E-TEXTBOOK FOR MATHEMATICAL INQUIRY: DESIGN of ENGAGEMENTS & BOUNDARIES

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Mathematical Inquiry
Engagement with empirical and formal aspects

- [Having] a primitive conjecture
- Proof (a rough thought, experiment, an argument)
- Global counterexamples to the primitive conjecture
- Proof re-examined
- Primitive conjecture improved
- Consequences and counterexamples turn into examples of a new inquiry

- Rules, norms & Guidance
- A curate exhibition
- A few central objects
- Many galleries
- (Too) Many exhibits & exhibited items
- Do-It-Yourself stations

A “simple pattern of mathematical discovery”
Lakatos Proofs & Refutations (p. 127)
Textbook Culture Characteristics

Love and Pimm’s (1996) “text on texts”

• A textbook is a message from the professional community to students about what they should learn
• It also represents the ideas of the author about how the content should be taught and learned
• It plays a central role in school pedagogy and classroom norms, and its authoritative image has been the dominant aspect of the common classroom culture often identified as textbook culture

• Textbooks are linear and demand “linear textual flow of reading” (p. 381);
• Textbooks are closed -- have been created in the past
• Traditionally, it includes problems for exercising but not aim at questioning the content
Multiple Resources is the key aspect of current eTextbooks

Source: Korea Education and Research Information Service (KERIS), 2007
What’s between “collection” & eTextbook?

My contribution aims at exploring design principles of Interactive textbook when *authored to be delivered* by technology (as opposed to *adopted for* technology)

- I would focus on the challenge of designing *non-sequential textbook* to turn by the teacher into instructional material
- I would zoom into the challenge of designing and analyzing *multimodal sequence* of interactive tasks
- A visual-semiotic framework has been developed to serve as the tool for analyzing students’ learning with *interactive tasks*
- I would illustrate how this framework could serve designers and teachers
- Implications & Redesigning: questioning the stability of design principles
Design Non-sequential Textbook

• The flexible structure and the features designed to create reader engagement challenge key functions of the textbooks as a complete presentation of “truth” ordered in the past into a single possible logical sequence.

• I argue for the importance of:
  - taking a view to mathematic subject matter
  - having tools reflecting this view making it transparent to the capacity to design textbook for inquiry curriculum
Nonlinear texts: Designing for Engagement

Writing about the meanings of composition, Kress and van Leeuwen (The Grammar of Visual Design 1996) observed non-linear texts imposing paradigmatic

“They [texts] select the elements that can be viewed and present them according to a certain paradigmatic logic... but leave it to the reader to sequence and connect them"

<table>
<thead>
<tr>
<th>Linear Text</th>
<th>Non Linear Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>“an exhibition in which the paintings are hung in long corridors through which the visitors must move, following signs, to eventually end up at the exit,”</td>
<td>“exhibition in a large room which visitors can traverse any way they like... It will not be random that a particular major sculpture is placed in the center of the room, or that a particular major painting has been hung on the wall opposite the entrance, to be noticed first by all visitors entering the room”</td>
</tr>
</tbody>
</table>

“Linear and non-linear texts thus constitute two modes of reading and two regimes of control over meaning” (p. 223)
# Taking a view & Reflecting this view:

An example of a design tool for Algebra

<table>
<thead>
<tr>
<th>Function</th>
<th>Representation</th>
<th>re-Representation</th>
<th>Transformations</th>
<th>Unary operations</th>
<th>Binary operations</th>
<th>Comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Power</td>
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<td>Polynomial</td>
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<tr>
<td>Quadratic</td>
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<tr>
<td>Rational</td>
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<tr>
<td>Irrational</td>
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<tr>
<td>Periodic</td>
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<tr>
<td>Exponential</td>
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</tbody>
</table>
VisualMath eBook
Structure that reflects a view
Forming Trajectories

The Equations tour

The Rate of Change tour

The Algebraic Structure tour
Design for Coherence: Multimodal Instructional Sequence

**Traditional aspects**
1. Mathematical processes: Modeling, manipulating, Reasoning, communicating
2. Type, Size, modes of communication, openness, difficulty: exercise, investigation, essay
3. Scaffolding

**Multimodal aspects**
- Traditional text page offers clear way to follow
- The interactive page requires from the reader to order & often it takes its logic from visuals

Tools’ space
Balanced Sequence of Tasks

Graph Transformations: The Vertex Form

This activity involves operations which change a function by changing its graph; these operations are called transformations of the graph. Using transformations, we can start from one function and generate a family of functions which share some common features.

The dynamic figures below are meant to give a “feel” for transformations. In these figures you can translate and stretch graphs to get new graphs. In the first figure, when you point to the graph and the cursor changes from "arrow" to "hand", you can translate the graph in various directions. In the second figure, when you point to the graph and the cursor changes from "arrow" to "hand", you can stretch the graph horizontally by making horizontal motions with the cursor, or vertically by making vertical motions.

The following dynamic figure presents families of functions which are generated by translations or stretching. By choosing the type transformation and using the "transform" button, you can generate different examples of such families.
Scaffolding practice

**Definition**

The dynamic figure below presents graphic examples of linear functions and of their product function.

How can you find solutions to equations? You can use graphs and value-tables. You can also employ algebraic methods: modify an equation to obtain another equation with the same solutions as the original (an equivalent equation), but with an easier-to-read solution. For example, the linear equation $3x+1=4x+6$ is equivalent to the equation $2x=5$, which has an easy-to-read solution. This activity is about solving quadratic equations using algebraic methods.

Some of the tasks in this activity suggest ideas related to finding the solutions of quadratic equations.

Other tasks present several problems that can be solved more easily by constructing quadratic equations and solving them.

Prepare a report on solving quadratic equations

- Explain how to solve various quadratic equations.
- Give examples of equations and show how you solve them. In your examples, display cases that are different both in the form of the equation and in the number of solutions.
- Use the tasks and exercises in this activity if you need ideas for various forms of quadratic equations.
- Show how you check and verify your results.
- Do you think that you have a general method for solving any quadratic equation? If you think you do, explain what it is. If not, give examples of equations that you find problematic.
- Compare methods of solving equations by algebraic processes with methods of solving equations using graphs or value tables.
Tools’ space

The graph above represents a journey taken by a group of bike riders who rode at a constant speed for some time then increased their speed and continued at a constant speed for the remainder. The graph is schematic; it gives a general description but does not contain quantitative information about time intervals, distances, or speeds.

The figure also presents various segments. Display them and try to understand the meaning of the length of each segment in terms of the bicycle journey.

Among the many different journeys that the schematic graph can describe are each of the three days of the camping trip taken by the group of riders:

Day 1: The ride lasted 3 hours. During the first hour, they rode at 15 kph, then increased their speed to 24 kph.
Day 2: The route was 90 km long. In the first 3 and a half hours, they rode at 18 kph, then increased their speed to 20 kph.
Day 3: The route was 60 km long. The first hour was full, the next 2 hours were half, and the last hour was full of a total of 30 km.

Construct descriptions of the rides

The schematic graph can represent each of the rides above, but more precise descriptions are needed to show the differences between rides on different days.

- Present functions that describe the dependence of the distance on time for each of the three days. Use the Journeys in two parts and the Representing motion at several speeds functions.

Function definitions on intervals

The functions defined on the intervals are:

\[ f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 & \text{for } x \geq 0 \end{cases} \]

These functions show the dependence of the distance on time for different intervals.
A semiotic approach for the Design of Tasks: Interactive Diagrams

A diagram is a representation, visual and other, containing clarifying or demonstrating information.

A diagram that presents specific information presents a point of view thus implicitly engaging the viewer in meaningful interpretations.

An interactive diagram (ID) is built around a pre-constructed example.

The Interactive Diagram requires from the viewer to take action and change the diagram within given limitations.

An ID differs from an interactive tool in that it is built for a specific task and contains a complete example.
The framework: Visual semiotic analysis of ID functions

I adopted a framework developed by semiotic research of text and visuals and provided categories that would allow an orderly discussion of the differences between the traditional page in math textbooks and the new page that derives its principles of design and organization from the screen and the affordances of technology (Yerushalmy 2005)

- **The presentational function**
  - Random examples
  - Specific examples
  - Generic examples

- **The orientational function**
  - The IDs is designed to be a sketch and/or a source of accurate information

- **The organizational function**
  - Illustrating IDs
  - Elaborating IDs
  - Narrating IDs
Design decisions: The 3 organizational functions

The design of an *Illustrating IDs* one representation and the minimal necessary control for operating with it

The boundaries that we used in *Narrating IDs'* design were:
(a) a well articulated examples
(b) support for the preferred solution path
(c) small number of representations and possibilities of elaboration

The important components in the design of the *Elaborating IDs* rich tools and linked representations that enable various directions in the search for a solution
### Comparative micro analysis of three series of activities

<table>
<thead>
<tr>
<th></th>
<th>Manipulating</th>
<th>Modeling</th>
<th>Formulating</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Illustrating ID</strong></td>
<td><img src="image1" alt="Illustrating Manipulating" /></td>
<td><img src="image2" alt="Illustrating Modeling" /></td>
<td><img src="image3" alt="Illustrating Formulating" /></td>
</tr>
<tr>
<td><strong>Elaborating ID</strong></td>
<td><img src="image4" alt="Elaborating Manipulating" /></td>
<td><img src="image5" alt="Elaborating Modeling" /></td>
<td><img src="image6" alt="Elaborating Formulating" /></td>
</tr>
<tr>
<td><strong>Narrating ID</strong></td>
<td><img src="image7" alt="Narrating Manipulating" /></td>
<td><img src="image8" alt="Narrating Modeling" /></td>
<td><img src="image9" alt="Narrating Formulating" /></td>
</tr>
<tr>
<td><strong>Paper diagram</strong></td>
<td><img src="image10" alt="Paper Manipulating" /></td>
<td><img src="image11" alt="Paper Modeling" /></td>
<td><img src="image12" alt="Paper Formulating" /></td>
</tr>
<tr>
<td><strong>Video clip</strong></td>
<td><img src="image13" alt="Video Manipulating" /></td>
<td><img src="image14" alt="Video Modeling" /></td>
<td><img src="image15" alt="Video Formulating" /></td>
</tr>
</tbody>
</table>
Modeling: Comparative Microanalysis of Interaction components

<table>
<thead>
<tr>
<th></th>
<th>Illustrating diagram</th>
<th>Elaborating diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Animation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motion control</td>
<td>“Run”</td>
<td>“Run”</td>
</tr>
<tr>
<td></td>
<td>“Stop”</td>
<td>“Stop”</td>
</tr>
<tr>
<td></td>
<td>“Initialize”</td>
<td>“Initialize”</td>
</tr>
<tr>
<td><strong>Information choice</strong></td>
<td>Not available</td>
<td>Choice of activated runners</td>
</tr>
<tr>
<td><strong>Representations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1D graph of activated</td>
<td>Not available</td>
<td>Discrete traces on running track</td>
</tr>
<tr>
<td>runners</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2D graph of activated</td>
<td>Not available</td>
<td>Hotlink animated motion to graph</td>
</tr>
<tr>
<td>runners</td>
<td></td>
<td>Manual dragging and marking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time and distance information</td>
</tr>
<tr>
<td>Numeric information</td>
<td>Non-linked table of distances at 6</td>
<td>Table of distances linked to traces</td>
</tr>
<tr>
<td></td>
<td>positions</td>
<td>Time and distance created and displayed along the run</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hot-linked timer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hot-linked active row</td>
</tr>
<tr>
<td><strong>Links</strong></td>
<td>Not available</td>
<td>2D graph and animation</td>
</tr>
<tr>
<td>Links between</td>
<td></td>
<td>1D graph and animation</td>
</tr>
<tr>
<td>representations</td>
<td></td>
<td>1D and 2D graphs, color coded</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Table, 2D graph, and animation</td>
</tr>
</tbody>
</table>
### Formulating: Comparative Microanalysis of Interaction components

<table>
<thead>
<tr>
<th></th>
<th>Illustrating ID</th>
<th>Elaborating ID</th>
<th>Narrating ID</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appearance of the example</td>
<td>Example appears as a graph line</td>
<td>Example appears as a graph line and its table of values</td>
<td>Example appears as two lines: a target graph (red graph) line and another graph (blue graph) line that reflects the changing parametric expression</td>
</tr>
<tr>
<td><strong>Interaction with the example</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Representations and tools</td>
<td>View</td>
<td>View and add your own</td>
<td>View, change, and compare</td>
</tr>
<tr>
<td>Graph</td>
<td>A sketch that can be made accurate graph</td>
<td>A sketch that can be made accurate graph</td>
<td>A sketch that can be made accurate graph</td>
</tr>
<tr>
<td>Graph tools</td>
<td>Revealing the coordinates of points</td>
<td>Revealing the coordinates of points Change scale</td>
<td>Revealing the coordinates of points Change scale</td>
</tr>
<tr>
<td>Algebraic expression</td>
<td>Not available</td>
<td>Function expressions in any format hot-linked to a graph and a table of values (up to 3 simultaneous expressions)</td>
<td>Function expression and parametric expression in the form: ( f(x) = a(x - c) + m ) hot-linked to a graph</td>
</tr>
<tr>
<td>Algebraic expression tools</td>
<td>Free syntax input of function expressions</td>
<td>Free syntax input of function expressions</td>
<td>Changing values of parameters of the given parametric expression</td>
</tr>
<tr>
<td>Table of values</td>
<td>Not available</td>
<td>A given table of ((x, f(x))). Hot-linked to point in the graph. Values spread homogeneously according to a specified scale and specified ( \Delta x )</td>
<td>Not available</td>
</tr>
<tr>
<td>Table of values tools</td>
<td>Control ( \Delta x )</td>
<td>Control ( \Delta x )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Input any ( x ) value and read ( f(x) ) values</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Research goals & design

- What are the characteristics of solving tasks which are presented as text with IDs?
- How do the characterizations of processes vary in accordance to the three designed organizational functions of IDs?

<table>
<thead>
<tr>
<th>Stage</th>
<th>Student A</th>
<th>Student B</th>
<th>Student C</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage A</td>
<td>The preliminary task</td>
<td></td>
<td></td>
<td>The same task</td>
</tr>
<tr>
<td>Stage B</td>
<td>illustrating ID</td>
<td>narrating ID</td>
<td>elaborating ID</td>
<td>One of 3 comparable tasks</td>
</tr>
<tr>
<td>Stage C</td>
<td>Students assembled in a group and were asked to describe the technique they used in their solution, to present their use of the diagram, to reflect upon their moves and to be involved in a conversation regarding other students' techniques</td>
<td></td>
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</tbody>
</table>
How do the characterizations of processes vary in accordance to the three designed organizational functions of IDs?

**Illustrating IDs**
Can be helpful in consolidating relevant knowledge

**Narrating IDs**
Can be a form of instruction toward development of new scientific concepts

**Elaborating IDs**
Leading to different problem-solving processes and a variety of solutions

Wang is third place, who is starting to slow down.
How do the characterizations of processes vary

**Illustrating IDs**
Can be helpful in consolidating relevant knowledge

**Narrating IDs**
Can be a form of instruction toward development of new scientific concepts

**Elaborating IDs**
Leading to different problem-solving processes and a variety of solutions

Rina: [changed again the parameter c, enlarged and diminished back and forth, this time by saving her tracks]
What are the characteristics of activity consisted of reading and solving tasks which are presented as text with IDs?

**Major Finding**

- Personalization of the text
- Unfolding the representativeness of the ID's example as carriers of the general meaning
- Development of mathematical principal ideas within the boundaries designed in the IDs
Three patterns of personalization of the text:

(a) attention to details in the given example resulting in the construction of additional details to the original example

(b) changing the given example, generating new examples

(c) rephrasing the question of the task, posing new question
Two settings in which an example served to start the investigation and the processes of unfolding the representativeness of the examples:

(a) Changing a given example by generating similar or new examples
(b) Interacting with components of the given examples (the representations, as well as the linking and control tools)

The study highlights the common features that support the processes in each setting and highlights differences and makes distinctions in the processes which are rooted in part in the compared design variations
(a) Narrating IDs can be a form of instruction toward the development of new mathematical knowledge for students

(b) Narrating ID designs limit the student's action and so support guidance, and at the same time remain an open space for student ideas

(c) The NID organized and directed the process of development of the students’ knowledge but the students controlled the task and were empowered by the changes they chose to make in the presentation of the task and by formulating new questions
## Designing Interactive Textbook or Transparent Coherence of Digital Resources

### Summary

<table>
<thead>
<tr>
<th>Non sequential Textbook</th>
<th>Multimodal Sequence</th>
<th>Interactive Task</th>
</tr>
</thead>
</table>
| • Choosing a view of the subject  
• Identify objects, operations & representations which are central to this view  
• Design a conceptual tool to map the mathematical occurrences  
• Design the textbook sections to correspond to the map | Tools reflect the curricular agenda: Purpose, Tone, Difficulty, Type of Reasoning | • The visual-semiotic analysis is a useful instrument for the design of interactive tasks  
• The organizational function of interactive task [illustrating, elaborating or narrating] corresponds to pedagogical purposes, instruction goals & guidance  
• Interactive diagram has noticeable effect on problem solving processes |
Challenges for Research: Textbook in the near future

- Stability of the finding & views upon new technology (personal tablet, touch interface)
- Learning Management System – The era of the Big Data
- Collaborative learning setting
- Formative & Summative assessment
- Teacher’s designed textbooks

To be continued....