TSG 34: Research and Development in Task Design and Analysis

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Designing the Topic Study Group

In our initial plans and reading for the TSG, we discerned a wide range of task types, and several different categorisations: problem; extended; real; group; technology-rich; realistic; practising; exercising; non-routine; application; non-routine tasks, and tasks which create uncertainty; tasks which provoke cognitive conflict; tasks scaffold conceptual understanding; tasks which reveal mathematical structure, and many more. It became clear that to move forward in our understanding of the importance of task design we needed to think about tasks not merely as ‘things to be done’ but as crucial in affording subsequent mathematical activity. We formed the view that conjectured relations between task structure, tool use, and mathematical activity inform the design and analysis of tasks, and also give insight into the nature of engagement while tasks are being carried out. It was our hope that vague descriptions like ‘problems’ or distinctions such as ‘open/closed questions’ would give way to thinking instead about what mathematical activity a task can prompt, given an appropriate pedagogical context.

We narrowed the focus of the group to tasks which enable learners to make shifts in understanding which are central to secondary mathematics. For example, we were interested in shifts from additive to multiplicative (and exponential) thinking. We were also interested in how learning mathematics at secondary level involves combining earlier conceptual understandings. We restricted the work of the group to invited contributions from a range of international researchers who have worked substantially and consistently in the field, with the aim of developing a coherent, emerging story that reflects the ´state-of-the-art´ in this domain. By not putting out a call for papers and relying instead on networks and publications we may have missed some significant developments, but we achieved a coherent working community with sustained attendance and discussion.

We solicited eight papers and asked the authors both to describe and to demonstrate theories and principles of task design. Presenters worked in pairs to compare their approaches, and two reactors carried out the mathematical tasks and provided pre-conference comments on the tasks and principles. Some of the contributors focused on tool use and technology; some focused on the nature of learning; some focused on how tasks reflect the nature of mathematics. None of the contributors assumed that tasks somehow cause learning in a direct fashion, and the role of pedagogy, or some other structures, in framing the activity that the tasks afford was seen as critical. We report on the principles but not on individual tasks for reasons of space. Because all the presenters refer to work that is ongoing we shall use the present tense.

The principles

Swan emphasises the importance of the mathematical purpose of tasks combined with relevant theories of learning and tested design principles. He focuses on constructing materials and methods which expose significant conceptual obstacles and generate surprise, tension, conflict which can be resolved through discussion. From these considerations Swan has devised five task-types: classifications which focus on properties, discrimination, definition and naming; relating different representations to coordinate and extend meanings and images; evaluating the truth value and domains of truth for mathematical statements and providing explanations;

creating problems; analysing reasoning and solutions and identifying errors within them. The activity prompted by these tasks has to be collaborative and the atmosphere non-judgemental; students are treated as serious thoughtful learners trying to sort out meanings.

Use of comparison and classification tasks is also described by Zaslavsky who justifies this approach as engaging learners directly with distinctions between objects whose properties relate in some way, or where visual similarities and differences might not be co-terminus with conceptual similarities and differences. Such tasks focus learners on those properties which are discernible using mathematical criteria. Objects, problems and methods can be sorted and classified according to their domains of application, relative power, membership of certain classes, and so on. The adoption and use of new classifications is an indicator of learning, and the use of new categories can develop mathematical awareness and the ability to seek new distinctions. She also highlights the importance of collaboration in such tasks.

Ainley and Schmittau are known for their work in the primary phase, but their contribution to our understanding of what kinds of learning are possible, given certain tasks, is of value for secondary education as well. Ainley, with her colleagues has developed tasks which have both purpose and utility. The purpose of a task is what the learners perceive it to be, the goals they construct for themselves, and a well-designed task helps them understand the more general utility of a mathematical idea. The designer starts by thinking about how to help students understand the utility of a mathematical idea through a task they will perceive to be purposeful, rather than by thinking about how to structure mathematical concepts and procedures for learners. Such tasks will:

- have an explicit end product that the pupils care about;
- be focused, but contain opportunities for meaningful decision-making;
- involve optimisation.

Optimisation imposes constraints that may require reasoning at a more general level than the task context, and subtle scaffolding of new ways of reasoning through the requirements of the task is also a feature of Swan's and Zaslavsky's work. Schmittau's approach is an enactment of Davydov's principle, which is to identify a genetic basis for a mathematical idea which is adequate to support its extension to more sophisticated abstractions. Thus the starting point for designing the teaching of exponentiation, for example, is to devise a concrete manifestation which supports a schema which is general enough to extend to include rational, irrational and negative exponents. Starting with exponents seen as repeated multiplications, and multiplications as repeated additions, will not support the development of appropriate schema since these understandings do not extend beyond positive integers. The concrete starting point she offers is plant growth, and the scaffolding is through sub-questions which focus learners away from the plant towards graphical representations, away from discrete points towards interpolation, and away from actual points to relations inherent in the patterns of growth.

Durand-Guerrier also starts her design by analysing mathematical structures, this time in a logical sense by focusing on their objects, properties, relationships, variables, connectors and quantifiers with the aim of relating what one does in mathematics to what it means, the syntactic to the semantic. She proposes constructing tasks to explore these logical elements which involve back and forth movements between familiar objects and the theories that arise from their manipulation and use. Learners have to construct certain understandings in response to prompts, but their understandings may be limited so the next prompt invites them to think again and refine their ideas further. The logical thread is not lost (students construct their own counter-examples when other approaches failed) and new mathematical understandings develop through the extended enquiry.

So far in our report, the only contributor to use 'real' contexts for enquiry is Ainley with a situation designed to be authentically purposeful for learners. In line with this, Tabach and Friedlander suggest design principles which exploit the potential of realistic contexts to facilitate learning processes by providing real or concrete meaning, to provide points of reference for work at an abstract level, and to raise motivation. However, they also recognise the concerns held by many educators that open-ended tasks and students' choice may lead to unproductive or low-level application of mathematical methods. They offer principles for structuring sequences of open tasks which enable mathematical exploration of a situation, increasing sophistication of mathematics, but still allow for extension, choice and enquiry. This involves sequencing the subtasks in ways which reflect the complexity of the situation being explored, complexity of cognitive demand, and complexity of the nature of the task. In this way, there is a balance between mathematical activity, general problem-solving activity, and activity which is directly relevant to the context.

The particular task offered involves relations between, and interpretation and transformation of, various representations. The power of representation and comparisons between representations is also reflected in the tasks offered by other authors above. In Arthur Lee and Allen Leung's work, representation of dynamic geometry is key to their task design principles, since it gives visual access to the constraints imposed on figures by essential geometrical relations, and also access to co-variational data from which learners might conjecture about such relations. In their words, the focus of the learner is on how measurements behave, rather than how they are found. Their principles are based on variation theory which is ideally suited for mathematics task design, since it characterises learners as deriving conjectures, expectations and theories by perceiving simultaneous variation. Dynamic geometry depicts theorems by this means.

Finally, Goddjin reported on the long process of design of a task which is intrinsically interesting for large numbers of students but is used for competition between teams. Since the object here is not the individual learning of new mathematics (although this undoubtedly takes place during such events) we would not expect to compare principles and methods directly, but nevertheless there are some interesting features. Their design process involves "a collective mathematical discovery trip" of teachers, mathematicians and mathematics educators exploring mathematical ideas for themselves in order to experience the kinds of obstacles and insights that might occur for others.

Some overarching points emerge which we recommend for further thought.

- Relation between task design and teaching: some approaches are about designing tasks *for* others to use; some about designing tasks *with* others who use them; some about designing task *by* those who use them.
- Pedagogic purpose of tasks: in all tasks learners are expected to learn some new-to-them mathematical ideas or methods, not merely to apply existing understandings or develop certain ways of working or reasoning.
- Mathematical enthusiasm: all the design methods exhibit, explicitly or implicitly, the mathematical enthusiasm and knowledge of the authors, and the final one made this particularly clear – that good design might depend on alertness for productive ideas and personal engagement with mathematics. In their case the team involves several mathematicians, teachers and didacticians, but on a smaller scale we suggest this still holds true.
- Representation: all the tasks offered depend on appropriate representations to draw learners' attention to key features of mathematical objects; in several cases deciphering the representation gives access to, and requires engagement with, underlying mathematical relations. Comparison of representations leads to deductive reasoning to distinguish between examples, or analogical reasoning to identify variables.
- Manipulation: all the tasks require some kind of manipulation, either of objects or variables, either physically or virtually. Learners have to discern, act out, initiate, or track changes; or they have to construct, match, combine static objects which require some tinkering to get right.

- Collectivity: all tasks provoke discussion in groups, or whole classes, the points of discussion being precisely those places where mathematical understanding needs to change.

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