

1. General issues

P. Your position as President of the IMU enables you to appreciate many important issues related to mathematics from "above". What would you highlight, good or bad, of the activity of mathematicians around the world?

Mathematicians have much to be proud of as regards the development and applications of our subject. One problem area is the often negative image of mathematics among the general public, which discourages students from studying the subject and reduces the number of good teachers to provide them with inspiration. Another is the uneasy relation which has developed between research mathematicians and those working in mathematics education.

P. What impression do you have of the mathematical community as a whole?

I have very good feelings about our community. It has rigorous scientific standards, high integrity, and a fine record of service.

P. What are the main challenges for mathematics and the mathematical community today in our society?

Mathematics can contribute understanding to almost every aspect of human existence. As subjects become better understood they become more susceptible to mathematical treatment. We see this happening now with biology and medicine, and the life sciences will certainly be a major growth area for mathematics over the coming decades. Mathematical biology is of course already an established branch of applied mathematics, but it is probably fair to say that it is not yet regarded by most in the life sciences as an essential part of their subject. More mathematicians need to invest the time to learn relevant parts of biology, and young researchers in the life sciences need to become convinced of the value of mathematical ways of thinking. In this process of interaction there can be resistance and misunderstanding, but the rewards are potentially immense.

P. What about education? What are the main issues in this important area, both in higher education and schools?

The first priority must be to encourage more young people to pursue further studies in mathematics and to increase the number of high quality mathematics teachers in schools. Of course the situation varies from country to country. In those countries in which mathematicians have various attractive careers to choose from, there seems little alternative to improving the pay and conditions for teachers of mathematics, and of other key science subjects in which high quality teacher recruitment is a problem. Such action requires political will, and thus an appreciation of the importance of mathematics and science among politicians and opinion formers.

In developing countries the opportunities for schoolchildren to study mathematics are often extremely limited, due to the lack of resources and trained teachers, while in universities an important goal is to improve the education of the teachers themselves.

P. The importance of higher education for the technological development of our society is clear. How do you feel that higher education could and should cooperate with programmes in innovation, research and development?

Most research leaders I have met from industry see the main role of universities as providing highly trained and flexible graduates and carrying out fundamental research. Fundamental research has much to contribute to and gain from industry, and it is important that structures are in place that make interaction between universities and industry as easy as possible, including appropriate schemes for undergraduates and postgraduates. Universities should ensure that their approach to intellectual property rights and internal structures give the necessary incentives for academics to found start-up companies to exploit their work. But universities should not be places where routine development work is carried out that could better be done in industry, detracting from the fundamental university research mission.

P. What is your opinion about the future of science in Europe? What do you think of the European Research Council?

My personal experience of European Union science initiatives, such as research networks and individual Marie Curie fellowships, has been rather positive. I think that they have contributed significantly to the personal development of young researchers and to forging a European unity for research. The funds are quite flexible, although the bureaucracy is heavy. On the other hand there are concerns, first about the representation of basic science, including mathematics, in the programmes, and second about the position of mathematics itself (for example, there is no mathematician on the 45 member European Research Advisory Board). The European Research Council offers some hope of responding to these concerns, and we will have to wait to see how it evolves.

2. IMU

P. Can you explain a bit for our readers the role and leadership of IMU?

IMU is the scientific union for mathematics, and the principal body representing mathematics on the world stage. Originally, its aims were mainly concerned with the organization of the ICMs, and the award of the Fields Medals, but now they include other issues of importance for mathematics, for example those connected with electronic publishing and with the development of mathematics in less advantaged parts of the world.

The members of IMU are countries. Each country is represented by an Adhering Organization, which is in turn advised by a Committee for Mathematics. The member countries are divided into five groups; for example Spain is currently in Group IV. The higher the number of the group the more votes the country has, and the more dues it pays. The dues finance almost all the activities of IMU.

The IMU Executive Committee consists of ten mathematicians, elected every four years by the IMU General Assembly. The Executive Committee is responsible for all policy matters, and for such tasks as choosing the members of the ICM Program Committee and various prize committees. It meets physically for two days once a year, but carries out most of its business by email. There is no permanent office, and the IMU administrative base is at the institution of the Secretary (currently Phillip Griffiths of the Institute for Advanced Study in Princeton).

P. Could you indicate some specific projects in which IMU is now involved?

The current Executive Committee has made a big effort to improve the service IMU provides for developing countries. It commissioned a report from an *ad hoc* group of experts, completed in 2003, to advise IMU on how to proceed, and most of the recommendations of this report have been implemented. These include the formation of a Developing Countries Strategy Group, the appointment of a half-time administrator for developing countries based at the International Centre for Theoretical Physics (ICTP) in Trieste, the merging of IMU conference support with its developing countries budget, and the proposed creation of a new category of Associate Membership of IMU suitable for poorer developing countries. We are fortunate to have received a generous grant of \$50,000 from the Abel Fund, which is likely to be renewed annually, and the award by the Nuffield Foundation of a substantial project grant of £105,000 (pounds sterling) to IMU and the London Mathematical Society to support mathematics and its teaching in the Anglophone countries of sub-Saharan Africa has just been announced. A new prize for young mathematicians from developing countries, the Ramanujan Prize, has been founded by ICTP with prize money donated by the Abel Fund and support from IMU.

The present printed World Directory of Mathematicians has become outdated, and through IMU's Committee for Electronic Information and Communication (CEIC) is being replaced by an Electronic World Directory of Mathematicians (EWDM), and a Federated World Directory of Mathematicians (FWDM) that simultaneously searches the EWDM and other databases such as the AMS/SIAM/MAA Combined Membership List.

A new prize for applications of mathematics having a real impact in the world, the Carl Friedrich Gauss Prize, will be awarded for the first time at ICM 2006, endowed by the German Mathematical Union from profits from the 1998 Berlin ICM.

IMU has also embarked on a process of electoral reform. If approved by the 2006 General Assembly, the slates for the IMU Executive Committee and Commissions of IMU will henceforth be proposed by independent Nominating Committees, a system that is being operated informally for the present cycle. The proposal is that in future the Officers and Executive Committee for the International Commission on Mathematical Instruction

(ICMI) will be elected by the ICMI General Assembly, and not by the IMU General Assembly as at present.

You can learn more about these various initiatives by subscribing to the (free) new IMU electronic newsletter IMU-Net via the IMU webpages.

P. What is your impression on the project of digitization of the mathematical literature?

You refer to the World Digital Mathematics Library (WDML) proposal, to make the entire mathematical literature digitally available in fully searchable and linked form, in particular via retro-digitization of older journals and books. Mathematics differs from other sciences in the generally much longer period over which research papers retain their value, which makes retrodigitization particularly attractive.

IMU strongly supports the WDML vision, and in particular the idea of a 'moving wall', a negotiated period – say 5 years – after which all journals are made electronically available free of charge by publishers. IMU cannot itself fund digitization, but can help with the adoption of common standards (see, for example, the digitization best practices document available on the IMU/CEIC webpages). For the WDML to become a reality will require a gradual process of persuasion of society and commercial publishers that it is in their interest to participate. There is already a considerable amount of retro-digitized material available, due to the splendid and continuing efforts of various projects, and some optimism that substantial new projects will soon emerge to increase this material. However, major obstacles, especially related to copyright, need to be overcome before the dream can be realized.

P. In what ways could IMU improve?

IMU could do more if it had more funds at its disposal. I already mentioned that the majority of these funds come from the dues of member countries, which are low in comparison with other scientific unions. For example, it has been difficult to justify financially the recent appointment of a half-time administrator for developing countries, despite the advantages of this post for furthering the aims of IMU. Given the funds, we could increase the scale of our activities on behalf of developing countries and in the areas covered by CEIC.

I also believe that IMU could take a more professional approach to relations with the media, and the task of influencing public and political opinion on behalf of mathematics. IMU should find occasions to say how important mathematics is for the world, and to draw attention to key problems (e.g. numbers of students and teachers) facing the subject. At the moment we do not have the mechanisms or experience for doing this in an effective way. In particular this means engaging with the issues of mathematics education, and helping to bring together the mathematics and mathematics education communities.

3. ICM

P. What is the significance of the next ICM being held in Spain?

The award of the ICM for the first time to Spain shows the respect and trust of mathematicians worldwide in Spanish mathematics and mathematicians.

P. What opinion do you have of the Organizing Committee and their capacity to organize this great event?

I have every confidence in their ability to organize a fine Congress.

P. What are the benefits that mathematics in Spain can derive from this event?

The ICM will be an unparalleled opportunity for press coverage of mathematics in Spain, and the build-up to it an important period for focussing the attention of the government on the needs of the Spanish mathematical community. I am sure that the preparations for the Congress will also bring together Spanish mathematicians in a way that will have lasting benefits.

4. Mathematics

P. What is, or should be, the role of mathematics in science, technology and society?

Mathematics is the language of science and technology, and is used by all scientists and engineers in one way or another. Today's standard mathematical tools – calculus, Fourier series, matrices, stress and so on –

with their incalculable economic and scientific impact, were once the latest mathematical research. This is rarely understood by the general public, or by politicians. Some important mathematical tools may take decades or longer to develop to the stage when they are of practical importance, the result of the combined efforts of many mathematicians. The development of such tools results often from a combination of ideas addressing questions posed by the world around us, together with progress on such ideas based on the intrinsic structure and elegance of mathematics. Sometimes the applications of mathematics can be very surprising, for example the turning of a mathematical theorem on the Radon transform found fifty years before into the life-saving medical tool of X-ray tomography.

P. Would your answer be any different about the role of mathematicians in those areas of human endeavour?

I think that mathematicians, and those who have studied mathematics in the past, can and do also contribute greatly by their ability to apply the mathematical way of thinking to life in general, for example through the discipline of careful problem formulation, and the logical analysis of problems starting from clear hypotheses.

P. What is the role of applied mathematics within mathematics itself?

I do not much like dividing mathematics into pure and applied. Many of the great mathematicians that we revere, for example Archimedes, Newton, Gauss, Euler, Cauchy, Poincaré, von Neumann, apparently regarded mathematics as a seamless whole. The modern tendency to make a division has, I think, been damaging, and is happily now being eroded. Understanding the world has always needed and stimulated the discovery of new mathematical techniques and theories. These theories are often then organized and developed on the basis of the natural structure and beauty of the subject, and these developments then in turn influence applications.

P. Would you single out some parts of mathematics as having a high impact either in science or technology?

Because many of the governing equations of science and engineering are partial differential equations, of course the understanding of such equations, both theoretically and numerically, is of paramount importance. But every

area of science and technology has its own special bits of mathematics that are crucial.

P. Your expertise in the Calculus of Variations is very well known to everyone. Is there a particular contribution of yours which is most dear to you?

Yes, finding a way to prove the existence of energy-minimizing deformations of elastic bodies under realistic assumptions on the material, since it set the scene for much of my future work. I was a young postdoc at Brown University, and had tried for months to make progress on this problem, and had several times considered giving it up as too hard. The issue of what properties it was reasonable to assume about the stored-energy function was puzzling, and the literature confusing. One afternoon while lying on my bed (it was too hot to do anything else) I suddenly realized that the determinant of the deformation gradient must be weakly continuous on an appropriate Sobolev space. I knew immediately that this would unlock the problem, and a lot of things that had been puzzling me fell into place. In fact I learnt a year or two later that this weak continuity was not a new discovery and had been previously proved by Reshetnyak. Also, the much earlier work of Morrey on quasiconvexity, which I had been reading but not understanding, effectively proved the same. So I could then go back to Morrey's work, which had gone almost unnoticed for 25 years, understand it, and recognize that perhaps its most important application was to elasticity. It was an exciting period, and it gave me the confidence not to stop working on other hard problems later in my career, even if I have only been able to make progress on a few.

P. What is the future of the Calculus of Variations as a mathematical discipline?

Like other branches of mathematics, the calculus of variations has undergone periods of rapid growth, of consolidation and of stagnation. Stagnation often comes when a subject becomes introverted, and exciting developments when contact is made with a different part of mathematics or with new problems generated by the outside world. I am sure that variational techniques will continue to be of very great importance for the foreseeable future, but whether the calculus of variations continues to be regarded as a

separate branch of mathematics may change as it coalesces with other areas or splits into more specialized parts.

5. Spain

P. What can you say about the development of mathematics in Spain seen from the outside?

I do not have a perspective on Spanish mathematics as a whole, but in the areas which I am familiar with there has been a steady improvement in quality over the last 20 years.

P. There are two natural communities to which Spain belongs, Europe and the Spanish speaking world that comprises Spain and countries of South America. How do you see the role of Spanish mathematics in these two communities?

I hesitate to comment on this! Perhaps I could just say that any help Spain can offer to mathematicians in less prosperous Spanish speaking countries would be valuable.

P. What recommendations do you have for the Spanish mathematical community to keep improving?

I am sure that the Spanish mathematical community is well aware of the kinds of things that are needed. Of course it is very important to have flexible funding mechanisms for retaining talented young people and nurturing good research groups.

P. Would you like to add any further comments?

Just to say that we all look forward with great anticipation to ICM 2006, and I send everyone involved with its organization the best wishes of IMU.