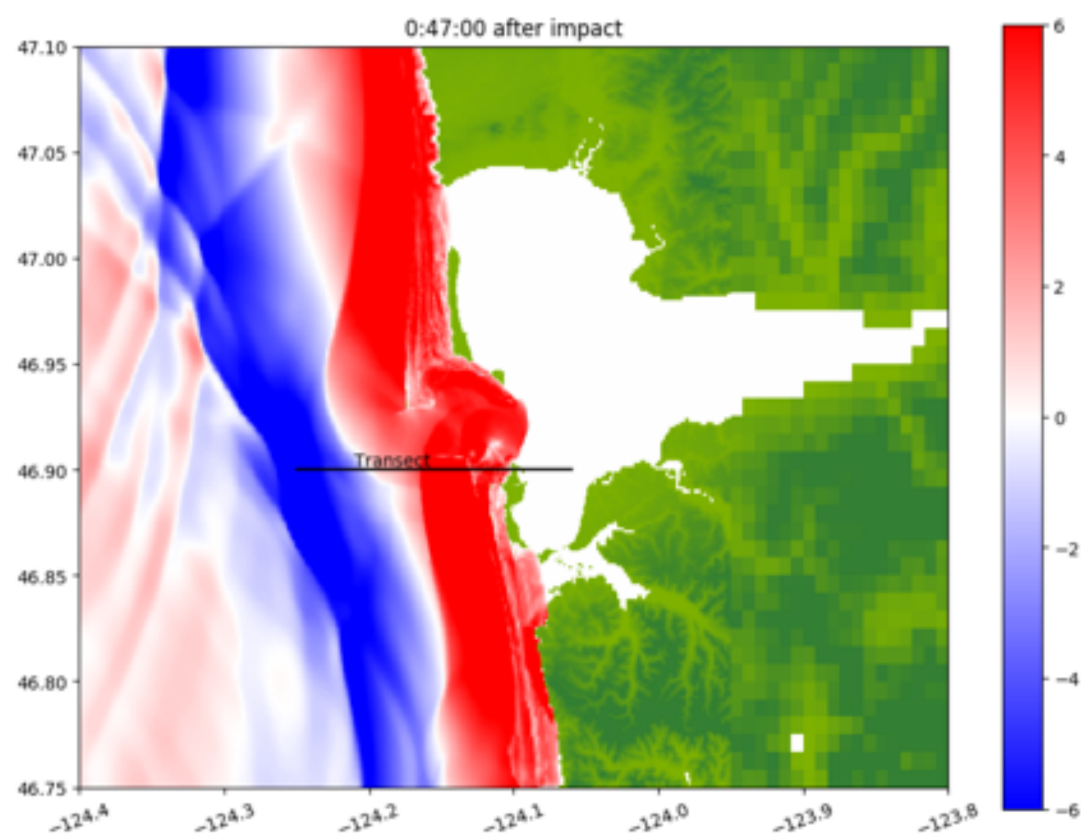
Towards Adaptive Simulations of Dispersive Tsunami Propagation



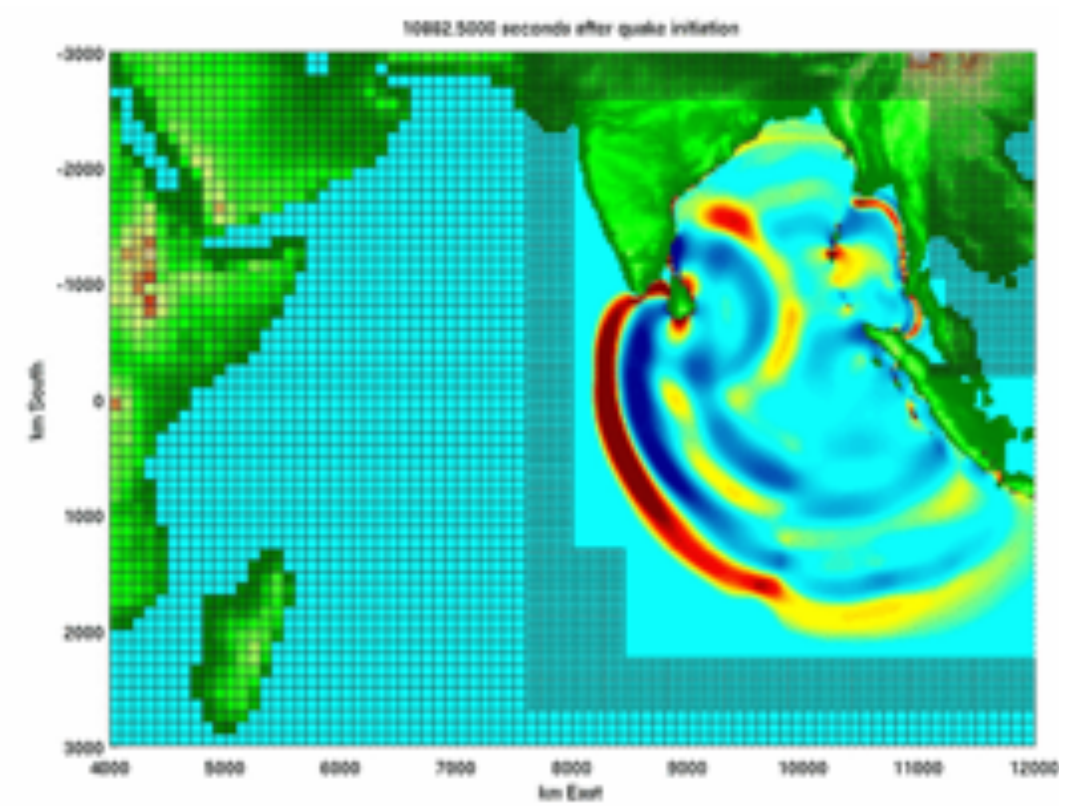
Marsha Berger, Courant Institute, Flatiron Institute, ATAP project (NASA Ames)

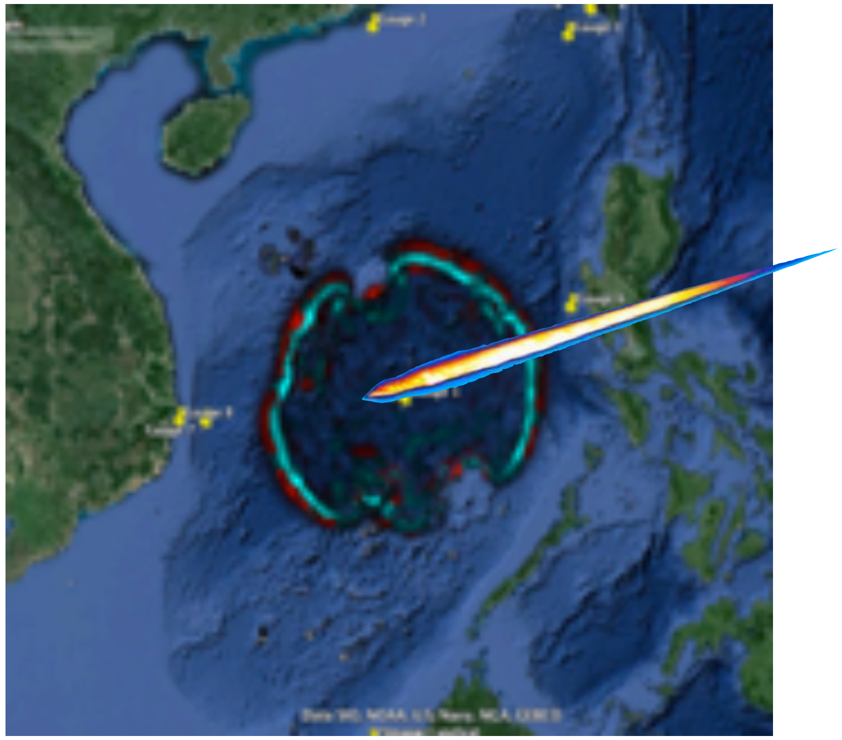
joint with: Randy Leveque, Univ. Washington, ATAP;
(Also Michael Aftosmis, NASA Ames; Jonathan Goodman, NYU)

Tsunami Modeling

Tsunamis:

- Arise from earthquakes, landslides, weather fronts, volcanic explosions, **asteroids**
- Simulations require resolution varies by orders of magnitude (**Adaptive Mesh Refinement)**



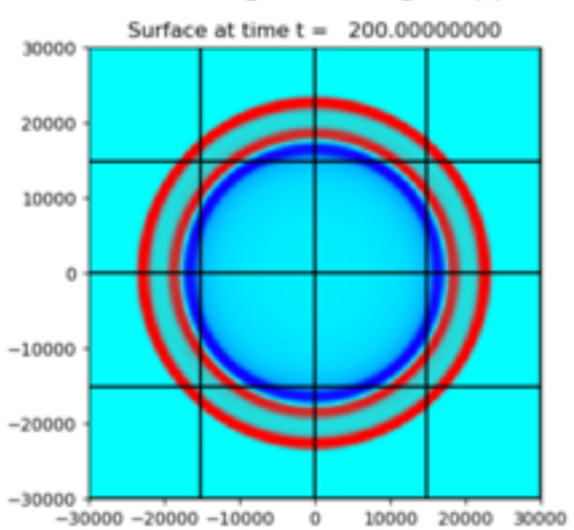


Tsunami Modeling

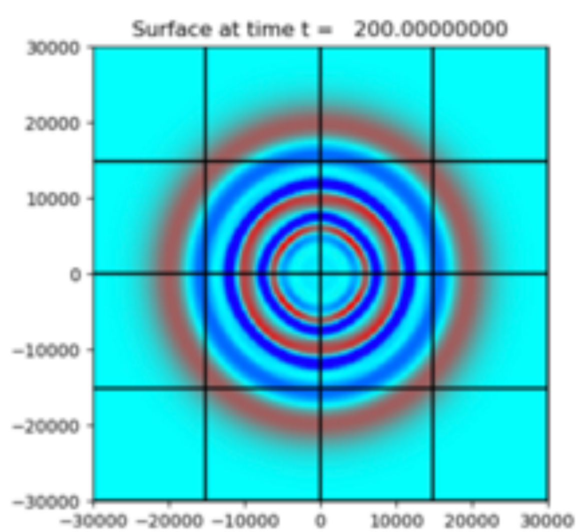
Shallow Water Equations vs. Dispersive Equations:

- SWE work well if wavelength long relative to water depth (earthquake-generated tsunamis)
- Nonlinear SWE work well for on-shore inundation, wetting/drying
- SWE not adequate for short-wavelength tsunamis, where dispersive depth-averaged equations perform better
- Several possible dispersive equations: how do they compare
- Dispersive equations involve 2nd (and 3rd) derivatives -> need implicit solvers

SWE - long wavelength approx



Boussinesq - allows dispersion



- dispersion breaks waves into wave trains
- how much does this affect inundation?
- using open source software GeoClaw

Outline of Talk

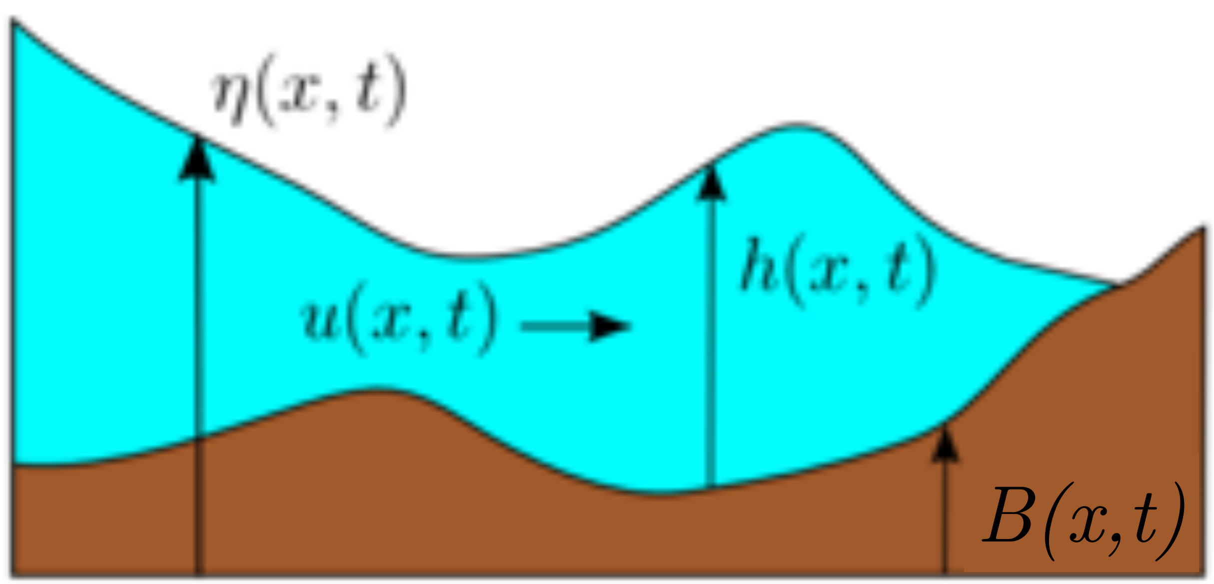
- **Review of Shallow Water Equations**
  - Introducing open source software GeoClaw
  - A good example using SWE for earthquake-generated tsunamis
- **Asteroid Airburst-generated Tsunamis**
  - A poor example using SWE
  - Model 1D problem explains several features found in simulations
  - Compare to linearized Euler to see effects of compressibility and dispersion
- **Adaptive Mesh Refinement with Implicit Scheme for Dispersion**
  - Algorithm overview
  - Tsunami simulation from ocean impact from asteroid off Washington coast
- **Conclusions and Future Research**

Shallow Water Equations

Solve depth-averaged shallow water equations with bathymetry B(x,y,t)

$$
\begin{aligned}
h_t + (hu)_x + (hv)_y &= 0 \\
(hu)_t + (hu^2 + \frac{1}{2}gh^2)_x + (huv)_y &= -ghB_x \quad -Du \\
(hv)_t + (huv)_x + (hv^2 + \frac{1}{2}gh^2)_y &= -ghB_y \quad -Dv
\end{aligned}
$$

Removes free surface and vertical dimension, but lots of numerical issues
(need well-balanced scheme, moving shoreline, robust handling of drying/wetting)



Some numbers:

wave speed	$c = \sqrt{gh}$
deep water $h \approx 4\text{km}$, c ≈ 200 m/sec	
shallow water $h \approx 10\text{m}$, c ≈ 10 m/sec	

Sketch of SWE Derivation

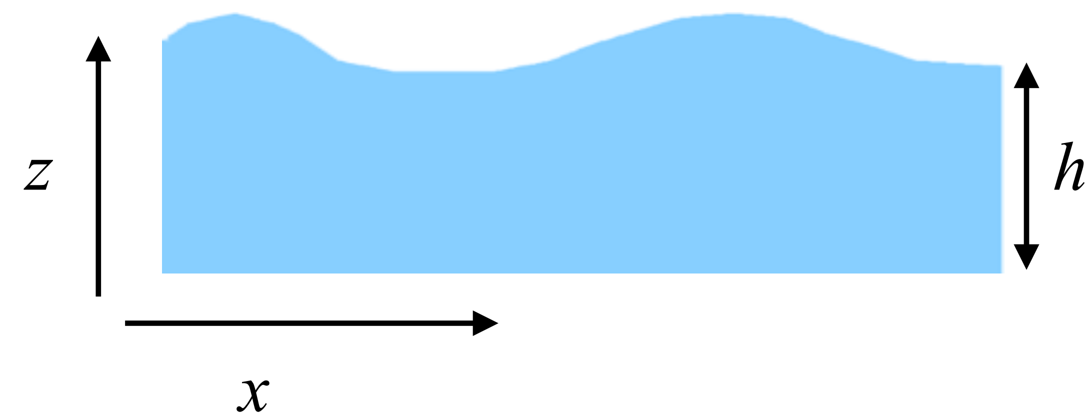
Start with incompressible, divergence-free, irrotational Euler eqs on flat bottom:

$$u_t + uu_x + wu_z + p_x = 0$$
$$w_t + uw_x + ww_z + p_z = -g$$
$$u_x + w_z = 0$$
$$u_z - w_x = 0$$

u = horizontal velocity

w = vertical velocity

p = pressure (setting ρ = 1)



Boundary conditions (flat bottom):

$w(z=0) = 0$ no flow through bottom

$p = 0$ pressure at top (take = 0 since only gradient enters)

$h_t + uh_x = w$ h is water height above bottom (kinematic bc - relates motion of free surface to velocity)

Long wave length approximation: $\epsilon = \frac{h}{L} << 1$ L horizontal length scale

h ~ 4 km in ocean (deepest is 11 km; continental shelf < 1 km)

L ~ for earthquakes could be 50 - 500 km

Sketch of SWE Derivation

$$u_t + uu_x + wu_z + p_x = 0$$
$$w_t + uw_x + ww_z + p_z = -g$$
$$u_x + w_z = 0$$
$$u_z - w_x = 0$$

Asymptotic solution of form

$$u = u_0(\epsilon x, z, \epsilon t) + \epsilon u_1(\epsilon x, z, \epsilon t) +$$
$$w = w_0(\epsilon x, z, \epsilon t) + \epsilon w_1(\epsilon x, z, \epsilon t) +$$
$$p = p_0(\epsilon x, z, \epsilon t) + \epsilon p_1(\epsilon x, z, \epsilon t) +$$
$$h = h_0(\epsilon x, \quad \epsilon t) + \epsilon h_1(\epsilon x, \quad \epsilon t) \quad +$$

Leading terms:

$$u_x + w_z = 0 \implies w_0 = 0$$

u_x: $O(\epsilon)$ $\qquad$ w_z: $O(1)$

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Leading terms:

$$u_x + w_z = 0 \implies w_0 = 0$$

u_x: $O(\epsilon)$ $\quad$ w_z: $O(1)$

$$u_z = w_x \implies$$ u_0, u_1 independent of z

u_z: $O(1)$ $\quad$ w_x: $O(\epsilon^2)$

Sketch of SWE Derivation

$$u_t + u u_x + w u_z + p_x = 0$$
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Leading terms:

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$O(\epsilon)$ $O(1)$

$$u_z = w_x \implies$$ u_0, u_1 independent of z

$O(1)$ $O(\epsilon^2)$

$$\frac{\partial p_0}{\partial z} = -g \implies p_0(z) = g(h_0(\epsilon x, \epsilon t) - z)$$ ← hydrostatic pressure

Sketch of SWE Derivation

$$u_t + uu_x + wu_z + p_x = 0$$
$$w_t + uw_x + ww_z + p_z = -g$$
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Asymptotic solution of form

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$$w = w_0(\epsilon x, z, \epsilon t) + \epsilon w_1(\epsilon x, z, \epsilon t) +$$
$$p = p_0(\epsilon x, z, \epsilon t) + \epsilon p_1(\epsilon x, z, \epsilon t) +$$
$$h = h_0(\epsilon x, \epsilon t) + \epsilon h_1(\epsilon x, \epsilon t) +$$

Leading terms:

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u_x: $O(\epsilon)$, w_z: $O(1)$

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u_z: $O(1)$, w_x: $O(\epsilon^2)$

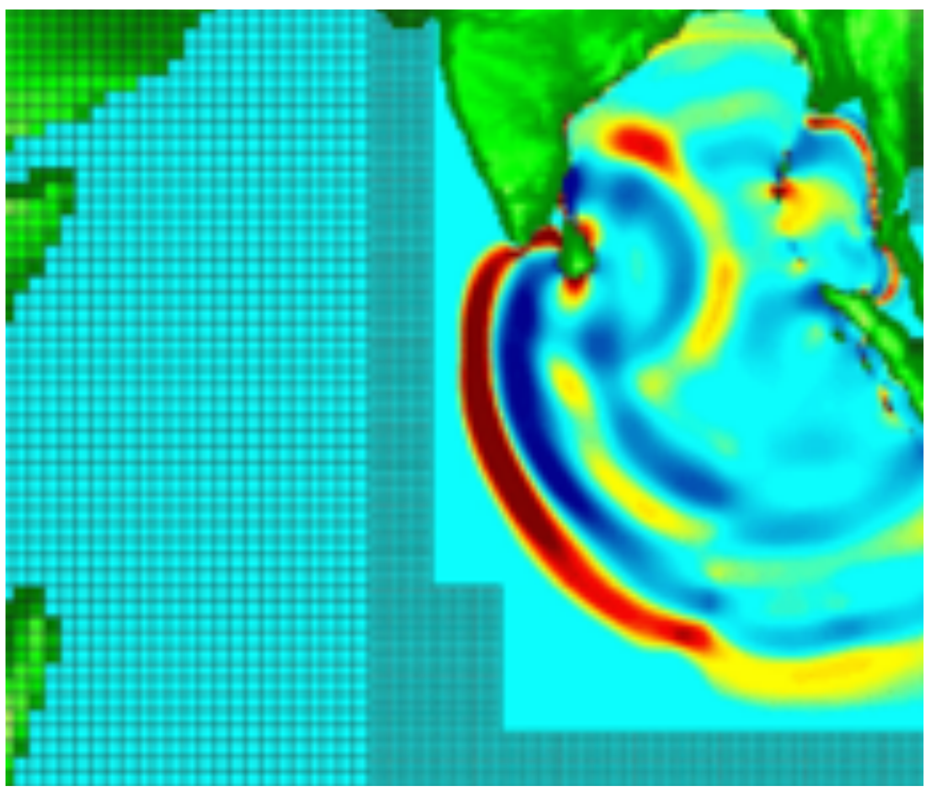
$$\frac{\partial p_0}{\partial z} = -g \implies p_0(z) = g(h_0(\epsilon x, \epsilon t) - z)$$ ← hydrostatic pressure

O(ε) terms give Shallow Water Equations

$$h_{0t} + (h_0 u_0)_x = 0$$
$$u_{0t} + u_0 u_{0x} + g h_{0x} = 0$$

Patch-based AMR with GeoClaw

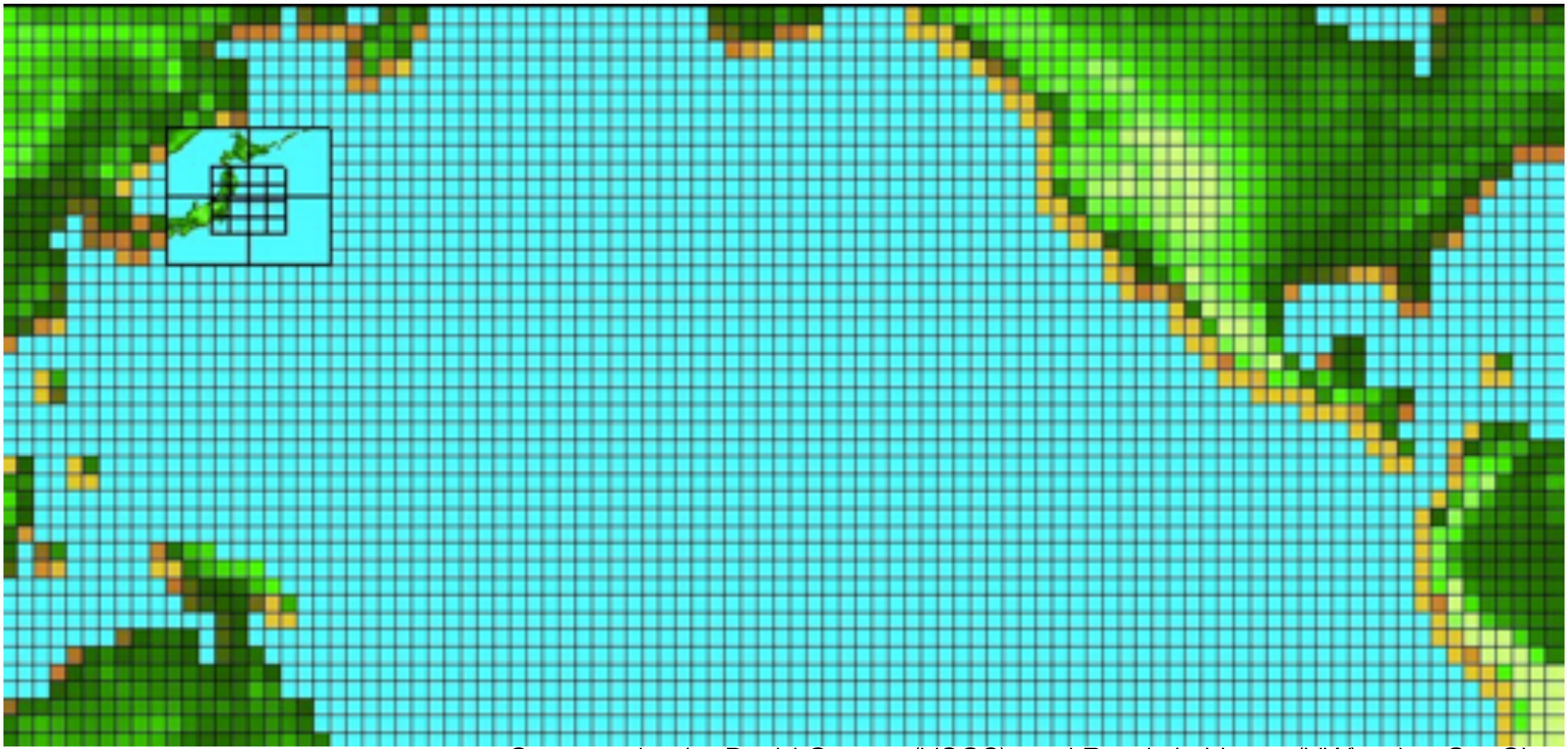
Patch-based mesh refinement to follow important features



- Adaptive in space and time - bridge scales from hundreds of kilometers to meters
- Refines automatically to follow waves and other features (LTE, wave height, adjoint-based)
- Interpolate between patches automatically preserving sea-level (i.e. well-balanced)
- Grid generation for patches handled automatically
- GeoClaw part of open source software project Clawpack (specialized for bathymetry)

(Approved by US National Tsunami Hazard Mitigation Program)

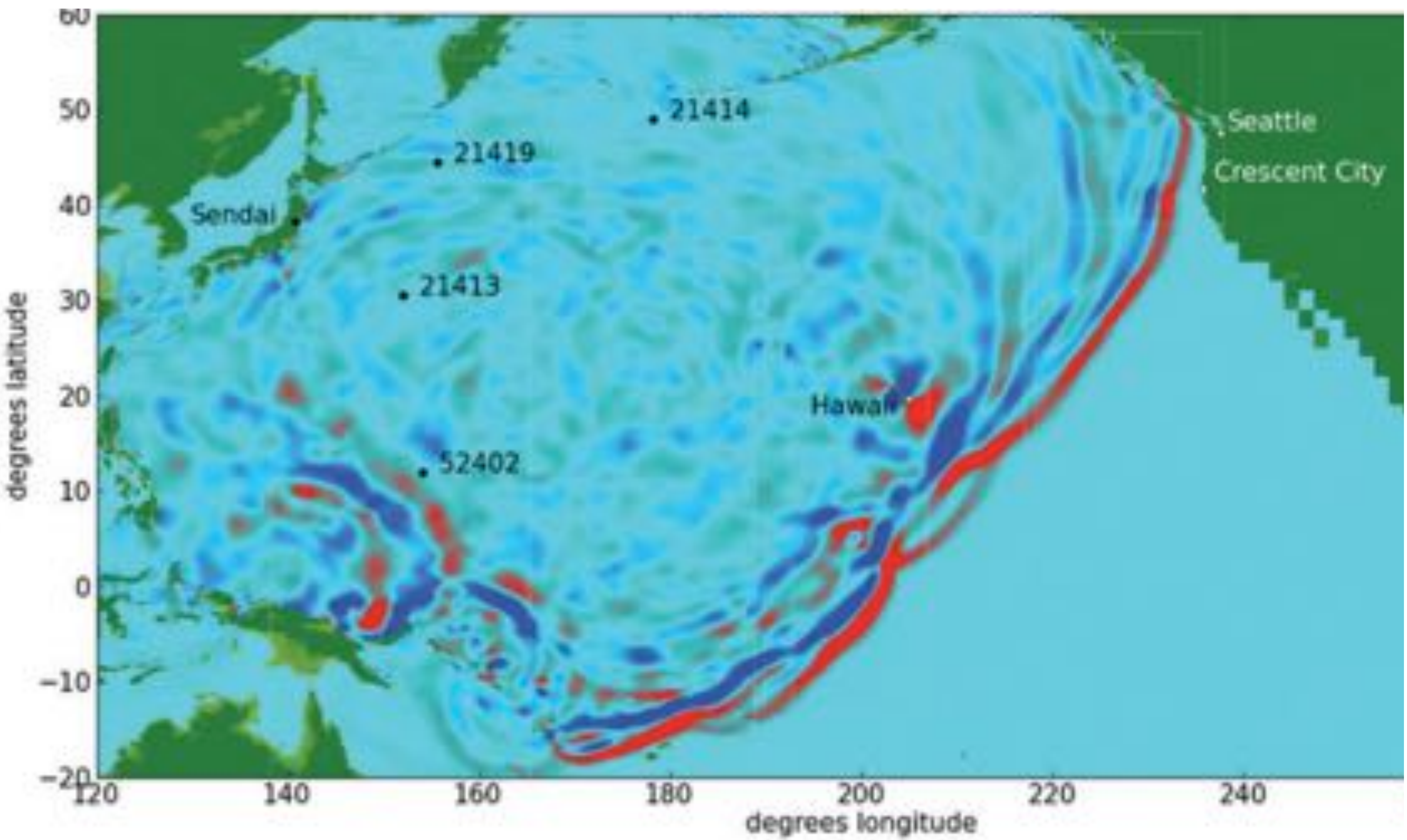
Tohuko Simulation



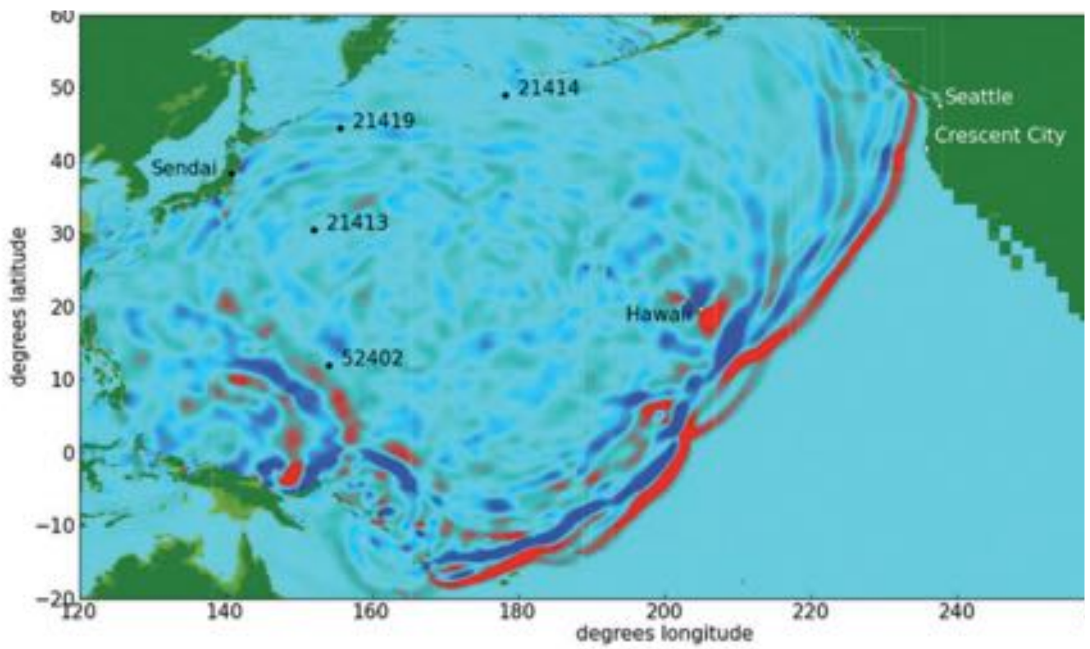
Computation by David George (USGS) and Randy LeVeque (UW) using GeoClaw

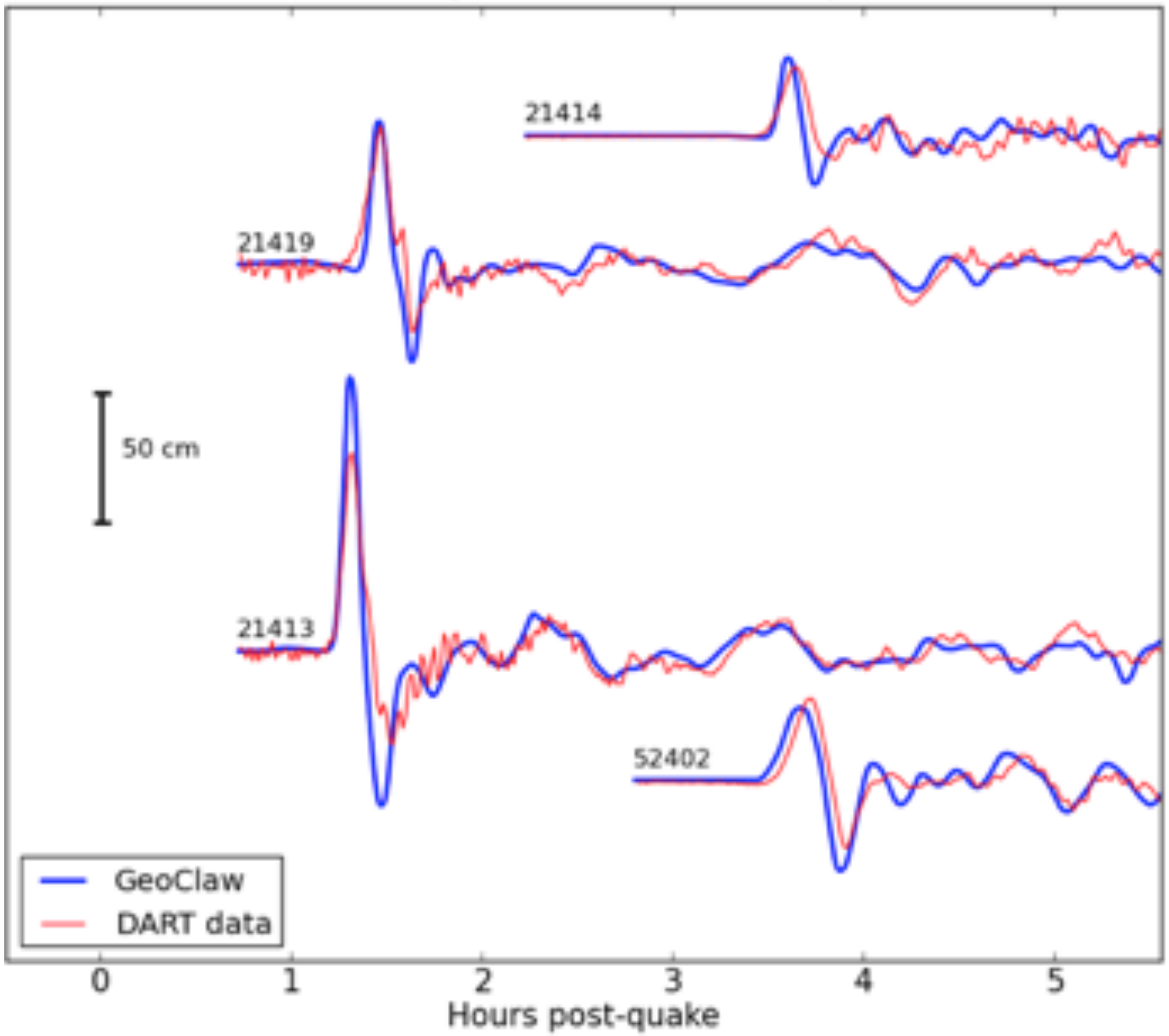
- 5 levels of refinement: 8,4,16, 32 (2^{14} in each spatial direction!)
- Resolution: 160 km on level 1; 10 m on level 5;
- Many V&V benchmarking studies

Tohuko Simulation



Tohuko Simulation





Wave height at selected DART (Deep Ocean Assessment and Reporting of Tsunamis) buoys from NOAA.

(Approved by US National Tsunami Hazard Mitigation Program)

Asteroid Airbursts (ATAP)

Chelyabinsk, Russia, February 2013

Size: ~20 m diameter

Speed: ~ 20 km/sec

Energy: ~500 kilotons of TNT (2.2 x 10^{15} joules)

Energy equivalent to magnitude 7.0 earthquake

Damage to ~7300 structures over 20,000 km^2

Tunguska, Russia, June 1908

Size: 50-80 m diameter

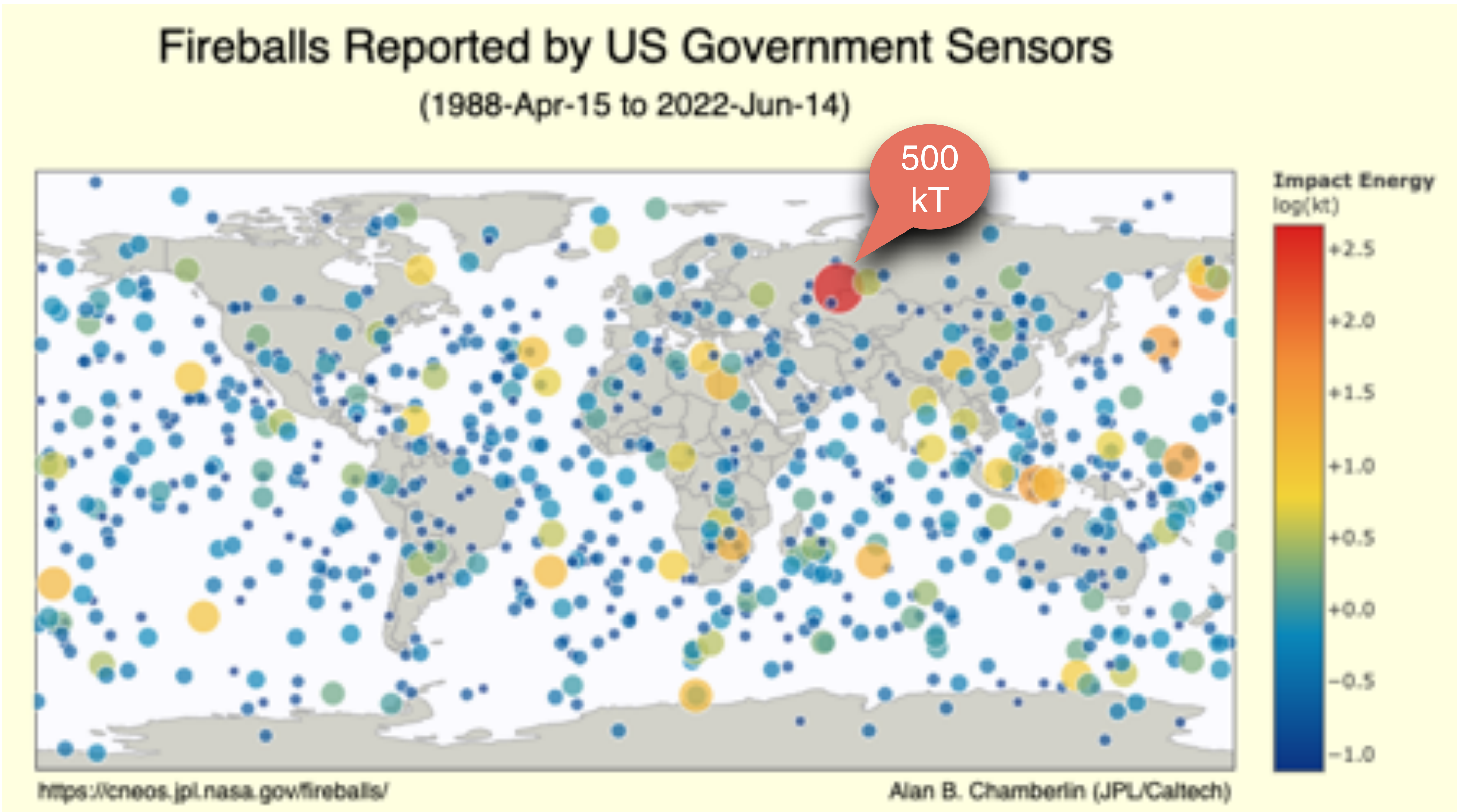
Air burst estimated at 10-15 km altitude

Energy: ~15-20 Mt TNT

Flattened 2000 km^2 of forest

- ATAP goal is to provide quantitative risk assessment for particular near earth objects *includes both high-fidelity simulations at specific locations as well as fast engineering model for simulations with millions of realizations*

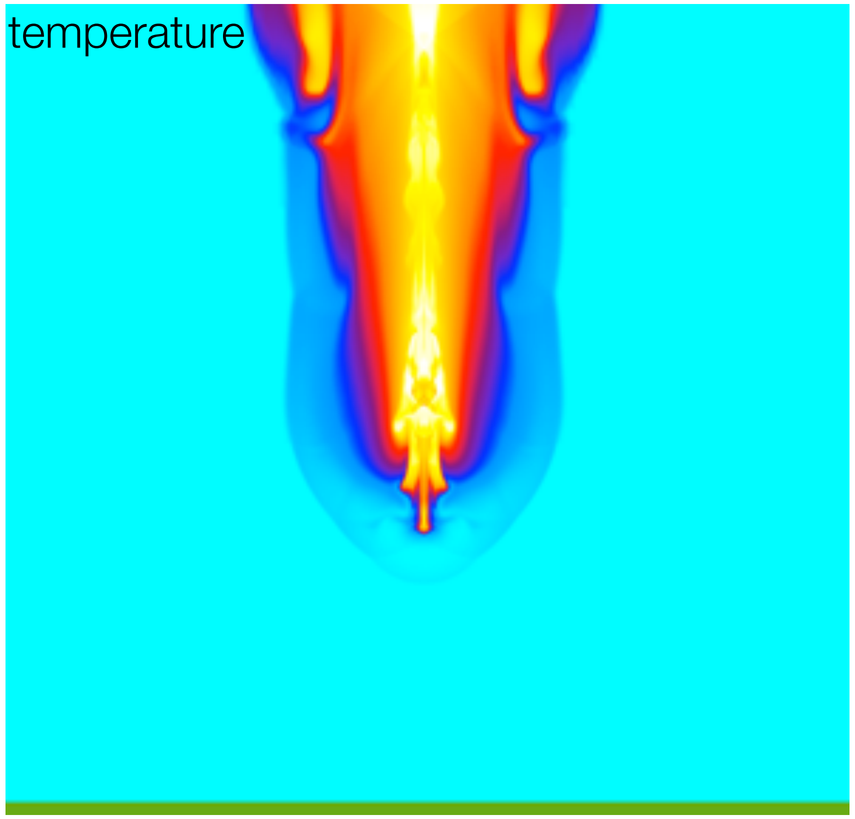
Asteroid Threat Assessment Project (ATAP)



- Earth is bombarded by relatively energetic objects with disturbing regularity
- Could an asteroid airburst or impact in the ocean cause a tsunami that endangered coastal populations far away?

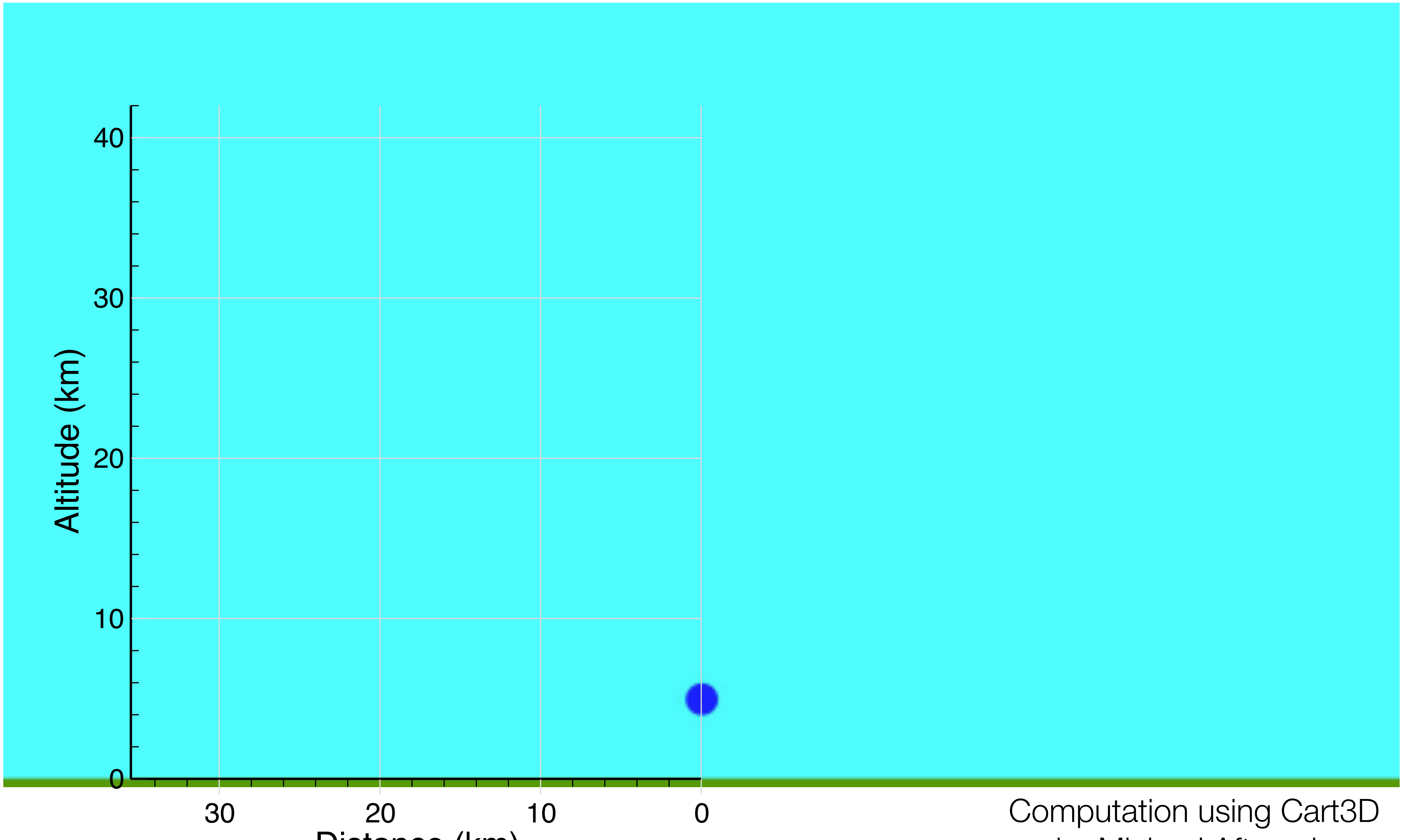
Air-Burst Generated Tsunamis

- Bolide enters atmosphere, ‘explodes’ in air over water.
- Initial shock wave reaches water, followed by longer rarefaction (plus more)
- Modify shallow water equations: atmospheric pressure gradient is forcing function on rhs.



$$
\begin{aligned}
h_t + (hu)_x + (hv)_y &= 0 \\
(hu)_t + (hu^2 + \frac{1}{2}gh^2)_x + (huv)_y &= -ghB_x(x,y) - hp_{x_{\text{air}}}/\rho_{\text{water}} \quad -Du \\
(hv)_t + (huv)_x + (hv^2 + \frac{1}{2}gh^2)_y &= -ghB_y(x,y) - hp_{y_{\text{air}}}/\rho_{\text{water}} \quad -Dv
\end{aligned}
$$

Simulation of 25 MT static spherical charge

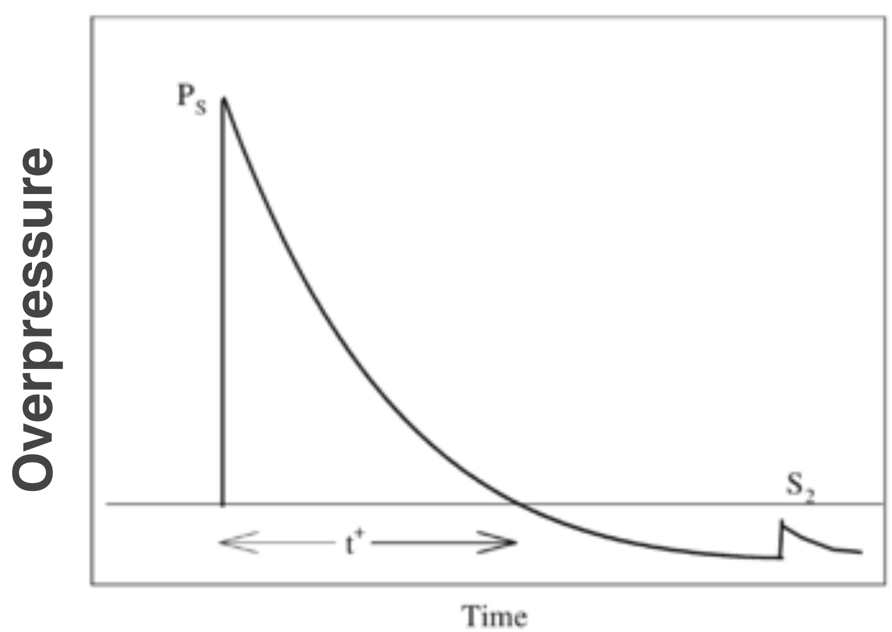


Computation using Cart3D
by Michael Aftosmis

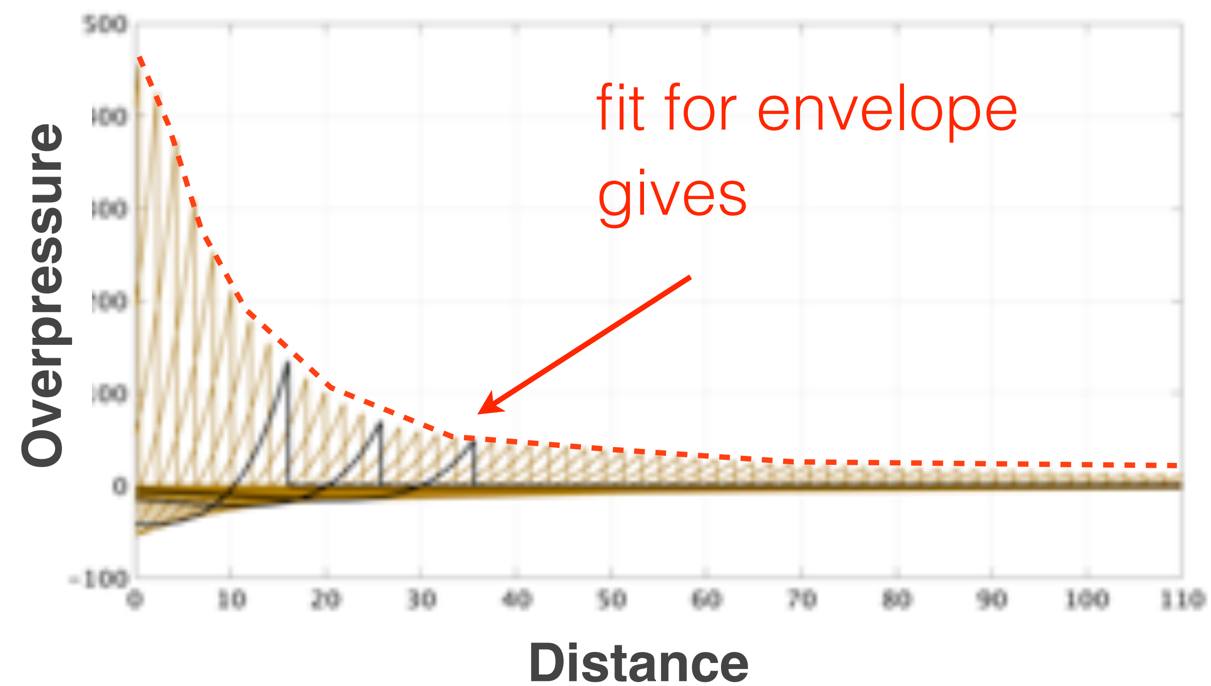
Contours of Mach number, buoyancy effects included

Air-Burst Generated Tsunamis

- Model of air-burst pressure wave based on ground footprint. Derived from 3D entry simulation by M. Aftosmis (NASA Ames, ATAP).
- 250Mt has max 450% overpressure; (100Mt has 230% overpressure)



Friedlander profile

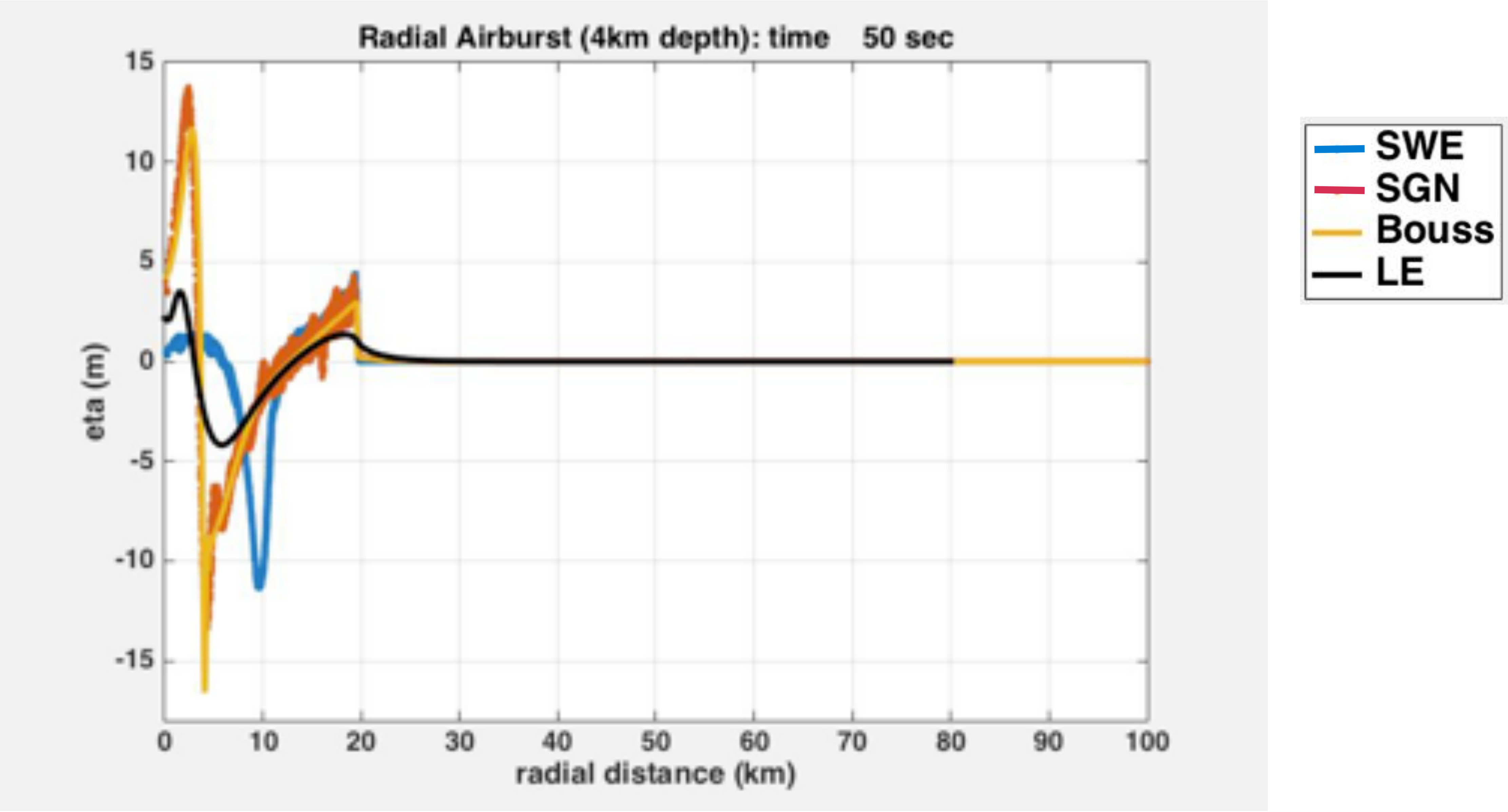


- Friedlander profiles 5 seconds apart, traveling at 391 m/sec
- Analytic model fed to GeoClaw as source term in momentum eq.

- For context, Mt. St. Helens: 25-35 Mt; Mt. Tamboura: 10-20 GT (global effects)

Comparison of 4 Airburst Models in 2D

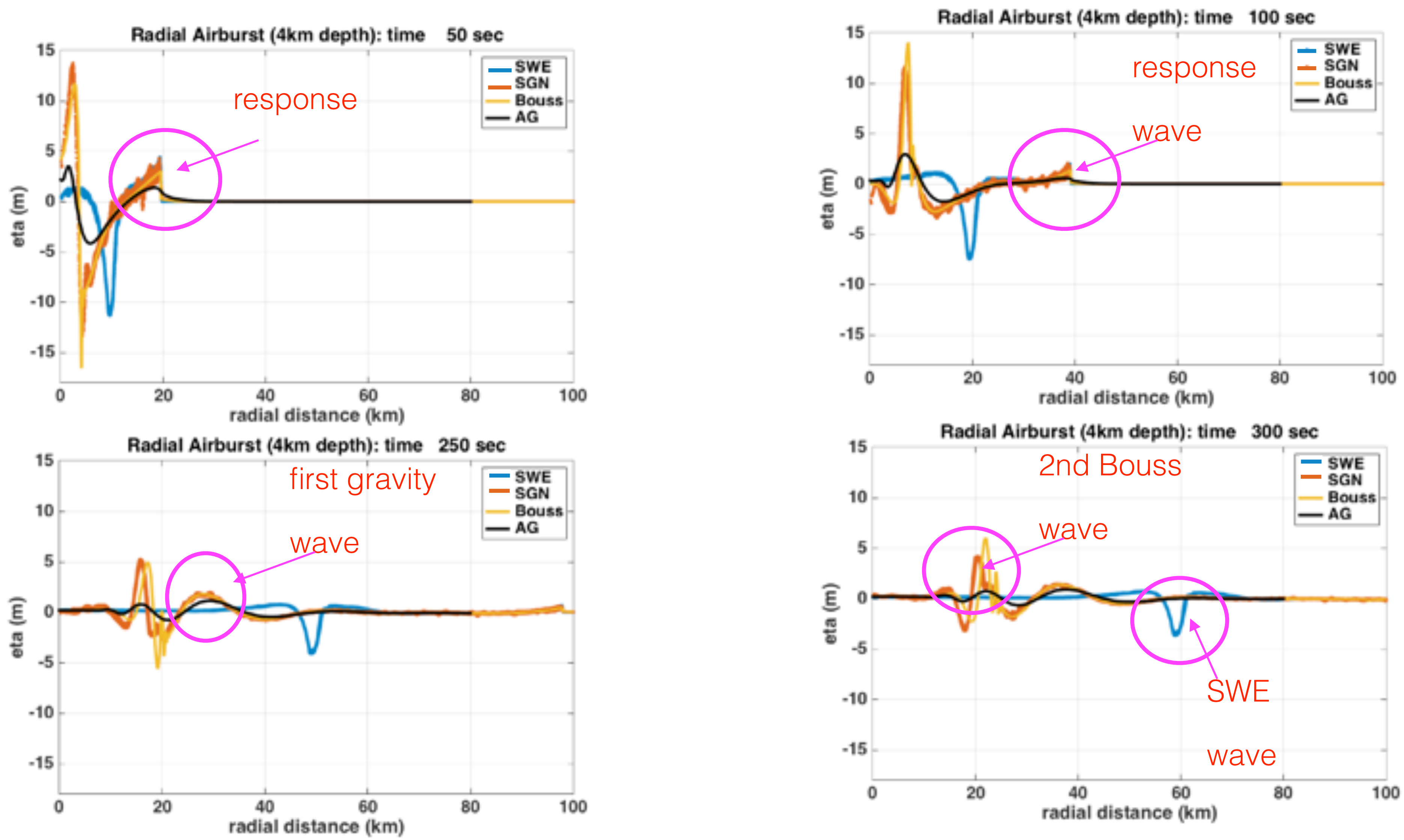
2 horizontal dimensions, 4 km deep ocean, 250MT airburst profile



- SWE (GeoClaw results).
- LE (linearized Euler)
- SGN (using Basilisk)
- Madsen Schaffer (using BoussClaw)

Boussinesq eqs: keep next terms in asymptotic expansion

Airburst Simulations (4 km depth)



- Leading response wave similar in all codes, but of different amplitudes
- By 300 seconds, AG and both Bouss. codes agree well on first gravity wave
- Not true for second wavefront, but both Bouss. codes agree with each other
- AG has leading wave largest, not true for either Boussinesq sim.
- SW gravity wave travels too fast and does not decay properly

1D Model Problem

Simplify to 1d without bathymetry:

$$
\begin{aligned}
h_t + (hu)_x &= 0 \\
(hu)_t + (hu^2 + \frac{1}{2}gh^2)_x &= -h\, p_{x_{\text{air}}}/\rho_{\text{water}}
\end{aligned}
$$

Look for traveling wave solutions:

$$p(x,t) = p(x - st)$$ for pressure wave speed s

Turns pde into ode: define $m = x - st$ $\quad \frac{\partial}{\partial x} = \frac{\partial}{\partial m} \quad \frac{\partial}{\partial t} = (-s)\frac{\partial}{\partial m}$

$$
\begin{aligned}
-sh_m + (hu)_m &= 0 \\
-s(hu)_m + (hu^2 + \frac{1}{2}gh^2)_m &= \frac{-hp_{m_{\text{air}}}}{\rho_{\text{water}}}
\end{aligned}
$$

Odes can be solved exactly:

1D Model Problem

Solving and linearizing gives: h_0 initial depth, h_r response wave

s = *air speed*

$c = \sqrt{g h_0}$

$h = h_0 + h_r$

$$h_r(x) = \frac{h_0}{(s^2 - c^2)\rho_{\text{water}}} \Delta p(x)_{air}$$

Implications:

- Effect stronger in deeper water (response ~ depth)
  Ex: $h_0 = 4\ km$, $\Delta p_{air} = 1 atm$, $s = 391\ m/sec \implies h_r = 3.5m$
- If s > c and pressure increases, water height positive ($h_r > 0$)!
- If s < c and pressure increases, water height decreases ($h_r < 0$)
- If s ~ c ⇒ Proudman resonance (regime for meteo-tsunamis)

Linearized soln:
(impulsive start)

$$h(x,t) = h_r(x - st) - \frac{1}{2}\left(\frac{s}{c} + 1\right) h_r(x - ct) + \frac{1}{2}\left(\frac{s}{c} - 1\right) h_r(x + ct)$$

1D Model Problem (Linearized Euler)

- Bring in effects of compressibility and dispersion

Euler equations:

$$\rho_t + (\rho u)_x + (\rho w)_z = 0$$
$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u w)_z = 0$$
$$(\rho w)_t + (\rho u w)_x + (\rho w^2 + p)_z = -\rho g$$

Boundary conditions: (flat bottom)

$$w(x, z=0, t) = 0$$
$$h_t + u h_x = w(x, h(x,t), t)$$
$$p(x, h(x,t), t) = p_{atm} + p_e(x,t)$$

Linearizing: $\rho = \rho_0 + \widetilde{\rho} \quad u = \widetilde{u} \quad w = \widetilde{w} \quad p = p_0 + c_a^2 \widetilde{\rho}$

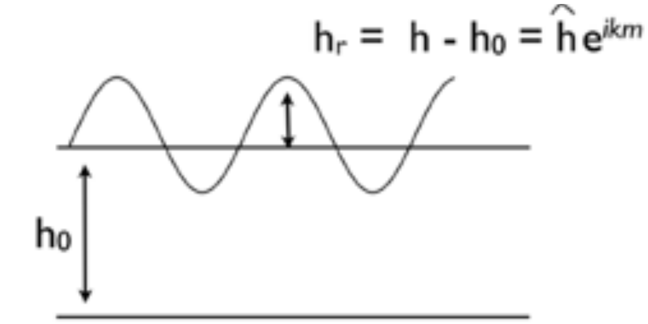
$$\widetilde{\rho}_t + \rho_w \widetilde{u}_x + \rho_w \widetilde{w}_z = 0$$
$$\rho_w \widetilde{u}_t + c_a^2 \widetilde{\rho}_x = 0$$
$$\rho_w \widetilde{w}_t + c_a^2 \widetilde{\rho}_z = -\widetilde{\rho} g$$

$$\widetilde{w}(x, z=0, t) = 0$$
$$\frac{\partial h_r(x,t)}{\partial t} = \widetilde{w}(x, h_0, t)$$
$$c_a^2 \widetilde{\rho}(x, h_0, t) - \rho_w g\, h_r(x,t) = p_e(x,t)$$

- Let $p_e(m) = A_k e^{ikm}$, compute *response* coefficients: $h_r(m) = \widehat{h}_r\, e^{ikm}$, $\widetilde{\rho}(m,z) = \widehat{\rho}(z) e^{ikm}$ etc.

- Soln depends on wave number k, length scale $L = 2\pi / k$

1D Model Problem (Linearized Euler)



- Traveling wave analysis for linearized Euler equations: $m = x - st$

- System becomes:

$$
\begin{aligned}
-s\widetilde{\rho}_m + \rho_w \widetilde{u}_m + \rho_w \widetilde{w}_z &= 0 \\
-s\rho_w \widetilde{u}_m + c_a^2 \widetilde{\rho}_m &= 0 \\
-s\rho_w \widetilde{w}_m + c_a^2 \widetilde{\rho}_z &= -\widetilde{\rho} g
\end{aligned}
$$

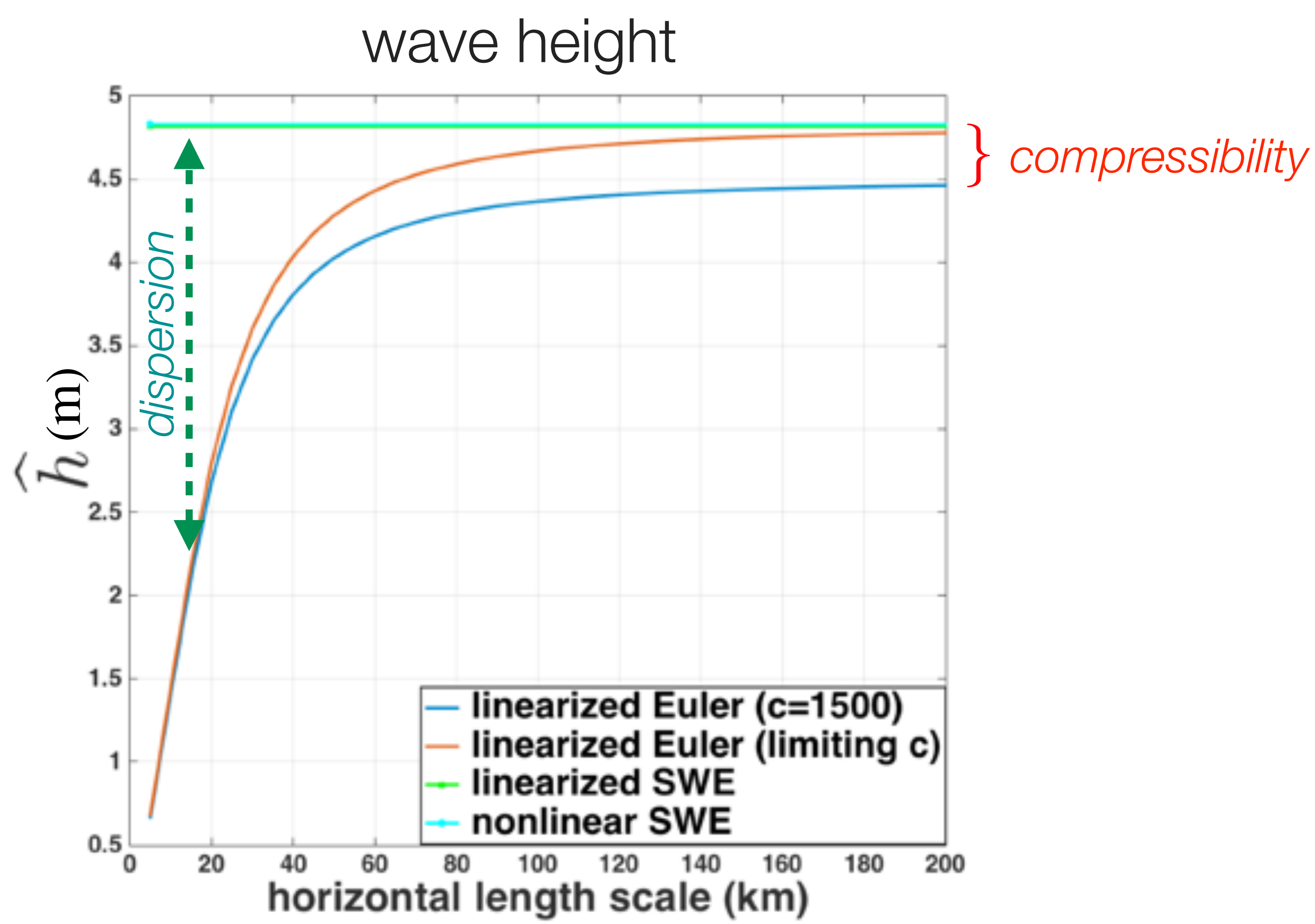
- which gives an ode (can be solved as fn. of z):

$$
\begin{pmatrix} \widehat{u} \\ \widehat{w} \end{pmatrix}_z = \begin{bmatrix} -g/c_a^2 & i\,k \\ -i\,k\,(1 - s^2/c_a^2) & 0 \end{bmatrix} \begin{pmatrix} \widehat{u} \\ \widehat{w} \end{pmatrix}
$$

- when use bc, gives expressions for $\widehat{h_r}, \ \widehat{u(z)}, \ \widehat{w(z)}$ for each k

1D Model Problem (Linearized Euler)

Single frequency results: for A_k=1 atm, h_0 = 4 km, c_a=1500 m/sec (and 10^8 m/sec), $k = 2\pi/L$



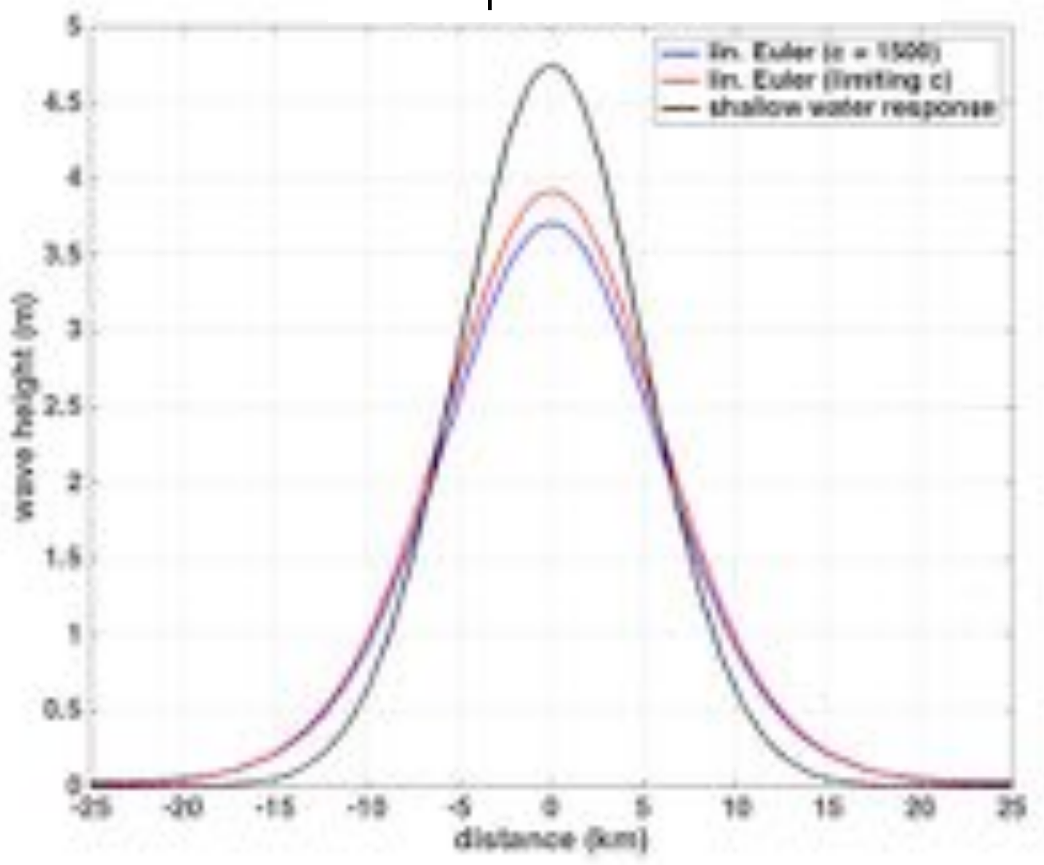
- Compressibility is ~ 10% effect at long length scales
- Dispersion reduces amplitude by 50% for short length scales
  (For L ≈ 15 km, amplitude is half the SWE amplitude)

1D Model Problem

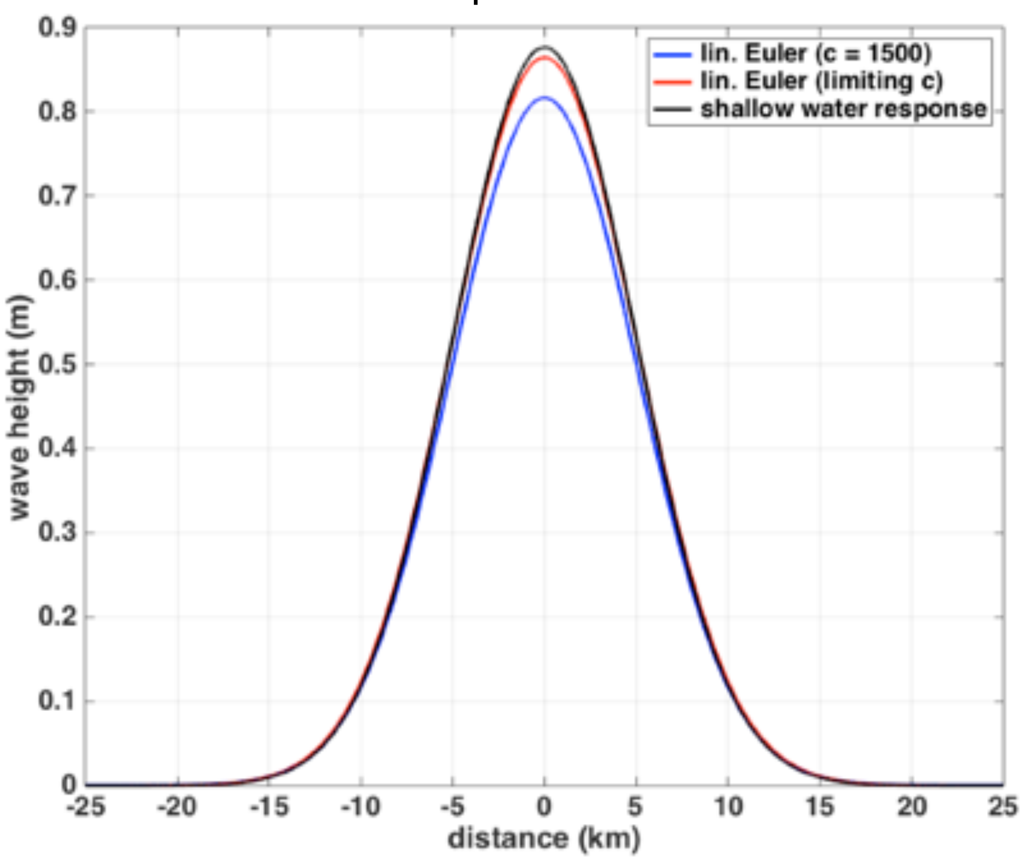
What does wave with all frequencies look like?

Look at Gaussian pressure pulse ($e^{-\frac{1}{2}(\frac{x}{5})^2}$, length scale 10 km) and forced wave response:

depth 4 km



depth 1 km



Broader LE peak due to dispersion; shorter wavelengths filtered

LE results for 1km depth much closer to SW

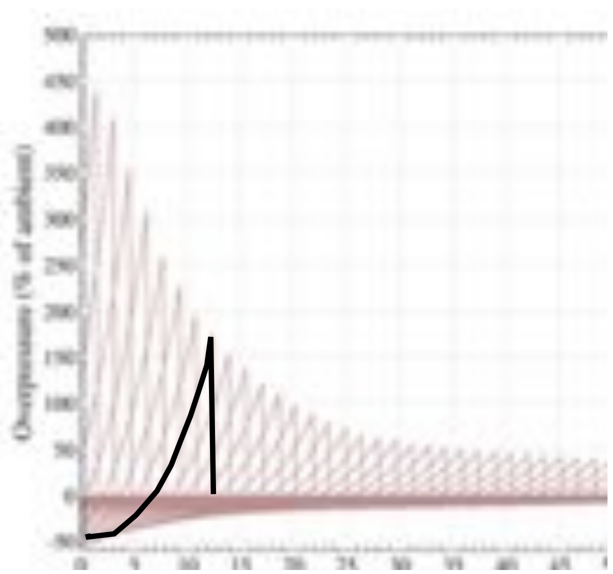
Discrepancy grows with higher frequency forcing

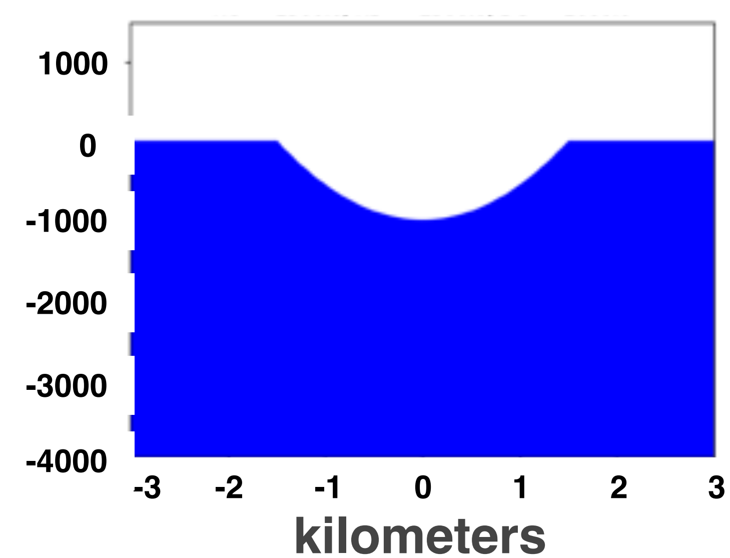
Modeling Summary

- Length scale for air burst pressure forcing is ~10-15 km, compared to >100 km for earthquake-generated tsunamis. Even worse for ocean impacts.

For airburst in 4 km water, ratio is ~0.25, at very edge of applicability

Shallow water waves decay ~ $1/\sqrt{distance}$,   Boussinesq waves ~ 1/distance





static crater test case with no lip

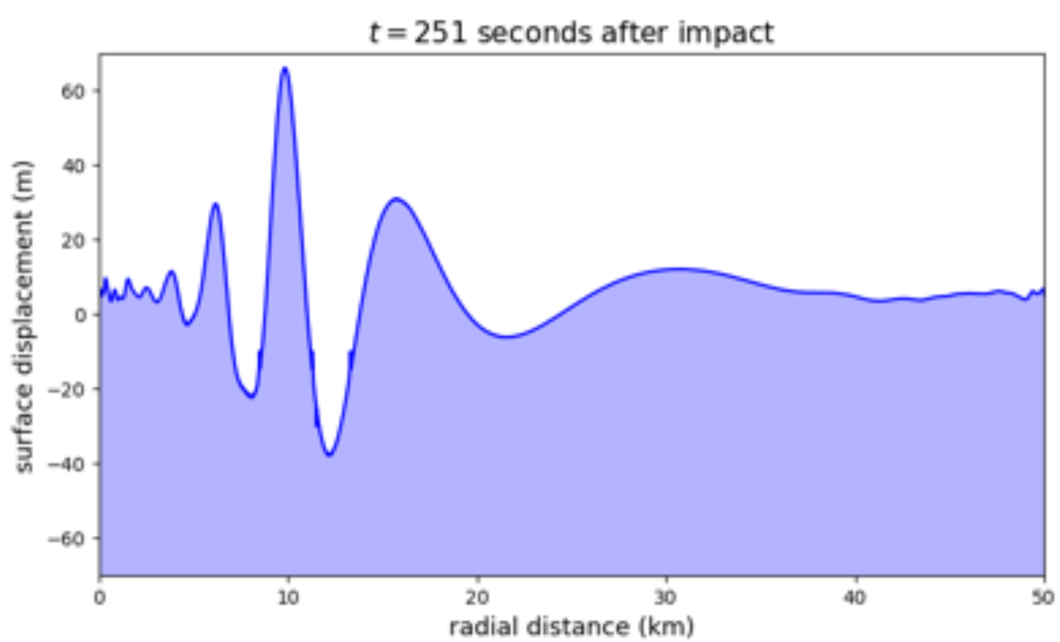
crater depth 1km

crater radius 1.5km

- Want depth-averaged equations to remove vertical dimensions. Euler equations expensive for trans-oceanic simulations.
- Impact-generated tsunamis cannot be initialized using depth-averaged equations

Asteroid Impact Simulation Strategy (1)

1. Initial conditions from 3D ALE3D hydrocode run in 2D axisymmetric mode to generate initial surface. Surface at time 251sec is transferred to GeoClaw assuming radial symmetry. Velocity initialized using outgoing SWE eigenvector.



Place solution in ocean for depth-averaged solution.
(Will show results of impact ~ 150 km off Washington coast)

*Thanks to ATAP team member Darrel Robertson (NASA Ames) for providing the initial conditions

Asteroid Impact Simulation Strategy (2)

2. Boussinesq equations of form:

$$h_t + (hu)_x = 0$$
$$(hu)_t + \left(hu^2\right)_x + gh\eta_x = \psi$$

where ψ is solution of implicit system of equations

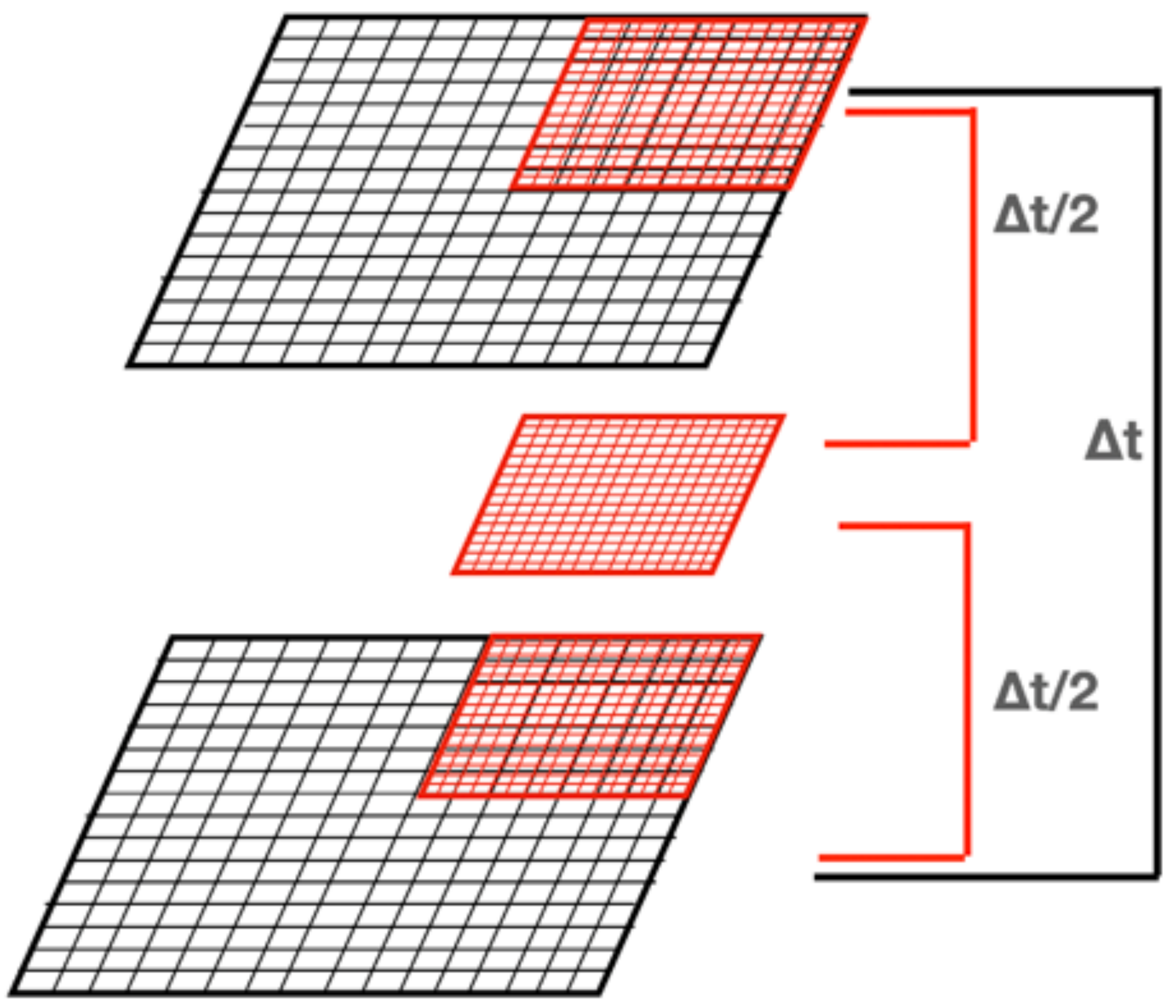
(we use either Madsen-Schaffer, or Serre Green Naghdi)

Solve by splitting:

- Solve elliptic equation for ψ
- Update momentum $(hu)_t = \psi$ using e.g. Forward Euler
- Take step with homogeneous SWE (already in GeoClaw)

3. Switch to Shallow Water Equations near coastline to compute inundation

What about Adaptive Mesh Refinement?



Coarse Grid Step

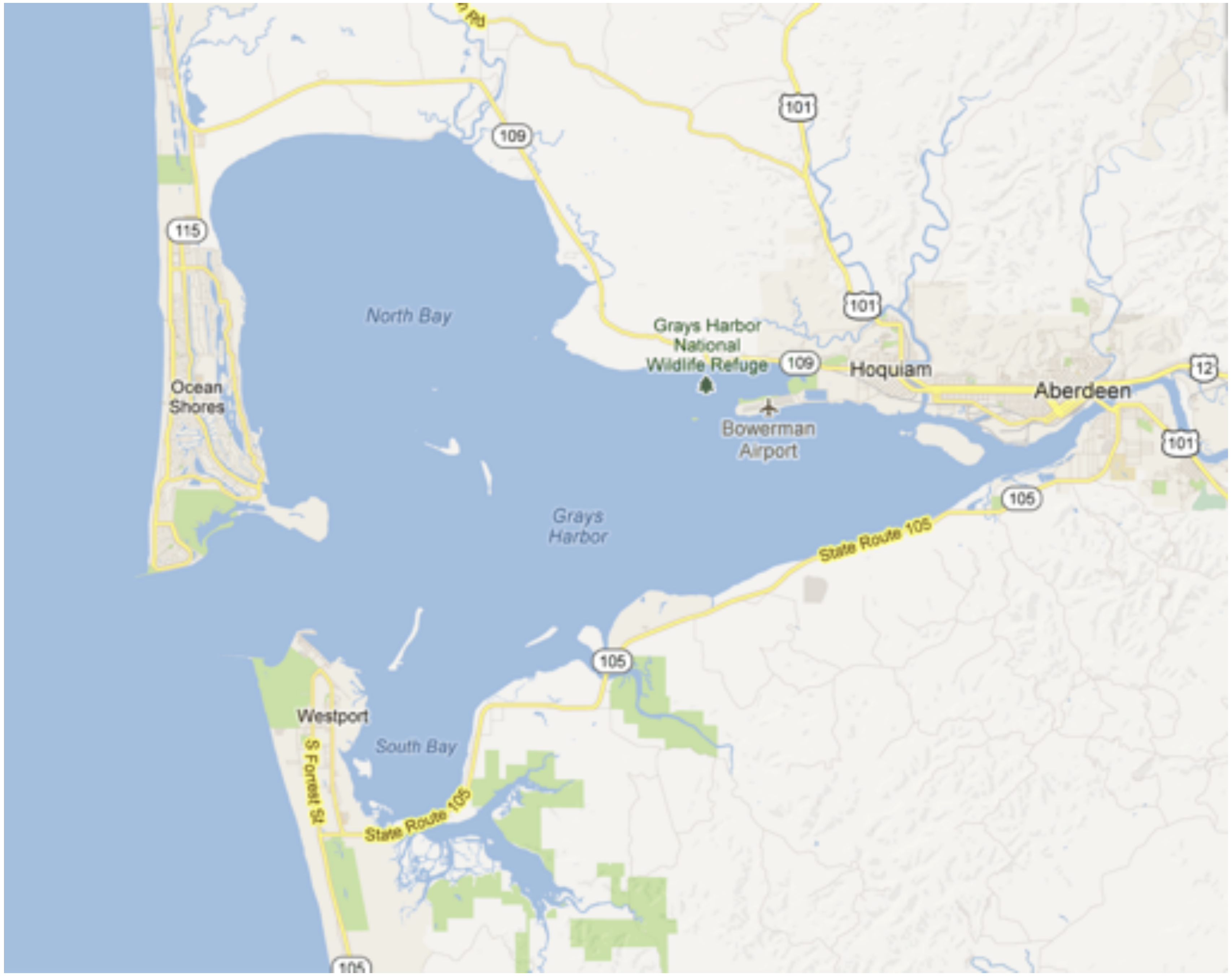
1. Coarse grid at time t_N takes time step **Δt**
2. Solve at new time for ψ_{N+1}

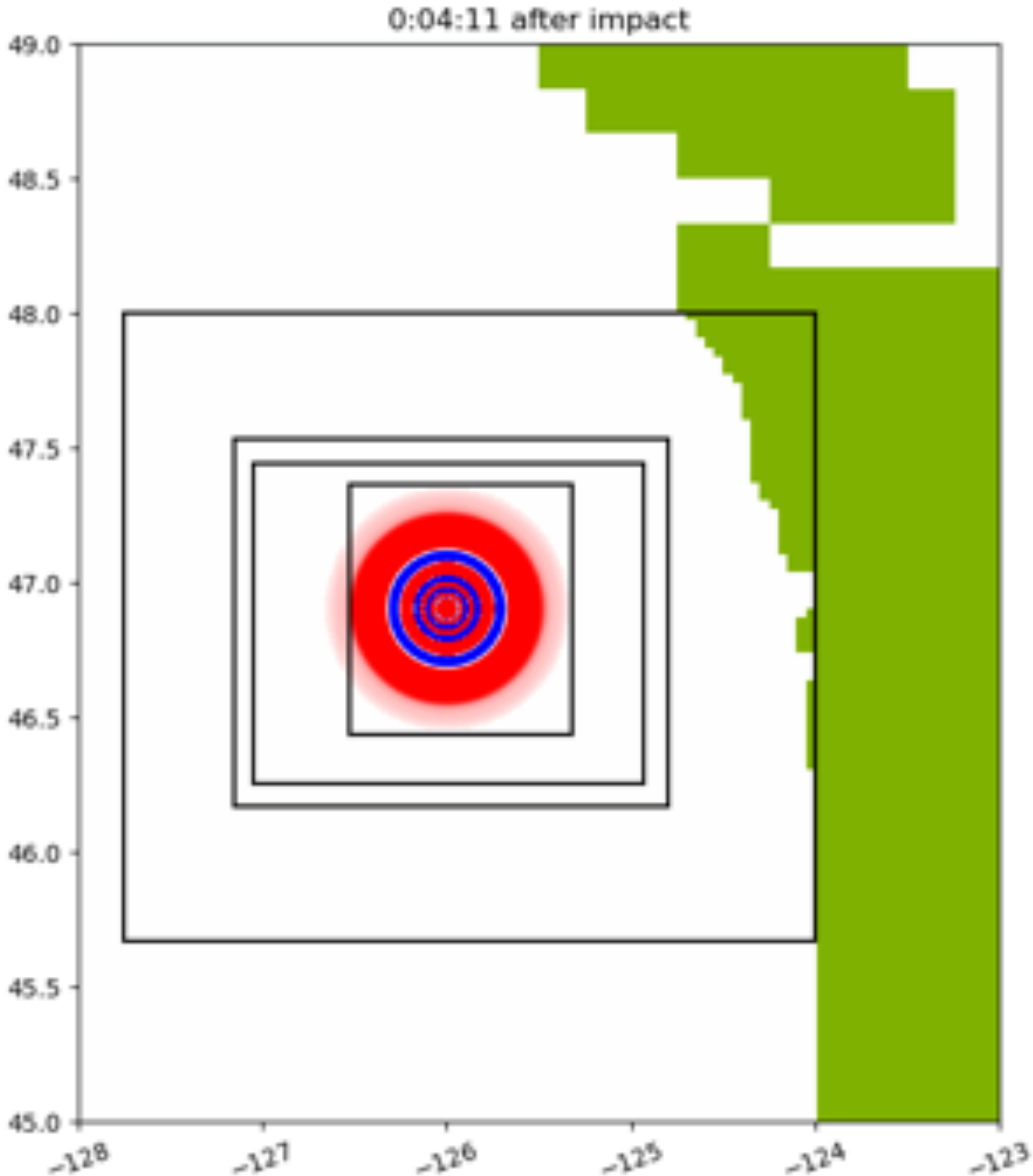
Fine Grid Step

1. Fine grid interpolates all variables from coarse grid including ψ to take two time steps of size **Δt/2**
2. At time t_{N+1} fine grid updates coarse grid

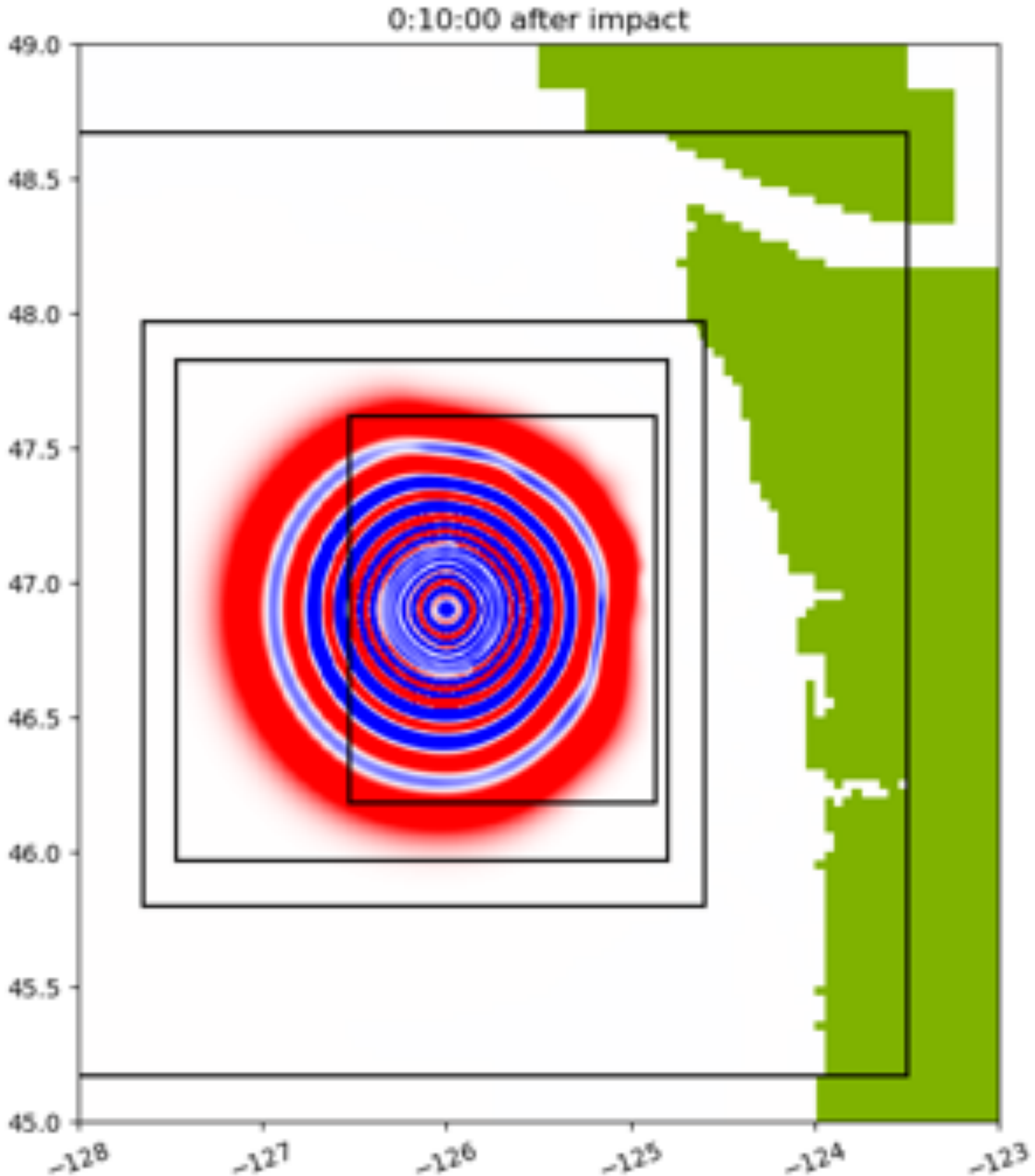
Repeat recursively for more levels

Grays Harbor

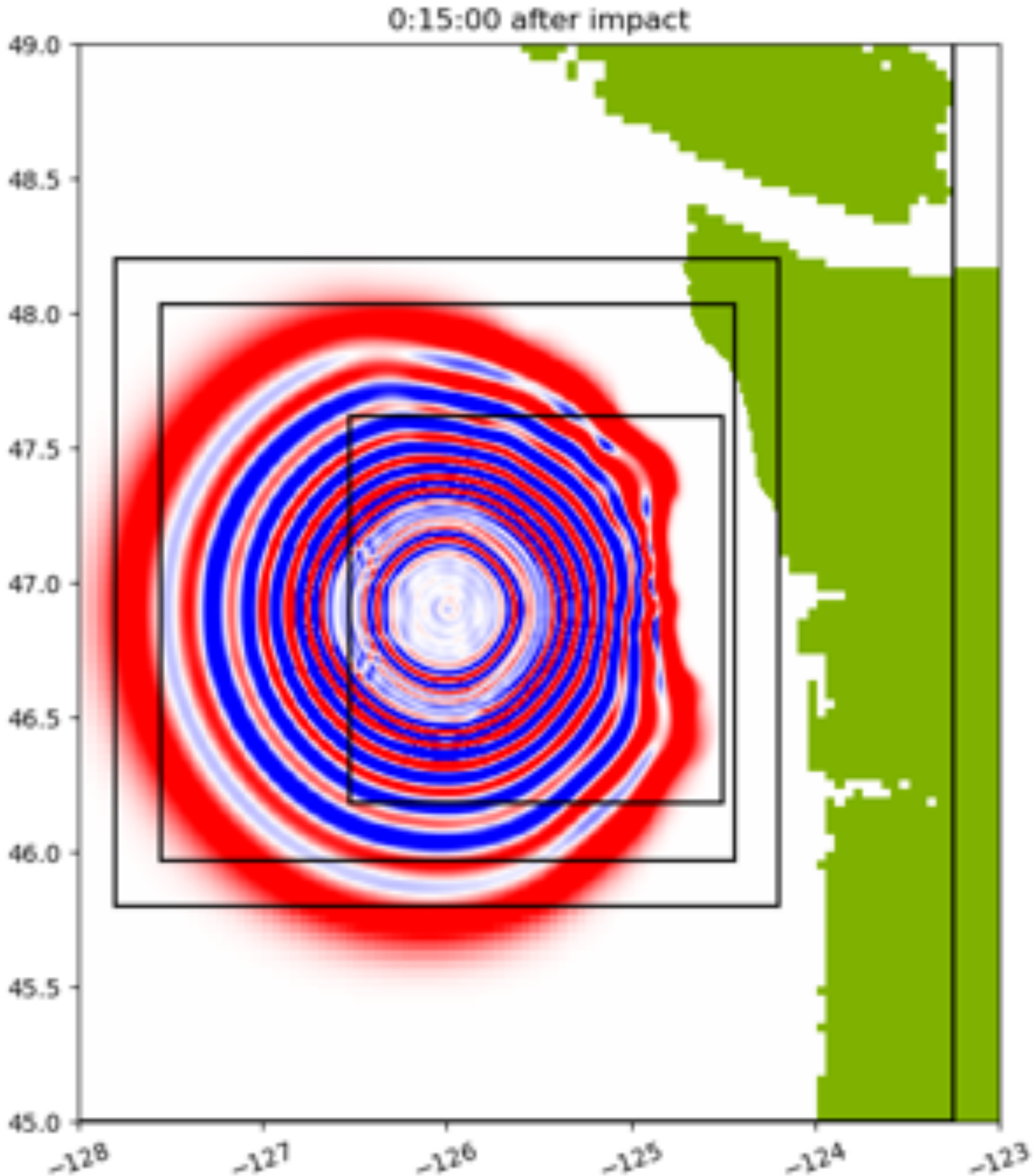




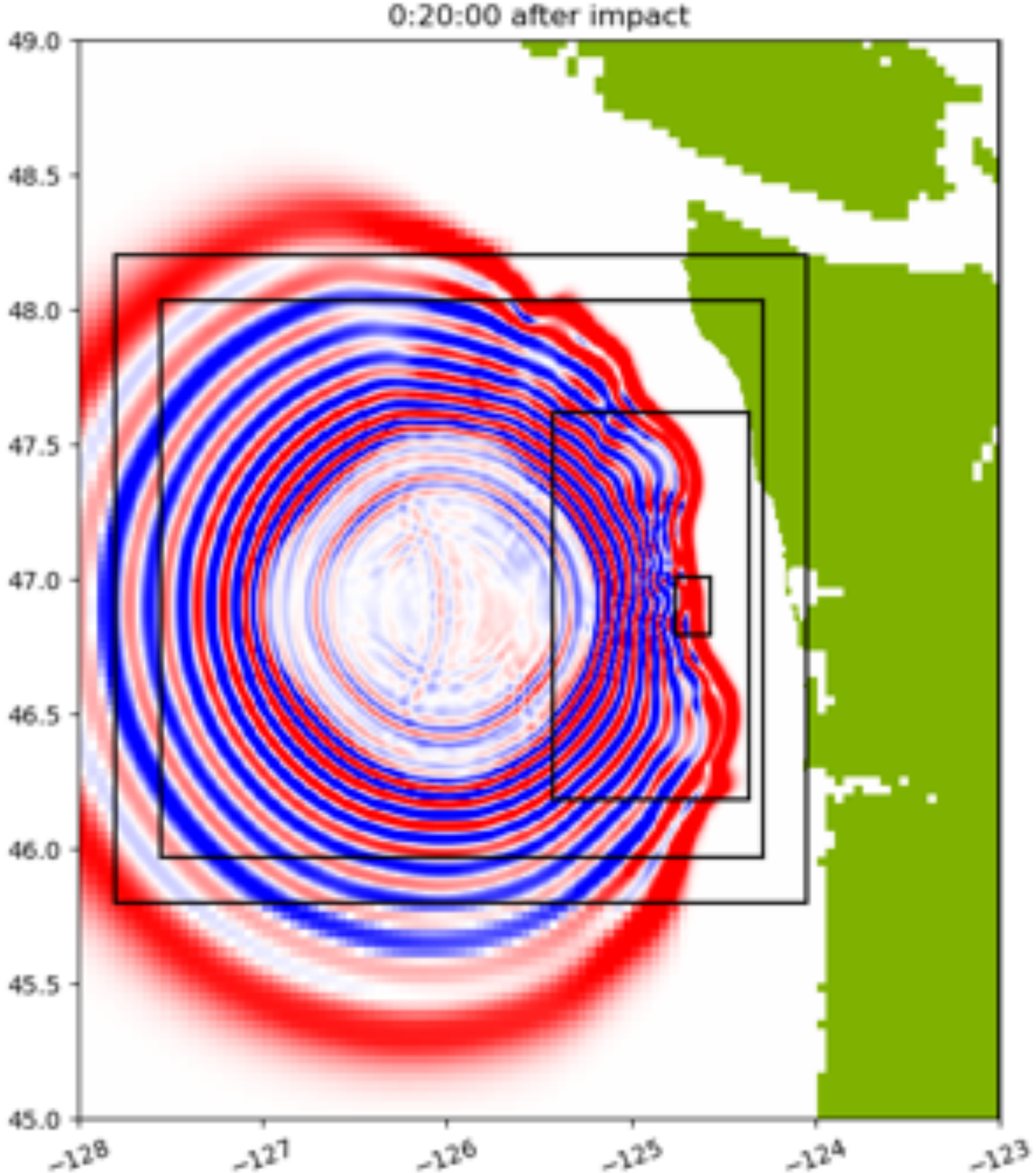
0:04:11 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123

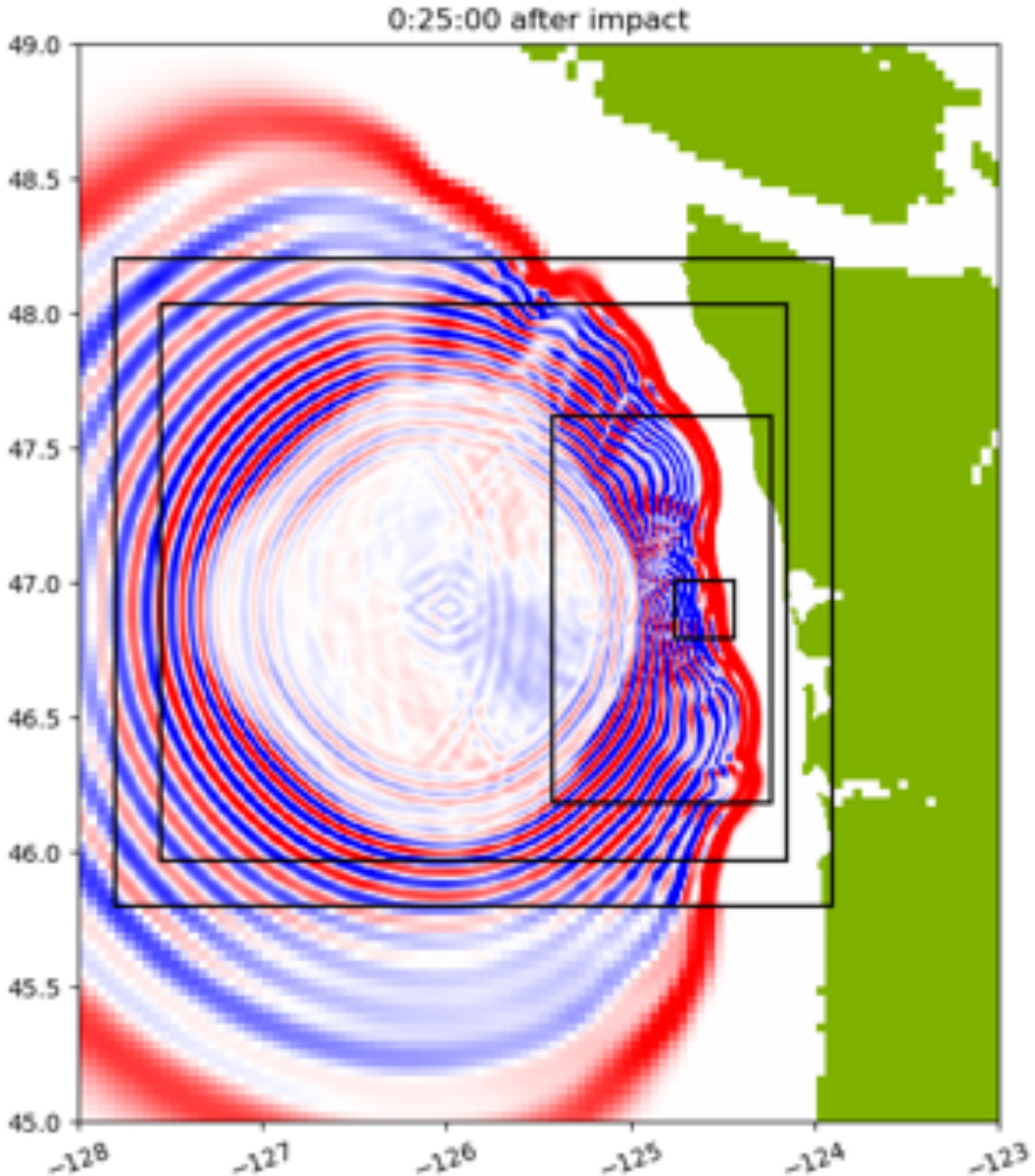


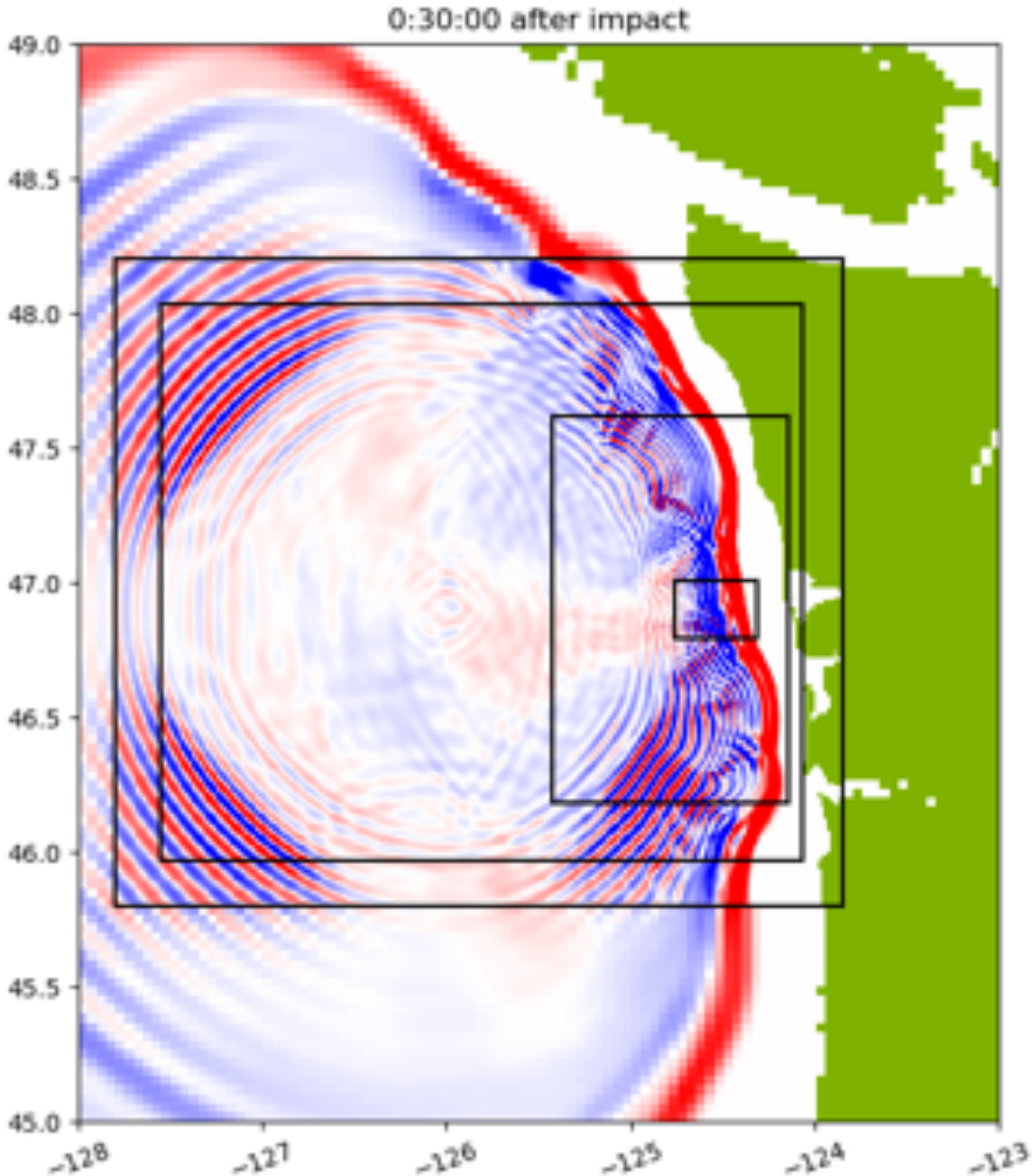
0:10:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123

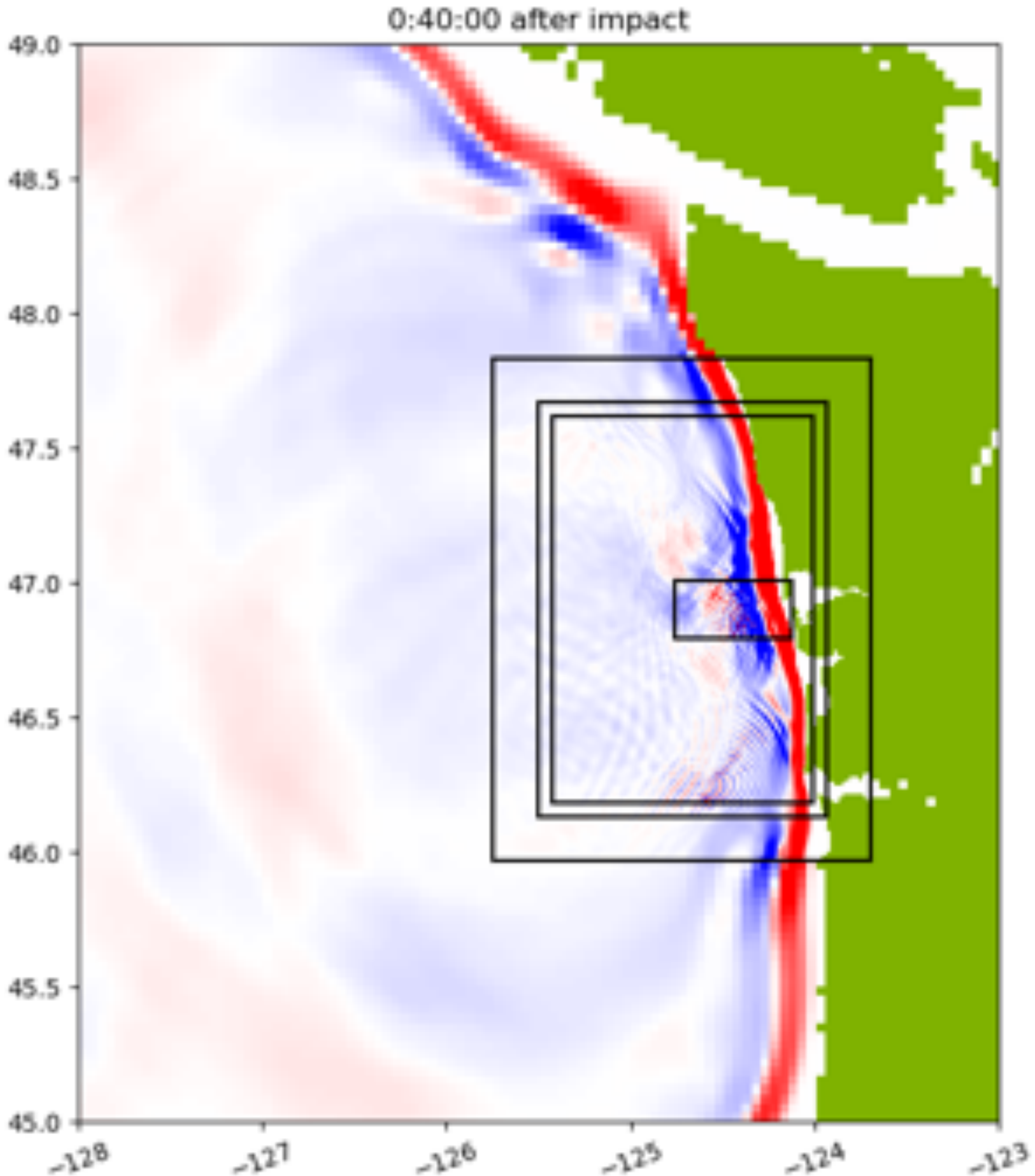


0:15:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123

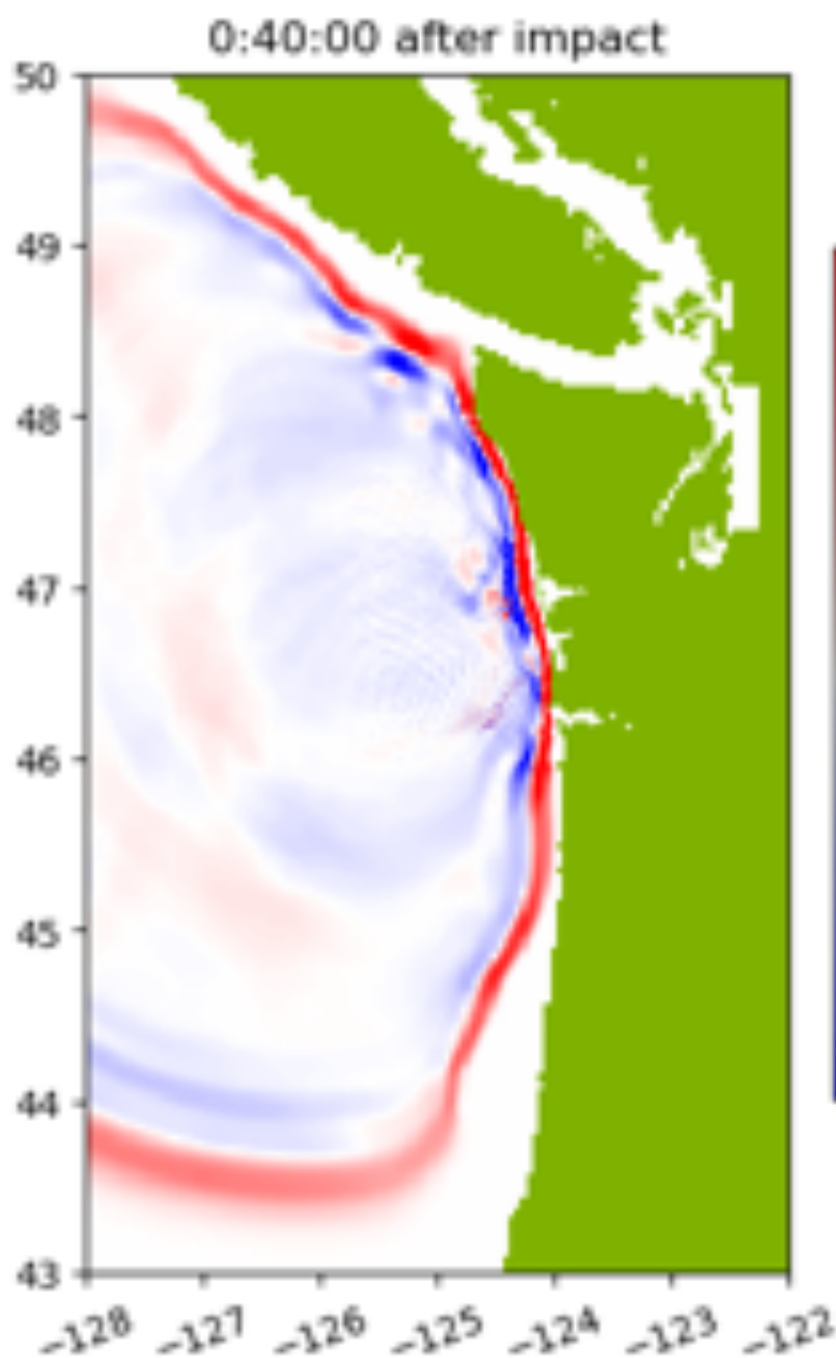

0:20:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123

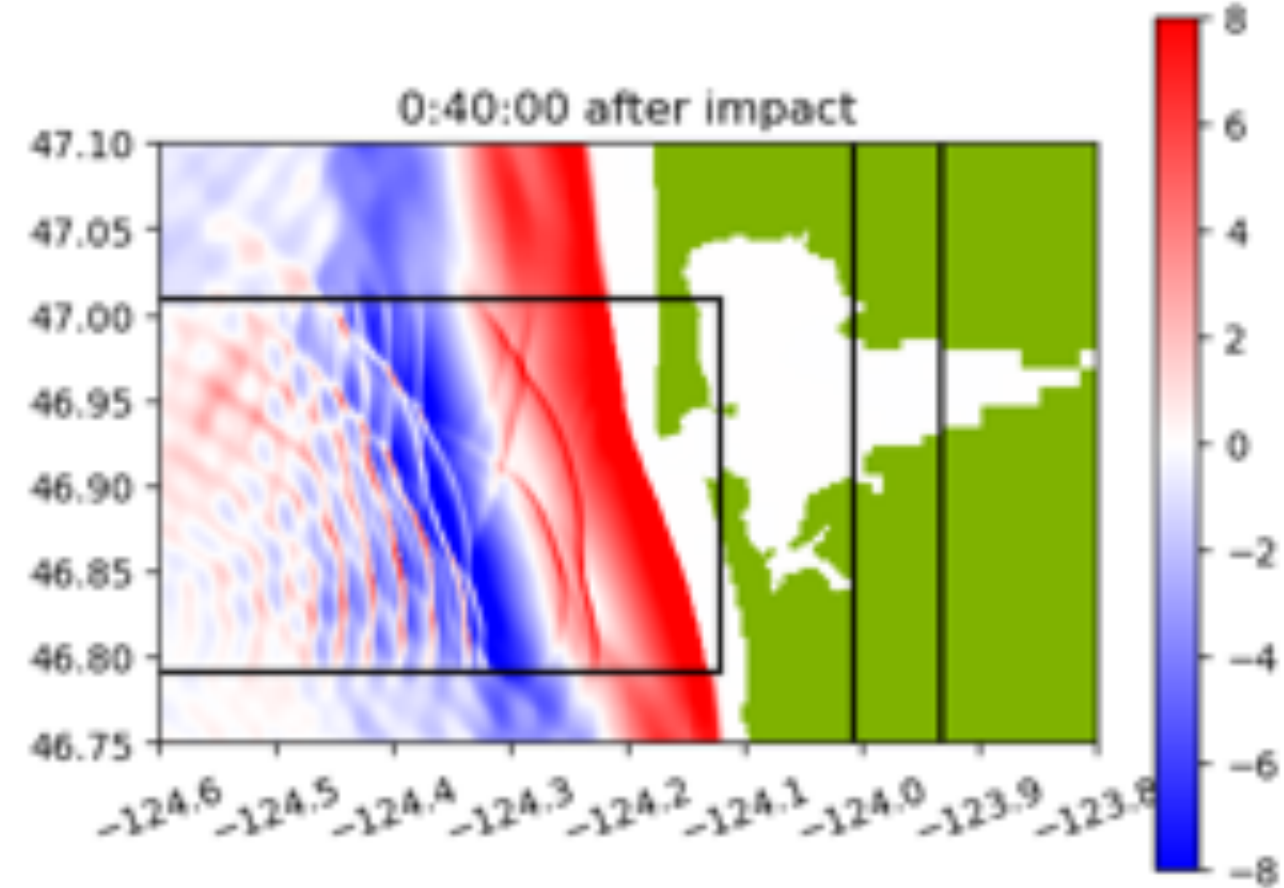

0:25:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123


0:30:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123


0:40:00 after impact
49.0
48.5
48.0
47.5
47.0
46.5
46.0
45.5
45.0
−128
−127
−126
−125
−124
−123

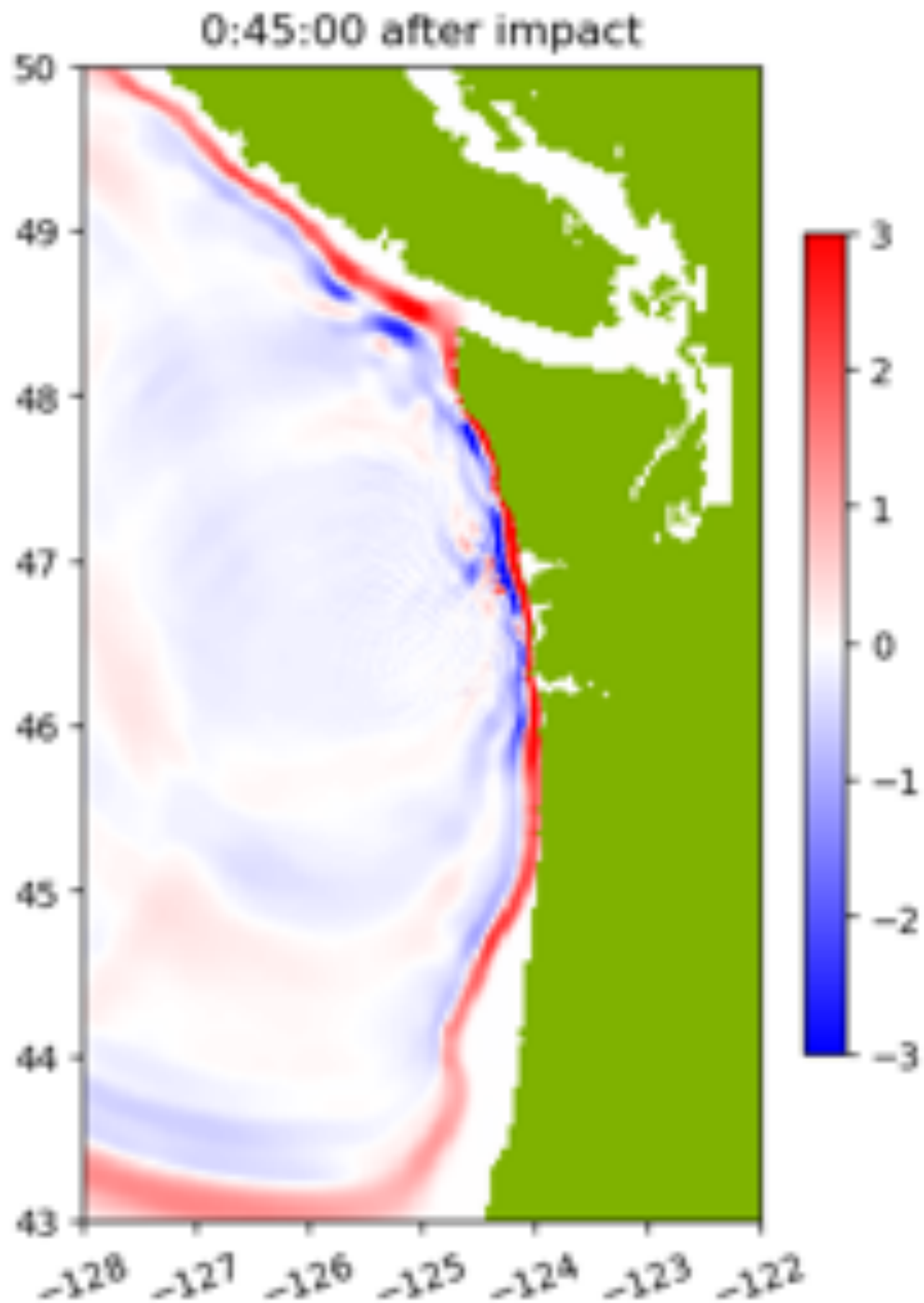
Boussinesq Simulation - zoom

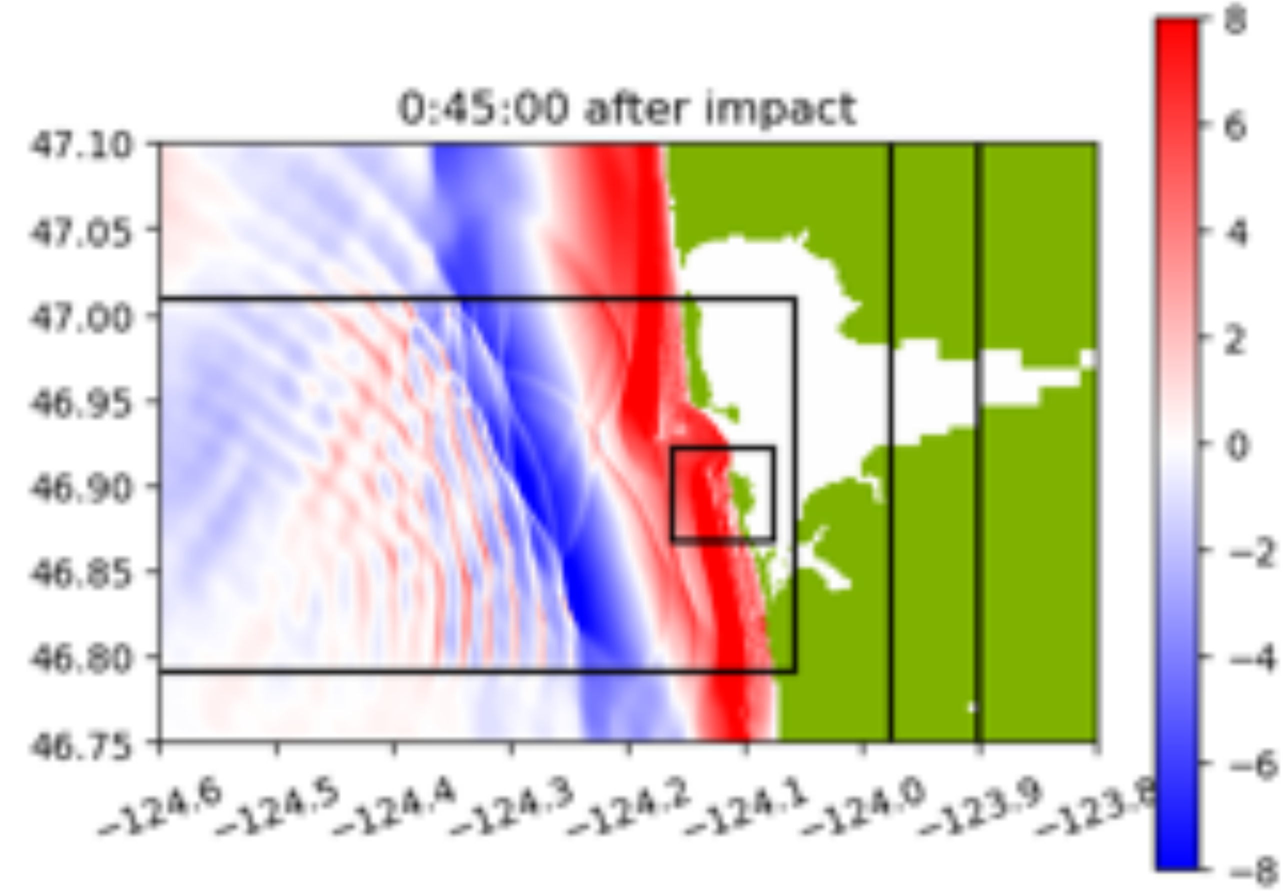




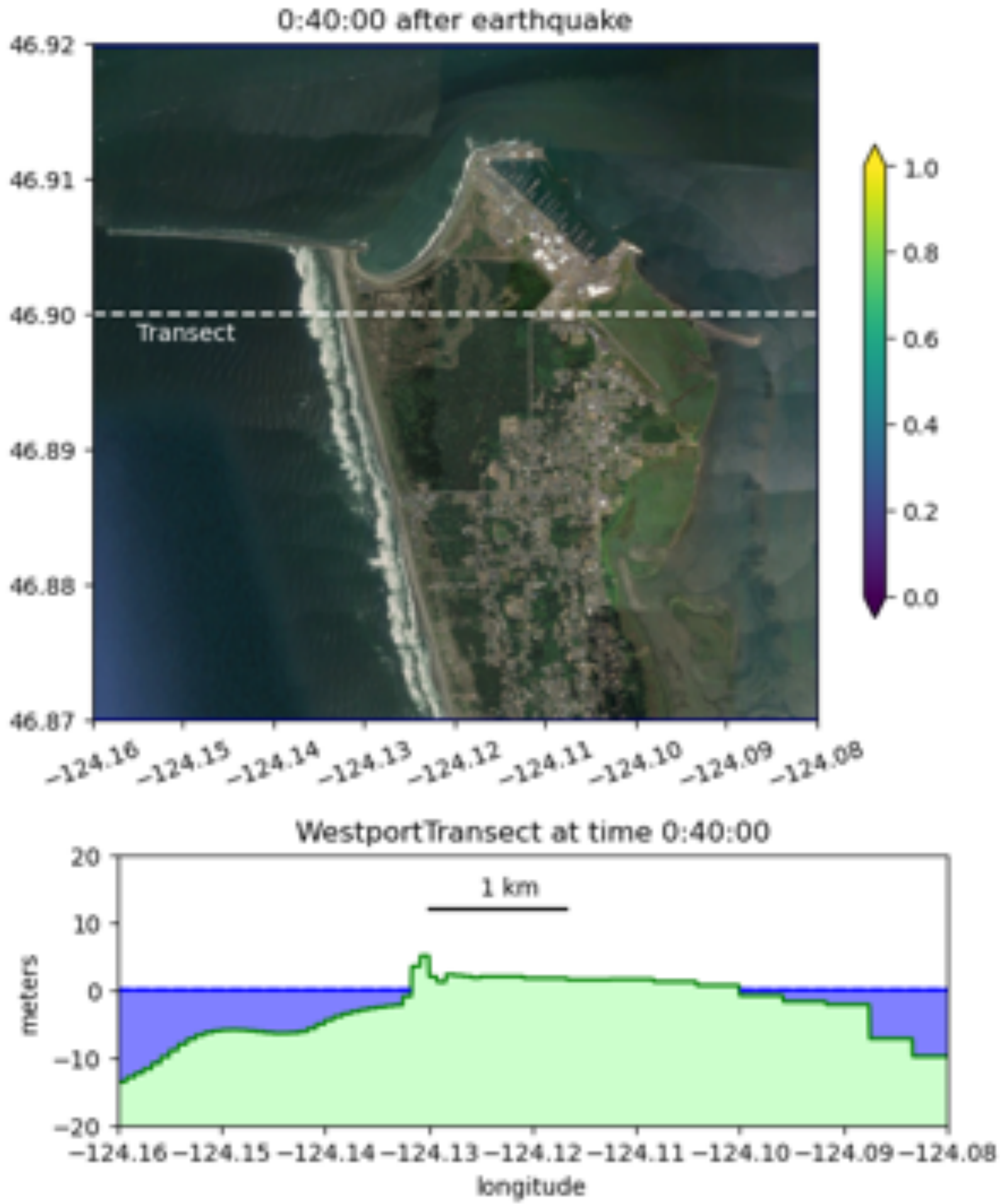
Dispersion leads to “soliton fission” near coast

Boussinesq Simulation - zoom

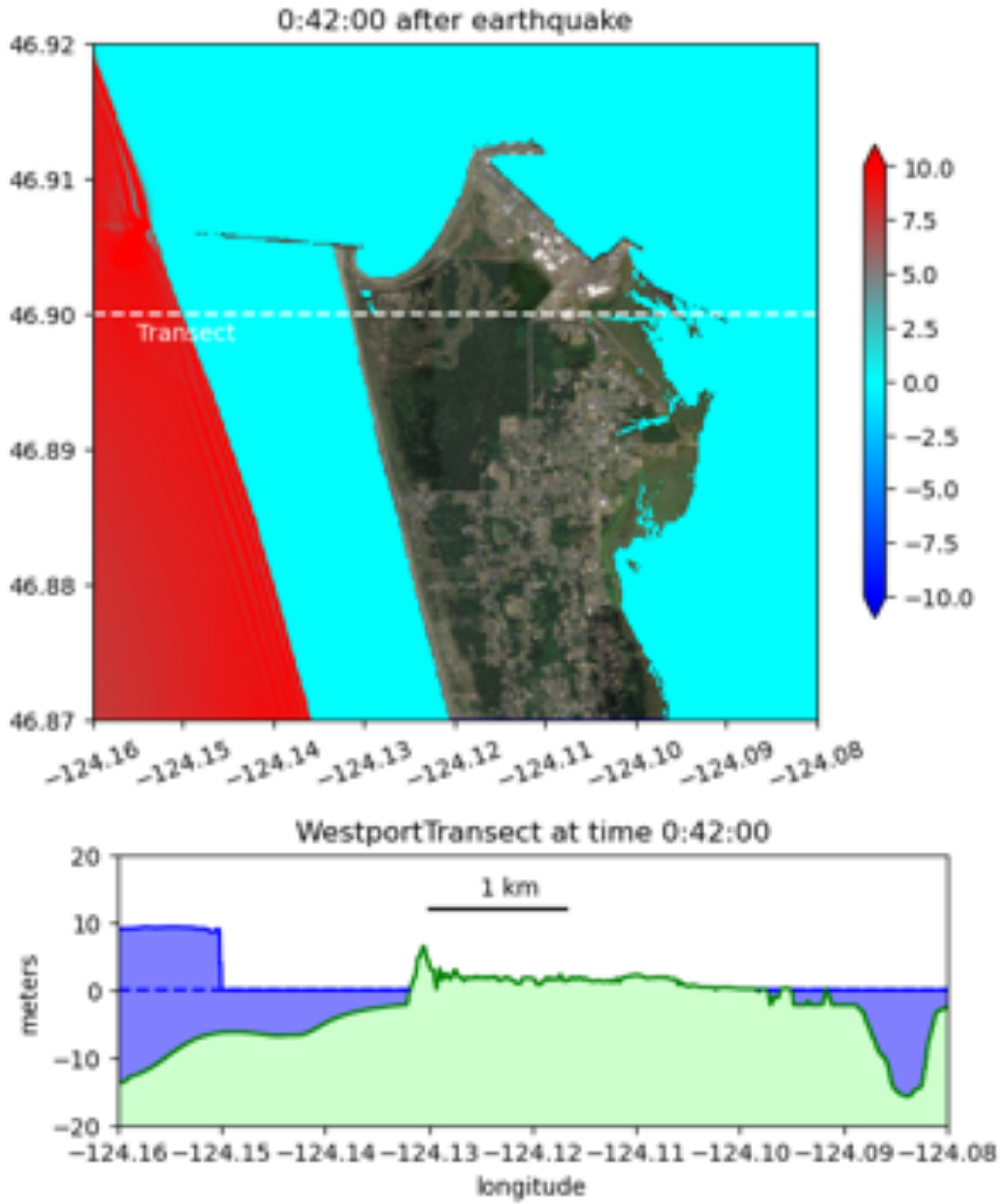




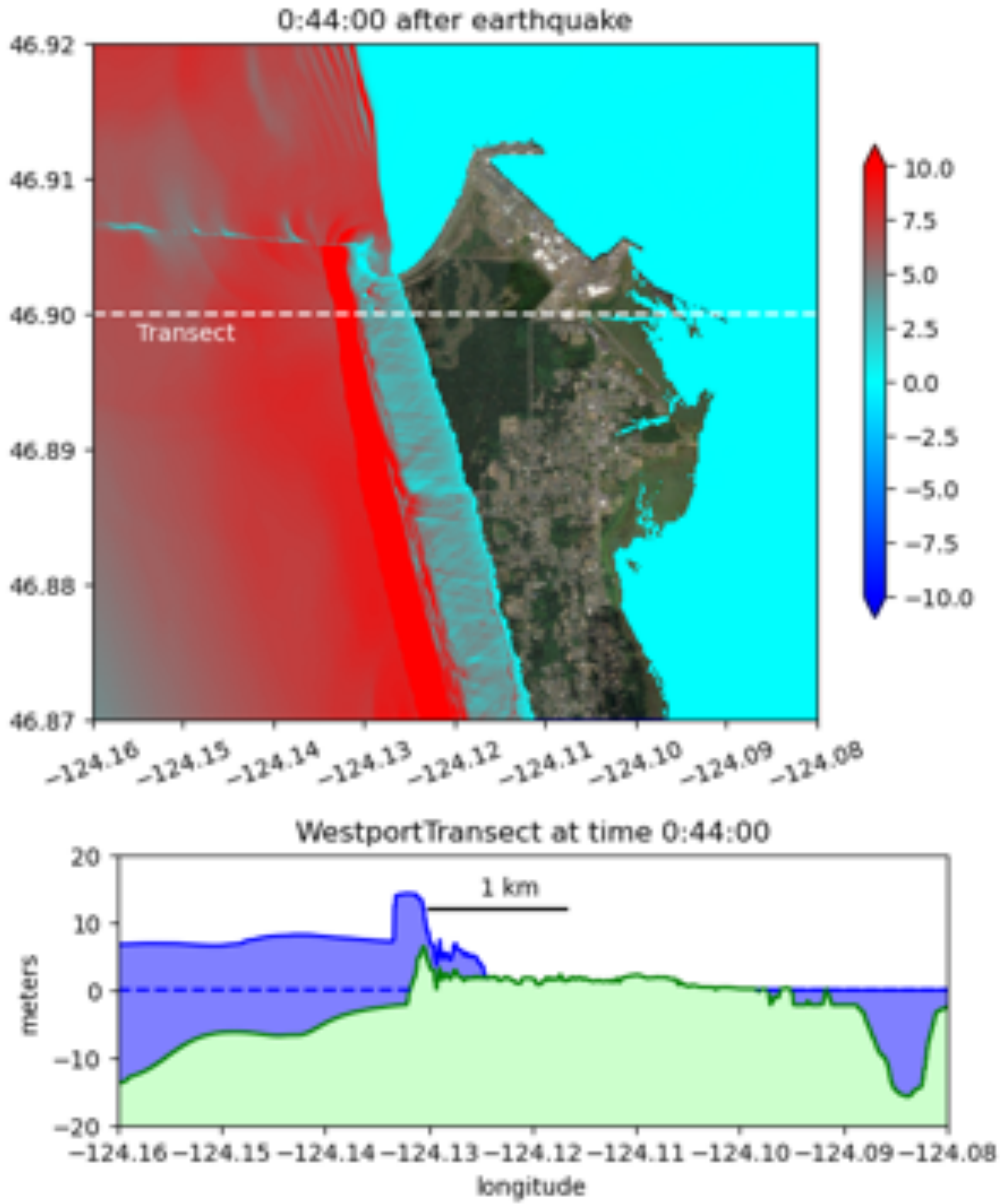
Dispersion leads to “soliton fission” near coast



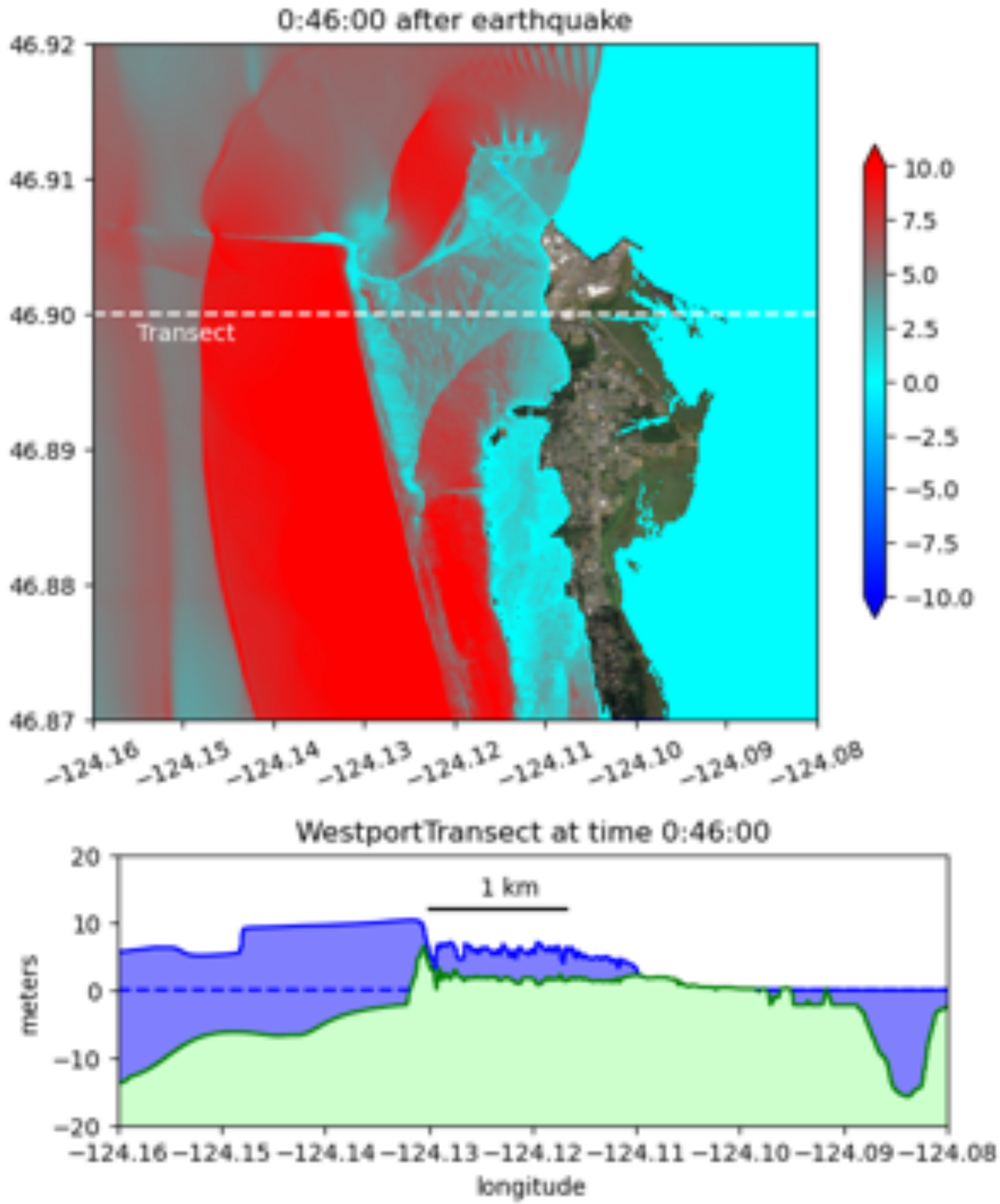
0:40:00 after earthquake
46.92
46.91
46.90
46.89
46.88
46.87
Transect
1.0
0.8
0.6
0.4
0.2
0.0
−124.16
−124.15
−124.14
−124.13
−124.12
−124.11
−124.10
−124.09
−124.08
WestportTransect at time 0:40:00
1 km
20
10
0
−10
−20
meters
longitude



0:42:00 after earthquake
46.92
46.91
46.90
46.89
46.88
46.87
Transect
−124.16
−124.15
−124.14
−124.13
−124.12
−124.11
−124.10
−124.09
−124.08
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
WestportTransect at time 0:42:00
1 km
20
10
0
−10
−20
meters
longitude



0:44:00 after earthquake
46.92
46.91
46.90
46.89
46.88
46.87
Transect
−124.16
−124.15
−124.14
−124.13
−124.12
−124.11
−124.10
−124.09
−124.08
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
WestportTransect at time 0:44:00
1 km
20
10
0
−10
−20
meters
longitude



0:46:00 after earthquake
46.92
46.91
46.90
46.89
46.88
46.87
Transect
−124.16
−124.15
−124.14
−124.13
−124.12
−124.11
−124.10
−124.09
−124.08
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
WestportTransect at time 0:46:00
1 km
20
10
0
−10
−20
meters
longitude

Conclusions

- We can do simulations of asteroid-generated tsunamis from air-bursts using the SWE
  - SWE not right model for high fidelity simulations. Can we use for parameter studies of inundation?
- Model problems in 1D to qualitatively understand airburst results
  - For SWE, response proportional to depth; seen in all codes
  - Linearized Euler, bringing in compressibility and dispersion, show reduced amplitude for both
- Simulations with Boussinesq equations using AMR, for impact craters

Future work

- Need better initialization to prevent small waves moving in wrong direction
- Solve Boussinesq equations via splitting. is it more accurate to solve coupled system when patch levels at same time? Can second solve be eliminated? What is right model?
- Switch to SWE when close to shore. How much impact on inundation is switching criteria?
- Tonga data for validation?