Fields Medal

Ngô Bao Châu

Citation: "For his proof of the Fundamental Lemma in the theory of automorphic forms through the introduction of new algebro-geometric methods."

Ngô Bao Châu removed one of the great impediments to a grand, decades-long program to uncover hidden connections between seemingly disparate areas of mathematics. In doing so, he provided a solid foundation for a large body of theory and developed techniques that are likely to unleash a flood of new results.

The path to Ngô's achievement began in 1967, when the mathematician Robert Langlands had a mind-bogglingly bold vision of a sort of mathematical wormhole connecting fields that seemed to be light-years apart. His proposal was so ambitious and unlikely that when he first wrote of it to the great number theorist André Weil, he began with this sheepish note: "If you are willing to read [my letter] as pure speculation I would appreciate that; if not — I am sure you have a waste basket handy." Langlands then laid out a series of dazzling conjectures that have proven to be a roadmap for a large area of research ever since.

The great majority of those conjectures remain unproven and are expected to occupy mathematicians for generations to come. Even so, the progress on the program so far has been a powerful engine for new mathematical results, including Andrew Wiles' proof of Fermat's Last Theorem and Richard Taylor's proof of the Sato-Tate conjecture. The full realization of Langlands' program would unify many of the fields of modern mathematics, including number theory, group theory, representation theory, and algebraic geometry.

Langlands' vision was of a bridge across a division in mathematics dating all the way back to Euclid's time, that between magnitude and multitude. Magnitudes are the mathematical form of butter, a continuous smear of stuff that can be divided up into pieces as small as you please. Lines and curves, planes, the space we live in, and even higher-dimensional spaces are all magnitudes, and they are commonly studied with the tools of geometry and analysis. Multitudes, on the other hand, are like beans, discrete objects that can be put in piles but can't be split without losing their essence. The whole numbers are the canonical example of multitudes, and they are studied with the tools of number theory. Langlands predicted that certain numbers that arise in analysis – specifically, the eigenvalues of certain operators on differential forms on particular Riemannian manifolds, called automorphic forms – were actually a code that, if unraveled, would classify fundamental objects in the arithmetic world.

One of the tools developed from the Langlands program is the "Arthur-Selberg trace formula," an equation that shows precisely how geometric information can calculate arithmetic information. That is valuable in itself, and furthermore, is a building block in proving Langlands' Principle of Functoriality, one of the great pillars of his program. But

Langlands ran across an annoying stumbling block in trying to use the trace formula. He kept encountering complicated finite sums that clearly seemed to be equal, but he couldn't quite figure out how to show it. It seemed like a straightforward problem, one that could be solved with a bit of combinatorial fiddling, so he called it a "lemma" – the term for a minor but useful result – and assigned it to a graduate student.

When the graduate student couldn't prove it, he tried another. Then he worked on it himself. Then he consulted with other mathematicians. At the same time as everyone continued to fail to prove it, the critical need for the result became increasingly clear. So the problem came to have a slightly grander title: the "Fundamental Lemma."

After three decades of work, only a few special cases had yielded to proof. The lack of a proof was such a roadblock to progress that many mathematicians had begun simply *assuming* it was true and developing results that depended upon it, creating a huge body of theory that would come crashing down if it turned out to be false.

Ngô Bao Châu was the one to finally break the problem open. The complicated identities in the Fundamental Lemma, he realized, could be seen as arising naturally out of sophisticated mathematical objects known as Hitchen fibrations. His approach was entirely novel and unexpected: Hitchen fibrations are purely geometric objects that are close to mathematical physics, nearly the last thing anyone expected to be relevant to this problem in the purest of pure math.

But it was instantly clear that he'd made a profound connection. His approach turned the annoying, fiddly complexity of the Fundamental Lemma into a simple, natural statement about Hitchen fibrations. Even before he'd managed to complete the proof, he'd achieved something even more impressive: He'd created genuine understanding.

Furthermore, by putting the problem in this much bigger framework, Ngô gave himself powerful new tools to assault it with. In 2004, he proved some important and difficult special cases working with his former thesis advisor Gérard Laumon, and in 2008, using his new methods, he cracked the problem in its full generality.

Ngô's methods are so novel that mathematicians expect them to break open a number of other problems as well. A prime target is another piece of Langlands' program, his "theory of endoscopy."

His techniques might even point the way toward a proof of the full Principle of Functoriality, which would be close to a full realization of Langlands' original vision. Langlands himself, who is now over 70 years old and still hard at work, has developed a highly speculative but enticing approach to the problem. It is still far from clear that these ideas will lead to a proof, but if they do, they will have to rely on the kinds of geometric ideas that Ngo has introduced.

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