International
Mathematical
Union

The Carl Friedrich Gauss Prize 2022

Short citation:

The Gauss prize is awarded to Elliott H. Lieb for deep mathematical contributions of exceptional breadth which have shaped the fields of quantum mechanics, statistical mechanics, computational chemistry, and quantum information theory.

Long citation:

Elliott H. Lieb is a mathematical physicist who has made outstanding contributions to physics, chemistry, and pure mathematics.

Reminiscent of Gauss and other 18th and 19th century giants, Elliott H. Lieb, driven by problems in and applications to physics, has unraveled elegant and fundamental mathematical structures, vastly transcending the original motivations.

In doing so, Lieb has introduced concepts which have shaped whole fields of research in mathematics even beyond his original area, while having a transformative impact on physics and chemistry.

His work from the 60s discovering the exact solvability of some fundamental models (square ice, Lieb–Liniger, Lieb–Mattis, Temperley–Lieb) contributed to the definition of much of modern statistical mechanics, and to the foundations of the modern field of integrable probability. His exact solutions of percolation and coloring problems, introducing what is now called the Temperley–Lieb algebra along the way, also had long-lasting algebraic and combinatorics implications, for knot invariants, quantum groups, braid groups and braid statistics, and also conformal field theory.

Lieb developed a whole program of the analysis of “stability of matter” and “existence of the thermodynamic limit” putting on firm mathematical grounds the study of quantum particles interacting through physically realistic potentials. An essential ingredient in this analysis is the celebrated Lieb–Thirring inequality estimating the sum of negative eigenvalues of Schrödinger operators. It has had an enormous impact over the years on spectral estimates, in particular on semi-classical analysis.

One of the most important computational tools of modern chemistry is density functional theory (DFT). Lieb provided a mathematically sound formulation of it, the now famous Lieb or Levy–Lieb functional most widely used in theoretical chemistry. A very important concept in DFT is to understand what is known as the indirect Coulomb energy. Lieb, partly in collaboration with Oxford, gave rigorous bounds on it, providing an ultimate benchmark for all suggested forms of the indirect energy. These very important bounds have been abundantly used and cited by chemists.

Lieb has proved several celebrated results in the area of matrix analysis including what became known as Lieb's concavity theorem. As a consequence of the latter, he and Ruskai were able to prove the strong subadditivity of the von Neumann entropy, a result which decades later has become the foundational result in the very active modern area of quantum information theory.

Lieb has made fundamental contributions to the areas of functional analysis and functional inequalities. He has both proved a long list of new functional inequalities, some of which now bear his name, as for instance the Brascamp–Lieb inequality, and brought existing ones into their sharp form, as for instance the Young and the Hardy–Littlewood–Sobolev inequalities. He has developed and masterly applied symmetrization and compactness methods. These works of Lieb are widely used by analysts, probabilistic and mathematical physicists and continue to have a tremendous impact in mathematics.

Elliott H. Lieb's work has a truly outstanding combination: as mathematical work, his contributions have a hard-to-rival impact on other sciences, and as applied work, his contributions have a hard-to-rival mathematical depth. It continues the tradition of a dialogue at the highest level between mathematics and physics, and beautifully demonstrates the power of mathematics as a theoretical and practical tool to understand nature.