

## **TSG 38: The History of the teaching and learning of mathematics**

Team chairs: Renaud d'Enfert, xxxxxxxx, France, renaud.denfert@u-psud.fr

Ángel Ruiz, xxxxxx, Costa Rica, angelruizz@racsa.co.cr

Team members: Luis Carlos Arboleda, Colombia xxxxxxx

Rodrigo Cambray, Mexico xxxxxx

Wann-Sheng Horng, Taiwan xxxxxxx

In 2004, for the first time, a TSG on *The history of the teaching and learning of mathematics* was held at ICME 10. Its principal promoter, Gert Schubring, in *Paedagogica Historica*, published the proceedings of this TSG in *Paedagogica Historica*<sup>1</sup>; an international bibliography on this subject compiled by Schubring in cooperation with the other TSG 29 team members was also published on line<sup>2</sup>.

This initiative was continued with TSG 38 with a more specific theme: The reforms of mathematics education in primary and secondary schools in the 19th and 20th centuries. Mathematics teaching reforms represent moments of significant changes, or even a breakdown, which redraw the contents, the methods, and the goals of teaching. Consequently, they are the subject of debates about the social, pedagogical or epistemological stakes of mathematics teaching that the historical enquiry can fruitfully investigate.

By focusing on mathematics teaching reforms during the last two centuries, the TSG aimed to analyse, within an international perspective: the conceptions and the motivations of the various actors—individual or collective—involved in the reform process; the transformations of the mathematical curricula established by the reforms in relation to the educational, social and political context; how they were received by teachers, pupils, and public opinion. It also studied ways to ensure their success as well as repercussions on teacher education. Another aim was to widen historical research further than the national frameworks, and to highlight the international importance of reflecting on mathematics teaching.

The following papers were delivered during the four sessions<sup>3</sup>:

### **Session 1. – Reforming mathematics teaching: commitments and resistances**

This session examined the involvement and reforming ambitions of mathematics teachers gathered in associations or study group, and also their opposition to changes.

T. Fujita (UK) & K. Jones (UK): *The process of redesigning the geometry curriculum – A case study of the UK Mathematical Association activity in the early 20th Century*.

This paper analyses the proposals of the 1900s UK Mathematical Association teaching Committee with the object of renewing geometry teaching in England. It outlines the defining factors which caused a reduction in the significance of the project: the variety of the positions as well as the lack of consensus within the teaching Committee, due essentially to the attachment of certain teachers to Euclidean geometry as the basis of school geometry, but also to the following argument: the vast majority of mathematics teachers do not seem to be ready for a radical reform.

L. Giacardi (Italy): *The School as a “Laboratory”. Giovanni Vailati and the Project for the Reform of the teaching of mathematics*.

The author examined the way Giovanni Vailati, an Italian mathematician and secondary school teacher, viewed mathematics, his proposals of reform of the mathematics teaching for the Italian

---

<sup>1</sup> G. Schubring (dir.), *History of teaching and learning mathematics*, Special Issue, *Paedagogica Historica*, vol. 42, n° 4 & 5, August 2006.

<sup>2</sup> See the web pages <http://www.tc.columbia.edu/centers/ijhmt/> and <http://www.icme-organisers.dk/tsg29/BiblTSG.pdf>

<sup>3</sup> Several papers presented at TSG 38 were published in the *International Journal for the History of mathematics education*, vol. 4, n° 1, 2009.

Secondary School system in the first decade of the 20th Century, and the criticism these proposals provoked. Vailati was influenced by other European mathematics reform movements, in particular in Germany with Felix Klein, and France with Jules Tannery and Emile Borel. He was in favor of Scientific Humanities (as were the French reformers) and defended the educational value of mathematics. Vailati advocated experimental and active geometry, and the interactions with algebra instead of traditional Euclidean geometry.

E. Zardo Bùrigo (Brazil): *Modern mathematics in Brazil: the promise of efficient and democratic teaching.*

The paper considers Brazilian teachers' involvement in the modernisation of mathematics teaching, fifty years later. It describes the activities of the GEEM, Grupo de Estudos em Ensino de Matemática in Sao Paulo, an organisation that took part in the "Modern mathematics" (or "New Math") movement in Brazil. GEEM was founded by Osvaldo Sangiorgi in 1961, under the influence of the Study Group in mathematics teaching of the USA, which played an important role in the New Math reform in that country and Latin America. This Brazilian group, like others in that country, became voluntary militant agents for a mission that provided them with a collective identity, and it was instrumental in developing the reform.

## **Session 2. – Changing mathematics teaching with textbooks**

N. F. Ellerton (USA) & M. A. Clements (USA): *The process of decolonizing school mathematics textbooks and curricula in the United States.*

This paper summarises key factors that influenced the generation and the transformation of mathematics textbooks in the United States during the 18th and 19th centuries. The impoverished fledgling nation wanted to avoid expenses associated with importing English Arithmetic. It also wanted to show that, as a nation, it was capable of generating texts superior in quality to those from England. However, Pike's Arithmetic (1788), the first mathematics text written in English by a US citizen, followed a similar sequence of topics and used a similar approach to texts used during the colonial period. The process of decolonising school mathematics textbooks and curricula evolved over the next 50 years.

• K. Bjarnadóttir (Iceland): *The history of public education in mathematics in Iceland and its Relations to Secondary education.*

The paper considers several official decisions with severe consequences for public mathematics education in Iceland and its relation to secondary education. Legislation was enacted in 1880 concerning education in writing and arithmetic and in 1907 concerning free public education within the responsibility of the local communities. These were landmarks in the history of public education in mathematics and spurred creation of mathematics textbooks for the general public. However, several official decisions made in the period from the 1920s to 1940s ensuring the right to education for all children, had grave consequences and hindered the development of public mathematics education. This paper shows, in particular, the longevity of the use of O. Danielsson's and E. Bjarbnason's textbooks during the 20th century, until the New Math period in the 1960s and 1970s.

I. Safuanov (Russian Federation): *History of teaching of the concept of a function in Russia.*

In this paper, definitions of functions in Soviet school and undergraduate textbooks used in the 20th Century are traced. Mathematicians such as Luzin, Kolmogorov and Aleksandrov were the first to realise the necessity to introduce the modern definition of a mapping into the scientific and educational literature. Already in the 1960s the general concept of a mapping (function) began to enter into curricula of secondary and tertiary schools. Appropriate steps have been undertaken for the preparation of school teachers. Nevertheless, in the 1980s, the general concept of a mapping has been eliminated from the school curricula.

M. C. Araújo de Oliveira (Brazil): *Modern mathematics teaching proposals as seen in published textbooks in Brazil.*

In Brazil, Professor Osvaldo Sangiorgi was one of principal supporters and advocates of the Modern Mathematics Movement. He has acted as a teacher at secondary and tertiary teaching levels, and was author of many textbooks adopted between the 1950's and 1970's. This paper expounds an analysis of professor Sangiorgi's use of updating proposals for mathematics teaching—as propagated by his textbooks *Matemática, curso moderno*. It includes the study of the contents of numerical sets and operations in the Gymnasium initial series (11-12 years old), as presented in Sangiorgi's textbooks, and focuses on methodological proposals as well.

### **Session 3: mathematics teaching reforms and counter-reforms**

H. Gispert (France): *Two mathematics reforms in their context in 20th Century France: similarities and differences.*

This paper compares two French reforms in mathematics teaching: the 1902 reform and the Modern Mathematics reform in the 1960s, the similarities and differences between them, in relation to the organisation and the goals of the school system, and the epistemological stakes or the pedagogical methods. One major difference is the way mathematicians consider their own discipline and its teaching, and more particularly geometry. In 1902, geometry teaching had to be experimental and concrete; in the 1960s, mathematicians consider geometry as a deductive science (and not an experimental one) and distinguish the real world from its mathematical model. But in both cases, they wanted to break away from the traditional Euclidian construction. Modern mathematics reformers did not consider that giving priority to a deductive approach contradicted the promotion of active methods for teaching and learning.

M. C. Leme Da Silva (Brazil) & W. Rodrigues Valente (Brazil): *Students' notebooks as a source of research. On the mathematics education history.*

Like Gispert's for France, this paper examines two reforms carried out during the 20th century, but in the case of Brazil: the reform promoted by Euclides Roxo (and influenced by Felix Klein) in the 1930s and the Modern mathematics reform in the 1960s. They consider students' notebooks to examine how these two reforms were received. The paper shows, for example, in the case of Modern mathematics, how a hybrid is found in the classroom which integrates the guidelines and topics of the reform simultaneously with methods and pedagogical approaches that dominated during many previous decades: that is to say, there was a distance between the formal intentions and the reality of the classroom. It also points out the existence of differences in the use of notebooks in both reforms: from being, in the first case, a means to almost crystallize an individualized teacher/student relationship to becoming, in the second case, decades later, just a classroom resource to complement the written specific textbook used with the spirit of the New Math.

A. Karp (USA): *Back to the Future: the Conservative Reform of mathematics education in the Soviet Union during the 1930s-1940s.*

The author describes how a mathematics teaching reform can take the opposite view to a previous one. That is the case with the Soviet reform of mathematical education during the 1930s and 1940s in the Soviet school system. Karp describes it as a “conservative reform”—one could say a “counter-reform”, that is, a kind of return to mathematics teaching as carried out before the 1917 Russian revolution, which was in complete contrast to the 1920s school programmes. He outlined several fundamental changes from 1931, mainly attributable to the ideological context: the simplification of mathematics programmes and examinations, which aimed to provide pupils with well-established knowledge instead of general education; an increased surveillance of mathematics teachers, asked to have a good command of their subjects and to apply specific teaching methodology. According to Karp, this “conservative reform” established a high standard of mathematics teaching traditions in the country.

#### **Session 4: teaching practice**

F. Soares (Brazil): *Defining the teachers' knowledge: a discussion about examinations for primary and secondary school teachers in Brazil in the nineteenth century.*

The authors reveal that, during the XIXth Century in Brazil, in order to be hired, teachers had to take examinations in public contests. The paper describes aspects of the knowledge held by Brazilian educators of primary and secondary schools in mathematics. By 1850, the subjects of the examinations concerned were: for primary schools, arithmetic – emphasizing problem solving and practical knowledge – and the Imperial System of weights and measures; for secondary schools, arithmetic, geometry, algebra and trigonometry. However, the candidates were not only examined on mathematical content, but also on methodological aspects.

C. A. N. Kurz (USA): *The struggle of mathematics education for the deaf during the late 19<sup>th</sup> century.*

The paper provides a review of the debate on how to teach mathematics to deaf people in the United States at the end of the 19th Century. The debate took place between two extreme positions. On the one hand, a pedagogical methodology where mental arithmetic was favored: it emphasised drill and rote, which, it was argued, provokes accuracy and rapidity for a reduction in the time spent on those tasks. On the other hand, to enable conceptual understanding by means of visual illustrations, object teaching and discovering activities proposed before they go to drill and rote for accuracy, that is, the learning process being more important than memorisation of facts. With time, what prevailed was a combination of approaches affirming a “mastery of arithmetic” which became a necessity for deaf graduates. A back-and-forth movement persisted between both approaches.

G. Schubring (Germany): *Comments on the presented papers and on recent developments in related research.*

Concluding TSG 38, Schubring insisted on the need to find new sources concerning the history of mathematics education, and to reinvestigate sources which have been already studied: in particular, the study of teachers' examinations and notebooks seems to be relevant to grasp the reality of the teaching and the learning of mathematics and its historical transformations.

#### **Conclusions**

By focusing on teaching reforms (or transformations) and, consequently, on the interactions of the various actors concerned, the TSG 38 sessions, which illustrate the vitality of research on this subject in Brazil, showed the social, cultural and political dimensions of mathematics education and the multiplicity of its determining factors. It appears, in particular, that the epistemological bias often cohabits with pedagogical intentions, so that it would be very fruitful to develop international research on the links between mathematics teaching reform movements and pedagogical ones, especially in the case of the New Math reform projects, which it seems were strongly influenced by perspectives of the New Education.