E-TEXTBOOK FOR MATHEMATICAL INQUIRY : DESIGN of ENGAGEMENTS & BOUNDARIES

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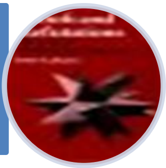
Mathematical Inquiry

Engagement with empirical and formal aspects

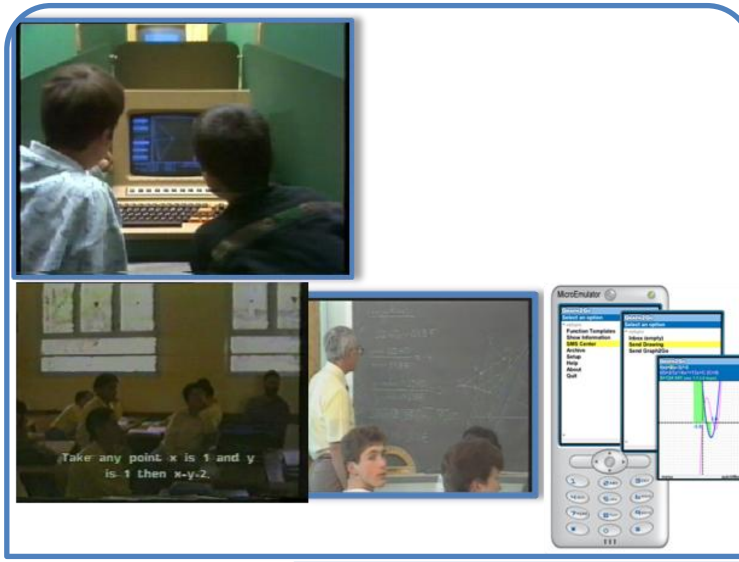
- [Having] a primitive conjecture
- Proof (a rough thought, experiment, an argument)
- Global counterexamples to the primitive conjecture
- Proof re-examined
- Primitive conjecture improved
- Consequences and counterexamples turn into examples of a new inquiry

A “simple pattern of mathematical discovery” Lakatos Proofs & Refutations (p. 127)



- Rules, norms & Guidance
- A curate exhibition
- A few central objects
- Many galleries
- (Too) Many exhibits & exhibited items
- Do-It-Yourself stations

Museum Image



In the classroom

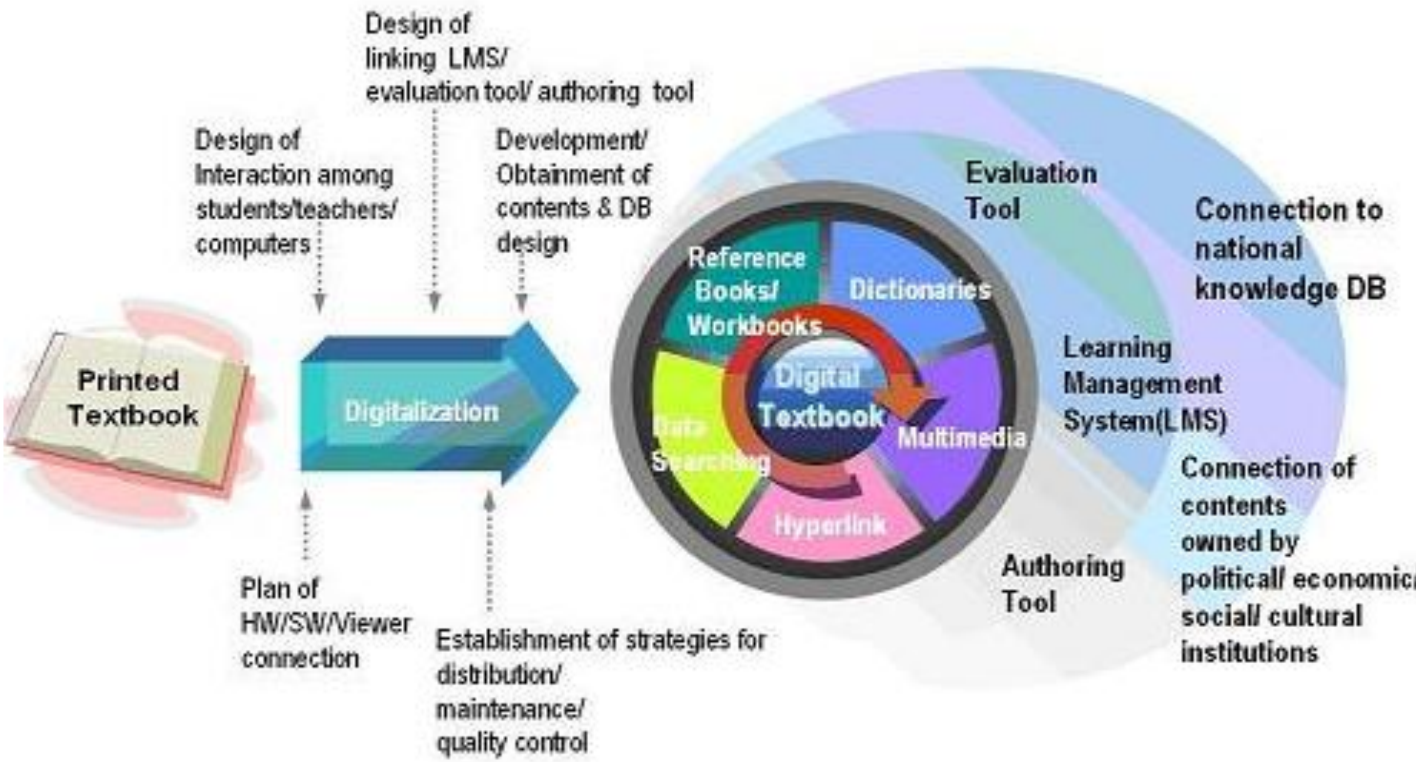
Textbook Culture Characteristics

Love and Pimm’s (1996) “text on texts”

- A textbook is a message from the professional community to students about what they should learn
- It also represents the ideas of the author about how the content should be taught and learned
- It plays a central role in school pedagogy and classroom norms, and its *authoritative image* has been the dominant aspect of the common classroom culture often identified as *textbook culture*

- **Textbooks are linear** and demand “linear textual flow of reading” (p. 381);
- Textbooks are closed -- have been **created in the past**
- Traditionally , it includes problems for exercising but **not aim at questioning the content**

Multiple Resources is the key aspect of current eTextbooks



Source: Korea Education and Research Information Service (KERIS), 2007

What's between "collection" & eTextbook?

My contribution aims at exploring design principles of Interactive textbook when *authored to be delivered* by technology (as opposed to *adopted for* technology)

- I would focus on the challenge of designing **non-sequential textbook** to turn by the teacher into instructional material
- I would zoom into the challenge of designing and analyzing **multimodal sequence** of interactive tasks
- A visual-semiotic framework has been developed to serve as the tool for analyzing students' learning with **interactive tasks**
- I would illustrate how this framework could serve designers and teachers
- Implications & Redesigning: questioning the stability of design principles

Design Non-sequential Textbook

- The flexible structure and the features designed to create reader engagement challenge key functions of the textbooks as a complete presentation of “truth” ordered in the past into a single possible logical sequence
- I argue for the importance of

  - taking a view to mathematic subject matter
  - having tools reflecting this view making it transparent

  to the capacity to design textbook for inquiry curriculum

Nonlinear texts: Designing for Engagement

Writing about the meanings of composition, Kress and van Leeuwen (The Grammar of Visual Design 1996) observed non-linear texts imposing paradigmatic

> "They [texts] select the elements that can be viewed and present them according to a certain paradigmatic logic... but leave it to the reader to sequence and connect them"

Linear Text	Non Linear Text
"an exhibition in which the paintings are hung in long corridors through which the visitors must move, following signs, to eventually end up at the exit,"	"exhibition in a large room which visitors can traverse any way they like… It will not be random that a particular major sculpture is placed in the center of the room, or that a particular major painting has been hung on the wall opposite the entrance, to be noticed first by all visitors entering the room"

"**Linear and non-linear texts thus constitute two modes of reading and two regimes of control over meaning**" (p. 223)

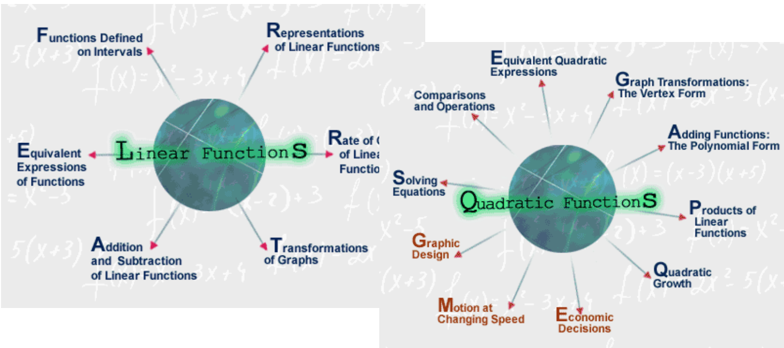
Taking a view & Reflecting this view:

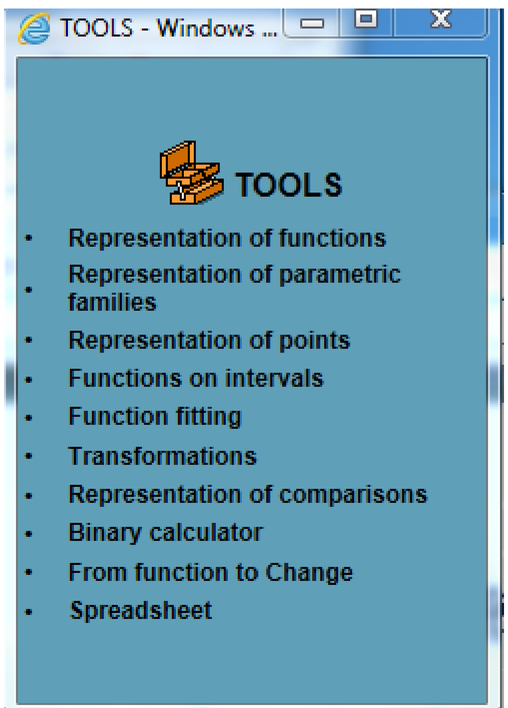
An example of a design tool for Algebra

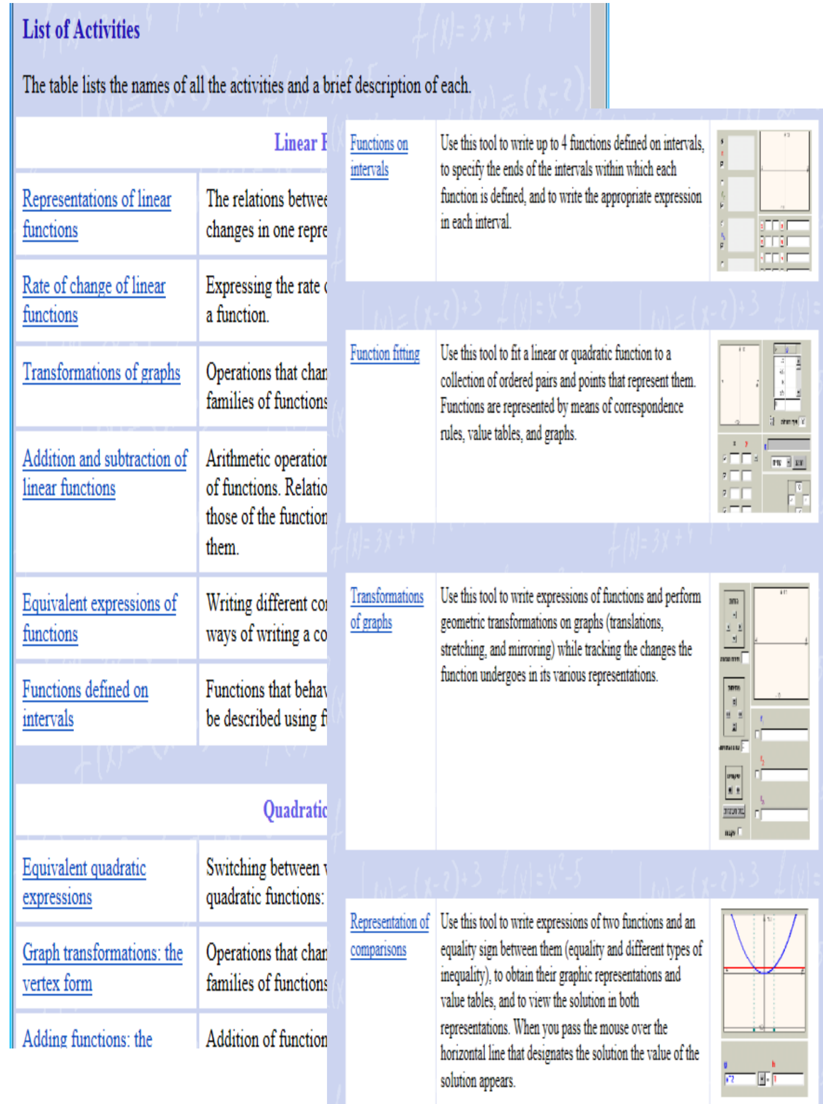
Function	Representation	re-Representation	Transformations	Unary operations	Binary operations	Comparisons
Linear						
Power						
Polynomial						
Quadratic						
Rational						
Irrational						
Periodic						
Exponential						

VisualMath eBook

Structure that reflects a view







List of Activities

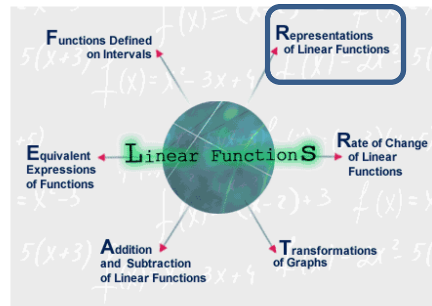
The table lists the names of all the activities and a brief description of each.

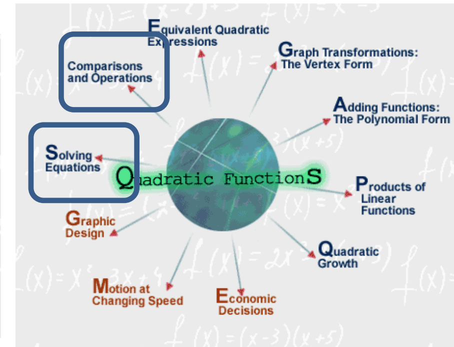
Linear F	
Representations of linear functions	The relations betwee changes in one repre
Rate of change of linear functions	Expressing the rate c a function.
Transformations of graphs	Operations that char families of functions
Addition and subtraction of linear functions	Arithmetic operatio of functions. Relatio those of the function them.
Equivalent expressions of functions	Writing different co ways of writing a co
Functions defined on intervals	Functions that behav be described using f
Quadratic	
Equivalent quadratic expressions	Switching between v quadratic functions:
Graph transformations: the vertex form	Operations that char families of functions
Adding functions: the	Addition of function

Functions on intervals	Use this tool to write up to 4 functions defined on intervals, to specify the ends of the intervals within which each function is defined, and to write the appropriate expression in each interval.
Function fitting	Use this tool to fit a linear or quadratic function to a collection of ordered pairs and points that represent them. Functions are represented by means of correspondence rules, value tables, and graphs.
Transformations of graphs	Use this tool to write expressions of functions and perform geometric transformations on graphs (translations, stretching, and mirroring) while tracking the changes the function undergoes in its various representations.
Representation of comparisons	Use this tool to write expressions of two functions and an equality sign between them (equality and different types of inequality), to obtain their graphic representations and value tables, and to view the solution in both representations. When you pass the mouse over the horizontal line that designates the solution the value of the solution appears.

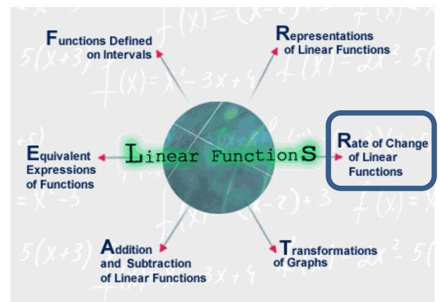
Forming Trajectories

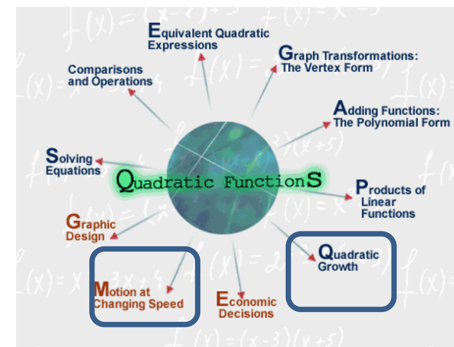
The Equations tour



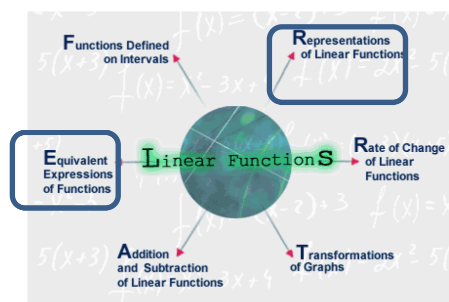


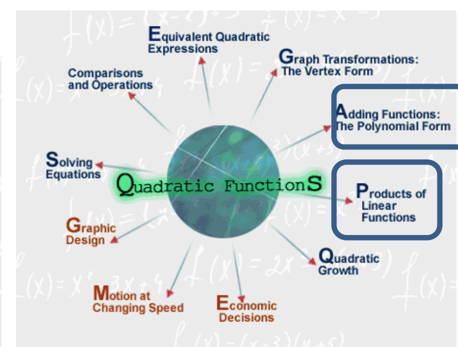
The Rate of Change tour





The Algebraic Structure tour





Design for Coherence: Multimodal Instructional Sequence

Traditional aspects

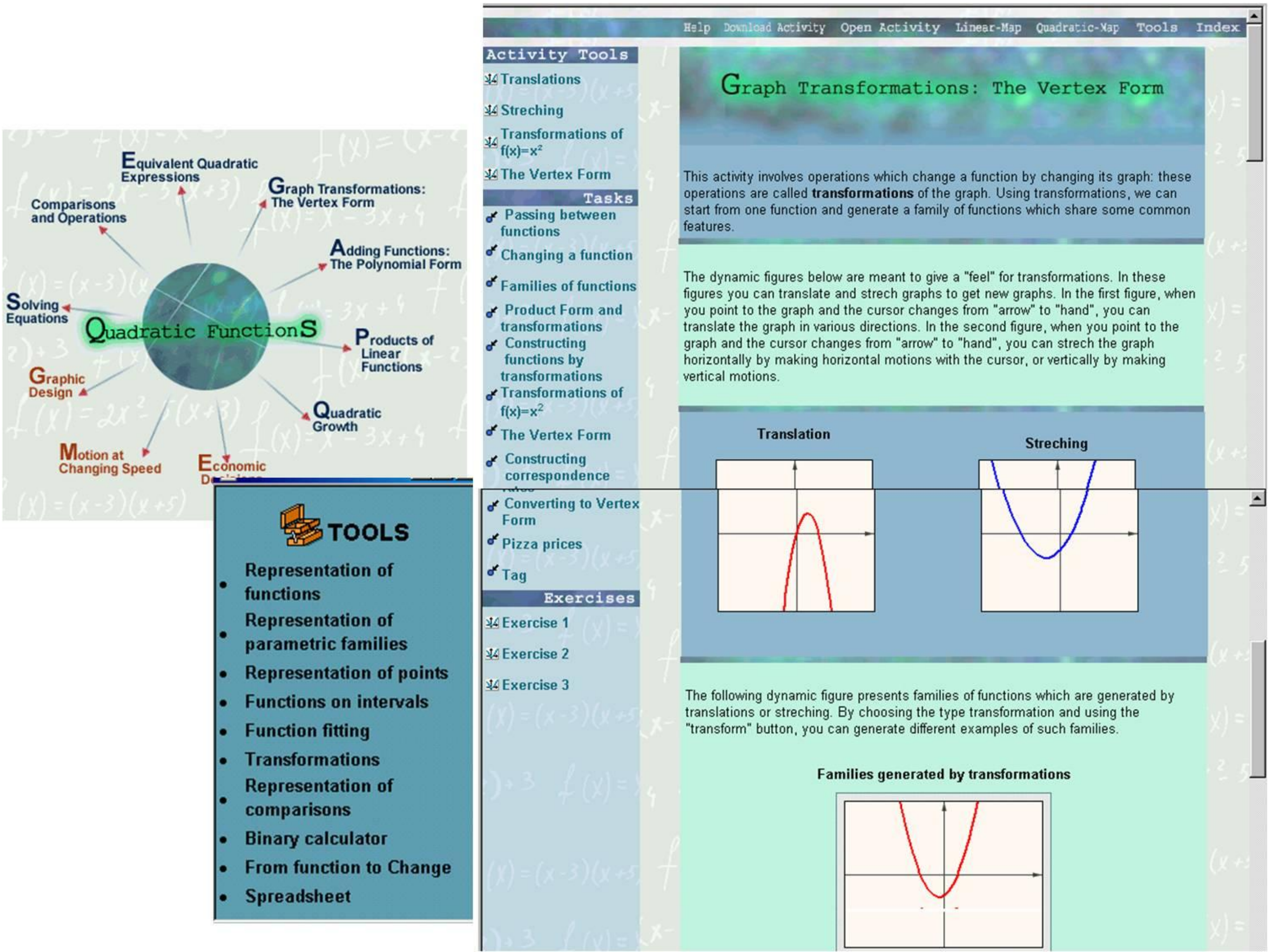
1. Mathematical processes: Modeling, manipulating, Reasoning. communicating
2. Type, Size, modes of communication, openness, difficulty: exercise, investigation, essay
3. Scaffolding

Multimodal aspects

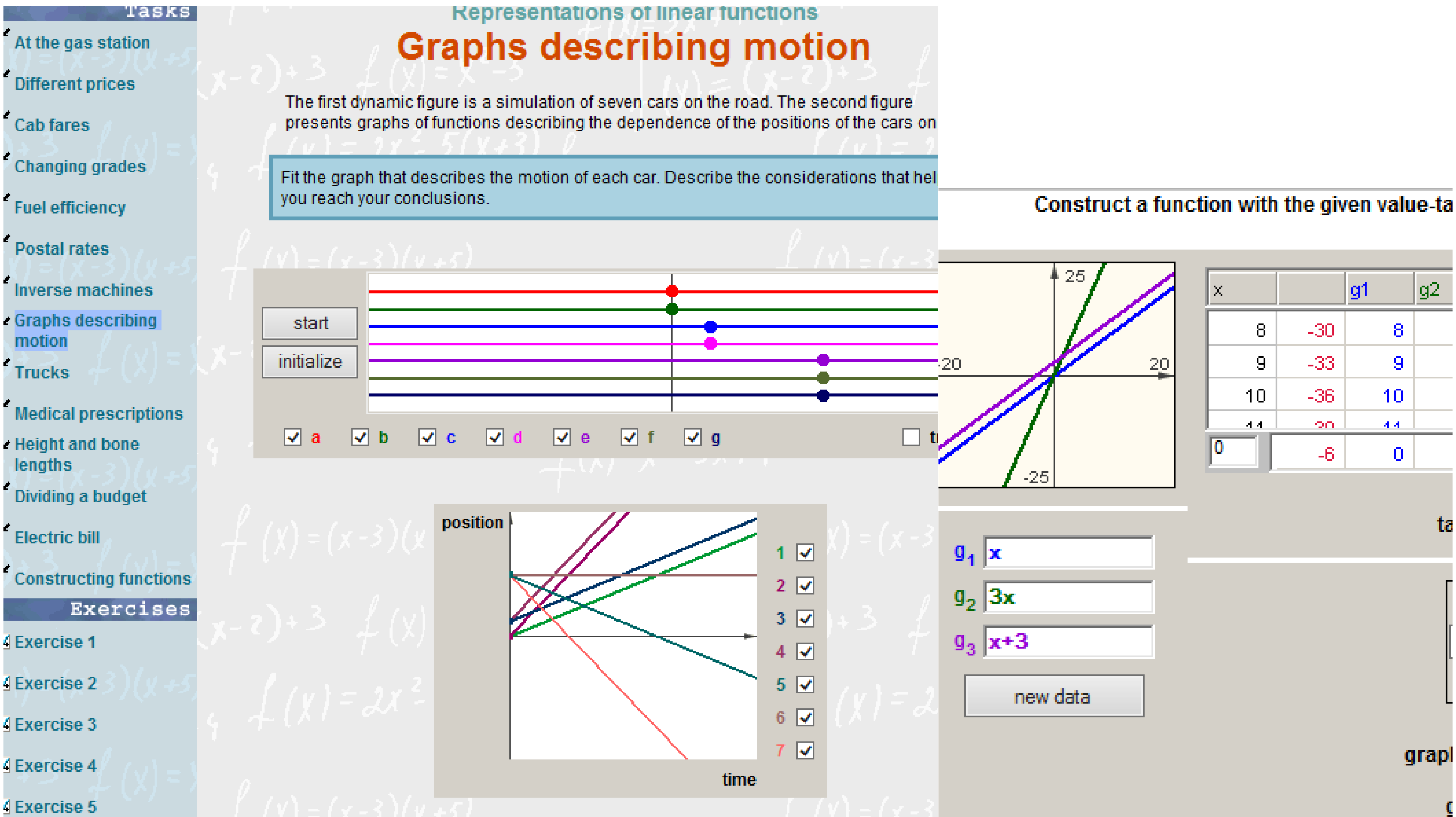
- Traditional text page offers clear way to follow
- The interactive page requires from the reader to order & often it takes its logic from visuals

Tools' space

Balanced Sequence of Tasks

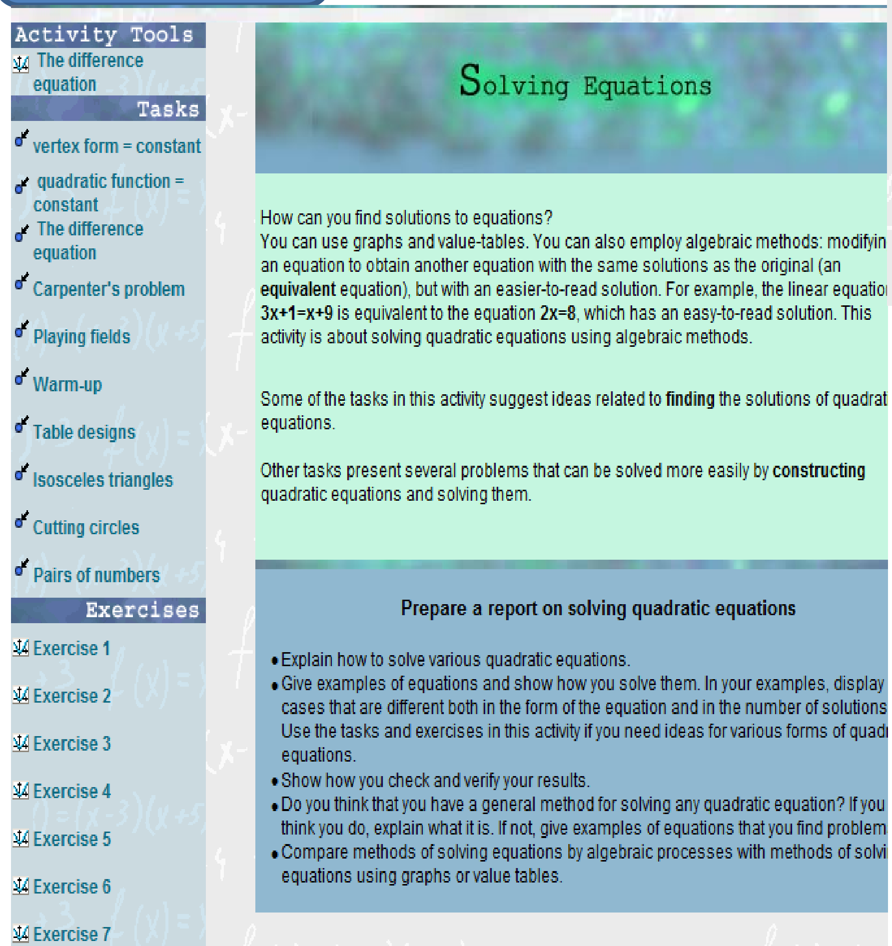


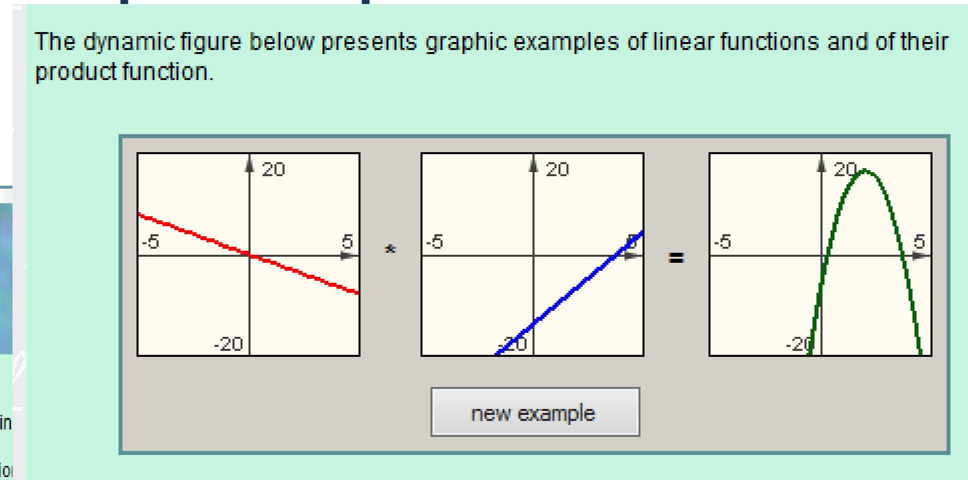
Design of Representation/Feedback



Choice of Examples' space

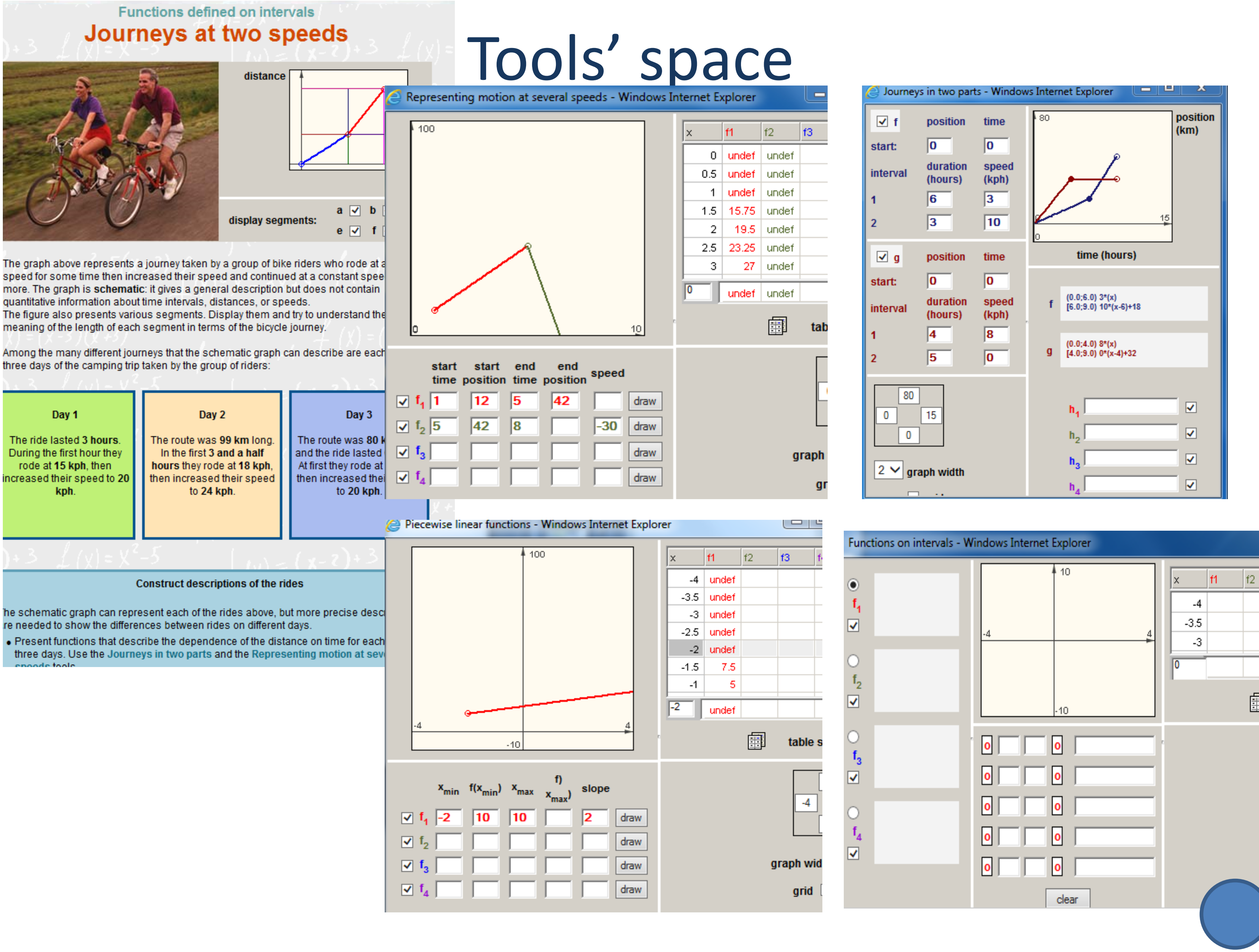
Scaffolding practice





Definition

Tools' space



Functions defined on intervals
Journeys at two speeds
distance
display segments:
a b e f
The graph above represents a journey taken by a group of bike riders who rode at a speed for some time then increased their speed and continued at a constant spee more. The graph is schematic: it gives a general description but does not contain quantitative information about time intervals, distances, or speeds.
The figure also presents various segments. Display them and try to understand the meaning of the length of each segment in terms of the bicycle journey.
Among the many different journeys that the schematic graph can describe are each three days of the camping trip taken by the group of riders:
Day 1
The ride lasted 3 hours. During the first hour they rode at 15 kph, then increased their speed to 20 kph.
Day 2
The route was 99 km long. In the first 3 and a half hours they rode at 18 kph, then increased their speed to 24 kph.
Day 3
The route was 80 k and the ride lasted At first they rode at then increased thei to 20 kph.
Construct descriptions of the rides
he schematic graph can represent each of the rides above, but more precise desc re needed to show the differences between rides on different days.
Present functions that describe the dependence of the distance on time for each three days. Use the Journeys in two parts and the Representing motion at sev
Representing motion at several speeds - Windows Internet Explorer
100
0
10
x f1 f2 f3
0 undef undef
0.5 undef undef
1 undef undef
1.5 15.75 undef
2 19.5 undef
2.5 23.25 undef
3 27 undef
0 undef undef
start time start position end time end position speed
f1 1 12 5 42 draw
f2 5 42 8 -30 draw
f3 draw
f4 draw
graph
Journeys in two parts - Windows Internet Explorer
f position time
start: 0 0
interval duration (hours) speed (kph)
1 6 3
2 3 10
g position time
start: 0 0
interval duration (hours) speed (kph)
1 4 8
2 5 0
80
position (km)
15
0
time (hours)
f (0.0;6.0) 3*(x) [6.0;9.0) 10*(x-6)+18
g (0.0;4.0) 8*(x) [4.0;9.0) 0*(x-4)+32
80 0 15 0
2 graph width
h1 h2 h3 h4
Piecewise linear functions - Windows Internet Explorer
100
-4
4
-10
x f1 f2 f3
-4 undef
-3.5 undef
-3 undef
-2.5 undef
-2 undef
-1.5 7.5
-1 5
-2 undef
table s
xmin f(xmin) xmax f(xmax) slope
f1 -2 10 10 2 draw
f2 draw
f3 draw
f4 draw
-4
graph wid
grid
Functions on intervals - Windows Internet Explorer
f1 f2 f3 f4
10
-4
4
-10
x f1 f2
-4
-3.5
-3
0
clear

A semiotic approach for the Design of Tasks: Interactive Diagrams

A **diagram** is a representation, visual and other, containing clarifying or demonstrating information

A diagram that presents specific information presents a point of view thus **implicitly engaging** the viewer in meaningful interpretations

An **interactive diagram** (ID) is built around a pre-constructed example

The Interactive Diagram **requires from the viewer to take action** and change the diagram within given limitations

An ID differs from an **interactive tool** in that it is built for a specific task and contains a complete example

The framework:
Visual semiotic analysis of ID functions

I adopted a framework developed by semiotic research of text and visuals and provided categories that would allow an orderly discussion of the differences between the traditional page in math textbooks and the new page that derives its principles of design and organization from the screen and the affordances of technology (Yerushalmy 2005)

The presentational function

Random examples

Specific examples

Generic examples

The orientational function

The IDs is designed to be a sketch and/or a source of accurate information

The organizational function

Illustrating IDs

Elaborating IDs

Narrating IDs

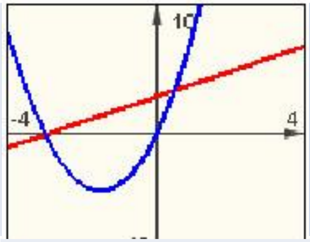
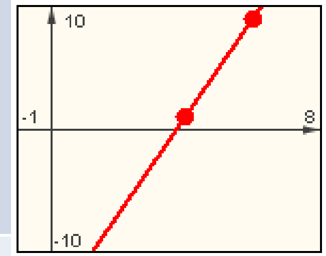
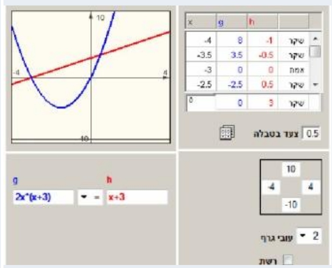
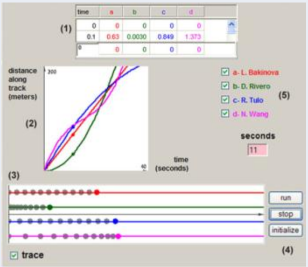
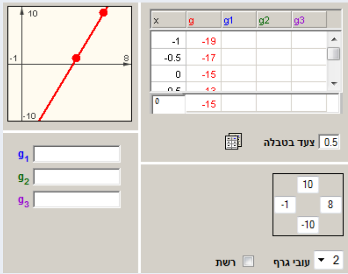
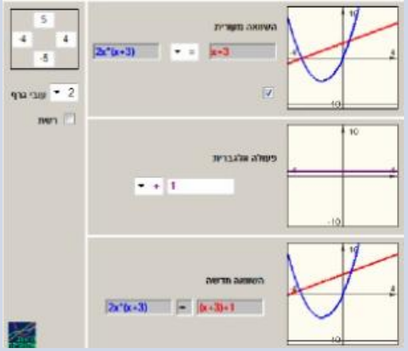
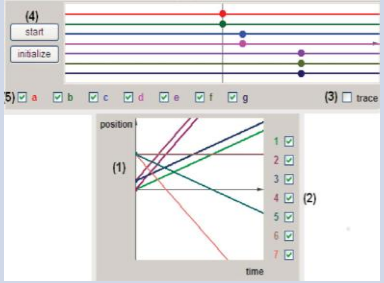
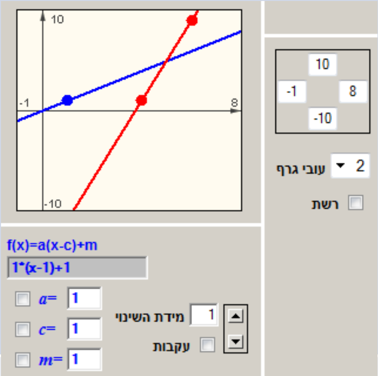
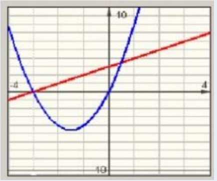
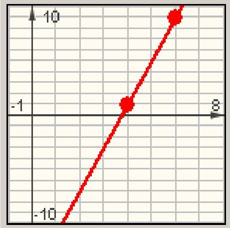
Design decisions:
The 3 organizational functions

The design of an *Illustrating IDs*
one representation and the minimal necessary control for operating with it

The boundaries that we used in *Narrating IDs'* design were:
(a) a well articulated examples
(b) support for the preferred solution path
(c) small number of representations and possibilities of elaboration

The important components in the design of the *Elaborating IDs*
rich tools and linked representations that enable various directions in the search for a solution

Comparative micro analysis of three series of activities

	Manipulating	Modeling	Formulating
Illustrating ID			
Elaborating ID			
Narrating ID			
Paper diagram Video clip			

Modeling: Comparative Microanalysis of Interaction components

Table 1 Comparative view of the interaction components of the two IDs

	Illustrating diagram	Elaborating diagram
Animation		
Motion control	"Run" "Stop" "Initialize"	"Run" "Stop" "Initialize"
Information choice	Not available	Choice of activated runners
Representations		
1D graph of activated runners	Not available	Discrete traces on running track
2D graph of activated runners	Not available	Hotlink animated motion to graph Manual dragging and marking Time and distance information
Numeric information	Non-linked table of distances at 6 positions	Table of distances linked to traces Time and distance created and displayed along the run Hot-linked timer Hot-linked active row
Links		
Links between representations	Not available	2D graph and animation 1D graph and animation 1D and 2D graphs, color coded Table, 2D graph, and animation

Formulating: Comparative Microanalysis of Interaction components

	Illustrating ID	Elaborating ID	Narrating ID
Examples			
Appearance of the example	Example appears as a graph line	Example appears as a graph line and its table of values	Example appears as two lines: a target graph (red graph) line and another graph (blue graph) line that reflects the changing parametric expression
Interaction with the example	View	View and add your own	View, change, and compare
Representations and tools			
Graph	A sketch that can be made accurate graph	A sketch that can be made accurate graph	A sketch that can be made accurate graph
Graph tools	Revealing the coordinates of points	Revealing the coordinates of points Change scale	Revealing the coordinates of points Change scale
Algebraic expression	Not available	Function expressions in any format hot-linked to a graph and a table of values (up to 3 simultaneous expressions)	Function expression and parametric expression in the form: $f(x) = a(x - c) + m$ hot-linked to a graph
Algebraic expression tools		Free syntax input of function expressions	Changing values of parameters of the given parametric expression
Table of values	Not available	A given table of $(x, f(x))$. Hot-linked to point in the graph. Values spread homogeneously according to a specified scale and specified *delta x*	Not available
Table of values tools		Control *delta x* Input any x value and read $f(x)$ values	

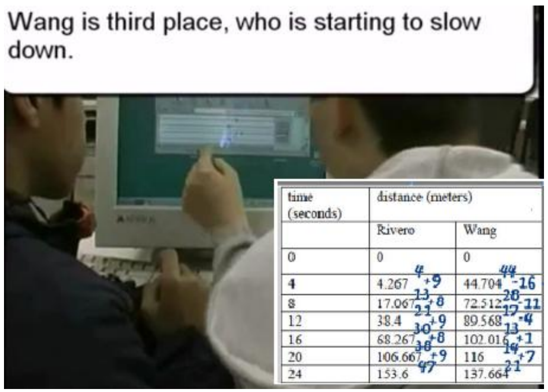
Research goals & design

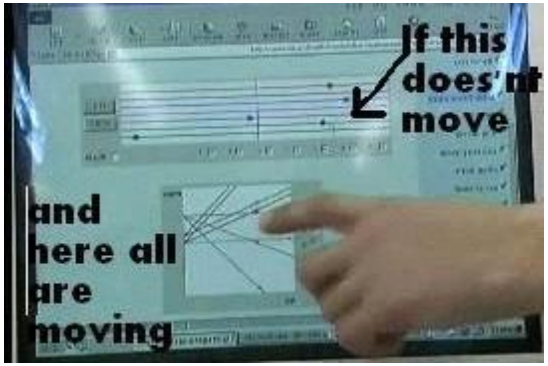
- What are the characteristics of solving tasks which are presented as text with IDs?
- How do the characterizations of processes vary in accordance to the three designed organizational functions of IDs?

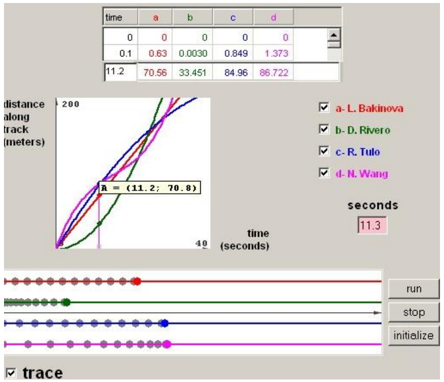
	Student A	**Student B**	**Student C**	Notes
Stage A		The preliminary task		The same task
Stage B	illustrating ID	narrating ID	elaborating ID	One of 3 comparable tasks
Stage C	Students assembled in a group and were asked to describe the technique they used in their solution, to present their use of the diagram, to reflect upon their moves and to be involved in a conversation regarding other students' techniques			

How do the characterizations of processes vary in accordance to the three designed organizational functions of IDs?

Illustrating IDs	Narrating IDs	Elaborating IDs
Can be helpful in consolidating relevant knowledge	Can be a form of instruction toward development of new scientific concepts	Leading to different problem-solving processes and a variety of solutions







How do the characterizations of processes vary

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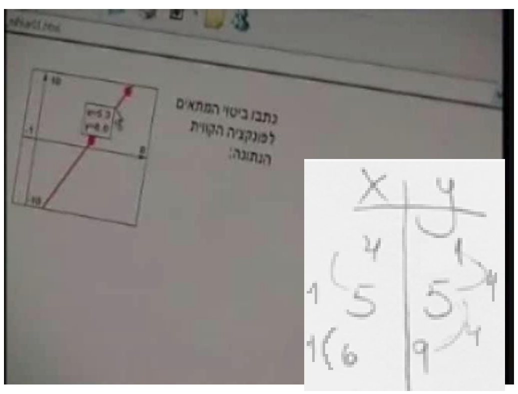

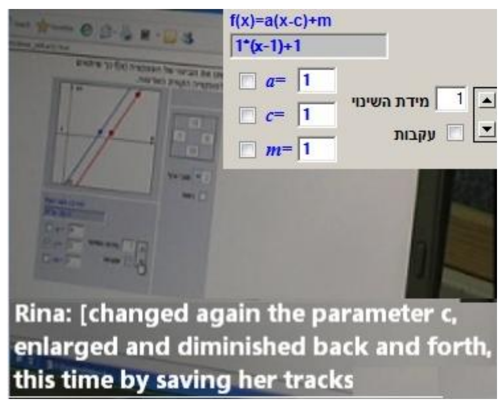

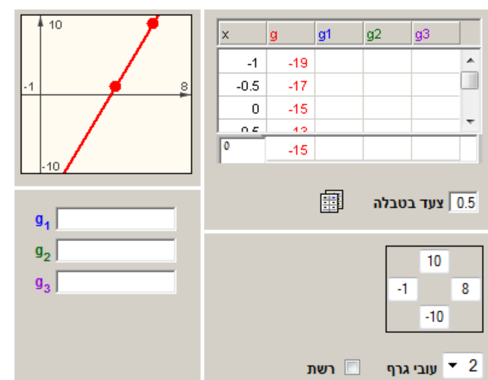

What are the characteristics of activity consisted of reading and solving tasks which are presented as text with IDs?

Major Finding

Personalization of the text

Unfolding the representativeness of the ID's example
as carriers of the general meaning

Development of mathematical principal ideas within
the boundaries designed in the IDs

Personalization of the text

Three patterns of personalization of the text:

(a) attention to details in the given example resulting in the construction of additional details to the original example

(b) changing the given example, generating new examples

(c) rephrasing the question of the task, posing new question

Unfolding the representativeness of the ID's example as carriers of the general meaning

Two settings in which an example served to start the investigation and the processes of unfolding the representativeness of the examples:

(a) Changing a given example by generating similar or new examples
(b) Interacting with components of the given examples (the representations, as well as the linking and control tools)

The study highlights the common features that support the processes in each setting and highlights differences and makes distinctions in the processes which are rooted in part in the compared design variations

Development of mathematical principal ideas within the boundaries designed in the Narrating IDs

(a) Narrating IDs can be a form of instruction toward the development of new mathematical knowledge for students

(b) Narrating ID designs limit the student's action and so support guidance, and at the same time remain an open space for student ideas

(c) The NID organized and directed the process of development of the students' knowledge but the students controlled the task and were empowered by the changes they chose to make in the presentation of the task and by formulating new questions

Designing Interactive Textbook
or
Transparent Coherence of Digital Resources
Summary

Non sequential Textbook	Multimodal Sequence	Interactive Task
• Choosing a view of the subject • Identify objects, operations & representations which are central to this view • Design a conceptual tool to map the mathematical occurrences • Design the textbook sections to correspond to the map	Tools reflect the curricular agenda: Purpose, Tone, Difficulty, Type of Reasoning	• The visual-semiotic analysis is a useful instrument for the design of interactive tasks • The organizational function of interactive task [illustrating, elaborating or narrating] corresponds to pedagogical purposes, instruction goals & guidance • Interactive diagram has noticeable effect on problem solving processes

Challenges for Research: Textbook in the near future

- Stability of the finding & views upon new technology (personal tablet, touch interface)
- Learning Management System – The era of the Big Data
- Collaborative learning setting
- Formative & Summative assessment
- Teacher's designed textbooks

To be continued….