TSG 22: New Technologies in the Teaching and Learning of Mathematics

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1. Aims, themes and organisation of the Topic Study Group

TSG 22 was intended to serve a dual role:

- overview of the current state of art in the topic and expositions of outstanding recent contributions to it, as seen from an international perspective
- sharing of ongoing work and perspectives.

The issues addressed focused on three broad themes:

Theme.1. Integration of technology into school and other learning environments: This theme refers both to research studies and the innovative development of technology-based curricula or units. It includes the study of learning processes with technology and the impact of technology on the learning of mathematics. It also addresses the issue of assessing mathematics with and through technology. Connectivity and virtual networks for learning mathematics is also part of the topic.

Theme.2. Issues related to the use of technology by teachers. How do teachers cope with perturbations introduced by technology? How do they succeed in the ordinary types of usage? What are teachers' conceptions of the use of technology? How do these conceptions evolve? How are they taken into account to promote changes in practice? The topic also includes issues about mathematics teacher preparation and professional development in the use of technology.

Theme.3. Design of technology for the learning and teaching of mathematics. This theme refers to the design choices of technology environments related to epistemological and cognitive aspects of mathematics and of the learning of mathematics, as well as to the features aimed at assisting teaching.

We received 70 submissions. We decided to subdivide the group into subgroups for 3 sessions and to have two types of presentations:

- - an oral presentation in a subgroup devoted to the theme of the paper, and
- - a poster or a demo presentation in a subgroup devoted to the theme of the paper.

Each paper was reviewed by two members of the Organising Team. From the reviews, agreement was reached on a final list of presentations, posters and demonstrations, leading to 39 oral presentations and 24 posters or demonstrations. Finally, due to cancellations, 35 presentations and 22 posters or demonstrations coming from 25 different countries took place during the sessions. Their distribution by themes was as follows.

	Theme 1	Theme 2	Theme 3
Presentation	19	6	10
Poster or demo	8	5	9
Total number	27	11	19

The report of the analogous group at ICME-10 mentioned that there was a need for more research that places the teacher as central focus. After four years, the number of research studies on teacher' use of technology is only slowly progressing and the same recommendation could be stated.

The Organising Team decided on several types of sessions to favor different types of interactions among participants: a plenary session (1 hour) introducing the work of the group by presenting the critical issues and questions, two sessions of 1.5 hours in groups of 20-30 people devoted to the presentations and discussion, one poster and demonstration session of 1 hour in three groups, one group per theme. In this latter session, attendants could visit the three adjacent rooms and have a global view of the content as well as discuss with presenters.

2. Content of the sessions

The poster session was very much appreciated: people enjoyed the free structure of this session that allowed them to interact directly with the presenters, to ask questions that could not be asked in a large discussion, and test themselves the demonstrations. In particular, a lot of interest was drawn by how technology is implemented in different countries. Examples were available from the Philippines, India and Mexico. The format of this session seems to be very appropriate for technology.

The sessions reflected the growing diversity of technologies and of mathematical content mediated by these technologies as follows.

Technology	Mathematical content	School level
CAS	Algebra, modeling, calculus	Early secondary, Secondary, University
Dynamic geometry 2D, 3D	Geometry, Multiple representations	Early secondary, Secondary, University, Pre- and in-service teacher education
Digital manipulatives Digital interactive representations	Numbers, Algebra	Primary, Middle school
Automatic assessment		Early secondary, Secondary, University
Hand held devices	Algebra, geometry, graphs, data processing, statistics	Secondary
Interactive whiteboard	any	any
Online, digital courses	any	Teacher prof development University

Theme 1 - Integration of technology into school and other learning environments

The presentations can be subdivided into four subthemes:

- Innovative development of technology based curricula or units dealing with the pragmatic aspect of the integration of technology; this development is needed to actualise the pedagogical potentials of technology and to expand individual case studies into class collective contexts (Karrer & Jahn, Burril, Or & Leung).
- Studies of learning processes with technology and its impact on the learning of mathematics: these studies are based on theoretical frameworks (Jahn & Flores Salazar, Psycharis & Kynigos, Soury-Lavergne, Geiger & Faragher); a key research focus is the epistemological value of technology in mathematical knowledge acquisition. New teaching learning paradigms as well as new mathematical knowledge may be constructed (Moreno, Armella, Gravina).
- Assessing mathematics with and through technology: What to assess and how to assess via techno-pedagogical tools pose challenges that might require re-conceptualisations of our assessment practices. Is there a new didactic contract when students are given computer based tasks (Pineau & Caron). Should technology serve to make traditional assessment modes more efficient or should technology facilitate to benchmark a different kind of mathematical knowledge? How can technology help to support or even create a continuous process of assessment for learning and teaching of mathematics? (Kortenkamp, Maffei et al., Wong)

- Connectivity and virtual networks for learning mathematics: Virtual Networking has become a pedagogical environment that plays an emerging role in forming communities of learning and practices contributing to the construction of social and even institutional knowledge. How can mathematical knowledge be communicated and even constructed in the Internet Highway? What are the roles/identities of teachers and students in a virtual network of learning? (Acelejado, Button, Karadag, Owens et al., Reyes, Wong)

The discussion after the presentations addressed the balance and interaction between using ICT environments and paper and pencil. Visualisation was a major topic of discussion for several ICT tools that are supposed to help visualisation. It appears that visualisation differs in some environments like 3D geometry environments and needs to be investigated further. Another question deals with the link between visualisation and internalisation. How to help students internalise what they visualise? Experimentation made possible by ICT and related feedback provided by ICT was also an aspect of technology addressed by the presentations and discussed in the groups. The collaborative dimension is a new element brought by the connectivity and should give rise to further studies in the future.

The dynamic geometry environments (DGE) gave rise to specific questions and discussion. In the design of tasks, how to make choices related to the "dragability" of components of the figure? How and what to assess in DG tasks? Construction activities offer a window on abstract thinking of students. The deductive dimension of a properly constructed figure in a DGE needs to be investigated further.

Theme 2 - Issues related to the use of technology by teachers

The questions addressed by the presentations were: How are teachers coping with perturbations introduced by technology; succeeding in the ordinary types of usage; conceiving and evolving their use of technology; promoting changes in their practice; preparing and continuing their professional development in the use of technology?

A common conclusion of several presentations is the need for assistance for teachers starting to use technology, either technically or pedagogically. The communities of practice as well as the preparation or the mentoring relationships may play a decisive role on teachers' attitudes toward technology as well as on the ways teachers use technology (Amado). It seems very difficult for teachers without adequate preparation or interaction with more experienced teachers to view new technologies as:

- changing the approach to problems or creating new problems to be solved by students; or
- modifying the learning and teaching processes (Faggiano, Garcia Campos & Rojano).

One presentation stressed that even if offsite training and virtual presentations may be satisfactory for some teachers, observations suggest that teachers who are tentative about technology need substantial onsite support (Sinclair et al.). This is related to the claim that distance workshops or on line courses were successful mostly because they became a means of creating a community of practice (Evans et al.).

As in Theme 1, the complexity of the teaching process explains the difficulty teachers are faced with when resorting to technology. Several theoretical frameworks were proposed to analyse this complexity, such as a multidimensional analysis structured around the three dimensions: cognitive, epistemological and didactic (Garcia Campos & Rojano), or an approach based on a Vygotskian perspective, or on activity theory (Juarez & Ramirez, Sinclair).

2.5. Theme 3 - Design of technology for the learning and teaching of mathematics

The presentations dealt with new digital products by making explicit their design choices and discussing the advances and challenges of such products for e-learning. The diversity of products ranged from panoramic works involving the participation of communities of practice around compiling and constructing digital tools for mathematics classrooms (see Montessinos & Kuntz re Sesamath), to developing digital resources addressing specific needs (see, for example, Borba, Leite & Gomes' software). Other important outputs were the development of web-based systems as Mathdev (Dibut & Leon, Cuba); Mathenpoche (Montessinos & Kuntz, France); WME (Wang et al., USA); and the design of digital manipulation artifacts that integrate different educative approaches (Prank et al.; Yeo Shu Mei et al.).

Some authors (Laborde, Trgalova & Chaachoua) discussed design issues concerning educational software, in particular specific characteristics or problems required by teachers or students to confront mathematically complex topics, such as 3D geometry. According to Laborde, if one takes the educational design principle that the computer representation of a mathematical object should be less complex than the object itself, it turns out to be an open non-trivial question how to represent 3D objects and interactivity with them.

An important issue addressed by several presentations is the choice of the types of representations and of metaphors. Several pieces of software combine several languages and representational systems. For example, in the case of Borba, Leite & Gomes' software, they combine Portuguese, LIBRAS (Brazilian sign language), iconic representations, diagrams, and algorithms. In this way it is possible to create an educational social space in classes that includes students of different needs, providing them with occasions and means for a joint approach to additive problems. The choice of metaphors is made at various levels: metaphors in the activities proposed to the students to make them attractive (Confrey, Maloney & Nguyen asked the students to design graphics and animation), metaphors for manipulating the provided representations, for example moving, shrinking or enlarging 3D objects (Laborde). Some systems propose representations used by computer scientists: tree representations (Trgalova et al.) for conceptualising algebraic expressions or diagrams of visual programming language like Prograph (Siller) for functional modeling. To what extent can mathematical knowledge be acquired via tree-like of Prograph-like thinking?

The collaborative dimension of the systems is another issue to be explored: what kind of collaborative work do they allow between students, between students and teachers, or between teachers? This issue seems to be critical for systems devoted to in-service teacher education, as noted in Theme 2.

Trgalova & Chaachoua pointed out that educational software cannot be effectively developed without a synergy between computer scientists, educational researchers and users (teachers and students). Making explicit the design choices is a necessary part of the communication and integration process within the design team.

The feedback issue was addressed. How should systems feedback to students with respect to errors (or inputs) of different kinds that will have implications on students' autonomy? The discussion contrasted the choices of T algebra (Prank et al.) and Aplusx (Trgalova & Chaachoua). These choices affect the ways algebra can be learned and understood.

One of the conclusions of this theme was that the time is ripe to look for specific methodologies or collaboration to allow qualitative assessing or comparative studies that delimit advantages and impact of digital systems for mathematics teaching and learning.

3. Conclusion

As the theme of the Topic Study Group was very broad, it is not surprising that the group received many contributions from numerous countries and that most of the sessions were well attended. One can expect that with the increasing development of technology in several directions, the scope of the theme new technology will grow in the future and it could give rise to more than one group in the next ICME. The sharing of results from a wide range of experiences, from many countries, and from different actors within the educational system (teachers, teacher educators, researchers) was appreciated.