to Social Text titled *Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity*, which proposed that quantum gravity is a linguistic and social construct and that quantum physics supports postmodernist criticisms of scientific objectivity. Later, Sokal exposed his article, as an experiment testing the intellectual rigor of an academic journal that would “publish an article liberally salted with nonsense if it sounds good and it flatters the editors’ ideological preconceptions”. The Sokal Affair thrust the academic world’s in-house scientific objectivity wars into the public eye.

Weak but popular science was not only on the postmodernist side. Also mathematics produced a good dose of it. Indeed, recall the situation during the 1980s with catastrophe theory. This is one of the few areas of mathematics research that have surfaced from the mathematics community and caught the fancy of the press and the general public. The theory and its application were the subject of the first article on mathematics published in Newsweek in at least seven years, were subsequently the subject of a Scientific American article, and have been praised by a number of mathematicians as well as by many investigators in other fields of science, including the social sciences. The mathematics behind these applications describe how solutions to a system of equations vary when certain parameters that appear in the equations are perturbed. When the parameters vary, the solutions can be pictured as jumping from one value to the next. These jumps, or discontinuities, are said to be *catastrophes*. The absurd situation arises when completely different phenomena, such as the crash of a stock market or the sudden attack of an angry and frightened dog, lend themselves to descriptions in terms of the same qualitative “jump” situations which, in principle, describe phenomena ruled by strict mathematical equations, when in fact nothing specific is said, or known, on the quantitative aspect of the phenomena “modeled”.

Certainly the worst pseudo-mathematics had its moment when postmodernists met areas of knowledge that lend themselves to loose applications. As explained by one of the proponents of such fusion[1]:

> New-wave postmodernists thinkers are likely to draw from chaos theory, Godel’s theorem, catastrophe theory, quantum mechanics and topology theory. Novel conceptions of space, time, causality, subjectivity, the role of discourse, desire, social structure, roles, social change, knowledge and the nature of harm, justice and the law...The call is for the abandonment of the center, privileged reference points, fixed subjects, first principles and the origin…Accordingly, this model starts with far from equilibrium conditions as being the more “natural” state, and places a premium on flux, nonlinear change, chance, spontaneity, intensity, indeterminacy, irony, and orderly disorder…Due to inherent uncertainties of initial conditions, iterative practices produce the unpredictable. Here, the focal concern is on tolerance and support for the incommensurable. Assumed is the existence of perpetual fragmentation, deconstruction and reconstruction…

The problem with this line of “science” is that it builds its discourse on ideas having no connection with reality, using a particular, not well specified language, science in the Aristotelian style. The sad and most damaging effect arises when philosophers, who should be giving illuminating views on nature and its processes, in fact do not understand the elements of modern science. In conclusion, we can follow physicist Steven Weinberg when he says: “. . . we should not expect philosophy to provide

[1] Dragan Milovanovic: *Dueling paradigms: modernist versus postmodernist thought*. Humanity and Society 19, 1 (1995), 1-22.

today's scientists with any useful guidance about how to go about their work or about what they are likely to find. After surveying three decades of professional writing in the philosophy of science, the philosopher George Gale concludes that 'these almost arcane discussions, verging on the scholastic, could have interested only the smallest number of practicing scientists.' Even Wittgenstein (1980) wrote that 'nothing seems to me less likely than that a scientist or mathematician who reads me should be seriously influenced in the way he works.' "

4. Mathematics for the 21st century.

Probably the most paradigmatic conditions in a rapidly changing world are the huge amount of information available to many people and the efficiency of communications systems. The new conception of this information-communication revolution started with the advent of the internet. Initially designed as an academic communication system it became, in few years, widely used, giving access to huge amounts of information to a number of people without precedent. As a consequence of this revolution, huge networks of social contacts have spread throughout the world, with uses ranging from the most pedestrian daily life messages to the most sophisticated exchanges, communications addressed to very specific recipients or messages seen by millions of people.

The development of new mathematics which helps understand social networks and the flow of information within them is a central topic with huge potential social applications. Intelligent widespread communication among people is probably the best tool society may find to assure the future of freedom and democracy.

A social network is a social structure made up of a set of actors (such as individuals or organisations) and the dyadic ties between these actors (such as relationships, connections, or interactions). A social network perspective is employed to model the structure of a social group, how this structure influences other variables, or how structures change over time. The study of these structures uses methods in social network analysis to identify influential nodes, local and global structures, and network dynamics. Social networks are distinct from information, biological, or electrical networks, but theories and methods generalising to all of these complex networks are studied in the field of network science.

Social networks and the analysis of them is an inherently interdisciplinary academic field which emerged from social psychology, sociology, statistics, and graph theory. Many different types of relations, singular or in combination, form into a network configuration, in such a way that network analytics are useful to a broad range of research enterprises. In social science, these fields of study include, but are not limited to anthropology, biology, communication studies, economics, geography, information science, organisational studies, social psychology, sociology, and sociolinguistics. Scholars in these and other areas have used the idea of "social network" loosely for almost a century to connote complex sets of relationships between members of social units across all scales of analysis, from the local to the global. Georg Simmel authored early structural theories in sociology emphasising the dynamics of triads and "web of group affiliations." Jacob Moreno is credited with developing the first sociograms in the 1930s to study interpersonal relationships as structures in which people were points and the relationships between them were drawn as connecting lines. These approaches were mathematically formalised in the 1950s and theories and methods of social networks became pervasive in the social and behavioral sciences by the 1980s.

As part of the recent surge of research on large, complex networks and their properties, a considerable amount of attention has been devoted to the computational analysis of social networks—structures whose nodes represent people or other entities embedded in a social context, and whose edges represent interaction, collaboration, or influence between entities. Natural examples of social networks include the set of all scientists in a particular discipline, with edges joining pairs who have coauthored articles; the set of all employees in a large company, with edges joining pairs working on a common project; or a collection of business leaders, with edges joining pairs who have served together on a corporate board of directors. The increased availability of large, detailed datasets encoding such networks has stimulated extensive study of their basic properties, and the identification of recurring structural features.

Social networks are highly dynamic objects; they grow and change quickly over time through the addition of new edges, signifying the appearance of new interactions in the underlying social structure. Identifying the mechanisms by which they evolve is a fundamental question that is still not well understood. We define a basic computational problem underlying social-network evolution, the *link-prediction problem*: Given a snapshot of a social network at time t, we seek to accurately predict the edges that will be added to the network during the interval from time t to a given future time t. In effect, the link-prediction problem asks: To what extent can the evolution of a social network be modelled using features intrinsic to the network itself? Consider a co-authorship network among scientists, for example. There are many reasons exogenous to the network why two scientists who have never written an article together will do so in the next few years: For example, they may happen to become geographically close when one of them changes institutions. Such collaborations can be hard to predict. But one also senses that a large number of new collaborations are hinted at by the topology of the network: Two scientists who are “close” in the network will have colleagues in common and will travel in similar circles; this social proximity suggests that they themselves are more likely to collaborate in the near future. This intuitive notion of *proximity* can be made precise through the introduction of adequate measures in a network. We find that a number of proximity measures lead to predictions that outperform chance by factors of 40 to 50, indicating that the network topology does indeed contain latent information from which to infer future interactions. Moreover, certain fairly subtle measures—involving infinite sums over paths in the network—often outperform more direct measures such as shortest-path distances and numbers of shared neighbors.

Although mathematical tools have been developed in order to understand networks, a full comprehension is far from reached. Two recent situations show the level or lack of understanding of the phenomenology of networks. Namely, the recent global economic crisis and the social uprising of Arab nor African countries which in days overthrew long lasting dictatorships. Let us look more closely to these events.

(1) The economic crisis.

The current financial crisis began in what is called the “structured credit products” market. The story here went something like this:

There was a greater demand for funding in the economy than what could be provided solely by bank lending. Therefore, bankers set up legal entities to provide funding by issuing notes to investors. Some investors, such as hedge funds, were willing to tolerate high risks of loss while other investors, such as pension funds, demanded low risk. To appeal to this wide range of investors, bankers pooled several assets, such as loans, and then structured the cash flows from these assets in such a way that they were able create “safe” and “risky” notes to sell to different types of investors.

This is where mathematics was needed. Mathematical and statistical models were required to assess the risk of the notes issued to investors. Specifically there was a need to model the correlation of defaults between assets in the pool that provided cash flows to the investors. The problem was that correlation estimates are notoriously unstable when modeling rare events such as defaults. Nevertheless, mathematicians happily provided bankers with their best efforts at modeling correlations. This went on for years until in 2007, unfortunately, the real world began to behave differently than what was predicted by the models. This led to even investors in “safe” notes to lose money, which in turn eventually led to the current financial crisis. “Quants” (as Mathematicians are known in the industry) and their esoteric models apparently misled the market.

The models failed to predict reality. Of course, the responsibility for failure lies with all market participants who directly or indirectly relied on these models. Mathematicians simply provided a tool. What was the problem? Classical determination of prices and risks is made assuming the performance of a single trade at a time with a fixed background economic environment. In the present situation, thousands of exchanges and trades are performed at the same time, sometimes via email, with a background which is far from being fixed. The mathematics of such massive-simultaneous trades has not been developed.

On the other hand, the crisis has accentuated the critics against the excessive formalism of economy. Criticism of the reliance of economy on abstract models, on its use of too much mathematics, has, in fact, been a constant for the past 150 years. Some of those attacks have come from knowledgeable insiders. More often, however, the attacks have come from outsiders - from journalists, political crusaders, and so on.

Of course, the gain of intuition due to the use of mathematical models sometimes comes at a cost: the modeler can become a prisoner of the assumptions embodied in his or her model. One often hears accusations, in particular, that model-based economics inevitably biases the field toward standard economic assumptions like constant returns, perfect competition, and perfect information. Yet this need not be true. In fact, economists who have worked at length on imperfect markets have found modeling an essential discipline in the process of exploring new territory. As was pointed out by Marshall, a reputed economist, "Economic doctrine ... is not a body of concrete truth, but an engine for the discovery of concrete truth ..."

(2) The Arab spring.

In 2011, the dictator of Tunisia was overthrown in less than one month after being in power for 23 years. He was the first of a series of deposed dictators in the Arab world. There is no question about how opponents of his regime were able to topple it. Two words describe it: Facebook, Twitter. These two social networking sites enabled protesters to take to the streets, organise the opposition, recruit new protesters, and overcome the police force and the military.

There is no question that if the government had chosen to use machine guns to cut down the protesters, it probably would have succeeded in suppressing the revolt. If it had combined machine guns with switching off the Internet, it would have been able to cut the protest down, both literally and digitally. But to do that, the regime would have had to act extremely fast, and it would have risked coming under international condemnation. It would also have created a permanent opposition, ready to revolt again.

The opposition forces are now connected, yet not organised. This has never happened before in recorded history. The masses can communicate with like-minded people for the price of a computer and an Internet connection. It is the power of the communications networks, when coupled with a willingness on the part of protesters to gather in the streets, that spells a period of crisis for every autocratic regime on earth. The autocrats have seen in January 2011 that it is difficult to put a lid on any unorganised protest. The organising did not come from some little group that can be infiltrated or arrested. This was as close to a spontaneous protest as anything we have seen in modern times.

The ability of the social networks to organise a protest almost overnight because people of similar beliefs and commitments are in close communication with others, has completely changed the nature of political resistance and revolution.

The governments of the world are caught in a disjunction. If they allow the Internet to stay up, and if the social networking systems continue to recruit people to go into the streets, a corrupt government will face a rapidly escalating crisis. Its legitimacy is being called into question, and the only way to restore order under these conditions is to begin to shoot people. On the other hand, trade and economy is every day more internet dependent. Shutting down the internet in the long run alienates the development of a modern state.

5. A proposal.

Our proposal to enhance mathematical sciences, in order to build the adequate conditions for a more equalitarian and democratic society, go in three complementary directions:

i. Educational. Mathematics education can give meaning to the idea of people empowerment. Good mathematics education can contribute to the creation of a critical citizenship and support democratic ideals. It can promote inclusion, but also exclusion and discrimination:

> Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an 'objective' judge, in order to decide who in the society 'can' and who 'cannot'. It therefore serves as the gate-keeper to participation in the decision making processes of society. To deny some access to participation in mathematics is then also to determine, a priori, who will move ahead and who will stay behind.

This statement by John Volmink can be read as a dramatic description of the role of mathematics education in marking a division between those who become included in and those who become excluded from society. There is no lack of suggestions of how to interpret mathematics education as serving questionable socio-political roles.

Better training in mathematics and mathematical methodologies should be promoted in all school levels hand in hand with a solid civic education which promotes critical independent thinking. Consequently, a more solid formation in mathematics should be compulsory for new teachers as well as development for in-service teachers.

ii. Research. New ideas and mathematical tools have to be designed in order to understand the evolution of real social networks. Social networks are highly dynamic objects; they grow and change quickly over time through the addition of new connections, signifying the appearance of new interactions in the underlying social structure. Identifying the mechanisms by which they evolve is a fundamental question that is still not well understood. A main problem is to predict accurately the evolution of a social network using features intrinsic to the network itself.

iii. Access to information resources. The amount of information accessible by digital means is so big and dispersed that it is becoming difficult to use, at least for non-experts. Mathematicians and computer scientists should help designing platforms where information is systematically classified, evaluated and presented to potential users.