The Critical Role of Task Design in Lesson Study

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Principles of task design are described in the context of Japanese Lesson Study. Then, through analyzing tasks used for research lessons, and analyzing teachers' discussions during lesson planning, this paper identifies some critical activities in designing tasks in Japanese Lesson Study. It is shown that task design and evaluation are based on *kyozaikenkyu*, a critical but under-appreciated element of Japanese Lesson Study, and address broad educational values. Furthermore, tasks are created and designed with consideration of anticipated student thinking and solutions, and are evaluated through the post-lesson discussion that follows a research lesson.

Keywords: task design, kyozaikenkyu, lesson study, problem solving

Introduction

Lesson Study, the Japanese approach to improving classroom teaching, came to the attention of educators outside of Japan primarily through the publication of *The Teaching Gap* (Stigler & Hiebert 1999). Though most of the book focuses on findings from the 1995 TIMSS Video Study, Chapter 7 of the book, based on work by Makoto Yoshida (Fernandez & Yoshida 2004), describes Lesson Study in detail. Since then, many mathematics teachers and teacher educators have been involved in Lesson Study, and many books and research papers have been written on various aspects of Lesson Study and the typical structure of Japanese problem-solving mathematics lessons.

But it is becoming clear that there are aspects of Lesson Study that are implicitly understood by Japanese teachers that have not transferred easily to other countries. For Lesson Study to be successful these aspects should be made explicit. This paper tries to clarify an embedded key aspect of Lesson Study: task design. In particular, the paper discusses how a task for a lesson is designed and evaluated in the context of Lesson Study.

Japanese Lesson Study

The history of Lesson Study in Japan spans more than a century. For Japanese educators, Lesson Study is like air, felt everywhere because it is implemented in everyday school activities, and so natural that it can be difficult to identify the critical and important features of it.

Catherine Lewis (2002, 2011) characterized the Lesson Study cycle as follows (see Figure 1):

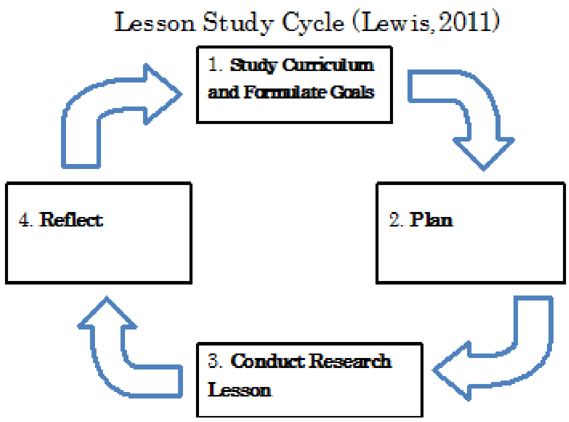


Figure 1: The Lesson Study cycle (Lewis, 2011)

1. Study curriculum and formulate goals: Consider long-term goals for student learning and development. Study curriculum and standards, identify topic of interest.
2. Plan: Select or revise research lesson. Write instruction plan that includes: Long-term goals, Anticipated student thinking, Data collection, Model of leaning trajectory, Rationale for chosen approach.
3. Conduct the research lesson: One team member teaches the lesson, others observe and collect data.
4. Reflect: In a formal lesson colloquium, share data from the lesson to illuminate student learning, discrepancies in content, lesson, and unit design, and broader issues in teaching-learning. Document the cycle to consolidate and carry forward learnings as well as new questions into the next cycle of lesson study.

Lesson Study in Japan takes place at three different levels: the individual school level, the district or regional level, and the national level. The Lesson Study cycle is basically the same at each level and usually spans one year. At the school level, the typical Lesson Study cycle begins at the end of one academic school year—i.e. in February or March—when the faculty decides upon a research theme for the next school year, which starts in April. Several research lessons are scheduled from, say, May to November. Each research lesson and its post-lesson discussion occupy only one day, but the teachers reflect on what they learned at the research lessons and usually write a booklet or long summary report by the end of school year.

Although the Lesson Study cycle is the same at all levels, the purposes are different, and these different purposes impact the task design. National-level Lesson Study is usually research-oriented: an academic or veteran teacher may take primary responsibility for the Lesson Study and teach the research lesson. It emphasizes the use of materials or tasks never seen before, and the goal is usually to demonstrate that the materials or task has a good mathematical and educational value. The goal of school-based lesson study, on the other hand, is usually to accomplish the school theme or mission. School-based teams usually use familiar tasks from a textbook, perhaps with slight revision.

In any case, Japanese teachers involved in Lesson Study spend at least a few months, but sometimes more than half a year designing a task and planning a lesson. The long-term period of planning a lesson crystallizes into a detailed lesson proposal or lesson plan. The lesson proposal includes the task for the lesson and the reason why it is used, described in detail. Therefore we can analyze the framework of a lesson proposal in order to gain insights into the nature of task design in Lesson Study in Japan. We will do that in the next section. Then, in the following section, we will discuss the common policy of focusing a research lesson on a single task.

The detailed lesson proposal

One of the characteristics of Japanese research lessons is that they are based on a *gakushushido-an*. *Gakushusido-an* is usually translated as "lesson plan." Since "plan" misleadingly implies a fixed script, however, the term "proposal" is better.

Japanese teachers spend a lot of energy and time crafting a lesson proposal. The contents of the typical lesson proposal give clues about the task design process. Although the details vary from school to school and even from teacher to teacher, Lewis (2002) notes that a typical proposal for a research lesson in Japan consists of the following:

1. Name of the unit
2. Unit objectives
3. Research theme
4. Current characteristics of students
5. Learning plan for the unit, which includes the sequence of lessons in the unit and the tasks for each lesson
6. Plan for the research lesson, which includes:
    - Aims of the lesson,
    - Teacher activities
    - Anticipated student thinking and activities
    - Points to notice and evaluate
    - Materials
    - Strategies
    - Major points to be evaluated
    - Copies of lesson materials (e.g., blackboard plan, student handouts, visual aids.)
7. Background information and data collection forms for observers (e.g. a seating chart)

The explicit inclusion in the proposal of the tasks for each lesson in the unit indicates how important the tasks are believed to be, and that the authors of the proposal think carefully about their sequence within the unit. Their role or function in the unit or even their position in the whole curriculum are studied by teachers and clearly stated in the lesson proposal. In other words, the lesson proposal shows that task design involves the explicit linking among tasks within the unit and across units in Lesson Study. Connections among tasks are revealed also when a research lesson is implemented: Shimizu (2010) showed evidence that tasks are connected to each other within the teaching unit through the teacher's explicit efforts to link students' ideas and experiences.

To link tasks within the unit and across related units in previous and later grades, teachers need to understand the scope and sequence of the curriculum. This requires a reasonably precise curriculum. Teachers also have to consider the learning trajectory of students, considering the mathematical and educational value of each task not only for the current lesson but also for the future. The learning trajectory is a critical consideration in constructing the detailed lesson proposal and, therefore, is critical in task design.

Structured problem solving

Almost every research lesson in mathematics follows a certain form, referred to by Stigler and Hilbert (1999) as "structured problem solving." A structured problem solving lesson focuses on a single task and contains four phases:

1. Presenting the problem for the day (5 to 10 minutes)
2. Problem solving by the students (10 to 20 minutes)
3. Comparing and discussing (*neriage*) (10 to 20 minutes)
4. Summing up by the teacher (*matome*) (5 minutes)

This type of lesson imposes certain demands on the task design. In Japan, "presenting the problem" means helping students understand the context of the task and what it will mean to solve the task, but it specifically excludes any exposition by the teacher about how to solve the task. Instead, students are expected to work independently on the task for 10 to 20 minutes, during which time at least some students should solve it. The third phase, *neriage*, assumes that students will arrive at different solution methods, which are then compared and discussed for the purpose of helping all students learn new mathematics and ways of thinking. Thus the task should be understandable by the students with minimal teacher intervention; it should be solvable by at least some students (but not too quickly), and it should lend itself to multiple strategies.

In the fourth phase, *matome*, the teacher may say something about which strategy may be the most sophisticated and why, but it should go beyond that to include comments by the teacher concerning the mathematical and educational values of the task and lesson (Fujii, 2008b).

Designing the task as part of *kyozaikenkyu*

The activities or factors involved in creating a research lesson proposal can be categorized based on whether they relate primarily to 1) the curriculum, 2) the students, 3) mathematics, or 4) tasks. Ultimately, however, the lesson requires a task, and so all activities eventually focus on investigating appropriate tasks consistent with the aim of the lesson. Watanabe et al. (2008) identified four core steps involved in constructing an instruction plan for a lesson: 1) Understand the scope and sequence; 2) Understand children's mathematics; 3) Understand mathematics; and 4) Explore possible problems, activities and manipulatives (Figure 2). Japanese teachers routinely do this as part of preparing a detailed lesson proposal; the process is called *kyozaikenkyu*.

The meaning of kyozaikenkyu

Kyozaikenkyu literally means the study of, or research on, teaching materials. For Japanese educators, designing the task is the essential activity of *kyozaikenkyu*. The word *kyozaikenkyu* and the activity to which it refers may be unfamiliar to non-Japanese, but it is a common educational term used in academic journals Japan. In fact, the *Journal of the Japanese Society of Mathematics Education* has a section devoted to *kyozaikenkyu.*

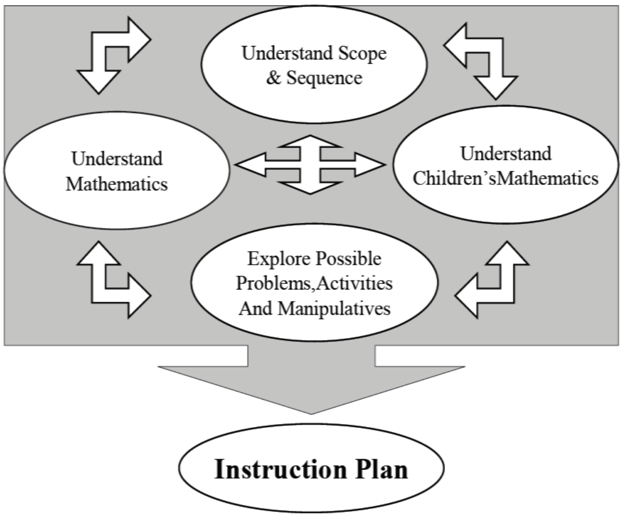


Figure 2: *Kyozaikenkyu* process (Watanabe et al., 2008)

Kyozaikenkyu involves examining teaching materials and tasks from mathematical and educational points of view as well as from the students' point of view. Moreover, Japanese teachers also investigate ways to encourage students to solve a task by themselves. Although *kyozaikenkyu* is recognized as a critical part of Lesson Study by Japanese educators, teachers outside Japan often neglect it:

> Japanese educators place a strong emphasis on task selection, [but] this effort is largely ignored by non-Japanese adapters of Lesson Study, possibly because the effort involved may be almost invisible, in the way that 90% of an iceberg is invisible, with all of our attention going to its visible tip. (Doig et al. 2011, p.182)

Task design principles for structured problem solving lessons

Japanese educators distinguish between "teaching how to solve the task" and "teaching mathematics through solving the task." This is why most structured problem solving lessons focus on a single task. If chosen well, a single task allows for the important new mathematical ideas to emerge in the discussion, and additional tasks are unnecessary.

But the ultimate aim of a structured problem solving lesson is not just to promote students' mathematical understanding or skill; the aim is also to deepen and widen their wisdom and thinking as human beings. This might sound strange or unrealistic, but consider the following problem:

> Squares are made using matchsticks as shown in the picture. When the number of squares is eight, how many matchsticks are there?

This problem lends itself to many solution strategies, including these:

(a) Consider a C-shape (3 match sticks) as a unit 8 times, and add the final side: 3*8 + 1
(b) Consider a C-shape as a unit 7 times, and finish with a full square: 3*7 + 4
(c) Draw the entire figure and count.

A comparison and discussion of these strategies might help students see the merits of strategy (a) relative to (b), because that solution (based on counting 8 C shapes) is more directly related to a condition given in the problem (8 squares). If the condition is changed to 100 squares, adapting the solution is very simple. Meanwhile, strategy (c), though mathematically primitive, is nonetheless quite powerful: you are certain to arrive at the answer. So this problem makes it possible for students to gain at least two general bits of wisdom: 1) one should think about the conditions of a problem and look for a solution in terms of those conditions; and 2) even if you cannot come up with a "clever" solution, you may still be able to solve a problem through hard work.

Thus we have the following principles for an ideal task:

- It is appropriate and mathematically valuable in terms of the aims of the lesson;
- It interests the students;
- It is at the appropriate level of difficulty;
- It can be solved in several ways.
- It can apply to other mathematical problems or real life problems.
- It has a potential to elicit valuable basic wisdom.

Designing tasks using *kyozaikenkyu* in Lesson Study

Doig et al. (2011) illustrate four types of tasks typically used in Lesson Study: tasks that either

1) directly address a concept;
2) develop mathematical processes;
3) are chosen based on a rigorous examination of scope and sequence; or
4) address a common misconception.

In this section we will focus on an example of designing a task based on a rigorous examination of scope and sequence, using *kyozaikenkyu* in Lesson Study in Japan.

The topic: Subtraction with regrouping

Japanese first grade textbooks contain a unit concerning subtraction of one digit numbers from two digit numbers (less than 20) using regrouping. There are a total of 36 such possible subtractions: 18 – 9, 17 – 9, 17 – 8, 16 – 9, 16 – 8, 16 – 7, …, 11 – 3 and 11 – 2. This is regarded as an important area of content, and which of these 36 subtractions should be the first for children to learn is hotly contested.

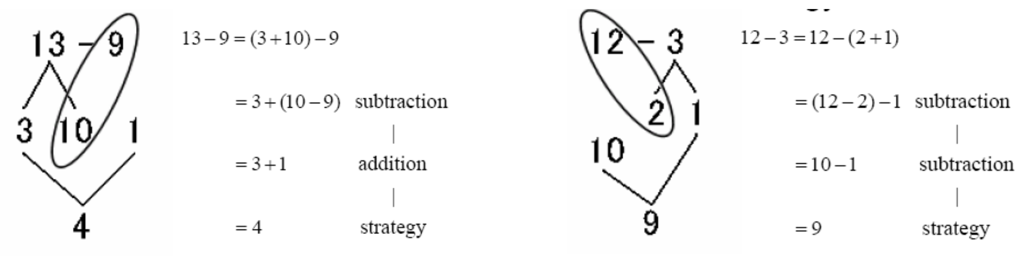
Teachers know there are reasons for the numbers used

Chapter 7 of The Teaching Gap follows a teacher team as they engage in Lesson Study focusing on this specific unit. Upon examining different textbooks, the teachers realize that almost all textbooks start with 13 – 9 or 12 – 9, and after reading the teacher's manuals, they understand why.

This activity, that is, investigating and studying textbooks and teacher's manuals, is a typical early step in the design task for teachers engaged in Lesson Study. Teachers may decide to use a task that is in one of the textbooks, or they may not. But they know that the specific choice of numbers influences students' solutions, and that there are reasons for the numbers in the textbooks. Therefore the decision to deviate from the textbooks, or not, is made carefully.

In the textbooks, the reason why 13 – 9 is the first subtraction problem with regrouping is that the subtrahend, 9, is close to 10. It is easy for the student to separate 13 into 10 and 3, subtract 9 from 10, and then add the difference to 3: 13 – 9 = (10 + 3) – 9 = (10 – 9) + 3. This strategy is referred to as the "subtraction-addition strategy" (see Figure 3a). Consistent with this sequence of tasks for subtraction, the addition part of the textbook uses 9 + 4 as the first task.

In contrast, with the subtraction 12 – 3, because 2 and 3 are close to each other, it is easy to break 3 down to 2 and 1 and subtract them sequentially: 12 – 3 = 12 – (2 + 1) = (12 – 2) – 1. This strategy is called the "subtraction-subtraction strategy" (see Figure 3b).



(a) (b)

Figure 3: The subtraction-addition strategy (a), and the subtraction-subtraction strategy (b).

With a subtraction like 14 – 8 the two strategies, *subtraction-addition* and *subtraction-subtraction* tend to be used by students with approximately equal likelihood. Therefore, teachers who wish to promote argumentation in their classes sometimes use a problem like 14 – 8 as the first task for children, while textbook companies adopt a more conservative stance based on their desire to make it easy for teachers to anticipate student responses and to be sure that there will be enough children who use the subtraction-addition strategy.

The teachers in *The Teaching Gap* decided not to use 12 – 9 from the textbooks because "it's not very interesting." One teacher suggested 15 – 8 or 15 – 7, then a teacher suggested 11 – 6: "Because kids can conceptualize in their heads about up to the number 6 at this age, I thought we should go with numbers like 11 – 6." Another teacher proposed 12 – 7, because "one of her students, who was a low achiever, happened to have seven family members. Everyone agreed that this was a good idea" (p.118). So the teachers decided to use 12 – 7, which seemed likely to provoke the subtraction-addition and subtraction-subtraction strategies equally, allowing for a discussion that would compare the relative merits of these two methods.

Such careful scrutiny of the sequencing of tasks is unusual by Western norms. "Western observers are often astonished… by the order of presentation being the subject of so much study and debate. However, Japanese Lesson Study is frequently used to investigate sequences of tasks that are different from those traditionally used." (Doig et al., 2011, p.194)

Why teachers begin **kyozaikenkyu** *with textbooks*

Kyozaikenkyu with textbooks is a typical activity of Japanese teachers in Lesson Study. Japan has a National Course of Study, and textbooks must be authorized by the Ministry of Education, Culture, Sports, Science and Technology. So all textbooks treat the same topics in each grade. But the six publishing companies that publish mathematics textbooks each have their own philosophy. Therefore, it is natural for teachers to compare each textbook's treatment of the same content. Investigating the textbooks often includes study of the teacher's manuals, which include not only suggestions about how to teach the content but also the reason for teaching the content and the reason behind the textbook's approach.

Exploring possible manipulatives

In the case of the Lesson Study on subtraction, the teachers implemented two research lessons focused on the subtraction problem 12 – 7. In the first lesson, children seemed to struggle with decomposing 12 into 10 and 2. Therefore at the second lesson, teachers modified the manipulative from a single piece of tape representing 12 to two pieces: a longer tape representing 10 and a shorter piece representing 2, and scissors were available for cutting the tape representing 10. The teachers spent a lot of time coming up with the new manipulatives. This is a good example of *kyozaikenkyu* in terms of exploring possible and appropriate manipulatives. And it is a good example of how task design in Lesson Study includes consideration of the materials and manipulatives that should be provided to students.

In the case presented above, one can see that, both in terms of choosing specific numbers to use and in terms of choosing suitable manipulatives to provide, good task design must involve considerations of likely student thinking and strategies, which is why anticipating student responses to a task is a standard part of Lesson Study.

Evaluating the task in action

In Lesson Study, the quality or functionality of the task is evaluated through the research lesson and the post-lesson discussion. At the research lesson, observers collect evidence from students' activities of whether the task worked well or not in terms of aims of the lesson. During the post-lesson discussion, teachers talk about the effect of the task on students' learning in accordance with the aims of the lesson by citing concrete evidence from the research lesson.

Since the role of the task and anticipated solutions are described in the lesson proposal, observers will typically watch to see if the anticipated solutions emerge or not. The proposal for the subtraction lesson identified four approaches that students might use to subtract 7 from 12:

1. counting-subtraction, i.e. starting with a group of 12 objects, or a group of 2 and 10 objects, and eliminating 7 objects while counting one-by-one;
2. supplement-addition, i.e counting on from 7 to 12 while keeping track of the number of counts ("8, 9, 10, 11, 12");
3. subtraction-addition (Figure 3a); and
4. subtraction-subtraction (Figure 3b).

At the research lesson, four children presented their methods at the blackboard to whole class. The four solutions included two that were anticipated: counting-subtraction and subtraction-subtraction. The supplement-addition method and the subtraction-addition method were not presented, but two unanticipated methods were presented. One was to subtract 2 from 12 and 7, and then subtract 5 from 10. This method could be expressed (not for first graders) as $12 – 7 = (12–2) – (7–2) = 10–5 = 5$. Only one child used this strategy in the class, although the whole class eventually seemed to understand it. The other was to partition the number 12 as 5, 5 and 2. Then, as the student explained, “because seven is five and two, I moved the five and two of the number twelve.” Only one child used this strategy although “many of the students said that her solution was good” (Fernandez & Yoshida 2004, p.165).

Taking a high-level view of the discussion that followed the research lesson, the school faculty raised 23 points of discussion. Ten of them, or about half, concerned the task:

- two concerned the specific numbers used in the task, such as “12-8 or 13-7 would be better”;
- four concerned the manipulatives, such as “if all 12 tiles had been lined up in one straight line, students might have cut the 10-strip into 7 and 3 to use subtraction-addition strategy” (p.176);
- the other four were: 1) the way the problem was presented and how to present word problems in general; 2) the reason why only one child—who was not asked to present his solution—used the subtraction-addition strategy; 3) how the handout and manipulatives had been improved; and 4) why the supplement-addition method was unlikely to emerge in the lesson.

These 10 points of discussion provide examples of how a task is evaluated in Lesson Study.

Taking a closer look at the post-lesson discussion, the teacher who implemented the research lesson with the problem 12 – 7 confessed that she was very disappointed because she could not get a variety of student solutions on the board; in particular, she had hoped to see the subtraction-addition method presented (Fernandez & Yoshida, 2004, p.171). These comments at the post-lesson discussion show that a task cannot be evaluated solely on its mathematical merits, but should be judged based on its actual effects on student thinking and learning. This is characteristic of task design and task evaluation in the context of Lesson Study in Japan.

The faculty did not discuss the unexpected solution method that used the subtraction rule (i.e. 12 – 7 = (12–2) – (7–2)). This was reasonable, since the focus of the lesson was on using regrouping. But the rule is interesting: another useful variant is (12+3) – (7+3) = 15 – 10 = 5. The task 12 – 7 created an opportunity to learn the rule, and the teachers could have discussed the possibility of including the subtraction rule in the elementary school curriculum.

Discussion

The case study from *The Teaching Gap* points to two important features of task design in Lesson Study. First, task design involves anticipating students’ solutions. Second, the task is evaluated in the post-lesson discussion based on concrete evidence collected during the lesson.

Task design in Lesson Study always involves anticipating students’ solutions

For the subtraction lesson, the teachers seriously considered which numbers to use, because they know that the choice of numbers will affect which strategies the students will use when solving the task. Furthermore, the teachers recognized that each strategy has both mathematical and educational values.

Such close attention to the specific numbers does not mean that teachers are sticking to a concrete level of thinking and encouraging students to think about things concretely. On the contrary: teachers consider the general aspect of the number—its *quasi-variable* aspects. A *quasi-variable* is a number deliberately used in a general way so that it serves as a representative of many numbers, just as a variable would (Fujii & Stephens, 2001, 2008a). Numbers are often chosen, then, based on their quasi-variable power, or how well they demonstrate a general truth.

For instance, the tasks 13–9 and 12–9 are likely to lead to the subtraction-addition strategy; thus they are not mere calculation problems, but lead to a particular general procedure for subtracting with regrouping in the base-ten system. Appreciating the base-ten system and place value notation system and its benefit for calculation is more important than getting an answer and gaining skill at calculating 13–9. To get such appreciation, however, students need to see alternative strategies such as the subtraction-subtraction strategy or counting down, neither of which depends on the base-ten notation system. Therefore, a structured problem solving lesson includes a *neriage* phase for students to compare or experience friends' methods and discuss similarities and differences among strategies in a whole class setting. Thus, when designing the task, there needs to be consideration of whether the task will elicit the alternative approaches needed for an effective *neriage*.

Task design in Lesson Study goes with task evaluation

The second feature of task design in the context of Lesson Study is that task evaluation is an inherent part of the process, wrapped into the evaluation of the lesson. The task is not judged based on some abstract determination about whether it is good or not for teaching a certain skill or concept, but based on concrete evidence from the lesson about how the students responded to it.

That evaluation often goes beyond the specific content of the lesson. For example, at the subtraction lesson, teachers discussed whether the task was appropriate or not and whether the manipulatives functioned well or not. But they also discussed more general issues. In fact, in 13 out of the 23 points of discussion, teachers discussed how to develop guidelines for fostering student's presentation skills. This was a concern not just for first-graders but all students. Sample comments included: "The skill like speaking in public and explaining what they think in a logical manner are the important things"; and "These skills are needed to do well beyond the subject of math" (p.181). These show that the teachers evaluate a lesson and task in terms of a broader educational aim.

The final commentator also addressed broader educational values. He said that in order for students to pit solution strategies against each other, they must be given the opportunity to evaluate them based on their own attempts to solve the problem. He pointed out that there were no comments from students such as "Teacher, I found that this method is more convenient," or "This method is much faster," because each student had experienced only one way to solve the problem. (p.187) He gave specific examples from the lesson in proposing improvements to the lesson. But he seemed to be suggesting that this activity could make students think about classroom values such as the importance of listening carefully to friends' opinions, of expressing ideas clearly to friends, of moving beyond "wrong or correct answers", of not underestimating friends' ideas, etc. Here is a good example from Lesson Study in Japan of how structured problem solving can be a context in which to nurture students as human beings.

The final commentator explicitly addressed broad educational values in the very beginning of his comments. "He urged teachers to think carefully about what were the most important 'skills for living' that students should be learning from their mathematics instruction." (p.182) Using as an example the formula for finding the area of a trapezoid, he said that "teachers should help students realize that moving from complicated to more simple forms is a convenient and a clever thing to do." (p. 183) This is an example of how Japanese Lesson Study concerns educational values.

This notion is related to the structured problem solving type of lesson. A common misconception about such lessons is that the solving the task is the main point. Such misconception leads to a focus on goals such as "students can do X" or "students understand X." But a structured problem solving lesson is not just about finding the solution to a problem. It is well and good that students can do X, but X should contain some value, and what that value is needs to be considered. To identify the educational values, the final commentator urged the teachers to do *kyozaikenkyu*.

Thus we see that the flat model of *kyozaikenkyu* from Watanabe et al. (2008) needs to be extended to a 3-dimensional model as shown in Figure 4. In this revision of the model, the four goals of *kyozaikenkyu* collectively serve a larger goal, which is to develop tasks and lessons that bring broad educational values to life in the classroom.

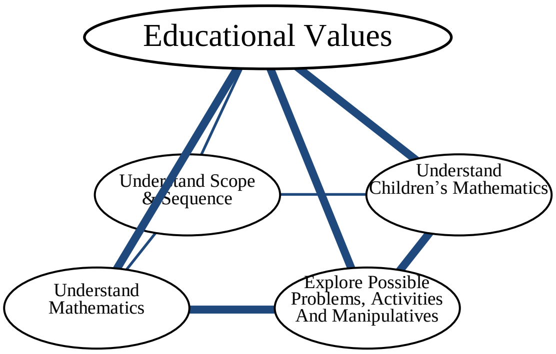


Figure 4: A pyramid model of *kyozaikenkyu*, showing how the process aims to serve broad educational values.

It is hard to actually implement an ideal lesson with a rich task and a discussion that addresses broad educational values. Accomplishing this is therefore a lifelong goal of teachers. Lesson Study, an on-going activity of Japanese teachers, both helps them develop such lessons and provides a testing ground for teachers.

Conclusion

Task design is the essential activity of *kyozaikenkyu*, which for Japanese educators is a critical part of Lesson Study. There are two sides to task design: anticipating students' solutions when writing the lesson proposal, and evaluating the task during the post-lesson discussion in light of the actual students' responses in the research lesson.

We hope that by making the task design activity more visible, we can help teachers understand the *kyozaikenkyu* process more profoundly. Designing tasks as part of *kyozaikenkyu* will strengthen teachers' content knowledge, improve instruction, and deepen their understanding of Lesson Study itself.

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References

Becker, J.P., E. Silver, M.G. Kantowski, K.J.Traversand, and J.W. Wilson. (1990). Some observation of mathematics teaching in Japanese elementary and junior high schools. *Arithmetic Teacher,* October, pp.12-21

Christiansen, B. & Walther, G. (1986). Task and activity. In B. Christiansen, G. Howson & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 243–307). Dordrecht: Reidel.

Doig, B., Groves, S., and Fujii, T (2011). The critical role of task development in lesson study, in Hart, Lynn C.; Alston, Alice S. and Murata, Aki (eds.), *Lesson study research and practice in mathematics education,* pp. 181-199, Dordrecht: Springer.

Fernandez, C. and Yoshida, M. (2004). *Lesson Study: A Japanese Approach to Improving Mathematics Teaching and Learning.* Routledge, Taylor & Francis Group, New York.

Fujii, T. and Stephens, M. (2001). Fostering an understanding of algebraic generalisation through numerical expressions: The role of quasi-variables. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra* (pp. 258-264). Melbourne: University of Melbourne.

Fujii, T. and Stephens, M. (2008a). Using Number Sentences to Introduce the Idea of Variable. *Algebra and Algebraic Thinking in School Mathematics.* National Council of Teachers of Mathematics, Seventeenth Yearbook, pp.127-140

Fujii,T(2008b). Knowledge for teaching mathematics. Plenary Talk at the 11th International Congress on Mathematical Education, Monterrey, Mexico, July 6-13.

Groves, S., & Doig, B. (2002). Developing conceptual understanding: The role of the task in communities of mathematical enquiry. In *Proceedings of the 26th conference of the International Group for the Psychology of Mathematics Education.* Norwich: University of East Anglia.

Lewis, C. (2002). *Lesson Study: A Handbook of Teacher-Led Instructional Change.* Research for Better Schools, Inc. Philadelphia, PA.

Lewis, C., & Hurd, J. (2011). Lesson Study step by step: How teacher learning communities improve instruction. Portsmouth, NH: Heinneman

Stigler, J. & Hiebert, J. (1999). *The Teaching Gap: Best ideas from the world's teachers for improving education in the classroom.* New York: The Free Press.

Watanabe, T., Takahashi, A., & Yoshida, M. (2008). Kyozaikenkyu: A critical step for conducting effective lesson study and beyond. In F. Arbaugh & P. M. Taylor (Eds.), *Inquiry into Mathematics Teacher Education.* Association of Mathematics Teacher Educators (AMTE) Monograph Series, Volume 5.

Yoshida, M.(1999). *Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development.* Unpublished doctoral dissertation, University of Chicago, Department of Education.