ABEL
PRISEN

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2007 to

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“for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations.”

Probability theory is the mathematical tool for analyzing situations governed by chance. The law of large numbers, discovered by Jacob Bernoulli in the eighteenth century, shows that the average outcome of a long sequence of coin tosses is usually close to the expected value. Yet the unexpected happens, and the question is: how? The theory of large deviations studies the occurrence of rare events. This subject has concrete applications to fields as diverse as physics, biology, economics, statistics, computer science, and engineering.

The law of large numbers states that the probability of a deviation beyond a given level goes to zero. However, for practical applications, it is crucial to know how fast it vanishes. For example, what capital reserves are needed to keep the probability of default of an insurance company below acceptable levels? In analyzing such actuarial “ruin problems”, Harald Cramér discovered in 1937 that standard approximations based on the Central Limit Theorem (as visualized by the bell curve) are actually misleading. He then found the first precise estimates of large deviations for a sequence of independent random variables. It took 30 years before Varadhan discovered the underlying general principles and began to demonstrate their tremendous scope, far beyond the classical setting of independent trials.

In his landmark paper “Asymptotic probabilities and differential equations” in 1966 and his surprising solution of the polaron problem of Euclidean quantum field theory in 1969, Varadhan began to shape a general theory of large deviations that was much more than a quantitative improvement of convergence rates. It addresses a fundamental question: what is the qualitative behaviour of a stochastic system if it deviates from the ergodic behaviour predicted by some law of large numbers or if it arises as a small perturbation of a deterministic system? The key to the answer is a powerful variational principle that describes the unexpected behaviour in terms of a new probabilistic model minimizing a suitable entropy distance to the initial probability measure. In a series of joint papers with Monroe D. Donsker exploring the hierarchy of large deviations in the context of Markov processes, Varadhan demonstrated the relevance and the power of this new approach. A striking application is their solution of a conjecture of Mark Kac concerning large time asymptotics of a tubular neighbourhood of the Brownian motion path, the so-called “Wiener sausage”.

Varadhan’s theory of large deviations provides a unifying and efficient method for clarifying a rich variety of phenomena arising in complex stochastic systems, in fields as diverse as quantum field theory, statistical physics, population dynamics, econometrics and finance, and traffic engineering. It has also greatly expanded our ability to use computers to simulate and analyze the occurrence of rare events. Over the last four decades, the theory of large deviations has become a cornerstone of modern probability, both pure and applied.

Varadhan has made key contributions in several other areas of probability. In joint work with Daniel W. Stroock, he developed a martingale method for characterizing diffusion processes, such as solutions of stochastic differential equations. This new approach turned out to be an extremely powerful way of constructing new Markov processes, for example infinite-dimensional diffusions arising in population genetics.
Another major theme is the analysis of hydrodynamical limits describing the macroscopic behaviour of very large systems of interacting particles. A first breakthrough came in joint work with Maozheng Guo and George C. Papanicolaou on gradient models. Varadhan went even further by showing how to handle non-gradient models, greatly extending the scope of the theory. His ideas also had a strong influence on the analysis of random walks in a random environment. His name is now attached to the method of “viewing the environment from the travelling particle”, one of the few general tools in the field.

Varadhan’s work has great conceptual strength and ageless beauty. His ideas have been hugely influential and will continue to stimulate further research for a long time.

The prize amount is NOK 6,000,000 (USD 875,000, GBP 475,000, EUR 710,000) and was awarded for the first time in 2003 to Jean-Pierre Serre.