

2022 Leelavati Prize: Nikolai Andreev

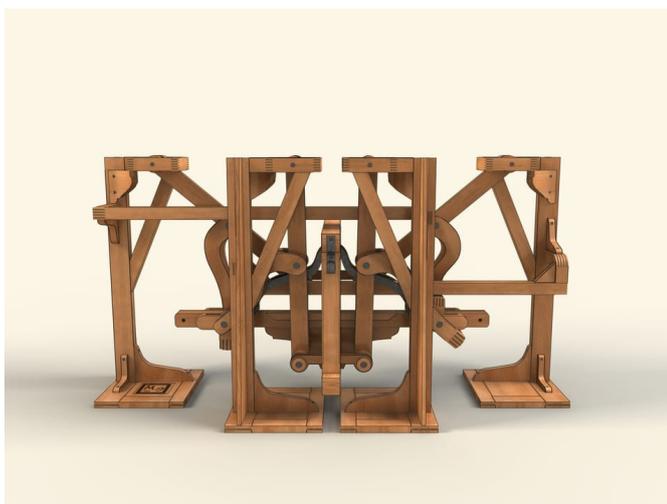
by Allyn Jackson

On the screen is a schematic diagram showing three rods—one long, one medium, one short. Their arrangement looks a bit like the Greek letter lambda, λ . The background is blank, revealing nothing. Touches of color give hints: red for fixed ends of the rods, blue for free ends, and gray for joints.

As you start to wonder, What is this thing?, it all starts to move. The free end of the short rod traces out a circle. That motion drives the long rod to trace out a closed curve shaped like a mushroom cap, flat on the bottom, domed on the top. Nothing goes by too fast, so you absorb without effort the equality of the phase of the circular and mushroom-cap motions.

Now a new rod takes its place, one end attached so as to trace out the mushroom curve, the other sporting a flat block looking vaguely like a foot. A mirror-image duplicate of the ensemble joins the first, linked to coordinate the phases. Now there are two feet, and it dawns on you that this thing is *walking*.

It ambles out of the picture, leaving the screen momentarily blank. That was the skeleton. Enter now the *real* walking machine, physically rendered in wood. As it executes its precise gait, the circle and the mushroom curve hover like ghosts in your mind. The wooden creature—it somehow has life to it—turns away and continues its journey off into the distance. The impression is quietly electrifying.



Wooden model of Tchebyshev's walking machine.

The movie just described is one of the many works of Nikolai Andreev, head of the Laboratory of Popularization and Promotion of Mathematics at the Steklov Mathematical Institute. This particular movie is based on the original mechanism designed and built by the mathematician Pafnuty Tchebyshev (1821–1894) for the 1878 World’s Fair in Paris. When the movie was posted on the web in 2007, it garnered more than one hundred thousand views on its first day. Since then has been shared and reproduced countless times.

An Unusual Approach

Trained as a mathematician, Andreev completed his PhD degree in 2000 in the Faculty of Mechanics and Mathematics of Moscow State University. That same year he joined the staff at the Steklov Institute and began to work on popularization. He became head of the laboratory when it was founded in 2010, and today he has a team of three: Roman Koksharov (illustrator and web-developer), Alexander Leshchinsky (woodworker and creator of mathematical models), and Nikita Panyunin (mathematics researcher). Together they are inventing new forms through which to bring the rich and distinctive Eastern European culture of mathematics to a wide audience.

The team’s approach to popularization is unusual. They are not trying to explain the latest mathematics, nor even to focus mainly on topics important in the history of the subject. They do not rely on the usefulness of mathematics as a lure. Their materials are not primarily didactic, though they have been used in education. There is no exciting packaging, no attempt to dazzle or divert. In shedding preconceptions about what makes popularization successful, Andreev’s group allows the mathematics to shine.

Their main activity is production of animations, which now number close to 200 and all of which are freely available on the internet. In today’s video-saturated world, where expositions about mathematics often feature complicated graphics like pictures of fractals or even cute cartoon characters, Andreev’s movies stand out for their minimalist approach.

Without background sound, the mathematics unfolds in serenity. The three-dimensional graphics are state-of-the-art and purposely spare. Exactly which details are included, the proportion of the figures, the choice of the colors, the pacing of the action—everything conspires to evoke the mathematics, so much so that one often has the sense of seeing mathematics in its native land. Nevertheless the movies are not somber affairs. They strike a contemplative tone but with a touch of whimsy. One senses the smile of the artist behind each one.

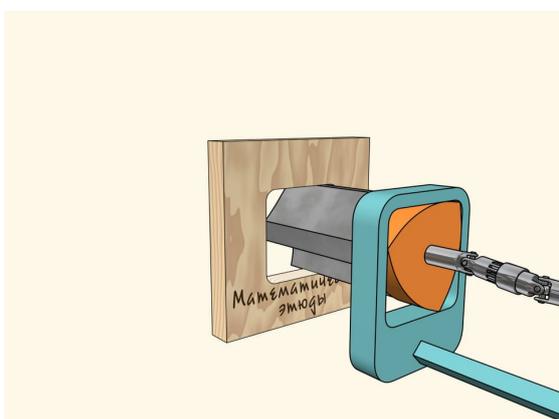
A Potato Chip, A Sausage, A Sheet of Paper

If you cut a straight slit in the lid of a potato chip can, could you slide a chip through? You could if the chip belongs to the class of surfaces known as ruled surfaces, as shown in the animation “Chips as Hyperbolic Paraboloid”. How can you create a perfect sine wave? In “Sine wave: cylinder net”, a knife slices a sausage crosswise at an angle of 45 degrees, then slices the casing along the length of the sausage. The sausage disappears, leaving just the casing, which unrolls to reveal that one of its edges is now a sine wave. Viewers can use the animations as a basis to craft physical models of their own.



A hyperbolic paraboloid passes through a slit in the lid of a potato chip can.

Folding of paper is a running theme in the animations. “Piecewise Linear Embedding of a Polyhedron” creates out of a flat piece of paper a three-dimensional shape that is curved everywhere and nowhere flat. In another video, a paper-folding technique illustrates a proof that the infinite sum of reciprocals of powers of two equals 1. Another theme in the animations is the Reuleaux triangle, a two-dimensional figure that rolls as smoothly as a circle and can perform all sorts of amazing feats, like drilling a square hole in wood.



The cross-section of the orange piece is a Reuleaux triangle, which allows one to drill a square hole.

A few of the videos touch on unsolved problems. For example, one video invites viewers to enter sequences of digits and locates those sequences in the infinite, nonrepeating decimal expansion of π . This leads to the question: Is π a *normal* number? That is, do all finite sequences of digits have equal likelihood of appearing in the decimal expansion of π ? Although the question has been around for more than a century, no one knows the answer.

One group of animations, coming under the rubric “Tchebyshev’s Mechanisms”, provide on-screen elucidation

of the ingenious mechanical devices, like the walking machine discussed above, that Tchebyshev designed and had built. Andreev is the curator for some of Tchebyshev's original wooden models, while others are housed in various museums. Those models served as source material for the "Tchebyshev's Mechanisms" animations.

From Multimedia to Print

Andreev's lab has a few other historical models; one is a model of inscribed Platonic solids that the mathematician V.I. Arnold (1937–2010) made as a child from pieces of wooden ski poles. In addition, the lab has continued the tradition of creating wooden models that "do math" by crafting many of its own. Team member Alexander Leshchinsky is a master woodworker who has developed deep intuition for creating wooden models that are both artistic and mathematically precise.

In his two decades as a mathematics popularizer, Andreev has made around a thousand presentations: talks for schoolchildren, seminars for schoolteachers, conference lectures, and master classes. In these presentations he often uses physical models to illustrate the ideas, and he has worked closely with schoolteachers to explain how the models are designed and produced. The teachers then help their students to build their own models. If the students successfully present a model in their school, they can earn a small prize from Andreev's lab.



In English, the book's title might be A Mathematical Take on Things.

The prize is a book, which stands as yet another distinctive work of the lab. After spending a decade on multimedia materials, the members of the lab returned to old roots in the written word. There were several reasons for this, one being that reading a book often leads more easily to deep contemplation than does staring at a computer screen. The members of the lab served as editors for the book and solicited contributions by thirty-two leading mathematicians, including three Fields Medalists.

The book, written in Russian, has a title that is difficult to translate adequately into English; possibilities include *Mathematical Component*, *Mathematical Essence*, or perhaps even *A Mathematical Take on Things*. The book is close in spirit to *Kvant*, the legendary mathematics magazine that began in the Soviet Union in 1970 and remained highly popular until it ceased publication in 2011. The aim of the book is to evoke the mathematics that is present all around us, in the great achievements of human civilization as well as in the more modest setting of everyday life.

In addition to the thirty-two solicited essays, the lab team wrote a few dozen of their own. The essays run from two to perhaps five or six pages and cover topics from cryptography to mechanics, from language to pattern formation. As with the animations, the choice of topics is highly original; nothing is trite. The first edition appeared in 2015 and is one-third the size of the second edition, which came out in 2019. The book has been very popular and won two awards for literature about science aimed at a general audience. Hopes are high for translations into other languages.

Reaching Across Barriers

The work of Nikolai Andreev and his team constitutes an outstanding artistic and scientific achievement. Reaching across barriers of geography, language and culture, it brings the delight of mathematics to people of all ages. Perhaps more importantly, it inculcates respect for truth and rational thought. Their work is a positive force for unity the world over.

Curriculum vitae of Nikolai Andreev:

<http://www.mathnet.ru/eng/person9017>