2026 Chern Medal: Graeme Segal

by Allyn Jackson

Graeme Segal is one of the preeminent mathematicians of the past half-century. So it is something of a surprise that he says of himself, “Honestly, I don’t really know how I ever became a mathematician, because I don’t have the usual skill set at all.”

What does he mean? He explains that he’s “hopeless at arithmetic” and gets “turned off by equations.” Prime numbers, which drive a great deal of mathematics research, do not particularly interest him. In principle he finds a topic like knot theory intriguing, but not the intricacies of complicated knots. “I’m really quite content with a trefoil knot”—meaning the rock-bottom, simplest knot there is.

That last statement gives an inkling of what Segal brings to mathematics: an affinity for the simplest objects that animate mathematical phenomena. He wants to understand what is at the root of mathematics—its fundamental objects and the most basic relations among them. His limpid curiosity brings clarity and elegance to the way he communicates about mathematics. Through those communications he has inspired generations of mathematicians and illuminated new paths to pursue. While Segal’s work focuses mainly on geometry and topology, he has also had a major influence as a builder of bridges between mathematics and physics.

At the center of his long career—he earned his doctorate at Oxford University in 1967—has been the drive to understand space. In the text of his 2013 Presidential Address to the London Mathematical Society, he wrote: “The idea of space is central in the way we think. We organize our perceptions in physical space, we think of time strung out as a line, and we carry spatial notions over to any number of our conceptual constructs... [S]pace is ‘our’ technology, wonderfully evolved for dealing with our experience of the physical world.”

The desire to understand space has driven a long intellectual development, as a few milestones suggest. Two thousand years ago, Euclid’s *Elements* probed the nature of space with axioms about points, lines, and planes. One hundred and fifty years ago, Bernhard Riemann gave curved geometric spaces a life independent of the Euclidean space surrounding them. One hundred years ago, Einstein’s theory of general relativity showed how physical phenomena depend crucially on the geometry of space.

And in the mid-20th century Shiing-Shen Chern—after whom the Chern Medal is named—proved his celebrated generalization of the Gauss-Bonnet Theorem, which provides a profound link between the small, local details of a space and its overall, global properties. It’s Segal’s “all-time favorite theorem.”

Central to these developments is the concept of a smooth manifold, which captures both the local and the global structure of space. Over the course of the 20th century, mathematicians developed various lenses through which to examine these spaces. The lens of algebraic geometry offers a precise, detailed view of local twists and turns, cusps and singularities. The lens of differential geometry centers on local features of curvature and smoothness. More broad-brush is the lens of diffeomorphism, which focuses on what remains unchanged when spaces are deformed without tearing or collapsing.

The simplest, most basic lens is that of homotopy theory. This lens sees a solid ball as same as an isolated point, because the former can be collapsed into the latter, and sees the solid torus, which can be collapsed into a circle, as fundamentally different from the ball. What the homotopy lens brings into focus are, as Segal put it, "the non-local properties of spaces: it records how they are connected-up globally—what we still need to know when we know everything about the local structure near every point of the space."

In his doctoral thesis, Segal observed that any category has a "realization" as a topological space, with functors between categories becoming maps between the realizations. Most importantly, transformations between functors become homotopies between the maps. He generalized this framework to topological categories, whose sets of objects and morphisms themselves have topologies. He was then able to show that the classifying spaces of various kinds of fiber bundles, one of the central objects of study in algebraic topology since the 1950s, were structures of this kind. For example, the realization of the category of complex vector spaces is the classifying space for complex vector bundles.

This framework helps most in describing the maps between classifying spaces that correspond to natural category-theoretical operations like the direct sum or the tensor product. Segal's lasting achievement is to have hit upon the most successful way of expressing in homotopy theory how operations such as these both are and are not "exactly" associative and commutative. The method is often used in describing less rigid—meaning "up to homotopy"—versions of familiar mathematical structures. But its first real impact was in Quillen's development of algebraic K-theory, which is based on the idea of the "space" of a category.

One of Segal's most important contributions is a method known as "scanning." This method allows one to gather information about a space, as if scanning it with a magnifying glass, and then to use that information to determine the global homotopy type. Segal was able to use the method to great effect in other contexts, and, more generally it has been widely used across geometry and topology. Not only is it a powerful method for proving theorems, but it also offers a visual guide for developing geometric intuition in abstract settings.

Given his fascination with the nature of space, it is no surprise that Segal has a longstanding interest in physics. When he left his native land of Australia to become a graduate student in the UK in 1962, the physics research that he encountered was mainly centered on reams of data from particle accelerators. "It was very complicated," he recalled. "It didn't seem to shed any light on anything."

Later, in the mid-1970s, he came into contact with physicists making the initial steps toward string theory, which took a more conceptual, geometric approach to understanding the behavior of the fundamental constituents of matter. This resonated with Segal's way of thinking. One of his early works in this area came out of his recognition of an uncanny similarity between a complicated conjectural formula in string theory and a known formula in homotopy theory. The relation between the formulae remains a mystery to this day, but proving the string theory formula led him into the representation theory of loop groups, and that in turn, led into the theory of integrable systems. After the completion of his book *Loop Groups* (written with his student Andrew Pressley and published in 1986), string theorists saw how deeper facts about loop group representations become intelligible when their true home is recognized as 2-dimensional conformal field theory.

One of Segal's enduring contributions has been to build bridges between the frontier of mathematics and that of theoretical physics. Quantum field theory, which began in the early part of the 20th century, seeks to construct mathematical conceptualizations for understanding elementary particles. For several decades the contributions of mathematicians to quantum field theory centered on development of axiomatic foundations, with partial success. Then, in the 1970s, as physicists began exploring new, more-geometric quantum field theories, excitement brewed among mathematicians who saw how the new theories could shed light on purely mathematical questions.

Among the quantum field theories being explored at this time was 2-dimensional conformal field theory, which connects to various parts of physics, including string theory, and has inspired a great deal of new mathematics. One of Segal's most important contributions is a paper he wrote in 1987 that circulated for a long time as a preprint and was published only in 2004. The paper, titled "The Definition of Conformal Field Theory," presented a geometric, axiomatic framework for the theory.

This was a very different kind of contribution from that of most mathematics papers, in that it did not solve an open problem or prove a new theorem. In signature Segal style, it offered new understanding and a clarifying perspective pointing the way to new developments. The paper made accessible to physicists mathematical ideas that they might not otherwise have known about and introduced ideas from physics in such a way that mathematicians could work with them. The influence of the paper was amplified as mathematicians rapidly adapted the framework to other quantum field theories.

Segal's oeuvre goes far beyond what this brief sketch suggests. In fact, his published works can tell only part of the story of his influence, which has permeated mathematics through interactions with other researchers. He is curious about the work of others, generous with his own ideas, and exceptionally effective at communicating. His mastery of his subject is commanding, but on a personal level he is unpretentious. Colleagues who talk to him often come away with clearer understanding of their own ideas and a new way forward in their own work.

If mathematics were a tree, Graeme Segal would be found at its taproot. The nourishment he has provided to the tree has brought much new growth.

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