The tasks

1. Distances and Voronoi Diagrams
2. From Rope Puzzles to Algebra
3. Productive Practice in Algebra
4. Drug Level
5. Basketball and Proportional Reasoning
6. Expressions and Formulas

- Work on some of these tasks in collaboration with your colleagues.
- Discuss them both from a student and a designer perspective.
- Do you recognize task overarching design principles?

ICMI Study 22 Task Design in Mathematics Education

Task Design from a Realistic Mathematics Education Theory Perspective

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Michiel Doorman m.doorman@uu.nl
Paul Drijvers p.drijvers@uu.nl

http://www.fisme.science.uu.nl/~doorm101/ICMI22_RME_FI_booklet.pdf

July, 24th, 2013

Universiteit Utrecht

[Faculteit Bètawetenschappen
FIsME Freudenthal Institute for Science and Mathematics Education]

This session’s goals

To …

- … have hands-on experience with tasks designed from an RME perspective

- … reflect on the RME principles for task design

This session’s agenda

1. Introduction (5’)

2. Group work on tasks (25’)

3. Plenary reflection on... (25’)
   - RME Principles
   - Relation with the tasks

4. Discussion (5’)

2. Group work on tasks

1. Distances and Voronoi Diagrams

Distances and Voronoi diagrams

Thirsty in the desert
Below you see part of a map of a desert. There are five wells in this area. Imagine you and your herd of sheep are standing at *J*. You are very thirsty and you only brought this map with you.

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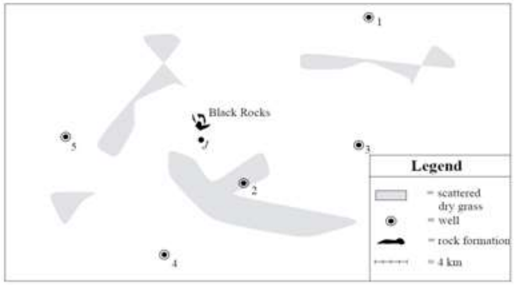

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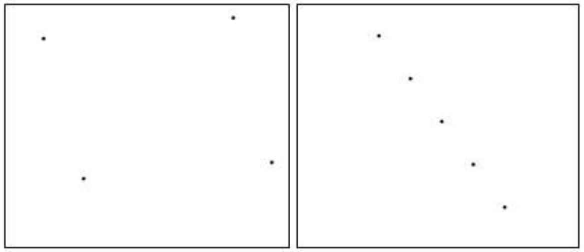

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(adapted from Goddijn, Kindt, & Reuter, 2004)

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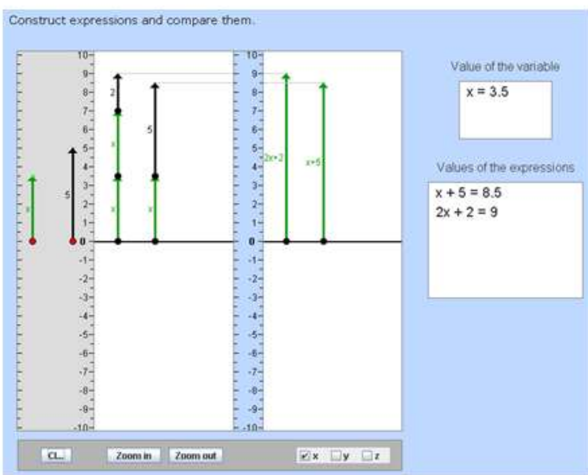

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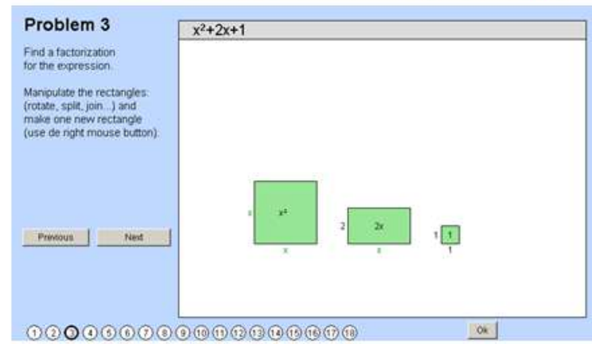
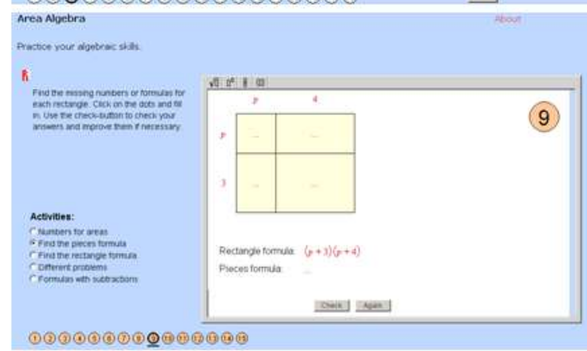
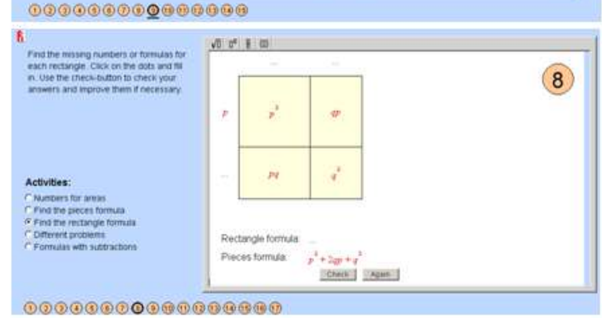


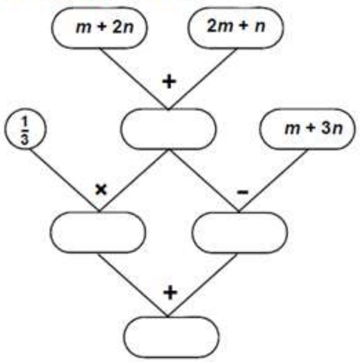




3. Productive Practice in Algebra

Productive Practice in Algebra

Operating with expressions



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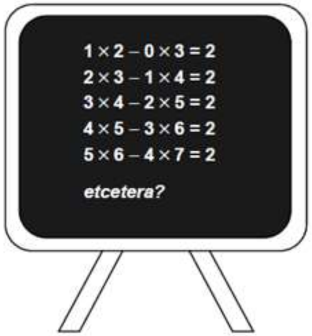

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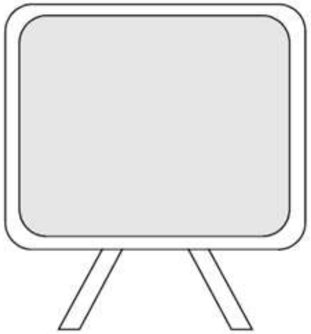

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Solution 1

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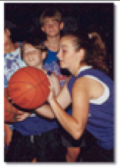


Ernst	●●●●● ●●●●● ●●●●● ●●●●● ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○
Julia	●●●●● ●●●●● ○○○○○ ○○○○○
Tom	●●●●● ●●●●○ ○○○○○ ○○○○○ ○○○○○
Kim	●●●●● ●●○○○ ○○○○○ ○○○○○
Bram	●●●●● ●●●●● ●●●●● ●●●●● ●●●●○ ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○
Paul	●●●●● ●●●●● ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○
Tess	●●●●● ●●●●● ●○○○○ ○○○○○
Liz	●●●●● ●○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○ ○○○○○

Start of a lesson plan

The teacher introduces the problem and makes sure that the students understand the situation and understand what is asked.

A prediction of student responses

Reasoning with absolute numbers: students may say, for example, that Bram is the best, because he has the most black dots.

Referring to proportions: students may say that it is the proportion of the black dots that matter. For example: 'Bram scores 24 times, but he has tried more often than others', or: 'Ernst has scored 20 *out of* 50, but Julia 10 *out of* 20.'

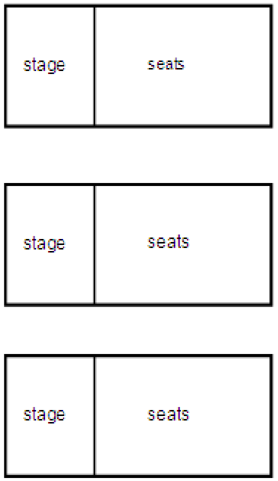
Actions of the teacher

The teacher stimulates a discussion about proportional reasoning. In a basketball game counts how many times someone might score when there is a chance. In other situations a reasoning in absolute terms might be more appropriate.

If all students immediately interpret the situation in proportional terms, the teacher may ask a question like: 'In another group a student said that Bram was the best, because he has scored 24 times, what do you think about that?'

Activity 2

Three performances will take place in the school theater. How busy will the theater be during each performance? Color the part of the hall that is occupied and write down the percentage of the seats that is occupied.



a. A pop concert

b. An historical play

c. A fashion show

6. Expressions and Formulas

Expressions and Formulas (MIC)

Home Repairs

Jim is a contractor specializing in small household repairs that require less than a day to complete. For most jobs, he uses a team of three people. For each one of the three people, Jim charges the customer $25 in travel expenses and $37 per hour. Jim usually uses a calculator to calculate the bills. He uses a standard form for each bill.

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People often call Jim to ask for a price estimate for a particular job. Because Jim is experienced, he can estimate how long a job will take. He then uses the table to estimate the cost of the job.

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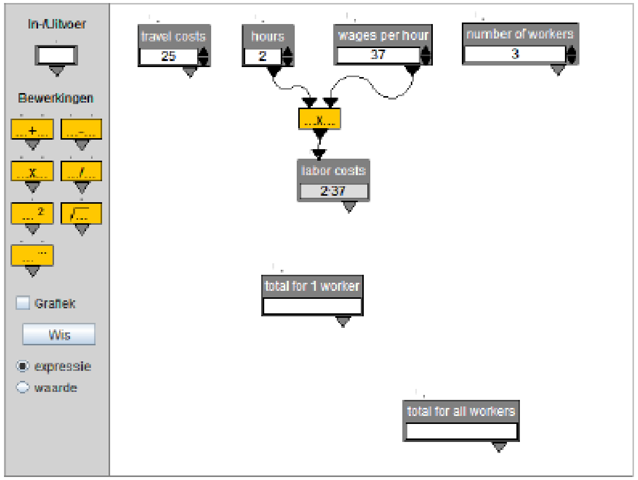
4.
   a. Draw an arrow string that Jim could use to make more rows for the table.
   b. Use your arrow string to make two more rows (for 6 and7 hours) on the table.

The calculations within this task can be structured in a way that prepares for dealing with functions:



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3. Plenary reflection: tasks

Revisiting the tasks

?

Remarks, reactions?

Do you recognize task overarching design principles?

3. Plenary reflection: RME theory

Realistic Mathematics Education

Local RME instruction theories for topics (fraction, function, ...)

Local instruction theory

Realistic Mathematics Education

Domain specific theory

General notions on learning and teaching

e.g., constructivism

Hans Freudenthal (1905-1990)

Mathematics as human activity:

- construct content from reality
- organize phenomena with mathematical means

Guiding ideas:

- Didactical phenomenology
- Guided reinvention
- Mathematizing (instead of transmitting mathematics)

What means ‘realistic’?

A double, somewhat confusing meaning!

1. ‘real world’ contexts, applications, problem situations

2. Zich REALISEren’ in Dutch: To be aware of, to realise, to imagine, to give meaning,….

Van den Heuvel-Panhuizen, M., & Drijvers, P. (in press). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. xxx-xxx). Dordrecht, Heidelberg, New York, London: Springer.

RME teaching principles

- Activity principle (social)
- Reality principle (in both senses, didactical phenomenology, mathematization)
- Level principle (horizontal and vertical mathematization, emergent modeling)
- Intertwinement principle
- Guidance principle (guided reinvention)

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RME design principles

- Use of meaningful contexts to support students' intuitive reasoning as starting points (scaffolds)
- Use of didactical models that fit students' reasoning and offer opportunities for vertical mathematization
- Intertwine mathematical ideas, strategies, topics
- Goal: extending 'common sense reasoning' instead of developing isolated pieces of knowledge and skills

RME and Didactic Engineering

- Both started as oposing against new math
- Both experience based and theory driven

- DE: phenomenotechnique (study didactical phenomena) <?> RME: didactical phenomenology
- DE: fundamental situations <?> RME: situations that beg to be organised

4. Revisiting the tasks

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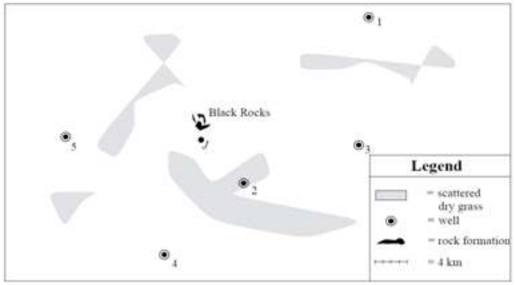

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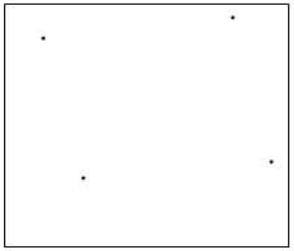

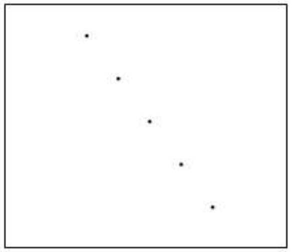

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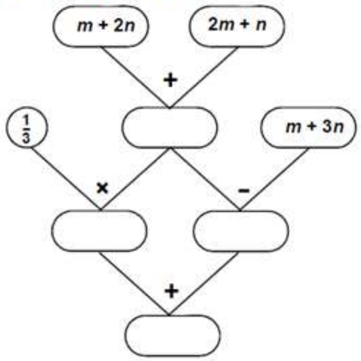

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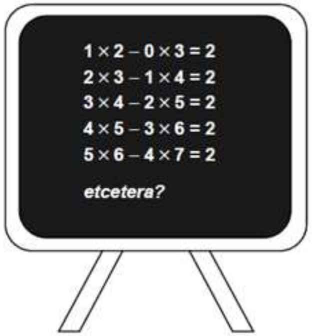

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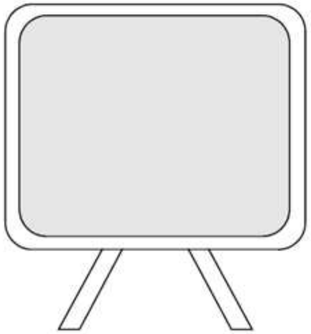

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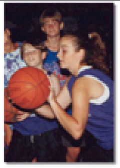

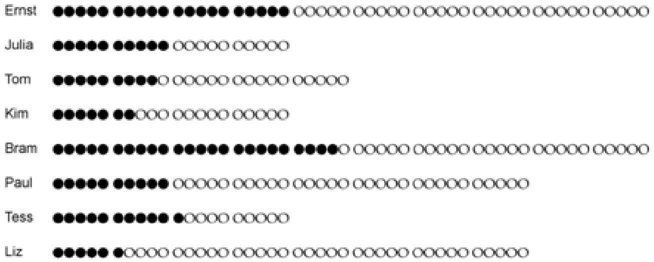

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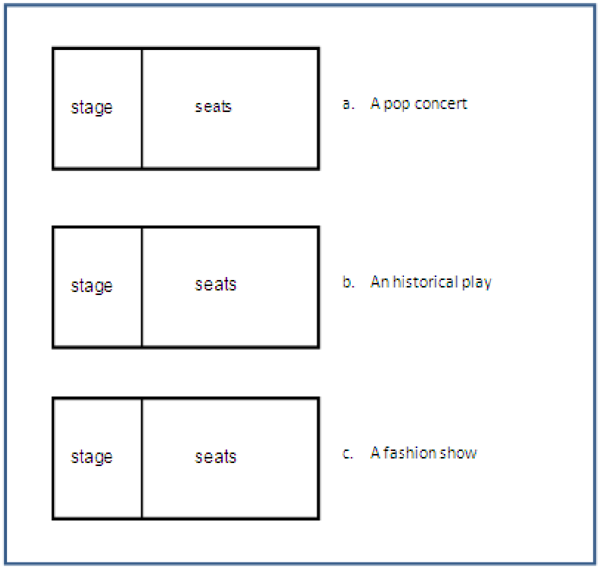

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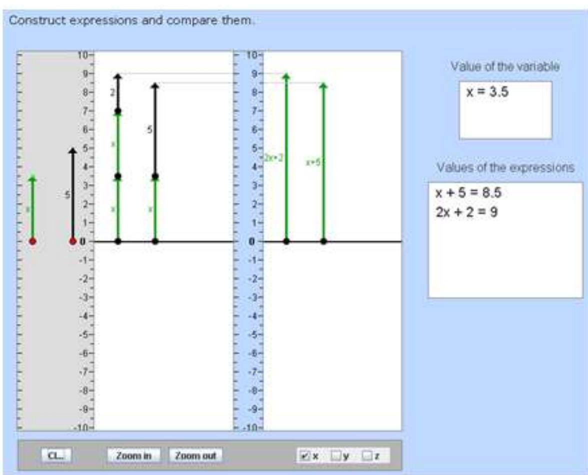

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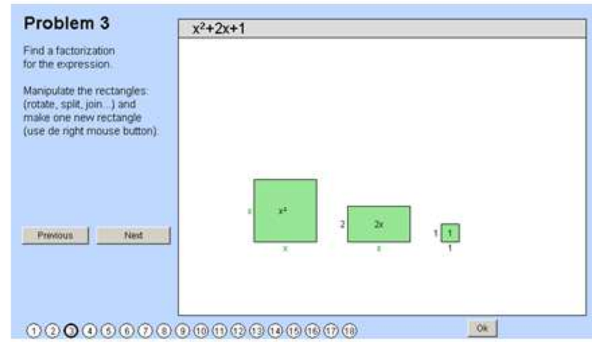

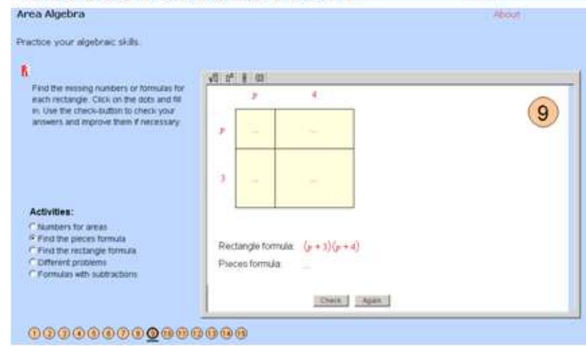

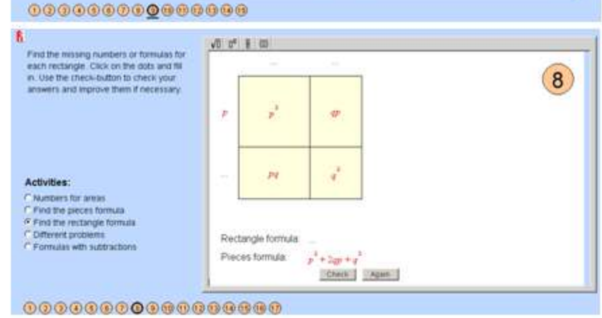

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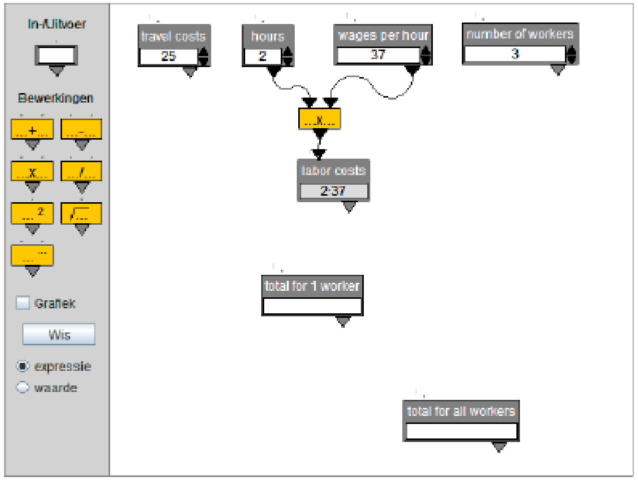
4.
   a. Draw an arrow string that Jim could use to make more rows for the table.
   b. Use your arrow string to make two more rows (for 6 and7 hours) on the table.

The calculations within this task can be structured in a way that prepares for dealing with functions:



No problems?

- Whole class progress *versus* variation in student thinking is essential for productive discussions
- Ideal learning processes *versus* making progress with a class towards attainment targets in time
- Focus on mathematizing (developing concepts) *versus* acquiring confidence, sustainable, flexible knowledge
- Real life contexts *versus* meaningful mathematics
- High demands on the role of the teacher

ICMI Study 22 Task Design in Mathematics Education

Thank you!

http://www.fisme.science.uu.nl/~doorm101/ICMI22_RME_FI_booklet.pdf

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