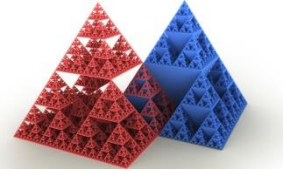# Thoughts on Variation and Mathematics Task Design

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Concept of Variation in Ancient Chinese Thought

Variation is an ancient theme in human's intellectual endeavour in extracting meanings out of experiences. The ancient Chinese classic I-Ching (the Book of Change) was an attempt to categorize variations of intertwining nature-human situations. The "I" (pronounce e) in title I-Ching is a transliteration of a Chinese character that embodies a three-fold meaning: the easy (obeying the laws of nature), the changing (variation) and the constant (invariant).

The sixty-four invariant hexagrams in I-Ching represent the potential phenomenological variations in the observable world that the ancient Chinese experienced.
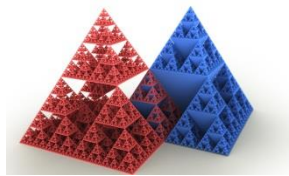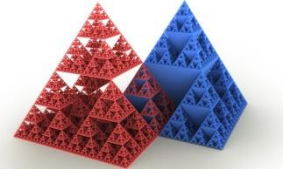
# Ference Marton's Theory of Variation in a Nutshell

"As we always act in relation to situations as we see them, effective actions spring from effective ways of seeing. Seeing a situation in a certain way amounts to discerning those aspects which are critical for engaging in effective action and taking all of them into consideration (focusing on them) at the same time. In order to discern a certain aspect, one must have experienced variation in those aspects. There is no discernment without variation. The only way we can prepare for the un-definable variation in the future is by experiencing variation in the present and by having experienced variation in the past."

(The abstract of a seminar given by Ference Marton at The University of Hong Kong, 21 Nov 2006).

Ference Marton's Theory of Variation in a Nutshell
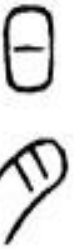
Microscopic Dimension (Discernment)
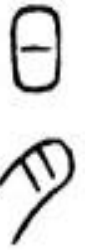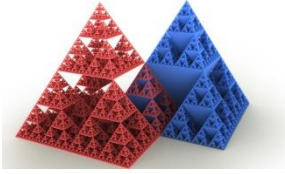Patterns of Variation
(Contrast • Separation• Generalization• Fusion)

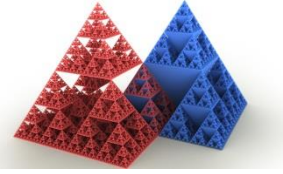**Macroscopic Dimension (Classroom Teaching/Learning)**

Variation in teachers' and students' understanding

Use of variation in pedagogical arrangement
Intended, Enacted and Lived Object of Learning

My Thoughts on Variation

A fundamental idea of variation is simultaneity. When we are simultaneously aware of (intentional focusing our attention on) different aspects of a phenomenon, we notice differences and similarities. By strategically observing variations of these differences and similarities, critical features of the phenomenon may be brought out.
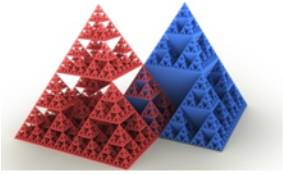
These critical features, when given interpretations, may become invariants that can be used to conceptualize the phenomenon. We could observe by strategically *contrasting* and *comparing*, *separating* out critical features, *shifting focus of attention* and *varying features together* to see whether invariant patterns emerge.
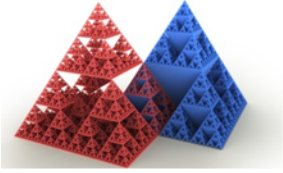
- **Variation Interaction**: strategic way to observe a phenomenon focusing on variation and simultaneity. They are interactions in the sense that the acts of observing may involve direct or indirect manipulation of the object being observed.
- **Variation Task**: task that makes use of variation interactions to organize pedagogical activities
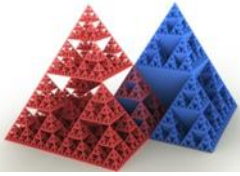
Principles of Variation
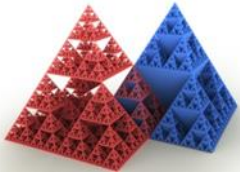
Difference and Similarity Principle

Contrasting and comparing in order to perceive possible invariant features

Sieving Principle

Separating under prescribed constraints/conditions to reveal ("make visual") critical invariant features or relationships

Principles of Variation
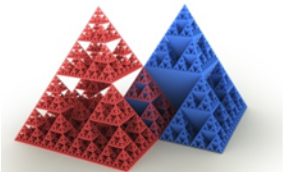
Shifting Principle
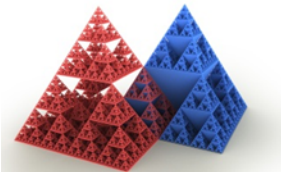
Focusing and paying attention to different aspects or critical features at different time in order to discern/perceive invariant

Co-variation Principle

Co-varying multiple features at the same time to perceive possible emergent patterns or invariant relationship between the features
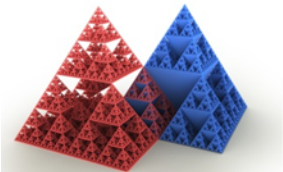
Mathematics Task Design: A Variation Approach

An Example from Dynamic Geometry

Leung, A., Baccaglini-Frank, A. & Mariotti, M. A. (2013). Discernment in dynamic geometry environments. *Educational Studies in Mathematics*. Online First. Berlin: Springer. (DOI) 10.1007/s10649-013-9492-4

Construction and Practice Mode

Variation tasks to bring about awareness of different and similar aspects/features in a mathematical situation that lead to observable invariants

TASK 1: Construction

Construct three points A, B, and C on the screen, the line through A and B, and the line through A and C. Construct a line l parallel to AC through B, and the line perpendicular to l through C. Label the point of intersection of these two lines D. Consider the quadrilateral ABDC.

TASK 2:   Contrasting and Comparing
(Differences and Similarities)

1.   Drag A, B, C to different positions to make different quadrilaterals

2.   How many different or similar types of quadrilateral ABCD can you make?

3.   Describe how you drag a point to make it changes into different types of quadrilateral
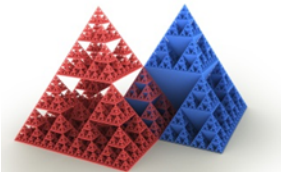
Critical Discernment Mode

Variation tasks to bring about awareness of   critical (causal) relationship among the observed invariants

TASK 3:   Separating out Critical Features
(Sieving and Shifting)

1.   Activate the Trace function for point A. Drag A to maintain ABCD to look like a rectangle.

2.   Describe your experience and what you observe

3.   Make a guess on the shape of the path that A follows when maintaining ABCD as a rectangle. How do you make this guess? Call this path a maintained path.
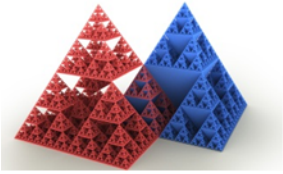
Discourse Mode

Variation tasks to bring about awareness of a connection between critical relationship observed and possible mathematical discourses (causal condition, formal/informal conjecture, concept, pattern, mathematical proof, etc.)

TASK 4:   Simultaneous Focusing
       (Co-variation)

1.   When A is being dragged to vary, vertices B, C and D either vary or not vary as consequence. Observe the behavior of B, C and D while A is varying to maintain ABCD as a rectangle.

2.   Find a possible condition to relate the maintained path and the changing configuration of B, C and D.

3.   Use the condition found in 2, to construct the maintained path

TASK 5:   Conjecturing and Proof
(Development of Theoretical Reasoning)

1. Write a conjecture on what you have discovered in the form

    GIVEN the construction as in TASK 1

    IF   (certain geometrical condition being
                      maintained during dragging)
    THEN   (certain geometrical configuration
                      appears to be maintained)

2.   Drag A along the constructed maintained path and
        observe how different aspects of the figure vary
        together. Explain what you observe and formulate a
   logical argument to prove/explain your conjecture
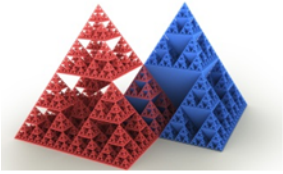
These three modes

Construction/Practice • Critical Discernment • Discourse

can be designed to constitute an evolving process (not necessarily linear) that merges gradually from dominate perceptive, "experiential thinking" to dominate conceptual, "theoretical thinking".

Leung, A. (2011). An epistemic model of task design in dynamic geometry environment. ZDM - The International Journal on Mathematics Education, 43, 325-336.
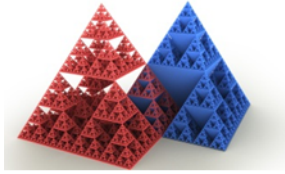
A Unity of Perceptive and Conceptual Mathematical Realms

Thank You