1. Logic and Foundations (3–5 lectures)

Model theory. Set theory. Recursion theory. Proof theory. Applications.

Connections with sections 2, 3, 13, 14.

2. Algebra (3–6 lectures)

Groups (finite, infinite, algebraic) and their representations. Rings (both commutative and noncommutative), fields and modules. General algebraic structures, algebraic K-theory, category theory. Computational aspects of algebra and applications.

Connections with sections 1, 3, 4, 5, 6, 7, 13, 14.

3. Number Theory (9–12 lectures)

Algebraic number theory. Galois groups of local and global fields and their representations. Arithmetic of algebraic varieties and Diophantine equations. Geometry of numbers, Diophantine approximation, and transcendental numbers. Modular and automorphic forms, modular curves, and Shimura varieties. Langlands program. p-adic analysis. Zeta and L-functions. Analytic number theory. Probablistic method in number theory. Number theory and physics. Computational number theory and applications.

Connections with sections 1, 2, 4, 7, 11, 12, 13, 14.

4. Algebraic and Complex Geometry (9–12 lectures)

Algebraic varieties, their cycles, cohomologies, and motives. Schemes and stacks. Geometric aspects of commutative algebra. Arithmetic geometry. Rational points. Low-dimensional and special varieties. Singularities. Birational geometry and minimal models. Moduli spaces and enumerative geometry. Transcendental methods and topology of algebraic varieties. Complex differential geometry, Kähler manifolds and Hodge theory. Relations with mathematical physics and representation theory. Computational methods. Real algebraic and analytic sets. Rigid and p-adic analytic spaces. Tropical geometry. Derived categories and non-commutative geometry.

Connections with sections 2, 3, 5, 6, 7, 8, 11, 13, 14.

5. Geometry (8–12 lectures)

Local and global differential geometry. Geometric PDE and geometric flows. Geometric structures on manifolds. Riemannian and metric geometry. Kähler geometry. Geometric aspects of group theory. Symplectic and contact manifolds. Convex geometry. Discrete geometry.

Connections with sections 2, 4, 6, 7, 8, 9, 10, 11, 12, 13.

6. Topology (8–11 lectures)

Algebraic, differential and geometric topology. Stable and unstable homotopy theory. Operands and higher categories. K-theory. Motivic homotopy theory. Floer and gauge theories. Low-dimensional manifolds including knot theory. Aspects of Teichmüller theory. Symplectic and contact manifolds. Topological quantum field theories.

Connections with sections 2, 4, 5, 7, 8, 9, 11.

7. Lie Theory and Generalizations (7–10 lectures)

Algebraic and arithmetic groups. Structure, geometry, and representations of Lie groups and Lie algebras. Related geometric and algebraic objects, e.g. symmetric spaces, buildings, vertex operator algebras, quantum groups. Non-commutative harmonic analysis. Geometric methods in representation theory. Discrete subgroups of Lie groups. Lie groups and dynamics, including applications to number theory.

Connections with sections 2, 3, 4, 5, 6, 8, 9, 11, 12, 13.

8. Analysis and Operator Algebras (10–14 lectures)

Classical analysis. Real and Complex analysis in one and several variables, potential theory, quasiconformal mappings. Harmonic analysis. Linear and non-linear functional analysis, operator algebras, Banach algebras, Banach spaces. Non-commutative geometry, spectra of random matrices. Asymptotic geometric analysis. Metric geometry and applications. Geometric measure theory.

Connections with sections 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16.

9. Dynamical Systems and Ordinary Differential Equations (9–12 lectures)

Topological and symbolic dynamics. Geometric and qualitative theory of ODE and smooth dynamical systems, bifurcations and singularities. Hamiltonian systems and dynamical systems of geometric origin. One-dimensional and holomorphic dynamics. Strange attractors and chaotic dynamics. Multidimensional actions and rigidity in dynamics. Ergodic theory including applications to combinatorics and combinatorial number theory. Infinite dimensional dynamical systems and PDE.

Connections with sections 5, 7, 8, 10, 11, 12, 13, 15, 16.

10. Partial Differential Equations (9–12 lectures)

Solvability, regularity, stability and other qualitative properties of linear and non-linear equations and systems. Asymptotics. Spectral theory, scattering, inverse problems. Variational methods and calculus of variations. Optimal transportation. Homogenization and multiscale problems. Relations to continuous media and control. Modeling through PDEs. SPDEs.

Connections with sections 5, 8, 9, 11, 12, 15, 16, 17.

11. Mathematical Physics (9–12 lectures)

Quantum mechanics. Quantum field theory including gauge theories. General relativity. Statistical mechanics and random media. Integrable systems. Supersymmetric theories. String theory. Fluid dynamics.

Connections with sections 4, 5, 6, 7, 8, 9, 10, 12.

12. Probability and Statistics (10–13 lectures)

Stochastic processes, Interacting particle systems, Random media, Random matrices, conformally invariant models, Stochastic networks, Stochastic geometry, Statistical inference, High-dimensional data analysis, Spatial methods.

Connections with sections 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17.

13. Combinatorics (8–11 lectures)

Combinatorial structures. Enumeration: exact and asymptotic. Graph theory. Probabilistic and extremal combinatorics. Designs and finite geometries. Relations with linear algebra, representation theory and commutative algebra. Topological and analytical techniques in combinatorics. Combinatorial geometry. Combinatorial number theory. Additive combinatorics. Polyhedral combinatorics and combinatorial optimization.

Connections with sections 1, 2, 3, 4, 7, 9, 12, 14.

14. Mathematical Aspects of Computer Science (6-8 lectures)

Complexity theory and design and analysis of algorithms. Formal languages. Computational learning. Algorithmic game theory. Cryptography. Coding theory. Semantics and verification of programs. Symbolic computation. Quantum computing. Computational geometry, computer vision.

Connections with sections 1, 2, 3, 4, 12, 13, 15.

15. Numerical Analysis and Scientific Computing (5-7 lectures)

Design of numerical algorithms and analysis of their accuracy, stability, convergence and complexity. Approximation theory. Applied and computational aspects of harmonic analysis. Numerical solution of algebraic, functional, stochastic, differential, and integral equations.

Connections with sections 8, 9, 10, 12, 14, 16, 17.

16. Control Theory and Optimization (5–7 lectures)

Minimization problems. Controllability, observability, stability. Robotics. Stochastic systems and control. Optimal control. Optimal design, shape design. Linear, non-linear, integer, and stochastic programming. Applications.

Connections with sections 9, 10, 12, 15, 17.

17. Mathematics in Science and Technology (7–10 lectures)

Mathematics and its applications to physical sciences, engineering sciences, life sciences, social and economic sciences, and technology. Bioinformatics. Mathematics in interdisciplinary research. The interplay of mathematical modeling, mathematical analysis, and scientific computation, and its impact on the understanding of scientific phenomena and on the solution of real life problems.

Connections with sections 9, 10, 11, 12, 14, 15, 16.

18. Mathematics Education and Popularization of Mathematics (2 lectures plus 3 panel discussions)

All aspects of mathematics education, from elementary school to higher education. Mathematical literacy and popularization of mathematics.

19. History of Mathematics (3 lectures)

Historical studies of all of the mathematical sciences in all periods and cultural settings.