— Figalli short citation —

For contributions to the theory of optimal transport and its applications in partial differential equations, metric geometry and probability.

— Figalli long citation —

Alessio Figalli has made multiple fundamental advances in the theory of optimal transport, while also applying this theory in novel ways to other areas of mathematics. Only a few of his numerous results in these areas are described here.

Figalli's joint work with De Philippis on regularity for the Monge-Ampère equation is a groundbreaking result filling the gap between gradient estimates discovered by Caffarelli and full Sobolev regularity of the second derivatives of the convex solution of the Monge-Ampère equation with merely bounded right-hand side. The result is almost optimal in view of existing counterexamples. It has direct implications on regularity of the optimal transport maps, and on regularity to semigeostrophic equations.

Figalli initiated the study of the singular set of optimal transport maps and obtained the first definite results in this direction: he showed that it has null Lebesgue measure in full generality. He has also given significant contributions to the theory of obstacles problems, introducing new methods to analyze the structure of the free boundary.

Figalli and his coauthors have also applied optimal transport methods in a striking fashion to obtain sharp quantitative stability results for several fundamental geometric inequalities, such as the isoperimetric and Brunn-Minkowski inequalities, without any additional assumptions of regularity on the objects to which these inequalities are applied; the methods are also not reliant on Euclidean symmetries, extending in particular to the Wulff inequality to yield a quantitative description of the low-energy states of crystals.