Mathematics in Africa: Challenges and Opportunities

A Report to the John Templeton Foundation

from the
Developing Countries Strategies Group
International Mathematical Union

February 2009

Preface

The John Templeton Foundation requested this study from the Developing Countries Strategy Group of the International Mathematical Union (IMU). The purpose was to generate a brief report on the current state of mathematics in Africa and opportunities for new initiatives to support mathematical development. The International Mathematical Union wishes to thank the John Templeton Foundation for the opportunity to prepare and submit this report on *Mathematics in Africa: Challenges and Opportunities*.

For many years the IMU has endeavored to support our colleagues on the African continent in their efforts to advance the work of mathematics and mathematics teaching. Traditionally this support took the form of travel research grants and some modest support for mathematics conferences held in the developing world.

In recent years IMU support for mathematics in Africa has increased due to the formation of a special IMU committee, the Committee for Developing Countries, whose duty is to distribute funds and human resources which IMU has been able to attract and dedicate to this effort. On the African continent, the chief focus of IMU in recent years has been to identify and support local mathematical leadership, both in mathematics research and in mathematics teaching. On the mathematics research side, IMU has identified the mathematics network called the African Mathematics Millennium Science Initiative as a vehicle for 'on-continent' leadership well aligned with IMU's norms and goals. The Ad Hoc Committee that oversaw the preparation of this report had almost equal representation from IMU and AMMSI leadership. This Committee, which takes full responsibility for the content of this report, is made up of:

John Ball, Mathematical Institute, Oxford University, Oxford, United Kingdom
David Bekollé, Department of Mathematics, University of Yaoundé, Cameroon, appointed since 2005 as Vice-Rector, University of Ngaoundere;
Herbert Clemens, Developing Countries Strategy Group, International Mathematical Union
Arlen Hastings, Executive Director, Science Initiative Group, Princeton New Jersey, USA
Edward Lungu, Department of Mathematics, University of Botswana, Gabarone, Botswana
Wandera Ogana, School of Mathematics, University of Nairobi, Kenya

On the mathematics education side, IMU's work in Africa has been handled largely by its International Commission on Mathematical Instruction (ICMI). In the area of mathematics teaching, which has been of special interest to the Templeton Foundation, we called upon the expertise of ICMI Vice-President, Professor Jill Adler, of South Africa. Indeed, the information about mathematics teaching in a representative cross-section of African countries prepared by her group at the University of Witwatersrand is a major component of this report.

The period agreed with Templeton for completion of this study was only five months, so the observations and suggestions below are neither comprehensive nor fully representative. However, in the preparation of this report, the Ad Hoc Committee cast as wide a net as it could, given the constraints of time and budget under which it did its work. In short, the IMU has attempted to make the report as balanced and objective as possible within this limited framework.

The information collected by IMU for this report comes from 19 institutions and individuals invited by IMU to contribute. We did not seek elaborate or comprehensive reports from our advisors, but rather brief papers based on their own experience and from their own or their organization's point of view. They were asked to select topics or questions from a comprehensive set we provided (see "Background Information" at the end of this report). Our advisors were asked to write about only those topics in which they had experience and/or expertise.

The majority of these advisors are drawn from the ranks of university faculty. As mentioned above, our comments on primary and secondary education rely heavily on a report submitted by M. K. Mhlolo, L. Nyaumwe, and J. Adler of the Marang Centre for Mathematics and Science Education, University of the Witwatersrand, South Africa.

The advisors who contributed to this effort include:

Leif Abrahamsson, Department of Mathematics, Uppsala University, Sweden; International Science Programme, mathematics
Jacek Banasiak, Professor/Head of School, School of Mathematical Sciences, University of KwaZulu-Natal, South Africa
Augustin Banyaga, Professor of Mathematics, Penn State University, USA, and Adjunct Professor, University of Abomey-Calavi, Benin
Jacob (Jaap) Bregman, Lead Education Specialist, Africa Region, The World Bank, Washington DC, USA.
Wilson Mahera Charles, Department of Mathematics, University of Dar es Salaam, Tanzania
Etienne Desquith, Institut de Recherches Mathématiques d'Abidjan, Côte d'Ivoire
Wilfrid Gangbo, School of Mathematics, Georgia Institute of Technology, Atlanta, GA, USA
Abba B. Gumel, Department of Mathematics, University of Manitoba, Canada
M. N. Hounkonnou, International Chair in Mathematical Physics and Applications (ICMPA-UNESCO Chair), Cotonou, Benin; TWAS Research Professor, University of Zambia, Lusaka
M. K. Mhlolo, L. Nyaumwe, and Jill Adler, Marang Centre for Mathematics and Science Education, University of the Witwatersrand, South Africa; Prof. Adler is also affiliated with King's College, London
Joseph Y. T. Mugisha, Professor of Biomathematics, Department of Mathematics, Makerere University, Kampala, Uganda

A.-S. F. Obada, Mathematics Department, Faculty of Science, Al-Azhar University, Cairo, Egypt
Marie-Françoise Roy, Institute of Mathematical Research, University of Rennes, France; Scientific Advisor for Sub-Saharan Africa, CIMPA/ICPAM; co-coordinator, RAGAAD
Daouda Sangare, Editor-in-Chief, *Afrika Matematika*
Temba Shonhiwa, School of Mathematics, University of the Witwatersrand, Johannesburg, South Africa
Precious Sibanda, School of Mathematical Sciences, University of KwaZulu-Natal, South Africa
George Thompson, Office of External Affairs, International Center for Theoretical Physics (ICTP), Trieste, Italy
Hamidou Touré, University of Ouagadougou, Burkina Faso
Nouzha el Yacoubi, Secretary General, African Mathematical Union

The fact that, of the 65 institutions/individuals invited to contribute to this report, only 18 were able to provide information is due at least in part to the short turn-around time of four to six weeks and the uneven performance of electronic communication with Africa. Although the observations of our advisors vary somewhat according to their geographic and disciplinary affiliation, their comments and emphases were surprisingly consistent. This lends confidence to the accuracy of the remarks offered in this report. The IMU wishes to formally thank these experts for their time and for their willingness to share their considerable knowledge. Any conclusions or recommendations in this study are however the responsibility of the IMU Ad Hoc Committee and do not necessarily represent the view of individual advisors.

The report was drafted by Mr. Alan Anderson, a writer for the Science Initiative Group and the U.S. National Academy of Sciences, and revised by the Ad Hoc Committee. Information collection was managed by Ms. Sharon Berry Laurenti, Administrator of the IMU Developing Countries Strategy Group.

Executive Summary

A continent of contrasts
The 50-plus nations of Africa exhibit huge variability in size, population level, wealth and culture. These variations are also seen in educational development, specifically in mathematics. For example, according to figures extracted from the database of MathSciNet,[1] the number of PhD holders in mathematics in South Africa, as of 2007, was 502; the number in nearby Tanzania – a country of comparable area and population – was only 38. Similarly, of the population aged 10 to 18 years in Mozambique and Burkina Faso, fewer than 10% attend

[1] MathSciNet is an electronic publication of the American Mathematics Society (AMS) that continues the tradition of the paper publication *Mathematical Reviews*, which was first published in 1940. The items in the MathSciNet database are classified in accordance with the Mathematics Subject Classification (MSC) established by the AMS. It should be noted that MathSciNet may well underestimate numbers of publications by mathematicians because it does not cover many science, engineering, education, and regional journals in which mathematicians publish. Nonetheless, the database is unique in its global reach and has some comparative value for the present discussion.

secondary school, while in South Africa and Swaziland more than 50% of that age group attend secondary school.

Another way to summarize the disparities in mathematical development is to look at the continent as a whole. We might take five different (and approximate) sections of the "mathematical geography." (1) North Africa is a relatively advanced region, partly due to governments' broad commitment to research and education at all levels and sustained support from nearby southern Europe. (2) Francophone Africa, reflecting its colonial past and continued support from the world-wide *Francophonie*, is relatively advanced in secondary and tertiary mathematics, but its strength in pure mathematics is not matched by activities in applied fields. This limits the ability of students to see mathematics-related career opportunities beyond teaching. (3) The Anglophone countries of East Africa are generally weaker in pure mathematics, with most emphasis directed toward applied fields. (4) Some of the smaller countries in Southern Africa offer exciting models of collaboration, primarily Tanzania, Botswana, and Zimbabwe, as well as the other countries of the Southern African Development Community (SADC). (5) Some South African schools, notably private schools and formerly all-white universities, have achieved relative strength in mathematics. Its capacity nationwide, however, is very low – a continuing effect of the neglect suffered by most institutions during apartheid.

Table 1: Regional groupings of 53 African countries

Region (no. of countries)	Countries
North Africa (5)	Algeria – Egypt – Libya – Morocco – Tunisia
East Africa (10)	Burundi – Djibouti – Eritrea – Ethiopia – Kenya – Rwanda – Somalia – Sudan – Tanzania – Uganda
Central Africa (8)	Cameroon – CARep – Chad – DRCongo – Equatorial Guinea – Gabon – Rep. Congo – Sao Tome & Principe
West Africa (16)	Benin – Burkina Faso – Cape Verde – Côte d'Ivoire – Gambia – Ghana – Guinea – Guinea-Bissau – Liberia – Mali – Mauritania – Niger – Nigeria – Senegal – Sierra Leone –Togo
Southern Africa (14)	Angola – Botswana – Comoros – Lesotho – Madagascar – Malawi – Mauritius – Mozambique – Namibia – Seychelles – South Africa – Swaziland – Zambia – Zimbabwe

United Nations, 2008.

Yet while African countries differ from one another in many features, they are broadly similar in the issues that concern our advisors – institutional and national conditions that hinder mathematical development. From their reports, it seems clear that these conditions are virtually the same throughout the continent. For example:

- In most African countries, mathematical development is limited by low numbers of secondary school teachers and mathematicians at the masters and PhD levels. With few professors to train the next generation of leaders in the field, countries cannot meet the growing demand for mathematicians with advanced, up-to-date training.

- Faculty and graduate students suffer from professional and geographic isolation. Lacking colleagues to share ideas and partnerships with, they have few opportunities to advance professionally.
- Talented students are dissuaded from careers in mathematics by low salaries, a poor public image, and a shortage of mentors engaged in exciting mathematical challenges.
- Record numbers of students clamor for a college education throughout Africa, overwhelming the teaching capacity of established public universities. New private colleges spring up to meet this demand – even though they lack qualified faculty in mathematics and many other fields. These colleges often hire professors away from public universities by offering higher salaries.
- Buildings, classrooms, and other infrastructure of most African universities were built more than 50 years ago and are inadequate for today's demand. Mathematics lecture rooms designed for classes of 30-50 students are commonly packed with 200-400 students per class who jostle and compete for space.
- Instrumentation for science courses commonly dates from the colonial days of the 1960s, limiting the ability to teach applied mathematics or interdisciplinary courses. Similarly, few mathematics students have access to even rudimentary computing facilities.

Many of these weaknesses result from low public support for education and research. For example, few governments in Africa provide funding for graduate students, common practice in most developed countries. Nor do they create national science foundations to support the research of faculty, who cannot pay graduate assistants and must often forego the work for which they were trained. Instead they spend their energy and time coping with heavy teaching loads, often at multiple institutions. With faculty ranks spread thin, institutions rely heavily on less experienced teachers, many of whom may lack tenure or even postgraduate degrees.

Teaching weakness begins in the primary schools, where mandatory education laws have caused enrollments to soar. Secondary school education suffers from a lack of teachers with mathematics training and low student participation. Dr. Bregman estimates that only about 30% of the secondary-school age group in sub-Saharan Africa attend junior secondary school and around 15-20% of the group attend senior secondary school.

Overall, the story of mathematical development in Africa is one of potential unfulfilled. Based on the outstanding achievements of some individuals and institutions, it is clear that no African country lacks talented potential mathematicians. But without a stronger educational structure at all levels, few of them are able to reach their potential.

What is needed?

So far, there are few models available to those who seek to strengthen mathematics development in Africa. But the suggestions of this report's advisors, who did not collaborate or consult with one another, centered around the same themes – more support for those who wish to become educators and researchers in mathematics, and more collaboration among institutions and people seeking to make this happen. Necessary steps include stronger teaching of primary and secondary students; more government support for teachers, faculty, and infrastructure; scholarships for graduate students and fellowships for faculty; and a clearer path to rewarding mathematics-based careers.

Each of these needs, of course, represents an enormous challenge for countries with multiple competing needs. Where can they – and those who wish to help them – begin the task?

Our advisors agree that there is no single solution, and they caution that statistics concerning numbers of mathematicians, numbers of publications, and other indicators of capacity and activity are not always complete. Nonetheless, they agree on many thoughtful suggestions about where to begin. Most of their suggestions touch on two primary needs – one that can be fully addressed only by the countries themselves, and another that offers many entry points for those who wish to donate assistance.

The first, and more expensive, is to *provide direct support for the mathematical enterprise at all levels.* This includes better teaching at the primary and secondary levels; scholarship support for postgraduates who can earn advanced degrees and expand the professoriate; fellowships for faculty who need time for research; and upgraded infrastructure, including expanded internet and computing facilities, to allow better linkages with resources and the worldwide academic community. These measures can be achieved only by a committed political leadership, including at the highest levels of government, who are fully aware of the value of the mathematical enterprise.

Given the enormity of this challenge, one might ask whether there are individual steps that offer exceptional leverage to jump-start the enterprise as a whole. For example, one step might be a program to support students of exceptional talent, identified perhaps by their participation in the mathematics Olympiads. While sending such students to top international universities, for example, is likely to produce great benefits for individuals, it was not suggested by our advisors. They felt unanimously that no "magic bullet" or quick fix could solve a problem that is systemic and institutional. Such a program might raise the visibility of mathematics among secondary school students, but this benefit could be reduced should privileged students decide to remain abroad rather than return home to unrewarding positions.

The second suggestion is to *strengthen and expand successful training and research activities, especially regional networks of people and institutions.* There are several reasons our advisors highlight this option. First, successful networks by definition involve leaders of demonstrated talent and institutions capable of supporting creditable mathematics programs. Second, supporting a network helps build a critical mass of students and faculty who are otherwise likely to be professionally isolated. Third, by building on institutions and people already in place, networks use tools that are relatively inexpensive in relation to their power, such as partnerships, mentoring, distance learning, and internet-based collaboration.

There are multiple approaches to building the "critical mass" mentioned above. Due to the relatively low cost of tuition and fees in Africa, one could support a dozen graduate students on even a small grant ($30,000-$50,000), a giant step in strengthening one or even two regional networks. Currently, the African Mathematics Millennium Science Initiative (AMMSI) offers $12,000 for scholarships to each of five AMMSI regions. Even in the Southern Africa region, where tuition is expensive, about 10 graduate students are offered supplements to their existing scholarship. These amounts have been used to support and increase the number of students at a number of graduate programs in sub-Saharan Africa.

A network, once established, can become a powerful agency for change because it focuses the authority of multiple institutions and leaders. A network might choose to use this authority to help educate the government on the need for stronger mathematics. It might also, through faculty and student exchanges, bring fresh ideas and capacities to teaching at the tertiary and postgraduate levels. It can enrich educational and research experiences by sponsoring joint conferences, mini-courses, sandwich programs, and special symposia, which can also be done at modest cost. Finally, by tapping the networks of contacts of members, networks can promote high-level international partnerships and collaborations that can raise the general level of scholarship in Africa.

Regenerating mathematics in African countries requires simultaneous support at all levels, from elementary school to postgraduate research. To concentrate on primary education alone will be futile if there are no qualified teachers; there can be no qualified teachers without skilled mentors to teach the teachers. These capacity-building activities are essential for every country because mathematics provides the fundamental underpinning of science and engineering. Every nation, even the smallest, must have sophisticated mathematicians and statisticians to help support the technology, health care, food production and economic growth to which they aspire.

More specific conclusions

Our conclusions here are provisional, suggesting further examination, as befits a 'snapshot' study of this type. Given this caution, our advisors agree the conclusions are framed by the conviction, almost universally held by mathematicians and mathematics educators, that each mathematical level of learning is grounded pyramid-like in the previous ones, and that lack of quality or capacity at any level of a country's mathematical infrastructure weakens all the levels above. Conversely the absence of some kind of pinnacle deprives the lower levels of leadership, training and context. In this sense, some of the main points of this study are the following:

a) Many, if not most, African countries lack an organized means of identifying gifted students of mathematics. Currently, the only systematic programs to identify mathematical talent are the mathematics Olympiads, either at the national or pan-African levels. National competitions seem to be in place in some countries, in many others not. Only a handful of African countries participate regularly in the pan-African Mathematics Olympiad (PAMO), sponsored by the African Mathematical Union. Thus talent is typically identified only upon reaching the university, this being the primary source of the few but talented and energetic graduate students of mathematics in some fields. Perhaps the most identifiable group of gifted students currently is the small network of African postgraduates studying mathematical biology.

b) School mathematics education in many African countries is compromised by lack of qualified teachers, caused partly by the otherwise laudable progress toward universal primary education. Even at the secondary level, the shortage of qualified, let alone highly qualified teachers is acute. Contributing factors are low salaries and low prestige of the teaching professions. Highly qualified students, especially in quantitative fields, tend toward government and the private sector, where pay and prestige are higher.

c) At the tertiary level, the shortage of mathematicians will be documented in some detail in the body of the report. Class sizes of 500 to 1000 are not uncommon in introductory university mathematics courses.

d) By history, size, and economic importance, South Africa, Egypt and Nigeria would seem to be the candidates for leadership in developing the mathematical sciences and in improving mathematics education. From the information we have received and our own experience, South Africa is currently exercising this leadership role, both in advanced mathematical training and research. Egypt, though very active mathematically, did not have international programs known to our non-Egyptian advisors. Nigeria has great potential, a relatively rich mathematical history, and considerable material resources. However, in the current period, its role on the stage of African mathematics seems to us somewhat diminished.

e) The African continent-wide mathematical organization is the African Mathematical Union. Although it was influential in earlier periods, its current activities seem confined to the organization of mathematics Olympiads and a quadrennial Pan-African Congress of Mathematicians (PACOM). (In fact, the last PACOM did not take place as scheduled.) In our view, it would be appropriate for the AMU to sponsor a spectrum of professional activities patterned on those of other national and international societies of mathematicians.

f) One exciting recent contribution to African mathematical development has been the establishment of the African Institute for Mathematical Sciences (AIMS) in South Africa. This institute provides a one-year program in various topics of advanced mathematics to postgraduate students from all over Africa. A goal is to prepare and stimulate graduates to go on to master's and PhD work at degree-granting institutions. AIMS has recently launched a Next Einstein Initiative to develop funding for AIMS, with the premise that well-designed programs can help unlock the enormous potential of African talent in mathematics and science (see http://www.nexteinstein.org/).

The mathematics in AIMS courses and seminars tends toward applications, engineering, and interdisciplinary topics with some attention to pure mathematics. If AIMS, in keeping with its name, is to fulfill its potential role in the broader mathematical community, we believe that the mathematics of its courses should be broadened considerably so as to further emphasize the fundamentals underlying the mathematics that students will require in applications.

AIMS has also proposed an Africa-wide network called AMI-Net consisting of "nodes" modeled on AIMS; the first of these opened in Abuja, Nigeria, in 2008. Our information on AMI-Net is limited; however we believe that AMI-Net has the potential to make a significant contribution to mathematics in Africa by offering support but encouraging its individual nodes to maintain full programmatic and degree-granting autonomy. More generally AIMS' relationship with AMI-Net nodes might well be modeled after that between the International Center for Theoretical Physics (ICTP)[2] and the institutions it serves.

[2] The ICTP was renamed the Abdus Salam International Center for Theoretical Physics several years ago, but the original acronym is still used.

g) Another recent development has been the establishment of the African Mathematics Millennium Science Initiative (AMMSI), a distributed network of mathematics promotion, with five regional offices in sub-Saharan Africa. To date AMMSI has concentrated on awarding fellowships, enabling staff to visit other institutions in Africa and participate in research and postgraduate training, and awarding partial scholarships for postgraduate students to complete their studies. It has also supported organization of regional mathematics conferences and workshops on various themes. AMMSI further supports a project that promotes collaboration between mentors in European and African institutions of mathematics. If AMMSI is to truly fulfill its potential, there is need for increased grant awards and for extension of its activities to promotion of interdisciplinary research.

h) To create greater awareness and support for mathematics, the mathematical community can take more active steps through educational campaigns in the media, public lectures, more forceful lobbying, and creation of programs that demonstrate the value of mathematics research to the country and its political leaders. Mathematics Olympiads and award schemes to talented students could also help to stimulate interest in mathematics in schools and universities.

An additional summary of recommendations by the IMU steering committee is presented at the end of this report.

The Current State of Higher Mathematics in Africa

Our request from the Templeton Foundation included the following questions:

Which countries in Africa have a strong university education in mathematics? In which mathematical areas are their strengths?

The responses of our advisors are summarized below in alphabetical order:

- Algeria: Mainly applied mathematics
- Benin: Differential geometry, Riemannian geometry, ordinary differential equations, partial differential equations, cosmology, mathematical physics
- Botswana: Fluid mechanics, functional analysis, numerical analysis, mathematical biology, mathematical epidemiology, algebraic topology
- Burkina Faso: Algebra, ordinary differential equations, partial differential equations
- Cameroon: Harmonic analysis, geometry/topology, differential geometry, general relativity
- Côte d'Ivoire: Algebra, functional analysis, harmonic analysis, ordinary differential equations, partial differential equations, differential geometry, probability-statistics, mechanics
- Mali: Numerical analysis, pure mathematics
- Morocco: Mathematical modeling, mathematical biology
- Nigeria: Algebra, ordinary differential equations, partial differential equations, group theory, mathematical physics

- Senegal: Algebra, differential geometry, partial differential equations, applied mathematics (modeling), probability/statistics, control theory
- Tanzania: Operations research, epidemiology (modeling of diseases), environmental modeling

General challenges

The following table illustrates the range of strengths among African nations that is relevant to mathematical development. The relatively advanced mathematical capacity of North Africa (region) and South Africa (country) appears clearly, as does the acute need for development elsewhere. Regional totals appear in bold type.

The third column makes use of data from the United Nations' Human Development Report, which classifies nations into three groups, according to the value of their Human Development Index (HDI): High (between 0.8 and 1.0), Medium (between 0.5 and 0.8), and Low (less than 0.5). It can be seen that only three African countries have High HDIs: Libya, Mauritius, and Seychelles. The average for countries in East and West Africa is in the Low range, where societies are unlikely to devote sufficient resources to developing a science and technology base or to build a dynamic mathematics community.

Table 2. Population, wealth, human development index (HDI), number of universities, doctoral degree holders, and mathematics doctoral degrees awarded through 2007.

Country	Population 2007 estimates (million)	GDP (PPP) per capita (USD) 2008 estimate	Human Development Index (HDI)	Number of universities	Number of doctorate holders in maths	Number of maths doctoral degrees awarded
Algeria	33.3	8,649	0.733	25	91	38
Egypt	80.3	5,643	0.708	15	500	85
Libya	6.0	15,041	0.818	13	2	-
Morocco	33.8	6,406	0.646	15	232	173
Tunisia	10.3	10,269	0.766	7	104	38
North Africa	**163.7**	**9,202**	**0.734**	**75**	**929**	**334**
Burundi	8.4	744	0.413	1	10	-
Djibouti	0.5	2,797	0.516	-	1	-
Eritrea	4.9	1,018	0.483	1	10	-
Ethiopia	76.5	1,346	0.406	22	48	-
Kenya	36.9	1,550	0.521	12	71	21
Rwanda Somalia	9.9 9.1	1,590 600 (2007 est.)	0.452 ..	2 3	12 2	- -
Sudan	39.4	3,395	0.526	33	36	3
Tanzania	39.4	932	0.467	9	38	7
Uganda	30.3	1,807	0.505	10	24	5
East Africa	**255.3**	**1,578**	**0.477**	**91**	**252**	**36**
Cameroon	18.1	2,362	0.532	7	113	26
CA Republic	4.4	1,317	0.384	1	7	-
Chad	9.9	1,836	0.388	1	5	1
DR Congo	65.8	957	0.411	5	41	-

Equatorial Guinea	0.6	21,316	0.642	1	-	-
Gabon	1.5	7,985	0.677	3	7	-
Rep. Congo	3.8	1,582	0.548	1	20	-
Sao Tome & Principe	0.2	3,708	0.654	-	-	-
Central Africa	**104.3**	**5,133**	**0.530**	**19**	**193**	**27**
Benin	8.1	1,507	0.437	2	56	29
Burkina Faso	14.3	1,576	0.370	2	28	12
Cape Verde	0.4	8,481	0.736	1	1	-
Côte d'Ivoire	18	1,795	0.432	3	48	15
Gambia	1.7	2,524	0.502	1	-	-
Ghana	22.9	3,142	0.553	5	46	-
Guinea	9.9	2,530	0.456	2	37	-
Guinea-Bissau	1.5	787	0.374	1	-	-
Liberia	3.2	500	..	1	4	-
Mali	12	1,438	0.380	1	31	16
Mauritania	3.3	2,626	0.550	1	37	-
Niger	12.9	1,052	0.374	2	24	3
Nigeria	135	1,373	0.470	45	233	88
Senegal	12.5	2,192	0.499	3	67	17
Sierra Leone	6.1	1,018	0.336	1	21	-
Togo	5.7	1,592	0.512	2	24	-
West Africa	**267.5**	**2,133**	**.465**	**73**	**657**	**180**
Angola	12.3	5,436	0.446	3	2	-
Botswana	1.8	18,402	0.654	1	13	3
Comoros	0.7	2,133	0.561	-	-	-
Lesotho	2.1	2,500	0.549	1	14	-
Madagascar	19.4	1,078	0.533	6	46	10
Malawi	13.6	786	0.437	2	16	-
Mauritius	1.3	14,954	0.804	2	12	-
Mozambique	20.9	1,706	0.384	3	15	-
Namibia	2.1	9,653	0.650	1	5	-
Seychelles	0.1	23,294	0.843	0	-	-
South Africa	44	14,529	0.674	18	502	432
Swaziland	1.1	5,544	0.547	1	6	-
Zambia	11.5	1,218	0.434	2	13	1
Zimbabwe	12.3	2,395	0.513	5	27	4
Southern Africa	**141.1**	**7,402**	**0.574**	**45**	**671**	**450**

Source: Population data (column 2): Wikipedia (http://en.wikipedia.org/wiki/List_of African countries by population); GDP (column 3): Wikipedia (http://en.wikipedia.org/wiki/List_of_African_countries_by_GDP); HDI values (column 4): UN Development Program (2007/2008) (http://hdr.undp.org/en/); the last three columns from MathSciNet. Additional data from El Tom (2008) and Gerdes (2007). Dashes indicate no known values.

Major countries and regions

This section offers brief summaries of the level of mathematical development in countries familiar to our advisors. Conditions to bear in mind include the small number of full professors, the scarcity of graduate students, and the heavy teaching loads that hinder the efforts of faculty to advance in their careers through research and other activities.

Southern Africa

With the exception of South Africa and the much smaller Botswana, the Southern Africa region is generally underdeveloped in mathematics. Except for the University of Botswana (26 mathematics staff) and several South African Universities (including University of the Witwatersrand, 48 mathematics staff and University of Cape Town, 48 mathematics staff), university mathematics departments are seriously under-resourced and understaffed. The University of Zimbabwe was a strong focal point for mathematics during the 1980s and early 1990s, but during the current political turmoil most mathematicians have left the country.

South Africa

South Africa seems to have both the strongest and some of the weakest universities providing education in mathematics. Four universities are ranked by one worldwide survey among the top 500.[3] Five universities, according to Prof. Banasiak, have departments of mathematics/applied mathematics large enough to employ 30-60 academic staff, representing a wide range of basic and applied disciplines. At the same time, many South African universities have departments with only five or six staff members, usually without a PhD, who do mostly undergraduate and service teaching.

Mathematical research in South Africa is concentrated in the 17 public universities and, to a lesser degree, in about five universities of technology. A small number of research mathematicians (mainly in applied fields) work for governmental agencies, such as Council for Scientific and Industrial Research (CSIR), and for the military. The current structure is relatively new, dating back to 2004-2005 when the government executed a nationwide program of mergers aimed at erasing the apartheid-based map of South African higher education. It did so partly by pairing a number of ‘historically white’ and ‘historically black’ universities. Today all universities are funded centrally and governed by councils on which the majority of members are external appointees. Nevertheless, the educational quality is highly uneven and still reflects historical patterns.

In mathematics, South Africa has several significant advantages over other African countries. First is the relatively high level of industrial development in some regions which supports the research enterprise, attracts international scholars, and provides employment for applied mathematicians. Second, opportunities for industrial consultancies allow some faculty to supplement their incomes and do research. Third, the National Research Foundation provides funding for research and for postgraduate students, boosting mathematics development and drawing many postgraduate students from elsewhere in sub-Saharan Africa.

However, the relatively poor salaries at most universities, compared to the private sector and even administrative posts within the same universities, make academic careers unattractive for young South Africans. Programs in mathematics and applied mathematics are very small in comparison with other disciplines, and most postgraduates and staff are foreign-born. A significant number of mathematics publications from South Africa are actually produced by

[3] Academic Ranking of World Universities, Shanghai Jiao Tong University

Zimbabweans, Zambians, Tanzanians, Malawians, and others who have immigrated to take advantage of the facilities and environment the country offers.

In addition, tertiary education, especially in applied mathematics, is strongly influenced by the British paradigm, with little emphasis on the fundamental mathematical underpinnings of the formal techniques and methods taught. Thus students may graduate without being able to properly formulate definitions or provide rigorous proofs of simple theorems.

North Africa

Egypt

Among African countries, Egypt has a uniquely strong mathematical community. As an example of this strength the country has 19 universities with more than a dozen mathematics PhD holders on staff, according to Prof. Obada; Ain Shams University alone has 206 mathematicians with doctorates. Members of the Egyptian mathematical community are estimated to submit some 800 papers a year to journals. Because of this strength, Egypt is not considered to have the same need for mathematical development as most of Africa and will not be highly featured in this report.

The Maghreb

The Maghreb countries (Morocco, Tunisia, and Algeria), like Egypt, have relatively high levels of training and research – in some places comparable to those of Europe. Many students are attracted to mathematics, and most universities have high-caliber staffs.

Because of geography, many links exist between the Maghreb and France, Italy and Spain. Accordingly, southern European research institutes have supported the establishment and maintenance of applied mathematics laboratories in the Maghreb countries. Many mathematicians from Europe have spent time in North African universities, doing joint research and teaching courses, while North African students do post-doctoral work in European universities. For example, Fields Medal winner Laurent Schwartz taught for some period of his career in Algeria.

Because of its relatively high level of development, North Africa today has begun to play a role in training students from sub-Saharan Africa. In the Francophone RAGAAD network, for example, several PhD students have advisors from Tunisia or Morocco.

One criticism of the northern academic approach is that while Tunisia, for example, has done much to improve its mathematics education in the past decade, it remains tightly bound to the French "quality conundrum" that values "academic" mathematics over useful "daily and applied" mathematics. This pedagogic bias is kept in place by strong teacher and parent opinions that equate basic mathematics with high-quality mathematics.

West Africa

Nigeria

Nigeria today seems to have good potential in mathematics, and may eventually assume a leadership role commensurate with its size and population. It has launched the National Mathematics Center north of Abuja, which has ambitious plans to raise the level of both Nigerian and regional mathematics and mathematics education. For the present, however, there seems to be a relatively modest amount of mathematical research underway in Nigeria, and little academic communication with other countries of Central and West Africa. Progress may have to await further organizational measures and consolidation of early steps.

Francophone countries

In the sub-Saharan portion of Francophone Africa, notably the Republic of Benin, Cameroon, and Senegal, university programs in mathematics are relatively advanced, due largely to strong secondary education and public support. Undergraduate programs are modeled on the French system, and many students who graduate with a bachelor's degree are comparably equipped to those graduating from French universities, where they often go for postgraduate study.

However, the number of people earning a mathematics degree in these countries is very low, with the exception of Cameroon. Curricula tend to be outdated and to lack applied and interdisciplinary topics, although some specialists in ODE and PDE apply their techniques to modeling water supplies, epidemics, and other phenomena. Applications to industry are virtually non-existent because most large industrial concerns, which are owned by expatriates, prefer to do their technical studies in their own metropolises.

In the view of Dr. Bregman, some countries overemphasize a style of teaching that rewards repetition and rote learning. In this view, there is insufficient value given to creativity and practical applications.

Cameroon

Cameroon, strong in mathematics relative to the other Francophone countries, still faces an acute shortage of faculty. Of 47 established positions for professor, for example, there are only five professors in place; out of 95 established positions for associate professor, only three are in place. Virtually all teaching is done by low-level staff – 451 lecturers, 117 assistant lecturers, and 44 teaching assistants.

Benin

The Republic of Benin has a long tradition in mathematics, a heritage of French colonial times and a traditional belief that education is the best route to professional success. In July 2008, the government increased salaries of higher- and secondary-education professors, which is likely to attract more bright students to academic careers. Mathematics has one of the largest departments at the main University of Abomey Calavi (UAC), and some of its graduates do better in advanced degree programs than chemists or biologists. Most doctoral studies are carried out at the Institut de Mathématiques et Sciences Physiques (IMSP), Porto Novo.

Senegal

In the past half-decade of leadership by the current president, Senegal has raised its commitment to higher education in general, including mathematics. The government, like that of Benin, has increased compensation for university professors, making the country more attractive to many young Senegalese who wish to return home after earning a PhD abroad.

Mali

During the summer of 2008, Mali organized a forum to review its educational system. Officials resolved to better organize and support the sciences, including mathematics. Many of the principal organizers of this forum were mathematicians.

East Africa

Most universities in East Africa offer an organized mathematics degree program, but most students with aptitude in mathematics choose other options, such as computer science, engineering, statistics, and economics.

Kenya

Kenya has relatively strong bachelor's programs in pure mathematics, applied mathematics, statistics and computer science. The majority of students opt for statistics courses which, they believe, are more likely to assure them of jobs on graduation. In recent years programs have been introduced in industrial mathematics and actuarial science, with the consequence that the departments are admitting more students who perform well in mathematics at the secondary school examinations. The poor performance in mathematics in primary and secondary schools remains an issue of great concern. The National Council for Science and Technology (NCST) is currently developing a project designed to find out the root causes and suggest corrective measures.

Tanzania

Prof. Charles describes the state of mathematics in Tanzania as poor, especially at the primary and secondary levels. He cites statistics of the National Examination Council of Tanzania (NECTA) showing that in the 2007 national examinations, only 17.4% of candidates passed mathematics. He attributes this partly to the insufficient number of competent mathematics teachers, and partly to a tumultuous history since the country was a German colony. After World War I, the educational system was disrupted when teachers had to switch to the British system, and mathematics was not even taught at the tertiary level until 1965, when the University of Dar es Salaam (UDSM) established its department of mathematics. According to one advisor, that department is now on a par with other East African universities.

Uganda

In Uganda, the five public universities all have departments of mathematics, as do some private universities, but nationwide there are only two full professors of mathematics and 10 senior lecturers. As Prof. Mugisha notes, "This number cannot in any way teach mathematics nationally to a good acceptable standard." Another advisor feels that the mathematics program at Makerere is strong, as is the country's research in disease modeling.

The level of research

One measure of mathematical strength is the number of PhD holders. For the whole of sub-Saharan Africa, according to MathSciNet figures in Table 2, this number is 1,773. Table 2 also suggests that the total number of PhD holders in Africa is 2,702, with a disproportionate number (929) living in the Maghreb and Egypt.[4] The IMU and the African Mathematical Union's Commission on the History of Mathematics in Africa calculated that about 250 doctorate holders

[4] As mentioned above, these figures from MathSciNet almost certainly understate the size of mathematical communities. According to Prof. Obada, for example, Egypt alone has more than 900 mathematics PhD holders, including those on leaves of absence and those in faculties of engineering or education.

are women, representing 11% of the total. In Egypt, Algeria and Tunisia, the proportion is about 20%.

Publications

Data on the mathematical research output for each of the five African regions during the period 2000-2007 was available from only one of our advisors, and then only based on an informal survey. This data should not be taken as definitive, but rather only as an informal indication of where increased attention might well be directed, either in obtaining more complete and precise information about actual activity, or in attending to a relative lack thereof. The data show that North and East Africa respectively have the highest and lowest level of mathematical research output. North Africa's share of total world research output in mathematics during the period 2000-2007 is about 0.87% per year, and the corresponding share for East Africa is 0.01%. The corresponding shares for Central, West and Southern Africa are 0.03%, 0.08%, and 0.39%, respectively.

It is worth noting that research output in Central, West and Southern Africa is largely concentrated in one country: Cameroon (more than 80% of the region's total publications), Nigeria (between 71.4% [2007] and 92.5% [2000]), and South Africa (more than 90%). Burkina Faso and Senegal account for almost all publications of Francophone West Africa. In Southern Africa, Botswana is the only country besides South Africa that has sustained a modest level of research through the period.

The structure of mathematical research in Africa is somewhat biased toward the field of analysis, according to Prof. El Tom. The only exception to this general observation is South Africa, where analysis contributes about 21% of total output (the average proportion worldwide in 2007 was about 25%). Classical applied mathematics is a second area emphasized by most African countries. Mathematical logic and foundations, combinatorics, number theory, algebraic geometry, algebra, geometry, topology, probability theory and stochastic processes, and statistics were not adequately represented in 2000-2007. There were a few exceptions: in Tunisia and South Africa, algebra and combinatorics, respectively, were overemphasized.

Only a few mathematical journals are based in sub-Saharan Africa: *Afrika Matematika; IMHOTEP: African Journal of Pure and Applied Mathematics; Questiones Mathematicae;* and *The Journal of the Nigerian Mathematical Society*. According to Prof. Banyaga, these journals usually publish high-level research papers and most of the authors are African mathematicians.

Inequities in publication

Like other features of the mathematical enterprise, publications are distributed unevenly across the continent. This can be seen by the following sampling from MathSciNet collected by Prof. El Tom[5]:

1. Egypt 370
2. South Africa 334
3. Algeria 173
4. Morocco 169
5. Nigeria 60

[5] Total publications on mathematical topics from 2000-2007.

6. Cameroon 20
7. Senegal 12
8. Burkina Faso 8
9. Botswana 7
10. Tanzania 6
11. Benin 3
12. Ghana 0

Another form of inequity, writes Prof. Charles, is that 84.2% of collaborative research involving France is with the three *Maghreb* countries, while 87.3% of USA collaborative research is with Egypt and South Africa.

Primary and Secondary School Mathematics[6]

Both primary and secondary level mathematics education are weak in most African countries, reducing the potential population of talented students who choose mathematics majors at the university level. In particular, our advisors were blunt in their assessment of primary mathematics. In responding to the question of how many countries have strong development in primary education, Prof. Roy answered, "To the best of my knowledge, none." She attributed this condition at least in part to the spread of laws requiring universal elementary education, which has increased crowding in the primary schools and brought "a severe drop in quality."

Prof. Sibanda answered the same question as follows: "At the school level, mathematics education in South Africa is in a state of crisis. This is characterized by a widespread shortage of qualified teachers and poor delivery of instruction…."

Profs. Gangbo and Mugisha both elaborated that at the elementary level instruction was especially poor in rural areas, with many schools lacking any staff trained in mathematics. Prof. Gangbo identified a "substantial gap between urban centers and schools in the countryside in general." He also noted that in the past few years, the government of Benin seems to be committed to fill that gap, "but this is still very far from a reality."

Teacher recruitment is one of the most critical factors to ensure that students have access to high-quality mathematics education in both primary and secondary schools. And yet teacher recruitment is difficult because few students want to be mathematics teachers. Students do not enjoy their first brush with mathematics in primary school, seek to avoid it in secondary school, and choose not to become teachers themselves. This vicious circle perpetuates teacher shortages.

According to Mhlolo et al, all countries surveyed report serious problems in recruiting qualified mathematics teachers. The difficulty is compounded by gross secondary school enrollment ratios of less than 50%. Even if more than 50% of the potential student population did enroll, there would not be enough teachers to accommodate them. Table 3 below shows that in some sub-Saharan countries, fewer than 10% of the secondary age population is enrolled in school; in the

[6] This chapter is based on "The state of Mathematics Education for gifted children in some selected African Countries: A report for the IMU Project," by M. K. Mhlolo, L. Nyaumwe, and J. Adler of the Marang Centre for Mathematics and Science Education, University of the Witwatersrand, South Africa.

rest for which figures are available to us, only South Africa and Swaziland have gross enrollment ratios greater than 50%.

Table 3. Summary of Gross Enrollment Ratios as % of the secondary age population (2005)

Below 10%	10 – 24%	25 – 49%	50%+
Burkina Faso	Kenya	Ghana	South Africa
DRC	Nigeria	Lesotho	Swaziland
Mozambique	Rwanda	Malawi	
Tanzania	Uganda	Zambia	
		Zimbabwe	

Source: Mhlolo et al.

At every stage of education, more students drop out of mathematics. In addition to low primary-school enrollment, there are high drop-out rates between primary and secondary school, during secondary school, and between lower secondary and higher secondary school years. This effectively prevents education systems from producing their own future labor force.

For example, the following 2005 enrollment figures for Zambia have been used to project the recruitment pool for "lower secondary teachers":

Table 4. Projection for Zambian recruitment pool 2005-2015

Enter Primary	Complete Primary	Enter Secondary	Complete Secondary	Enter Sec TTC	Complete TTC	Enter Sec Profession
467,306	428,990	316,681	123,894	6,938	6,661	5,994

Source: Mhlolo et al.

As shown in the table, of the 467,306 students who entered grade 1 in 2005, only 123,894 are projected to complete secondary school, and only 6,938 to enter secondary school teacher training. Using this model, which Prof. Mhlolo et al call conservative, the pool of potential secondary teachers (123,894) shrinks to 6,661 teacher training graduates and 5,994 teachers who actually enter the profession. A 2006 World Bank report confirmed the poor quality of instruction and the poor performance of students in secondary education systems in Africa, stating that the ability to produce large numbers of highly qualified teachers is limited.[7]

Another example shows the dramatic fall-off in student enrollment in mathematics. In Niger, only 200 out of a total of 16,000 students were specializing in mathematics at the end of secondary school, in a country of 15 million inhabitants. In France, by contrast, 45,000 students

[7] Prof. Mhlolo et al note that official statistics often do not reflect enrollments for specific subjects or show teacher deployment for specific subjects. In most cases these ratios are calculated as averages from total enrollments in the secondary schools divided by the number of teachers in those schools without considering the subject specialization for both learners and teachers. Therefore, teacher-pupil ratios often fail to distinguish between teachers qualified simply to teach and those qualified in the subject being taught. Even where teacher-pupil ratios are given for specific subjects, country profiles do not guarantee that teachers do in fact have adequate content mastery of the subject they are teaching.

specialize in mathematics out of a total of 400,000 students at the end of secondary school, in a country of 60 million inhabitants.

This gloomy picture extends to many other countries. For example, in Tanzania the spread of universal elementary education has sharply increased the number of students and size of classes, particularly for those studying mathematics. Investigation has shown that large class size prevents teachers from interacting with students, attending to those who need special attention, or practicing learner-centered techniques.

Countries with strong development in secondary mathematics education

In Francophone Africa, secondary-school teaching is relatively strong, although class sizes are generally too large in the sub-Saharan countries and instructors are overloaded. A few other regions have strength in the secondary schools, but these are primarily countries of higher-than-average GDP per capita: Botswana, Mauritius, and, until the current phase of political unrest, Zimbabwe, as well as the dominant country of South Africa. These are exceptions, however, within a generally gray landscape.

West Africa

In West Africa, countries with strong secondary-school mathematics include the Republic of Benin (with a long tradition in mathematics, partially inherited from France), Burkina Faso, Côte d'Ivoire, Mali, Senegal, and Togo.

At the same time, the strong mathematics tradition in these countries is being strained by demographic and other changes. Teacher training has not kept up with rapid population growth in West African countries, where 40 to 50% of the population is now under age 18. Prof. Touré notes that teaching in rural areas is especially weak, as the best teachers cluster in the towns where salaries are higher.

South Africa

South Africa has some excellent secondary schools in the wealthier cities and suburbs, but this anomaly is created by inequities in spending and culture. For example, the government pays for teachers at a ratio of about 35 learners per teacher per class, but governing bodies can set higher tuition fees to attract more teachers. This generally happens in schools in more affluent areas. Meanwhile, the situation is dismal in rural areas, where poor salaries, weak or nonexistent infrastructure, and security concerns rob the teaching profession of its appeal. The vast majority of teachers do not have bachelor's degrees, nor is it a requirement (although the government has launched an effort to upgrade teachers' qualifications through in-service training). Within districts, strong teachers' trade unions block efforts to reward better teachers with merit pay.

Most high school education places emphasis on algorithmic procedures rather than an understanding of basic principles. According to Prof. Banasiak, "It borders on impossibility to force students to embark on independent thinking." On average, therefore, the preparation of students entering university is weak.

South Africa, despite its relative wealth, has been the worst-performing country on recent TIMSS (Trends in International Mathematics and Science Study) ratings.[8] This is attributed, noted Prof. Sibanda, to poor showings in the township and rural schools (i.e., the majority of all schools in South Africa), where only a small minority of students leave high school with a pass in mathematics. At his university, more than 90% of all current black mathematics postgraduate students are from outside South Africa.

Teacher-pupil ratios

According to the study led by Prof. Adler, high teacher-pupil ratios and large class sizes are a major cause of poor teaching in the primary and secondary mathematics classrooms of Africa. These ratios run counter to the aims of equitable provision of education and learner-centered education, common goals in mathematics education in most developing countries.

Dr. Bregman agrees with this assessment, offering a general estimate for teacher-pupil ratios of about 1:50-100 for primary classes. These ratios decrease sharply at grade 6, due to examinations and selection for entry into the first SE cycle and continue to fall through the grades. In grade 7-8, he estimates, the ratio is about 1:30-40, and in grades 11-12 about 1:15-25. The ratios are thought to decrease because of repetition (the "holy grail of quality quest by teachers") and the selection mechanisms for university entry.

Like school attendance, however, student-teacher ratios are difficult to compare numerically because of variations in primary and secondary school classification. In most countries, learners start their primary school at the age of five or six. If a school system uses a 6+3+3 model, such a learner would turn 11 or 12 in primary school and exit the secondary school at 16 or 17. In some systems, like the 7+4+2 used in Uganda, learners may exit secondary school at the age of 18.

In addition, primary school teachers do not specialize in subject areas, so it may not be possible to determine a teacher-pupil ratio that is specific to mathematics. It is with such complexities in mind that the following teacher-pupil ratios are presented:

Table 5

Country & Grade Level	**Teacher/Pupil Ratio**
<u>Burkina Faso</u>	
(10-18 YEARS)	
2003-2004	1:301
2004-2005	1:337
2006-2007	1:309
<u>Ghana</u>	
Secondary Lower	1:18
Secondary Higher	1:22
<u>Kenya</u>	
Secondary	1:160

[8] See, for example, http://nces.ed.gov/timss/.

Country & Grade Level	Teacher/Pupil Ratio
Lesotho	
Primary Urban	1:50
Primary Rural	1:75
Secondary Urban	1:30
Secondary Rural	1:50
Malawi	
Standard 1-8 (5-12 years)	1:84
Forms 1-4 (13-18 years)	1:70
Nigeria	
Primary Private Schools	1:30
Primary Public Schools	1:55
Junior Sec Sch. JSS 1-3	1:50
Senior Sec Sch. SSS 1-3	1:40
Rwanda	
Grades 1-7 (6-12 years)	1:45
Forms 1-4 (13-18 years)	1:233
Uganda	
10-12 years	1:120
13-16 years	1:80
17-18 years	1:30
Zambia	
Grades 1-7 (7-13 years)	1:57
Grades 8-9 (14-15 years)	1:32
Grades 10-12 (16-18 years)	1:21
Zimbabwe	
Grades 1-7 (6-12 years)	1:45
Forms 1-2 (13-14 years)	1:33
Forms 3-4 (15-16 years)	1:30
Forms 5-6 (17-18 years)	1:20

Clearly, there are large differences in teacher-pupil ratios by country, as well as within the countries. The higher levels compromise mathematics education in a number of ways, primarily because teachers cannot give individual attention to learners. One effect of the higher levels is that teachers try to avoid working in these schools, further exacerbating the shortage.

Data seem to suggest that certain countries have adequate numbers of mathematics teachers at the secondary level, such as Zambia (1:21), South Africa (1:21); Ghana (1:22), and, until recently, Zimbabwe (1:20). But official statistics usually paint an optimistic picture, according to Mhlolo et al, that conceals the realities of teacher shortages. Most African countries, for example, use a system of centralized teacher deployment, in which it is common for a secondary school teacher's contract to require deployment anywhere in the country where teacher services are needed. Centralized deployment is praised as being transparent, fair, and generally free of local pressures. However, these systems also tend to become bureaucratic and unresponsive, jeopardizing the quality of education.

Education level of secondary school teachers
The normal educational level for secondary mathematics teachers varies from country to country, ranging from a bachelor's or master's degree at the high end to various certificates and diplomas in education. The following list is indicative of this range:

- *Republic of Benin:* Many secondary school teachers in mathematics have a bachelor's degree, others do not; many have special training in education.
- *Botswana:* Bachelor's and master's degrees are common.
- *Burkina Faso:* For the first four years, a two-year degree followed by two years of teacher training; for the senior secondary level, a master's in maths.
- *Democratic Republic of the Congo:* Normally, a license after five years of post-secondary school training.
- *Côte d'Ivoire:* A minimum is the DEUG (Diplôme d'Etudes Universitaires Générales), awarded after two years of university.
- *Mauritius:* Bachelor's and master's degrees are common.
- *Nigeria: A* bachelor's is normal, but some schools accept NCE (National Certificate in Education) teachers.
- *Rwanda:* A bachelor's degree, but some have only a Cert.Ed. or a Dip.Ed.
- *South Africa:* Many do not have the normal bachelor's, but the government is attempting to upgrade teachers' qualifications through in-service training.
- *Tanzania:* A diploma is the norm, but insufficient.
- *Uganda:* A BSc/Ed degree, with a bachelor's increasingly an advantage.
- *Zimbabwe:* Most have held a certificate or diploma in education, with a BEd or BSc.

Although these summaries give an impression of specificity and uniformity, official requirements are not always met or carried out on the ground. This is often due to inherent bottlenecks and the incapacity of governments to provide the necessary training resources.

In some cases, stringent requirements unintentionally worsen the situation. Teacher shortages may be eliminated *artificially* by a process of "inferior substitution": that is, surplus teachers (in other subjects) and temporary teachers are assigned to teach mathematics, even though they are not qualified to do so. Because of disagreements on appropriate teacher qualifications, it is not easy to quantify the full effects of this bureaucratic sleight-of-hand. But it certainly contributes to poor performance in mathematics and the vicious circle alluded to earlier. Such practices do not augur well for efforts to develop a mathematically literate society.[9]

[9] The report by Mhlolo et al offers substantial evidence of inferior substitution. In Nigeria, for example, the Director General and Chief Executive of the Mathematics Improvement Project (MIP), Prof. Sam Ale, reported that the problem of poor performance in mathematics was a result of non-qualified teachers teaching mathematics. In South Africa, although 85% of mathematics teachers in the schools in 2005 were said to be qualified, only 50% had specialized in mathematics. South Africa's poor performance at TIMSS 2003 is consistent with a lack of qualified teachers. In Zimbabwe, secondary

Identifying and tracking gifted students

The purpose of identifying exceptionally gifted mathematics students is to ensure that mathematically gifted learners reach their maximum potential to the benefit of their countries as well as themselves. Unfortunately, according to Dr. Bregman, in sub-Saharan Africa there are few structured systems for identifying highly gifted mathematics students – apart from the traditional examinations for grading purposes and competitions such as the Olympiads – and no systems for tracking them once identified. Such systems, were they available, might make a small but concrete contribution to the economic development of African countries.

Special schools for gifted students are not common, but most sub-Saharan countries have private secondary schools that include many students of excellent local reputation, some of international quality, and many of "in-between" levels. According to Dr. Bregman, the identification and tracking of gifted students in these or other schools does not appear in education policies or statistics.

Although not structured specifically for following mathematically gifted learners, the only attempts at tracking are seen in Zambia, Zimbabwe, and Malawi (see below). In these countries, students who do exceptionally well in mathematics are encouraged to apply for Beit Trust funding so they can further their studies in England, Ireland, or South Africa.

It appears that the most common method of identifying mathematically gifted learners is through their participation in mathematics Olympiads. Participation is also considered useful in promoting public awareness of mathematics and encouraging more students to take up the subject. The Pan African Mathematics Olympiad (PAMO) is organized by the African Mathematical Union (AMU) to encourage youthful talent in mathematics and to exchange information on curricula and teaching methods. These Olympiads have gained international recognition and support with a grant from the John Templeton Foundation to support the Pan African Mathematical Olympiad Symposium on Training and Research (PAMOSTAR 2007), the Pan African Mathematics Olympiad (PAMO 2008), and participation in the International Mathematics Olympiad (IMO) thereafter.

The following summaries give an indication of how African countries attempt to identify the most mathematically gifted students.

<u>North Africa</u>

In *Algeria* and *Morocco*, screening for talented pupils in mathematics begins at age 15. National competitions are organized and the best students are followed, trained, and selected for the Pan African Mathematics Olympiads (PAMO) or International Mathematics Olympiads (IMO). In addition, countries hold national competitive examinations at the bachelor level and the winners are awarded Excellence Scholarships to study abroad. In *Tunisia*, at the end of both primary and lower secondary schools, students take written examinations, and the best of them go to a more advanced level.

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mathematics education cannot be evaluated at present, but inferior substitution is certainly taking place in the country's secondary schools, especially with regards to mathematics teaching.

Egypt has no Olympiad infrastructure, has not participated in PAMO since 1991, and has not taken part in the IMO. For the first time, Egypt sent an observer to PAMO in 2008.

Central Africa
Cameroon has a structure similar to those of North African countries. It takes part regularly in PAMO and the team earns good scores, but Cameroon does not yet participate in IMO.

West Africa
Benin has the same screening infrastructure as Morocco and regularly participates in PAMO, in which it scores second only to South Africa. An observer from Benin was able to attend the IMO in 2008, supported by the John Templeton Foundation. Benin will compete in the IMO in 2009 in Germany.

Benin also makes an effort to identify talented women in mathematics. It has created a Miss Mathematics contest for eighth-grade students, which expands in 2008 with an extra initiative for 15-year-old girls called Miss MPC (Mathematics, Physics, Chemistry). The aim is to identify and motivate young girls toward scientific studies.

Burkina Faso participates in the PAMO and has a more organized National Mathematics Olympiad as well. The best three students in such Olympiads are rewarded during a public ceremony by the Ministry of Education and offered an award or scholarship.

In *Côte d'Ivoire*, talented learners are detected via the PAMO and other competitions, directed to the Preparatory Classes of Morocco, and may subsequently apply for entry into France's Grandes Ecoles. The final screening of talent is done in the various master's programs in mathematics, the best students receiving scholarships to study in France, the USA, or Canada.

Nigeria presently has no screening system that targets gifted mathematics students. It does have general performance screening in the form of its college entrance examination. High performers usually find their way into Suleja Academy, where competition is very stiff. There is no screening for entrance into private secondary schools.

Nigeria's newly formed National Mathematical Centre (NMC), however, has put into place many projects aimed at both stimulating enthusiasm and identifying young talent in the mathematical sciences. One is the Mathematics Improvement Project (MIP), which organizes national Olympiad activities and other competitions.

Senegal has the same screening infrastructure as Benin.

Southern Africa
Malawi has no formal screening systems, but schools have various activities for those who are gifted in mathematics, such as mathematics clubs.

In the southern region, *South Africa* is unique in the size of its mathematical enterprise. Talented learners are actively sought from an early age through several mechanisms. The Department of Science and Technology, for example, has a national plan for Science, Technology, Engineering and Mathematics (STEM), part of which is a mandate to organize national Olympiads, mathematics competitions and mathematics camps. South Africans have usually brought the top team to the PAMO, but the country has not fared well at the IMO. According to Prof. Sibanda, there is no system to encourage and develop mathematical talent beyond the Olympiads.

Swaziland began to participate in PAMO several years ago.

Although *Zambia* has not taken part in the PAMO, it has its own national Olympiads and the Zambia Association of Mathematics Education (ZAME) national mathematics quiz. These screening tools are used mainly for advancing to the next grade and for entry into special courses.

Zimbabwe has used various forms of screening to identify exceptionally gifted mathematics learners, such as a national examination at the end of primary school. Most reputable secondary boarding schools enroll learners on the strength of their results, setting their own entrance tests in mathematics.

<u>East Africa</u>
Kenya has not taken part in the PAMO since 1991 and has not yet entered the IMO. Other East African countries have yet to enter Olympiads.

Rwanda has a screening system specifically targeting mathematically gifted learners. The country supports scientific schools with mathematics-physics or biology-chemistry as major subjects for the scientifically gifted. At the end of high school those with high marks in mathematics are sponsored for higher education in technology and science, sometimes in South Africa, America, or Europe.

In *Tanzania* the country's mathematics association annually conducts a contest that is useful in stimulating mathematics within the country. Despite the incentives of an award certificate and sometimes cash, Prof. Charles reported, this effort does not lead to placement of highly gifted students in special accelerated schools.

Regrettably, very few countries take part in the Mathematics Olympiad. On average about seven African countries out of 52 take part in the PAMO every year.

Educational career opportunities for gifted students
Few hard data are available on career opportunities in education for mathematically gifted students in sub-Saharan Africa. As the examples below indicate, academic jobs are often low in prestige and pay, so that most students with mathematical aptitude leave the field for more rewarding opportunities in telecommunications, mining, banking, information and communications technologies, and other industries.

Among the <u>Francophone</u> countries, highly gifted mathematics learners in *Tunisia* prefer to study in schools that prepare them to become engineers rather than mathematicians, and many of them can do this in France or Germany.

In Central Africa (e.g., the *Democratic Republic of the Congo*), the usual career path for highly gifted mathematics students is a teaching post, which may not provide a viable income.

In East Africa, specifically in *Uganda*, the most gifted mathematics students end up in the most prestigious secondary schools, from which they usually enter government universities under government sponsorship. In *Kenya*, the most gifted mathematics students choose engineering, computer science and commerce as subjects of first priority at university and only join mathematics as a second choice. This situation has changed slightly in recent years when introduction of actuarial science increased enrollment of gifted mathematics students in departments of mathematics.

In West Africa, opportunities vary with the socio-economic background of the student. In *Nigeria* for example, if a child is highly gifted in mathematics and the parents cannot afford fees, the child may not attend school at all. A gifted student whose parents are wealthy may end up at Cambridge University or another top-flight university.

In Southern Africa, the academic career path for mathematically gifted learners is clearer and more open. In *Lesotho,* for example, mathematically gifted students often choose to pursue a BSc in mathematics, chemistry, physics, statistics, biology, technology, construction, or information technology.

In *Malawi,* the best students may expect (limited) scholarships to enroll in the universities for further studies. Some hope for scholarships to study in the United Kingdom. Very few gifted mathematics students end up prematurely in industry.

In *South Africa*, mathematically gifted learners have many academic career opportunities, including majors in computer science, mathematical sciences, epidemiological modeling, applied mathematics, marine population dynamics, mathematics of finance, and engineering, among others.

In *Zambia* those who perform exceptionally well in mathematics at university level are trained as Staff Development Fellows and may be sponsored for postgraduate degrees within or outside the country, depending on availability of scholarships. Then they are employed as university staff members when a vacancy exists or at a research center.

In *Zimbabwe,* mathematically gifted students at the secondary level traditionally do not go on for a BSc in mathematics at university, preferring a path to a career that is more rewarding financially.

In seeking advanced educational opportunities, African students tend to be influenced by their location:

- North African students prefer European universities.
- West Africans prefer either European or American universities, and sometimes Russian universities.
- Central African students prefer European or American universities.

- East African students prefer American, European, or Russian universities.
- Southern African students prefer American universities.

Career opportunities beyond the educational arena

Beyond teaching in secondary school or university, career opportunities for mathematicians broaden considerably and become more lucrative. If present understanding is correct, the demand for mathematicians will grow even greater in the future as the vital role of mathematics in multidisciplinary research, modeling, and powering the economy becomes ever more evident.

As with educational career opportunities, there are few data on the commercial or specialized career opportunities for mathematicians in Africa. Several researchers are doing labor-market studies and household surveys, and may soon extract some conclusions from those. But already the anecdotal evidence, some of which is summarized below, gives a consistent picture: because of low university budgets, the seniority system at most departments, and low public image of mathematics as a career, the most talented mathematics students overwhelmingly seek nonacademic opportunities once they complete their studies. These include banking, engineering, statistics, mining, telecommunications, and ICT-related industries.

Specific countries from which our participants drew examples:

In *Zambia,* mathematically gifted students find commercial career opportunities as engineers, accountants, agricultural officers, geologists, veterinary doctors, medical doctors, chemists, physicists, biologists, mathematicians, statisticians, economists, bankers, computer programmers, teachers and mechanics.

In *Lesotho* the commercial opportunities open for the mathematically gifted include careers in engineering, finance/accounting, statistics and information technology.

In *Malawi* the commercial or specialized career opportunities favored by gifted mathematics students include computer sciences, engineering, commerce, accounting and banking.

In *Zimbabwe* the commercial or specialized career opportunities for highly gifted mathematics students is currently very limited. At present, most mathematically gifted students become part of the Diaspora.

In *Nigeria,* highly gifted mathematics graduates have a good chance of employment in the private sector. This is true partly because firms are able to hire on the basis of merit, whereas university hiring is usually restricted by seniority rules.

In the *Democratic Republic of the Congo,* few companies hire mathematics graduates, largely because the application of quantitative activities in the jobs market is not well developed.

Conclusion

Several trends are seen in the education of mathematically gifted students in African countries. From the information of our advisors, one is that some ad hoc efforts are made to identify students with exceptional mathematical talent, but *little is done to track and support them*. An example is organization of mathematics Olympiads by many countries and participation in the Pan African Mathematics Olympiads (PAMO). Although Olympiads undoubtedly raise the profile of mathematics among students and provides valuable learning experiences, those who have succeeded in these competitions must still return afterward to educational environments that may be far less challenging or rewarding.

The Olympiads certainly provide a positive stimulus for participants, but they may have only marginal effects on national educational systems. Both the teacher corps and student populations are far too small, given gross enrollment ratios of less than 50% in secondary schools for all countries surveyed except South Africa and Swaziland. Because educational systems must produce the labor forces of tomorrow by inspiring the student body of today, it is disheartening to note that the best students in mathematics appear to take up teaching as a last resort. As a result, the next generation of mathematics teachers will be drawn from students who have done poorly in their subject.

According to Mhlolo et al, the success of the South African team at the PAMO deserves special mention. While its good performance may reflect the country's efforts to develop mathematics education, other indicators point to broader challenges. For example, South Africa performed poorly in the most recent TIMSS Study, with a few very high scores more than offset by a preponderance of very low scores. Further study of the performances by ex-racial departments revealed the harsh consequences of apartheid rule: Those who performed extremely well were from the previously advantaged schools – the same few schools that are actively involved in the PAMO.

Africa's Weaknesses and Strengths in Mathematics at the Tertiary Level

As suggested in the previous section, there are many causes of weakness in African mathematics, but underlying them is the scarcity of skilled teachers and professors at all levels. Prof. Touré suggests that the problem begins "with the initial formation and continues with the training of qualified mathematicians in sufficient numbers to assure the formation of a base of young people." This scarcity is blamed partly on poor remuneration and working conditions for teachers, students' knowledge of these conditions, and the generally low opinion among the public of mathematics as a career.

The student experience

<u>Poor infrastructure</u>

Those few students who arrive at the university aspiring to a career in mathematics find daunting and pervasive problems. These begin with the learning space as they push into classrooms originally built to hold 30 or 40 students. Today several hundred students sit in that same room, jammed elbow to elbow, balancing on window sills, or standing along walls. Many skip classes entirely when they cannot gain entry. Even those who arrive early enough to find a seat find it difficult to take notes, ask questions, or find tutorials or thesis supervision.

Infrastructure problems extend beyond the classrooms. Most buildings are old, dating from the colonial era of 50 or more years ago. Few have adequate electrical wiring, let alone the internet access that is taken for granted in modern universities. Students seldom have access to textbooks or journals, and libraries built decades ago to hold several thousand students are now expected to serve 20,000. The few functional public computers are kept in tiny labs, where students must share, peer over shoulders or wait their turn; few students can afford their own. Inadequate bandwidth hinders internet use and downloading even when one succeeds in going on line.

Outdated teaching methods

Themes of research often focus exclusively on traditional branches of pure mathematics such as algebra, geometry and analysis. Francophone courses, according to Prof. Roy, offer poor guidance in probability and statistics, and, adds Prof. Touré, in applied mathematics. Writes Prof. Sibanda, "There are no modern applied mathematics and industrial mathematics. Institutions are working with outdated curricula, not integrative and multidisciplinary topics to interest mathematics students. Very few lectures have typeset lecture notes due to limited computers and software. Teaching methodology [is] non-participatory…."

In universities with good internet access, however, opportunities can expand quickly. Mathematicians are able to find current papers posted on various sites, and older papers can be read on such sites as JSTOR. Many journals and websites, such as ScienceDirect, can be accessed online free or for a nominal charge, and other sites, such as Wikipedia, contain numerous mathematical definitions, theorem statements, and articles. According to Prof. Banyaga, "The combination of these online resources can give a serious mathematician in a developing country access very rapidly to most of mathematical literature he/she needs for his/her research. (This was not the case ten years ago!)"

Teaching does not map well with students' career needs

Seldom is the university curriculum in sync with career realities. Prof. Mugisha notes that curricula often include no career guidance for the student, who simply finishes as a "math major." Universities and sometimes whole countries tend to emphasize either pure mathematics or applied mathematics, but seldom both. An emphasis on pure mathematics does have advantages, such as preparing students for deeper understanding and for a variety of applications: "pure" today may prove to be "practical" tomorrow. More importantly, pure research develops a rigor of thinking and assessment. However, as in most Francophone countries, an overemphasis on pure mathematics may map poorly with students' career needs. One's career vision is usually limited to university teaching, which attracts only a few and, as Prof. Roy points out, leads to a high drop-out rate.

Strong emphasis on applied mathematics brings the opposite situation, where students may be well prepared to apply their skills in government, industry, or multidisciplinary work, but lacking in strength in pure mathematics. A neglect of basic approaches may leave students without the theoretical tools with which to understand their subject in depth.

The faculty experience

Excessive teaching loads affect faculty as well as students. While students receive too little attention, most faculty have too little time, no incentives to pursue research, and inadequate salaries. Faculty seldom have teaching assistants to lessen the load. They experience the dual frustration of not being able to do their best for themselves or for their students. Their institutions lack resources to correct these shortcomings, so that many staff are unable to attend professional conferences and therefore have slight exposure to the people and ideas of their field.

At a more practical level, lecturers are poorly paid compared with their counterparts in government and the private sector, while still facing high costs of living. In Benin, for example, a country of small size but high mathematical achievement, the salary for a full professor is about US$1000 per month.[10] Other countries, such as Kenya, Senegal, and Benin, have raised faculty salaries recently, giving hope that this particular source of stress may ease.

One consequence of the poor teaching conditions in these countries is that the institutions are unable to fill faculty slots. As one example, Makerere University recently had four positions vacant for PhD holders in mathematics which had been advertised for a year without drawing any applications. In Côte d'Ivoire, according to Prof. Desquith, professors are also pinched by housing costs. Mortgage rates in summer 2008 were around 17%, while interest paid on savings was 3 to 4%.

One unfortunate but common result is a sense of pessimism and resignation. As Prof. Banyaga notes, many lecturers have received master's and PhD degrees with very high marks from good European and American universities. But when they return home, the lack of career support saps their enthusiasm and energy. Their interest in research evaporates as they settle into supplementing their low compensation with part-time jobs, accepting the negative incentive of automatic promotion for years of service.

Another result of the thin faculty ranks is professional isolation. Most faculty and graduate students have only a few or no colleagues in their field of specialty at their own university or nearby, and few can afford to travel to professional events to meet others. Classes for mathematics majors are small (5-10 students) compared to the required entry-level classes (200-400 students). Prof. Desquith notes that his Institut de Recherches Mathématiques in Abidjan, Côte d'Ivoire, despite the high quality of researchers, performs poorly because of its "lack of adequate government financing and the small number of researchers whose specialties are increasingly remote from one another." In some countries language differences bring about additional isolation, as between Francophone and Anglophone countries, and within multi-language countries such as Cameroon (French-English) and Tanzania (English-Swahili).

<u>Teaching does not map well with African needs</u>
Just as teaching does not map well with students' career needs, it does not map well with the needs of African countries. The traditional mathematics curricula of many programs does not offer the applied and multidisciplinary options that would point them in the direction of technology-intensive careers. Even though quantitative skills are sought by businesses and industries world-wide, the industrial base is weak in most African countries and unprepared to make use of modern mathematical skills.

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[10] By contrast, the median monthly salary in South Africa for a university professor with 20+ years' experience is about $3000. In the U.S., the median annual salary for a full professor of mathematics at a public university was $112,340 (*Notices of the AMS*, March 2008).

Low image of mathematics and insufficient public support

Virtually all of our advisors report that the field of mathematics suffers from a low public image throughout sub-Saharan Africa. Common, mistaken views are that mathematics is for only a few select students, or only for male students. Many students have difficulty seeing the value of mathematics in real life and consequently have a low opinion of mathematics as a career. As Prof. Charles notes, poor early preparation leaves secondary students with a poor understanding of mathematics and a general aversion to the subject. And Prof. Sibanda writes that a degree in mathematics is regarded by many parents and students as a luxury they cannot afford. In Prof. Touré's view, the negative public attitude toward mathematics is accompanied by few policies that might counteract it.

Some Features of Strength

African countries, like every country, have talented students with the potential to succeed in mathematics. However, in the face of the difficulties described above, those few students who succeed in graduating with a mathematics major from a good university must do so with very little support. Some of this support is attributable to several features of the mathematical enterprise that gain little public recognition. Nonetheless, they promote activities essential to modern mathematical scholarship: collaboration, exposure to potential mentors and partners, and exposure to allied specialties that might not be available at one's own university. These tools help develop and support talent in many ways, and form the basis for the underlying optimism of our advisors. They are:

- Research and education centers: There are many viable centers on the continent, various in size and structure, but focusing the energies of research and education in a single location.
- Regional networks: Africa has a relatively long and encouraging tradition of using academic networks to multiply the abilities of single institutions and extend the "reach" and exposure of students and faculty. While these networks vary in size and effectiveness, they bring a potentially powerful solution to the problem of professional isolation at relatively little cost.
- Outside partnerships and support: For many years, donor organizations based outside Africa, including the International Center for Theoretical Physics (ICTP) in Italy, the International Science Program (ISP) in Sweden, and the Norwegian Program for Development, Research, and Education (NUFU) have offered a mix of scholarships, fellowships, exchange programs, and modest instrumentation grants to promote mathematics and the other sciences. Programs include sandwich exchanges, regional doctoral programs, and South-South cooperation.

Research and education centers of excellence

The viable research centers established in many countries are one symptom of strength in African mathematics. Most of these centers have outreach programs, some of them significant, while maintaining a strong single base. The following countries are known to our advisors as having one or more such centers: Algeria, Benin, Botswana, Burkina Faso, Cameroon, Côte d'Ivoire, Mali, Morocco, Nigeria, Senegal, South Africa, Swaziland, Tanzania, and Tunisia.

Some African centers are doing world-class research, and others have the potential to do so. Our advisors agreed that it is more cost-effective to optimize existing centers than to build additional ones. Some existing centers are young ones, such as the National Mathematics Center near Abuja, Nigeria; the African Institute for Mathematical Sciences (AIMS), near Cape Town, South Africa; the International Chair in Mathematical Physics (ICPMA) in Cotonou, Benin; and the Southern Africa Program in Mathematical Modeling in Dar es Salaam, Tanzania. Others, such as the Institut de Mathématiques et Sciences Physiques (IMSP) in Porto-Novo, Benin, and the International Center for Theoretical Physics (ICTP) in Trieste, Italy, have been supporting graduate students and faculty for several decades. The ICTP is unusual in both bringing many Africans to its main campus in Italy and supporting numerous initiatives across Africa through its Office of External Activities (OEA). One concern of the OEA is to fill the "gap" left for former colonies of the UK, notably in Eastern and Southern Africa, which are not as actively supported by the former colonial power as the Francophone countries.

So significant have been the activities of these centers to the mathematical enterprise that our advisors suggested the creation of additional centers of three basic types:

- *Stand-alone African center*: Such a center, would be supported by member countries and various donors, modeled after UNESCO's ICTP in Italy. It might offer scholarships, fellowships, and on-site programs for mathematicians, as well as a well-equipped library, computing facilities, efficient staff, and democratic procedures for determining programs. It might run research semesters in one of the countries where a critical mass has been attained. A small, local mathematics staff would complement a larger corps of visiting experts, with standards at the highest level. This is a financially ambitious project that might inspire jealousies over location and management.
- *African/international center*: This center would be a large institution located in Africa but implemented as a collaboration between an African university and a prestigious non-African university. It would offer strong graduate programs in both mathematics and mathematics education. Like the previous proposal, it would be expensive and vulnerable to jurisdictional debates.
- *Regional centers*: Smaller, multiple centers, located at existing facilities, might achieve less individually than more ambitious centers, but they would probably be less contentious and provide more diversity. The infrastructure (libraries, computing facilities, staff) would be supported by member countries, institutions, and other donors. Such centers might sponsor off-site "research semesters" in member countries where a critical mass of professors and students exist. Examples offered for potential Francophone sites were Benin (Porto-Novo and Cotonou), Cameroon (Yaoundé), and Senegal (Dakar).

Morocco and Tunisia are well known for their good research centers, particularly in Tunis and Marrakech. In sub-Saharan countries, Senegal, Benin, and Cameroon have small but high-level centers. They provide mathematicians with a critical mass of colleagues in supporting up to several dozen active mathematicians in various specialties, such as algebra, geometry, analysis, and numerical analysis. In general, according to Prof. Roy, probability theory and statistics are underdeveloped.

Benin

Benin's Institute of Mathematics and Physical Sciences (IMSP) was established in 1988 in Porto-Novo as a joint venture of the International Center for Theoretical Physics (ICTP) in Trieste and the Benin government. This is probably the oldest mathematics institute in West Africa, training master's and PhD students and specializing in Riemannian geometry, cosmology, and partial differential equations; IMSP has also trained successful computer scientists. The Benin government has recently obtained financial support from abroad for large new buildings that will host not only mathematics but also related areas.

Benin has a new center called the International Chair in Mathematical Physics (ICPMA), in Cotonou, which also does work of high quality. One challenge for these two centers is to avoid redundancies. They have similar research programs and scientific activities, such as PhD programs in computer sciences, water resource management, and other fields. Unfortunately, they compete for funding from the same sources, and the numbers of scientific staff and students in each institution are small. Some mathematicians advocate a merger of the two to provide greater overall strength for the country's mathematics.

Senegal

In Senegal, the University of Dakar has a strong center specializing in algebra (led by Prof. Mammadou Sangare), with a small group doing differential geometry. The University of Saint-Louis has groups specializing in partial differential equations and applied mathematics (modeling).

Cameroon

Cameroon has a strong research center at the University of Yaoundé, with work being done in harmonic analysis, around the team of Prof. D. Bekollé; in geometry/topology around Prof. Bitjong Ndombol; in general relativity, around Prof. Noutchegueme; and some differential geometry, with Prof. Wouafo Kamga.

Côte d'Ivoire

In Côte d'Ivoire, there are strong teams in algebra (with Prof. Daouda Sangare) and harmonic analysis, with Prof. Saliou Touré and his students.

Several of these Francophone centers, considered to be especially strong, have been designated "Affiliated Centers" by the Office of External Affairs (OEA) of the ICTP. These are:

- IMSP, Benin (Porto Novo): pure mathematics
- ICMPA, Benin (Cotonou): applied mathematics
- Abidjan, Côte d'Ivoire: pure mathematics
- WATS (West African Training Schools), Senegal (Saint Louis): pure mathematics

Nigeria

The principle behind the African University of Science and Technology (AUST), being set up in Abuja by the Nelson Mandela Institution and the World Bank Institute, is to establish world-class centers of excellence in science and technology in Africa. The original objective was to network with other institutions in training African graduates to use their knowledge in science and technology with entrepreneurial skills in building sustainable knowledge-based economies in Africa. The AUST commenced activities in June 2008, offering graduate programs in:

- Applied mathematics and computer science
- Materials science
- Petroleum engineering

Approximately 50 graduate students have enrolled, about half of them in applied mathematics and computer science. Programs are planned for minerals, water and environmental engineering, power engineering, and biotechnology, although differences of vision and strategy have slowed the center's development. For the time being, according to Prof. Gumel, the center remains a fledgling enterprise with ambitious objectives.

South Africa
The African Institute for Mathematical Sciences (AIMS), based near Cape Town, South Africa, offers a nine-month postgraduate diploma in the mathematical sciences. The curriculum includes primarily applied mathematics, multidisciplinary math-science topics, and other mathematics-related fields; pure mathematics, as only one of ten topic areas offered, is not a focus, and PhDs are not offered. Its website (http://www.aims.ac.za/) states that the goals of AIMS are "to promote mathematics and science in Africa, to recruit and train talented students and teachers, and to build capacity for African initiatives in education, research, and technology."

The institute supports deserving students from all over the continent through bursaries. Lecturers are drawn from the region and abroad. On graduation, some students register for graduate/postgraduate degrees with South African or overseas universities. According to Prof. Banasiak, AIMS is well regarded and has considerable potential. Currently, AIMS is recruiting staff and postdocs from many countries to its new research wing, which offers first-class facilities.

Zimbabwe
One strong model is the Southern African Postgraduate Training Program in Mathematical Modeling, established in 1996 at the University of Zimbabwe with support from the University of Oslo and NUFU, the Norwegian Program for Development, Research and Education. Its two-year MSc course provided training in advanced mathematical techniques and their applications to epidemiology, industry, economics, and finance. Because the program had the flexibility to allow pooling of resources, the NUFU scholarship fund could be used to support travel for staff to teach at the University of Zimbabwe, to allow students to study at the University of Zimbabwe, and to support student travel to professional conferences. By the end of the funding cycle in 2006, about 100 MSc and 20 PhD students had graduated, most of them absorbed into academic and industrial jobs in the region.

One direct result of this network was to increase the number of papers published during the period 1998 to 2006 in Southern Africa. A significant number of the publications in South Africa are published by former NUFU students who are now professors in South Africa. According to Prof. Shonhiwa, the modeling program also did much to improve cooperation among mathematicians of the region and to establish links with industry.

East Africa
The Southern Africa Program in Mathematical Modeling is a new program, based at the University of Dar es Salaam. It is the direct descendant of the successful network described above under Zimbabwe. Operations began in August 2008, with support from NORAD. Initially, it has accepted students from Tanzania and other countries, and its small PhD staff will be supplemented by visiting lecturers. It has extensive support in the region from the many active graduates of the Zimbabwe-based NUFU program, whose experience and participation form an invaluable resource for the new network.

Also in East Africa, a Center for Biomathematics and Modeling was proposed in 2005, by Prof. Mugisha, to "spearhead the development of applied mathematics research as a center for international activities." It would also endeavor to identify and help exceptionally bright students and introduce them to applied mathematics. This center has not yet been funded.

Regional networks
Prof. El Tom observes that many countries have a mathematical enterprise that may be adequate for the nation as a whole, but is conspicuously weak for each individual institution. This weakness can be mitigated through collaborative networks of institutions at the national, intra-regional, and inter-regional levels. Our advisors agree that the web of dispersed but related partnerships and networks could be the most effective response to the particular problems of Africa.

More specifically, these would be regional training and research networks linking countries and institutions to generate greater effectiveness than any single institution can have by itself. They would pool resources to support much-needed activities: conferences, personnel exchanges, mentoring arrangements, research partnerships, special symposia, instrument sharing. They would link people electronically, programmatically, and face to face, stimulating collaboration. They would allow individuals and institutions to form partnerships and reach out electronically to others for the complementary expertise or partners they lack. A multi-country network would be likely to have greater reach and authority than any single institution or country in fund raising, public education, political action, and recruitment of young people to science.

The idea has already been implemented and tested in many locations over the past two decades. The following models illustrate the breadth of approaches already functioning:

- International North-South networks funded by entities outside Africa, such as CIMPA (France), the IMU, ICTP (Trieste, Italy), NORAD (Norway), the International Science Program (Sweden), the IDRC (Canada), and others. These have the virtue of funding levels not available in Africa, some degree of sustainability, and local design and management by Africans.
- The small multi-country networks in Francophone Africa, supported in many cases by members of the global Francophonie. These offer many sandwich and other exchange opportunities, as well as coursework and conferences. Areas emphasized at some institutions might be considered old-fashioned (algebra, analysis), but in other cases are expanding into newly relevant areas (e.g., modeling).

- In a new intra-African model, a single dominant country – currently South Africa – assumes a strategic goal of pan-African development. The African Institute for Mathematical Sciences (AIMS), initiated in 2003 and well respected by our advisors, provides postgraduate courses in applied mathematics and related sciences for students from many countries (including non-African countries). The program does not offer basic mathematics or PhD degrees, but seeks to serve as a "beacon" to draw bright Africans toward the mathematical sciences.
- Another new model is the African Mathematics Millennium Science Initiative (AMMSI), a network of five regions. Administered from the University of Nairobi, AMMSI was funded initially by the Mellon Foundation and additionally by the Nuffield Foundation, the Leverhulme Trust, and the International Mathematical Union. AMMSI was initiated in 2004 and has, to date, concentrated on fellowship and partial postgraduate scholarship grants, and support for regional conferences and workshops. To date more than 20 university staff and 200 postgraduate students have benefited from the grants. In collaboration with the International Mathematical Union and the London Mathematical Society, AMMSI has implemented a project called Mentoring African Research in Mathematics (MARM), which promotes linkages between mentors in Europe and mathematicians in African institutions. To date nine African universities participate in the project. AMMSI's website (http:www.ammsi.org) states that its mission is "to nurture the next generation of African mathematicians and mathematical leadership."
- South-South networks where African mathematicians are offered scholarships and fellowships by other countries, notably India, Brazil, and China. This is a relatively new model that has attracted widespread interest.
- SACEMA (South African Center for Epidemiological Modeling and Analysis) is a new research center of excellence at Stellenbosch that offers research facilities to South Africans as well as other Africans and scientists from outside Africa. The center supports research in disease modeling, offering scholarships to young African graduate students to pursue master's and PhD degrees.

A purely self-generated model – a network that is planned, administered, and funded wholly from within Africa – awaits implementation, partly because sources of funding within Africa are extremely limited. At present, an effective compromise may be a network that is supported by an international entity but planned and managed by Africans.

Examples of regional networks

One simple example is the partnership of the University of Brazzaville (Republic of the Congo) and the University of Dschang (Cameroon), which decided to conduct a PhD program in common. Another is the effort of the IMSP in Benin to use expertise in Abidjan (Côte d'Ivoire) to help expand its focus on Riemannian geometry to group representations.

Our advisors are familiar with many other networks, some still new and small, others well established. Prof. Roy wrote that with improvements to the research programs, it should be possible to create international research centers based on these networks. They include:

- Réseau EPIMATH: Central Africa (Cameroon, Congo, DRC), Gabon, Central African Republic, Chad)

- RAGTAAC (Réseau d'Analyse Géometrie, Topologie, et Applications en Afrique Centrale)
- Geonet: located in Porto-Novo, Benin, with members in 7 countries.

- GIRAGA: Benin and Cameroon universities; organized to support geometry, analysis, and applications.
- RAMA (Réseau International d'Analyse Mathématique et Applications): a network organized in Abidjan
- EAUMP (Eastern African Universities Mathematics Program): promotes development of mathematics and its applications in East Africa; aimed at alleviating the poor state of mathematics in East Africa. This network has been particularly instrumental in promoting short courses in algebra and analysis, attended by postgraduate students from eastern African countries, including Burundi.
- RAMAD (African Network for Applied Mathematics for Development); founded in 1999 in Burkina Faso
- RAGAAD (Reseau Africain de Géometrie et Algèbre Appliquées au Développement): about 200 members from 13 countries, including the three Maghreb nations; key node in Yaoundé, Cameroon; created at ICTP in 2003; attempts to bring to countries new postgraduate subjects.
- SAMSA (Southern Africa Mathematical Sciences Association): an active group of mathematicians in the Southern African region. SAMSA holds annual research meetings to discuss teaching and research in mathematics.

There are several types of regional networks. Those with an Affiliated Center as the central core offer workshops or meetings for students and support collaborative research between institutes. The students are typically doing their Masters or PhD in the featured field. The network serves to reinforce scholarship and to advance the research field itself. RAGAAD, on the other hand, emphasizes research less than it attempts to broaden the exposure of students and staff to fields of interest.

A dimension is added to these networks when they are sanctioned and supported by another institution. The networks cited above as Affiliated Centers of ICTP gain access to additional personnel, structures, and funding. Other such networks include:

- Math Sciences Network for Africa (African Mathematical Union, built around affiliated centers, with the key node in Cotonou, Benin)
- Network on PDE, Modeling and Control; key node: Ouagadougou, Burkina Faso
- Partnerships between RAGAAD and teams from Guinea, Mauritania, and Congo.
- Center for Mathematics, University of Ghana, focusing on applied mathematics

<u>Imbalances in collaboration</u>

Our advisors cite one problem that networks might help to address; a pronounced imbalance in African countries whose researchers collaborate with their peers in their own or other countries. For example, nearly 85% of African-French collaborations during the period 2000-2007 involved partners from the three Maghreb nations. Similarly, 87.3% of all African-USA collaborations in the same period involved partners from Egypt (29.2%) and South Africa (58.1%). According to Prof. El Tom, the number of research collaborations within countries was particularly small.

Outside partnerships and support
In addition to African centers and regional networks, a variety of other programs support mathematics in Africa, mostly from outside the continent. Some have been effective over time, offering steady support for African colleagues.

ICTP
The ICTP in Trieste is notable in its support for many training activities (diploma courses, schools, workshops, fellowships), both at ICTP and in Africa. Thanks to its firm commitment to outreach, the ICTP's Office of External Activities (OEA) has promoted mathematics research in Africa for over 20 years. In addition to the OEA's Visiting Scholar and sandwich programs in Trieste, the institute has supported mathematics activities in some 27 of the more than 50 countries in Africa.

Other support programs
- CIMPA/ICPAM: an international organization based in Nice, France, supported by UNESCO and the French Ministry of Foreign Affairs. It has supported and organized research and education programs in Africa for more than 25 years.
- West African Training Schools (WATS): at Gaston Berger University, Saint Louis, Senegal; sponsored by ICTP (Italy), CIMPA (France), and University Gaston Berger.
- ICTP Special Project in Algebra for East Africa: One of three planned schools for Masters and PhD-level students, with a different topic each year. The project at present lacks a home base. Also supports the organization of conferences and workshops in Africa.
- The ICTP Diploma in Mathematics program brings 10 students from developing countries to the ICTP each year for intensive courses meant to bridge the gap between the scholarship level of the home university and that required to enter graduate school.
- Southern Africa Program in Mathematical Modeling: Initiated at the University of Dar es Salaam, funded in August 2008 by NORAD. Will initially fund students from Tanzania and other countries, then augment small PhD staff with visiting lecturers. (Although the predecessor of this program was based in Zimbabwe, in the Southern region, political conditions there necessitate an alternative venue for the time being.)
- DIMACS (Discrete Mathematics and Advanced Computer Science). DIMACS' African Initiative leads a project, in collaboration with the US Mathematical Biosciences Institute (MBI), aimed at identifying key bio-mathematical challenges arising from problems of Africa. Most of the project's activities take place in Africa; funded by the NSF and other US-based sources.
- MITACS' African Initiative: A Canada-Africa Biomathematics project that collaborates with DIMACS; launched in 2007.
- International Mathematical Union; its Commission on Development and Exchanges supports conference and research travel and some projects.
- ANSTI (African Network of Scientific and Technological Institutions): Promotes math research in 35 sub-Saharan countries since 1980; supported by UNESCO, UNDP, and DAAD (Deutscher Akademischer Austausch Dienst).

- African Mathematical Union (AMU): African affiliate of the IMU; its current activities seem to be confined to its support for the Pan-African Mathematics Olympiad, in which several African countries participate.
- Agency of Francophone Universities (AUF): A worldwide network that supports conference travel, doctoral scholarships, cooperative projects, centers of excellence, networks, and other activities in Africa.
- SARIMA (Soutien aux Activités de Recherche Informatique et Mathématique en Afrique): Support for regional centers in Senegal, Cameroon, Maghreb, Niger, Madagascar, and Lebanon; funded from 2004 to 2008 by the French Ministry of Foreign Affairs; emphasis on information sciences and applied mathematics.
- Sida/SAREC: A small, active funding group supported by the Swedish Agency for Research Cooperation (SAREC) and the University of Uppsala; supports mathematics as well as physics and chemistry in Africa and other regions.
- TEAMS (Teachers Education Assistance in Mathematics and Science): a collaboration between the Netherlands and three universities in Tanzania.
- Schlumberger Foundation: Supports research visits by African mathematicians to IHES (Institut des Hautes Etudes Scientifiques) in France and the organization of mathematics conferences in Africa.

The effects of centers and networks

With the exception of institutions in Botswana and South Africa, universities in sub-Saharan countries lack resources and adequately trained staff to run sustainable programs on their own. Pooling resources in shared centers and networks offers an alternative structure to sustain high-level activities.

One model of success is the Southern African Postgraduate Training Program in Mathematical Modeling, established in 1996 at the University of Zimbabwe. This was supported by the University of Oslo and funded by NUFU, the Norwegian Program for Development, Research and Education. That two-year MSc course provided training in advanced mathematical techniques and their applications to epidemiology, industry, economics and finance. By the end of the funding cycle in 2006, about 100 MSc and 20 PhD students had graduated from the program and most of them were absorbed into academic and industrial jobs in the region. As mentioned above, the modeling program did much to improve cooperation between mathematicians in the region and to establish links with industry.

The status of AIMS and AMMSI

We offer an additional word about the two networks that seem to have the potential for pan-African reach: AIMS, especially through its plans for an “AMI-Net” network in many countries, and AMMSI, a distributed network that supports mathematics research and training in five regions.

A potential danger for any network with a relative abundance of resources is that of over-centralization. In our opinion, networks should avoid a dynamic leading to a single institutional model for all the network's nodes. They best serve sustainable progress by cultivating a diverse network of nodes with locally autonomous leadership, finances and scientific programs. In particular, content of graduate studies, subject balance, and fiscal management should remain local responsibilities. As mentioned above, ICTP is a good model, offering as it does targeted funding and high international standards for the universities and research institutes it serves.

AMMSI, as a distributed network of mathematics research and training, has achieved promising early steps, in delivering postgraduate scholarships and faculty fellowships; in sponsoring much-needed symposia on important topics; and in developing the mentoring program (MARM) mentioned above. At the same time, despite the endorsement of the International Mathematical Union and African Mathematical Union, it has yet to generate long-term support from African governments or their significant organizations, especially the African Union.

At this point, it would seem that AIMS has won some support at the governmental level, particularly from the South African government, while AMMSI, with limited funds, emphasizes the support of local programs and leadership. If these virtues could be unified within a single effort with strong leadership, the resulting program might be equipped to effectively support diversification and modernization of mathematical research across Africa and to create local doctoral training of good quality across many countries. It could also help to develop South-South exchanges with mathematically more advanced African countries, such as South Africa and several countries of the Maghreb and West Africa, and to build North-South exchanges throughout the international community.

Our advisors conclude that *the effects of all these programs have been generally positive.* These effects include:

- improvements in university facilities and infrastructures
- enhanced training of mathematics teachers and instructors
- bursaries for talented students who could not otherwise afford postgraduate education.

At the same time, there is work to be done.

- More coordination would make these centers and networks more effective. Without well-planned interaction, they can do little to overcome the pervasive problem of isolation.
- More research centers and more scholarship opportunities for postgraduate studies are urgently needed.
- Mathematics clubs need to be established and promoted in secondary schools to help develop student participation in learning and problem solving. Clubs can also conduct academic exercises in relevant workplaces, such as banks, industries, and insurance companies, to raise interest and help answer a common question: "Mathematics for what?"

The importance of colonial traditions

Africa, like many of the poorer regions of the world, has been shaped by its colonial experiences in both geographic and cultural ways. The practice and teaching of mathematics is also shaped by colonial traditions. The advantages of continued ties with a former colonial power, suggests Prof. Mugisha, is that it may offer substantial support in the form of scholarships, grant programs, and exchanges for its former colonies. A disadvantage is that the best African scholars are often tempted to remain in the former colonial country, draining scarce local talent.

The former French colonies, primarily those in West Africa, experience many advantages from continued relationships with France and, to a lesser extent, with Belgium and Germany. For example, French institutions formulate and provide modern research tools and facilities to former colonies and invite mathematicians from the developing countries to spend time at European institutes. This is especially valuable for African faculty, according to Prof. Desquith, whose time and intellectual energy are drained by the many responsibilities of institutional life at home. A disadvantage is that the French-based curricula of some countries are considered outdated, underemphasizing modeling and interdisciplinary studies. Some faculty feel that continuing dependence on colonial powers reduces local incentives to think independently and solve African problems, and others point out that few mathematicians from the Francophone countries are well trained in English, the predominant language of mathematics and sciences.

In South Africa, the structure of the mathematics degree continues to be based on the Scottish system, in the view of Prof. Banasiak. Tertiary education, especially in applied mathematics, is also influenced by the British paradigm, which gives little emphasis to theory. Here, students' inadequate preparation often leads to a "caricature" of the classical British degree, he added, where many students are taught "only methods and techniques without understanding of the underlying theory and graduate without being able to properly formulate a definition or provide a formal proof of a simple theorem."

Some advisors comment that relations with former colonial powers should be designed to encourage intellectual, as well as political, independence. For Prof. Yacoubi, aid is welcome when "the strategy is to assist the country but ensure the replacement of the colonial experts by the locally trained ones." Offered in this spirit, she said, the linkages with Europe can be an advantage.

In the same vein, Dr. Thompson affirms the value of training African mathematicians primarily in Africa: "It is essential, I believe, that excellent African mathematicians are not just formed in the developed countries but that they are formed in Africa as well. The ICTP experience of the Diploma program shows that with good teaching and a great deal of effort from the staff students can reach heights that were previously unimaginable to them."

In conclusion, while it is true that colonial traditions played a great part in shaping education policies and trends, situations have changed appreciably in the last several decades. What is happening in African countries today is defined by a wide variety of factors and interaction with new partners.

The place of ethno-mathematics

Ethno-mathematics is the formal study of the relationship between mathematics and culture. Prof. Paulus Gerdes of Mozambique is perhaps Africa's leading specialist in ethno-mathematics, and he has detailed its evolution and significance in Africa.[11] In his book *Women, Art and Geometry* (Africa World Press, Trenton, New Jersey), he has shown that women, in particular,

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[11] Prof. Gerdes has also published a list of African PhD's in mathematics; see *African Doctorates in Mathematics: A Catalogue,* Lulu.com, March 9, 2007. Also, see Paulus Gerdes and Ahmed Djebbar, *Mathematics in African History and Culture: An Annotated Bibliography,* African Mathematical Union, Cape Town, 2004.

have developed numerous activities involving mathematics, teaching mathematics to children in their mother tongue. Each region has developed its own techniques, along with local particularities of the systems of numeration, logic, and language. Prof. Gerdes further develops these ideas in *Ethnomathematics and Education in Africa* (Institute of International Education, University of Stockholm, 1995).

There is also a rich tradition of mathematical games, puzzles and contests in African culture. Even today, some people like to practice competitive mathematics as a high-level sport; national and international mathematics Olympiads could be considered an extension of this trend. In Africa, notes Prof. Roy, such activities should be better developed and supported. One finds in Niger, for example, an association of volunteers who organize contests and publish mathematical problems in the press.

At the same time, ethno-mathematics is not, in the opinion of our advisors, generally well developed as an academic subject in Africa, beyond the work of Prof. Gerdes and perhaps one or two others. For example, Prof. Banasiak writes that the concept is not well known in South Africa. Our advisors did suggest one other worker in the field, Mme. Salimata Doumbia of Côte d'Ivoire, author of *les Mathématiques dans l'environnement socioculturel Africain (MESCA)*, a study that explores such topics as educational games and the uses of mathematical ideas in society. However, the subject is generally perceived as a difficult one, according to Prof. Mugisha, which leads to low participation among students.

A Vision for Developing Africa's Mathematical Future

In response to the question "What must be done to develop Africa's future?," the responses of our advisors can be summarized as follows: "Train the young well and teach them good practice."

There are encouraging signs that this process has begun. For example, more and more countries are starting to organize national mathematics competitions and to participate in the Pan African Mathematics Olympiads (PAMO); some have reached the level of the International Mathematics Olympiads (IMO). Talented young African mathematicians who have found productive careers in many countries stand ready to assist in developing capacity in their home countries. In many African countries, an increasing number of mathematics books are written by nationals of those countries. And in more and more African countries, the internet is accessible to mathematics students, allowing them access to journals, problems, and online courses, such as MIT's OpenCourseWare.

Most encouraging has been the gradual change in the "standard model" of international assistance to African countries and institutions. The strategy formulated in the 1960s was to send promising students abroad for postgraduate education – despite the fact that many of them would not return home to contribute first-hand to Africa's development. The new model is one of capacity development, of gradually strengthening the research and education infrastructures *within Africa*. In this model, advanced mathematics students may do some study abroad, but they are awarded their master's and PhD degrees by their home institution. They help train those coming up the ladder after them, and seek to find rewarding employment not only in academia but also in government and industry. As resident Africans, they are uniquely positioned to train people in their own institutions and regions, build relationships with other institutions, understand local problems, and become contributors to new solutions. This, as Prof. El Tom indicates, requires a matching commitment "on the ground" to support these young leaders with strong institutions, stimulating colleagues, and the sustained commitment of government.

Building the capacity of existing centers
Improving the level of mathematical development depends first and foremost on building capacity in mathematical education and research. There is agreement among our advisors that the best approach to capacity development is to increase the value of existing programs, specifically those of centers and networks. "Existing international institutions like CIMPA/ICPAM and ICTP," states Prof Roy, "should be supported rather than launching new ones." Other existing centers include WATS in Senegal and AIMS in South Africa, both of which train students beyond the bachelor's degree to prepare for a master's degree.

Some of the goals of capacity building are the following:

- *Revitalize teacher training at all levels.* Teaching should move beyond the old emphasis on computation to include practical exercises and a vision of the broad usefulness of mathematics. To achieve this requires teachers and professors who have been exposed to the exciting problem-solving potentials of mathematics. Teachers with the basic tools of functional computers, software, and access to on-line journals can keep up with the techniques and debates of their profession worldwide. Faculty motivation should include promotion on the basis of merit rather than the current criterion of seniority.
- *Make mathematics teaching more relevant to exciting problems.* Modern teaching of applied and industrial mathematics should accompany pure mathematics as a focus of every institution. Applied mathematics is more expensive to teach than pure mathematics, but essential to prepare students for the real world and justify the role of mathematics in the curriculum.
- *Raise the public image of mathematics.* Communicate the value of mathematics to the general public by demonstrating its importance to science, culture, and economic development. For mathematics to be seen as relevant, teachers and professors must be involved in this effort. They can take advantage of public talks, writings, and TV presentations to explain the importance of their subject to modern life. They can also reach out to the private sector to demonstrate the value of scientific thinking for firms small and large.

New regional centers
Although the majority of our advisors advocated the strengthening of existing centers, several spoke out for new regional centers in Africa, modeled perhaps on the ICTP (Trieste) or IHP (France). Such a center, suggests Prof. Roy, would run research semesters in one of the countries that already has a critical mass of mathematicians, such as Benin, Cameroon (Yaoundé), or Senegal (Dakar). The scientific directors and topic of the programs would change every semester while the infrastructure (libraries, computing facilities, efficient staff, clear procedures for making decisions) would remain fixed. "It would combine the idea of a center with excellent

permanent equipment," writes Prof. Roy, "with the idea of having excellent regional/continental networks."

Dr. Thompson offers the vision of an "African PhD school" with a small local staff and many visiting experts. It would offer good lecture courses, starting with the basics and ending with research-level material in most major fields. Contact between lecturers and students would lead to sandwich programs, new mentoring relationships, conference plans, and other projects. "The ICTP experience of the Diploma programme," he wrote, "shows that with good teaching and a great deal of effort from the staff, students can reach heights that were previously unimaginable to them."

Other examples suggested were a center for mathematical modeling in *East Africa*, a center for pure mathematics in *West Africa*, a center for dynamical systems in *North Africa*, a center for applied mathematics in *Central Africa*, a center for industrial mathematics in *Southern Africa.* Such a plan of dispersed centers, writes Prof. Mugisha, "could catapult the regional development of mathematics."

In South Africa, Prof. Banasiak has advocated a National Research Institute, similar to Brazil's Institute of Pure and Applied Mathematics and the Indian Tata Institute of Fundamental Research, to drive mathematical research and provide inspiration for young people. A proposal, presented to the government by the South African Mathematical Society, is under review.

Another model, suggested by Prof. El Tom, is a "pan-African center" for graduate studies in mathematics and mathematics education. This would be implemented as a collaborative project between an African university and a prestigious non-African university with strong graduate programs in both mathematics and mathematics education. While the non-African university would award doctoral degrees, the African university would host the project.

Networks

In addition to revitalizing existing universities and other institutions, our advisors favor the use and improvement of existing networks. They prefer the strategy of working from existing networks over the creation of new ones. Existing initiatives have the advantage of experience, which has allowed participants to learn essential lessons about support, management, and collaboration.

Some other advantages already demonstrated by regional mathematics networks:

- The most obvious is that they create a critical mass of students and researchers in overcoming isolation. This makes it possible to improve the quality of postgraduate training by increasing the intellectual richness of the environment.
- Researchers and educators who are part of a critical mass have greater authority in demonstrating the value of mathematics and educating governments on the needs and value of the mathematical enterprise.
- Networks increase program flexibility by bringing to bear more diverse skills. A network-based program can develop both fundamental and applied aspects of mathematics, which is not always possible in single institutions with firm traditions.

- More than single institutions, networks can support the vital conferences, poles of excellence, staff and student exchanges, and co-supervision that provide the life-blood of advanced learning.
- The web of contacts in a network allows for greater "reach" in developing South-South exchanges; international South-South cooperation with India, China, and South America; and North-South exchanges.
- Network members can share essential infrastructure, including hardware and software, instrumentation, and bandwidth, as well as expertise.
- Network members can share career skills in creating proposals, budgets, cooperative projects, and English/French language programs.
- Networks can address inequities. In each country, mathematics research is usually dominated by one or two universities; through linkages, networks can extend such skills to multiple institutions.

Networks could also be strengthened by helping to close the cultural and linguistic gap between Francophone and Anglophone countries. Prof. Touré advocates "a point of support to develop regional and inter-African cooperation to move beyond the barriers of the colonial heritage." If Francophone and Anglophone mathematicians could learn more of each others' language, wrote Prof. Roy, they could more easily build and share networks, personnel, contacts, and resources. This could be assisted by a combination of linguistic and mathematical workshops built around the common language of mathematics.

Prof. Sibanda notes his personal experience with one small regional network: "The one truly regional initiative that we had in this region [the Southern African Postgraduate Training Program in Mathematical Modeling] had a far greater impact than what could have been achieved if the resources were placed at the national level. For example, almost the entire current academic staff of the Department of Mathematics at the University of Swaziland are products of that regional training initiative. Indeed a significant number of regional universities, including those in South Africa, are beneficiaries of that particular initiative. Of interest also was the level of research collaboration at the regional level that resulted from this broad-based initiative."

Internet assets

No network can achieve its potential without robust and reliable internet resources, including hardware, software, and sufficient bandwidth. Given adequate training in access techniques and research traditions, the problem of scarce learning materials can to a large extent be solved. Equally important, students and faculty alike can interact with colleagues around the world and teachers can gain useful material unavailable at home. AMI-Net is one program that aspires to tackle this problem, but substantial resources will be required both from donors and from African institutions and companies.

The role of the mathematically stronger countries

Although many African countries have strengths in sub-specialties, a handful of countries have developed broad capacity across the discipline. These include South Africa, several Francophone countries, Egypt, and the Maghreb countries. Some of these have the potential to catalyze and support mathematical development in other countries, whether by strengthening existing centers or networks or creating new ones. In Francophone Africa, Benin, Senegal, and Côte d'Ivoire have already been active in supporting networks that benefit neighboring countries, in large part because of their continuing support from the *Francophonie*.

For Nigeria, the recent establishment of the National Mathematics Center in Abuja indicates the country's strong potential for a leadership role. There are many obstacles to overcome, including the continuing struggle for political stability and the relatively low level of mathematical development in relation to Nigeria's potential. Should the NMC resurrect its earlier emphasis on mathematics research and grow in effectiveness, the country could become a regional leader.

South Africa has already become a destination of choice for thousands of students from sub-Saharan Africa wishing to pursue postgraduate studies. The country's institutions make available modern research tools and facilities to other African countries, helping to boost mathematical development. South Africa in turn benefits by gaining excellent foreign talent, who now dominate the faculties of some universities there. At the same time, there are concerns about a new process of intra-African brain drain.

Prof. Touré notes that any kind of assistance can have a lasting impact only if it is sustained over long periods. He cites the example of the ISP in Sweden, which has been supporting physics in Africa for three decades, and began its support for mathematics and chemistry more recently. He does note that funding levels from ISP, as from AMMSI, have been very low, and that higher levels are required to have a strong impact.

A more active role for governments

Because salaries, scholarships, and much capital spending in African countries are determined by federal governments, political leaders have unique ability to accelerate mathematical development. They also have the responsibility to do so, in that every country, including its agriculture, public health, and economic growth, would benefit from increased mathematical and technical skills among its people. Governments could begin, for modest expense, by providing support for graduate students, as is the custom elsewhere, and undertaking to support all students who perform well in mathematics from the early levels of education. They should also bring faculty salaries more closely in line with need. In Senegal, for example, the government a few years ago raised the salary for university instructors by over 50%, making the country more attractive to young Senegalese who wish to return home after completing a PhD abroad. In Benin, the government increased salaries for professors in higher and secondary education in July 2008. As a consequence, our advisors already see an overall rise in the quality of instruction across Francophone universities.

Summary: Meeting the Need

In most African countries, mathematical development is limited by low numbers of well-qualified educators. More participants are needed at virtually every level: primary and secondary school teachers; undergraduate faculty at the masters and especially the PhD levels; research mathematicians with expertise in modeling, interdisciplinary topics, and industrial mathematics. With too few professors in place to train the next generation of teachers, researchers, and leaders, countries cannot harness the power of mathematics to support modern science-based activities, including efforts to improve food security, combat diseases, use information tools, and power the economy.

While record numbers of students in Africa clamor for a college education, many of the most talented avoid mathematics careers because of low salaries, crushing teaching loads, a poor public image, and a shortage of mentors engaged in exciting challenges. Given their low numbers, faculty and graduate students suffer from professional and geographic isolation and lack of resources to pursue the research essential to professional advancement. Faculty and students alike endure overcrowded lecture rooms, inadequate computer and bandwidth resources, and instrumentation dating from the colonial days of the 1960s.

Advisors to this report, all of whom have long experience in the research and training conditions of Africa, uniformly point to educational improvement as the overwhelming need facing the mathematics enterprise today.

There are several approaches to meeting this need. The most obvious is increased *direct support for the mathematical enterprise*, which is primarily the responsibility of the national government and president of each country. The government pays the teachers, supports the institutions, and determines education practices. But increased government spending, though essential, is restrained by limited budgets and the strong competition from other urgent needs.

The mathematical community can take more active steps to promote government support. These include educational campaigns, more forceful public speaking and lobbying, and programs that demonstrate value of research to the country. Certainly, one kind of program is the mathematics Olympiads that raise the public visibility of mathematics and inspires talented students to compete.

More realistically, our advisors made the case that no single program or "magic bullet" can be sufficient to invigorate the mathematics enterprise. Instead, they advocated *strengthening and expanding successful existing mechanisms, most notably regional training and research networks*. Such networks have gradually spread across sub-Saharan Africa because of their ability to leverage the limited resources of individual institutions. Such networks are possible largely because of the internet, which allows isolated mathematicians or departments to communicate with distant colleagues and to create a virtual "critical mass" of faculty and students working toward similar goals of training and knowledge discovery.

Regional networks bring several advantages for mathematicians. They are relatively inexpensive, at a time of acute financial uncertainty, because many of their activities are generated by people and institutions already in place. They increase the magnitude and power of projects by adding the expertise of multiple nodes. They allow students access to more role models, mentors, and career possibilities than are visible from a single location. And they add the authority of increased mass to each of the member nodes, allowing mathematicians to speak with a more powerful and unified voice.

A step recommended by many is to focus support on postgraduate study. Because of the relatively low cost of tuition and fees in Africa, a donor could support a dozen or more graduate students on even a small grant. This, in turn, would strengthen the regional networks to which they belong.

A second, parallel step – also relatively low in cost – would be to provide fellowships and career training for faculty, most of whom have little or no support for the research or advanced degree work necessary for career satisfaction. This, too, raises the quality of the network to which the lecturer belongs.

There are many other network-related steps suggested by our advisors that would benefit individuals, institutions, and the mathematical enterprise:

- *Support more research collaborations and partnerships.* Modern mathematics is no longer a solo endeavor; more and more research has become collaborative, especially in multidisciplinary fields. Collaboration can be stimulated in many ways, including regional conferences, staff and student exchanges, and co-supervision.
- *Support science academies.* Africa's academies of science are becoming stronger, more numerous, and better positioned to advocate for public support for science. The Network of African Scientific Academies (NASAC), assisted by the global InterAcademy Panel on International Issues (IAP), can assist individual academies to communicate with authority to political and private-sector leaders, sponsor activities for students, and help develop sources of support.
- *Support and expand mathematics competitions.* National and international competitions, such as the various Olympiads, raise the visibility of mathematics and convey the enjoyment of intellectual achievement. They also provide a forum for meeting mathematicians and visualizing a career path.
- *Promote regional and international partnerships.* These now include South-South cooperation with institutions in India, China, and Brazil. Other possibilities include partnerships with the mathematically more advanced African countries, such as South Africa, the Maghreb countries, Benin, Cameroon and Senegal; with the U.S.; and with several supportive European countries.

In short, there are many ways to support mathematics development in Africa, especially when the mathematics enterprise is viewed as the joint endeavor of many partners. Partnerships in the form of regional networks have already developed in response to the distinctive geographic challenges of this immense continent, and our advisors have endorsed this path as much by their actions as by their recommendations: most of those who presently live in Africa are themselves involved in regional networks, some of them as founders and leaders. The concept is supported as well by those who are part of the Diaspora, and who share the same dream of a stronger mathematical enterprise across Africa.

IMU Recommendations

The informal advisory committee listed above has contributed a valuable range of pertinent and stimulating information that forms the basis of this report. The IMU deliberately sought out such a range of responses so as to reflect the true diversity of mathematical development across the many countries of Africa.

At the same time, the IMU steering committee, which has helped edit this report, feels it would be helpful to present a more succinct summary of this information in the form of recommendations. These recommendations may be useful for institutions, governments, and other organizations desiring “action items” to help strengthen mathematics in Africa. We emphasize that these recommendations may not necessarily represent the views of each individual advisor. Instead, they are proposed by the IMU Steering Committee for this report as a composite view that may reasonably be drawn from this exercise and may help point to concrete steps and even programs of considerable benefit.

1) Enhancing mathematics and its contributions to development requires simultaneous attention at all levels. To concentrate on primary education alone is futile without qualified and motivated teachers; producing such teachers requires effective secondary and tertiary education, wise career counseling, and skilled mentors. Such programs and mentors must be linked to and sustained by graduate education, mathematics research, and specialists in mathematics education. This professional infrastructure, then, can generate scientific literacy throughout society: an educated workforce, quantitatively sophisticated scientists and engineers, government policy-makers and technocrats, and industrial and governmental leadership.

2) One effective way to enhance mathematics development is through research and education networks. Most centers in Africa do not have the critical mass necessary for mathematics research and education at internationally competitive levels. At least in the short run, these centers can leap ahead in capacity by sharing people, programs, and resources with other institutions and countries. Through such networking, institutions and individuals have access to talented students, partners, mentors, and infrastructure not available at home. At the outset, the focus of these networks should be on supporting the mathematical growth of the next generation of mathematicians and mathematics educators, with the goal of creating the “critical mass” necessary for mathematics research and education at internationally competitive levels. Only then can mathematics bring its potential power to bear on problems relevant to Africa.

3) Mathematically stronger countries in Africa, and governments and organizations outside Africa desiring to contribute to continental development, should support, rather than manage, the activities of participating centers in Africa. Programs that are designed and managed from within are most likely to be sustainable. (The ICTP, in its work with scientists throughout the developing world, provides a good model for effective support.) This does not mean that the mathematically stronger countries should not lend their expertise to smaller partners, but that leadership should be shared evenly among all participating countries.

4) The support of mathematical development should take account of broader national and regional realities. Different strategies are required depending on national and regional socioeconomic context. Institutional support is most likely to be cost-effective and of lasting impact if it is directed toward institutions whose need is clear but which also show the ability to take advantage of targeted assistance. In countries with supportive governments, external support for local institutions can quickly strengthen the processes of teaching and learning. But, for example, the internet offers opportunities for helping mathematical development even in more problematic situations. Sound and flexible strategies to develop human capacity, especially in mathematics and the sciences, are essential steps toward a self-sustaining knowledge economy in Africa.

Mathematics in Africa: Challenges and Opportunities

Background Information

The John Templeton Foundation recently asked the International Mathematical Union to conduct for them a survey on Mathematics in Africa: Challenges and Opportunities.

The survey should provide the following information:

1) A brief account of the current situation of mathematical development on the African continent:
   i) by region (North Africa, Central Africa, East Africa, West Africa, Southern Africa)
   ii) by mathematical specialty
   iii) by educational infrastructure

Basic questions:

1. Which countries in Africa have a strong university education in mathematics? In which mathematical areas are their strengths?
2. Which countries have viable centers for research in mathematics? What mathematical areas are represented?
3. Which countries have a strong development in secondary mathematics education?
4. Which countries have a strong development in elementary mathematics education? What is the balance between urban centers and schools in the countryside?
5. To what extent is mathematics education informed by traditions from colonial periods?
6. What is the existing network of relationships as regards mathematics teaching and research?
7. What is the role of ethno-mathematics?

2) An analysis of strengths and weaknesses:
   i) by region (North Africa, Central Africa, East Africa, West Africa, Southern Africa)
   ii) by mathematical specialty
   iii) by educational infrastructure

Basic questions:

1. Which areas of mathematics are strongest on the African continent? How do these map to African needs?
2. What are the advantages and disadvantages for mathematical development of continuing ties with former colonial powers?
3. Given their size and population, what are the appropriate roles of countries like Nigeria and South Africa in mathematical development on the continent?
4. What is the role of the countries of North Africa? Has the proximity of Western Europe served to enhance their mathematical development?

3) School mathematics:
   i) by region (North Africa, Central Africa, East Africa, West Africa, Southern Africa)
   ii) by educational policy

Basic questions:

1. On a per-country basis, what is the number of ten-year-old to eighteen-year-old students for each mathematics teacher?
2. What is the normal education level for secondary school math teachers in Africa? Do they need bachelor's degrees?

4) What are the opportunities for talented/high achieving students?
   i) by region (North Africa, Central Africa, East Africa, West Africa, Southern Africa)
   ii) by educational policy.

Basic questions:

1. Are there screening systems in African countries to identify exceptionally gifted math students?
2. If such screening tools exist, to what use are they put? Do such screening procedures lead to placing these highly-gifted students in special accelerated schools or courses?
3. Is there a system of educational tracking of highly talented math students to assure their entry into topflight universities with strong mathematics departments?
4. What are the educational career opportunities for highly-talented math students in Africa? How does this vary by specific countries within Africa?
5. What are the commercial or specialized career opportunities, such as research, engineering, etc, for talented and well-trained math students in Africa?

5) Highlight a selection of opportunities for highly leveraged investment in Africa's mathematical future.

Basic questions:

1. Which are the greatest opportunities for improvement in mathematical development and training?
2. How are these challenges affected by the spread of universal elementary education in an increasing number of African countries?

6) Provide a vision for cohesive, sustainable development through networking and building on existing nodes of quality.

Basic questions:

1. Single nation strategies vs. regional or continental approach.
2. Building on the strengths of Francophone and Anglophone traditions in mathematical training and development .

7) Provide a vision of the mechanics of investment in Africa's mathematical development.

Basic questions:

1. What are the current initiatives to encourage the development of mathematics and mathematics education on the African continent?
2. To what extent is cooperation with these initiatives, as opposed to initiating new ones, the more effective and highly leveraged strategy?

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The members of the team of experts who will oversee the preparation and submission of this study are the following:

Professor Sir John Ball, Mathematics Department, Oxford University, UK. Professor Ball is a research mathematician specializing in the area of Nonlinear Analysis and its applications to materials science. He is the recipient of several mathematical honours and prizes. He is the former President of the International Mathematical Union (IMU) and a current member of the IMU Executive Committee.

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Professor Herbert Clemens, Professor of Mathematics, Ohio State University, Columbus, Ohio, USA. He co-chairs the Developing Countries Strategy Group of the IMU and is a consultant for the African Mathematics Millennium Science Initiative.

Ms. Arlen Hastings, Executive Director of the Science Initiative Group (SIG), Institute for Advanced Study, Princeton, NJ, USA (www.ias.edu/sig). SIG is an international team of scientific leaders and supporters dedicated to fostering science in developing countries through projects including the Millennium Science Initiative, conceived of and implemented in partnership with the World Bank, and the African Regional Initiative in Science and Education (RISE), supported by the Carnegie Corporation of New York.

Professor Edward Lungu is a Professor of Mathematics at the University of Botswana, Gaborone, Botswana. He is a research mathematician in the area of mathematical modelling especially as it applies to biological problems. He chairs the Advisory Board of the African Institute of Mathematical Sciences, and is a member of the Programme Committee of AMMSI. He is former Chair of the Mathematics Department of his university and former President of the Southern Africa Mathematical Sciences Association.

Professor Wandera Ogana, Professor of Mathematics of the University of Nairobi, Kenya, is a research mathematician in the area of Computational Fluid Dynamics and Mathematical Modelling. He is former Chair of the Department of Mathematics and former Dean of the Faculty of Sciences of the University of Nairobi. Professor Ogana is the Chair of the Programme Committee of AMMSI.

Ms. Sharon Berry Laurenti, Administrative Secretary of IMU's Developing Countries Strategy Group, based at the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, will serve as Secretary for the project.