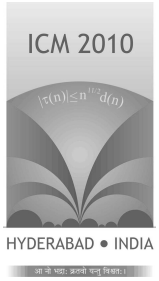

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Abstracts

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Panel Discussions

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Plenary Lectures

Dynamics of Renormalization Operators

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2000 Mathematics Subject Classification. 37E20

It is a remarkable characteristic of some classes of low-dimensional dynamical systems that their long time behavior at a short spatial scale is described by an induced dynamical system in the same class. The *renormalization operator* that relates the original and the induced transformations can then be iterated, and a basic theme is that certain features (such as hyperbolicity, or the existence of an attractor) of the resulting "dynamics in parameter space" impact the behavior of the underlying systems. Classical illustrations of this mechanism include the Feigenbaum-Coullet-Tresser universality in the cascade of period doubling bifurcations for unimodal maps and Herman's Theorem on linearizability of circle diffeomorphisms. We will discuss some recent applications of the renormalization approach, focusing on what it reveals about the dynamics at typical parameter values.

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Exchangeability and Continuum Limits of Discrete Random Structures

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2000 Mathematics Subject Classification. 60C,05C

Exchangeable representations of complex random structures are useful in several ways, in particular providing a moderately general way to derive continuum limits of discrete random structures. I shall give an old example (continuum random trees) and a newer example (dense graph limits). Thinking this way about Google map routes suggests challenging new problems in the plane.

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❖ ❖ ❖

Highly Composite

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2000 Mathematics Subject Classification. 11Mxx, 97A30

Partly owing to the legend of Ramanujan, generations of Indian mathematicians after him have been fascinated with analytic number theory. We provide account of the varied Indian contribution to this subject from Ramanujan to relatively recent times.

❖ ❖ ❖

Endoscopy Theory of Automorphic Forms

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2000 Mathematics Subject Classification. Primary 22E; Secondary 11F, 14G

Keywords: Automorphic forms, endoscopy, transfer conjecture, fundamental lemma, Hitchin fibration.

Historically, Langlands has introduced the theory of endoscopy in order to measure the failure of automorphic forms from being distinguished by their L-functions as well as the defect of stability in the Arthur-Selberg trace formula and ℓ-adic cohomology of Shimura varieties. However, the number of important achievements in the domain of automorphic forms based on the idea of endoscopy has been growing impressively so far. Among these, we will report on Arthur's classification of automorphic representations of classical groups and recent progress on the determination of ℓ-adic Galois representations attached to Shimura varieties originating from Kottwitz's work. These results have now become unconditional; in particular, due to recent progress on local harmonic analysis. Among these developments, we will report on Waldspurger's work on the transfer conjecture and the proof of the fundamental lemma.

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On the Controllability of Nonlinear Partial Differential Equations

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2000 Mathematics Subject Classification. 93B05, 93C10, 93C20

A control system is a dynamical system on which one can act by using controls. A classical issue is the controllability problem: is it possible to reach a desired target from a given starting point by using appropriate controls?

If the starting point and the desired target are both close to some equilibrium, one starts by looking at the linearized control system at this equilibrium. Of course, if this linearized control system is controllable, one expects that the nonlinear control system is locally controllable around this equilibrium, and therefore one can indeed move from the given starting point to the desired target if they are both close to the equilibrium. This indeed follows from the standard inverse mapping theorem in finite dimension. Due to some "loss of derivatives", this might be more difficult to prove in infinite dimension for control systems modeled by partial differential equations. Yet, one can usually indeed get the local controllability of the nonlinear system by using some suitable fixed point method.

Unfortunately, for many interesting applications, the linearized control system is not controllable and one cannot prove anything with this method. To deal with this case, in finite dimension, there is a quite useful tool, namely "iterated Lie brackets". Iterated Lie brackets give also interesting results in infinite dimension. However, for many control systems modeled by partial differential equations, iterated Lie brackets are not well defined (or do not live in a good space). In this talk, we survey methods to handle some of these systems. We illustrate these methods on control systems coming from fluid mechanics (Euler equations of incompressible fluids, shallow water equations) and quantum mechanics. We show how these methods can also be useful to handle the case where the linearized control system is controllable but one looks for global controllability (i.e. when the starting point and the desired target are not close to the equilibrium). We give an application of this situation to the global controllability of the Navier-Stokes equations.

A lot remains to be done on the controllability of nonlinear partial differential equations and we also present some challenging open problems.

Probabilistically Checkable Proofs and Codes

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2000 Mathematics Subject Classification. 68Q17

NP is the complexity class of problems for which it is easy to check that a solution is correct. In contrast, finding solutions to NP problems is widely believed to be hard.The canonical example is the problem SAT: given a Boolean formula, it is notoriously difficult to come up with a satisfying assignment, whereas given a proposed assignment it is trivial to plug in the values and verify its correctness. Such an assignment is an "NP-proof" for the satisfiability of the formula.

Although the verification is simple, it is not *local*, i.e., a verifier must read (almost) the entire proof in order to reach the right decision. In contrast, the landmark PCP theorem [2, 1] says that there are proofs (PCPs) that are probabilistically checkable: they can be verified by a randomized procedure that reads only a *constant* (!) number of bits from the proof.

In this talk we will describe, in terms understandable to the layperson, how any NP proof can be mapped to a new locally checkable proof, the so called PCP, via a gap amplifying encoding.

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❖ ❖ ❖

Ergodic Structures and Non-Conventional Ergodic Theorems

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2000 Mathematics Subject Classification. 37Axx

A well known theorem of Szemeredi asserts that a set of integers of positive upper density contains arbitrarily long arithmetic progressions. This is equivalent

to a multiple recurrence theorem for a measure preserving transformation which can be formulated as:

if $T : X \to X$ is a measure preserving transformation on a measure space $(X, \mathcal{B}, \mu)$, $f \geq 0$ a bounded measurable function with $\int f d\mu > 0$, then for any $k, \exists n$ with $\int f(x)f(T^n x)f(T^{2n}x)\cdots f(T^{kn}x)d\mu(x) > 0$. Setting $A_n^{(k)}$ as the latter integral, the earliest ergodic theoretic approaches to Szemeredi's theorem established this fact by showing that $\liminf \frac{1}{N} \sum_1^N A_n^k > 0$. One now knows that this limit exists, and more specifically, one has a mean "non-conventional" ergodic theorem asserting that in $L^2(X, \mathcal{B}, \mu)$

$$\lim \frac{1}{N} \sum_{n=1}^{N} f_1(T^n x) f_2(T^{2n}{}_x) \cdots f_k(T^{kn}x)$$

exists for bounded measurable $f_1, f_2, \cdots, f_k$. This is shown by linking these averages with the corresponding averages taken for a factor system of a special type (a nil-system). Current investigations are directed to more general averages of functions $f_1(T_1^{P_1(n)}x)f_2(T_2^{P_2(n)}x)\cdots f_k(T_k^{P_k(n)}x)$ where the $P_i(n)$ are integer valued polynomials and $T_1, T_2, \cdots, T_k$ are commuting measure preserving transformations. Here one finds that in addition to factor systems it is useful to consider extensions of a system.

Isogeometric Analysis

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2000 Mathematics Subject Classification. Numerical Analysis

Computational geometry has until very recently had little impact upon the numerical solution of partial differential equations. The purpose of this talk is to explore Isogeometric Analysis, in which NURBS (Non-Uniform Rational B-Splines) and T-Splines are employed to construct exact geometric models [1, 2] of complex domains. I will review recent progress toward developing integrated Computer Aided Design (CAD)/Finite Element Analysis (FEA) procedures that do not involve traditional mesh generation and geometry clean-up steps, that is, the CAD file is directly utilized as the analysis input file. I will summarize some of the mathematical developments within Isogeometric Analysis that confirm the superior accuracy and robustness of spline-based approximations compared with traditional FEA. I will present applications to problems

of solids, structures and fluids, and a modeling paradigm for patient-specific simulation of cardiovascular fluid-structure interaction.

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Eigenfunctions and Coordinate Systems on Manifolds

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2000 Mathematics Subject Classification. 35P05, 58J50, 58J65

A common idea in spectral theory is to study the behavior of eigenfunctions, arising from Laplace like operators, on manifolds and graphs. A more recent idea, sometimes called Diffusion Geometry, is to use a certain number of eigenfunctions as coordinate systems on data sets. While this method has proven to be effective in practice, the reasons for its success have not been clear. We present joint work with Mauro Maggioni and Raanan Schul that explains why this method works for sufficiently smooth manifolds. with finite volume. One of our results is that on a D dimensional manifold, with volume equal to one, for any embedded ball there is a choice of exactly D eigenfunctions that provides a "good" coordinate system on a large portion of the ball. We also explain the history of results of this type for eigenfunctions and heat kernels.

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The Global Behavior of Solutions to Critical Non-linear Dispersive Equations

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2000 Mathematics Subject Classification. 35L70, 35Q55

We will discuss some recent developments in the area of non-linear dispersive and wave equations, concentrating on the long time behavior of solutions to critical problems. The issues that arise are global well-posedness, scattering and finite time blow-up. In this direction we will discuss a method to study such problems (which we call the "concentration compactness/rigidity theorem" method) developed by the author and Frank Merle. The ideas used here are natural extensions of the ones used earlier, by many authors, to study critical non-linear elliptic problems, for instance in the context of the Yamabe problem and in the study of harmonic maps. They also build on earlier works on energy critical defocusing problems. Elements of this program have also proved fundamental in the determination of "universal profiles" at the blow-up time. This has been carried out in recent works of Duyckaerts, the author and Merle. The method will be illustrated with concrete examples, from works of several authors.

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New Algorithms in Image Science

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The past few years have seen an incredible explosion of new (or revival of old) fast and effective algorithms for various imaging and information science applications. These include: nonlocal means, compressive sensing, Bregman iteration, as well as relatively old favorites such as the level set method and PDE based image restoration. I'll give my view of where we are and what's left to do.

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Arithmetic of Linear Algebraic Groups over Two-dimensional Fields

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2000 Mathematics Subject Classification. 11E72, 11E57, 20G10

Kneser, in the early 60's, posed the Hasse principle conjecture for number fields: every principal homogeneous space under a semisimple simply connected linear algebraic group over a number field has a rational point if it has rational points at all real completions. Essentially around the same time, Serre posed a conjecture, now referred to as Conjecture II, which states that principal homogeneous spaces under semisimple simply connected linear algebraic groups over perfect fields of cohomological dimension two have rational points. Conjecture II includes Kneser's conjecture for totally imaginary number fields. The Hasse principle conjecture for number fields was settled by Kneser (1969 TIFR lecture notes) for classical groups, by Harder (1965) for exceptional groups other than type E_8 and by Chernousov (1989) for groups of type E_8. The first major breakthrough concerning Conjecture II was for groups of inner type A_n by Merkurjev and Suslin (1984). In this talk, after summarising the status of Conjecture II over fields of cohomological dimension two, we shall discuss progress concerning the study of homogeneous spaces under linear algebraic groups over function fields of two-dimensional schemes: surfaces over algebraically closed fields, strict henselian two-dimensional local domains and arithmetic surfaces that are relative curves over p-adic integers.

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Representations of Higher Adelic Groups and Arithmetics

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2000 Mathematics Subject Classification. 11F, 11G

We will consider the following issues:

1. n-dimensional local fields and adelic groups (general survey: cohomology of sheaves, residues, class field theory, intersection theory, algebraic groups).

2. Harmonic analysis on local fields and adelic groups for two-dimensional arithmetical schemes (functional spaces, Fourier transform, Poisson formula).

3. Representations of discrete Heisenberg groups. Holomorphic theory vs. unitary theory. Moduli spaces of representations as complex-analytical manifolds. Characters of induced representations as modular forms.

4. Heisenberg adelic groups and their representations arising from two-dimensional schemes. Characters of the representations and L-functions of the schemes.

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Backward Stochastic Differential Equations, Nonlinear Expectations and Their Applications

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2000 Mathematics Subject Classification. 60H, 60E, 62C, 62D, 35J, 35K

We give a survey of the developments in the theory Backward Stochastic Differential Equations (BSDE) during the past 20 years, including existence, uniqueness, comparison theorem, nonlinear Feynman-Kac formula and many other important results in BSDE theory and their applications to dynamic pricing and hedging in a incomplete financial market (see [1, 2, 3]).

We also present our new framework of nonlinear expectation and their applications to financial risk measure under uncertainty of probabilities and distributions. Our new law of large numbers and central limit theorem under

sublinear expectation shows that its limit distribution is a sublinear one, called G-normal distribution. We present a new type of Brownian motion, called G-Brownian motion, which is a continuous stochastic process with independent and stationary increments under a sublinear expectation. The corresponding robust version of Ito's calculus is also very useful for problems of risk measure in finance (see [3, 4, 5]).

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❖ ❖ ❖

"Indian" Rules, "Yavana" Rules: Foreign Identity and the Transmission of Mathematics

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2000 Mathematics Subject Classification. 01

Numerous ideas and methods derived from Indian mathematics became familiar in the west long before European scholars began systematically studying Sanskrit scientific texts. The name "Indian" was attached to many mathematical concepts and techniques in West Asia/North Africa and Europe starting at the beginning of the medieval period, from the "Indian numbers" and "Indian calculation" adopted by Arab mathematicians to the "Hindoo method" for solving quadratic equations in nineteenth-century algebra textbooks. Likewise, the Sanskrit term "Yavana", originally a transliteration of "Ionian (Greek)" but later applied to other foreigners as well, was applied by Indian scholars to various foreign importations in the exact sciences. This talk explores the historical process of adoption and assimilation of "foreign mathematics" both in and from India.

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On Mathematical Problems in Quantum Field Theory

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2000 Mathematics Subject Classification. 57R56, 81T10, 83C47

The goal of this talk is to survey the recent progress in mathematical understanding of quantum field theory and some important unresolved problems in this direction.

Quantum field theory is a framework for mathematical models describing the dynamics of elementary particles. It was initially designed by physicists, and it remained a subject of theoretical physics for some time. But over the last two to three decades it gradually transformed into a formidable tangle of mathematical problems. In the process of resolving these problems a number of new areas in mathematics emerged: quantum groups, many aspects of the representation theory of affine Kac-Moody algebras, vertex algebras and their representation theory, invariants of knots and 3-manifolds, mirror symmetry, and many others.

Roughly, mathematical problems arising in such models can be divided into, first, formulating the model in mathematically acceptable terms, and then, extracting meaningful information from such a model. Quantum field theory also shares many common structures with statistical mechanics. Making sense of path integrals, developing non-perturbative methods, and the renormalization problem are examples of problems of the first type. Computing correlation functions, and expectation values of observables are examples of problems of the second type.

Among recent developments in topological quantum field theory is a better understanding of the Chern-Simons topological quantum field theory, and particularly the theory related to complex simple Lie groups. Computation of correlation functions and the dependence of the partition function on boundary conditions are other examples of rapidly developing directions. The structure of the quantum Yang-Mills theory (one of the Clay problems) remains one of the main outstanding unresolved problems.

Riemannian Manifolds of Positive Curvature

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2000 Mathematics Subject Classification. 53, 35

The study of positive sectional curvature is one of the oldest pursuits in Riemannian geometry, but despite the efforts of many outstanding researchers, basic questions remain unanswered. In this lecture we will briefly summarize the state of knowledge in this area and outline the techniques which have had success. These techniques include geodesic and comparison methods, Hodge theory, minimal surface methods, and Ricci flow. We will then describe our recent work (see [1], [2]) with S. Brendle which uses the Ricci flow to resolve the differentiable sphere theorem; that is, the complete classification of manifolds whose sectional curvatures are 1/4-pinched.

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On the Cohomology of Algebraic Varieties

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2000 Mathematics Subject Classification. 14F25, 14F40, 32J25, 32J27

The central object in this talk is the de Rham complex. It allows to compute the cohomology of a manifold and to understand the interplay between geometry and topology. There are several avatars of it, namely the holomorphic de Rham complex for complex manifolds, and the algebraic de Rham complex for smooth algebraic varieties.

We will first of all explain how to use Hodge theory in Kähler geometry to exhibit topological restrictions on compact Kähler manifolds, some very classical, the others being new and related to the notion of "Hodge structure on a cohomology algebra". More surprisingly, we will use it to exhibit further topological restrictions on the topology of complex projective manifolds (see

[3]). The later are based on the notion of a "polarized Hodge structure on a cohomology algebra".

Our second main topic will be the description of extra data, complementing Hodge theory, on the cohomology of a complex projective manifold. The topology in the usual sense of a complex projective manifold can be partially computed using only the data of the corresponding abstract algebraic variety defined over a subfield K of $\mathbb{C}$. One can use for this (following Grothendieck [2]) the above mentioned algebraic de Rham complex. The extra data consist of a K-structure on Betti cohomology with complex coefficients.

From the point of view of topology, there is the natural Betti $\mathbb{Q}$-structure on cohomology, but the two have almost nothing to do together. This is crucial to understand better in the algebrogeometric context the Hodge conjecture [1], which may seem to be a conjecture in complex differential geometry and can be stated in the Kähler context as well, but in fact fails there [4].

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Strong Axioms of Infinity and the Search for V

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2000 Mathematics Subject Classification. 03E45

The axioms ZFC do not provide a concise conception of the Universe of Sets. This claim has been well documented in the nearly 50 years since Paul Cohen established that the problem of the Continuum Hypothesis cannot be solved on the basis of these axioms.

Gödel's Axiom of Constructibility, $V = L$, provides a conception of the Universe of Sets which is perfectly concise modulo only large cardinal axioms which are strong axioms of infinity. However the axiom $V = L$ limits the large cardinal axioms which can hold and so the axiom is *false*. The Inner Model

Program which seeks generalizations which are compatible with large cardinal axioms has been extremely successful, but incremental, and therefore by its very nature unable to yield an ultimate enlargement of L. The situation has now changed dramatically and there is for the first time a genuine prospect for the construction of an ultimate enlargement of L.

Emmy Noether Lecture

Cluster Categories

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2000 Mathematics Subject Classification. 16G20; 16G70

Cluster algebras were introduced by Fomin-Zelevinsky in 2002 in order to give a combinatorial framework for phenomena occurring for algebraic groups. The cluster algebras also have links to a wide range of other subjects, including the representation theory of finite dimensional algebras, as first discovered by Marsh-Reineke-Zelevinsky. Modifying module categories over hereditary algebras, the cluster categories were introduced in work with Buan-Marsh-Reineke-Todorov in order to "categorify" the essential ingredients in the definition of cluster algebras in the acyclic case. They were shown to be triangulated by Keller. Related work was done by Geiss-Leclerc-Schroer using preprojective algebras of Dynkin type. In work by many authors there have been further developments, leading to feedback on cluster algebras, new interesting classes of finite dimensional algebras, and the investigation of categories of Calabi-Yau dimension 2.

Abel Lecture

Large Deviations

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2000 Mathematics Subject Classification. 60F10

The theory of Large Deviations deals with techniques for estimating probabilities of rare events. These probabilities are exponentially small in a natural parameter and the task is to determine the exponential constant. To be precise, we will have a family $\{P_n\}$ of probability distributions on a space X and asymptotically

$$P_n(A) = \exp[-n \inf_{x \in A} I(x) + o(n)]$$

for a large class of sets, with a suitable choice of the function $I(x)$. This function is almost always related to some form of entropy. There are connections to statistical mechanics as well as applications to the study of scaling limits for large systems. The subject had its origins in the Scandinavian insurance industry where it was used for the evaluation of risk. Since then, it has undergone many developments and we will review some of the recent progress. References [1], [2] and [3] provide a window to the subject.

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Invited Lectures

In case of abstracts with several authors, the invited speakers are marked with *.

Section 1

Logic and Foundations

The Proper Forcing Axiom

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2010 Mathematics Subject Classification. Primary 03E57; Secondary 03E75.

Keywords. Forcing axiom, Martin's Axiom, OCA, Open Coloring Axiom, PID, P-ideal Dichotomy, proper forcing, PFA

The Proper Forcing Axiom is a powerful extension of the Baire Category Theorem which has proved highly effective in settling mathematical statements which are independent of ZFC. In contrast to the Continuum Hypothesis, it eliminates a large number of the pathological constructions which can be carried out using additional axioms of set theory.

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Interactions of Computability and Randomness

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2010 Mathematics Subject Classification. 03D15, 03D32.

Keywords. Algorithmic randomness, lowness property, K-triviality, cost function.

We survey results relating the computability and randomness aspects of sets of natural numbers. Each aspect corresponds to several mathematical properties. Properties originally defined in very different ways are shown to coincide. For instance, lowness for ML-randomness is equivalent to K-triviality. We include some interactions of randomness with computable analysis.

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Tame Complex Analysis and o-minimality

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2010 Mathematics Subject Classification. Primary 03C64, 32B15, 32C20; Secondary: 32B25, 14P15, 03C98

Keywords. o-minimality, real closed fields, non-Archimedean analysis, complex analytic sets, Weierstrass function, theta functions, Abelian varieties

We describe here a theory of holomorphic functions and analytic manifolds, restricted to the category of definable objects in an o-minimal structure which expands a real closed field R. In this setting, the algebraic closure K of the field R, identified with R^2, plays the role of the complex field. Although the ordered field R may be non-Archimedean, o-minimality allows to develop many of the basic results of complex analysis for definable K-holomorphic functions even in this non-standard setting. In addition, o-minimality implies strong theorems on removal of singularities for definable manifolds and definable analytic sets, even when the field R is $\mathbb{R}$. We survey some of these results and several examples.

We also discuss the definability in o-minimal structures of several classical holomorphic maps, and some corollaries concerning definable families of abelian varieties.

Section 2

Algebra

Tensor Triangular Geometry

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2000 Mathematics Subject Classification. Primary 18E30; Secondary 14F05, 19G12, 19K35, 20C20, 53D37, 55P42.

We shall survey a relatively new subject, called "tensor triangular geometry", which is dedicated to the study of tensor triangulated categories as they appear in various areas of mathematics, from algebraic geometry to noncommutative topology, via homotopy theory, motives, or modular representation theory of finite groups. In all those examples, although objects themselves can almost never be classified, it is remarkable that one can always classify so-called thick tensor-ideal subcategories, i.e., one can classify object modulo the elementary operations available in the structure. This classification is done via suitable subsets of an interesting topological space, called the spectrum of the tensor triangulated category under inspection. This space opens the door to algebro-geometric techniques, like gluing, which have interesting applications beyond algebraic geometry. More generally, the abstract platform of tensor triangular geometry allows us to transpose results and methods between the various areas under its roof. We shall try to illustrate this philosophy and indicate some open problems.

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Modules for Elementary Abelian p-groups

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2010 Mathematics Subject Classification. Primary: 20C20; Secondary: 14F05

Keywords. Modular representations, elementary abelian groups, constant Jordan type, vector bundles.

Let $E \cong (\mathbb{Z}/p)^r$ $(r \geq 2)$ be an elementary abelian p-group and let k be an algebraically closed field of characteristic p. A finite dimensional kE-module M is said to have constant Jordan type if the restriction of M to every cyclic shifted subgroup of kE has the same Jordan canonical form. I shall begin by discussing theorems and conjectures which restrict the possible Jordan canonical form. Then I shall indicate methods of producing algebraic vector bundles on projective space from modules of constant Jordan type. I shall describe realisability and non-realisability theorems for such vector bundles, in terms of Chern classes and Frobenius twists. Finally, I shall discuss the closely related question: can a module of small dimension have interesting rank variety? The case p odd behaves throughout these discussions somewhat differently to the case $p = 2$.

Total Positivity and Cluster Algebras

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2010 Mathematics Subject Classification. Primary 13F60, Secondary 05E10, 05E15, 14M15, 15A23, 15B48, 20F55, 22E46.

Keywords. Total positivity, cluster algebra, chamber minors, quiver mutation.

This is a brief and informal introduction to cluster algebras. It roughly follows the historical path of their discovery, made jointly with A. Zelevinsky. Total positivity serves as the main motivation.

Canonical Dimension

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2010 Mathematics Subject Classification. Primary 14L17; Secondary 14C25.

Keywords. Algebraic groups, projective homogeneous varieties, Chow groups and motives.

Canonical dimension is an integral-valued invariant of algebraic structures. We are mostly interested in understanding the canonical dimension of projective homogeneous varieties under semisimple affine algebraic groups over arbitrary

fields. Known methods, results, applications, and open problems are reviewed, some new ones are provided.

Essential Dimension

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2010 Mathematics Subject Classification. Primary 14L30, 20G10, 11E72.

Keywords. Essential dimension, linear algebraic group, Galois cohomology, cohomological invariant, quadratic form, central simple algebra, algebraic torus, canonical dimension

Informally speaking, the essential dimension of an algebraic object is the minimal number of independent parameters one needs to define it. This notion was initially introduced in the context where the objects in question are finite field extensions [BuR97]. Essential dimension has since been investigated in several broader contexts, by a range of techniques, and has been found to have interesting and surprising connections to many problems in algebra and algebraic geometry.

The goal of this paper is to survey some of this research. I have tried to explain the underlying ideas informally through motivational remarks, examples and proof outlines (often in special cases, where the argument is more transparent), referring an interested reader to the literature for a more detailed treatment. The sections are arranged in rough chronological order, from the definition of essential dimension to open problems.

References

[BuR97] J. Buhler and Z. Reichstein, *On the essential dimension of a finite group*, Compositio Math. **106** (1997), no. 2, 159–179.

Quadratic Forms, Galois Cohomology and Function Fields of p-adic Curves

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2010 Mathematics Subject Classification. Primary 11E04, 11R34; Secondary 11G35, 14C25.

Keywords. Quadratic forms, Galois cohomology, u-invariant, p-adic curves.

Let k be a p-adic field and K a function field of a curve over k. It was proved in ([PS3]) that if $p \neq 2$, then the u-invariant of K is 8. Let l be a prime number not equal to p. Suppose that K contains a primitive l^{th} root of unity. It was also proved that every element in $H^3(K, \mathbb{Z}/l\mathbb{Z})$ is a symbol ([PS3]) and that every element in $H^2(K, \mathbb{Z}/l\mathbb{Z})$ is a sum of two symbols ([Su]). In this article we discuss these results and explain how the Galois cohomology methods used in the proof lead to consequences beyond the u-invariant computation.

References

[PS3] Parimala, R. and Suresh, V., *The u-invariant of the function fields of p-adic curves*, to appear in Annals of Mathematics.

[Su] Suresh, V., *Bounding the symbol length in the Galois cohomology of function field of p-adic curves*, to appear in Comm. Math. Helv.

Section 3

Number Theory

The Emerging p-adic Langlands Programme

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2010 Mathematics Subject Classification. Primary 11S80; Secondary 22D12.

Keywords. p-adic Langlands programme, p-adic Hodge theory, $\mathrm{GL}_2(\mathbb{Q}_p)$, (φ,Γ)-modules, completed cohomology.

We give a brief overview of some aspects of the p-adic and modulo p Langlands programmes.

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Selmer Groups and Congruences

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2010 Mathematics Subject Classification. Primary 11G05, 11R23; Secondary 11G40, 11R34.

Keywords. Selmer groups, Iwasawa invariants, Root numbers, Parity conjecture.

We first introduce Selmer groups for elliptic curves, and then Selmer groups for Galois representations. The main topic of the article concerns the behavior of Selmer groups for Galois representations with the same residual representation. We describe a variety of situations where this behavior can be studied fruitfully.

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Artin's Conjecture on Zeros of p-adic Forms

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2010 Mathematics Subject Classification. Primary 11D88; Secondary 11D72, 11E08, 11E76, 11E95

Keywords. Artin's conjecture, p-adic forms, Quartic forms, Systems of quadratic forms, u-invariant

This is an exposition of work on Artin's Conjecture on the zeros of p-adic forms. A variety of lines of attack are described, going back to 1945. However there is particular emphasis on recent developments concerning quartic forms on the one hand, and systems of quadratic forms on the other.

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Relative p-adic Hodge Theory and Rapoport-Zink Period Domains

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2010 Mathematics Subject Classification. Primary 14G22; Secondary 11G25.

Keywords. Relative p-adic Hodge theory, Rapoport-Zink period domains.

As an example of relative p-adic Hodge theory, we sketch the construction of the universal admissible filtration of an isocrystal (ϕ-module) over the completion of the maximal unramified extension of $\mathbb{Q}_p$, together with the associated universal crystalline local system.

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Serre's Modularity Conjecture

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2010 Mathematics Subject Classification. Primary 11R39; Secondary 11F80.

Keywords. Galois representations. Modular forms.

We state Serre's modularity conjecture, give some hints on its proof and give some consequences.

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The Structure of Potentially Semi-stable Deformation Rings

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2010 Mathematics Subject Classification. 11F80

Keywords. Galois representations

Inside the universal deformation space of a local Galois representation one has the set of deformations which are potentially semi-stable of given p-adic Hodge and Galois type. It turns out these points cut out a closed subspace of the deformation space. A deep conjecture due to Breuil-Mézard predicts that part of the structure of this space can be described in terms of the local Langlands correspondence. For 2-dimensional representations the conjecture can be made precise. We explain some of the progress in this case, which reveals that the conjecture is intimately connected to the p-adic local Langlands correspondence, as well as to the Fontaine-Mazur conjecture.

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The Intersection Complex as a Weight Truncation and an Application to Shimura Varieties

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2010 Mathematics Subject Classification. Primary 11F75; Secondary 11G18, 14F20.

Keywords. Shimura varieties, intersection cohomology, Frobenius weights

The purpose of this talk is to present an (apparently) new way to look at the intersection complex of a singular variety over a finite field, or, more generally, at the intermediate extension functor on pure perverse sheaves, and an application of this to the cohomology of noncompact Shimura varieties.

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Wild Ramification of Schemes and Sheaves

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2010 Mathematics Subject Classification. Primary 14F20; Secondary 11G25, 11S15.

Keywords. Conductor, ℓ-adic sheaf, wild ramification, Grothendieck-Ogg-Shafarevich formula, Swan class, characteristic class.

We discuss recent developments on geometric theory of ramification of schemes and sheaves. For invariants of ℓ-adic cohomology, we present formulas of Riemann-Roch type expressing them in terms of ramification theoretic invariants of sheaves. The latter invariants allow geometric computations involving some new blow-up constructions.

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Quantum Unique Ergodicity and Number Theory

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2010 Mathematics Subject Classification. Primary 11F11, 11F67, 11M99, 11N64.

Keywords. Quantum unique ergodicity, modular surface, Hecke operators, subconvexity problem, L-functions, multiplicative functions, sieve methods.

A fundamental problem in the area of quantum chaos is to understand the distribution of high eigenvalue eigenfunctions of the Laplacian on certain Riemannian manifolds. A particular case which is of interest to number theorists concerns hyperbolic surfacess arising as a quotient of the upper half-plane by a discrete "arithmetic" subgroup of $SL_2(\mathbb{R})$ (for example, $SL_2(\mathbb{Z})$, and in this case the corresponding eigenfunctions are called Maass cusp forms). In this case, Rudnick and Sarnak have conjectured that the high energy eigenfunctions become equi-distributed. I will discuss some recent progress which has led to a resolution of this conjecture, and also on a holomorphic analog for classical modular forms

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Statistics of Number Fields and Function Fields

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2010 Mathematics Subject Classification. 11R47.

We discuss some problems of arithmetic distribution, including conjectures of Cohen-Lenstra, Malle, and Bhargava; we explain how such conjectures can be heuristically understood for function fields over finite fields, and discuss a general approach to their proof in the function field context based on the topology of Hurwitz spaces. This approach also suggests that the Schur multiplier plays a role in such questions over number fields.

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Section 4

Algebraic and Complex Geometry

The Tangent Space to an Enumerative Problem

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2010 Mathematics Subject Classification. Primary 14M17, 14N15, 14D20; Secondary 14L24, 14N15.

Keywords. Intersection theory, homogeneous spaces, theta functions, invariant theory, Horn conjecture, saturation conjecture, strange duality.

We will discuss recent work on the relations between the intersection theory of homogeneous spaces (and their quantum, and higher genus generalizations), invariant theory, and non-abelian theta functions. The main theme is that the analysis of transversality in enumerative problems can be viewed as a bridge from intersection theory to representation theory. Some of the new results proved using these ideas are reviewed: multiplicative generalizations of the Horn and saturation conjectures, generalizations of Fulton's conjecture, the deformation of cohomology of homogeneous spaces, and the strange duality conjecture in the theory of vector bundles on algebraic curves.

Boundedness Results in Birational Geometry

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2010 Mathematics Subject Classification. Primary 14E05; Secondary 14J40

Keywords. Pluricanonical map, boundedness, minimal model program.

We survey results related to pluricanonical maps of complex projective varieties of general type.

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Hyperkähler Manifolds and Sheaves

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2010 Mathematics Subject Classification. Primary 14F05, 53C26; Secondary 18E30,14J28.

Keywords. Hyperkähler manifolds, moduli spaces, derived categories, holomorphic symplectic manifolds.

Moduli spaces of hyperkähler manifolds or of sheaves on them are often non-separated. We will discuss results where this phenomenon reflects interesting geometric aspects, e.g. deformation equivalence of birational hyperkähler manifolds or cohomological properties of derived autoequivalences. In these considerations the Ricci-flat structure often plays a crucial role via the associated twistor space providing global deformations of manifolds and bundles.

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Motivic Structures in Non-commutative Geometry

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2010 Mathematics Subject Classification. 14F05, 14F30 and 14F40.

Keywords. Motivic, non-commutative, cyclic, p-adic, Hodge-de Rham.

We review recent theorems and conjectures saying that periodic cyclic homology of a smooth non-commutative algebraic variety carries all the additional structures the usual de Rham cohomology has in the commutative case, such as a mixed Hodge structure, and a structure of a filtered Dieudonné module.

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Gromov-Witten Theory of Calabi-Yau 3-folds

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2010 Mathematics Subject Classification. 14N35

Keywords. Gromov-Witten invariants, Calabi-Yau 3-folds

We describe some recent progress and open problems in Gromov-Witten theory of Calabi-Yau 3-folds, focusing on the quintic 3-fold and toric Calabi-Yau 3-folds.

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Flips and Flops

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2010 Mathematics Subject Classification. Primary 14E30

Keywords. Flips, Flops, Minimal model program, Mori theory.

Flips and flops are elementary birational maps which first appear in dimension three. We give examples of how flips and flops appear in many different contexts. We describe the minimal model program and some recent progress centred around the question of termination of flips.

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Quantitative Extensions of Twisted Pluricanonical Forms and Non-vanishing

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2010 Mathematics Subject Classification. 14C30, 32J25, 32QXX.

Keywords. L^2 estimates, extension theorems, non-vanishing, closed positive currents, metrics with minimal singularities.

We will discuss here a few recent applications of the analytic techniques in algebraic geometry.

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Cohomological Hasse Principle and Motivic Cohomology of Arithmetic Schemes

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2010 Mathematics Subject Classification. 19F27, 19E15, 14C25, 14F42

Keywords. Hasse principle, motivic cohomology, zeta function, higher class field theory

In 1985 Kazuya Kato formulated a fascinating framework of conjectures which generalize the Hasse principle for the Brauer group of a global field to the so-called cohomological Hasse principle for an arithmetic scheme X. He defined an invariant $KH_a(X)$ $(a \geq 0)$, called the Kato homology of X, that reflects the arithmetic nature of X. As a generalization of the classical Hasse principle, Kato conjectured the vanishing of $KH_a(X) = 0$ for $a > 0$, when X is a proper smooth variety over a finite field, or a regular scheme proper and flat over the ring of integers in a number field or in a local field. The conjecture turns out to play a significant rôle in arithmetic geometry. We will explain recent progress on the conjecture and its implications on finiteness of motivic cohomology, special values of zeta functions, a generalization of higher dimensional class field theory, and a geometric application to quotient singularities.

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Betti Numbers of Syzygies and Cohomology of Coherent Sheaves

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2010 Mathematics Subject Classification. Primary 13D02; Secondary 14F05.

Keywords. Betti numbers, free resolutions, syzygies, cohomology of coherent sheaves, multiplicity

The Betti numbers of a graded module over the polynomial ring form a table of numerical invariants that refines the Hilbert polynomial. A sequence of papers

sparked by conjectures of Boij and Söderberg have led to the characterization of the possible Betti tables up to rational multiples—that is, to the rational cone generated by the Betti tables. We will summarize this work by describing the cone and the closely related cone of cohomology tables of vector bundles on projective space, and we will give new, simpler proofs of some of the main results. We also explain some of the applications of the theory, including the one that originally motivated the conjectures of Boij and Söderberg, a proof of the Multiplicity Conjecture of Herzog, Huneke and Srinivasan.

Algebraic Cycles on Singular Varieties

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2010 Mathematics Subject Classification. 14C17, 14C30, 14B05.

Keywords. Chow ring, singular varieties.

We discuss algebraic cycles on singular varieties, in relation to the Grothendieck group of vector bundles. This theory, which is still not fully worked out, seems to admit some surprises. On the other hand, conjectured aspects of the refined structure of cycle groups of nonsingular varieties, predicted by motivic considerations, seem to have plausible extensions to singular varieties, which can be verified in some nontrivial examples.

An Exercise in Mirror Symmetry

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2010 Mathematics Subject Classification. Primary 14J33; Secondary 53D37, 57M27, 53C26.

Keywords. Mirror symmetry, Khovanov cohomology.

This expository article is an attempt to illustrate the power of Kontsevich's homological mirror symmetry conjecture through one example, the heuristics of which lead to an algebro-geometric construction of knot invariants.

Invariants Entiers en Géométrie Énumérative Réelle

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2010 Mathematics Subject Classification. Primary 53D45; Secondary 14N35.

Keywords. Enumerative geometry, rational curve, real algebraic variety, holomorphic discs.

Je rappelle les divers problèmes de géométrie énumérative réelle desquels j'ai pu extraire des invariants à valeurs entières, fournissant un pendant réel aux invariants de Gromov-Witten. Je discute l'optimalité des bornes inférieures fournies par ces invariants ainsi que certaines de leurs propriétés arithmétiques. Je présente enfin davantage de résultats garantissant la présence ou l'absence de disques pseudo-holomorphes à bord dans une sous-variété lagrangienne d'une variété symplectique donnée.

Section 5

Geometry

Poisson-Furstenberg Boundaries, Large-scale Geometry and Growth of Groups

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2010 Mathematics Subject Classification. Primary 20F69, 60B15; Secondary 43A05, 43A07, 60G50, 60J50, 30F15.

Keywords. Random walks on groups, boundary, harmonic function, amenable groups, growth of groups.

We give a survey of recent results on the Poisson-Furstenberg boundaries of random walks on groups, and their applications. We describe sufficient conditions for random walk to have non-trivial boundary, or, on the contrary, to have trivial boundary. We review recent progress in description of the boundary for random walks on various groups, including wreath products. We describe how the Poisson-Furstenberg boundary can be used to obtain lower bounds for the growth function of the groups of intermediate growth. We also discuss relation between properties of the boundary with other asymptotic properties of groups, including isoperimetry and various characteristics of random walks.

On non-Kähler Calabi-Yau Threefolds with Balanced Metrics

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2010 Mathematics Subject Classification. 53.

Keywords. Calabi-Yau manifold, Balanced metric, Strominger system, hermitian-Yang-Mills metric, Monge-Ampère equation, form-type Calabi-Yau equation.

The solution of the Strominger system can be viewed as a canonical structure on non-Kähler Calabi-Yau threefolds with balanced metrics. In this talk, we review the existence of balanced metrics on non-Kähler complex manifolds and the existence of solutions to the Strominger system.

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Locally Homogeneous Geometric Manifolds

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2010 Mathematics Subject Classification. Primary 57M50; Secondary 57N16.

Keywords. Connection, curvature, fiber bundle, homogeneous space, Thurston geometrization of 3-manifolds, uniformization, crystallographic group, discrete group, proper action, Lie group, fundamental group, holonomy, completeness, development, geodesic, symplectic structure, Teichmüller space, Fricke space, hyperbolic structure, Riemannian metric, Riemann surface, affine structure, projective structure, conformal structure, spherical CR structure, complex hyperbolic structure, deformation space, mapping class group, ergodic action.

Motivated by Felix Klein's notion that geometry is governed by its group of symmetry transformations, Charles Ehresmann initiated the study of geometric structures on topological spaces locally modeled on a homogeneous space of a Lie group. These locally homogeneous spaces later formed the context of Thurston's 3-dimensional geometrization program. The basic problem is for a given topology Σ and a geometry $X = G/H$, to classify all the possible ways of introducing the local geometry of X into Σ. For example, a sphere admits no local Euclidean geometry: there is no metrically accurate Euclidean atlas of the earth. One develops a space whose points are equivalence classes of geometric structures on Σ, which itself exhibits a rich geometry and symmetries arising from the topological symmetries of Σ.

We survey several examples of the classification of locally homogeneous geometric structures on manifolds in low dimension, and how it leads to a general study of surface group representations. In particular geometric structures are a useful tool in understanding local and global properties of deformation spaces of representations of fundamental groups.

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Metaphors in Systolic Geometry

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2010 Mathematics Subject Classification. Primary 53C23

Keywords. Systole, filling radius, isoperimetric inequality.

We discuss the systolic inequality for n-dimensional tori, explaining different metaphors that help to organize the proof. The metaphors connect systolic geometry with minimal surface theory, topological dimension theory, and scalar curvature.

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Volume Comparison via Boundary Distances

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2010 Mathematics Subject Classification. Primary 53C23; Secondary 53C60.

Keywords. Filling volume, minimal filling, boundary distance rigidity.

The main subject of this lecture is a connection between Gromov's filling volumes and a boundary rigidity problem of determining a Riemannian metric in a compact domain by its boundary distance function. A fruitful approach is to represent Riemannian metrics by minimal surfaces in a Banach space and to prove rigidity by studying the equality case in a filling volume inequality. I discuss recent results obtained with this approach and related problems in Finsler geometry.

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Geometric Quantization on Kähler and Symplectic Manifolds

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2010 Mathematics Subject Classification. Primary 53D; Secondary 58J, 32A.

Keywords. Bergman kernel, Dirac operator, Geometric quantization, Index theorem.

We explain various results on the asymptotic expansion of the Bergman kernel on Kähler manifolds and also on symplectic manifolds. We also review the "quantization commutes with reduction" phenomenon for a compact Lie group action, and its relation to the Bergman kernel.

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Scalar Curvature, Conformal Geometry, and the Ricci Flow with Surgery

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2010 Mathematics Subject Classification. Primary 53C21; Secondary 83C05.

Keywords. Scalar curvature; Yamabe problem; Ricci flow with surgery.

In this note we will review recent results concerning two geometric problems associated to the scalar curvature. In the first part we will review the solution to Schoen's conjecture about the compactness of the set of solutions to the Yamabe problem. It has been discovered, in a series of three papers, that the conjecture is true if and only if the dimension is less than or equal to 24. In the second part we will discuss the connectedness of the moduli space of metrics with positive scalar curvature in dimension three. In two dimensions this was proved by Weyl in 1916. This is a geometric application of the Ricci flow with surgery and Perelman's work on Hamilton's Ricci flow.

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Constant Mean Curvature Surfaces in 3-dimensional Thurston Geometries

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2010 Mathematics Subject Classification. 53A10, 53C42

Keywords. Constant mean curvature surfaces, homogeneous spaces, Thurston geometries, harmonic maps, minimal surfaces, entire graphs.

This is a survey on the global theory of constant mean curvature surfaces in Riemannian homogeneous 3-manifolds. These ambient 3-manifolds include the eight canonical Thurston 3-dimensional geometries, i.e. $\mathbb{R}^3$, $\mathbb{H}^3$, $\mathbb{S}^3$, $\mathbb{H}^2 \times \mathbb{R}$, $\mathbb{S}^2 \times \mathbb{R}$, the Heisenberg space Nil_3, the universal cover of $\mathrm{PSL}_2(\mathbb{R})$ and the Lie group Sol_3. We will focus on the problems of classifying compact CMC surfaces and entire CMC graphs in these spaces. A collection of important open problems of the theory is also presented.

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Morse Landscapes of Riemannian Functionals and Related Problems

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2010 Mathematics Subject Classification. Primary 53C23, 58E11, 53C20; Secondary 03D80, 68Q30, 53C40, 58E05.

Keywords. Non-computability, geometric calculus of variations, best Riemannian metrics, algorithmic unsolvability, quantitative topology, Riemannian functionals, the length functional, thick knots, curvature-pinching, loop spaces.

The subject of this talk is Morse landscapes of natural functionals on infinite-dimensional moduli spaces appearing in Riemannian geometry.

First, we explain how recursion theory can be used to demonstrate that for many natural functionals on spaces of Riemannian structures, spaces of submanifolds, etc., their Morse landscapes are always more complicated than what

follows from purely topological reasons. These Morse landscapes exhibit non-trivial "deep" local minima, cycles in sublevel sets that become nullhomologous only in sublevel sets corresponding to a much higher value of functional, etc.

Our second topic is Morse landscapes of the length functional on loop spaces. Here the main conclusion (obtained jointly with Regina Rotman) is that these Morse landscapes can be much more complicated than what follows from topological considerations only if the length functional has "many" "deep" local minima, and the values of the length at these local minima are not "very large".

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Constant Scalar Curvature and Extremal Kähler Metrics on Blow ups

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2010 Mathematics Subject Classification. Primary 32J27; Secondary 53C21.

Keywords. Extremal metrics, Kähler geometry, perturbation methods.

Extremal Kähler metrics were introduced by E. Calabi as best representatives of a given Kähler class of a complex compact manifold, these metrics are critical points of the L^2 norm of the scalar curvature function. In this paper, we report some joint works with C. Arezzo and M. Singer concerning the construction of extremal Kähler metrics on blow ups at finitely many points of Kähler manifolds which already carry an extremal Kähler metric. In particular, we give sufficient conditions on the number and locations of the blown up manifold points for the blow up to carry an extremal Kähler metric.

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Reconstruction of Collapsed Manifolds

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2010 Mathematics Subject Classification. Primary 53C20; Secondary 58J50.

Keywords. Gromov-Hausdorff convergence, collapsing, three-manifolds, four-manifolds, essential coverings, Betti numbers, inverse spectral problem

In this article, we consider the problem of reconstructing collapsed manifolds in a moduli space by means of geometric or analytic data of the limit spaces. The moduli space of our main interest is that consisting of closed Riemannian manifolds of fixed dimension with a lower sectional curvature and an upper diameter bound. In this moduli space, we can reconstruct the topology of three-dimensional or four-dimensional collapsed manifolds in terms of the singularities of the limit Alexandrov spaces. In the general dimension, we define a new covering invariant and prove the uniform boundedness of it with an application to Gromov's Betti number theorem. Finally we discuss the reconstruction and stability problems of collapsed manifolds by using analytic spectral data, where we assume an additional upper sectional curvature bound.

Section 6

Topology

Fukaya Categories and Bordered Heegaard-Floer Homology

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2010 Mathematics Subject Classification. 53D40 (53D37, 57M27, 57R58)

Keywords. Bordered Heegaard-Floer homology, Fukaya categories

We outline an interpretation of Heegaard-Floer homology of 3-manifolds (closed or with boundary) in terms of the symplectic topology of symmetric products of Riemann surfaces, as suggested by recent work of Tim Perutz and Yankı Lekili. In particular we discuss the connection between the Fukaya category of the symmetric product and the bordered algebra introduced by Robert Lipshitz, Peter Ozsváth and Dylan Thurston, and recast bordered Heegaard-Floer homology in this language.

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A Geometric Construction of the Witten Genus, I

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2010 Mathematics Subject Classification. 58J26, 81T40

Keywords. Elliptic genera, quantum field theory

I describe how the Witten genus of a complex manifold X can be seen from a rigorous analysis of a certain two-dimensional quantum field theory of maps from a surface to X.

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Hyperbolic 3-manifolds in the 2000's

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2010 Mathematics Subject Classification. Primary 57M50; Secondary 20F65, 30F40, 51M10, 51M25, 57N10, 57S05.

Keywords. Hyperbolic 3-manifold, generalized Smale conjecture, tube, tameness, volume, Weeks' manifold, ending lamination

The first decade of the 2000's has seen remarkable progress in the theory of hyperbolic 3-manifolds. We report on some of these developments.

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The Classification of p–compact Groups and Homotopical Group Theory

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2010 Mathematics Subject Classification. Primary: 55R35; Secondary: 55R37, 55P35, 20F55.

Keywords. Homotopical group theory, classifying space, p–compact group, reflection group, finite loop space, cohomology ring.

We survey some recent advances in the homotopy theory of classifying spaces, and homotopical group theory. We focus on the classification of p–compact groups in terms of root data over the p–adic integers, and discuss some of its consequences e.g., for finite loop spaces and polynomial cohomology rings.

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Actions of the Mapping Class Group

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2010 Mathematics Subject Classification. Primary 30F60, Secondary 20F28, 20F65, 20F69

Keywords. Mapping class group, isometric actions, geometric rigidity

Let S be a closed oriented surface S of genus $g \geq 0$ with $m \geq 0$ marked points (punctures) and $3g-3+m \geq 2$. This is a survey of recent results on actions of the mapping class group of S which led to a geometric understanding of this group.

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Embedded Contact Homology and Its Applications

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2010 Mathematics Subject Classification. Primary 57R58; Secondary 57R17.

Keywords. Embedded contact homology, contact three-manifolds, Weinstein conjecture, chord conjecture

Embedded contact homology (ECH) is a kind of Floer homology for contact three-manifolds. Taubes has shown that ECH is isomorphic to a version of Seiberg-Witten Floer homology (and both are conjecturally isomorphic to a version of Heegaard Floer homology). This isomorphism allows information to be transferred between topology and contact geometry in three dimensions. In this article we first give an overview of the definition of embedded contact homology. We then outline its applications to generalizations of the Weinstein conjecture, the Arnold chord conjecture, and obstructions to symplectic embeddings in four dimensions.

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Finite Covering Spaces of 3-manifolds

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2010 Mathematics Subject Classification. Primary 57N10, 57M10; Secondary 57M07.

Keywords. Covering space; hyperbolic 3-manifold; incompressible surface; subgroup growth; Cheeger constant; Heegaard splitting; Property (τ)

Following Perelman's solution to the Geometrisation Conjecture, a 'generic' closed 3-manifold is known to admit a hyperbolic structure. However, our understanding of closed hyperbolic 3-manifolds is far from complete. In particular, the notorious Virtually Haken Conjecture remains unresolved. This proposes that every closed hyperbolic 3-manifold has a finite cover that contains a closed embedded orientable π_1-injective surface with positive genus.

I will give a survey on the progress towards this conjecture and its variants. Along the way, I will address other interesting questions, including: What are the main types of finite covering space of a hyperbolic 3-manifold? How many are there, as a function of the covering degree? What geometric, topological and algebraic properties do they have? I will show how an understanding of various geometric and topological invariants (such as the first eigenvalue of the Laplacian, the rank of mod p homology and the Heegaard genus) can be used to deduce the existence of π_1-injective surfaces, and more.

K- and L-theory of Group Rings

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2010 Mathematics Subject Classification. Primary 18F25; Secondary 57XX.

Keywords. K- and L-theory, group rings, Farrell-Jones Conjecture, topological rigidity.

This article will explore the K- and L-theory of group rings and their applications to algebra, geometry and topology. The Farrell-Jones Conjecture characterizes K- and L-theory groups. It has many implications, including the Borel

and Novikov Conjectures for topological rigidity. Its current status, and many of its consequences are surveyed.

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Moduli Problems for Ring Spectra

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2010 Mathematics Subject Classification. Primary 55P43, Secondary 14B12.

Keywords. Structured ring spectra, deformation theory, derived algebraic geometry.

In algebraic geometry, it is common to study a geometric object X (such as a scheme) by means of the functor $R \mapsto Hom(SpecR, X)$ represented by X. In this paper, we consider functors which are defined on larger classes of rings (such as the class of *ring spectra* which arise in algebraic topology), and sketch some applications to deformation theory.

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On Weil-Petersson Volumes and Geometry of Random Hyperbolic Surfaces

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2010 Mathematics Subject Classification. Primary 32G15; Secondary 57M50

Keywords. Moduli space, Weil-Petersson volume form, simple closed geodesic, hyperbolic surface

This paper investigates the geometric properties of random hyperbolic surfaces with respect to the Weil-Petersson measure. We describe the relationship between the behavior of lengths of simple closed geodesics on a hyperbolic surface and properties of the moduli space of such surfaces. First, we study the asymptotic behavior of Weil-Petersson volumes of the moduli spaces of hyperbolic surfaces of genus g as $g \to \infty$. Then we apply these asymptotic estimates to study the geometric properties of random hyperbolic surfaces, such as the length of the shortest simple closed geodesic of a given combinatorial type.

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A New Family of Complex Surfaces of General Type with $p_g = 0$

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2010 Mathematics Subject Classification. Primary 14J29; Secondary 14J17, 53D03.

Keywords. $\mathbb{Q}$-Gorenstein smoothing, rational blow-down, surface of general type

In this article we review how to construct new families of simply connected complex surfaces of general type with $p_g = 0$ and $2 \leq K^2 \leq 4$ using a rational blow-down surgery and $\mathbb{Q}$-Gorenstein smoothing theory. Furthermore, we also explain that this technique is a very powerful tool to construct many other interesting families of complex surfaces.

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Ozsváth-Szabó Invariants and 3-dimensional Contact Topology

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2010 Mathematics Subject Classification. 57R17; 57R57

Keywords. Contact 3-manifolds, tight contact structures, Heegaard Floer theory, Ozsváth–Szabó invariants, Legendrian and transverse knots

We review applications of Ozsváth–Szabó homologies (and in particular, the contact Ozsváth–Szabó invariant) in 3-dimensional contact topology.

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Section 7

Lie Theory and Generalizations

Quasi-isometric Rigidity of Solvable Groups

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2010 Mathematics Subject Classification. Primary 22E25; Secondary 20F65.

Keywords. Quasi-isometry, rigidity, polycyclic groups.

In this article we survey recent progress on quasi-isometric rigidity of polycyclic groups. These results are contributions to Gromov's program for classifying finitely generated groups up to quasi-isometry [Gr2]. The results discussed here rely on a new technique for studying quasi-isometries of finitely generated groups, which we refer to as *coarse differentiation*.

We include a discussion of other applications of coarse differentiation to problems in geometric group theory and a comparison of coarse differentiation to other related techniques in nearby areas of mathematics.

References

[Gr2] Gromov, Mikhael. Infinite groups as geometric objects. Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Warsaw, 1983), 385–392, PWN, Warsaw, 1984.

Rational Cherednik Algebras

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2010 Mathematics Subject Classification. Primary 16G, 17B; Secondary 20C, 53D.

Keywords. Cherednik algebra, symplectic singularity, hamiltonian reduction.

We survey a number of results about the rational Cherednik algebra's representation theory and its connection to symplectic singularities and their resolutions.

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Tensor Product Decomposition

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2010 Mathematics Subject Classification. 20G05, 22E46

Keywords. Semisimple groups, tensor product decomposition, saturated tensor cone, PRVK conjecture, root components, geometric invariant theory.

Let G be a semisimple connected complex algebraic group. We study the tensor product decomposition of irreducible finite-dimensional representations of G. The techniques we employ range from representation theory to algebraic geometry and topology. This is mainly a survey of author's various results on the subject obtained individually or jointly with Belkale, Kapovich, Leeb, Millson and Stembridge.

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Some Applications of the Trace Formula and the Relative Trace Formula

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2010 Mathematics Subject Classification. Primary 11F72; Secondary 11F70, 58C40.

Keywords. Trace formula

The trace formula is a major tool in the theory of automorphic forms. It was conceived by Selberg and extensively developed by Arthur. Among other things it is applicable to the study of spectral asymptotics as well as to (special cases of) Langlands functoriality conjectures. An important variant invented by Jacquet – the relative trace formula – is used to study period integrals and invariant functionals.

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Finite W-algebras

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2010 Mathematics Subject Classification. Primary 16G99, 17B35; Secondary 53D20, 53D55.

Keywords. W-algebra, semisimple Lie algebra, nilpotent orbit, universal enveloping algebra, primitive ideal, Whittaker module.

A finite W-algebra is an associative algebra constructed from a semisimple Lie algebra and its nilpotent element. In this survey we review recent developments in the representation theory of W-algebras. We emphasize various interactions between W-algebras and universal enveloping algebras.

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Dynamics on Geometrically Finite Hyperbolic Manifolds with Applications to Apollonian Circle Packings and Beyond

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2010 Mathematics Subject Classification. Primary 37A17, Secondary 37A40

Keywords. Circles, Apollonian circle packings, geometrically finite groups, Patterson-Sullivan density

We present recent results on counting and distribution of circles in a given circle packing invariant under a geometrically finite Kleinian group and discuss how the dynamics of flows on geometrically finite hyperbolic 3 manifolds are related. Our results apply to Apollonian circle packings, Sierpinski curves, Schottky dances, etc.

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Equidistribution of Translates of Curves on Homogeneous Spaces and Dirichlet's Approximation

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2010 Mathematics Subject Classification. Primary 22E40; Secondary 11J83.

Keywords. Equidistribution, homogeneous flow, unipotent flow, Ratner's Theorem, Dirichlet's approximation, hyperbolic manifold, geodesic flow

Understanding the limiting distributions of translates of measures on submanifolds of homogeneous spaces of Lie groups leads to very interesting number theoretic and geometric applications. We explore this theme in various generalities, and in specific cases. Our main tools are Ratner's theorems on unipotent flows, nondivergence theorems of Dani and Margulis, and dynamics of linear actions of semisimple groups.

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Schur-Weyl Dualities and Link Homologies

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2010 Mathematics Subject Classification. Primary 17B10, 17B37, 57M27, 32S55

Keywords. Reshetikhin-Turaev invariants, knots, TQFT, general Lie supergroup, diagram algebras, Koszul algebras, 3j-symbols, Hecke algebra.

In this note we describe a representation theoretic approach to functorial functor valued knot invariants with the focus on (categorified) Schur-Weyl dualities. Applications include categorified Reshetikhin-Turaev invariants, an extension of Khovanov homology and a diagrammatical description of the category of finite dimensional $GL(m|n)$-modules.

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Cohomology of Arithmetic Groups and Representations

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2010 Mathematics Subject Classification. Primary 11F75; Secondary 22E40, 22E41

We give a survey of results on restriction of cohomology classes on locally symmetric spaces to smaller locally symmetric spaces; these results are closely connected with cohomological representations of semi-simple Lie groups associated with the locally symmetric spaces and we describe the connection.

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Section 8

Analysis

Differentiability of Lipschitz Functions, Structure of Null Sets, and Other Problems

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2010 Mathematics Subject Classification. Primary 26B05; Secondary 28A75.

Keywords. Lipschitz, derivative, tangent, width, unrectifiability

The research presented here developed from rather mysterious observations, originally made by the authors independently and in different circumstances, that Lebesgue null sets may have uniquely defined tangent directions that are still seen even if the set is much enlarged (but still kept Lebesgue null). This phenomenon appeared, for example, in the rank-one property of derivatives of BV functions and, perhaps in its most striking form, in attempts to decide whether Rademacher's theorem on differentiability of Lipschitz functions may be strengthened or not.

We describe the non-differentiability sets of Lipschitz functions on $\mathbb{R}^n$ and use this description to explain the development of the ideas and various approaches to the definition of the tangent fields to null sets. We also indicate

connections to other current results, including results related to the study of structure of sets of small measure, and present some of the main remaining open problems.

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Asymptotic Analysis of the Toeplitz and Hankel Determinants via the Riemann-Hilbert Method

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2010 Mathematics Subject Classification. Primary 47B35, 15B52; Secondary 35Q15, 34M55 .

Keywords. Toeplitz determinants, Riemann-Hilbert problem, Painlevé equations

The basic features of the asymptotic analysis of Toeplitz and Hankel determinants via the Riemann-Hilbert method including the fundamental connections to the theory of Painlevé equations are outlined. Some of the most recent results obtained in the field are discussed.

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Regularity of the Inverse of a Sobolev Homeomorphism

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2010 Mathematics Subject Classification. Primary 30C65; Secondary 46E35.

Keywords. Sobolev mapping, bounded variation, homeomorphism, inverse, finite distortion

We give necessary and sufficient conditions for the inverse of a Sobolev homeomorphism to be a Sobolev homeomorphism and conditions under which the inverse is of bounded variation.

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Multiple Orthogonal Polynomials in Random Matrix Theory

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2010 Mathematics Subject Classification. Primary 42C05; Secondary 15B52, 31A15, 60C05, 60G55.

Keywords. Multiple orthogonal polynomials, non-intersecting Brownian motion, random matrices with external source, two matrix model, vector equilibrium problems, Riemann-Hilbert problem, steepest descent analysis.

Multiple orthogonal polynomials are a generalization of orthogonal polynomials in which the orthogonality is distributed among a number of orthogonality weights. They appear in random matrix theory in the form of special determinantal point processes that are called multiple orthogonal polynomial (MOP) ensembles. The correlation kernel in such an ensemble is expressed in terms of the solution of a Riemann-Hilbert problem, that is of size $(r+1) \times (r+1)$ in the case of r weights.

A number of models give rise to a MOP ensemble, and we discuss recent results on models of non-intersecting Brownian motions, Hermitian random matrices with external source, and the two matrix model. A novel feature in the asymptotic analysis of the latter two models is a vector equilibrium problem for two or more measures, that describes the limiting mean eigenvalue density. The vector equilibrium problems involve both an external field and an upper constraint.

Quasiregular Mappings, Curvature & Dynamics

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2010 Mathematics Subject Classification. 30C65, 37F10, 37F30 and 30D05.

Keywords. Quasiconformal, Rational mapping, conformal dynamics.

We survey recent developments in the area of geometric function theory and nonlinear analysis and in particular those that pertain to recent developments linking these areas to dynamics and rigidity theory in dimension $n \geq 3$. A self mapping (endomorphism) of an n-manifold is *rational* or *uniformly quasiregular* if it preserves some bounded measurable conformal structure. Because of Rickman's version of Montel's theorem there is a close analogy between the dynamics

of rational endomorphisms of closed manifolds and the classical Fatou-Julia theory of iteration of rational mappings of $\hat{\mathbb{C}}$. The theory is particularly interesting on the Riemann n-sphere $\overline{\mathbb{R}}^n$ where many classical results find their analogue, some of which we discuss here. We present the most recent results toward a solution of the Lichnerowicz problem of classifying those manifolds admitting rational endomorphisms. As a by product we discover interesting new rigidity theorems for open self maps of closed n-manifolds whose fundamental group is word hyperbolic.

Nodal Lines of Random Waves

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2010 Mathematics Subject Classification. 30B20, 33C55 and 60G55.

Keywords. Gaussian entire functions, random complex zeroes, random waves, random nodal lines.

In the talk, I will introduce random spherical harmonics and random plane waves, and will describe recent attempts to understand the mysterious and beautiful structure of their nodal lines. The talk is based on a joint work with Fedor Nazarov.

Potential Analysis Meets Geometric Measure Theory

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2010 Mathematics Subject Classification. Primary 28A33; Secondary 31A15.

Keywords. Elliptic measure, Harmonic measure, Ahlfors regular.

A central question in Potential Theory is the extent to which the geometry of a domain influences the boundary regularity of solutions to divergence form elliptic operators. To answer this question one studies the properties of the corresponding elliptic measure. On the other hand one of the central questions in Geometric Measure Theory (GMT) is the extent to which the *regularity* of a measure determines the geometry of its support. The goal of this paper is to present a few instances in which techniques from GMT and Harmonic Analysis come together to produce new results in both of these areas. In particular, the

work described in section 3 makes it clear that for this type of problems in higher dimensions, GMT is the right alternative to complex analysis in dimension 2.

Section 9

Functional Analysis and Applications

Orbit Equivalence and Measured Group Theory

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2000 Mathematics Subject Classification. Primary 37A20; Secondary 46L10.

Keywords. Orbit equivalence, Measured group theory, von Neumann algebras

We give a survey of various recent developments in orbit equivalence and measured group theory. This subject aims at studying infinite countable groups through their measure preserving actions.

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Group Actions on Operator Algebras

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2010 Mathematics Subject Classification. Primary 46L40; Secondary 46L35.

Keywords. Operator algebras, group actions, K-theory

We give a brief account of group actions on operator algebras mainly focusing on classification results. We first recall rather classical results on the classification of discrete amenable group actions on the injective factors, which may serve as potential goals in the case of C^*-algebras for the future. We also mention Galois correspondence type results and quantum group actions for von

Neumann algebras. Then we report on the recent developments of the classification of group actions on C^*-algebras in terms of K-theoretical invariants. We give conjectures on the classification of a class of countable amenable group actions on Kirchberg algebras and strongly self-absorbing C^*-algebras, which involve the classifying spaces of the groups.

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L_1 Embeddings of the Heisenberg Group and Fast Estimation of Graph Isoperimetry

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2010 Mathematics Subject Classification. 46B85, 30L05, 46B80, 51F99.

Keywords. Bi-Lipschitz embeddings, Sparsest Cut Problem, Heisenberg group.

We survey connections between the theory of bi-Lipschitz embeddings and the Sparsest Cut Problem in combinatorial optimization. The story of the Sparsest Cut Problem is a striking example of the deep interplay between analysis, geometry, and probability on the one hand, and computational issues in discrete mathematics on the other. We explain how the key ideas evolved over the past 20 years, emphasizing the interactions with Banach space theory, geometric measure theory, and geometric group theory. As an important illustrative example, we shall examine recently established connections to the the structure of the Heisenberg group, and the incompatibility of its Carnot-Carathéodory geometry with the geometry of the Lebesgue space L_1.

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Non-asymptotic Theory of Random Matrices: Extreme Singular Values

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2010 Mathematics Subject Classification. Primary 60B20; Secondary 46B09

Keywords. Random matrices, singular values, hard edge, Littlewood-Offord problem, small ball probability

The classical random matrix theory is mostly focused on asymptotic spectral properties of random matrices as their dimensions grow to infinity. At the same time many recent applications from convex geometry to functional analysis to information theory operate with random matrices in fixed dimensions. This survey addresses the non-asymptotic theory of extreme singular values of random matrices with independent entries. We focus on recently developed geometric methods for estimating the hard edge of random matrices (the smallest singular value).

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Free probability, Planar algebras, Subfactors and Random Matrices

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2010 Mathematics Subject Classification. Primary: 46L37, 46L54; Secondary 15B52.

Keywords. Free probability, von Neumann algebra, random matrix, subfactor, planar algebra.

To a planar algebra $\mathcal{P}$ in the sense of Jones we associate a natural non-commutative ring, which can be viewed as the ring of non-commutative polynomials in several indeterminates, invariant under a symmetry encoded by $\mathcal{P}$. We show that this ring carries a natural structure of a non-commutative probability space. Non-commutative laws on this space turn out to describe random matrix

ensembles possessing special symmetries. As application, we give a canonical construction of a subfactor and its symmetric enveloping algebra associated to a given planar algebra $\mathcal{P}$. This talk is based on joint work with A. Guionnet and V. Jones.

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Rigidity for von Neumann Algebras and Their Invariants

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2010 Mathematics Subject Classification. Primary 46L36; Secondary 46L40, 28D15, 37A20.

Keywords. Von Neumann algebra, II_1 factor, measure preserving group action, fundamental group of a II_1 factor, outer automorphism group, W^*-superrigidity.

We give a survey of recent classification results for von Neumann algebras $\mathrm{L}^\infty(X) \rtimes \Gamma$ arising from measure preserving group actions on probability spaces. This includes II_1 factors with uncountable fundamental groups and the construction of W^*-superrigid actions where $\mathrm{L}^\infty(X) \rtimes \Gamma$ entirely remembers the initial group action $\Gamma \curvearrowright X$.

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Section 10

Dynamical Systems and Ordinary Differential Equations

Green Bundles and Related Topics

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2010 Mathematics Subject Classification. Primary 37E40, 37J50, 37C40; 70H20; Secondary 70H03 70H05 37D05 37D25

Keywords. Twist maps, Tonelli Hamiltonians, minimizing measures, Aubry-Mather sets, Lyapunov exponents, hyperbolic sets, non uniform hyperbolic measures, C^1-regularity, weak KAM theory, Hamilton-Jacobi

For twist maps of the annulus and Tonelli Hamiltonians, two linear bundles, the Green bundles, are defined along the minimizing orbits.

The link between these Green bundles and different notions as: weak and strong hyperbolicity, estimate of the non-zero Lyapunov exponents, tangent cones to minimizing subsets, is explained.

Various results are deduced from these links: the relationship between the hyperbolicity of the Aubry-Mather sets of the twist maps and the C^1-regularity of their support, the almost everywhere C^1-regularity of the essential invariant curves of the twist maps, the link between the Lyapunov exponents and the angles of the Oseledec bundles of minimizing measures, the fact that C^0-integrability implies C^1-integrability on a dense G_δ-subset.

Arnold's Diffusion: From the *a priori* Unstable to the *a priori* Stable Case

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2010 Mathematics Subject Classification. 37J40, 37J50, 37C29, 37C50, 37J50.

Keywords. Arnold's diffusion, normally hyperbolic cylinder, partially hyperbolic tori, homoclinic intersections, Weak KAM solutions, variational methods, action minimization.

We expose some selected topics concerning the instability of the action variables in *a priori* unstable Hamiltonian systems, and outline a new strategy that may allow to apply these methods to *a priori* stable systems.

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Quadratic Julia Sets with Positive Area

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2010 Mathematics Subject Classification. Primary 37F50; Secondary 37F25.

Keywords. Holomorphic dynamics, Julia sets, small divisors.

We recently proved the existence of quadratic polynomials having a Julia set with positive Lebesgue measure. We present the ideas of the proof and the techniques involved.

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Variational Construction of Diffusion Orbits for Positive Definite Lagrangians

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2010 Mathematics Subject Classification. Primary 37Jxx; Secondary 70Hxx.

Keywords. Tonelli Lagrangian, Action minimizing, Arnold diffusion.

In this lecture, we sketch the variational construction of diffusion orbits in positive definite Lagrangian systems. Diffusion orbits constructed this way connects different Aubry sets, along which the action is locally minimized.

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Generic Dynamics of Geodesic Flows

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2000 Mathematics Subject Classification. Primary 53D25; Secondary 37D40.

Keywords. Geodesic flows, topological entropy, twist map, closed geodesic.

We present some perturbation methods which help to describe the generic dynamical behaviour of geodesic flows.

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Applications of Measure Rigidity of Diagonal Actions

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2010 Mathematics Subject Classification. Primary 37A45; Secondary 37D40, 11J13, 11J04.

Keywords. Invariant measures, entropy, homogeneous spaces, Littlewood's conjecture, diophantine approximation on fractals, distribution of periodic orbits, ideal classes, divisibility in integer Hamiltonian quaternions.

Furstenberg and Margulis conjectured classifications of invariant measures for higher rank actions on homogeneous spaces. We survey the applications of the

partial measure classifications result by Einsiedler, Katok, and Lindenstrauss to number theoretic problems.

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Measure Theory and Geometric Topology in Dynamics

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2010 Mathematics Subject Classification. Primary 37-02, 37Axx, 37Cxx, 37Dxx.

Keywords. Geometric structure, ergodicity, partial hyperbolicity, entropy, Lyapunov exponents.

In this survey we shall present some relations between measure theory and geometric topology in dynamics. One of these relations comes as follows, on one hand from topological information of the system, some structure should be preserved by the dynamics at least in some weak sense, on the other hand, measure theory is soft enough that an invariant geometric structure almost always appears along some carefully chosen invariant measure. As an example, we have the known result that in dimension 2 the system has asymptotic growth of hyperbolic periodic orbits at least equal to the largest exponent of the action in homology.

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Unique Ergodicity for Infinite Measures

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2010 Mathematics Subject Classification. Primary 37A40, Secondary 37A17

Keywords. Unique ergodicity, Infinite ergodic theory, Horocycle flows, Infinite genus

We survey examples of dynamical systems on non–compact spaces which exhibit measure rigidity on the level of infinite invariant measures in one or more

of the following ways: all locally finite ergodic invariant measures can be described; exactly one (up to scaling) admits a generalized law of large numbers; the generic points can be specified. The examples are horocycle flows on hyperbolic surfaces of infinite genus, and certain skew products over irrational rotations and adic transformations. In all cases, the locally finite ergodic invariant measures are Maharam measures.

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Richness of Chaos in the Absolute Newhouse Domain

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2010 Mathematics Subject Classification. Primary 37C20, 37D45; Secondary 37G25, 37J40, 37E20, 37D20, 37D25, 37D30, 37C70.

Keywords. Renormalization, homoclinic tangency, elliptic orbit, hyperbolic attractor, zero Lyapunov exponent, reversible system, Hamiltonian system

We show that universal maps (i.e. such whose iterations approximate every possible dynamics arbitrarily well) form a residual subset in an open set in the space of smooth dynamical systems. The result implies that many dynamical systems emerging in natural applications may, on a very long time scale, have quite unexpected dynamical properties, like coexistence of many non-trivial hyperbolic attractors and repellers and attractors with all zero Lyapunov exponents. Applications to reversible and symplectic maps are also considered.

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Conservative Partially Hyperbolic Dynamics

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2010 Mathematics Subject Classification. Primary 37D30; Secondary 37C40.

Keywords. Partial hyperbolicity, dynamical foliations, Lyapunov exponents, rigidity.

We discuss recent progress in understanding the dynamical properties of partially hyperbolic diffeomorphisms that preserve volume. The main topics addressed are density of stable ergodicity and stable accessibility, center Lyapunov exponents, pathological foliations, rigidity, and the surprising interrelationships between these notions.

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Section 11

Partial Differential Equations

A Hyperbolic Dispersion Estimate, with Applications to the Linear Schrödinger Equation

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2010 Mathematics Subject Classification. Primary 35P20; Secondary 37D99.

Keywords. Quantum chaos, Schrödinger equation, quantum unique ergodicity, hyperbolic dynamical systems, resonances, Strichartz estimates

On a Hilbert space $\mathcal{H}$, consider the product $\hat{P}_n \hat{P}_{n-1} \cdots \hat{P}_1$ of a large number of operators $\hat{P}_j$, with $\|\hat{P}_j\| = 1$. What kind of geometric considerations can serve to prove that the norm $\|\hat{P}_n \hat{P}_{n-1} \cdots \hat{P}_1\|$ decays exponentially fast with n ? In the first part of this note, we will describe a situation in which $\mathcal{H} = L^2(\mathbb{R}^d)$, and the operators $\hat{P}_j$ are Fourier integral operators associated to a sequence of canonical transformations κ_j. We will give conditions, on the sequence of transformations κ_j and on the symbols of the operators $\hat{P}_j$, under which we can prove exponential decay. This technique was introduced to prove results related to the quantum unique ergodicity conjecture. In the second half of this paper, we will survey applications in scattering situations, to prove the existence of a gap below the real axis in the resolvent spectrum, and to get local smoothing estimates with loss, as well as Strichartz estimates.

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Random Data Cauchy Theory for Dispersive Partial Differential Equations

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2010 Mathematics Subject Classification. Primary 35LXX; Secondary 35Q55.

Keywords. Random series, Wave equations, Schrödinger equations

In a series of papers in 1930-32, Paley and Zygmund proved that random series on the torus enjoy better L^p bounds than the bounds predicted by the deterministic approach (and Sobolev embeddings). The subject of random series was later largely studied and developed in the context of harmonic analysis. Curiously, this phenomenon was until recently not exploited in the context of partial differential equations. The purpose of this talk is precisely to present some recent results showing that in some sense, the solutions of dispersive equations such as Schrödinger or wave equations are better behaved when one consider initial data randomly chosen (in some sense) than what would be predicted by the deterministic theory. A large part of the material presented here is a collaboration with N. Tzvetkov.

Study of Multidimensional Systems of Conservation Laws: Problems, Difficulties and Progress

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2010 Mathematics Subject Classification. Primary 35L65; Secondary 35L67; 35L60; 76N15; 35M10.

Keywords. Conservation laws; characteristics; free boundary value problem; shock; transonic flow; mixed type equation.

In the study of multidimensional systems of conservation laws people confront more difficulties than that for one-dimensional systems. The difficulties include characteristic boundary, free boundary associated with unknown nonlinear waves, various nonlinear wave structure, mixed type equations, strong singularities, etc. Most of them come from the complexity of characteristics. We will give a survey on the progress obtained in the study of this topic with the applications in various physical problems, and will also emphasize some crucial points for the further development of this theory in future.

Finite Morse Index and Linearized Stable Solutions on Bounded and Unbounded Domains

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2010 Mathematics Subject Classification. Primary 35J61; Secondary 35J91.

Keywords. Nonlinear elliptic equations, stable solutions, finite Morse index solutions.

We discuss stable and finite Morse index solutions of nonlinear partial differential equations. We discuss problems on all of space, on half spaces and on bounded domains where either the diffusion is small or the solutions are large.

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Almgren's Q-valued Functions Revisited

Camillo De Lellis

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2010 Mathematics Subject Classification. Primary 49Q20; Secondary 35J55, 54E40, 53A10 .

Keywords. Area-minimizing currents , regularity theory , multiple-valued functions, analysis on metric spaces, higher integrability.

In a pioneering work written 30 years ago, Almgren developed a far-reaching regularity theory for area-minimizing currents in codimension higher than 1. Building upon Almgren's work, Chang proved later the optimal regularity statement for 2-dimensional currents. In some recent papers the author, in collaboration with Emanuele Spadaro, has simplified and extended some results of Almgren's theory, most notably the ones concerning Dir-minimizing multiple valued functions and the approximation of area-minimizing currents with small cylindrical excess. In this talk I will give an overview of our contributions and illustrate some possible future directions.

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New Entire Solutions to Some Classical Semilinear Elliptic Problems

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2010 Mathematics Subject Classification. Primary 35J60; Secondary 35B25, 35B33.

Keywords. Allen-Cahn equation, standing waves for NLS, Yamabe equation.

This paper deals with the construction of solutions to autonomous semilinear elliptic equations considered in entire space. In the absence of space dependence or explicit geometries of the ambient space, the point is to unveil internal mechanisms of the equation that trigger the presence of families of solutions with interesting concentration patterns. We discuss the connection between minimal surface theory and entire solutions of the Allen-Cahn equation. In particular, for dimensions 9 or higher, we build an example that provides a negative answer to a celebrated question by De Giorgi for this problem. We will also discuss related results for the (actually more delicate) standing wave problem in nonlinear Schrödinger equations and for sign-changing solutions of the Yamabe equation.

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The Solvability of Differential Equations

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2010 Mathematics Subject Classification. Primary 35A01; Secondary 35S05, 47G30, 58J40.

Keywords. Solvability, pseudodifferential operators, principal type, systems of differential equations, pseudospectrum.

It was a great surprise when Hans Lewy in 1957 presented a non-vanishing complex vector field that is not locally solvable. Actually, the vector field is the tangential Cauchy–Riemann operator on the boundary of a strictly pseudoconvex domain. Hörmander proved in 1960 that almost all linear partial differential equations are not locally solvable. This also has connections with the spectral instability of non-selfadjoint semiclassical operators.

Nirenberg and Treves formulated their well-known conjecture in 1970: that condition (Ψ) is necessary and sufficient for the local solvability of differential

equations of principal type. Principal type essentially means simple characteristics, and condition (Ψ) only involves the sign changes of the imaginary part of the highest order terms along the bicharacteristics of the real part.

The Nirenberg-Treves conjecture was finally proved in 2006. We shall present the background, the main ideas of the proof and some open problems.

Equilibrium Configurations of Epitaxially Strained Elastic Films: Existence, Regularity, and Qualitative Properties of Solutions

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2010 Mathematics Subject Classification. 74G55; 49K10.

Keywords. Epitaxially strained elastic films, shape instabilities, free boundary problems, second order minimality conditions, regularity

We consider a variational model introduced in the physical literature to describe the epitaxial growth of an elastic film over a thick flat substrate when a lattice mismatch between the two materials is present. We prove existence of minimizing configurations, study their regularity properties, and establish several quantitative and qualitative properties of local and global minimizers of the free-energy functional. Among the other results, we determine analytically the critical threshold for the local minimality of the flat configuration, we investigate also its global minimality, and we provide some conditions under which the non occurrence of singularities in non flat global minimizers is guaranteed. One of the main tools is a new second order sufficient condition for local minimality, which provides the first extension of the classical criteria based on the positivity of second variation to the context of functionals with bulk and surface energies.

Weak Solutions of Nonvariational Elliptic Equations

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2010 Mathematics Subject Classification. Primary 35J15, 35D30, 35D40, 35J60 Secondary 17A35, 20G41, 53C38, 60G46 .

Keywords. Fully nonlinear elliptic equations, viscosity solutions, stochastic processes, triality, division algebras, Hessian equations, Isaacs equation, special Lagrangian equation

We discuss basic properties (uniqueness and regularity) of viscosity solutions to fully nonlinear elliptic equations of the form $F(x, D^2u) = 0$, which includes also linear elliptic equations of nondivergent form. In the linear case we consider equations with discontinuous coefficients.

Section 12

Mathematical Physics

Topological Field Theory, Higher Categories, and Their Applications

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2010 Mathematics Subject Classification. Primary 57R56; Secondary 81T45, 18D05, 14D24, 14F05

Keywords. Topological field theory, 2-categories, monoidal categories, derived category of coherent sheaves, geometric Langlands duality

It has been common wisdom among mathematicians that Extended Topological Field Theory in dimensions higher than two is naturally formulated in terms of n-categories with $n > 1$. Recently the physical meaning of these higher categorical structures has been recognized and concrete examples of Extended TFTs have been constructed. Some of these examples, like the Rozansky-Witten model, are of geometric nature, while others are related to representation theory. I outline two applications of higher-dimensional TFTs. One is related to the problem of classifying monoidal deformations of the derived category of coherent sheaves, and the other one is geometric Langlands duality.

Origins of Diffusion

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2010 Mathematics Subject Classification. Primary 37L60; Secondary 82C05.

Keywords. Coupled map lattices, diffusion, hydrodynamic limit, renormalization group

We consider a dynamical system consisting of subsystems indexed by a lattice. Each subsystem has one conserved degree of freedom ("energy") the rest being uniformly hyperbolic. The subsystems are weakly coupled together so that the sum of the subsystem energies remains conserved. We prove that the long time dynamics of the subsystem energies is diffusive.

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Noncommutative Geometry and Arithmetic

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2010 Mathematics Subject Classification. 11M55.

Keywords. Noncommutative tori, real multiplication, Stark numbers, real quadratic fields, spectral triples, noncommutative boundary of modular curves, modular shadows, quantum statistical mechanics.

This is an overview of recent results aimed at developing a geometry of noncommutative tori with real multiplication, with the purpose of providing a parallel, for real quadratic fields, of the classical theory of elliptic curves with complex multiplication for imaginary quadratic fields. This talk concentrates on two main aspects: the relation of Stark numbers to the geometry of noncommutative tori with real multiplication, and the shadows of modular forms on the noncommutative boundary of modular curves, that is, the moduli space of noncommutative tori.

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Universality, Phase Transitions and Extended Scaling Relations

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2010 Mathematics Subject Classification. Primary 82B20, 82B27, 82B28; Secondary 81T16, 81T17

Keywords. Universality, lattice Ising systems, critical phenomena, Renormalization Group, nonperturbative renormalization.

The universality hypothesis in statistical physics says that a number of macroscopic critical properties are largely independent of the microscopic structure, at least inside a universality class of systems. In the case of planar interacting Ising models, like Vertex or Ashkin-Teller models, this hypothesis means that the critical exponents, though model dependent, verify a set of universal extended scaling relations. The proof of several of such relations has been recently achieved; it is valid for generic non solvable models and it is based on the Renormalization Group methods developed in the context of constructive Quantum Field Theory. Extensions to quantum systems and several challenging open problems will be also presented.

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Weak Solutions to the Navier-Stokes Equations with Bounded Scale-invariant Quantities

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2010 Mathematics Subject Classification. Primary 35Q30, Secondary 76D05.

Keywords. Navier-Stokes equations, regularity, weak Leray-Hopf solutions, suitable weak solutions, ancient solutions.

The main assumption of the so-called ε-regularity theory of suitable weak solutions to the Navier-Stokes equations is uniform smallness of certain scale-invariant quantities, which rules out singularities. One of the best results of ε-regularity is the famous Caffarelli-Kohn-Nirenberg theorem. Our goal is to understand what happens if the assumption on smallness of scale-invariant quantities is replaced with their uniform boundedness. The latter makes it possible to use blow-up technique and reduce the local regularity problem to the question of existence or non-existence of "non-trivial" ancient (backward)

solutions to the Navier-Stokes equations. There are at least two potential scenarios: the classical Liouville type problem for mild bounded ancient solutions and backward uniqueness for the Navier-Stokes equations. In this survey, we discuss sufficient conditions implying non-existence of "non-trivial" solutions and the corresponding sufficient conditions ensuring local regularity of original weak solutions.

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Weakly Nonlinear Wave Equations with Random Initial Data

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2010 Mathematics Subject Classification. Primary 82C05; Secondary 35Q55.

Keywords. Kinetic theory of wave equations

We discuss the derivation of the kinetic equation for the weakly nonlinear Schrödinger equation on the lattice $\mathbb{Z}^d$ and state a theorem, which establishes that the equilibrium time covariance is damped because of the nonlinearity. A more general space-time central limit theorem is discussed.

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On the Geometry of Singularities in Quantum Field Theory

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2010 Mathematics Subject Classification. Primary 14E15; Secondary 14E16, 14J28, 17B68, 32S30, 32S45, 81T40, 81T45.

Keywords. Conformal field theory; topological field theory; singularity theory.

This survey investigates the geometry of singularities from the viewpoint of conformal and topological quantum field theory and string theory.

First, some classical results concerning simple surface singularities are collected, paying special attention to the ubiquitous ADE theme. For conformal field theory, recent progress both on axiomatic and on constructive issues is discussed, as well as a well established classification result, which is also related to the ADE theme, but not complete. Special focus concerning constructive results is owed to superconformal field theories associated to $K3$ surfaces and

some of their higher dimensional cousins. Finally, for topological quantum field theories, their role between conformal field theory and singularity theory is reviewed, along with the origin of tt^* geometry, and some of its applications.

Section 13

Probability and Statistics

Random Planar Metrics

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2010 Mathematics Subject Classification. Primary 05C80; Secondary 82B41.

Keywords. First passage Percolation, Quantum gravity, Hyperbolic geometry.

A discussion regarding aspects of several quite different random planar metrics and related topics is presented.

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Growth of Random Surfaces

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2010 Mathematics Subject Classification. Primary 82C41; Secondary 60B10, 60G55, 60K35.

Keywords. Random growth, determinantal point processes, Gaussian free field

We describe a class of exactly solvable random growth models of one and two-dimensional interfaces. The growth is local (distant parts of the interface grow independently), it has a smoothing mechanism (fractal boundaries do not appear), and the speed of growth depends on the local slope of the interface.

The models enjoy a rich algebraic structure that is reflected through closed determinantal formulas for the correlation functions. Large time asymptotic analysis of such formulas reveals asymptotic features of the emerging interface in different scales. Macroscopically, a deterministic limit shape phenomenon

can be observed. Fluctuations around the limit shape range from universal laws of Random Matrix Theory to conformally invariant Gaussian processes in the plane. On the microscopic (lattice) scale, certain universal determinantal random point processes arise.

Patterned Random Matrices and Method of Moments

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2010 Mathematics Subject Classification. Primary 60B20; Secondary 60F05, 62E20, 60G57, 60B10.

Keywords. Moment method, large dimensional random matrix, eigenvalues, empirical and limiting spectral distributions, Wigner, Toeplitz, Hankel, circulant, reverse circulant, symmetric circulant, sample covariance and XX' matrices, band matrix, balanced matrix, linear dependence.

We present a unified approach to limiting spectral distribution (LSD) of patterned matrices via the moment method. We demonstrate relatively short proofs for the LSD of common matrices and provide insight into the nature of different LSD and their interrelations. The method is flexible enough to be applicable to matrices with appropriate dependent entries, banded matrices, and matrices of the form $A_p = \frac{1}{n}XX'$ where X is a $p \times n$ matrix with real entries and $p \to \infty$ with $n = n(p) \to \infty$ and $p/n \to y$ with $0 \leq y < \infty$.

This approach raises interesting questions about the class of patterns for which LSD exists and the nature of the possible limits. In many cases the LSD are not known in any explicit forms and so deriving probabilistic properties of the limit are also interesting issues.

Renormalisation Group Analysis of Weakly Self-avoiding Walk in Dimensions Four and Higher

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2010 Mathematics Subject Classification. Primary 82B41; Secondary 60K35, 82B28.

Keywords. Self-avoiding walk, Edwards model, renormalization group, supersymmetry, quantum field theory

We outline a proof, by a rigorous renormalisation group method, that the critical two-point function for continuous-time weakly self-avoiding walk on $\mathbb{Z}^d$ decays as $|x|^{-(d-2)}$ in the critical dimension $d = 4$, and also for all $d > 4$.

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Quantiles in Finite and Infinite Dimensional Spaces

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2010 Mathematics Subject Classification. Primary 60B11, 62H99; Secondary 46B10.

Keywords. Convexity, dual space, monotone operators, quantile-quantile plot, reflexive spaces, separable spaces, spatial quantile.

There have been several proposals in the literature for quantiles in finite dimensional spaces. We begin by demonstrating that most of those versions of

multivariate quantiles do not have any meaningful and natural extension for data or distributions in infinite dimensional spaces. Then we consider an extension of spatial quantiles in infinite dimensional spaces, and it is shown that this version of quantiles defined in infinite dimensional spaces retains many of the interesting and useful properties of univariate quantiles associated with univariate distributions. In particular, it can be shown that spatial quantiles possess some interesting monotonicity properties in some Banach spaces, and they characterize the probability distributions in some Hilbert spaces. Asymptotic consistency of empirical spatial quantiles for data in Banach spaces also holds under appropriate conditions. A very useful application of spatial quantiles in finite and infinite dimensional spaces is in the construction of quantile-quantile plots for data in such spaces. For data lying in some finite or infinite dimensional space, such plots can be used for assessing how well a specified probability distribution fits the observed data and also for checking whether two different sets of observations follow the same probability distribution or not.

A Key Large Deviation Principle for Interacting Stochastic Systems

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2010 Mathematics Subject Classification. Primary 60F10, 60G50, 60K35; Secondary 82C22, 82D60.

Keywords. Large deviation principle, quenched vs. annealed, interacting stochastic systems, variational formulas, phase transitions, intermediate phases.

In this paper we describe two large deviation principles for the empirical process of words cut out from a random sequence of letters according to a random renewal process: one where the letters are frozen ("quenched") and one where the letters are not frozen ("annealed"). We apply these large deviation principles to five classes of interacting stochastic systems: interacting diffusions, coupled branching processes, and three examples of a polymer chain in a random environment. In particular, we show how these large deviation principles can be used to derive variational formulas for the critical curves that are associated with the phase transitions occurring in these systems, and how these variational formulas can in turn be used to prove the existence of certain intermediate phases.

Time and Chance Happeneth to Them all: Mutation, Selection and Recombination

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2010 Mathematics Subject Classification. Primary 60G57, 92D15; Secondary 37N25, 60G55, 92D10.

Keywords. Measure-valued, dynamical system, population genetics, Poisson random measure, Wasserstein metric, equilibrium

Many multi-cellular organisms exhibit remarkably similar patterns of aging and mortality. Because this phenomenon appears to arise from the complex interaction of many genes, it has been a challenge to explain it quantitatively as a response to natural selection. We survey attempts by the author and his collaborators to build a framework for understanding how mutation, selection and recombination acting on many genes combine to shape the distribution of genotypes in a large population. A genotype drawn at random from the population at a given time is described by a Poisson random measure on the space of loci and its distribution is characterized by the associated intensity measure. The intensity measures evolve according to a continuous-time measure-valued dynamical system. We present general results on the existence and uniqueness of this dynamical system and how it arises as a limit of discrete generation systems. We also discuss existence of equilibria.

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Coevolution in Spatial Habitats

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2010 Mathematics Subject Classification. Primary 60K35; Secondary 82C22.

Keywords. Interacting particle systems, voter model, host-symbiont model, coevolution

Empirical and theoretical studies have implicated habitat coarseness and coevolution as factors in driving the degree of specialization of mutualists and pathogens. We review recent advances in the development of a framework for host-symbiont interactions that considers both local and stochastic interactions in a spatially explicit habitat. These kinds of interactions result in models with

large numbers of parameters due to the large number of potential interactions, making complete analysis difficult. Rigorous analysis of special cases is possible. We also point to the importance of combining experimental and theoretical studies to identify relevant parameter combinations.

Weakly Asymmetric Exclusion and KPZ

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2010 Mathematics Subject Classification. Primary 82C22; Secondary 60H15.

Keywords. Kardar-Parisi-Zhang equation, stochastic Burgers equation, stochastic heat equation, random growth, asymmetric exclusion process, anomalous fluctuations, directed polymers.

We review recent results on the anomalous fluctuation theory of stochastic Burgers, KPZ and the continuum directed polymer in one space dimension, obtained through the weakly asymmetric limit of the simple exclusion process.

Stein's Method, Self-normalized Limit Theory and Applications

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2010 Mathematics Subject Classification. Primary 60F10, 60F05, 60G50; Secondary 60F15, 62E20, 62F03, 62F05, 00B10.

Keywords. Stein method, normal approximation, non-normal approximation, self-normalized sum, Studentized statistics, limit theory, large deviation, moderate deviation, concentration inequality, Berry-Esseen inequality, false discovery rate, simultaneous tests

Stein's method is a powerful tool in estimating accuracy of various probability approximations. It works for both independent and dependent random variables. It works for normal approximation and also for non-normal approximation. The method has been successfully applied to study the absolute error of approximations and the relative error as well. In contrast to the classical limit

theorems, the self-normalized limit theorems require no moment assumptions or much less moment assumptions. This paper is devoted to the latest developments on Stein's method and self-normalized limit theory. Starting with a brief introduction on Stein's method, recent results are summarized on normal approximation for smooth functions and Berry-Esseen type bounds, Cramér type moderate deviations under a general framework of the Stein identity, non-normal approximation via exchangeable pairs, and a randomized exponential concentration inequality. For self-normalized limit theory, the focus will be on a general self-normalized moderate deviation, the self-normalized saddlepoint approximation without any moment assumption, Cramér type moderate deviations for maximum of self-normalized sums and for Studentized U-statistics. Applications to the false discovery rate in simultaneous tests as well as some open questions will also be discussed.

ℓ_1-regularization in High-dimensional Statistical Models

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2010 Mathematics Subject Classification. Primary 62G05; Secondary 62J07.

Keywords. High-dimensional model, ℓ_1-penalty, oracle inequality, restricted eigenvalue, sparsity, variable selection

Least squares with ℓ_1-penalty, also known as the Lasso [1], refers to the minimization problem

$$\hat{\beta} := \arg\min_{\beta \in \mathbb{R}^p} \left\{ \|\mathbf{Y} - \mathbf{X}\beta\|_2^2/n + \lambda \|\beta\|_1 \right\},$$

where $\mathbf{Y} \in \mathbb{R}^n$ is a given n-vector, and $\mathbf{X}$ is a given $(n \times p)$-matrix. Moreover, $\lambda > 0$ is a tuning parameter, larger values inducing more regularization. Of special interest is the high-dimensional case, which is the case where $p \gg n$. The Lasso is a very useful tool for obtaining good predictions $\mathbf{X}\hat{\beta}$ of the regression function, i.e., of mean $\mathbf{f}^0 := \mathbb{E}\mathbf{Y}$ of $\mathbf{Y}$ when $\mathbf{X}$ is given. In literature, this is formalized in terms of an oracle inequality, which says that the Lasso predicts almost as well as the ℓ_0-penalized approximation of $\mathbf{f}^0$. We will discuss the conditions for such a result, and extend it to general loss functions. For the selection of variables however, the Lasso needs very strong conditions on the Gram matrix $\mathbf{X}^T\mathbf{X}/n$. These can be avoided by applying a two-stage procedure. We will show this for the adaptive Lasso. Finally, we discuss a modification that takes into account a group structure in the variables, where both the number of groups as well as the group sizes are large.

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Bayesian Regularization

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2010 Mathematics Subject Classification. Primary 62H30, 62-07; Secondary 65U05, 68T05.

Keywords. Posterior distribution, nonparametric Bayes, Gaussian process prior, regression, classification, density estimation, rate of contraction, adaptation, sparsity.

We consider the recovery of a curve or surface from noisy data by a nonparametric Bayesian method. This entails modelling the surface as a realization of a "prior" stochastic process, and viewing the data as arising by measuring this realization with error. The conditional distribution of the process given the data, given by Bayes' rule and called "posterior", next serves as the basis of all further inference. As a particular example of priors we consider Gaussian processes. A nonparametric Bayesian method can be called successful if the posterior distribution concentrates most of its mass near the surface that produced the data. Unlike in classical "parametric" Bayesian inference the quality of the Bayesian reconstruction turns out to depend on the choice of the prior. For instance, it depends on the fine properties of the sample paths of a Gaussian process prior, with good results obtained only if these match the properties of the true surface. The Bayesian solution to overcome the problem that these fine properties are typically unknown is to put additional priors on hyperparameters. For instance, sample paths of a Gaussian process prior are rescaled by a random amount. This leads to mixture priors, to which Bayes' rule can be applied as before. We show that this leads to minimax precision in several examples: adapting to unknown smoothness or sparsity. We also present abstract results on hierarchical priors.

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Section 14

Combinatorics

Flag Enumeration in Polytopes, Eulerian Partially Ordered Sets and Coxeter Groups

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2010 Mathematics Subject Classification. Primary 06A11; Secondary 05E05, 16T30, 20F55, 52B11.

Keywords. Convex polytope, Eulerian poset, Coxeter group, Kazhdan-Lusztig polynomial, **cd**-index, quasisymmetric function, Hopf algebra

We discuss the enumeration theory for flags in Eulerian partially ordered sets, emphasizing the two main geometric and algebraic examples, face posets of convex polytopes and regular CW-spheres, and Bruhat intervals in Coxeter groups. We review the two algebraic approaches to flag enumeration – one essentially as a quotient of the algebra of noncommutative symmetric functions and the other as a subalgebra of the algebra of quasisymmetric functions – and their relation via duality of Hopf algebras. One result is a direct expression for the Kazhdan-Lusztig polynomial of a Bruhat interval in terms of a new invariant, the complete **cd**-index. Finally, we summarize the theory of combinatorial Hopf algebras, which gives a unifying framework for the quasisymmetric generating functions developed here.

Order and Disorder in Energy Minimization

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2010 Mathematics Subject Classification. Primary 05B40, 52C17; Secondary 11H31.

Keywords. Symmetry, potential energy minimization, sphere packing, E_8, Leech lattice, regular polytopes, universal optimality.

How can we understand the origins of highly symmetrical objects? One way is to characterize them as the solutions of natural optimization problems from discrete geometry or physics. In this paper, we explore how to prove that exceptional objects, such as regular polytopes or the E_8 root system, are optimal solutions to packing and potential energy minimization problems.

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Hurwitz Numbers: On the Edge Between Combinatorics and Geometry

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2010 Mathematics Subject Classification. Primary 05A15; Secondary 14H10, 14H30, 37K10.

Keywords. Hurwitz numbers, permutations, ramified covering, Riemann surface, KP hierarchy, moduli space of curves, Gromov–Witten invariants

Hurwitz numbers were introduced by A. Hurwitz in the end of the nineteenth century. They enumerate ramified coverings of two-dimensional surfaces. They also have many other manifestations: as connection coefficients in symmetric groups, as numbers enumerating certain classes of graphs, as Gromov–Witten invariants of complex curves. Hurwitz numbers belong to a tribe of numerical sequences that penetrate the whole body of mathematics, like multinomial coefficients. They are indexed by partitions, or, more generally, by tuples of partitions, which does not allow one to overview all of them simultaneously. Instead, we usually deal with some of their specific subsequences. The Cayley numbers N^{N-1} enumerating rooted trees on N marked vertices is may be the simplest such instance. The corresponding exponential generating series has

been considered by Euler and he gave it the name of Lambert function. Certain series of Hurwitz numbers can be expressed by nice explicit formulas, and the corresponding generating functions provide solutions to integrable hierarchies of mathematical physics. The paper surveys recent progress in understanding Hurwitz numbers.

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Cluster Algebras and Representation Theory

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2010 Mathematics Subject Classification. Primary 05E10; Secondary 13F60, 16G20, 17B10, 17B37.

Keywords. Cluster algebra, canonical and semicanonical basis, preprojective algebra, quantum affine algebra.

We apply the new theory of cluster algebras of Fomin and Zelevinsky to study some combinatorial problems arising in Lie theory. This is joint work with Geiss and Schröer (§3, 4, 5, 6), and with Hernandez (§8, 9).

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Subgraphs of Random Graphs with Specified Degrees

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2010 Mathematics Subject Classification. Primary 05C80; Secondary 05A16, 60B20

Keywords. Random graphs, vertex degree, subgraph, regular graph

If a graph is chosen uniformly at random from all the graphs with a given degree sequence, what can be said about its subgraphs? The same can be asked of bipartite graphs, equivalently 0-1 matrices. These questions have been studied by many people. In this paper we provide a partial survey of the field, with emphasis on two general techniques: the method of switchings and the multidimensional saddle-point method.

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Sparse Combinatorial Structures: Classification and Applications

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2010 Mathematics Subject Classification. Primary 0502; Secondary 05C75, 05C15, 05C83, 05C85, 03C13, 68Q19.

Keywords. Graphs, hypergraphs, structures, homomorphism, sparsity, model checking, bounded expansion, property testing, separators, complexity, structural combinatorics.

We present results of the recent research on sparse graphs and finite structures in the context of contemporary combinatorics, graph theory, model theory and mathematical logic, complexity of algorithms and probability theory. The topics include: complexity of subgraph- and homomorphism- problems; model checking problems for first order formulas in special classes; property testing in sparse classes of structures. All these problems can be studied under the umbrella of classes of structures which are Nowhere Dense and in the context of Nowhere Dense – Somewhere Dense dichotomy. This dichotomy presents the classification of the general classes of structures which proves to be very robust and stable as it can be defined alternatively by most combinatorial extremal invariants as well as by algorithmic and logical terms. We give examples from logic, geometry and extremal graph theory. Finally we characterize the existence of all restricted dualities in terms of limit objects defined on the homomorphism order of graphs.

Elliptic Analogues of the Macdonald and Koornwinder Polynomials

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2010 Mathematics Subject Classification. Primary 33D52, Secondary 14H52

Keywords. Macdonald polynomials, elliptic curves, special functions

Perhaps the nicest multivariate orthogonal polynomials are the Macdonald and Koornwinder polynomials, respectively 2-parameter deformations of Schur functions and 6-parameter deformations of orthogonal and symplectic characters, satisfying a trio of nice properties known as the Macdonald "conjectures". In recent work, the author has constructed elliptic analogues: a family of multivariate functions on an elliptic curve satisfying analogues of the Macdonald conjectures, and degenerating to Macdonald and Koornwinder polynomials under suitable limits. This article will discuss the two main constructions for these functions, focusing on the more algebraic/combinatorial of the two approaches.

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Percolation on Sequences of Graphs

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2010 Mathematics Subject Classification. Primary 05C80; Secondary 60C05.

Keywords. Inhomogeneous random graphs, phase transition, metrics on graphs

Recently many new random graph models have been introduced, motivated originally by attempts to model disordered large-scale networks in the real world, but now also by the desire to understand mathematically the space of (sequences of) graphs. This article will focus on two topics. Firstly, we discuss the percolation phase transition in these new models, and in general sequences of dense graphs. Secondly, we consider the question 'when are two graphs close?' This is important for deciding whether a graph model fits some real-world example, as well as for exploring what models are possible. Here the situation is well understood for dense graphs, but wide open for sparse graphs.

The material discussed here is from a variety of sources, primarily work of Bollobás, Janson and Riordan and of Borgs, Chayes, Lovász, Sós, Szegedy and

Vesztergombi. The viewpoint taken here is based on recent papers of Bollobás and the author.

Recent Developments in Extremal Combinatorics: Ramsey and Turán Type Problems

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2010 Mathematics Subject Classification. 05C35, 05C65, 05D10, 05D40

Keywords. Extremal combinatorics, Ramsey theory, Turán problems, Probabilistic methods

Extremal combinatorics is one of the central branches of discrete mathematics and has experienced an impressive growth during the last few decades. It deals with the problem of determining or estimating the maximum or minimum possible size of a combinatorial structure which satisfies certain requirements. Often such problems are related to other areas including theoretical computer science, geometry, information theory, harmonic analysis and number theory. In this paper we discuss some recent advances in this subject, focusing on two topics which played an important role in the development of extremal combinatorics: Ramsey and Turán type questions for graphs and hypergraphs.

Section 15

Mathematical Aspects of Computer Science

Quantum Computation and Mathematics

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Keywords. Quantum Computation, quantum algorithms, cryptography, error correcting codes, knot theory, braids, group representations, adiabatic evolution, spectral gaps, random walks, lattices.

Shor's 1994 ground breaking discovery of a polynomial quantum algorithm for factoring launched the field of quantum computation. This vibrant interdisciplinary area relies on the strong belief that quantum computers can be exponentially faster than their classical counterparts. This possibility has profound implications: On technology, on the foundations of the theory of computation, on cryptography, on quantum physics, even on philosophy of science.

Much has happened since 1994. New quantum algorithms and cryptographic protocols were found; quantum error correction was discovered; important connections between quantum complexity and condensed matter physics were drawn. Yet, we are still facing the most important challenges: Can we move to larger scale physical realizations? What other quantum algorithms, protocols, games are possible? What are the exact limits of the quantum computation model? and what are the implications of all this to Physics, and to the understanding of quantum entanglement?

In many of those questions, connections to various areas of Mathematics turn out to be crucial. Number theory and Combinatorics appear naturally; but intimate ties exist also to knot theory and braids; to group representations; to statistical physical models; to random walks and spectral gaps; and to many other seemingly unrelated areas such as lattices and differential geometry. In my talk I will try to explain some of those beautiful ideas, connections, and challenges, assuming only basic mathematical knowledge.

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Smoothed Analysis of Condition Numbers

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2010 Mathematics Subject Classification. 65H20, 65Y20, 68Q25, 90C31

Keywords. Condition number, distance to ill-posedness, analysis of algorithms, smoothed analysis, volume of tubes, convex conic feasibility problem, Renegar's condition number, interior point methods, polynomial equation solving, homotopy methods, polynomial time, Smale's 17th problem

We present some recent results on the probabilistic behaviour of interior point methods for the convex conic feasibility problem and for homotopy methods solving complex polynomial equations. As suggested by Spielman and Teng, the goal is to prove that for all inputs (even ill-posed ones), and all slight random perturbations of that input, it is unlikely that the running time will be large. These results are obtained through a probabilistic analysis of the condition of the corresponding computational problems.

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Privacy Against Many Arbitrary Low-sensitivity Queries

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2010 Mathematics Subject Classification. Primary 68Q99; Secondary 68P99.

Keywords. Privacy, private data analysis, differential privacy, boosting, learning theory

We consider privacy-preserving data analysis, in which a trusted curator, holding an n-row database filled with personal information, is presented with a large set $\mathcal{Q}$ of queries about the database. Each query is a function, mapping the database to a real number. The curator's task is to return relatively accurate responses to all queries, while simultaneously protecting the privacy of the individual database rows.

An active area of research on this topic seeks algorithms ensuring *differential privacy*, a powerful notion of privacy that protects against all possible linkage attacks and composes automtically and obliviously, in a manner whose worst-case behavior is easily understood. Highly accurate differentially private algorithms exist for many types of datamining tasks and analyses, beginning with *counting queries* of the form "How many rows in the database satsify

Property P?" Accuracy must decrease as the number of queries grows. For the special case of counting queries known techniques permit distortion whose dependence on n and $|Q|$ is $\Theta(n^{2/3}\log|Q|)$ [1] or $\Theta(\sqrt{n}log^2|Q|)$ [2]. This paper describes the first solution for large sets $\mathcal{Q}$ of *arbitrary* queries for which the presence or absence of a single datum has small effect on the outcome.

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Bridging Shannon and Hamming: List Error-correction with Optimal Rate

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2010 Mathematics Subject Classification. Primary 11T71; Secondary 94B35.

Keywords. Error-correction algorithms; Explicit constructions; Reed-Solomon codes; Algebraic-geometric codes; Shannon capacity; List decoding; Polynomial reconstruction.

Error-correcting codes tackle the fundamental problem of recovering from errors during data communication and storage. A basic issue in coding theory concerns the modeling of the channel noise. Shannon's theory models the channel as a stochastic process with a known probability law. Hamming suggested a combinatorial approach where the channel causes worst-case errors subject only to a limit on the number of errors. These two approaches share a lot of common tools, however in terms of quantitative results, the classical results for worst-case errors were much weaker.

We survey recent progress on list decoding, highlighting its power and generality as an avenue to construct codes resilient to worst-case errors with information rates similar to what is possible against probabilistic errors. In particular, we discuss recent explicit constructions of list-decodable codes with information-theoretically optimal redundancy that is arbitrarily close to the fraction of symbols that can be corrupted by worst-case errors.

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Inapproximability of NP-complete Problems, Discrete Fourier Analysis, and Geometry

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2010 Mathematics Subject Classification. Primary 68Q17.

Keywords. NP-completeness, Approximation algorithms, Inapproximability, Probabilistically Checkable Proofs, Discrete Fourier analysis.

This article gives a survey of recent results that connect three areas in computer science and mathematics: (1) (Hardness of) computing approximate solutions to NP-complete problems. (2) Fourier analysis of boolean functions on boolean hypercube. (3) Certain problems in geometry, especially related to isoperimetry and embeddings between metric spaces.

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Algorithms, Graph Theory, and Linear Equations in Laplacian Matrices

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2010 Mathematics Subject Classification. Primary 68Q25; Secondary 65F08.

Keywords. Preconditioning, Laplacian Matrices, Spectral Graph Theory, Sparsification.

The Laplacian matrices of graphs are fundamental. In addition to facilitating the application of linear algebra to graph theory, they arise in many practical problems.

In this talk we survey recent progress on the design of provably fast algorithms for solving linear equations in the Laplacian matrices of graphs. These algorithms motivate and rely upon fascinating primitives in graph theory, including low-stretch spanning trees, graph sparsifiers, ultra-sparsifiers, and local graph clustering. These are all connected by a definition of what it means for one graph to approximate another. While this definition is dictated by Numerical Linear Algebra, it proves useful and natural from a graph theoretic perspective.

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The Unified Theory of Pseudorandomness

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2010 Mathematics Subject Classification. Primary 68Q01; Secondary 94B99, 68R01, 68Q87, 68-02.

Keywords. Pseudorandom generators, expander graphs, list decoding, error-correcting codes, samplers, randomness extractors, hardness amplification

Pseudorandomness is the theory of efficiently generating objects that "look random" despite being constructed with little or no randomness. One of the achievements of this research area has been the realization that a number of fundamental and widely studied "pseudorandom" objects are all *almost equivalent* when viewed appropriately. These objects include pseudorandom generators, expander graphs, list-decodable error-correcting codes, averaging samplers, and hardness amplifiers. In this survey, we describe the connections between all of these objects, showing how they can all be cast within a single "list-decoding framework" that brings out both their similarities and differences.

Section 16

Numerical Analysis and Scientific Computing

The Hybridizable Discontinuous Galerkin Methods

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2010 Mathematics Subject Classification. Primary 65N30; Secondary 65M60.

Keywords. Convection, diffusion, incompressible fluid flow, discontinuous Galerkin methods, mixed methods, finite element methods

In this paper, we present and discuss the so-called hybridizable discontinuous Galerkin (HDG) methods. The discontinuous Galerkin (DG) methods were originally devised for numerically solving linear and then nonlinear hyperbolic problems. Their success prompted their extension to the compressible Navier-Stokes equations – and hence to second-order elliptic equations. The clash between the DG methods and decades-old, well-established finite element methods resulted in the introduction of the HDG methods. The HDG methods can be implemented more efficiently and are more accurate than all previously known DG methods; they represent a competitive alternative to the well established finite element methods. Here we show how to devise and implement the HDG methods, argue why they work so well and prove optimal convergence properties in the framework of diffusion and incompressible flow problems. We end by briefly describing extensions to other continuum mechanics and fluid dynamics problems.

Numerical Analysis of Schrödinger Equations in the Highly Oscillatory Regime

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2010 Mathematics Subject Classification. 65M06, 65M12, 65M70, 35Q41, 35Q83

Keywords. Schrödinger equation, Wigner measure, semiclassical asymptotics, discretisation schemes, spectral methods, Bloch decomposition

Linear (and nonlinear) Schrödinger equations in the semiclassical (small dispersion) regime pose a significant challenge to numerical analysis and scientific computing, mainly due to the fact that they propagate high frequency spatial and temporal oscillations. At first we prove using Wigner measure techniques that finite difference discretisations in general require a disproportionate amount of computational resources, since underlying numerical meshes need to be fine enough to resolve all oscillations of the solution accurately, even if only accurate observables are required. This can be mitigated by using a spectral (in space) discretisation, combined with appropriate time splitting. Such discretisations are time-transverse invariant and allow for much coarser meshes than finite difference discretisations.

In many physical applications highly oscillatory periodic potentials occur in Schrödinger equations, still aggrevating the oscillatory solution structure. For such problems we present a numerical method based on the Bloch decomposition of the wave function.

Why Adaptive Finite Element Methods Outperform Classical Ones

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2010 Mathematics Subject Classification. Primary 65N30, 65N50, 65N15; Secondary 41A25.

Keywords. Finite element methods, a posteriori error estimates, adaptivity, contraction, approximation class, nonlinear approximation, convergence rates.

Adaptive finite element methods (AFEM) are a fundamental numerical tool in science and engineering. They are known to outperform classical FEM in practice and deliver optimal convergence rates when the latter cannot. This paper surveys recent progress in the theory of AFEM which explains their success and provides a solid mathematical framework for further developments.

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Wavelet Frames and Image Restorations

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2010 Mathematics Subject Classification. Primary 42C15; 42C40; 94A08 Secondary 42C30; 65T60; 90C90.

Keywords. Tight wavelet frames, Unitary extension principle, Image restorations.

One of the major driven forces in the area of applied and computational harmonic analysis over the last decade or longer is the development of redundant systems that have sparse approximations of various classes of functions. Such redundant systems include *framelet* (tight wavelet frame), *ridgelet, curvelet, shearlet* and so on. This paper mainly focuses on a special class of such redundant systems: tight wavelet frames, especially, those tight wavelet frames generated via a multiresolution analysis. In particular, we will survey the development of the unitary extension principle and its generalizations. A few examples of tight wavelet frame systems generated by the unitary extension principle are given. The unitary extension principle makes constructions of tight wavelet frame systems straightforward and painless which, in turn, makes a wide usage

of the tight wavelet frames possible. Applications of wavelet frame, especially frame based image restorations, are also discussed in details.

Role of Computational Science in Protecting the Environment: Geological Storage of CO_2

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2010 Mathematics Subject Classification. 65N12, 65N15, 65N30, 65N08, 65N22, 65Z06, 76T30, 76V05, 35J15, 35J70, 35K61, 35Q86, 35L02; 86-08.

Keywords. CO_2 sequestration, parallel computation, multiscale and multiphysics coupling, multiphase flow, reactive transport, mixed finite element, discontinuous Galerkin, and a-posteriori error estimation.

Simulation of field-scale CO_2 sequestration (which is defined as the capture, separation and long-term storage of CO_2 for environmental purposes) has gained significant importance in recent times. Here we discuss mathematical and computational formulations for describing reservoir characterization and evaluation of long term CO_2 storage in saline aquifers as well as current computational capabilities and challenges.

Fast Poisson-based Solvers for Linear and Nonlinear PDEs

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2010 Mathematics Subject Classification. Primary 65N55 and 65N22; Secondary 65N30.

Keywords. Finite element, FASP, auxiliary space preconditioing, method of subspace correction, adaptivity, multigrid, domain decomposition, nearly singular systems, near-null space recovery condition, H(grad), H(curl), H(div), saddle-point, non-Newtonian models, MHD.

Over the last few decades, developing efficient iterative methods for solving discretized partial differential equations (PDEs) has been a topic of intensive research. Though these efforts have yielded many mathematically optimal solvers, such as the multigrid method, the unfortunate reality is that multigrid methods have not been used much in practical applications. This marked gap between theory and practice is mainly due to the fragility of traditional multigrid methodology and the complexity of its implementation. This paper aims to develop theories and techniques that will narrow this gap. Specifically, its aim is to develop mathematically optimal solvers that are robust and easy to use for a variety of problems in practice. One central mathematical technique for reaching this goal is a general framework called the *Fast Auxiliary Space Preconditioning* (FASP) method. FASP methodology represents a class of methods that (1) transform a complicated system into a sequence of simpler systems by using auxiliary spaces and (2) produces an efficient and robust preconditioner (to be used with Krylov space methods such as CG and GMRes)

in terms of efficient solvers for these simpler systems. By carefully making use of the special features of each problem, the FASP method can be efficiently applied to a large class of commonly used partial differential equations including equations of Poisson, diffusion-convection-reaction, linear elasticity, Stokes, Brinkman, Navier–Stokes, complex fluids models, and magnetohydrodynamics. This paper will give a summary of results that have been obtained mostly by the author and his collaborators on this topic in recent years.

Section 17

Control Theory and Optimization

Optimal Control under State Constraints

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2010 Mathematics Subject Classification. 49K15, 34A60, 47J07, 49N35, 49N60.

Keywords. Optimal control, state constraints, value function, optimal synthesis, normal maximum principle, smoothness of optimal trajectories, regularity of the adjoint variable.

Optimal control under state constraints has brought new mathematical challenges that have led to new techniques and new theories. We survey some recent results related to issues of regularity of optimal trajectories, optimal controls and the value function, and discuss optimal synthesis and necessary optimality conditions. We also show how abstract inverse mapping theorems of set-valued analysis can be applied to study state constrained control systems.

Submodular Functions: Optimization and Approximation

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2010 Mathematics Subject Classification. Primary 90C27; Secondary 68W25.

Keywords. Submodular functions, discrete optimization, approximation algorithms.

Submodular functions are discrete analogue of convex functions, arising in various fields of applied mathematics including game theory, information theory, and queueing theory. This survey aims at providing an overview on fundamental properties of submodular functions and recent algorithmic developments of their optimization and approximation.

For submodular function minimization, the ellipsoid method had long been the only polynomial algorithm until combinatorial strongly polynomial algorithms appeared a decade ago. On the other hand, for submodular function maximization, which is NP-hard and known to refuse any polynomial algorithms, constant factor approximation algorithms have been developed with applications to combinatorial auction, machine learning, and social networks. In addition, an efficient method has been developed for approximating submoduar functions everywhere, which leads to a generic framework of designing approximation algorithms for combinatorial optimization problems with submodular costs. In some specific cases, however, one can devise better approximation algorithms.

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Recent Advances in Structural Optimization

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2010 Mathematics Subject Classification. Primary 90C25; Secondary 90C06.

Keywords. Convex optimization, structural optimization, complexity estimates, worst-case analysis, polynomial-time methods, interior-point methods, smoothing technique.

In this paper we present the main directions of research in Structural Convex Optimization. In this field, we use additional information on the structure of specific problem instances for accelerating standard Black-Box methods. We

show that the proper use of problem structure can provably accelerate these methods by the order of magnitudes. As examples, we consider polynomial-time interior-point methods, smoothing technique, minimization of composite functions and some other approaches.

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Computational Complexity of Stochastic Programming: Monte Carlo Sampling Approach

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2010 Mathematics Subject Classification. Primary 90C15; Secondary 90C60.

Keywords. Stochastic programming, Monte Carlo sampling, sample average approximation, dynamic programming, asymptotics, computational complexity, stochastic approximation.

For a long time modeling approaches to stochastic programming were dominated by scenario generation methods. Consequently the main computational effort went into development of decomposition type algorithms for solving constructed large scale (linear) optimization problems. A different point of view emerged recently where computational complexity of stochastic programming problems was investigated from the point of view of randomization methods based on Monte Carlo sampling techniques. In that approach the number of scenarios is irrelevant and can be infinite. On the other hand, from that point of view there is a principle difference between computational complexity of two and multistage stochastic programming problems – certain classes of two stage stochastic programming problems can be solved with a reasonable accuracy and reasonable computational effort, while (even linear) multistage stochastic programming problems seem to be computationally intractable in general.

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A Cutting Plane Theory for Mixed Integer Optimization

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2000 Mathematics Subject Classification. Primary, 90C11; Secondary, 90C10.

Keywords. Mixed-integer, cutting plane, lattice point free convex sets

From a practical perspective, mixed integer optimization represents a very powerful modeling paradigm. Its modeling power, however, comes with a price. The presence of both integer and continuous variables results in a significant increase in complexity over the pure integer case with respect to geometric, algebraic, combinatorial and algorithmic properties. Specifically, the theory of cutting planes for mixed integer linear optimization is not yet at a similar level of development as in the pure integer case. The goal of this paper is to discuss four research directions that are expected to contribute to the development of this field of optimization. In particular, we examine a new geometric approach based on lattice point free polyhedra and use it for developing a cutting plane theory for mixed integer sets. We expect that these novel developments will shed some light on the additional complexity that goes along with mixing discrete and continuous variables.

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A Unified Controllability/Observability Theory for Some Stochastic and Deterministic Partial Differential Equations

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2010 Mathematics Subject Classification. Primary 93B05; Secondary 35Q93, 93B07.

Keywords. Controllability, observability, parabolic equations, hyperbolic equations, weighted identity.

The purpose of this paper is to present a universal approach to the study of controllability/observability problems for infinite dimensional systems governed by some stochastic/deterministic partial differential equations. The crucial analytic tool is a class of fundamental weighted identities for stochastic/deterministic partial differential operators, via which one can derive the

desired global Carleman estimates. This method can also give a unified treatment of the stabilization, global unique continuation, and inverse problems for some stochastic/deterministic partial differential equations.

Section 18

Mathematics in Science and Technology

Deterministic and Stochastic Aspects of Single-crossover Recombination

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2010 Mathematics Subject Classification. Primary 92D10, 34L30; Secondary 37N25, 06A07, 60J25.

Keywords. Population genetics, recombination dynamics, Möbius linearisation and diagonalisation, correlation functions, Moran model.

This contribution is concerned with mathematical models for the dynamics of the genetic composition of populations evolving under recombination. Recombination is the genetic mechanism by which two parent individuals create the mixed type of their offspring during sexual reproduction. The corresponding models are large, nonlinear dynamical systems (for the deterministic treatment that applies in the infinite-population limit), or interacting particle systems (for the stochastic treatment required for finite populations). We review recent progress on these difficult problems. In particular, we present a closed solution of the deterministic continuous-time system, for the important special case of single crossovers; we extract an underlying linearity; we analyse how this carries over to the corresponding stochastic setting; and we provide a solution of the analogous deterministic discrete-time dynamics, in terms of its generalised eigenvalues and a simple recursion for the corresponding coefficients.

BSDE and Risk Measures

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2010 Mathematics Subject Classification. 91G80

Keywords. BSDE, Risk Measures, Time Consistency, Quasi-linear PDE

The study of dynamic coherent risk measures and risk adjusted values is intimately related to the study of Backward Stochastic Differential Equations. We will present some of these relations and will also present some links with quasi-linear PDE.

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Novel Concepts for Nonsmooth Optimization and their Impact on Science and Technology

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2010 Mathematics Subject Classification. 35Q93, 46N10, 49K20, 65K10.

Keywords. Non-smooth optimization, semi-smooth Newton methods, optimal control, complementarity problems, ill-posed problems.

A multitude of important problems can be cast as nonsmooth variational problems in function spaces, and hence in an infinite-dimensional, setting. Traditionally numerical approaches to such problems are based on first order methods. Only more recently Newton-type methods are systematically investigated and their numerical efficiency is explored. The notion of Newton differentiability combined with path following is of central importance. It will be demonstrated how these techniques are applicable to problems in mathematical imaging, and variational inequalities. Special attention is paid to optimal control with partial differential equations as constraints.

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Modelling Aspects of Tumour Metabolism

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2010 Mathematics Subject Classification. 92C50

Keywords. Carcinogenesis – Glycolytic phenotype – Mathematical modelling

We use a range of mathematical modelling techniques to explore the acid-mediated tumour invasion hypothesis. The models make a number of predictions which are experimentally verified. The therapeutic implications, namely either buffering acid or manipulating the phenotypic selection process, are described.

On Markov State Models for Metastable Processes

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2010 Mathematics Subject Classification. Primary 65C50; Secondary 60J35.

Keywords. Markov process, metastability, transition path theory, milestoning, eigenvalue problem, transfer operator, propagation error, Markov state models, committor, Galerkin approximation

We consider Markov processes on large state spaces and want to find low-dimensional structure-preserving approximations of the process in the sense that the longest timescales of the dynamics of the original process are reproduced well. Recent years have seen the advance of so-called Markov state models (MSM) for processes on very large state spaces exhibiting metastable dynamics. It has been demonstrated that MSMs are especially useful for modelling the interesting slow dynamics of biomolecules (cf. Noe et al, PNAS(106) 2009) and materials. From the mathematical perspective, MSMs result from Galerkin projection of the transfer operator underlying the original process onto some low-dimensional subspace which leads to an approximation of the dominant eigenvalues of the transfer operators and thus of the longest timescales of the original dynamics. Until now, most articles on MSMs have been based on full subdivisions of state space, i.e., Galerkin projections onto subspaces spanned by indicator functions. We show how to generalize MSMs to alternative low-dimensional subspaces with superior approximation properties, and how to analyse the approximation quality (dominant eigenvalues, propagation of functions) of the resulting MSMs. To this end, we give an overview of the construction of MSMs, the associated stochastics and functional-analysis background, and its algorithmic consequences. Furthermore, we illustrate the mathematical construction with numerical examples.

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Second Order Backward SDEs, Fully Nonlinear PDEs, and Applications in Finance

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2010 Mathematics Subject Classification. Primary 60H10; Secondary 60H30.

Keywords. Backward stochastic differential equations, stochastic analysis, non-dominated mutually singular measures, viscosity solutions of second order PDEs.

The martingale representation theorem in a Brownian filtration represents any square integrable r.v. ξ as a stochastic integral with respect to the Brownian motion. This is the simplest Backward SDE with nul generator and final data ξ, which can be seen as the non-Markov counterpart of the Cauchy problem in second order parabolic PDEs. Similarly, the notion of Second order BSDEs is the non-Markov counterpart of the fully-nonlinear Cauchy problem, and is motivated by applications in finance and probabilistic numerical methods for PDEs.

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Data Modeling: Visual Psychology Approach and $L_{1/2}$ Regularization Theory

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2010 Mathematics Subject Classification. 6IH30, 68T10, 62-07, 94A12.

Keywords. Data modeling, sparse signal recovery, visual psychology approach, L_1 regularization, $L_{1/2}$ regularization.

Data modeling provides data analysis with models and methodologies. Its fundamental tasks are to find structures, rules and tendencies from a data set. The data modeling problems can be treated as cognition problems. Therefore, simulating cognition mechanism and principles can provide new subtle paradigm and can solve some basic problems in data modeling.

In pattern recognition, human eyes possess a singular aptitude to group objects and find important structure in an efficient way. I propose to solve a clustering and classification problem through capturing the structure (from micro to macro) of a data set from a dynamic process observed in adequate scale spaces. Three types of scale spaces are introduced, respectively based on the

neural coding, the blurring effect of lateral retinal interconnections, the hierarchical feature extraction mechanism dominated by receptive field functions and the feature integration principle characterized by Gestalt law in psychology.

The use of L_1 regularization has now been widespread for latent variable analysis (particularly for sparsity problems). I suggest an alternative of such commonly used methodology by developing a new, more powerful approach – $L_{1/2}$ regularization theory. Some related open questions are raised in the end of the talk.

Mathematicalising Behavioural Finance

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2010 Mathematics Subject Classification. Primary 91G10; Secondary 91C99.

Keywords. Behavioural finance, cumulative prospect theory, Yaari's criterion, SP/A theory, portfolio selection, continuous time, reference point, S-shaped function, probability distortion, Choquet integral, quantile formulation

This article presents an overview of the recent development on mathematical treatment of behavioural finance, primarily in the setting of continuous-time portfolio choice under the cumulative prospect theory. Financial motivations and mathematical challenges of the problem are highlighted. It is demonstrated that the solutions to the problem have in turn led to new financial and mathematical problems and machineries.

Section 19

Mathematics Education and Popularization of Mathematics

Professional Knowledge Matters in Mathematics Teaching

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2010 Mathematics Subject Classification. 97C60, 97C70 and 97D99

Keywords. Mathematics for teaching; Mathematics teacher education; mathematical reasoning; mathematical objects and processes.

In this paper, I argue that mathematics teachers' professional knowledge matters, and so requires specific attention in mathematics teacher education. Two examples from studies of mathematics classrooms in South Africa are described, and used to illustrate what mathematics teachers use, or need to use, and how they use mathematics in their practice: in other words, the substance of their mathematical work. Similarities and differences across these examples, in turn, illuminate mathematics teachers' professional knowledge, enabling a return to, and critical reflection on, mathematics teacher education.

Section 20

History of Mathematics

History of Convexity and Mathematical Programming: Connections and Relationships in Two Episodes of Research in Pure and Applied Mathematics of the 20th Century

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2010 Mathematics Subject Classification. Primary 01A60; Secondary 52-03, 90-03.

Keywords. History of 20th century mathematics, the theory of convexity, postive definite quadratic forms, convex sets, the lattice point theorem, mathematical programming, linear programming, nonlinear programming, the Kuhn-Tucker theorem, Minkowski, Fenchel, Tucker, Kuhn, the military-university complex, the Second World War.

In this paper, the gradual introduction of the concept of a general convex body in Minkowski's work and the development of mathematical programming, are presented. Both episodes are exemplary for mathematics of the 20th century, in the sense that the former represents a trend towards a growing abstraction and autonomy in pure mathematics, whereas the latter is an example of the many new disciplines in applied mathematics that emerged as a consequence of efforts to develop mathematics into a useful tool in a wider range of subjects than previously. It will be discussed, how and why these two new areas emerged and developed through different kinds of connections and relations; and how they at some point became connected, and fed and inspired one another. The examples suggest that pure and applied mathematics are more intertwined than the division in 'pure' and 'applied' signals.

Rewriting Points

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2010 Mathematics Subject Classification. Primary 01A55, 01A60; Secondary 03-03, 11-03, 12-02, 14-03.

Keywords. History of mathematics, abstract Riemann surface, Intuitionism, Foundations of Algebraic Geometry

A few episodes from the history of mathematics of the 19^{th} and 20^{th} century are presented in a loose sequence in order to illustrate problems and approaches of the history of mathematics. Most of the examples discussed have to do with some version of the mathematical notion of point. The Dedekind-Weber theory of points on a Riemann surface is discussed as well as Hermann Weyl's successive constructions of the continuum, and the rewriting of Algebraic Geometry between 1925 and 1950. A recurring theme is the rewriting of traditional mathematics, where 'rewriting' is used in a colloquial, non-terminological sense the meaning of which is illustrated by examples.

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Islamic Astronomical Handbooks and their Transmission to India and China

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2010 Mathematics Subject Classification. Primary 01A30; Secondary 01A25, 01A32.

Keywords. Islam, India, China, astronomy, mathematical astronomy, transmission

Islamic mathematical astronomy was built on the foundations of the *Almagest*, the main astronomical work of the Greek scholar Ptolemy (Alexandria, ca. 140 C.E.). Starting in the early ninth century, Muslim scholars improved the parameter values underlying Ptolemy's planetary models by means of systematic observations, increased the efficiency and accuracy of the calculations of trigonometric functions and spherical astronomical quantities, and compiled at least 250 different astronomical handbooks with mathematical tables. Using such works the practising astronomer or astrologer could conveniently perform all necessary calculations of planetary positions, lunar visibility, solar and lunar eclipses, etc. In this lecture some of the main characteristics of Islamic

astronomical handbooks will be discussed. Furthermore, two case studies will be presented of the transmission of Islamic astronomical handbooks to other cultural areas, namely India and China, and it will be shown how modern mathematics may be helpful in analysing such cases of transmission.

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Panel Discussions

Ethnomathematics, Language & Socio-cultural Issues

Symbolic Power and Mathematics

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2000 Mathematics Subject Classification. 03A05

Symbolic power will be discussed with reference to mathematics. Two distinctions are pointed out as crucial for exercising such power: one between appearance and reality, and one between sense and reference. These distinctions include a nomination of what to consider primary and what to consider secondary. They establish the grammatical format of a mechanical and formal world view. Through an imposition of such world views symbolic power is exercised through mathematics. This power is further investigated through different dimensions of mathematics in action: (1) Technological imagination which refers to the possibility of constructing technical possibilities. (2) Hypothetical reasoning which addresses consequences of not-yet realised technological initiatives. (3) Legitimation or justification which refers to possible validations of technological actions. (4) Realisation which signifies that mathematics itself comes to constitute part of reality. And (5) evaporation of responsibility which might occur when issues about responsibility are eliminated from the discourse about technological initiatives and their implications. Finally, it is emphasised that whatever form symbolic power may take it cannot be addressed along a single good-evil axis.

Modelling and Ethno-Mathematics for Mathematics Education

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2000 Mathematics Subject Classification. 97C60

We seek in this article to approach the ideas of modeling and ethnomathematics, and to provide considerations about the use of these methods in formal education. To illustrate, we present an experience using mathematical modeling and ethno-mathematics for teachers through courses of continuing education. Mathematical modeling is geared towards the design of a mathematical model for the solution of a problem and as a support for other applications and theories. Ethno-mathematics seeks to know, understand and explain how a person or a group from a social culture elaborates a mathematical model, or how they make use of this model in their practical activities. Research shows that modeling and ethno-mathematics integrated for teaching can produce in teacher as in student new perceptions and interpretations of mathematics. And, more than knowledge of mathematical rules, the student learns cultural values and some general principles as individuals responsible for the reality that surrounds us.

Relations Between the Discipline & School Mathematics

A Continuous Path from School Calculus to University Analysis

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2000 Mathematics Subject Classification. 97-XX

It is common to describe university-level mathematics as virtually a different subject from school-level mathematics, even when their subject matter overlaps. The difference is particularly keenly felt in analysis, where there is a big contrast between a typical first course in calculus and the more rigorous epsilon-delta approach that one encounters at university.

I shall argue that this appearance is misleading, and that the epsilon-delta definitions and proofs are more intuitive than they might at first appear. I shall focus in particular on the treatment of the real number system, the definition of continuity, and the proof of the intermediate value theorem.

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Relations Between the Discipline & School Mathematics

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More than half of the students in the Latin American and the Caribbean region are below Pisa level 1 which means that the majority of the students in our region cannot identify information and carry out routine procedures according to direct instructions in explicit situations.

There have been some good experiences in each country to reverse the depicted situation but it is not enough and this is not happening in all countries.

I will talk about these experiences. In all of them professional mathematicians need to help teachers to have the necessary knowledge, and become more effective instructors that can raise the standard of every student.

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Excavating School Mathematics

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2000 Mathematics Subject Classification. 97D30

The school curriculum can be viewed as an archeological record of the history of mathematics and of previous efforts at reforming school instruction, eroded in places by the winds of policy and covered in others by the sands of neglect. Some topics in the school curriculum are like encrusted relics, difficult to make sense of because they are no longer connected to a larger structure that once gave them meaning. Other areas may be more coherent, but no longer serve the originally intended purpose. In order to make decisions about where and whether to implement changes in the curriculum, some reconctruction and analysis is needed, both tasks in which research mathematicians can offer expertise. In this paper we give some examples of a type of mathematical excavation that we believe could contribute to building a coherent architecture of school mathematics.

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Knowledge in Processes of Teaching and Learning at School - Its Specific Nature and Epistemological Status

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2000 Mathematics Subject Classification. 97C60, 97D20

Mathematical knowledge as object of teaching-learning processes undergoes changes in its epistemological status. In primary and secondary schools:

- mathematics teaching does not aim at training mathematical experts but contributes to the students' general education to become politically mature citizens (expert knowledge vs. knowledge in everyday settings)
- mathematical knowledge cannot be conveyed as a ready made product but it develops in a genetic manner by students' own activities (Mathematics as Product vs. Process, Hans Freudenthal)

- mathematical concepts (e.g. number, probability) cannot be introduced by formal definitions, consistent axioms or defining equations, but receive their meaning by referring to (different embodiments of) structures, patterns and relationships

The epistemological particularities of mathematical knowledge in teaching-learning processes will be elaborated by using elementary examples of basic mathematical concepts.

Communicating Mathematics to Society at Large

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What image does "the public" have of mathematics? Why and how should research mathematicians be involved in communicating mathematics and mathematical research to the public? Which "general audience" can we expect to reach (media, kids, general public, learned public, etc.)? How do we reach them? What can we expect them to learn, to understand?

The panelists will briefly present and discuss their experiences in communicating with the public, both from the perspectives of mathematicians in academia, and from the perspectives of science journalists. They will highlight

the importance of the scientific message, the vocabulary of mathematics, the creative use of different formats to reach diverse audiences, and the wide range of mathematics the public can be stimulated to take an interest in.

The subsequent discussion will enlarge on these themes and, with comments from the audience, provide a basis for suggesting strategies for communicating effectively with society at large. The panel will conclude by discussing options and opportunities for international collaboration.

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