

Masaki Kashiwara

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Citation: “The Chern medal is awarded to Masaki Kashiwara for his outstanding and foundational contributions to algebraic analysis and representation theory sustained over a period of almost 50 years.”

Masaki Kashiwara is a builder of bridges, a creator of tools, and a visionary who opens new mathematical worlds to explore. Over his nearly 50 years in mathematics, he has opened a new field and proven astonishing theorems with methods no one had imagined.

Kashiwara’s earliest, enormous contribution was his development of a tool known as a “D-module.” D-modules are an elaborate structure woven out of differential equations, which are one of the most fundamental mathematical tools used throughout science. Differential equations themselves belong to a branch of mathematics known as analysis, while the loom Kashiwara built for them belongs to a branch known as algebra. Therefore D-modules live in both these mathematical worlds, creating a bridge between the two fields that allows objects and methods in one field to be carried over to the other. He developed this bridge so much that it became the foundation of a new field, algebraic analysis.

D-modules were initially created by Kashiwara’s professor in graduate school, Mikio Sato. And they worked together to fully understand D-modules, discovering all the different types and how those types relate to one another.

Then Kashiwara put D-modules to work, demonstrating how fantastically powerful they could be. He used them to prove a longstanding, extremely important conjecture, the Riemann-Hilbert correspondence, which asks a fundamental question about differential equations. These equations are sometimes not defined at every point; for example, $y=1/x$ is not defined at the point $x=0$. These points are called “singularities,” and the equations can behave in different ways near them. Kashiwara showed how to find all the differential equations that behave in a particular way near those points. (The simplest case, when the equation is in a single variable, had been solved in the 1960s.)

But the methods he used accomplished far more than simply proving that conjecture. He proved it by building yet another bridge between fields, this time between algebraic analysis and a field known as topology. He showed that there was a perfect, one-to-one correspondence between a particular type of D-module and a topological object known as a “perverse sheaf.”

He then applied this to a third field, representation theory. Representation theory asks a particular type of question about symmetry. One of the most fundamental questions in mathematics is about all the different types of symmetries there might be. In the physical

world, we generally experience only a few fundamental types: the mirror symmetry of a face, the rotational symmetry of a snowflake, the translational symmetry of a frieze pattern in wallpaper, and combinations of those, like that of a corkscrew. But in higher dimensions, there are infinitely many possibilities. The question of representation theory is: What are all the different mathematical objects that exhibit any particular type of symmetry?

Together with Jean-Luc Brylinski (and simultaneously with another group, Alexandre Beilinson and Joseph Bernstein), he astounded specialists in representation theory by proving something known as the Kazhdan-Lusztig conjecture, which lies at the juncture of algebra, analysis and geometry. The method of the proof struck most mathematicians as nearly magical because it was so brilliant and unexpected. Kashiwara went on, together with Toshiyuki Tanisaki, to prove an even more general and powerful form of the conjecture. The proof revolutionized representation theory, transforming it into its modern form.

Kashiwara also invented another key tool in representation theory known as “crystal bases.” There are often many, many different mathematical objects that exhibit a particular symmetry, and these objects can be related in complex ways that are very difficult to understand. A crystal basis is a kind of skeleton showing how these objects are interrelated and revealing those relationships in a simple graph. Suppose, for example, that the underlying symmetry is composed of two different symmetries, the way the symmetry of a corkscrew is a combination of rotation and translation. Then the crystal basis will be a graph that has two unconnected pieces. Crystal bases are now used throughout representation theory.

His accomplishments continue to this day. He is still developing the theory of crystal bases and of D-modules, proving an extension of the Riemann-Hilbert correspondence in 2016. He has also built bridges to other fields as well, including symplectic geometry.

In addition, he has inspired many other mathematicians through his ideas. He has written several books that have become the bibles of their fields. He was also director of the Research Institute for Mathematical Sciences at Kyoto University and vice president of the International Mathematics Union.