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ICMI 1908/2008

The first century of the International
Commission on Mathematical Instruction
(1908-2008)

Reflecting and shaping the world
of mathematics education

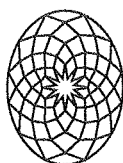


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Collana coordinata da:
Massimo Bray



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THE FIRST CENTURY
OF THE INTERNATIONAL
COMMISSION ON MATHEMATICAL
INSTRUCTION
(1908-2008)
REFLECTING AND SHAPING THE WORLD
OF MATHEMATICS EDUCATION

edited by

Marta MENGHINI, Fulvia FURINGHETTI,
Livia GIACARDI, Ferdinando ARZARELLO

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Introduction

The Symposium

The IV International Congress of Mathematicians (ICM), which took place in Rome from 6 to 11 April 1908 was memorable. An exceptional chronicler, the French mathematician Henri Poincaré, wrote:

The number of participants was the highest of any of the preceding Congresses, which is doubtless due to the attraction of the Eternal City, but this is not the only reason. ... France was brilliantly represented ... there were also very distinguished representatives of German science ... No country was absent. ... It goes without saying that Italy had the most and most brilliant representatives ... The sessions were held at the Palazzo Corsini, home of the Accademia dei Lincei ... a beautiful palace in Trastevere (*Le Temps*, 21.4.1908).

It was during this Congress that ICMI, the International Commission on Mathematical Instruction (formerly Commission Internationale de l'Enseignement Mathématique, Internationale Mathematische Unterrichtskommission, Commissione Internazionale dell'insegnamento matematico, International Commission on the Teaching of Mathematics) was created. Its first president was Felix Klein, an eminent mathematician and promoter of an important reform for teaching of mathematics in Germany. A substantial role in establishing the Commission was played by David Eugene Smith, a professor at Teachers College of New York, who was deeply interested in education and in the history of mathematics. Thus the commission was born of the closest collaboration between mathematicians and educators.

The original aim was to make an inquiry and publish a general report on the current trends in the secondary teaching of mathematics in various countries. From then on, the Commission cycled through successive periods of greater or lesser activity until the late 1960s, when it had a genuine Renaissance. From that time the activities of the ICMI as well

as the wide lines of research it promoted have broadened enormously and have contributed step by step to the creation and the growth of the new discipline *Mathematics Education*.

To celebrate the centennial of the foundation of ICMI, an international symposium was held in Rome from 5 to 8 March 2008. As it did a hundred years ago, Palazzo Corsini, home of the Accademia Nazionale dei Lincei, provided the splendid venue for the congress. The final session was held in Palazzo Mattei di Paganica, home of the *Enciclopedia Italiana*. The Logo of the Symposium is inspired by Michelangelo's Piazza del Campidoglio, where the ICM was inaugurated in 1908.

The Symposium was attended by about 180 participants from 43 countries. The program consisted of nine plenary lectures, a session of eight short talks coordinated by *Alan Bishop* and *Lee Peng Yee*, five working groups and a panel discussion.

The website www.unige.ch/math/EnsMath/Rome2008

The permanent Symposium website, managed by Daniel Coray, provides the full documentation of the Symposium (the Symposium program, the papers presented in the Working Groups, and photos).

Moreover, on the occasion of the Symposium, a website was created on the *history of the ICMI*, under the direction of Fulvia Furinghetti and Livia Giacardi (www.icmihistory.unito.it/, also accessible from the main page). Its aim is to delineate the most significant events and the key figures through documents, images and interviews. The site is divided into various sections: Timeline, Portrait Gallery, Documents, The Affiliated Study Groups, The International Congresses on Mathematical Education, and Interviews and Film Clips. The Timeline presents the main moments in the history of the ICMI, with references to the original sources. The Portrait Gallery contains biographical cameos of the ICMI officers who have passed away. The focus of the cameos is on the officers' contributions to ICMI activities and on their publications expressly dedicated to mathematics education/instruction. When possible the authors were chosen in the countries of the officer treated in the cameo.

The opening

The Symposium opened on Wednesday, 5 March. Opening addresses were given by Ferdinando Arzarello, chair of the International Program Committee; Edoardo Vesentini, president of Accademia Nazionale dei Lincei; Alessandro Figà-Talamanca, head of the Mathematics Department of the Sapienza University of Rome, on behalf of the Sapienza; Franco Brezzi, president of the Italian Mathematical Union; Lazlo Lovasz, president of the International Mathematical Union; Ramadas Ramakrishnan from ICTP (International Centre for Theoretical Physics)

on behalf of UNESCO; Mario Dutto, direttore generale per gli ordinamenti scolastici of Ministero della Pubblica Istruzione; and Michèle Artigue, president of the International Commission on Mathematical Instruction.

The institutions that contributed to the organization of the Symposium were represented as well by Gerardo Marotta, president of the Istituto Italiano di Studi Filosofici; Claudio Procesi, vicepresident of IMU; Lucia Ciarrapico, of the Ministry for Public Instruction; Francesco Paolo Casavola, Istituto della Enciclopedia Italiana; Renato Serafini, Banca d'Italia; Gabriele Anzellotti, Commissione Italiana per l'Insegnamento della Matematica; Carlo Sbordone, Italian Mathematical Union; Hartmut Retzlaff, Goethe Institut Italien; Paolo Maroscia, Dipartimento di Metodi e Modelli Matematici of the Sapienza, Rome; and Gianni Labella of the University of Modena. Sergei Nikolskii, an *old* friend of ICMI, sent a message of greetings.

A letter of the President of the Italian Republic, Mr Giorgio Napolitano, read to the assembly, expressed appreciation for ICMI's involvement in mathematical instruction and wished a good work.

The Italian Afternoon

During the afternoon spent by participants in the visit to Campidoglio and the Capitoline Museums (courtesy of the City of Rome), a special event dedicated to Italian mathematics teachers was organized. This 'Italian afternoon' consisted of the following talks:

- *L'intelligence du calcul* (Michèle Artigue)
- *Matematica e interpretazione* (Giorgio T. Bagni)
- *Matematica e software di geometria dinamica* (Mario Barra)
- *Breve storia di ICMI* (International Commission on Mathematical Instruction) (Fulvia Furinghetti)
- *What can we learn from assessment?* (Gilah Leder)
- *Il rapporto tra teoria e pratica visto 'dalla parte della pratica'* (Francesca Morselli)

Morselli was the winner of a competition among young Italian researchers concerning the relationship between theory and practice in mathematics education. About 100 teachers attended the event; a further 50 Italian schools all over the country were connected in videoconference. All together more than 1000 additional teachers were reached. The texts of the talks, translated when necessary, are published in the Italian journal for mathematics teachers *Progetto Alice* (2008, n. 25).

Apart from the value of informing a number of Italian teachers of some recent results of research in mathematics education, we deem it important that they were given the chance to become aware of being part of the valuable international project behind ICMI.

The excursion to Tivoli: Villa d'Este and Villa Adriana

An excursion to Tivoli closed the symposium, as it had when the International Congress of Mathematicians took place in 1908.

The Tivoli Rotary Club was the main organizer for the arrangements of the excursion. In Tivoli, the participants visited Villa d'Este and Hadrian's Villa, both UNESCO World Heritage sites, representing remarkable examples of the ancient Roman and Renaissance cultures.

The guided visit and information packets were provided courtesy of the Soprintendenza per i Beni Archeologici del Lazio and of Soprintendenza per i Beni Architettonici e Paesaggistici del Lazio. At the end of the tour, participants enjoyed a rich buffet at Ristorante Adriano, near Hadrian's Villa. The buffet was provided courtesy of the Mayor of the town of Tivoli.

This volume

The present volume contains the proceedings of the Symposium. Taking as a point of departure the analysis of the principle themes connected to the activities of the ICMI in the course of its one hundred year history, the Symposium attempted to identify the future directions of research in education and possible initiatives for improving the level of mathematics culture in the various countries.

The Symposium was focused on three themes: the past, the present and the future. Based on our main conviction that history is a powerful and appealing means not only of giving an account of the past but also of building the future, all the activities scheduled in the Symposium were pervaded by a sense of history.

Against this background, the Working Groups addressed the following topics: disciplinary mathematics and school mathematics (*Bill Barton, Frédéric Gourdeau*); the professional formation of teachers (*Deborah Ball, Barbro Grevholm*); mathematics education and society (*Hilary Povey, Robyn Zevenbergen*); resources and technology throughout the history of ICMI (*Maria G. Bartolini Bussi, Marcelo C. Borba*); mathematics education in an ICMI perspective (*Gilah Leder, Luis Radford*).

Among the talks, some were specifically formulated to reconstruct the history of ICMI.

In his opening essay *Hyman Bass* provides a vivid picture of some moments of the life of the ICMI from the perspective of some key figures of the past. The recent past was sketched through quotations from some of the interviews present in the Symposium website.

The early days of ICMI and the troubled period between the two World Wars are outlined by *Gert Schubring*, with particular reference to the important roles of Felix Klein and David Eugene Smith in constituting ICMI. Schubring's work is based on archives that have been largely unexplored until now. Complementing Schubring's contribution, *Eileen*

F. Donoghue provides a behind-the-scenes glimpse of early efforts to organize the Commission through Smith's published papers, non-published memoranda, notes, and correspondence.

The period from the 1950s onwards is dealt with in the contribution by the *Italian researchers*. They analyse the emerging of new arenas for discussing problems of mathematics education/instruction and the background and the circumstances that fostered and shaped the emergence of new issues in mathematics education. During those years the new discipline of mathematics education was slowly formed and, as *Alan Bishop* also observes, in the late 1960s the accent was moved from the word 'instruction' to that of 'education'. In the decades that followed the new structure of the ICMI fostered the creation of the ICMEs and of the ICMI studies, the development of international Study Groups and of regional international groups. ICMI Bulletins fostered international communication. The significance and impact of these initiatives are illustrated in the short talks of *Derek Holton*, *Claudi Alsina* and *Frederick K. S. Leung*. Holton gives an account of the tasking work of carrying out an ICMI Study from initial planning to the final stage of producing the Study volume. Leung refers to the ICMI Study on *Mathematics education in East Asia and the West* and stresses the need of interpreting educational problems in the light of different cultures. Using the example of ICME 8 held in Spain, Alsina analyzes the impact that the organization of an ICME has on the host country.

The development of mathematics education as a field of study and practice as distinct from mathematics is traced by *Jeremy Kilpatrick* and in the reaction by *Jean-Luc Dorier*. They discuss different aspects on how ICMI has both mirrored that development and stimulated new directions. The contributions by *Dina Tirosh and Pessia Tsamir* and by *Mogens Niss* deal with themes that have been a main concern of ICMI since its early days. Tirosh and Tsamir, together with the reaction by *Aldo Brigaglia*, focus on the relationship between intuition and rigour and present the development of this issue in the work of important mathematicians of the twentieth century and in the work of *Efraim Fischbein*.

Niss and *Toshikazu Ikeda* look at mathematical applications and modelling in the context of the varying ultimate ends and goals of mathematics education at large. Their main concern is to establish the balance between application and modelling and 'pure' mathematics.

Discussing the issue of the gap between research and practice, *Jo Boaler* considers the impact of research on teaching practice, but she also pays considerable attention to the impact of research on the curriculum. In addition to these levels, *João Pedro da Ponte* focuses also on other aspects such as students' learning and teacher education.

Since its early days, ICMI has been attentive to non-Western countries.

The increasing communication has promoted ICMI initiatives at a global level, as described in the contributions by *Jill Adler* (for Africa), *Ubiratan d'Ambrosio* (for Latin America), and *Lim-Teo Suat Khoh* (for the Far East). *Bienvenido F. Nebres* looks at the impact of ICMI on East-Southeast Asian mathematics education and on mathematics education in developing countries in general and discusses what centres and peripheries can learn from each other. In her reaction *Gelsa Knijnik* shifts the attention on developing countries, whose problems are not only those of communication, but mainly depend on the social conditions.

The issues emerging when dealing with peripheries and centers will be among the main concerns of ICMI in the future. This is the point stressed in the talk of *Mamokgheti Setati*. In particular, she pushes ICMI to address primary mathematical literacy in developing countries and those goals which are indicated by UNESCO in the *Millenium Development Goals*. As for the future action of ICMI, *Morten Blomhøj* lists ten major challenges in mathematics education research and suggests how ICMI can contribute to face them.

The following crucial challenges now facing mathematics education and ICMI are discussed by the president of ICMI *Michèle Artigue*: improving and extending ICMI outreach; increasing accessibility and visibility of ICMI activities; understanding and benefiting from cultural diversity; consolidating the field of mathematics education as a field of research and practice; acting as a moral authority.

Artigue also reflects on the relationship between the community of mathematicians and mathematics educators throughout the first century of the history of ICMI. The last decade shows an increased collaboration among the two communities. ICMI secretary-general *Bernard Hodgson* stresses the recent fundamental revision in the infrastructure and governance of the ICMI, because of which responsibility for the election of the Executive Committee now falls to the ICMI General Assembly.

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We particularly wish to thank the Istituto della Enciclopedia Italiana which took care of the publication of the proceedings of the Symposium.

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With the collaboration of Daniela Spagnulo.

HYMAN BASS

Moments in the life of ICMI

What has the International Commission on Mathematical Instruction (ICMI) been about? What have been its struggles, and accomplishments? My aim is to provide some glimpses of this, over time, and from the perspective of some key figures in the history of ICMI. In one hour I can hardly do justice to this remarkable century long saga, so I apologize in advance for the many important events and people I must fail to mention. But you will hear much more of this as the week unfolds in this magnificent room that witnessed the birth of ICMI, in April, 1908, here in the Accademia dei Lincei, facing, as we do, the portrait of one of its founding members, Galileo Galilei.

1. The first two half-centuries of ICMI

ICMI, like most international organizations of the last century, was deeply affected by the two World Wars, resulting in a long middle period of quiescence. It is therefore natural to distinguish, in the history of ICMI, the following two eras.

1.1. 1908 – World War II: The ‘Klein Era’

This era in ICMI was dominated largely by mathematicians with a substantial, but peripheral interest in education, of whom Felix Klein (Figure 1) was by far the most notable example, plus some secondary teachers of high mathematical culture. The main focus was on examination and development of the secondary school curriculum and on international comparisons of educational systems (mainly in Europe and North America). The main public venue offered to ICMI for exposure of educational issues was one section of the quadrennial ICMs, in which education/teaching would share the stage with other themes, like history or philosophy (In the Paris ICM in 1900, the section on *Enseignement*

et méthodes featured a landmark talk by D. Hilbert, ‘Sur les problèmes futurs des mathématiques’). The official journal of ICMI during this era was *L’Enseignement Mathématique*.



Figure 1. Felix Klein



Figure 2. Hans Freudenthal

1.2. Post World War II – 2008: The ‘Freudenthal Era’

This period witnessed the emergence of mathematics education (didactics) as an international academic discipline, and of the corresponding scholarly community, for which ICMI was a major resource and agent, ICMI having been resuscitated as a sub-commission of IMU. The earlier interests in curriculum design and reform were now expanded to studies in psychology and cognition. In this period we see also the first significant examples of research mathematicians becoming professionally engaged with mathematics education even at the scholarly level. Hans Freudenthal (Figure 2), the first major example of this, launched two major and enduring institutions of ICMI – the International Congresses on Mathematical Education (ICMEs), and the journal *Educational Studies in Mathematics*, to serve the scholarly needs of the emerging educational research community, for which *L’Enseignement Mathématique* was no longer adequate in its evolved form. Also, toward the end of this period, ICMI started gestures of outreach – to Asia, Latin America, Africa, and developing countries – beyond its earlier Eurocentric character.

Since my presentation will draw largely on artifacts such as recorded interviews with *living* protagonists from the life of ICMI, I must necessarily concentrate mainly on this second, ‘Freudenthal Era’. I shall focus on some of the important personalities, and report events and reflections mainly through their reminiscences.

2. Klein, and the Klein Era

Felix Klein remains to this day one of the noblest expressions of a mathematician's engagement with education. His deeply intertwined concerns for both the highest advancement and integrity of science, and for the quality and seriousness of mathematics education, were manifest throughout his life. Born in Düsseldorf 25 April, 1849 (he was fond of noting that his birthday, 25/4/1849 was formed of the squares of prime numbers), he was appointed Professor at Erlangen at the age of 23, already a brilliantly productive mathematician. The paper he presented on that occasion was his famous 'Erlangen Programme,' which transformed our basic conception of geometry in group-theoretic terms.

But this was not the subject of his inaugural lecture ('Antrittesrede') (Rowe, 1983). Instead, and contrary to academic tradition, he spoke «On the pedagogical principles and goals of my future academic work», a discourse on mathematics education! In this he emphasized the unity of knowledge, even across the sciences and humanities. He urged the cultivation of applied as well as pure mathematics, and connections with other disciplines. While acknowledging the importance of logical skills, he emphasized even more emphatically the role of intuition and imagination. And he strongly advocated the active and serious attention to the mathematical preparation of school teachers. These concerns and dispositions seem as fresh and relevant today as they were for Klein.

Klein properly exercised his expertise by contributing to deepening and enriching the mathematical knowledge of teachers. His book, *Elementary mathematics from an advanced standpoint*, an eloquent expression of that interest, can still be read with profit by educators and mathematicians today. At the same time, Klein never pretended to mastery of the pedagogical demands on teachers, toward whom he expressed a sensitive professional respect and a disposition of intellectual generosity.

When ICMI was founded, in 1908, Klein was the obvious and consensual choice for its first president, a position he actively exercised until 1920. Gert Schubring's talk (Plenary 5) offers vivid and poignantly sad details of the end of Klein's term, in the emotionally charged wake of World War I. The American mathematics educator, David Eugene Smith, who first initiated the proposal to form (what became) ICMI, emerges then as an important and sympathetic, perhaps underappreciated figure in this story. Following Klein, Smith served as president of ICMI (1928-32), the only mathematics educator to do so until Michèle Artigue's election in 2006.

3. Hans Freudenthal (1905-1990)

Let me quote from what I have written elsewhere about him (Bass (2005): «Freudenthal had remarkably broad mathematical and cultural interests. In mathematics he worked in topology, Lie groups, logic, and probability and statistics (for which he wrote a textbook). He also wrote a book on the popularization of mathematics, and he worked and published extensively in mathematics education. He was a gifted linguist, and even developed a proposed language for extraterrestrial communication. Howson (2004) reports that, during a heated argument with Dieudonné, Freudenthal protested, “Don’t shout at me, for I can shout louder than you—and in more languages”. Freudenthal was charming, mischievous, argumentative, autocratic, and an activist who accomplished many things.

Freudenthal viewed mathematics not primarily as a body of knowledge, but as a human activity, and he urged that mathematics education should do likewise. It should, he argued, be based in reality around phenomena that “beg to be organized”—a process he called “mathematization”, a form of mathematical modeling of real problems or of organizing and synthesizing mathematical ideas. He opposed deductive approaches and favored instead development from the concrete to the general. He was highly critical of most educational reform (‘New Math,’ or ‘Mathématiques Modernes’) and of the educational research of his day (both statistical and psychological). He believed in “mathematics for all”, and favored small heterogeneous classes (no differentiation). Freudenthal’s educational philosophy continues to guide much of the work of the Freudenthal Institute».

Among Freudenthal’s enduring cultural/institutional legacies in mathematics education are:

Launching of the ICME’s, with the inaugural congress held in Lyon in 1969.

Founding of the international journal *Educational Studies in Mathematics*.

Founding of what has come to be called the Freudenthal Institute at Utrecht University, which has had a pervasive influence on mathematics education, in Holland and more broadly.

Freudenthal’s bold and adventurous launching of ICME 1 (in Lyon) was essentially, and characteristically, a unilateral action, for which he sought approval and authorization from no one, not the Executive Committee of ICMI nor that of IMU. And this provoked some anger and concern about ICMI inside IMU. But ICME 1 was a great success, and its successors are healthy and still evolving.

4. The ICMEs

4.1. *The birth of the ICMEs and of the IREMs, through the eyes (and hands) of Maurice Glaymann.*

Born in Paris in 1931, Maurice Glaymann lived from the age of 4 in Lebanon, returning definitively to France only in 1963, for a teaching post at Lyon. In 1966 he was elected president of the Association des Professeurs de Mathématiques (APM) de France. The APM at that time was pushing towards math education reform and in 1967 obtained from the Ministry of Education the creation of what came to be called the Lichnérowicz commission, after the name of its president. The 'Lichnérowicz Commission' recommended that year the creation of 'Instituts de recherche sur l'enseignement mathématique' (IREMs). APM also proposed, in a text known as the Charte de Chambéry, the creation of IREMs. In 1968, at the time of the student demonstrations, Glaymann asked, and received an audience with the new Minister of National Education, Edgar Faure. Faure had decided to move things and Glaymann reiterates the APM proposition of creating IREMs. Faure, after one week of reflection, to evaluate the cost of such an operation, proposed to create an IREM in Paris and says that he has evaluated the cost to be 3 million francs. Glaymann answers him that with the same budget the APM thinks that three IREMs could be created. This came to pass, Glaymann was named first director of the IREM at Lyon, and this, just in time, put him in a position to offer to host the international congress on mathematics education proposed by Freudenthal. Let's listen (from the interview by Michèle Artigue) to Glaymann's own words, following his meeting with Faure:

We met again ten days later, and he says, all right then, we can create an IREM in Paris, with a budget that I estimate to be about 3 million (old) francs. I suggest to him to make three IREMs with the same amount. And it was thus that the IREMs of Paris, Strasbourg, and Lyon were created. This structure, about to be put in place in Lyon, and of which I suddenly found myself director, would then permit me to propose to ICMI, during a meeting that took place at that time, to hold the first (international) congress on mathematical education in Lyon. I found myself thus, with a structure that allowed us to stage such a congress (my transl.)

Michèle Artigue asks, «So you already had contacts with ICMI, and these took place during conferences [...] ?»

Well, yes, there were various *European* conferences – in Paris, in Hamburg, under the aegis of UNESCO, [...] and we thought seriously during this period about what a reform of mathematics education might be. And then, there were these important European meetings under the direction of Freudenthal,

who was the head of ICMI [MA: "He became the president of ICMI in '67."] Well yes, president. He suggested, among other things, the creation of a commission in order to put in place a first *international* congress on mathematics education. I jumped at the idea, and proposed that he do that at Lyon. [MA: "And that was accepted."] And that was accepted (my transl.).

4.2. Henry Pollak: Some american perspectives

Henry Pollak (Figure 3), an applied mathematician, directed the mathematics group at Bell Labs, long the premier Industrial Scientific Research Laboratory in the US. He was, throughout his career, interested in mathematics education, with a major focus on mathematical modeling, a field he did much to implant in the school curriculum. He was also an active participant in the School Mathematics Study Group (MSG) directed by Ed Begle, one of the main American expressions of the 'New Math' movement. He was active in ICMI affairs, serving on its Executive Committee 1971-74 and 1983-86, and as chair of ICME 4 in Berkeley, 1980. Following his retirement from Bell Labs, Pollak joined the faculty at Teachers College of Columbia University in New York, which is where I, together with Eileen Donoghue, interviewed him in June, 2007.

Pollak's view of the ICMEs:

I got invited to the first ICME, which was held in Lyon in 1969. I went to the first five congresses (Lyon 1969, Exeter 1972, Karlsruhe 1976, Berkeley 1980, Adelaide 1984). The congress in Lyon was very chummy, a wonderful congress in terms of the social interaction among the people [...] Talking to Ed Begle after the Exeter ICME, we asked ourselves, "What did we hear that we couldn't have heard four years ago? It's so different from ICMs." This had a huge influence on me in organizing the Berkeley Congress.

And this is how he addressed that concern in the Berkeley Program.

You can talk about areas of mathematics whose importance and whose *teachability* have grown to such an extent that they are (ready) to be considered seriously. Now three of these that went on at Berkeley I insisted on: One was coding theory, and J. Van Lint from Eindhoven organized three sessions telling people what *algebraic coding theory* was all about. Both from the point of view of its use [...]. What's the subject about? Why is it so useful? And how can you start to teach it? Those were the three things that you did. We did the same thing with *exploratory data analysis*. And other areas of discrete mathematics. Another thing that was already big, by Berkeley certainly, and continues to be, is *technology*. There are lots of things related to technology that you couldn't have done four years ago.

Pollak on the *centrality of teacher education*: In today's efforts to improve mathematics education, people are (re-)discovering that the central problem (much more than standards or curriculum) is the knowledge and skills of teachers to provide effective instruction. Pollak's 1960s experience with SMSG, principally a curriculum development project, led him already then to this conclusion. Here he reflects on the fact that SMSG, while it provided professional development for secondary teachers, did not do so for elementary teachers, for practical reasons of size.

Teacher education is really the solution to everything. More so than any single point. But [...] so what about elementary teachers? I mean the New Math was a revolution in elementary school, at least as much as it was in high school. I was there, and I can say that, in retrospect the attitude on this at SMSG in the early days was stupid. People said we are really trying to get to the high school teachers, but *there are ten times as many elementary teachers*. It can't be done. We'll give them teachers' guides. But nobody had brains enough to say, "Are you crazy!? Teachers don't have time enough to read teachers' guides". It may be that the number of elementary teachers is an order of magnitude bigger than secondary, which it is, but you have to provide for them.

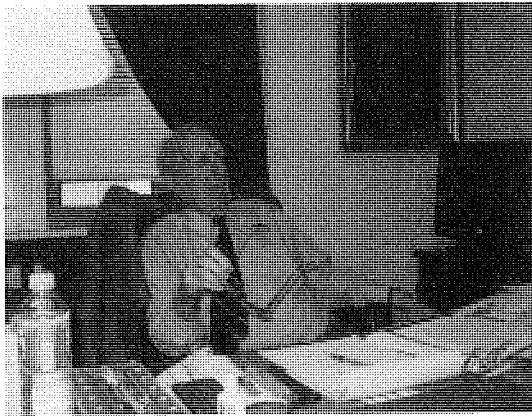


Figure 3. Henry Pollak

Pollak on the *governance and politics* of ICMI/IMU:

I was a delegate to the IMU General Assembly meeting before ICM Nice (1970). They nominated for the EC of ICMI a very active hard working man [...]. And a mathematician representing that man's country on the IMU got up to condemn the nomination:

"He's not a mathematician, he shouldn't be there. He's terrible [...] we are opposed to this, and [...] we wish you wouldn't do it."

It seemed to me that was an absurd thing to happen and I wanted to make sure

that we never got into that sort of situation in the U.S. So that when people were considered for ICMI, the U.S. National Committee on Mathematics and everybody involved would be on board, because it was painful to watch this.

And, following the Berkeley ICM (1986) Pollak was involved in a discussion that interestingly anticipates the recent changes in ICMI election procedures.

Now what really is ICMI? ICMI is a sub-commission of IMU. Well, okay, where is the input to this of the education side as distinct from the mathematics side? There isn't any. Why not? Well there is no such thing as an international educational union. Ideally, if this were a different world from the one that it is, there would be an international educational union as well as an international mathematical union, and nominations for ICMI would have input from both. But this doesn't exist. So nomination to ICMI has always been a very large percentage mathematics. And people have been very good about having people that also have had a great deal of interest in mathematics education, as is represented by the two of us here, and as was certainly true of most of the people, Freudenthal, Hassler, and so on. But it isn't quite the same thing as having the point of view education represented as such. And we had a discussion over this, and we couldn't find any way to think about it.

5. Sightings of the emergence of Mathematics Education as an academic subject

The main cultural transformation of the 'Freudenthal Era' is the emergence and continuing growth, and methodological evolution, of mathematics education (didactics) as a disciplined field of scholarly study. We sample here some testimonies of mathematicians that bear witness to this phenomenon.

Sir Bryan Thwaites, an applied mathematician from Southampton University (UK), was a member of the ICMI EC (Executive Committee) with Freudenthal at the time of ICME 1. He was prominently involved with the curriculum reforms of the 1960s, inspired by the Royaumont Seminar of 1959, organized under the auspices of the OEEC (now OECD). Thwaites organized and chaired a follow-up conference at Southampton, an outcome of which was the School Mathematics Project (SMP), still today based at Southampton.

Thwaites observes (in his interview by Bernard Hodgson) that, «As the end of the 60s came, and the 70s came, there was a greater emphasis on [...] what one might almost call 'individual scholarship', in the field of mathematical education. Mathematical education itself became a kind of aca-

demic subject [...] in a way that it wasn't certainly in the 50s or really in the early 60s». Hodgson elaborates, «In France they use the word 'didacticiens' or didacticians in English. So people whose first interest is research in math education, while it seems that prior to that it was more mathematicians having some personal interest in issues of education». And Thwaites continues,

Yes, well like myself, I like to think of myself as a good applied mathematician, a professional applied mathematician, and my interest in education was so-to-speak a sideline, even though, in fact, it turned out that it took a good deal of my life out. But you see Geoffrey Howson – I appointed Geoffrey Howson to the [...] editorship of the School Mathematics Project, in 1962 I think, and [...] he himself is a very good example of what we've just been talking about, because he had been a lecturer *in* mathematics, at Greenwich, which was the Royal Navy College, and so was a perfectly straightforward and good mathematician. And so he then embraced the subject of mathematical education, and he developed that as his sort of career move, and became one of the first of those who became so-to-speak a specialist in the subject of mathematical education.

Hans-Georg Steiner, of the University of Bielefeld, and one of the leading German didacticians, wrote the following in a personal review of the ICMEs (Steiner, 1997):

It was Professor Hans Freudenthal who, against strong opposition from influential mathematicians in the IMU, forcefully pressed the idea of having separate and genuine ICMEs. He started at Lyon, in 1969, a pacemaking movement, which was flanked by Freudenthal's initiation of a new very successful progressive international journal, *Educational Studies in Mathematics*. [...] ICME 7 in Québec (1992) showed an unprecedented level of achievement made internationally in mathematics education as a domain of interconnected research, development and practice. For the first time *didactics of mathematics showed itself in great clarity as a scientific discipline* which, under increasing theoretical orientation and empirical foundation is dynamically growing within an international frame of complex cultural, political, and interdisciplinary interrelations.

But recognition of this new field and professional community of mathematics education was, and continues to be, slow in coming in the mathematics research community. A small indication of this can be seen in the following observation by Geoffrey Howson (from his interview by Bernard Hodgson) about the program of an international conference on mathematics education in Japan, organized by S. Iyanaga while he was president of ICMI (1975-1978).

It typified everything I worried about ICMI at that time . . . It was an actual meeting of ICMI, that is it would have been a meeting announced under ICMI

sponsorship. And the principal speakers were Iyanaga, Marshall Stone, Lichnérowicz, Sobolev, Bernhard Neumann, (laughter) [...]. So you start off with five very very good mathematicians, who were not greatly concerned – they were *interested* in school mathematics, but not terribly involved - and then there was a Malaysian who was involved in school mathematics, and there was Hans-Georg Steiner and myself.

6. *The ICMI studies: Their launch, as recounted by Jean-Pierre Kahane*

(from the Interview by Michèle Artigue)

Michèle Artigue: «So this was the first of the studies. And can you tell us now ... how you chose the themes, why these themes appeared, . . . which themes were most important, the other themes that you could have treated at that particular moment, . . . how all of that was done, and how you established the organization of these studies, which continue still today?»

Jean-Pierre Kahane:

So the first study was called, *The influence of computers and informatics on mathematics and on its teaching*. There, certainly, you see the influence of Carleson. Carleson, as president of the International Mathematical Union, had introduced the Nevanlinna Prize, alongside the Fields Medals, in order to honor, and make better known, the contribution to theoretical computer science, or the mathematics of informatics, to mathematics itself. And it was clear that, with regard to teaching, one had to reflect on the role of informatics. But note that the *name* of the study had a significance (beyond this). First of all, we had to struggle over use of the term ‘informatics’. Because, *a priori*, it was about “the role of computers”. We had to insist that it be “the role of computers and informatics”. And then, in turn, we were the commission “on the teaching of mathematics”. But why “the role in *mathematics* [...] and its teaching?” And there I had to hold firm. At that time it was clear that (computers and informatics) should have some consequences for mathematics instruction, because informatics would have consequences everywhere. But what was not clear among French mathematicians was that there was an influence of informatics on the development of mathematics itself. Okay, for me this was self evident. But this was not a universal perception. I think that we had to drive the point home. And you know the remarks I was led to make twenty years later; it is now completely common place to observe that informatics has an influence on the development of mathematics. On the other hand, with regard to the perspectives, not the uses, but the *perspectives* that informatics opens for mathematics instruction, we are still at the beginning steps of understanding.

[...] The program committee was active. It met here, at the École Normale Supérieure, and we had two days of work, punctuated by some *cous-cous*, and,

well, everyone was quite satisfied. The committee was rather large. The French contribution was not as important as it could have been [...]. I think. On the other hand, this study, even before its proceedings were published, had a certain influence among scholars, as you no doubt know better than I.

[...] And so, this first study served me as a model for what followed. The principle of the studies was this, as stated in English in the prefaces of the first studies. It is a question of identifying subjects that are, if not 'on fire,' at least ripe for a serious international study. And that we preside over such a study, that we provide the state of the art, but that we never make recommendations. The principle is that the study should provide a survey of the problems, and possibly the elements of solutions, but never a solution with an 'ICMI' label.

7. Recent Leadership of ICMI

Bent Christiansen (1921-1996)

First a secondary teacher in Copenhagen, Bent Christiansen later became the first Professor in the Royal Danish School of Education. He was a plenary speaker at ICME 1 (Lyon, 1969). He was devoted to causes of social justice as well as to the improvement of mathematics education. He became a UNESCO program specialist in 1972, serving developing countries as a UNESCO expert, in Liberia in the late 1950s. Memorable achievements were his organization of a Conference on Language and Mathematics (Nairobi, 1974) and UNESCO's New Trends in School Mathematics (1979), planned in conjunction with ICME 3 (Karlsruhe, 1976). He was a Vice-President of ICMI (1975-1986). In 1980, Christiansen seemed poised to become ICMI's next President (and thus the first mathematics educator to hold that post since ICMI had become a sub-commission of the IMU, and only the second since ICMI was founded (following D.E. Smith 1928-31). But . . . as we learn from Bernard Hodgson's interview of Geoffrey Howson,

When I took over (as ICMI Secretary), let's say that it had been a 'rocky' period for ICMI, and IMU were very worried about whether it was still a viable organization, or what they should do about it. I think it's significant that when I was asked in Berkeley, in 1980, if I would be prepared to be secretary, the idea then was that Bent Christiansen would be the president. But, somewhere along the line that got changed, and I was asked if I would be still prepared to act as secretary, but now with Jean-Pierre Kahane, whom I had never met. And I remember on that first visit Bent and I went to – Bent came down from Copenhagen and we were joined by Ed Jacobsen from UNESCO – and we went to have breakfast at the Kahane's, and spent the morning with Jean-Pierre discussing plans with him, and then we had lunch with him at Orsay. And [...] then we realized, Bent and I, that really this person we hadn't met before was going to be *extremely* good. And, I remember about a year or two after this Bent saying, "Really, I'd have loved to have been president, but I think that I

couldn't have done what Jean-Pierre has managed to do. To use his influence to get funds, and to give us that standing with IMU.”

Jean-Pierre Kahane (President of ICMI 1983-1990)

Jean-Pierre Kahane is a French mathematician (analysis and probability) of the first rank, in the grand French tradition – a graduate of l'École Normale Supérieure, Professor at the Université de Paris-Sud à Orsay, member of the Académie des Sciences Française. But, more than that, he is a thinker and communicator of philosophical depth, elegance, and eloquence, qualities that made him an exemplary president of ICMI (1983-1990). This no doubt also led to his selection by the French Minister of Education to chair what came to be called the 'Commission Kahane', charged to bring recommendations for revision of the mathematics programs in the French schools. We have seen intimations of his qualities already in the above reminiscence of Geoffrey Howson. And we have heard this as well in Kahane's account of the first ICMI Studies. I personally have found the following observations on mathematics and mathematics education, that Kahane offered on his retirement as ICMI president, to be particularly insightful.

- In no other living science is the part of presentation, of the transformation of disciplinary knowledge to knowledge as it is to be taught (*transformation didactique*), so important at a research level.
- In no other discipline, however, is the distance between the taught and the new so large.
- In no other science has teaching and learning such social importance.
- In no other science is there such an old tradition of scientists' commitment to educational questions.

Miguel de Guzmán (1936-2004; President of ICMI 1991-1998)

Miguel de Guzmán (Figure 4), my predecessor as president of ICMI, passed away suddenly and prematurely on April 14, 2004. He was a distinguished harmonic analyst, passionate about geometry, and an inspiring teacher and mentor. He was an intellectual and spiritual leader of the current blossoming of mathematics and mathematics education in Spain. One of his major contributions to ICMI is the Solidarity Project, whose aim is outreach to support the improvement of mathematics education in developing countries. He was a man of broad culture, deep compassion, and an inspiring communicator and teacher. His passing is a sad loss for our Spanish colleagues, and for the many communities of mathematics and mathematics education worldwide.

Here are Jean-Pierre Kahane's reflections on the presidential succession. The preparation for my succession as president, as you know the mechanics –

it's a little bit complicated – it's a suggestion (to IMU) coming initially from the Executive Committee (of ICMI), but the nomination comes officially before the General Assembly of the International Mathematical Union. And so various different persons were considered, and finally the name suggested was Miguel de Guzmán [...]. And Miguel de Guzmán was an extraordinary man. *Extremely* seductive, [...] you knew him? (MA: “Oui, oui.”) And he was, I think, a much appreciated president of ICMI. He was very diplomatic [...]. Very nice and gentle, but at the same time with a very firm sense of direction. One of these directions was that we had to be concerned with what is happening in the poorer countries; and it was the initiative of the Solidarity Fund that permitted ICMI to take effective actions in Nicaragua, in Palestine, etc. He affirmed equally that we had to give a more important place to the didactics of mathematics, [...] publicly representing the didactics of mathematics. And then he was led to give an important place to women in mathematics education.

[...] In this regard, I remember a study that took place in Sweden. It was at the initiative of the organization of women in mathematics education, which had petitioned to become an adhering organization of ICMI. In fact, there had been the traditional affiliated groups, and during my presidency came the group of women in mathematics education. They had organized this colloquium session, with a stage, where were assembled the members of the ICMI Executive Committee who were present – in fact a majority of the Executive. Well, the hall there was filled with women, while on the stage there were only men, nothing but men.

Hyman Bass (President of ICMI 1999-2006)

My initiation into the world of ICMI, and my presidency of ICMI, coincided! This remarkable circumstance is perhaps worth recounting, for what it reveals about the organizational nature and culture of ICMI.

In 1997, before the Berlin ICM, I received phone calls from David Mumford and Jacob Palis (President and Secretary of IMU), two mathematicians whom I knew and greatly respected, inviting me to consent to have my name put into nomination as president of ICMI. I had been substantially involved in some of the mathematics reform efforts and debates *in the US*, and I had extensive international interactions in the world of *mathematics research*. But I knew hardly anything about ICMI, and had never participated in any ICMI related activities.

So I first treated this overture as an error of judgment; it seemed to make no sense. But they (David and Jacob) persisted, so I agreed to think about the proposition for a while, allowing myself meanwhile to find out more about ICMI. I knew from my experience in the US that mathematics education is a complex and highly multi-disciplinary field, in which mathematicians have a crucial, but still specialized role to play. Moreover, I learned that the dramatic growth in the professional community in mathematics education was a world wide, not just US phenomenon, and that ICMI

played a central role in supporting that diverse international professional community. So it seemed obvious to me that ICMI should most naturally be led by a professional mathematics educator, albeit one with a highly developed mathematical culture, because of the close ties of ICMI with IMU.

But this suggestion was not taken up. Moreover, it was clear from recent history that ICMI had been traditionally led by respected, educationally engaged, *research mathematicians*, not by professional educators. Thus, were I to decline the nomination, I would likely be replaced on the ballot by another mathematician, not an educator. Given my inexperience and lack of expertise about ICMI, I found this situation, and the culture that produced it, to be frankly somewhat bizarre. But, at the same time, this offered a piece of the agenda I might take on as president of ICMI, to see if I could influence the ICMI/IMU configuration to evolve toward something more professionally and intellectually sensible. So it was with that (personal and privately held) understanding that I finally consented to be put in nomination.

In fact, the various aspects of the structure of the ICMI/IMU relationship did occupy much of the time of my presidency of ICMI. This relationship was at a low point when I arrived. The program for the section of ICM-Berlin on *Teaching and Popularization of Mathematics* that had been drafted by the ICMI EC was pretty much discarded by the Program Committee of ICM, which constructed a new program of its own. This then was one of the perennial points of significant interface of ICMI and IMU - the representation of mathematics education in the ICMs. Another was the possibility of ICMI representatives being present at portions of meetings of the IMU EC that bear on the interests of ICMI. And, reciprocally, the participation of IMU representation in the ICMI EC. And, most importantly, the method of election of the leadership of ICMI, historically done at the General Assembly of the IMU, in the absence of representation of ICMI.

I am happy to report that, thanks to extensive and constructive discussions with the recent IMU presidents, Jacob Palis, John Ball, and now László Lovász, very dramatic progress has been made on all of these fronts, and that the ICMI/IMU relationship is now, I believe, in a very healthy and promising state.

The Heroic Secretaries-General of ICMI

The resources and infrastructure of ICMI are extremely frail, and the very functioning of ICMI rests heavily on its Executive Committee, particularly its officers, and most especially its Secretary-General. The Secretary-General is the one who carries the main burden of day-to-day functions, budget, etc., the one who 'makes the trains run'. The viability of the organ-

ization rides increasingly, and perhaps unreasonably, on the shoulders of the Secretary-General. ICMI has been blessed in the recent past with a succession of extraordinary Secretaries-General – *Geoffrey Howson*, *Mogens Niss*, and now *Bernard Hodgson* – who have been heroic in their levels of dedication, skill, and efficacy in advancing the work of ICMI. Each of them, in his own right, has been a major international figure in mathematics education. One cannot overestimate the debt that the ICMI community owes them, as each of the ICMI presidents will readily bear witness.

When Bernard Hodgson and I assumed office in 1999, there was only one EC member, Gilah Leder (Australia), continuing from the previous EC, so there was little built-in institutional memory. In support of the transition, Mogens Niss and Miguel de Guzmán led the previous EC to produce a ‘Will and Testament’, laying out the then current state of ICMI, and offering a careful analysis of what they perceived to be the major issues that the new EC would have to face. Mogens, Miguel, Bernard, and I met, in February of 1999, in my apartment in New York, to go through that document. Mogens and Miguel shared their ‘wisdom of practice’ and provided us with an invaluable initiation into the governance and culture of ICMI. This was a precious apprenticeship for Bernard and me, particularly with my lack of ICMI experience.

8. Looking ahead (*The ‘Artigue Era’*)

Michèle Artigue (Figure 5) ushers in a new era of ICMI. She is the first *mathematics educator* to be president of ICMI since ICMI became a sub-commission of IMU. And she is the first *woman* president of ICMI. And, most importantly, she combines the finest qualities of leadership that serve the needs of ICMI. She is a world-class scholar and intellect in many aspects of mathematics education. She is deeply enculturated in the world of disciplinary mathematics. She has been an active and supportive leader of education communities not only in Europe, but also in Latin America, Africa, and elsewhere. And she has a clarity of purpose and vision, and a generosity of spirit, that can guide the work of ICMI into the new century. You can see this in her closing plenary address here.

The main themes of the work ahead will surely include:

- Closer attention to the quality of teachers, and to the work of teaching
- Balancing the relationship of mathematics as a discipline with the work of mathematics education
- Developing the infrastructure and governance of ICMI
- Developing (and defining) an international mathematics education community, in particular one that embraces countries in all stages of development.



Figure 4. Miguel de Guzmán



Figure 5. Michèle Artigue

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JEREMY KILPATRICK

The development of mathematics education as an academic field

I trace the development of mathematics education as a field of study and practice distinct from mathematics. After 1908, but especially after 1969, ICMI has been both a mirror of that development and a stimulus for new directions. As the community of people who identify themselves as mathematics educators has grown, it has increased in diversity, in part because of the growing multi-disciplinarity of the field. There is also diversity across countries in the way mathematics education is institutionalized and how it is related to mathematics. How has ICMI dealt with these issues?

1. *Introduction*

The International Commission on Mathematical Instruction (ICMI) was established as a commission of the International Mathematical Union (IMU) at the 1952 meeting of the IMU general assembly in Rome. Both organizations were reincarnations: The ICMI was the successor to the Commission Internationale de l'Enseignement Mathématique (CIEM, anglicized as the International Commission on the Teaching of Mathematics), which had been organized at the 1908 international congress of mathematicians in Rome, functioned between congresses until the First World War, was revived in 1928, and then was dissolved in 1939 (although its secretary general, Henri Fehr, claimed it was technically still in existence in 1952, when it became officially attached to the IMU; Lehto, 1998, p. 316). The IMU had been founded in 1920, was suspended in 1932, and then was reconstituted in 1952 (see Figure 1). Before 1952, «the Commission was connected not with the IMU, but with the International Congresses. Each Congress gave it a mandate for the period between two Congresses, i.e., for four years, and appointed a Committee to coordinate its activities» (p. 65). In other words, the subordinate (ICMI) antedated the superordinate (IMU).

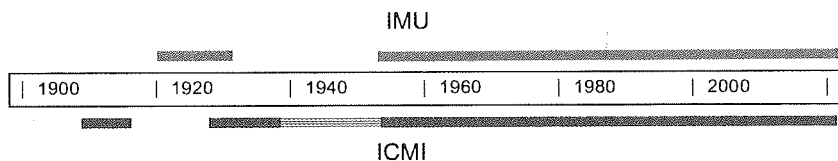


Figure 1. Timeline for ICMI and IMU

The idea for an international commission on mathematics education had first been put forward by David Eugene Smith (1905) in an article in *L'Enseignement Mathématique*. He was one of five respondents to a request expressed at the Third International Congress of Mathematicians in Heidelberg the preceding year for opinions regarding needed reforms in the teaching of mathematics in higher education. Addressing the question of how the teaching of pure mathematics might be improved, Smith argued for the creation of a commission to be named by an international congress to study the problem. At the Fourth International Congress in Rome in April 1908, he made a formal proposal that a commission be established to conduct «a comparative study on the methods and plans of teaching mathematics at secondary schools» (quoted in Lehto, 1998, p. 13). The scope of the study was eventually expanded to all types of schools, including primary schools, vocational schools, and universities.

The proposal for a commission was adopted by the Congress, and Felix Klein was appointed the founding president of the CIEM, serving until 1920 and beginning a tradition in which presidents of the commission were research mathematicians with an interest in mathematics education. The one exception was David Eugene Smith, president from the CIEM's revival in 1928 until 1932. Although a distinguished historian of mathematics and holder of a chair in mathematics education at Teachers College, Columbia University, Smith had studied mathematics only as an undergraduate, obtaining his doctorate in art history. He had studied to become a lawyer before beginning his teaching career (Swetz, 1987, pp. 299-304; Travers, 1983, pp. 381-382). He is credited (Jones & Coxford, 1970, p. 42) with being one of the two principal founders of mathematics education as an academic field in the United States (the other was Jacob William Albert Young of the University of Chicago).

Both 1908 and 1952 marked periods when the school mathematics curriculum was being faced with new demands, countries were attempting or about to attempt efforts to reform that curriculum, and an international exchange of views seemed appropriate (Howson, 1984). Before both periods, the gap between the mathematics taught in schools and mathematics as a scientific discipline had widened, and the views of mathematicians on how to close the gap seemed to set the terms of debate (Wheeler, 1989;

Wojciechowska, 1989). In each case, activities undertaken by the ICMI were to be decisive, not only in addressing issues of reform but also in shaping the field of mathematics education.

In what follows, I discuss the development of mathematics as an academic field over the past century or so, taking the ICMI as both a barometer of that development and a spur to greater change.

2. Mathematics Education enters the academy

Although mathematics itself as an academic subject can trace its lineage back at least to the *quadrivium* of Plato's academy and even to the Sumerian and Babylonian scribal schools (Davis & Hersh, 1981; Høyrup, 1994), the subject of mathematics education is both more recently and less firmly established in the academy. Throughout the 19th century, the modern scientific disciplines were emerging in higher education, beginning with the reforms of the Protestant universities of Prussia (Kilpatrick, 1992, p. 4). During the second half of the 19th century, mathematics succeeded in becoming an autonomous discipline in German universities although not yet in the technical colleges (Schubring, 1989, p. 175).

As teacher education began to move into institutions of higher education in various countries during the last decades of the 19th century and early decades of the 20th, education, too, began to be treated as a separate academic field. The first seminary for teachers had been established at Halle in 1704 and the first chair of education there in 1779 (Fleming, 1954), but as of 1910, there were only 13 staff members at German universities with teaching responsibilities in education (Husén, 1983). Other pioneering chairs in education were those established at the University of Iowa in 1873, at the University of Edinburgh in 1876, and at the University of Uppsala in 1910. At about the same time, school mathematics was gradually becoming an object of scholarly study and not just a field of practice (Jahnke, 1986; Schubring, 1988).

2.1. The Reform Program of Felix Klein

By the end of the 19th century, secondary teachers of mathematics were being prepared in universities, pedagogical seminars, and normal schools, but their preparation typically consisted almost entirely of lectures in mathematics with little or no instruction in teaching. Felix Klein, by devising and conducting courses on teaching methods at several universities in Germany, took the lead in addressing that problem. He wanted to establish applied mathematics in the university as a means of reorienting pure mathematics there, with his ultimate goal to make mathematics a foundational discipline in the universities and technical colleges. To achieve that

goal, he initiated a reform of secondary mathematics education so that it would include the calculus, thereby raising the level of mathematics in secondary and higher education (Schubring, 1989). In the process, Klein

began [...] to interest himself in the improvement of teacher education. By so doing, he hoped to reverse the trend toward one-sidedly formal, abstract approaches to mathematics instruction by promoting practical instruction and the development of spatial intuition (p. 184).

He started to address issues that concerned secondary mathematics teachers and hit upon «the key phrase that would hereinafter serve as the slogan for his reform program» (p. 188): *functional reasoning*.

Klein's strategy for change «was clearly to forge an extraordinarily broad and powerful alliance of teachers, scientists and engineers that would advocate a series of reforms for mathematics and science curricula» (Schubring, 1989, p. 188). He enlisted the international commission in his efforts, «and the organization began to function as an agent for curricular change» (p. 189). He saw that if secondary and higher education curricula were to be brought closer, a new approach to teacher education would be needed, and he continued to push for reforms in teacher education. During his lifetime, Klein published over 30 books and articles on mathematics education (Rowe, 1983).

Meanwhile, primary school teachers were being prepared in institutions at the secondary school level, whether those institutions were termed colleges, institutes, seminaries, or normal schools (Kilpatrick, 1992, p. 4). In general, we tend to know less about the mathematics taught and learned in those varied institutions than we do about the preparation of secondary school mathematics teachers. Although prospective primary school teachers were likely to receive some practice-teaching experience and attention to teaching methods as part of their preparation, most of it was apparently devoted to studying the primary school subjects (Butts, 1947, p. 424).

2.2. Professional education for teachers

As countries began to establish national school systems from the end of the 19th century through the beginning of the 20th, they began to require a larger supply of teachers with a professional education, and that need stimulated both the evolution of institutions at the secondary level into institutions of higher education and the establishment of academic units concerned with education within those evolving institutions. Those changes prompted the gradual development of mathematics education as a university subject. In the 1912 survey conducted in response to the mandate given the CIEM in Rome in 1908, four countries—Belgium, Germany, Great Britain, and the United States—reported that university lectures in math-

ematics education were being offered to supplement mathematics lectures (Schubring, 1988).

Throughout the 20th century, many of the special schools for training elementary or secondary teachers either became part of a university or attained university status for themselves, which has led to great differences across countries in the way teacher preparation is handled. By the 1930s in the United States, for example, most former public normal schools had become teachers colleges, and by the 1950s they had become departments, schools, or colleges of education in universities. Today, in some less economically developed countries, prospective teachers still receive only a short training course at the level of secondary education to prepare them to teach large classes of young children. In more economically developed countries, in contrast, most teachers are university graduates who began their teacher preparation after finishing secondary school. And between those extremes are many other arrangements.

Mathematics education as a practice has developed, therefore, within a variety of academic structures, whereas its major development as a field of study has been within universities. As practice and as field of study it is different from mathematics:

Mathematics education and *mathematics*, though obviously linked, are fundamentally different as domains of practice and scholarship. Their main historical intersection has been the induction and advanced mathematical preparation of mathematical researchers and scientists, a small but now growing fraction of the population served by school education, and this primarily at post-secondary levels. While most mathematicians teach, mathematics education treats teaching much more seriously as a professional practice, requiring dedicated training and certification (Bass & Hodgson, 2004, p. 640).

To see some of the differences, it is useful to consider how *mathematics* and *education* are to be understood in the phrase *mathematics education*.

3. *What is mathematics?*

A major difference between mathematicians and mathematics educators lies in the way they look at mathematics. To the mathematician, it is clear: Mathematics is the body of knowledge and the academic discipline that studies such concepts as quantity, structure, space, and change. It has been defined as the science of quantity and space, the science of relations, the science that draws necessary conclusions, and the science of patterns. Regardless of the definition they prefer, mathematicians have few doubts

as to what their subject is, whether they follow Courant and Robbins (1941) or Davis and Hersh (1981).

Mathematics educators are less sure. As Lynn Steen (1999) observes:

Mathematics educators cannot even agree on the nature of mathematics. Although mathematics is at the heart of mathematics education, it turns out that educators' mathematics is not mathematicians' mathematics [...]. To a mathematician, mathematics is singular—a Platonic paradigm in which there are simple, unquestionable criteria for distinguishing right from wrong and true from false. But to mathematics educators, mathematics is plural. Mathematics, among other things, offers a lens through which one can look at the world. In mathematics education the direction is reversed—one looks at mathematics through the lens of learners (and teachers) (pp. 236–237).

Mathematics educators see mathematics not simply as a body of knowledge or an academic discipline but also as a field of practice. Because they are concerned with how mathematics is learned, understood, and used as well as what it is, they take a comprehensive view. A mathematician might concede that, yes, applied mathematics is a branch of mathematics, but a mathematics educator looks beyond applications to ways in which people think about mathematics, how they use it in their occupations and daily lives, and how learners can be brought to connect the mathematics they see in school with the mathematics in the world around them.

A good example of the disparity in views is provided by ethnomathematics. To the mathematician, ethnomathematics is not mathematics at all. If anything, it is anthropology. What are the axioms or theorems of ethnomathematics? Martin Gardner (1998) reviewing books and videotapes reflecting efforts to reform U.S. school mathematics that he dubs «the new new math, whole math, fuzzy math, standards math, and rain forest math» (p. 9) defines *ethnomathematics* as «math as practiced by cultures other than Western, especially among primitive African tribes» (p. 9). He goes on to disparage attention to non-Western mathematics:

Knowing how pre-industrial cultures, both ancient and modern, handled mathematical concepts may be of historical interest, but one must keep in mind that mathematics, like science, is a cumulative process that advances steadily by uncovering truths that are everywhere the same (p. 9).

To the mathematics educator, in contrast, ethnomathematics is a highly popular subject for discussion and research even if it is not a branch of academic mathematics. There have been lectures, sessions, and working groups at International Congresses of Mathematics Education on ethnomathematics ever since Ubiratan D'Ambrosio (1986) gave his memorable

plenary address in 1984 at ICME-5 in Adelaide. The field has spawned newsletters, study groups, journals, doctoral dissertations, and books. International conferences on ethnomathematics have been held in Spain, Brazil, and New Zealand. Ethnomathematics may not be part of the mathematician's mathematics, but it is an important part of school mathematics because teachers understand the value of taking into account the mathematical systems of the cultures from which students come or in which they are interested (Gerdes, 1966; Skovsmose, 2006).

4. *What is education?*

Mathematics education has a blurred identity in the university. Across countries and institutions, it is not located in any one place. As universities took on the education of teachers of mathematics, mathematics education became a field that straddled the liberal arts and the professions. In some universities, the faculty members responsible for teacher education in mathematics tend to be in a mathematics department; in others, they tend to be in a department, school, or college of education. Either way, much of the education of mathematics teachers is seen as professional education.

4.1. *A profession?*

A profession is ordinarily understood to be an occupation that requires extensive specialized study, and its practitioners are viewed as possessing a body of arcane knowledge. Although «there is no authoritative set of criteria by means of which we can distinguish professions from other occupations» (Lieberman, 1956, p. 1), it is possible to identify some generally accepted characteristics (Figure 2).

Figure 2. Characteristics of a profession (from Lieberman, 1956, pp. 2–6)

- | | |
|----|---|
| 1. | A unique, definite, and essential social service. |
| 2. | An emphasis upon intellectual techniques in performing its service. |
| 3. | A long period of specialized training. |
| 4. | A broad range of autonomy for both the individual practitioners and for the occupational group as a whole. |
| 5. | An acceptance by the practitioners of broad personal responsibility for judgments made and acts performed within the scope of professional autonomy. |
| 6. | An emphasis upon the service to be rendered, rather than the economic gain to the practitioners, as the basis for the organization and performance of the social service delegated to the occupational group. |
| 7. | A comprehensive self-governing organization of practitioners. |
| 8. | A code of ethics which has been clarified and interpreted at ambiguous and doubtful points by concrete cases. |

Education's status as a profession suffers by comparison with that of established professions such as medicine, law, and engineering. In particular, teachers have little of the autonomy, independence, and power suggested by the characteristics listed in Figure 2. Teachers have relatively slight control over such matters as selecting their clients, determining their working conditions, and deciding who is allowed to practice their trade (Lieberman, 1956). As Lortie (1975) observed, «Teaching is only partly professionalized» (p. 23).

4.2. *Didactics, Pedagogy, or Education?*

In countries such as France and Germany, *education* is a term not much used by mathematics educators. They prefer to speak of *didactic* or *subject-matter didactics*. «In French, the term *didactique* does not mean *the art or science of teaching*. Its purpose is far more comprehensive: it includes teaching AND learning AND school as a System, and so on» (Douady & Mercier, 1992, p. 5). In many countries, including the Nordic countries, there is a long history of using *pedagogy* to designate education in the university, and mathematics educators have had to struggle to get subject-matter didactics recognized as a field of study in its own right—a part of pedagogy.

In English, both *didactics* and *pedagogy* have acquired negative—even pejorative—connotations, and neither is much used in educational discourse (Andrews, 2007; Kilpatrick, 2003). Didactic teaching is understood to be moralistic and somewhat rigid; the pedagogue is one who teaches in a pedantic or dogmatic fashion. Consequently, English-speaking countries tend to use *education*, and other countries use the other terms.

Although it can be argued that the avoidance of *didactics* (as well as *pedagogy*) may stem from an aversion to theory (Andrews, 2007), I see it as undermining the struggle to validate education as an academic field. Adopting the term *didactics of mathematics* has been very beneficial to mathematics educators in some countries who seek recognition for the field as a science—in fact, as one of the mathematical sciences.

Distinctions among *education*, *didactics*, and *pedagogy* are far from clear, however, even to insiders. Bertrand and Houssaye (1999) argue that in the Francophone world, researchers and theorists in didactics and pedagogy «operate in the same territory and use the same epistemological tools» (p. 34). However,

most Francophone didacticians are eager to differentiate themselves from pedagogists, theorists and practitioners of pedagogy. This differentiation is generally made as follows: Pedagogy is more general than didactics, less scientific, and therefore less status-enhancing. It's more attractive to define oneself as a didactician than a pedagogist, say the didacticians (p. 41).

After reviewing the literature, Bertrand and Houssaye conclude that didacticians tend to rely more on cognitive psychology than pedagogues do but that

when they leave behind research and take up classroom work, they inevitably face day-to-day pedagogical reality and find the learner is much more than a cognitive subject. Educational reality being highly complex, they are often led to take a larger variety of variables into account and do research that can be called pedagogical (p. 49).

It appears that the terminology we use may be chosen as much for political reasons as for academic ones. And it also appears, as one looks at patterns of usage, that *mathematics education* tends to be a broader, fuzzier term than *didactics of mathematics*.

4.3. *A discipline?*

Mathematics is a well-established discipline. It is a branch of knowledge with clearly defined objects of study and accepted methods for studying them. Whether education is a discipline is a more open question. Didactics of mathematics appears to have acquired disciplinary status, at least in the eyes of some of its practitioners (Adda, 1998; Biehler, Scholz, Sträßer, & Winkelmann, 1994), but mathematics education more broadly understood may be a different matter (Conant, 1963; Walton & Kueth, 1963; although see King & McLeod, 1999, for a contrasting view). Norma Presmeg (1998) describes mathematics education as «an emergent discipline», but Lynn Steen (1999) observes

Mathematicians tend to believe that only when mathematics education research produces a “theory of mathematical thinking that convincingly explains observed phenomena” [...] will it become a true academic discipline on a par with other sciences that produce robust theories with broad explanatory power (e.g., evolution in biology, thermodynamics in chemistry, relativity in physics) (p. 238).

4.4. *A science?*

Steen’s (1999) observation raises the related question of whether education in general, and mathematics education in particular, can be considered a science. According to James Bryant Conant (1963), there is no science of education; it is at best a practical art. After examining the history of education research in the United States, Ellen Lagemann (2000) claims that education research remains «an elusive science» and points out that late in his life John Dewey (1929/1984) called education a new science that was just beginning to achieve scientific status. Controversy over that status continues to rage (National Research Council, 2002) and has spilled over to mathematics education (Steen, 1999). The consensus seems to be that math-

ematics education is not itself a science but that at least some of its research fits the social sciences' criteria for being scientific.

5. A field of study and practice

A fair description might be that whereas mathematics is both a discipline and a profession, mathematics education is a field of study and practice whether or not it qualifies as a discipline or a profession. The terminology is less important than the recognition that each field has one side concerned with knowledge and another concerned with practice. Schubring (1989) observes:

The autonomy of an academic discipline depends upon the existence of specialized professional careers for the discipline's graduates. As a consequence, discipline and profession are functionally related. Changes that take place at one of these two poles will induce changes in the other (p. 174).

At the disciplinary pole, the 20th century saw an abundance of mathematics education research studies, whether they were conducted within the pedagogue tradition, the empirical scientist tradition, or the scholastic philosopher tradition (Bishop, 1992). The field has constantly struggled to define the research it conducts (McLeod & King, 1999; Sierpinska & Kilpatrick, 1998; Steen, 1999), and the ICMI has responded with sections at its International Congresses on Mathematics Education and regional meetings, its affiliated study groups (especially the International Group for the Psychology of Mathematics Education), and its ICMI Studies.

In a lecture at the first ICME in Lyons, E. G. Begle (1969) cited some findings from empirical research that contradicted the received wisdom of the field and pointed out that neither teachers, nor mathematicians, nor mathematics educators had «been in a position to gather, during the course of our ordinary activities, the kind of broad knowledge about mathematics education that we need» (p. 239). He called for mathematics education to be made an experimental science in which mathematics educators would «abandon our reliance on philosophical discussion based on dubious assumptions and instead follow a carefully correlated pattern of observation and speculation, the pattern so successfully employed by the physical and natural scientists» (p. 239). Although Begle's vision has not been realized, progress has been made in the empirical observations and theory building he called for.

At the other pole, mathematics education developed greatly throughout the century as a field of practice, and again, ICMI played a role. Its regional meetings have allowed countries in which mathematics education is institutionalized in a similar fashion to build a community and share ideas

regarding curriculum, teaching, and learning. Each ICMI Study has created, through the study conference, a forum where approaches to practice can be identified, and through the resulting study report, a platform for sharing those approaches. The affiliated study groups, through their own meetings and publications and at the ICMEs, have helped build communities of practice within the larger organization.

Skovsmose (2006) observes that mathematics educators need a broad view of practice, one that goes beyond the classroom to include out-of-school practice. Such a view inevitably raises socio-political issues:

When the different school-sites for learning mathematics as well as the many different practices that include mathematics are related, for instance by considering possible transitions between such practices, we enter the socio-political dimension of mathematics education. The principal issues are: Could a socio-political discrimination be acted out through mathematics education? How could mathematics education ensure an empowerment? (p. 268)

A hallmark of recent discussions of mathematics education, within ICMI activities and outside, has been the attention that increasing numbers of mathematics educators are giving to the socio-political dimension of our practice.

5.1. *Teaching as vehicle and constituter*

Jens Høyrup (1994) argues that «one aspect of mathematics as an activity [...] is to be a reasoned discourse; [...] as an organized body of knowledge [it is] the *product of communication by argument*» (p. 3). Consequently, «teaching is not only the *vehicle* by which mathematical knowledge and skill is transmitted from one generation to the next; it belongs to the *essential characteristics* of mathematics to be *constituted through teaching*» (p. 3). Just as certain algebraic techniques did not exist until the ancient Babylonians developed them in schools for training scribes, so the mathematics of the recent proof of Fermat's last theorem by Andrew Wiles did not exist until it had been 'taught' to others by appearing in print.

In a similar fashion, one can argue that an essential characteristic of mathematics education is to be constituted through teaching—in this case, through teaching teachers as well as students, and through teaching teachers to teach as well as to understand and do mathematics.

5.2. *Domains of knowledge and communities of practice*

Bass and Hodgson (2004) raise and partly answer two important questions regarding these academic fields:

How are mathematics and mathematics education, as domains of knowledge and as communities of practice, now linked, and what could be the most nat-

ural and productive kinds of connections? The ICMI represents one historical, and still evolving, response to those questions at the international level (p. 640).

It is tempting to treat the relation between mathematics and mathematics education as resembling that of parent and child (although, as noted at the outset, the child's international organization predates the parent's). The status of mathematics in the academy is both higher and more secure than that of mathematics education, and it seems natural to think of mathematics as the more mature, responsible endeavor. Nonetheless, I think a better metaphor than parent and child lies in the concept of yin and yang (Figure 3). Mathematics and mathematics education have a synergistic relation, and neither can exist without the other.

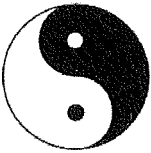


Figure 3. The yin yang symbol

In my view, mathematics education has not attained the status of a discipline, and it is not completely a profession. But as an academic field, it is linked to mathematics through a mutual concern with teaching (Figure 4). That mutual concern has been reflected in the many activities of the ICMI.

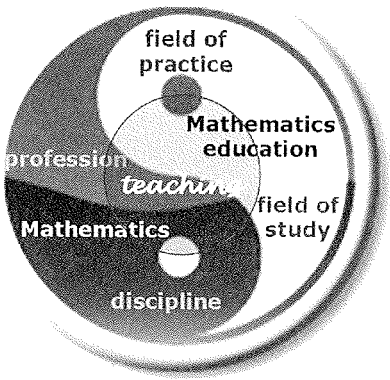


Figure 4. Mathematics and mathematics education as academic fields

Nineteen countries participated in the first international commission meeting in Cologne in September 1908, together with another 14 'associ-

ated countries' (Howson, 1984, p. 76). Few could have predicted that a century later, there would be 72 member states of ICMI or that in each of those member states a distinctive community of mathematics education would have developed, comprising people from schools, universities, government ministries and agencies, and other organizations and associations. Today an astonishing profusion of books, handbooks, proceedings, articles, research reports, newsletters, journals, meetings, and organizations is devoted to mathematics education in all its aspects. A search of the scholarly literature on the Web for the phrase *mathematics education* yields 125,000 hits; a search of the entire Web yields almost 9 times that number.

Mathematics education currently occupies a substantial if somewhat unclear position in the academy thanks to the many people, including mathematicians, who have devoted some or all of their careers to its furtherance. During the next century of ICMI's existence, mathematics and mathematics education will continue to require each other's support and assistance if mathematics education is to develop further as a field of study and practice.

Note

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JEAN-LUC DORIER

Reaction to J. Kilpatrick's plenary talk
The development of mathematics education as an academic field

1. *Academic? You said academic?*

I will start my task as non-native-English-speaker reactor by tackling the question of language, even though Jeremy, as a 'fin connaisseur' of Europe and France especially, has already raised this issue. Indeed, my first reaction, after reading Jeremy's paper was to look in a dictionary not for *mathematics education* or *didactic* but first for the definition of *academic*. Here is a brief synthesis of my search:

- In the *Concise Oxford English Dictionary* (2006)

Academic → adj.

1. relating to education and scholarship. • scholarly rather than technical or practical.
2. (of an art form) conventional in an idealized or excessively formal way.
3. not related to a real or practical situation and therefore *irrelevant*.

Scholarly → adj.

involving academic study or dedicated to academic pursuits.¹

- In the *Oxford American Thesaurus of Current English* (1999)

Academic adjective 1. academic considerations educational, scholastic, instructional, pedagogical. 2. of an academic turn of mind scholarly, studious, literary, well-read, intellectual, erudite, learned, cultured, bookish, highbrow, pedantic, professorial, cerebral; inf. brainy. 3. an academic rather than a practical solution theoretical, hypothetical, abstract, conjectural, notional, impractical, unrealistic, speculative, ivory-towerish.

Scholarly adjective learned, erudite, academic, well-read, intellectual, scholastic, literary, studious, bookish, lettered; inf. egghead, highbrow. opposites: illiterate; ignorant.

Therefore the issue of mathematics education being an academic field has to do not only with a structural aspect of research (as mentioned by Jeremy) but also with the question of the legitimacy of a theoretical positioning about issues within the scope of mathematics education.

¹ Interesting circular definitions.

From the definitions above, it turns out that ‘being academic’—like ‘being didactic’—is often judged as rather negative, ‘excessively formal’, ‘not related to real or practical situation’, even ‘pedantic’, ‘ivory-towerish’, ‘unrealistic’ and ‘impractical’—and this is not just specific to the English language. Therefore, why would mathematics education struggle to be an academic field, rather than just a practical matter relevant to the reality of teaching? Is there any space for theory in mathematics education, or is it just a field of action and practice?

2. Social relevance of mathematics education

In several countries today, education and teacher training are problematic political issues, subject to a crisis, whose importance is still difficult to evaluate, especially because these issues take different aspects throughout the planet. The democratization of education is one of the biggest challenges that education has had to face in decades and maybe even centuries.

Jeremy noted that the two crucial dates of 1908 and 1952 in the history of ICMI, both «marked the beginnings of periods when school curriculum was being faced with new demands». However, 1908 and 1952 also opened periods of hopes (dreams?) for a better and more democratic teaching of mathematics. Even if the period of ‘modern math’ was subject to sharp criticism afterwards, the counter-reform that started in the 1980s was still a period of enthusiasm that led to a large development of research work in all fields of education. In contrast, it seems that the turn of the 21st century was more pessimistic, and we are now facing this paradox that, while research in mathematics education has never been so developed and allows us to know much more about the teaching and learning of mathematics, it is politically attacked and accused of being irrelevant in many countries. Conservative viewpoints are getting more and more popular: «a good teacher is someone with a gift, a vocation, and (eventually) a good training in mathematics (there is no need for all the theoretical mess that trainee teachers are fed by pedantic scholars)»; «a child needs above all to know his or her fundamentals, like algorithms for basic operations»; «pocket calculators should be banned»; and so on.

Should we see here signs of the failure to make mathematics education a socially efficient academic field? Or are these negative comments just a side effect of a general distrust of academic intervention coupled with a global crisis touching the social status of school and education?

Mathematics education cannot answer this question, but the academic development of this field is somehow dependent on its ability to react to the new social challenges. This is not just a demagogical issue, but really an epistemological one. If we follow this argument, it is therefore interesting

to analyze the evolution of mathematics education as an academic field during the last century in terms of responses to the social evolution of education as well as the evolution of mathematics in society.

3. *The increasing importance of mathematics from the beginning of 20th century*

In 1908, our starting point, in most countries, education was mostly provided to boys coming from the upper levels of society. The curriculum changes happening at the beginning of the 20th century were the result of the evolution of sciences in the preceding century and their role in society.

Besides the claimed necessity to take into account the training of all kinds of elites, including those of industry and business, another reason is the importance of the positivist context of the end of the 19th century, when the status of the sciences was raised to that of the humanities (*my translation from Gispert, 2002, p. 159*).

In this movement, the place of mathematics in education was to be totally transformed. Carlo Bourlet's address at the meeting of the recently created CIEM during the *Exposition Universelle* in 1910 in Brussels gave a good overview of the situation:

Modern teaching cannot be limited to the training of faculties of the brain; it must show how to fill it with facts, numerous and precise. We do not have to train philosophers who will live like erudite hermits, but men of action who will have to contribute their share to human progress. That is why we can no longer just present to our students mathematical science in a purely speculative way. We must at all costs serve the whole of society, so that with each one of our students specifically, we do our best to make mathematical abstractions yield to the necessities of reality (*my translation from Bourlet, 1910, p. 374*).

As a consequence, Bourlet advocated a radical change in the curriculum, bringing a new face to mathematics teaching:

Let us put the antique Euclidean edifice, admirable for its harmony and perfection, away with historical monuments and build, according to a new schema, a homogeneous work consistent with today's necessities (p. 384).

However, no major changes occurred in the social structure of school. Therefore, the movement to which the creation of the CIEM

in 1908 gave an international dimension called for a general reflection about the nature of mathematics to be taught and how to organize its teaching. As a consequence, a need for teacher training followed. Several mathematicians from different countries and status participated in this reform, creating a first step towards the constitution of mathematics education as an academic field, within the mathematical circle, with contributions from the *Didaktik* or curriculum traditions depending on national traditions (see, e.g., Hopman, 2007, and Hudson, 2007).

4. Modern mathematics and the democratization of education

The second important event of the 20th century regarding mathematics education corresponds to the reform of modern mathematics, whose first signs appeared in the 1950s. This reform brought not only important changes in the content of mathematics teaching but also a first democratization of education as well as a reflection about new ways of teaching, initiated by several pedagogical movements and research work from psychologists in different countries. Mathematics and sciences were to be renewed in their content and objectives, but also to be taught to a larger portion of students as part of their training as citizens and not only to the elite. That is what André Lichnerowicz (president of CIEM, 1963-1966) expressed in the introduction of the report of the commission he directed that settled the reform in France in 1967:

In order to feel like a full citizen of human society, a man of the second half of the 20th century must know how to locate himself in space and time, must communicate with foreign communities, but must above all perceive some methods for thinking and acting in what constitutes the know-how of today's science and technique (*my translation from the quotation in Gispert, 2002, p. 160*).

Therefore mathematics education was to broaden its means and scope and not only reflect on curricula, a demand that Hans Freudenthal (president of CIEM 1967-1970) already expressed in 1963:

Since its origin at the beginning of this century, CIEM has mainly been busy with organizing mathematics teaching in different countries, including current and prospective curricula and examinations. History has proven the sterility of purely organizational issues (*my translation from Freudenthal, 1963, p. 29*).

He clearly put forward the issue of the social evolution of education and claims for a more didactical orientation, also referring to research in psychology:

The social implications of education have changed a lot during the last decades. The educational task evolves more and more from the transfer of cultural acquisitions to the initiation into cultural activities. ...The educational psychologists know how to show why understanding new subjects is deeper and more longer lasting if it is of a reinventing nature (p. 31).

Indeed, from the 1950s, research in mathematics education developed new ideas, paradigms, and theoretical frameworks. This evolution took different aspects throughout the world, but the general tendency was to develop research that took into account cognitive and psychological aspects in the teaching and learning processes. Piaget's and Vygotsky's work (to cite only the most famous) were important sources of inspiration. Nevertheless, mathematics education developed specific theoretical frameworks in which mathematics was still central. In France, for instance, both Brousseau's and Vergnaud's contributions were original in that sense.

The CIEM had an essential role in the international cohesion with, in particular, the first International Congress on Mathematical Education, held in Lyon in 1969, the formation of the Psychology of Mathematical Education affiliated group in 1976, and the ICMI Studies beginning in 1985. It would be too much here to give details, but the evolution in content of these different international meetings of the mathematics education community gives an overview of the large variety of subjects and methods developed and discussed since the 1960s.

5. *A more complex surrounding for mathematics education*

I do not have space here to analyze the key points in this evolution in the light of social changes in education and the role of mathematics in society. Important issues like teacher training, teaching in difficult social contexts or multicultural environments, the development of new technologies, the decreasing social value attached to mathematics, the competence-versus-content orientated curriculum reforms, and so forth led researchers in mathematics education to develop new theoretical frameworks in order to tackle these questions. Cultural differences between countries have also emerged that sometimes make communication within an international community difficult. Jeremy developed this point while discussing the difference between *didactique*, *Didaktik*, *mathematics education*, *pedagogy*, and so forth. Beyond language, these differences bring to light differences in

research paradigms and theoretical frameworks. There are also differences in national educational policies that international surveys like PISA reveal or even exacerbate from time to time.

In the meantime, research in education has led to the constitution of an academic field that is of greater or lesser importance in different countries. The relations that mathematics education hold with this field are varied, depending on different contexts in every country. In some countries, the possible space for mathematics education, as an independent academic field between education and mathematics may be problematic (in various ways) on both institutional and epistemological grounds. An interesting issue is that this situation can have different aspects for mathematics than for other school subjects (like literature, sciences, or history) even in the same country (see, e.g., Sarremejane, 2001). On the other hand, mathematics education has developed several types of cooperation with other academic fields in relation to education because the development of research shows that the complexity of the reality of education needs to be tackled from different viewpoints. However, in this diversity, it is also important that mathematics education be able to put forward the specificities of its objects, methods, and epistemology. In that sense, the relation to mathematics is essential, and the role of ICMI is thus vital in order to maintain and develop in all its variety an academic field specific to mathematics education that maintains a privileged relation with the mathematical community at large.

Issues of education are more and more subject to political, cultural, and social contexts. The traditional organization of school subjects, the role of mathematics for other sciences, in society in general and in education in particular, and the social constraints of education are among the crucial questions that mathematics education has to take into account. Since 1908, the situation has changed in many ways. Research in mathematics education has tremendously enlarged its scope and means, but mathematics education is also subject to more social pressure. A risk would be that it fails to qualify as an independent academic field because it cannot ensure a consistent and original domain and is absorbed in related fields. A barrier against this possible dilution remains the attachment of mathematics education to mathematics that ICMI can guarantee while encouraging cooperative work with other academic fields connected to education.

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Intuition and rigor in mathematics education

A recurrent theme in the work of ICMI is the relationship between intuition and rigor. The emphasis on either of these two dimensions has varied a lot throughout history. In the first part of this paper we focus on the development of this issue in the history of ICMI. In the second part we present the seminal work of Efraim Fischbein on intuition and rigor. In the last part we raise some issues for further consideration.

A perennial theme in mathematics education in general and in the work of ICMI in particular is the balance between, on the one hand, rigor¹, logic, axiomatic, and formalism and, on the other hand, intuition², insight, common sense, invention and heuristics. This pertains to concept formation and understanding as well as to reasoning and justification in mathematics. The emphasis on either of these two dimensions of mathematics education has varied a lot throughout history, sometimes even taking the form of tension, thus reflecting the difficulty of finding balance. In the first part of our plenary we briefly relate to the development of this issue in the history of ICMI and the way it has been addressed by various communities concerned with mathematics education. In the second part we present one of the seminal works on intuition, rigor and their relationship. In the last part we raise some issues for further consideration.

¹ These terms are clearly not synonyms. We chose to use the term rigor in our own writing in this paper. The other terms are used when mentioned in publications that we cite.

² These terms are clearly not synonyms. We chose to use the term intuition in our own writing in this paper. The other terms are used when they are mentioned in publications that we cite.

1. *Intuition and rigor in ICMI: 1908-2007*

One way of telling the story of intuition, rigor and their relationship in ICMI could be by tracing the work of various members of these communities and by immersing ourselves in the various, related documents. These documents, however, do not fully represent the vivid, often passionate debate among mathematicians, mathematics educators, educators, philosophers and psychologists of mathematics during various conferences, including for instance ICM, ICME, PME, HPM and TME. Such arguments have indeed accompanied ICMI from its birth until today. We shall describe two major periods of about 50 years each in ICMI with reference to the role of intuition and rigor in mathematics education. The first we call the *introspection period*, and during it the participants in ICMI meetings were almost solely mathematicians. The arguments related to the role of intuition and rigor in mathematics education were mainly based on personal experience and on descriptions of the mental processes that mathematicians went through when forming a concept, validating a theorem, or solving a mathematical problem. During the second, *research-based period*, the discussions on the role of intuition and rigor in mathematics education started to be based on scientific research. In each of these periods, some members of the community regarded intuition and rigor as being at odds while others argued that the two dimensions should have complementary roles in mathematics education. The occasion that signalled the end of the first period and the beginning of the second was the meeting organised by ICMI in Geneva in 1955, at which Freudenthal insisted on the necessity to promote scientific research to address the complex issues and debates regarding mathematics education. We should note that often in scientific domains, there are calls for reform and modification way before changes are made, and that such changes when they occur, are not global. Nonetheless, the meeting in Geneva marks the beginning of a new period in ICMI in general, and with respect to issues related to the role of intuition and rigor in particular. In this new period the nature of the evidence for supporting one view or another includes empirical, behavioural data. We shall briefly describe each of these periods.

2. *The retrospective period: 1908-1955*

Let us go back to the beginning of the 20th century, the years in which ICMI was founded. Mathematicians, at this time, largely differed in their opinions regarding the role and the importance of intuition and of rigor in mathematics (this was a key issue in the debate on the foundations of mathematics among logicism, intuitionism and formalism). It is widely docu-

mented that Felix Klein, a distinguished mathematician and the first president of ICMI, respected rigor, but favoured intuition and imagination. Klein's attitude to intuition is highlighted in both his own writings and in the work of other mathematicians, some of which was published before the foundation of ICMI (e.g., Poincaré, 1905) while others in recent years (e.g., Bass, 2005). One such example is quoted in a chapter on *Intuition and logic in mathematics*, in Poincaré's (1905) book on *the value of science*. In this chapter Poincaré attempted to classify mathematicians into two groups: those who think mainly in images (the geometers) and those he called the analysts, the pure conceptual thinkers. Poincaré included Felix Klein in the first category, demonstrating that the latter was content with publishing a solution that was not yet fully, rigorously developed. Poincaré discussed the different roles played by intuition and logic in mathematics. He argued that although formal logic is certain, it does not pave the way to a proof. It is intuition that helps mathematicians find the way to assemble basic inferences into a proof. Poincaré offered the following analogy: An unskilled chess player who watches a game can verify whether a move is legal, but he does not understand why players move certain pieces, for he does not see the plan which guides players' choices. In a similar way, a mathematician who uses only logical methods can verify every inference in a given proof, but he cannot find an original proof. In other words, every elementary inference in a proof is easily verifiable through formal logic, but the invention of a proof requires the understanding – grasped by intuition – of the general scheme, which directs mathematician's efforts towards the final goal.

An opposing opinion regarding the roles of intuition and logic in mathematics was expressed by Hans Hahn, a prominent, Austrian mathematician. Hahn worked, at the time that ICMI was founded, in seminars with leading mathematicians including Felix Klein. In an essay on *the crisis in intuition* Hahn severely criticized Kant's thesis that mathematics is founded on pure intuition, not on thought. Hahn explained, at the beginning of his lecture, that he would not discuss the opposition to Kant's thesis that arithmetic, the study of numbers, rests on pure intuition because Bertrand Russell had already proved «in complete contradiction to Kant's thesis, [that] arithmetic belongs exclusively to the domains of intellect and logic» (Hahn, 1956/1929, p. 1958). Hahn noted that he narrowed his subject to geometry and intuition and summarized by stating that:

Again and again we have found that in geometric questions, and indeed even in simple and elementary geometric questions, intuition is a wholly unreliable guide. It is impossible to permit so unreliable an aid to serve as the starting point or basis of a mathematical discipline. The space of geometry is not a form of pure intuition, but a logical construct. (1956/1929. p. 1974).

A significant indication of the disagreements among mathematicians regarding the role of intuition and rigor in mathematics is provided by Felix Klein himself, in the following quotation:

On this point Pasch does not agree with me, the point in question being the rigorous value of axioms. He believes – and this is the traditional approach – that it is possible, eventually, to give up completely intuition and base all the science on axioms only. As far as I am concerned, I am convinced that, in research, one should always combine intuition with axioms (Klein, 1898, in Fischbein, 1975, p. 5).

In the light of these, it is not surprising that issues related to the role of intuition and rigor in mathematics education, were among the first to be discussed at ICMI. Calls for significant reforms in the mathematics curriculum in various countries (e.g., the call for ‘utility’ rather than ‘rigor’, laboratory-based experience rather than abstraction, by John Perry in England at 1905), acted as catalysts and brought these issues to the fore. Thus, the question «What is the place of rigor in mathematics teaching?» was one of the three main themes that were dealt with in Milan, in 1911, in one of the first international meetings of ICMI. In 1912, at the ICMI meeting in Cambridge, the congress paid special attention to two major issues, on which it had instigated surveys. One issue was *intuition and experiment in mathematical teaching in the secondary schools*. Unfortunately, the First World War brought these international activities to an end, and, as stated by Geoffrey Howson (1984), although ICMI was formally reconstituted in 1928, the years between the two world wars were not particularly momentous ones for the commission. Howson (1984) clarified that it was not until 1952 that the commission resumed its activity as a sub-commission of the International Mathematical Union (IMU) and that new, major questions regarding the role of mathematics in contemporary life were raised.

A significant indication of the introspective nature of arguments that were regarded as validating one position or another in respect to the role of intuition and rigor in mathematics even toward the end of the first period is featured in Hadamard (1945)’s book: *An essay on the psychology of invention in the mathematical field*. In this book Hadamard, a distinguished French mathematician and a president of ICMI, used introspection to describe the role of insight in mathematical thought processes. He surveyed about 100 leading mathematicians and physicists of the day, asking them how they did their work. Many of them described their own mathematical thinking as largely wordless, often accompanied by mental images. Based on these introspective comments, Hadamard highlighted the essential role of insight, the sudden illumination, in mathematics invention.

As noted previously, we chose the meeting organised by ICMI in Geneva

in July, 1955 as a marker of the end of the first, introspective period and the beginning of a second, research-based period regarding the role of intuition and rigor in mathematics education. This meeting could be viewed, from many perspectives, as a key moment in the development of the issue discussed in our plenary and in the development of ICMI in general.

3. The research-based period: 1955-2007

The Geneva (1955) meeting was organized to elect officers and to determine a plan of work for ICMI for the next three years. Two major events that occurred during this meeting could be viewed as markers of the beginning of the research-based period regarding the role of intuition and rigor in mathematics education. These events are significant as there were clear calls, in both, for a need to take account of educational and psychological research when making decisions regarding school mathematics. The first is a suggestion made by Behnke, who in this meeting talked about: *mathematics education up to the age of fifteen years*. Behnke stressed the need to listen to psychologists and didacticians when discussing mathematics teaching up to the age of fifteen. Howson (1984) reported that Behnke's view received support, but that it was urged that the mathematician's voice should be heard as well. The second, significant event was the appearance of Hans Freudenthal, a new contributor, «who without doubt was to have the greatest influence on ICMI's development» (Howson, 1984, p. 42). Freudenthal expressed, right from the start, strong positions against the themes that were discussed in ICMI. He insisted on the need to promote scientific research, including psychological considerations, and suggested several subjects for inquiry, two of which are directly related to the focus of our plenary: the need to explore the role of a preliminary, intuitive approach in the teaching of geometry, and to study the role of logic in mathematics teaching.

The generally-agreed upon calls, during the Geneva meeting, for collaboration between mathematicians and psychologists did not have immediate results. Shortly after the meeting in 1955, significant changes began to take place in school mathematics. The effect of Bourbaki, the group that strove for utmost rigor, became evident in many countries including Belgium, England, France, Germany, and the USA. At the international meetings of ICMI between 1964-1965, many discussions addressed issues such as how to teach set theory and relations, in secondary schools 'with all possible exactness and rigour' (Dakar Conference Report, UNESCO, 1967, p. 71). Still, the call for more attention to pedagogical and psychological issues in ICMI did not vanish, and provoked some-

times bitter arguments. The following quotation, from a description of the Utrecht meeting at 1964, provides a sense of the disagreements:

The crucial point of the colloquium was a confrontation of the ideas provoked by the lectures and the interventions of Professor A. Wittenberg, who... underlined the dangers connected with a formal modernization (UNESCO, 1967, p. 378).

In 1967, Freudenthal, the then newly elected president of ICMI, continued his persistent efforts to stress the need to conduct scientific research, including psychological considerations. He suggested planning a special congress devoted solely to mathematics education. The first ICME (International Congress of Mathematics Education) was held in Lyon, in 1969. The importance of basing decisions regarding mathematics education on empirical observations was highlighted in many of the talks, including that of Begle, who argued:

I see little hope for any further substantial improvements in mathematics education until we turn mathematics education into an experiment science, until we abandon our reliance on philosophical discussions [...] (1969, p. 342)

Issues related to the relationship between intuition and rigor were discussed during all the meetings of ICME. At ICME2 (Exeter, 1972), for instance, Fischbein gave a talk on intuition, structure and heuristic methods in the teaching of mathematics. Notes from this conference reveal that at this conference the position that less emphasis should be put on rigour was not popular:

The most challenging talk was given by Professor René Thom (France) on *Modern Mathematics - does it Exist?* He was almost barracked as he made some rather controversial but highly interesting points. He is a distinguished mathematician who was awarded the Fields medal in 1958 [...] Professor Thom distinguished between two aspects of modern reforms in mathematical education: pedagogical renovation and modernization of the syllabus. [...] He derided the emphasis on structure in the teaching of mathematics because finer relations were very difficult to formalize. He suggested that rigour is not particularly important in mathematics, and that he would always choose meaning before rigour.

In the following ICMI and ICME conferences (e.g., ICME3, in Karlsruhe, 1976 and ICME4, in Berkeley, 1980, ICME7, in Quebec, 1992; ICME8, in Sevilla, 1996, and ICME10 in Copenhagen, 2004), the extreme positions for emphasising one dimension (either intuition or rigor) in mathematics education became moderated. As indicated by Erich Wittman, already

at ICME4, «one-sided or controversial considerations have been placed by attempts to reconcile the two positions». The proceedings of ICME4, for instance, include a chapter on *the complementary role of intuitive and analytical reasoning*. In 1992, Michael Otte, in his lecture at ICME7, discussed the role of intuition and logic in mathematics, claiming that «mathematics is the incarnation of intuitive knowledge» (Otte, 1994, p. 274). Otte immediately added that: «This idea is naturally very attractive. However it is connected with a dilemma. Although the conviction that results from the intuitive character of our mathematical insights may indeed be very strong, it is also very fragile» (p. 274). Howson (1998) at ICME8, asserted that:

Our mathematics teaching, like mathematics itself, must be founded upon common sense. However... we must ensure that students see mathematics not only as an extension of common sense, but as a structured, organized discipline that builds upon accepted, shared results and techniques (p. 268).

Finally, Hyman Bass, a former president of ICMI, related in his plenary address during the most recent ICME conference, to the work of the Felix Klein, the first president of ICMI, noting that:

In his inaugural lecture Klein emphasized the unity of knowledge, in particular emphasizing close ties between science and the humanities. In mathematics, he advocated focused attention on applied, as well as pure, mathematics and on connections with the other sciences. While attending to rigor and logical skills, at the same time he strongly urged the cultivation of intuition and imagination (Bass, 2005, p. 419).

We have related so far mainly to discussions on the roles of intuition and rigor in mathematics education that took place in ICMI and in ICME. However, this issue was also commonly addressed in the conferences of independent groups affiliated to ICMI. Many of the publications (proceedings and books) generated by one of these groups, the International Group for the Psychology of Mathematics Education (PME) describe attempts to study the role of some aspects of these dimensions (intuition, rigor, or both) in mathematics education. Issues related to intuition and rigor are discussed, for instance, in each of the chapters of the first book presenting the major findings of PME research related to mathematics education: *Mathematics and cognition: A research synthesis by the international group for the psychology of mathematics education* (Nesher & Kilpatrick, 1990). The chapter on advanced mathematical thinking (written by Tommy Dreyfus with the collaboration of Michèle Artigue, Theodore Eisenberg, David Tall and David Wheeler) describes three prevalent themes that were explored in this direction: concept image and concept definition

(e.g., Tall & Vinner, 1981), processes and objects (e.g., Douady, 1985, Dubinsky & Lewin, 1986, Sfard, 1987), and visualization (e.g., Eisenberg & Dreyfus, 1989). This chapter also describes a body of studies on proof (e.g., Alibert, 1988, Bell, 1976, Balacheff, 1988; Dubinsky & Lewin, 1986; Leron, 1985), and on problem solving (e.g., Schoenfeld, 1985; Rogalski & Robert, 1988). In many chapters, the roles of intuition and of rigor in both problem solving and concept formation are addressed. Since 1990, much of the research and the discussions at PME addressed, either explicitly or implicitly, the issues at hand. Accordingly, the chapters that discussed the cognitive aspects of learning and teaching mathematics in the recently published, second book offering an overview of the research done in the PME community (Gutierrez & Boero, 2006) relate to the need to pay attention to both the intuitive and the formal dimensions when dealing either with concept formation or with problem solving.

It is impossible, in one paper, to describe the amount of work that has been done in PME on issues related to intuition and rigor. We chose to focus, in the second part of our plenary, on presenting some of the work of the first president of PME, Fischbein, who devoted his academic life to the study of the role of intuition and rigor in mathematics education.

4. Intuition and rigor: The work of Efraim Fischbein

Efraim Fischbein (1920-1998) was born in Romania, where he started his professional career as a teacher of mathematics and philosophy in high school. In 1949 Fischbein became a lecturer at the Department of Educational Psychology at the University of Bucharest; he served as head of this department from 1959 to 1975. Fischbein's work in child development, cognitive psychology, and mathematics education during that period was distinguished for the innovative questions he posed, his carefully designed methodologies and insightful interpretations of the data. His publications attracted international interest, and he was frequently invited, from behind the Iron Curtain, to present his work in western countries. Fischbein left Romania for Israel in 1975. He was offered a professorship in mathematics education at Tel Aviv University where he founded the Department of Science Education.

In 1976 Fischbein made a major contribution to the international, mathematics education community by taking a leading role in the establishment of PME, The International Group for the Psychology of Mathematics Education. Fischbein was the first president and an honorary member of PME. He actively participated in almost all the annual meetings of PME, and significantly contributed to the ongoing development of the organization.

One of the main issues Fischbein addressed (e.g., 1993) was the role of, and the interaction between, intuition and rigor in mathematics education. Fischbein claimed that mathematics should be considered from two points of view: as a formal, deductive rigorous body of knowledge, and as a human activity. Elaborating on the latter, he explained:

The fact that the ideal of a mathematician is to get a strictly coherent, logically structured body of knowledge, does not exclude the necessity to consider mathematics also as a creative process: As a matter of fact, we want the student to understand that mathematics is, essentially, a human activity, that mathematics is invented by human beings... We want them to learn not only the formal, deductive sequence of statements leading to a theorem, but also to become able to produce, by themselves, mathematical statements, to build the respective proofs, to evaluate not only formally, but also intuitively the validity of mathematical statements (1993, p. 231).

Fischbein suggested considering three basic components of mathematics as a human activity: the formal, the algorithmic and the intuitive. The formal component of mathematical knowledge is one's knowledge about the mathematical realm, knowledge of what is 'right', 'permitted' and 'acceptable' versus what is 'wrong' and 'unacceptable' in mathematics. Fischbein stated that:

The formal aspect refers to axioms, definitions, theorems, and proofs... Understanding what rigor means in a hypothetic-deductive construction, the feeling of coherence and consistency, the capacity to think propositionally, independently of practical constraints, are not spontaneous acquisitions of the adolescent (p. 232).

Fischbein argued that a second component of mathematical knowledge is the algorithmic component. According to him, algorithmic knowledge is the knowledge of how to carry out a finite sequence of steps from a given mathematical problem to its solution, while being able to provide a valid mathematical explanation for each step. Fischbein emphasized the significance of the algorithmic component of the mathematical knowledge and its crucial interaction with the formal component within any mathematical activity.

It is a mere illusion to believe that by knowing axioms, theorems, proofs and definitions as they are exposed formally in textbooks, one becomes able to solve mathematical problems. Mathematical capabilities are also stored under the form of solving procedures, theoretically justified, which have to be actively trained. There is a spread misconception according to which, in mathematics, if you understand a system of concepts, you become spontaneously able to use

them in solving the corresponding class of problems. We need *skills* not only *understanding*, and skills can be acquired only by practical systematic training. The reciprocal is also, sometimes, forgotten. Mathematical reasoning cannot be reduced to a system of solving procedures (p. 233).

The third component of mathematical knowledge is the intuitive component. Fischbein is best known for his inspiring contribution to the understanding of the role of intuition in learning and teaching mathematics and science. His first offerings in this area concerned intuitions of probability and combinatorics, where he established a strong bridge between psychology and education (e.g., Fischbein, 1975). Freudenthal related warmly to this work, noting that in his opinion, Fischbein's work is a major breakthrough in mathematics education.

In *Intuition in Science and Mathematics* (Fischbein, 1987), Fischbein proposed a comprehensive theoretical, view of intuition. He identified and organized related research findings, and described and discussed their educational and didactic implications. In this book, as in other publications, he coined new and useful terminology (e.g., primary intuitions, secondary intuitions), raised problematic issues (e.g., the educational dilemma) and stimulated further related research, thus leading the field of mathematics education forward.

Fischbein's contribution to the educational approach to intuition in science and mathematics was formulated in response to basic contradictions in the domain of intuition which he identified in various publications. While intuition tends initially to be grasped as a self explanatory notion, a deeper look reveals a highly controversial concept which is variously addressed in science, in philosophy, in psychology and in daily life. For example, in some contexts, intuition is referred to as a source of ultimate truth (e.g., the work of Spinoza, 1967), while in others, it is mainly addressed as a source of misconceptions (e.g., Hahn, 1956). Usages of the notion of intuition are often connected to 'common sense', 'naïve reasoning', 'empirical interpretation', 'instinctive knowledge', 'an impression that something is the case', 'reasonable guess', 'perception', and 'insight'.

Fischbein defined intuition as «a knowledge imposing itself *subjectively* to the individual as absolute» (1987, p. 46). He explained that in his early analyses of the different approaches to intuition in the literature, he realized that underlying its inconsistent appearances, intuition expresses a basic and steady 'quest for certitude' of the human mind. This quest is evident, for example, in decision making, in problem solving, in evaluating chances and in predicting outcomes, and «as paradoxical as it may appear, it is mainly as an effect of the scientific endeavor towards rigour, in the history of science, that the rich implications of intuitive knowledge have been revealed and described» (p. 8).

Fischbein's definition of intuition refers to several characteristics: immediacy, self-evidence, intrinsic certainty, perseverance, coerciveness, implicitness, theory status, extrapolativeness and globality. In his publications, Fischbein explained each of these characteristics and illustrated their respective input to intuitive knowledge (see Fischbein, 1987). In his careful analysis of intuitive knowledge, Fischbein explained that the notion of intuition includes a vast variety of possible meanings, and he expressed the need for related classifications. He claimed that a first dichotomy may be distinguished between problem solving intuitions and affirmatory intuitions.

Problem solving intuitions - consist of two types of intuitions:

Anticipatory intuitions are the preliminary, global views which precede the analytically fully developed solution of a problem;

Conclusive intuitions are the post-solving-feelings that summarize a global unique, internally structured sense regarding the strategy that was used for the solution.

Affirmatory intuitions are cognitions which are directly and confidently accepted as being self evident. Fischbein presented two main types of affirmatory intuitions:

Primary intuitions are intuitions that «develop in individuals independently of any systematic instruction as an effect of their personal experience» (Fischbein, 1987, p. 202)

Secondary intuitions are intuitions «which are acquired, not through natural experience, but through some educational intervention» (Fischbein, 1987, p. 71).

Fischbein and his colleagues investigated students' intuitive knowledge of various mathematical notions in a large number of studies (e.g., Probability, Combinatory, geometry, set theory). The reported results usually related to students' primary intuitions and pointed to a wide range of alternative conceptions as well as to possible sources for their occurrences. Several follow-up studies suggested interventions aimed at promoting students' knowledge, and many showed the persistent nature of the primary intuitions, that are still apparent with gains of formal knowledge.

Fischbein put forward many didactical implications of his theory to mathematics education. Here we refer mainly to the section on didactical implications in the concluding chapter of Fischbein's 1987 book. In this chapter, he called for the promotion of formal and algorithmic knowledge, while being sensitive to intuitive ideas. He criticized a common instructional practice in mathematics education that tends to provide the learner with a certain amount of information (principles, laws, theorems, formulas) and to develop methods of formal reasoning adapted to the respective domains (1987, p. 206). He argued that such a 'formal-only' approach to instruction ignores the intuitions that deeply influ-

ence one's mathematical and scientific knowledge. He urged teachers to take account of these intuitive forces in the instructional process.

When discussing instructional approaches that are sensitive to intuitions, Fischbein posed a central question: «Should one encourage the use of intuitive means in science and mathematics education?» (1987, p. 211). He then explained that intuitive interpretations are «a sine-qua-non component of every productive reasoning endeavor» (1987, p. 211).

Fischbein advocated that mathematics education should not aim at eliminating intuitions but rather to develop new, adequate, secondary intuitive interpretations that are consistent with the formal structures of logical reasoning. He described several components of attempts to develop secondary intuitions, including, raising students' awareness of the role of intuition in their thinking processes; creating cognitive conflict and developing metacognitive abilities; experiencing practical activities, and introducing the formal meaning and the formal content of the concepts as early as possible.

In a posthumously published article, Fischbein summarized the main educational implications of his work:

Generally speaking, for the teaching of mathematics, it is very important that the teacher understands the interactions between the intuitive, the formal and the procedural aspects in the processes of understanding, remembering and problem solving. If the intuitive forces are neglected, they will, nevertheless, continue to influence the pupil's capacity of understanding and solving, but unfortunately, in an uncontrolled manner, usually disturbing the mathematical thinking process. If the formal aspect is neglected and one would tend to rely exclusively on intuitive arguments, that which will be taught will not be mathematics (1999, p. 28).

Fischbein's position regarding the role of research in mathematics education appears clearly at the end of his book (1987):

It seems to me beyond any doubt that it is inadmissible to introduce new concepts in school programmes, especially in science and in mathematics without a thorough preliminary psychological and psycho-didactical investigation. This is probably the main lesson that may be drawn from our analysis. Too much time and energy have been wasted, too many mistakes have already been made with new inadequate projects (1987, p. 214).

This lesson led to a series of studies that have been conducted in the last thirty years, many of which are aimed at studying the role of intuition and rigor in the learning and teaching of central mathematical concepts. One such example is our own work on the concept of infinity (e.g., Bills, Dreyfus, Mason, Tsamir, Watson, & Zaslavsky, 2006; Dreyfus & Tsamir, 2004; Fischbein, Tirosh, & Hess, 1979; Stavy & Tirosh, 2000; Tirosh, 1990; 1991;

Tiresh, Fischbein, & Dor, 1985; Tiresh & Tsamir, 2004; 2006; Tsamir, 1999; 2003; Tsamir & Dreyfus, 2005; Tsamir & Tiresh, 1992; 1994).

5. Summing up and looking ahead

We started our plenary with the telling story on the debates and agreements at ICMI regarding the roles that intuition and rigor should play in mathematics education. We then describe the significant contribution of Fischbein to the discussion of this issue. These parts of our plenary uncovered the importance and the complexity of the issue at hand. We would like to bring this plenary to a close by re-emphasizing the indispensable role that psychological and psycho-didactical research should play in mathematics education. The critical role of such research is beautifully expressed by Fischbein:

In order to cope successfully with the instructional problems, one has first to have a good, serious understanding of the psychological aspects of the concepts involved. Much more research is needed in this respect. We have to know what are the tacit interpretations the student attached to these concepts, what are the intuitive reactions, the intuitive models he produces, the impact they may have on the acquisition of the new concepts. On the other hand, we have to evaluate the effect of the various didactical means on the complex and labile relationships between the intuitive loading of the concepts taught and their formal structure. An inadequate strategy may destroy the productive interaction of these two components (1987, p. 214).

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Reaction to D. Tirosh's and P. Tsamir's plenary talk
The development of mathematics education as an academic field

Once, having announced to him that I didn't see why an assertion was true, [...] he stopped all of a sudden (we were in the middle of one of our habitual walks) and [...] he twirled his walking stick pointing it at a little dog on a windowsill, saying to me: "you don't see? For me it is as if you were telling to me that you don't see that little dog!"

My main reaction to the stimulating talk by Tirosh and Tsamir has been to recall how the problem of the relations between intuition and rigor in mathematics was a compelling one for Italian algebraic geometers during the first years of last century. So I decided to begin with a citation from one of the best students of Federico Enriques, Fabio Conforto (1947, p. 227), who recalled the many scientific interviews with Enriques.

No wonder that Italian algebraic geometers (mainly Cremona, Segre, Castelnuovo, Enriques, Severi and more recently Chisini, Campedelli and Brusotti), who worked on their deep research problems with a tight interplay between heuristic intuitive and logical methods, were fascinated by didactical problems and particularly by those educational issues linked to the main theme of this talk. To have a wider look on their didactical contributions I refer to (Giacardi, 2003; 2006), Gario (2004), and Menghini (2006).

I would like to stress now that we are confronted with a very subtle problem in epistemology of mathematics. Indeed, what exactly do we mean by 'intuition'?

I think that we must accurately distinguish between intuition in the mathematical sense and intuition in the common sense. In some way the main effort of mathematical education could be identified in shifting from 'common sense intuition' to 'scientific' intuition. This is a very old educational problem. Socrates in the dialogue *Meno* makes intuitive for Meno's slave what in the beginning was not intuitive: how to double the size of a given square.

In this famous dialogue, the boy presumes to know something, but he really doesn't. Socrates' aim is to *reverse* an intuitive (but wrong) idea, to construct new, 'truer' evidence. It is worth noting that Socrates does not go through a formal proof. He doesn't want to substitute intuition with logical reasoning. He wants to shift from common-sense intuition to a

new, 'truer' intuition of the relations between side and area of the square. He makes the well known drawing of the double square. The boy should now have reached a new, deeper intuition of the problem, not through a formal proof, but through a new way of looking at it.

Socrates' drawing is equivalent to Enriques's little dog. The student must see it as the teacher does, and this is possible only by looking at it in a different way. The didactical methodology of Socrates (and of Enriques) is at once transmissive and active.

We may look at this example bearing in mind Fischbein's point of view cited in the talk: «the mathematics education should not aim at eliminating intuitions but rather to develop new [...] intuitive interpretations that are consistent with the formal structures of logical reasoning». I find that this statement indicates a key concept for mathematics education. As I tried to show, it is a very old pedagogical problem, and one that is also very important epistemologically and historically. I want to remind you of another well known example, i. e., Galileo's mental experiment. Galileo was confronted with what we may call an 'intuitive misconception': the intuitive idea that we may always distinguish whether we are at rest or moving. His words are very famous, and I think that everybody remembers them. In these pages Galileo effects an epoch-making transition from a 'common sense' intuition to a 'scientific' one, which constitutes a precondition for the development of modern science. I would like to stress the strict analogy between physical 'mental experiments' (besides Galileo, I may also cite Einstein in this context) and the mathematical creation of Fischbein's figural concepts (Fischbein, 1993).

In many ways, mathematics becomes intuitive only by destroying 'common sense intuition', or as Fischbein said (cited by Tirosh and Tsamir), by passing from primary to secondary intuition. But, as he says, «the persistent nature of primary intuitions are still apparent with gains of formal knowledge». This is another key issue in didactics of mathematics: we want secondary intuitions become persistent, and to substitute completely the primary ones (when they are linked to misconceptions).

This is not an easy task. In many cases formal concepts simply superpose to primary intuitions without modifying them. Everyone knows many instances of that: even if a student can give the right definition of the height of a triangle, when required to draw it, he or she can fail and draw instead a simple vertical line. Formal definition has not yet become a mental image, has not attained the status of intuition. We can guess that if, some years after the dialogue with Socrates, Meno's boy were asked how to double a square, he would have answered again 'by doubling the side'.

If mathematics is also a language, we may learn something from the didactic of the linguistic disciplines. Learning grammar cannot come before speaking. One must speak mathematics correctly before studying its for-

mal structure. It is useless to try to teach formal mathematics without having created the need for it through correct intuitions.

There is a dialectical interplay between the development of formal reasoning and the creation of higher-level intuitions. We cannot pass to a higher standard in either of the two different kinds of mathematical knowledge without also advancing in the other.

A fundamental role of logical and formal mathematics is to shed light on concepts that lay hidden; to guide us to look (with fresh eyes) at them to discern their intimate intuitive essence. Mathematical (and more generally, scientific) ideas are not part of common-sense notions: they may become intuitive, but only through a guided path; and rigorous, formal language gradually becomes a fundamental tool for this aim.

As I am an historian of mathematics, I will refer to some historical examples of the introduction of new ideas.

One such example is the idea of a one-sided surface. The existence of this kind of surface has no intuitive background. But it would be very difficult to convey the very idea of it through formal reasoning. Instead, it is easy enough to build a Möbius strip and work with it. Only after grasping this new intuition of spatial relations may we reconsider our old intuitive idea of surface and construct a mental image and the formal machinery necessary to deal with this concept.

An interesting example in this sense comes to us through the history of the idea of hyperspace. Again, we start without any intuitive background for this idea. In this case, I think intuition comes directly from formalization and from analogy. I will be very sketchy in this context.

We may start from geometry. Geometrical problems may be algebraized in the style of Viète and Descartes to give rise to a new, more efficacious way of solving them. In a geometrical context, we suffer many limitations: if algebraic objects are intended only as disguised geometrical segments we have no need of negative numbers; if the multiplication of segments is intended as area and volume we have no need to exceed third-degree equations; if equations in n variables are intended as loci for curves we have no need for more than three variables.

But algebra is not only a tool for geometry: it has its own life, its own problems and applications. So it can develop its own formal methods, free from the geometrical interpretation.

We know that geometrical intuition, whenever possible, is a great aid to our algebraic thinking. So we may try to ape geometric language even when a geometric interpretation is not yet allowed. Cauchy made, even if timidly, the first step:

Let us imagine now that the number of variables [...] becomes more than three. Then each system of values of x, y, z [...] will determine what we will

call an analytic point [...]. The consideration of the analytic loci is highly suitable for guiding the calculator through the difficulties that I have just mentioned (Cauchy, 1847, pp. 885-886).

At first it is the analogy of the formalism that suggests the use of an analytical trick to guide our intuition. Some years later, Cayley goes beyond Cauchy:

The science presents itself in two ways: as a legitimate extension of the ordinary two and three dimensional geometry and as a need in these geometries and in analysis generally. To use such a representation we require the geometry of such space (Cayley, 1870, p. 51).

There are two important mathematical events taking shape during those years. As Cayley tells us, it is now completely legitimate to extend (by analogy?) ordinary geometry to higher spaces, and we must build a representation of this new geometry.

In some way mathematicians created a new geometrical world, but the wonderful thing is that they were also able to create new tools to grasp even on an intuitive plane this new 'artificial' world. Once again Poincaré explains clearly the important role played by intuition and analogy in the development of mathematical creativity:

Is that to say that, having left this restricted domain in which they seem to want to enclose us, we must only count on pure Analysis and that every geometry with more than three dimensions is void and without an object? But what can be the use of this? It is easy to see: it gives us above all a very convenient language, which expresses in concise terms what ordinary analytic language would express with longwinded phrases (Poincaré, 1908, p. 180).

Now historical considerations may be merged with the didactical. To get 'associations of fertile ideas' the mathematician – exactly in the same way as the student – must have not only formal definitions of the objects of his study, but intuitive mental representations of them as well. The story is well known. To get a mental image of hyperspace we must first understand the world of lower dimensions and how it is interrelated with our three-dimensional world. In a letter to Helmholtz, Beltrami expressed in 1869 a clear stand in this matter:

in this method [i.e., Gauss's theory of surfaces], the relationships between surface and the surrounding space escape entirely: the surface is considered in itself, as it would be if it were observed by someone who had no notion at all of the third dimension (Boi, Giacardi & Tazzioli, 1997, p. 204).

Mathematical imagination has now created not only new spaces but also new beings! Fiction and didactics will soon use these kinds of complex mental images. In fact in 1884 reverend Abbott published his famous romance *Flatland*.

It is worth noting that James Sylvester immediately understood the educational relevancy of such kind of fiction. In fact, in the same year, 1884, he wrote to his friend Cayley, «I recommended to my hearers to procure *Flatland* [...] in order to obtain a general notion of the doctrine of space of n dimensions» (Parshall, 1998, p. 255).

Thus, to transfer an highly abstract concept to his university students even a pure algebraist (a great ‘calculator’ we may say) feels the need to create a new kind of spatial intuition in his listeners!

I have, perhaps at too great a length, concentrated my attention on this case because it seems to me that the introduction during the 19th century of new geometries and of new mathematical objects (think of complex numbers, for example) shows very well the interrelations between intuition (geometrical or analytical, it doesn’t matter), formal methods and analogy in the development of mathematical ideas. This is true in scientific research as well as in didactical matters.

Basing themselves on those kinds of problems, Italian geometers were able to develop a complete analysis of the role of intuition in mathematical education.

Obviously it was not without harsh debates among mathematicians that these kinds of ideas went on. In Italy in the beginning of 20th century we had a bitter quarrel on the role of rigor and intuition in teaching mathematics, in many ways similar to the controversy between the advocates and the opponents of the ‘new math’. Giuseppe Peano was one of most distinguished advocates of rigor and formal language in teaching mathematics. I will not go on this discussion, but refer to the extant literature for more information about Peano’s stand on didactic of mathematics.

I have not spoken of the interesting point of view of Tirosh and Tsamir on students’ conception of infinity. I also found this part of the talk very stimulating, but I will react only saying that my personal experience largely agrees with their analysis. I want only to add another sense in which we could explore students’ conception of infinity. I refer to the idea of point at infinity, in connexion with the problems relating to perspective.

I have experimented with many students regarding the study of perspective, with and without the aid of computer software. I will only give a short qualitative summary of the remarks connected with the object of our conversation. In short, any approach with point at infinity as the ‘point where two parallel lines intersect’ was clearly unfruitful. Philosophical discussions were also useless. Students became acquainted with this abstract

notion only through the actual manipulation of images related to the problems of plane representation of a spatial scenery. The capacity to draw three-dimensional landscapes came before the actual understanding of the theoretical meaning of what they were doing. All this seems to me to confirm the ideas expressed in the talk. I will conclude by underlining also the role that psychological and educational research plays in mathematical education. Enriques was already well aware of this role and he wrote to his friend Castelnuovo in 1896:

Now I am engaged with another problem [...]: when you learn its name you will feel more horror than surprise. It refers to the philosophical problem of space [...] Books of psychology, logic, physiology, and epistemology pass through my table (Bottazzini, Conte & Gario, 1996, p. 260).

I conclude by saying that in educational work, the teacher's intuition also plays an important role. Educational research must help us to shift from primary to secondary intuitions and «should not aim at eliminating intuitions but must preserve their role». If not, the dissection of methodological issues could interfere with the human relations between teacher and student, impairing our effort to transmit mathematical knowledge and culture to new generations.

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*Perspectives on the balance between
applications and modelling and 'pure'
mathematics in the teaching
and learning of mathematics*

This paper looks at mathematical applications and modelling in the context of the varying ultimate ends and goals of mathematics education at large, as a function of time and place. The nature of mathematical applications and modelling, and their different purposes and roles in other fields and areas of practice, are presented and discussed. Against this background the dual role of applications and modelling - as an end or as a means - in mathematics education is analysed, as is the historical development from the use and study of models to the process of modelling. The paper concludes with remarks on applications and modelling as an area of research and development, in particular in relation to ICMI.

1. Changing goals of mathematics teaching and learning in relation to applications and modelling

The teaching of mathematics, at any level, has always been offered to its recipients in order to pursue certain goals. More often than not, these goals are implicit or taken for granted under the circumstances present. Sometimes, however, goals are formulated explicitly, e.g. in curriculum documents, in syllabi or in the introduction to a specific teaching unit. Oftentimes, the different stakeholders and agents in mathematics education - e.g. politicians, curriculum authorities, mathematicians, teachers, parents, employers, and, above all, students - advocate, pursue (or at least accept) different, if not conflicting, explicit or implicit goals of the teaching and learning of mathematics. Different educational levels give rise to different *kinds* of goals. The goals of the teaching and learning of mathematics in primary school are normally rather differ-

ent in nature from those of the teaching and learning of mathematics at the first year of university for, say, engineering students or mathematics majors.

Whether explicit or implicit, the goals of mathematics teaching and learning at a given time reflect the perceived actual or potential needs of different kinds of target groups at various educational levels. What are the projected educational trajectories and the societal places and roles of members of these different target groups, and what mathematical backgrounds and competencies should they, therefore, possess?

The answers to these questions vary with time and place in response to cultural, political, socio-economic, and technological conditions, trends and challenges, and with the evolution of the structure and organisation of the education systems and the institutions in which mathematics education has been institutionalised.

More specifically, in some periods - let us call them *utilitarian* periods, for short - the emphasis in mathematics education is on preparing students for societal practices and jobs. The pragmatic demands of society at large, of employers, of everyday life etc. are in focus. Mathematics is taught so as to be useful for various sorts of extra-mathematical purposes. In other words, the utilitarian aspects of mathematics education form the centre-piece of its justification, and exert an influence on the way curricula are designed, textbooks are written, teaching is planned and carried out etc. Of course, a utilitarian focus of mathematics education does not relegate mathematical substance, knowledge and skills to being unimportant - if they were, there wouldn't be any mathematical competence at all to put to external use. What it often does mean, however, is that in such contexts the importance of purely mathematical considerations are derived from the primary purpose of usefulness. What it also may mean is that mathematical subject matter which cannot be linked to external use comes under attack, if it is not simply being removed from the curriculum.

For an illustration, let us consider the development in the USA in some detail. In their chapter (9) *The Ascendancy of Practical and Vocational Mathematics, 1893-1945: Academic Mathematics under Siege*, in *A History of School Mathematics* (Stanic & Kilpatrick, 2003), Kliebard and Franklin (Kliebard & Franklin, 2003) describe the utilitarian tendencies which dominated US school mathematics education during the first half of the 20th century.

The end of the 19th century saw a beginning emphasis on the application of mathematics to commerce and business (p. 402) along with attacks on previous notions of mathematics as a means of fostering mental ability (p. 403). Doubts about the usefulness of mathematical knowledge beyond elementary arithmetic for 'the masses', were raised again and again (p. 403-404). While the period 1890-1910 saw a marked absolute and relative increase in the number of students taking high school mathematics, the utilitarian focus gained

momentum in the 1920's: «industrialization brought new urgency to the idea that schools, more than anything, needed to prepare citizens directly for their role as producers and consumers of goods» (p. 405). And «The curriculum as a whole was now being *vocationalized*, and vocationalization proved to be inimical to the status traditionally granted to subjects like algebra and geometry in the school curriculum. Those subjects now faced the daunting task of justifying their place in the curriculum according to their applicability to the workplace or their usefulness in the day-to-day lives of future citizens» (p. 405). Ideas such as these were also behind the so-called 'real problem movement', which was given particular momentum in and by the National Education Association. This movement emphasised the «functional value in dealing with practical problems of everyday life» (p. 407) for school subjects in general and for mathematics in particular. The focus on the practical use and functionality of mathematics education was seen to be consistent with democratic ideals as opposed to the favouring of intellectual and elitist perspectives, something which was referred to as 'scholastic Kaiserism' (pp. 410, 413, 414). Around 1920, calls were made for thoroughly reconsidering the mathematics curriculum so as to identify what mathematics adults actually use in their private and occupational activities (p. 415-416).

During the depression in the 1930's the voices requiring social efficiency became increasingly louder, especially as high school enrolments expanded substantially, simply because there were not enough jobs for the youth who in former times would never have found their way into upper secondary education (p. 420-421). Instead, large new segments of students now went to high school. As the majority of these students were not college-bound, practice and vocation oriented functional mathematics courses (or 'shop mathematics', p. 422, or 'life situations', p. 424) were introduced in a great many places: «practical and vocational mathematics courses were not simply being introduced alongside the traditional ones; the value of academic mathematics was being openly questioned, and this was demonstrably reflected in school practice» (p. 426). This development gained further momentum during World War II, in fact transgressing the borderlines to sheer anti-intellectualism (p. 427), where «Academic mathematics [...] was singled out for special criticism» (p. 428), even though voices stating that a substantial knowledge of mathematics was essential for military people of all ranks, even the lowest, were heard as well (p. 431).

The utilitarian focus of US mathematics education was maintained after World War II, but the emphasis was now slightly more of a rhetorical nature. Taking off in the late 1940's, and gaining momentum in the New Math reform movements of the 1950's and 1960's, calls for serious attention to mathematics proper (albeit still in different ways for different groups of recipients) were made. As Garrett and Davis (2003) put it, «criticism of the perceived lack of rigor in postwar American schools, including mathematics education, began

early following the war and continued throughout the remainder of the century» (p. 511) and «as early as in 1948, in fact, university mathematicians and mathematics educators had begun to complain about what they viewed as the problems of pre-collegiate mathematics education» (p. 513). In summary, the United States, in the first half of the 20th century, provides a prime example of a strong and most explicit utilitarian focus in school mathematics education.

Utilitarian views of mathematics education have returned onto the stage in several countries from the late 1970's and onwards. A possible explanation for this may be a combination of the worldwide economic recession that began in the 1970's and the subsequent ultra-neo-libertarian developments that took off around 1980 and gave way to the current form and state of economic and political globalisation. A common feature of these developments is that 'the market rules' and such a rule tends to favour things that are effective in the market place and to discard things that aren't. This tends to permeate education as well, as things deemed 'useful' prevail while the rest wither.

In other periods - let us call them *introvert* - the primary emphasis is on providing students with mathematical knowledge and skills in and of themselves, because mathematical insights proper are being perceived as major cultural and scientific accomplishments of mankind, the transmission of which, to various categories of recipients, is meant to maintain and further human civilization and culture. Moreover, the dissemination of mathematical knowledge and skills to students is supposed to foster their intellectual, personal, moral and aesthetic development as human beings. While the intrinsic qualities and values of mathematics as a discipline are in focus in such periods, this does not imply that no importance is being attached to the external usefulness of mathematics, either as an illustration of the widespread role of mathematics in the world or as a facilitation of learning. However, this is seen as a derivation of the primary purpose of transmitting mathematics to new generations.

For instance, the years from the mid-1950's throughout most of the 1960's were a period in which mathematics as a theoretical edifice, built on set theory and on 'mother structures' defined in set theoretical terms, was given massive attention in a great many countries that embarked on the New Math reform, including Belgium, Denmark, France, and the USA (cf. Damerow, 1977, and Skovsmose, 1984). Other countries (e.g. Norway, cf. Gjone, 1983, and Iceland, cf. Bjarnadottir, 2006) joined this reform movement only in the late 1960's or early 1970's, while still other countries (e.g. Hungary) never enlisted in it, and opinions were strongly divided between different quarters in others (e.g. the Soviet Union). It is worth noticing that despite the strong theoretical orientation of the New Math movement, its founders insisted that one of the points of the reform was to provide an ideal platform for dealing with the application of mathematics to matters extra-mathematical (as stated by the former President of ICMI, Marshall Stone, in OECD, 1961).

Within the teaching and learning of mathematics in itself, there is a delicate issue of balance between establishing a purely mathematical edifice of concepts, methods, techniques and results, on the one hand, and demonstrating the extra-mathematical use of mathematics, on the other hand, either to support students' gaining of experience and construction of interpretation and meaning of mathematical entities, or to motivate the study of mathematics. Taken together these two aspects give rise to *pendulum swings* between an emphasis on the pure, theoretical aspects of mathematics and an emphasis on the extra-mathematical use (fulness) of mathematics to other disciplines or areas of practice.

Mathematical applications and modelling in mathematics education are situated in these general movements, trends and pendulum swings at large and are influenced by the way balances are being struck at given times. It appears that the place and role assigned to applications and modelling will reflect such trends and waves. For instance, applications and modelling tend to occupy a more prominent position in the teaching and learning of mathematics in utilitarian periods than in introvert periods.

Looking at the role of applications and modelling in different countries, it is evident that there are - and always have been - major differences amongst countries. In some countries, such as the UK, Australia, Germany, the Netherlands, and Denmark, applications and modelling often have - but to varying degrees, and in very different ways - played a sizable part in the teaching and learning of mathematics, also at post-elementary educational levels. In other countries, such as the East Asian countries, Latin America, Italy, Spain, Finland, and Greece, applications and modelling have, traditionally, occupied a relatively minor position in mathematics education, even though individual institutions or programmes may provide counter-examples to the general picture. There is reason to believe that these differences have deep roots in the basic perceptions of the purpose, nature and role of mathematics as an educational subject in different countries, which in turn are likely to reflect varying societal and cultural circumstances.

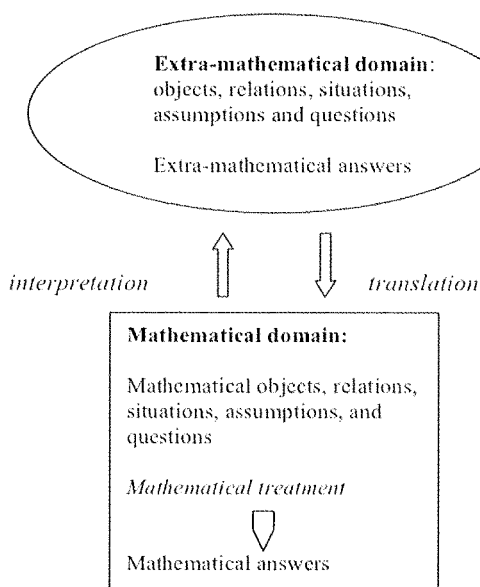
2. The nature of mathematical applications and modelling

In order to provide a foundation for the rest of this paper, in this section aspects of the nature of mathematical applications and modelling will be introduced and discussed.

Whenever mathematics is being put to use to deal with aspects of some extra-mathematical domain, we are faced with an *application* of mathematics. In any such application, a mathematical *model* is necessarily involved, either explicitly or implicitly. This is so because the very reason for involving mathematics in extra-mathematical contexts is to employ it as a means for

answering practical or academic questions pertaining to the extra-mathematical domain. And such an activation of mathematics to a domain outside mathematics itself requires selected objects, phenomena, situations, problems and processes pertaining to this domain to be given some kind of mathematical representation. In the literature one can find a multitude of structural diagrams, some of which are very elaborate, to depict what is supposed to be going on. For the purpose of this paper, the simplest possible diagram will suffice (see, e.g., Niss et al., 2007).

Suppose we are dealing with some extra-mathematical domain from nature, technology, culture, society, everyday life, or from some scientific or scholarly area (such as physics, biology, medicine, economics, geography, psychology, sociology, to name just a few). In this domain, we are interested in certain phenomena, issues or problems that involve particular objects, relations, situations, assumptions etc., as well as questions pertaining to them. The relevant items are often depicted within some drawn frame, for example an ellipse, representing the domain at issue. These items are then *translated* into mathematical objects, relations, situations, assumptions and questions chosen from some mathematical domain (e.g. numbers, geometry, functions, algebra, probability, calculus). We have chosen to represent this domain by a rectangle. As the crucial point is to employ mathematics to answer the extra-mathematical questions motivating the whole enterprise, these questions, too, have to be translated into mathematical questions. The full translation process is represented by the right-hand, downward arrow in the diagram below.



Once the mathematical questions to be answered have been identified, a *mathematical treatment* has to be undertaken in order to obtain answers to these questions. This will typically involve solving mathematical problems by activation of established results, symbolic manipulations and calculations, numerical computations, mathematical reasoning - including proving of statements - and so forth and so on. The treatment within the mathematical domain, giving rise to answers to the mathematical questions, is depicted by an arrow within the rectangle that represents the mathematical domain.

When answers to the mathematical questions have been obtained, they have to be *interpreted* - translated back - into answers to the extra-mathematical questions. The interpretation process is depicted by the left-hand upward arrow in the diagram.

What, then, do we mean by a *mathematical model* in this context? I propose to define a mathematical model as a triple (D, M, f) composed of the extra-mathematical domain D , the mathematical domain M , and the translation, f , that 'maps' (aspects of) the extra-mathematical domain into the mathematical domain (Niss, 1989). This conception makes the relationship between the extra-mathematical domain and the mathematical domain explicit. Sometimes, in everyday usage, people use the term 'mathematical model' to refer to the collection of mathematical entities only - ' $f(D)$ ', so to speak - even though this downplays the role of the extra-mathematical domain. The process of creating a mathematical model is traditionally called *mathematical modelling*. Accordingly, the flow represented by diagrams such as the one above is often referred to as the *modelling cycle*.

The origins of specific mathematical models are rather diverse. Sometimes a model is straightforward, obvious or even trivial. This is the case, for example, when the amount of money to be paid for the items in a shopping basket is determined by adding the individual prices, or when the number of chairs needed to arrange a lecture hall of 28 rows with 18 chairs in each row is determined by multiplying 28 and 18 - the underlying models are indeed trivial but, nevertheless, models they are. Models dealing with entities that are, so to speak, 'born' with mathematical features and structures, such as money, measurement, weight, calendars, geographical coordinates, train timetables, and so forth and so on, typically are of this category. The same is true of many models dealing with everyday physico-geometrical objects. Today, such established models - which are usually not made explicit in actual practice - belong to what is sometimes called 'cultural techniques'. When they were first introduced, centuries or millennia ago, they were indeed results of frontline research modelling, although that term was, of course, not used at that time. However, for us, today, the underlying modelling cycle is implicit or hidden.

In other contexts there is no established or straightforward model that covers the situation to which mathematics is being or may be applied. Thus,

in order to activate mathematics in the situation under consideration, a model has to be built for the specific context and purposes at issue. This involves the full modelling cycle to be run through by someone who wants to introduce a mathematical model to deal with the situation. It should be emphasised that building a model does not have to happen from scratch. Oftentimes, a model construction consists in borrowing, transposing, modifying or extending existing models from another domain or context, and importing it into the one under consideration.

So, whether we are in a situation where an established standard model applies, or in one not covered by any such a model - which then requires a modelling process to take place - the full modelling cycle is actually implicated. This does not mean that a person applying mathematics to the situation necessarily has to be engaged in, or even to be aware of, the full modelling cycle. Sometimes, it is possible to enter the scene at a stage when a model has already been introduced. This is especially true if we are talking about the teaching and learning of mathematics, where it is often the case that students are supposed to work solely on the mathematical treatment of a given model. Under such circumstances, the genesis of the model is not part of the educational agenda. In contrast, under circumstances where students are supposed to be engaged in the very generation of a model, things are, of course, very different.

In summary: Any application of mathematics relies on the introduction of a mathematical model, whether explicit or implicit. Any mathematical model has to be constructed by someone by means of mathematical modelling, in principle involving the full modelling cycle. People may enter the game either at a stage when a model is already in place, explicitly or implicitly, or at a stage where active modelling is required. It varies with time and place which of these aspects are on the agenda in the teaching and learning of mathematics. We shall return to this issue in Section 5.

3. The different purposes and roles of applications and modelling in other disciplines and areas of practice

The previous section briefly outlined what is meant by an application of mathematics, by a mathematical model, by the modelling process and by the modelling cycle. We also mentioned that the overarching purpose of constructing and employing mathematical models is to help seek answers to practical or academic questions, in a broad sense, concerning some extra-mathematical domain or context. In this section we shall take a closer look at the different purposes and roles of mathematical applications and modelling.

Basically, mathematics is put to use in other disciplines or areas of practice for three different kinds of purposes:

- in order to *understand* (represent, explain, predict) parts of the world, i.e. aspects of some extra-mathematical domain,
- in order to subject parts of the world to some kind of *action* (including decision making and problem solving),
- in order to *design* parts or aspects of the extra-mathematical world (creating or shaping artefacts, i.e. objects, systems, structures).

These purposes are pursued by constructing or utilising mathematical models that allow us to *describe* or *prescribe* aspects or parts of some domain.

For example, to illustrate the first purpose let us assume that we want to understand observed empirical data showing oscillations of two populations of predators and prey, respectively, and to predict whether both populations may remain in existence in the long run. A model involving a system of two differential equations representing the instantaneous growth rates of the two populations as functions of their current sizes can be a vehicle for understanding the mechanisms of interaction between the two populations, while an analysis of the stability properties of the solutions to the equations will lead to conclusions - modulo the model, of course - concerning the long term development of the two populations.

For an example of the second purpose, imagine that we were to organise an excursion for a large group of people and had to decide between a number of different transport and accommodation schemes, involving costs based on different combinations of rates. A cost optimisation model involving the different schemes may then be set up so as to allow for a comparison, and eventually a decision, between the schemes. Such a model may be supplemented with another model of, say, the duration of the rides involved, thus giving rise to fine-tuning of the cost-based decision making.

For a final example, illustrating the third purpose, assume that we want to design two straight cylindrical metal plate ventilation pipes that are to be joined at an angle of 120° , in such a way that the two pipes can be assembled just by welding them together along a common cross section; how should the plane metal plate be cut so as to meet the specifications indicated? A suitable combined 2D-3D geometrical model of plane cuts in cylinders leads to an answer to this design question.

The purpose of a model usually cannot be determined by looking at the model, i.e. (D, M, f) , itself. A given model can serve different purposes in different contexts. In fact it can serve different purposes at the same time. Nevertheless, the fact that mathematical applications and modelling serve multiple purposes implies that their roles vary across domains, contexts and situations. This is, in principle, reflected in the teaching and learning of mathematics.

However, usually in the teaching of applications and modelling, the various purposes of the entire exercise are not being articulated let alone discussed. When dealing with a specific case the purpose often seems to be taken for

granted, perhaps by being implicitly embedded in the questions or problems posed. Thus, the above-mentioned distinction between three different kinds of purposes of applications and modelling is mostly not being made in educational contexts. Nevertheless, the distinction is an important one as the purpose being pursued has a significant impact on the specific applications and modelling activities undertaken, also in the classroom, in particular as regards the attention paid to the different phases of the modelling cycle. For instance, if the purpose is to identify possible qualitative mechanisms responsible for a phenomenon observed, concrete numerical data may be largely unimportant, whereas they may well be highly important if the purpose of the activity is to guide actions or the design of material or immaterial artefacts.

4. The dual role of applications and modelling in the teaching and learning of mathematics

To the extent applications and modelling are present at all in the teaching and learning of mathematics, there are two different major reasons.

The first reason sees it as an *end* in itself that students should become acquainted with the use of mathematics in extra-mathematical contexts and/or become able, themselves, to put mathematics to use in such contexts in other disciplines or practice areas. As we have seen above this implies that students should become able to deal with at least some aspects of models and modelling.

The second reason sees applications and modelling as a *means* to support the learning of mathematics, either by providing interpretation and meaning to mathematical ideas, concepts, theories, methods, argumentation and proof, etc., or by providing motivation for the study of mathematics.

While the former reason is more likely to manifest itself in utilitarian periods than in mathematically introvert periods, the latter reason may prevail in both kinds of periods - in introvert periods, though, depending on the extent to which applications and modelling are in fact being perceived as an efficient means for learning.

This duality of the role of applications and modelling is present whenever applications and modelling are on the agenda of mathematics education. What we are faced with is a duality, not a contradiction. It is often the case that both reasons are involved in underpinning the presence of applications and modelling in the teaching and learning of mathematics. However, not only are the two reasons analytically distinct, in practice the duality between them gives rise to intrinsic dilemmas when striking concrete balances in mathematics education.

Thus, when applications and modelling constitute a goal, in its own right, of mathematics education, it follows that several extra-mathematical aspects

of the models or modelling processes at issue will necessarily have to be taken seriously, e.g. the validity and relevance of the extra-mathematical assumptions being made, gathering and cleaning of real data, identifying and scrutinising established knowledge and theories about the extra-mathematical domain, etc.. This requires time and effort to be spent on such aspects, which in turn will necessarily have to be taken from the total amount of time and effort allotted to mathematical activity, thus reducing the investment in purely mathematical work.

This fact implies that the non-negligible 'costs' entailed by making applications and modelling a goal in its own right may well lead to a rejection of the goal itself - along a line of reasoning like «it may be a very nice goal to pursue but it's so effort and time consuming that important mathematical goals cannot be pursued at the same time; hence, regrettably, we have to let go of a full-fledged version of the applications and modelling goal». Alternatively, and less drastically, instead of completely discarding applications and modelling as an independent end, the level of ambition for its pursuit may be lowered, for instance by paying greater attention to the mathematical aspects of the modelling cycle, such as mathematisation and the intra-mathematical treatment of the model(s), than to the other aspects.

The relative high cost of taking all aspects of the entire modelling cycle seriously, may further account for the fact that the second reason mentioned above seems to prevail in most of the educational contexts in which applications and modelling have gained a foothold. Although reliable statistical surveys of this matter are not available, it is only very seldom the case that the first-mentioned reason stands alone. Even if it predominates, the second reason comes in at least as a desirable additional reason, a side effect, so to speak. This is also reflected in recent research literature, e.g. Blum, 1998, and Lesh & Zawojewski, 2007. Moreover, this reason(ing) has always permeated the entire Realistic Mathematics Education tradition developed at the Freudenthal Institute in the Netherlands (see, for instance, Treffers, 1987, and de Lange, 1996).

On the other hand, when applications and modelling are primarily seen as a vehicle supporting the learning of mathematics, and not as a goal in itself, research evidence suggests that this focus does not suffice for students to learn how to apply mathematics, how to analyse models, and how to carry through modelling processes (see, e.g., Ikeda & Stephens, 1998).

5. From the use and study of models to the process of modelling

In former times, to the extent applications and modelling were admitted onto the agenda of mathematics education, for several decades the point of departure lay in already established standard models. Oftentimes the term model was not mentioned at all, and similarly no attention was paid to the

underlying assumptions involved in the foundation of the model being used or made an object of discussion or study. Simply put, in those days, in mathematics teaching and learning models were taken for granted and used, but not built, and the activity concentrated on working solely within a mathematical context derived from the model. This was also largely true of the well known British 'applied mathematics' tradition, which actually dealt with applied rational mechanics, where a number of standard models from classical mechanics were invoked and put to use in more or less complex physico-technical situations, sometimes of a rather contrived nature (Pollak, 2003).

Later on - mainly from the 1970's and onwards - the models themselves, including their foundations and properties, were, from time to time, considered in mathematics education in some places. From the same time, roughly speaking, students' active, independent construction of models - including revision and adaptation of existing models for new contexts or purposes - was made part of the teaching and learning of mathematics - again in some places. This means that also extra-mathematical aspects of the entire modelling process were given some attention in the mathematics classroom.

We may refer to this development as an evolution from the *application* of models, to the *investigation* of models, and through to the *building* of models (Niss, 2003). Henry Pollak (Pollak, 2003) has given a rather detailed account of the history of modelling in, primarily, the USA and the UK, ranging from the late 1950's to present days. In the USA an entire not-profit organisation, COMAP, the Consortium for Mathematics and Its Applications, led by Sol Garfunkel, has been active in this development since 1980. Internationally, this development has been reflected at all educational levels in the ICTMA (*International Conference on the Teaching and Learning of Modelling and Applications*) series of conferences, which was initiated by people in the so-called Polytechnics in the United Kingdom (Exeter) in 1983, in the beginning with a clear focus on the tertiary level but later with a wider scope. The conferences have been held every other year since then. The latest one, ICTMA-13, was held in Bloomington, Indiana, in 2007.

If we take a closer look at the case of Denmark, the application of mathematics has explicitly been a part of the primary, the lower secondary, and the upper secondary (high school) mathematics curricula throughout the 20th century. During the utilitarian periods of Danish mathematics education that occurred from time to time up until the mid-1970's, concrete applications, mostly in commerce, business, banking, but also in areas such as astronomy and geography were included in the curricula, in textbooks, and in set written national exams. As regards the primary and lower secondary levels, it is fair to say that a considerable amount of training for particular recurrent vocational uses of mathematics was on the agenda. However, it was never the case that any form of model-building was expected, let alone required. Furthermore, the term model was never used until around 1970. So, the underlying model

of the activities, e.g. computation of the weight of a certain amount of a given alloy, based on the weights of its components, or computation of elements in annuities, e.g. an accumulated capital, was always implicit. The focus was on the mathematical treatment of some aspect of the given model so as to obtain answers to the specific questions posed with regard to the extra-mathematical domain. In other words, all activity took place within the rectangular box in the diagram in Section 2, except that questions and answers would refer to units of the relevant quantities, thus in principle transgressing the borderlines of pure mathematics.

From around 1975, the situation in Denmark changed rather markedly, in particular at the upper secondary levels (grades 10-12), where modelling activities began to occupy prominent positions in many classrooms and were later - in the 1980's - made part of the official national curricula (Pilemann, 1996). Students performed projects or other kinds of large tasks, in which they were supposed to introduce, utilise, and analyse models to deal with extra-mathematical situations. Sometimes they worked on such tasks for several weeks, or even months. Usually, the models employed were to be selected and modified from a reservoir of known models that the students had gained experiences with. Often, students were asked or encouraged to compare the suitability of different kinds of models to a given context, e.g. some growth phenomenon accompanied by data.

Another country worth looking at is France, which used to have a rather pure orientation of its mathematics curricula. In recent years, an emphasis on mathematical modelling at the upper secondary level has been adopted in France, in particular in the so-called *Terminale S* (the science stream) (Ministère de l'Éducation Nationale, 2001). In a recent doctoral dissertation, Ruth Rodriguez Gallegos (Rodriguez Gallegos, 2007) has analysed the actual state of affairs in France with respect to differential equations modelling as reflected in three selected textbook systems and by the everyday teaching practice in a number of upper secondary classrooms. While she finds that only selected aspects of the modelling cycle are emphasised in the textbooks examined and in actual teaching practice, modelling is indeed meant to occupy a prominent position in the official curriculum, both in mathematics and physics.

6. Concluding remarks on applications and modelling as an area of research and development

There is a long tradition of research and development concerning the teaching and learning of arithmetic. As sense-making, through links between concept and operations, on the one hand, and real-life interpretation of them, on the other hand, has always been a crucial aspect of the learning of arithmetic, mathematics education researchers have taken the applica-

tional aspects of arithmetic into consideration since the beginning of the 20th century. But apart from singular exceptions in particular in Germany (Kaiser-Messmer, 1986), researchers have not made applications, let alone modelling, beyond the application of arithmetic a subject of research or reflective development until around 1970. For accounts of the earlier phases of the evolution see also Niss (2003) and Pollak (2003).

In the 1970's an internationally oriented research community interested in applications and modelling began to emerge. Applications and modelling were placed on the agenda of the ICMEs from ICME-3 (Karlsruhe 1976) and onwards, and have remained there, in various shapes, ever since. From the early 1980's the ICTMA conference series, mentioned in the previous section, have been in place. Along the road an increasing number of theoretical and empirical research papers on applications and modelling have been published in the proceedings of these conferences.

The relatively small international community gathering around the ICTMAs - a total of a few hundred people - gradually began to provide itself with an organisation. At first the organisational structure was a rather informal one, as the chairs of all past ICTMAs simply formed a body, the major task of which was to decide the location of the next ICTMA (ICTMA-14 will be held in Hamburg, Germany in 2009). When it became clear that this organisation was neither representative, efficient and democratic enough, it was decided to establish, on a more formal basis, an international community, also called ICTMA, with a membership structure and an Executive Committee, led by a Chair. Thus, today, the acronym stands for two different things, *The international Community of Teachers of Mathematical Modelling and Applications*, and the International Conference on the Teaching of Modelling and Applications, held under the auspices of the Community. Equipped with such an organisational platform, the Community decided to apply for affiliation as an ICMI Affiliated Study Group. The ICMI Executive Committee decided to grant ICTMA that status as of 2003, under the name of *The International Study Group for Mathematical Models and Applications (ICTMA)*.

A crucial step in the consolidation of applications and modelling as a sub-domain of mathematics education research was the launching of an ICMI Study (No. 14) on this theme - *Modelling and Applications in Mathematics Education* - along the now classical lines for such a study, including an International Programme Committee (chaired by Werner Blum), a Discussion Document, an ICMI Study Conference in Dortmund in 2004 (chaired by Hans-Wolfgang Henn), and finally the publication of a volume in the New ICMI Study Series (Blum et al., 2007).

It follows from the previous sections that the place and role of applications and modelling in mathematics education are rather complex and intricate. This is so because applications and modelling, by their very nature,

transgress the boundaries of mathematics as a self-contained theoretical edifice and reach out to other subjects and areas of practice. There are, therefore, at all times, dilemmas to be dealt with, balances to be struck, obstacles to be considered, and discussions to be had. The emergence and growth of research on the teaching and learning of applications and modelling in mathematics education have provided us with a much more solid ground for dealing with the intricacies outlined in this paper than were at our disposal just a few decades ago.

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Reaction to M. Niss's plenary talk
*Perspectives on the balance between
applications and modelling and 'pure' mathematics
in the teaching and learning of mathematics*

1. *Basic questions*

First, we should distinguish mathematics in school, from mathematics outside of school. 'Pure' mathematics and applications & modelling (below, 'a&m') exist outside of school. We think about the balance between a&m and 'pure' mathematics in school mathematics as shown Figure 1.

When we focus on this theme, it is important for us to ask the following basic questions:

- (1) What are the similarities and differences between the meanings of a&m and 'pure' mathematics?
- (2) Why do we need to pay attention to the balance between a&m and 'pure' mathematics?
- (3) What is the relationship between a&m and 'pure' mathematics in the teaching and learning of mathematics?
- (4) How can we set the balance between a&m and 'pure' mathematics in a mathematics curriculum?

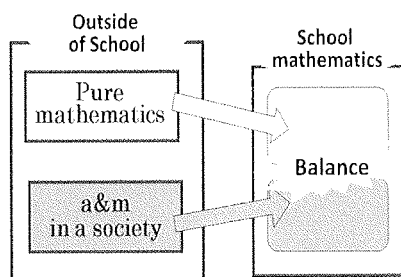


Figure 1. Image of this theme

2. *Suggestions from concepts of 'Place' and 'Time' about applications and modelling*

It is noted by Niss that any application of mathematics relies on the introduction of a mathematical model, whether explicitly or implicitly.

Therefore, any mathematical model has to be constructed by someone. Then, when and where was each mathematical model constructed by someone? It is suggested that concepts about 'time' and 'place' have an important role in the teaching and learning of a&m.

Concerning 'place', there seems to be two different aspects for where a mathematical model is constructed. The first aspect is a place where people are living, such as country, area, etc. It is obvious that the problem situations people face or are interested in differ between developing countries and developed countries. Further, even in the same country, problem situations also differ between urban areas and suburbs. Therefore, it is suggested for the teacher to select an appropriate modelling task so that students can make sense of a problem situation proposed by the teacher. The second aspect is a place where people are working on. Niss identified three different kinds of purposes to use mathematics in other disciplines or areas of practice: 1) *understand* (represent, explain, predict), 2) *action* (including making decisions, solving problems), 3) *design* (creating or shaping artifacts, i.e. objects, systems, structures). I think these three purposes are important in the following three points. The first is characterized by the question «What kinds of educational goals are emphasized?» Modelling is treated from a variety of goals, such as foundations of science, critical citizenship, vocational preparation and way of life. Therefore, it is important for us to clarify what relationships between goals and the three purposes are. Second, these three purposes imply that the modelling process depends on the purpose or area of other disciplines. For example, it seems that the diagram of the modelling process in the natural world is different from one in a society. When we introduce mathematical modelling for students, a particular diagram of modelling process is often used to let students understand roughly what modelling is. We have to pay more attention to the fact that the modelling process is different according to the purpose to use the mathematics or area of other disciplines, and we need to consider how we can introduce what is modelling. Third, these three kinds of purposes are also useful when we teach the usefulness of mathematics to students. When we teach how mathematics is used in a real world situation, one of the methods is to identify purposes to use mathematics in the real world. By tackling a series of modelling tasks, it is expected for students to reflect and find out the purposes to use mathematics in a variety of real world situations.

Concerning 'time', there also seems to be two different aspects when a mathematical model is constructed. The first aspect is problem situations people face or are interested in. Problem situations are different in past society and present society. For example, constructing a figure to measure the length or angle was important in past society, but as we now

have convenient instruments to measure, it is not important at present. It is suggested for the teacher to select an appropriate modelling task so that students can make sense from the problem situation proposed for the students. This point is similar to that presented in the first aspect of 'place'. The second aspect is the degree of acquisition of particular mathematics concepts and methods. The degree of acquisition is different between past society and present society. In the past, the particular mathematical methods which we used as a matter of fact were searched for by building up a mathematical model, but at present the mathematical methods are used with cultural techniques. This fact leads to two quite different approaches to teaching mathematical modelling. A first approach is to present a mathematical solution or mathematical model to students and let them interpret it. Students analyze given mathematical structures or models of existing real life products or phenomenon. For example, the reflector of a bicycle, different systems of interest, finding the focus of an earthquake, etc. The second is to ask students to construct a mathematical model themselves by taking account of the real problem situation. This approach is concerned with students needing to construct a mathematical model to solve a real life problem. It is important for the teacher to consider which approach is appropriate for students by taking account of the educational goals and students' surroundings.

3. Applications and modelling in Japan from a historical perspective

As Niss suggested, a&m have traditionally occupied a relatively minor position in mathematical education in Japan. This is true, but a&m played a sizable part in a teaching and learning of mathematics from the 1940's to the 1950's. We analyzed the historical aspects of teaching and learning a&m in Japan (Ikeda, 1998).

In 1943, Mathematics in Junior high school (grade 7 to grade 11) was integrated from algebra, geometry and trigonometry into *Mathematics Dai Ichirui* (type 1) and *Dai Nirui* (type 2). *Mathematics Dai Ichirui* focuses relatively more on algebra, while *Mathematics Dai Nirui* focuses relatively more on geometry. In this period, mathematics in junior high school was not taught to everyone, but was for elite students for 5 years after 6 years of elementary education. The objective to teach mathematics was to find out mathematical concepts and methods from real world situations. For example, the following problem is handled in this textbook.

I am standing on the ground and looking at the top of the tower and a shadow of it on the pond. There is a 30 degree angle between a line from my eye to the top

of the tower and a horizontal line, and there is a 40 degree angle between a line from my eye to the top of the tower of the shadow on the pond and a horizontal line. The height of the location of my eye from the surface of water is 3m. Find out the height of the tower from the surface of water to construct the figure.

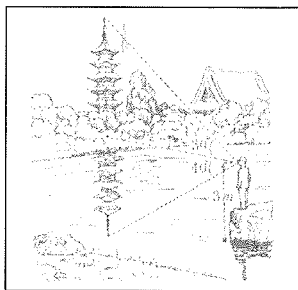


Figure 2. The height of the tower (Mathematics 3 (grade 9), Dai Nirui, 1943)

These textbooks were only used for a few years because of World War II. After World War II, compulsory education was extended from 6 years to 9 years, and the national course of study was first endorsed. This new curriculum was called the 'core curriculum'. In this curriculum, all subjects are integrated based on 'life unit study'. There were several opinions about the interpretation of 'life unit study' (Inagaki, 1994). In the opinion of Wada (1951) who was assigned to the Ministry of Education, Science and Culture at the time, the aim of 'life unit study' is to foster the ability of pupils/students to derive their own problems from the real world, then select appropriate conditions in the problem and get the conclusions which satisfy the conditions. It was he who distinguished the word *unit* from a real world problem. A real world problem in this context is one which concerns everyday life and the world around us, but is not concerned with the reason why pupils/students have to solve the problem. In 'life unit study', it was emphasized for pupils/students to clarify the reason why they have to solve the real world problem. Therefore, mathematical knowledge and skills were intended to be acquired through the students' own problem solving processes.

There were many arguments discussed to develop the curriculum based on 'life unit study'. For example, in junior high school, the following points were discussed (Shimada, et. al., 1949).

- a) What kinds of 'units' are selected in the textbook? The environment of pupils/students is quite different between urban areas and suburbs.
- b) Who sets up the initial situation (or problem), and how does the teacher develop the classroom teaching?
- c) How does the teacher distinguish the mathematical issues from non mathematical issues, and how does he/she handle them?

These issues are concerned with the methods of applying 'life unit study'. Regarding a), it was considered that the textbook was to be composed of exemplary topics, therefore teachers should select good materials according to the actual situation. Regarding b), it was considered desirable for teachers to present a good situation so that pupils/students can derive their own problem at first, and participate in the discussion after they had derived their own prob-

lem. Regarding c), various ideas were mentioned. Some said that it was difficult to treat mathematical issues in 'life unit study', and some said that teachers should present more structured situations so that students can derive the mathematical problems. Considering these arguments, a variety of approaches were tried, to create specific applications of 'life unit study'.

Several years after the endorsement of 'life unit study', it was reported that students' mathematical ability had declined, compared with students who had studied mathematics by the previous curriculum. For example, 'life unit study' was criticized in the following points (Okuno, 1953):

a) There were so many topics for the teacher to handle that he couldn't handle each topic to the full extent.

b) It was difficult for students to acquire the mathematical knowledge.

c) It was difficult for students to systematize the mathematical knowledge.

d) Students acquire only the knowledge available for a specific situation.

Eventually, it was concluded that 'life unit study' is unfit for teaching students because of the decline in the mathematical ability of students, and the curriculum was improved to target a systematic sequence of mathematics topics for students. Since then, namely from 1960's, the Japanese curriculum of mathematics has been based on a systematical sequence of mathematics topics for students (Fukumori, 1996). But there are several opinions stating that the intention of 'life unit study' was never really understood by teachers. For example, Suemitsu (1949) insisted that it was important for teachers to find the mathematical contents from students' activities, and not to guide students to the existing mathematics. He considered that teachers who criticized 'life unit study' regarded mathematics as an existing subject. Then he suggested the following points:

- Independent aspect

It is possible for teachers to foster the students' attitude to study mathematics willingly, autonomously and independently.

- Social aspect

It is possible for teachers to foster the students' ability to cooperate with each other.

- Scientific aspect

It is possible for teachers to foster the students' abilities to think about a problem scientifically.

The 'Scientific aspect' was also stressed by Miura (1949). He views mathematics education in core curriculum as three complementary activities (to mathematize, to solve mathematically, to interpret) which are related to the mathematical modelling process, and indicates the following points:

1) In the curriculum based on the systematical sequence of mathematical topics, students can study only a part of the problem solving process (mathematical modelling process). As all activities within the problem solving process (to mathematize, to solve mathematically, to interpret) are interrelated, it is

important for students to execute the full activity of problem solving (mathematical modelling).

2) With regard to pure mathematics, students should perform the problem solving process (mathematical modelling), which has an important role in developing and systematizing mathematics.

3) To realize the usefulness of mathematics in a society, it is required for students to perform the problem solving process (mathematical modelling).

From the historical perspective in Japan, as Niss noted, this is a case that non-negligible 'cost' entailed by making a&m a goal in its own right led to a rejection of the goal itself. Balance and relationship between teaching of a&m and teaching 'pure' mathematics knowledge should be considered, and teacher education is the central and commonly recognized issue.

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Bridging the gap between research and practice: International examples of success

This paper considers the pervasive gap that exists between research and practice in mathematics education and reports upon a small survey of research studies that had made an impact in seven different ICMI countries. An illustrative look around the world, with examples collected from Asia, North America, Europe, and Australasia reveals some interesting cases, both of research that has had a tremendous impact on practice, changing the experiences of students in thousands of school classrooms, and country systems that serve to encourage the transfer of research knowledge into practice. In conclusion I consider the importance of researchers paying careful attention to the issue of teacher learning in the design of research studies.

1. Introduction

An elusive and persistent gulf exists between research in mathematics education and the practices of mathematics classrooms, in many countries in the world. Indeed I would contend that mathematics is the subject with the largest gap between what we know works from research and what happens in most classrooms. For example, there is a large body of literature pointing to the value of engaging students *actively* in the learning of mathematics, teaching them in ways that require them to think and to problem solve. Yet in many mathematics classrooms across the world students are forced into a very passive role as they watch and then try to copy the methods teachers show them. Obviously there are country exceptions to this state of affairs but I think it is fair to say that in many countries the gap between the two versions of mathematics learning is striking and enduring.

As researchers we would all like to uncover something of great value to others, something that makes a real difference to children and adults in their experiences of mathematics; unfortunately few of us do. The reason is not,

I contend, because of the inadequacy of the research, or even the topic of the research but because of the relationships between the knowledge that researchers produce and the learning of teachers and other end-users of research. I will refer to teachers and mathematics classrooms – even though I recognise they are not the only sites for research in mathematics education.

This paper will report a small, purposeful survey of the issue in seven different ICMI countries. A small illustrative look around the world, with examples collected from Asia, North America, Europe, and Australasia shows that the problem varies quite considerably between different countries, as one might expect. This small-scale survey also reveals some interesting cases, both of research that has had a tremendous impact on practice, changing the experiences of students in thousands of school classrooms, and country systems that serve to encourage the transfer of research knowledge into practice.

2. A small survey

In June of 2007 I contacted several members of the ICMI international program committee and other senior colleagues and asked them to tell me of any research studies in mathematics education that had made an impact on practice in their countries. I received a very positive response, all but one of the members replied and all with very thoughtful answers. Interestingly the amount of impact that research has had promoted very different responses, with answers ranging from «I cannot think of any» to «there are so many I cannot tell you about all of them». This variety in the amount of impact research has had seemed to be due to the presence of particular policy structures and organisations in some countries, that support the connection between research and practice, two countries, Singapore and the Netherlands, were particularly interesting in this regard.

2.1. Influential institutions

Singapore has a single institution, called the ‘Centre for Research in Pedagogy and Practice’ that was established to conduct research to support education. The centre describes itself in this way:

The Centre for Research in Pedagogy and Practice is the largest educational research centre in the Asia Pacific. It was established in 2002 by the National Institute of Education, Singapore’s sole teacher education institution, and funded by the Singapore Ministry of Education. The Centre brings together researchers, educators and administrators to research, develop and implement new and innovative ways of teaching and learning. The Centre’s research will

provide the basis for educational policy and decision making in Singapore, to help our schools and students address the complex challenges of new economies, cultures and technologies.

The claim that «the centre's research will provide the basis for educational policy and decision making in Singapore» is supported by a ministry of education that seems highly receptive to the research conducted in the institute. As an example, the centre conducted a study of classrooms in 2004-5 during which they coded over 200 lessons in different subject areas, including mathematics. One of the results of the study was that researchers found that mathematics lessons were largely procedural (Mei & Yan, 2005) which prompted the ministry to implement a new approach to teaching, learning and assessment in schools. The policy initiative, called PETALS (Pedagogies, Experience of Learning, Tone of environment, Assessment for Learning, meaningful Learning content and Student-centred learning) was a direct response to the findings of the research. This direct link between a research conducting institution and government policy is unusual, and illustrates a system that few countries seem willing or able to emulate.

In the Netherlands a different type of institution exists that is also extremely influential in the practices of teachers. When I contacted Marja Van Den Heuvel she was very positive about the number of research projects that have impacted practice. This is probably not surprising given that the Netherlands is the home of the Freudenthal institute, a centre devoted to mathematics education that conducts research, designs classroom materials and implements and evaluates research findings in classrooms. The influence of research that emanates from the Freudenthal institute, founded and named after the mathematician Hans Freudenthal, who designed a teaching approach based on realistic mathematics education (RME), is widespread in and beyond the Netherlands. Van Den Heuvel gave one illustrative example of research that has impacted practice, that of Adri Treffers 'empty number line'. This is an interesting example as it is a *concept* rather than a data study that was developed from a paper given by Hassler Whitney at PME in 1985 and has now made its way into curriculum worldwide. The impact of this concept has probably been greatly enhanced by the respect of the researcher and the institute from which it emanated and which could serve to disseminate the concept through products and publications. The Freudenthal institute has links with universities across the world and research and ideas emanating from the institute have changed practice in countries as diverse as the US (for example, the Freudenthal Institute USA, at Wisconsin), and Indonesia (Zulkardi, Nieveen, van den Akker, & de Lange, 2001). Indeed it is hard to overestimate the impact of this energetic and generative centre devoted to work in mathematics education, upon practices in classrooms across the world. In some ways it seems that a cen-

tre, comprising a group of people working under the same roof, inspired by similar ideals and grounded in careful research and theory, such as the Freudenthal centre, is the ideal way for research to have an impact on practice as it is the place to which policy makers and practitioners turn for ideas on ways to improve classrooms. The reality for many researchers of mathematics education is quite different as many work as the single mathematics educator in an institution with many less opportunities to collaborate on research, to take research findings into schools, to place research ideas into curricula and other materials and generally to be inspired by the presence of colleagues with similar interests and concerns. Indeed the isolated nature of much research that does not build upon what has gone before it is one of the criticisms commonly leveled at research studies (e.g. Hargreaves, 1996).

2.2. *The curriculum route*

Probably the clearest way for researchers of mathematics education to influence policy and practice is through the design of curriculum. In some countries, such as the UK, curriculum are designed with limited if any recourse to research, but in others countries, including two of those I surveyed, research has had a substantial impact upon curriculum in very interesting ways. France and Italy provide two interesting examples because in both cases the research ideas that have influenced curriculum are extremely complex. In Italy, for example, the Scientific Committee of the Italian Mathematical Union appointed a group of experts to produce a new curriculum in mathematics from primary to high school, in July 2000. The committee was chaired by Ferdinando Arzarello, who provided me with information on this innovative curriculum. The experts, including school and university teachers and mathematics education experts, produced the 'mathematics for the citizen' curriculum. The curriculum incorporated a number of interesting elements including attention to both the instrumental and cultural role of mathematics, and the design of a *mathematics laboratory* to be used in classrooms. The laboratory is not a physical place but a set of activities based upon the theory of cognitive apprenticeship, with students learning by «doing and watching what is done in the laboratory, communicating with one another and with the experts» (Arzarello, 2007, p. 1). This is a particularly interesting use of research as cognitive apprenticeship, drawing from the theoretical perspective of situated learning, is a theory that has captured many researchers in mathematics education and is one that could have a profound impact on practice, but is also one that is fairly complex, often misunderstood and one that has rarely found its way into practice. The fact that it was influential in the design of the interesting Italian curriculum may be linked to the presence of Arzarello, an active researcher in mathematics education as well as a mathematician, as the chair

of the commission that produced the curriculum. He worked both with other researchers and with teachers who themselves work as researchers to produce the new curriculum for Italy.

France provides another interesting case of research influencing practice. Michèle Artigue and Catherine Houdement have written a paper, entitled: *Problem Solving in France: Research and Curricular Perspectives*, on the ways that research ideas have influenced the curriculum. Artigue and Houdement argue that there has been a *progressive infusion of didactic theory* into curriculum. Didactic theoreticians include Guy Brousseau, who proposed the influential 'didactic contract' (1997), Yves Chevallard (2006) who proposed an anthropological theory of didactics, and Gérard Vergnaud (1990) who has developed the theory of conceptual fields. In didactic theory:

Any problem set up in a classroom is explicitly or implicitly part of a situation seen as a system of relationships between the teacher, the group of students and mathematical knowledge, and the situation is considered the minimal unit of analysis for understanding what could be or is actually at stake from a cognitive point of view in the solving process (Artigue & Houdement, 2007, p. 366).

Artigue & Houdement give the example of the mathematical game known as 'Race to 20' – a game in which different students take it in turns to add numbers (for example, limiting the choice to 1 or 2) and the winner is the person who arrives at 20. Brousseau talks about the situation when students play the game and refine their strategy and argues that in didactical situations, students are expected to be able to test, reject, progressively adapt and refine their models and solutions thanks to the potential offered by the milieu of the situation in terms of action and feedback, without relying on the teacher's guidance, without trying to guess the teacher's expectations. A real classroom situation can generally be modelled as a complex intertwining of didactical and didactic episodes.

Artigue & Houdement argue that the move from information processing models of mathematical problem solving to more explicit references to didactic research in the 2002 revision of the curriculum give evidence of the growing influence of the research community in French policy making. The strong influence emanates, interestingly, from three researchers with highly coherent and connected theoretical perspectives.

In both France and Italy, innovative curricula, drawing directly from research results, have been developed. In both cases the presence of researchers in the groups organized to revise school curriculum enabled a positive and productive link between research and practice to take place. But researchers are not always influential in the reform of curricula, particularly when the process becomes overly politicized, as in the case of California (Becker & Jacob, 2000). And even when research influences curricula it is impossible

to assume that the research ideas find their way into classroom practices, although their presence in the curriculum is an important starting point.

I will now consider perhaps the most difficult route of all – the movement from research directly into practice.

2.3. Inputting research ideas directly into practice

Whilst there are undoubtedly many examples of research one could find from across the world that have impacted directly into schools, having been taken up by teachers themselves or through official documents, I will present here just three, from the UK, and the USA.

Informal methods of calculation - the UK

Julia Anghileri's work (e.g. Anghileri, 1989; 2006; Beishuizen & Anghileri, 1998) on informal and formal methods of calculation has had a substantial effect on children's learning in the UK. Until recently children in the UK were taught formal algorithms at an early stage. Dutch research (e.g. Treffers, 1991) in the Realistic Mathematics Education tradition, suggested that children benefit from developing their informal strategies into efficient methods with, where possible, flexibility in choice of method according to the problem set and the numbers contained in the problem, before (perhaps instead of) being taught any formal algorithms. Anghileri's used and developed this research and published it widely in the UK. In order to impact practice she worked directly with teachers and schools and published widely in practitioner journals. After some years Anghileri's findings were put into National policy in the UK. In 1999/2000 the National Numeracy Strategy (NNS) was brought into all primary schools in an effort to raise standards in all schools. It had been developed by a Task Group set up in 1997 which contained a small number of well known researchers in mathematics education (Brown, Askew, Millett, & Rhodes, 2003). Whilst the timeline set by Ministers did not allow for extensive consultation and discussion there is clear evidence that the results of a number of research studies had been incorporated into the programme for schools. The guidance in the NNS indicates that year 5 is a suitable stage in which pupils should develop written methods. This is indeed a profound change in the teaching of primary mathematics in the UK.

Mathematics Recovery – Australia

Based on a funded research project from 1992-1995 Bob Wright and colleagues developed a programme to support low achieving children in their learning of mathematics (e.g. Wright, Martland, & Stafford, 2006). The project drew on constructivist theory and a design approach to instruction and aimed to prepare teachers, through a professional development programme, to use one-to-one assessment and intervention, building on

hypothetical learning trajectories of children and encouraging autonomy and confidence. It has been taken up extensively in the USA (see www.math-recovery.org) and currently across the state of Victoria in Australia in 300 Catholic schools. As in the UK example, the research base is the RME work, but it draws also on the North American constructivist studies of the 1980s and 1990s.

I wrote to Wright and to Anghileri to ask them why they thought their research had succeeded in having a wide effect on school mathematics practice, when so many others do not. Julia described how she writes in both professional and research journals and gives talks to teachers and consultants across the country. She also offered the following:

My evidence is based on what children actually do and teachers can identify with this. It then takes them forward to what they might do if research findings are taken into account. As well as being interested in what children in their own and other schools do they are interested in what happens in other countries. Through collaboration with colleagues, particularly in the Netherlands, I am able to discuss the research backgrounds to using innovations in teaching... (Personal communication, July 23 2007).

Wright's comments on the success of the research in impacting on schools have some similarities but also some differences:

For the most part, my work involves working collaboratively with school systems rather than individual schools. NSW-DET (*New South Wales Department of Education and Training*) for example, is a system of 1,700 primary (elementary schools). I am currently working in collaboration with the Catholic Education Office, Melbourne – a system of about 300 elementary schools, on an Australian Research Council funded project on intervention at 3rd- and 4th-grade levels. A second point is that, in my view, early years teachers have both a major need to learn more about young children's number development and a significant unrealised capacity for such learning (Personal communication, July 23 2007).

Opinions about how mathematics should be taught, and what should be taught when, are quite strongly established and hard to shift, yet these two similar interventions have succeeded. Both innovations concern successful ways of enabling more children to acquire skills in number, the necessary foundation for any further mathematics development, and they are both the work of talented and able researchers who have remained in close touch with teachers and schools.

Cognitively Guided Instruction – The USA

Tom Carpenter, and Megan Franke are two of the authors of a highly successful North American study, nominated by Jeremy Kilpatrick and oth-

ers as being one that has had a great impact upon practice in the US. Intriguingly the research is another example from elementary education, which seems to be well represented as a field in the number of studies that have impacted practice. Cognitively Guided Instruction, commonly known as CGI, is a very interesting case worthy of some consideration, as the reason that the research had such a great impact upon practice seems to be that the researchers paid really careful attention to the ways teachers learn from research. The research in question identified children's mathematical strategies and placed these within frameworks that showed ways in which young children develop mathematical understandings. But the researchers did not give their frameworks to teachers for them to use in their classes, instead they organised professional development workshops in which teachers became familiar with the strategies and frameworks and were then encouraged to watch and listen to their own students in their own classrooms, in order to develop their own frameworks for student thinking. The CGI professional development workshops now take place across the United States and beyond. These programs have been well researched and there is evidence that they not only change teachers' practices in the short term, but that they substantively reorient teachers' attention towards students' mathematical thinking in ways that have a long lasting impact (Franke, Carpenter, Levi, & Fennema, 2001).

One of the interesting aspects of the CGI program of research and professional development is that the research had such a huge impact upon practice because of the researchers' careful attention to the question of teacher learning. In one of their seminal publications Franke, Carpenter, Levi, & Fennema (2001) argued that in order for teachers to learn from their studies of students' mathematical strategies they would need to *understand* and that teacher understanding, just like student understanding, was characterised by some important conditions. They proposed that teachers would need to develop knowledge from their research that was *rich in structures and connections*, so that new knowledge could connect with the teacher's prior knowledge; they said that the teacher's learning must be *generative* – and that teachers must be adding to their understanding when they were back in their classrooms, applying it to new topics and problems; and they said that teachers should be inspired to form their own inquiries, forming their own questions and learning from them. The CGI researchers considered the teacher's learning in ways that are similar to the way many of us consider students' learning of mathematics. The CGI researchers did not just try to teach teachers about students' mathematical strategies, instead they introduced teachers to their research findings and then set them upon their own paths of discovery. When 22 teachers who had attended CGI professional development workshops were studied 4 years later, researchers found that all 22 teachers had continued working with children's thinking

and that 10 of them had maintained a more profound change, creating and elaborating upon their understandings in and through their own practice (Franke, Carpenter, Levi, & Fennema, 2001).

The CGI researchers did something very interesting and highly effective. They took research findings on student thinking and they designed opportunities for teachers to learn from the research, encouraging teachers to become active inquirers. The researchers considered the teacher's learning very carefully and respectfully saying «If teachers can learn to talk to their students about their thinking, puzzle about what the responses tell them about student's understanding, decide how to use this knowledge in planning instruction and interacting with students, and figure out how to learn more about the students' thinking, then the teachers' own learning can become generative. Our goal then is to understand how teachers can make use of the opportunities provided within professional development focused on the development of children's mathematical thinking to become engaged in generative change» (Franke, Carpenter, Levi, & Fennema, 2001, pp. 656-657). In designing professional development opportunities for teachers that were aimed at creating inquiring learners the CGI researchers took their research to a level that is highly unusual, but that was clearly the reason for their great impact upon practice. The translation of their research into teacher learning opportunities no doubt required considerable time, effort and funding, beyond the levels that many researchers can access, but this does not take away from the important attention that they gave to teacher learning, that could be a feature of many more research projects than is currently the case.

3. Discussion: What can be learned from these cases?

If research is to make a difference, to impact practice in schools and other places, then I believe we have reached a point in the development of our field when we can make wiser decisions when we design research. Two issues that I will conclude with, drawing back from the seven examples I have reviewed, are those of *relevance* and the *nature of knowledge*.

3.1. Relevance

The issue of whether research is relevant to practice is obviously an important one. There has been a lot of concern about the relevance of research for teachers, and the lack of impact of research from researchers. In the UK for example, David Hargreaves, a prominent educational figure who has played a number of important roles in the UK, including a local education advisor, and university professor, issued a scathing attack on the relevance of research for teachers, in 1996, at the Teacher Training Agency (TTA). Hargreaves said

that teaching, as it is in Japan, should be a 'research-based' practice and that current research is neither sufficiently practical or cumulative to be relevant to teachers.

When I asked Julia Anghileri why she thought her research had made such an impact on practice she said something interesting, she said that: «My evidence is based on *what children actually do* and *teachers can identify* with this».

It was interesting to note that all three of the successful examples of research that had impacted practice concerned children's strategies and errors and methods in elementary classrooms. These are the things that 'children actually do' and Anghileri suggests that teachers found the work relevant because the research showed ways in which children's methods and strategies could be improved. But Anghileri goes further to say that teachers identified with the research. The word identity is important because teachers who engage with research – who take an idea and work with it to transform their practice – are undergoing a transformative process. Wenger has argued that learning doesn't only change what we know, it changes who we are and one could argue that learning from research has an effect that is equally profound. Important questions for researchers to consider as we design research may not only concern the subject matter, for example: is this important for students' learning? But also the way in which the content may engage teachers. Perhaps if we asked the question: Is this an issue with which teachers will identify? We would be more likely to impact practice when research results are produced.

When I conducted my small survey of research that has impacted practice, it was noticeable that most of the examples came from elementary education. This was surprising in some ways as much more research is done in secondary than elementary classrooms, yet the only way research impacted secondary classrooms, it seemed, was through curriculum design. This made me wonder whether elementary teachers are more willing to change their practice as a result of research findings, which may be the case as many elementary teachers are not as confident about mathematics and mathematics teaching as their secondary counterparts. When I asked Wright about why he thought his research had been taken up, he replied interestingly saying: «in my view, early years teachers have both a major need to learn more about young children's number development and a significant unrealised capacity for such learning». The 'significant unrealised capacity' he refers to, is probably quite different for secondary teachers of mathematics (or indeed teachers in higher education). I would argue, partly from this small survey, that the difficult task of encouraging teachers to think that research results would be helpful to their teaching is more difficult when the teachers are trained specialists in mathematics, and less likely to think that an outside authority is needed. This could suggest that one of the most pressing areas for reform is a change in culture in which teachers see themselves as needing to learn from research on a regular basis. An alter-

native would be the model that Hargreaves and others have called for, where the teaching profession is itself a research based profession in which teachers are encouraged to conduct their own research into matters that affect their practice.

As many writers suggest, research that involves teachers, or other practitioners from the outset, tends to be more successful at impacting practice. Coburn and Stein (*in press*) offer the example of design experiments as a type of research that is influential, as researchers usually work with or act as teachers in the change process. But the challenges of such research, while it clearly influences the teachers involved in it, are to influence other teachers who were not working on the design experiment themselves (de Corte, Verschaffel, & Depaepe, *in press*). Involvement of practitioners at the outset certainly helps with relevance, but may not be the key to widespread take-up of practice that we hope for if only a few practitioners are involved in the particular research. A more radical shift, in many countries, would involve changing the teaching profession to be one that conducts its own research. Countries that include a 'lesson study' approach to educational improvement, such as China and Japan, could be said to characterise such an approach (Lewis, Perry, & Murata, 2006). In such countries teaching is regarded as something that needs constant improvement but the initiatives and mechanisms for such improvement come from inside the classroom, rather than external policy makers or researchers. In a lesson study approach teachers work together, designing, watching and then critiquing a lesson before working together to improve it; «The observed lessons, called 'research lessons', are not regarded as an end in themselves but as a window on the larger vision of education shared by the group of teachers, one of whom agrees to teach the lesson while all the others make detailed records of the learning and teaching as it unfolds» (Murata, Lewis, & Perry, 2004).

I fully support moves to make teaching more of a research based profession, with teachers creating their own knowledge that will be highly relevant to them, although I would not support replacing university research with practitioner research as I know that many of the important studies that have changed education would not have taken place without university research. The research on situated learning, or cultural apprenticeship, for example, that has informed the Italian model of curriculum and made an incredibly important contribution that is probably still in its infancy, could not have taken place without university research. Although it seems that countries predominantly support university research or practitioner research, a mix of both approaches would seem to be a very productive way forward.

3.2. The nature of knowledge

One of the clearest insights that came through from my small survey of research that has impacted practice is that the most successful results came

from researchers who had paid very careful attention to teacher learning. Franke and Carpenter's research provides an excellent example as they turned their research results into professional development programs that encouraged teachers to become active inquirers and to continue learning from their own practice. Wright in Australia and Julia Anghileri also worked hard to make their research findings accessible to practitioners by writing for them and by working with teachers and schools intensively. More research in education would impact practice if researchers were encouraged, from the outset of research, to consider what they would design to encourage teacher (or other practitioner) learning. For example researchers could, at the outset consider whether research results could be converted into lesson plans, video records or other records of practice from which practitioners may learn.

For many years educational researchers have created new knowledge that is communicated in research journals – the gold standard for universities. But the problem with this model is that teachers not only have limited access and time to read such journals but they need a very different *form* of knowledge in their classrooms. For example, a teacher may learn from a study that it is good to ask questions rather than tell students information, but consider what she would need to do to make use of that knowledge in the classroom. At minimum she would need to decide upon the nature of the specific questions to ask, as no researcher could tell her that, also when and how to ask them, which children to ask, at what stage in the students' thinking and in association with which problems. When she was asking one student a question, she would also need to be considering and attending to the rest of the class. The knowledge that is required to handle such a complex intellectual and practical act is very different from the knowledge communicated in a journal. Teaching is a complex mix of judgments that happen in the moment, and teachers are constantly using a kind of knowledge that is both practical and theoretical, it is also both instant (it happens in the moment) and historical – drawing from a long history of ideas and knowledge. It is not easy to see a direct route from a research finding in a journal to the actions of teachers in classroom situations. Lee Shulman (1987) made a seminal contribution to education when he identified a new form of knowledge – *pedagogical content knowledge* – the knowledge that is unique to teachers and that blends practice and content. A consideration of the nature of the knowledge that Shulman defines gives important insights into the limitations of journal articles reporting research findings for teachers. Deborah Ball and others have called for researchers and professional developers to move from research knowledge to the creation of 'records of practice' from which teachers may learn (Ball & Cohen, 1999). In order to make teaching a learning profession they argue that we produce such records as sites for teacher investigation and learning. But they point out that the

records themselves are not enough, they need to be part of a kind of an inquiry curriculum.

A persistent feature of research studies that have impacted practice is a careful focus upon teacher *learning* and recognition that research doesn't need to be disseminated, it needs to be *learned from*. One of the interesting aspects of the CGI program of research and professional development is that the research had such a huge impact upon practice because of the researchers' careful attention to the question of teacher learning. In mathematics education many researchers are fluent in theories of learning and recognize the inadequacy of historical theories of learning, such as behaviourism that did not acknowledge the active role played by learners in constructing understanding (Greeno, 1997). Yet whilst researchers pay careful attention to the agency that mathematics learners have in the learning process and the inadequacy of teaching approaches that embody a *teaching by telling* mode of instruction, we have paid less attention to the role that teachers need to play when they learn from research. Indeed many researchers, myself included, who would be critical of teachers who simply present mathematics to students and expect them to learn from it without active involvement, have only presented research results as written text for teachers.

The CGI researchers considered the teacher's learning in ways that are similar to the way many of us consider students' learning of mathematics. The CGI researchers did not just try to teach teachers about students' mathematical strategies, instead they introduced teachers to their research findings and then set them upon their own paths of discovery. Theories of learning suggest that research in mathematics education will have a much greater impact upon practice if researchers, or others in the system, pay attention to the ways teachers learn and the opportunities that are provided for engagement around the research in important practices through which they will come to understand them. This task is not straightforward, not least because many of us work for universities and are judged by the research publications we produce. Indeed many universities not only do not encourage attempts to impact practice, they frown upon them, regarding writing for the media or the 'trade press' as somehow of a lesser quality. Nevertheless, if we are to create education systems that are more effective than researchers also need to create more opportunities for teachers to learn from research.

4. In conclusion

As part of this centennial celebration of the work of ICMI it is important to note that there are a number of ways that ICMI does and could help with the important task of impacting practice. ICMI is currently performing an important survey to consider the questions that teachers and policy

makers consider important, the results of which will be reported at ICME 11 in Mexico. It will be critical to give these much thought and to consider what they mean for our field. This could be a regular form of information that is collected and analyzed. In addition to this sort of survey, ICMI could also develop forums for the consideration and development of records of practice and other forms of records and actions that encourage teacher learning and teacher understanding.

Jim Greeno *et al* (1999) were critical of the model that is often offered, of research *dissemination*, a process by which expert research knowledge comes from above and is given to teachers to use. They propose that «expert knowledge is better seen as a working hypothesis that must enter a community of practice and jostle apparent knowledge until it takes root in a reorganization of what people can do with each other» (p. 302). ICMI is one of the few organisations that brings different people together in mathematics. The quadrennial meetings of ICME bring together teachers, researchers, mathematicians, school inspectors, curriculum designers, policy makers – and many other people from across the education system. These are global meetings but there are also regional meetings in some parts of the world. It may well be that an extension of the programme of ICMI regional meetings can play an important role in facilitating a jostling of good research knowledge and practice to the benefit of all the communities involved.

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Research and practice: *Bridging the gap or changing the focus?* *Reaction to J. Boaler's plenary talk*

This is a timely paper on a pressing issue: How to deal with the gulf that tends to persist between research in mathematics education and mathematics teaching practice? Boaler reviews some previous articles on this issue, discusses a number of successful cases, and contrasts them with her own experience. She provides several interesting and important reflections on which I wish to comment.

1. *Teachers' practices, researchers practices, and students' practices*

I find particularly important Boaler's suggestion that in order to change

teachers' teaching practices it is absolutely necessary that something changes in their professional culture – teachers need to become more adept at continuing learning throughout their professional lives. Another suggestion that I wish to underline is that something also needs to change in the practices and culture of researchers, who should be more concerned with doing more relevant research (from the point of view of the needs of teachers and policy makers), with publicizing in new ways the ideas and results from research (in written form and using other media such as video), and with working in new ways with teachers and school systems.

In addition, I wish to emphasise that students' learning practices are also important when discussing the real impact of research on students' learning. On a large scale, students' learning practices are difficult to document. However, in small scale teaching experiments we have many examples of successful research that have contributed to our knowledge about students' learning practices and to the emergence of new learning practices. That has certainly been the case with a number of teaching experiments conducted in many countries. I can document this in the case of Portugal, where, under the heading of students' exploring and investigating mathematics we have seen considerable change taking place with remarkable influence on students' learning (see Ponte, 2001, 2007 for some examples). In fact, students' exploring and investigating mathematics is a perspective that resonates with Boaler's (1997) former research as well as with the stress she puts in her paper on the role of research as a process of constructing knowledge.

2. Impact of research on teaching practices

Boaler discusses in her paper the shortcomings of the 'linear model' of dissemination of research results and of several alternative models. She emphasises the idea that a proper use of research requires learning. She also indicates several features of research programs that had strong influence on teacher's practices and policy documents: (i) providing living examples, in attractive form, (ii) valuing at least some aspects of what teachers already do, and (iii) providing continuous support to teachers who are committed to change. These are all quite reasonable and resonate with my own experience as well as with my views about teachers' practices and processes of changing (Ponte & Chapman, 2006). On another level, Boaler indicates several examples of successful research projects that had a significant influence on practice. It would be useful, perhaps, to use the distinction between 'fundamental oriented research', that seeks to produce knowledge and results that are presented and convincingly argued on standard research papers (and this is the kind of research that is currently most valued by universi-

ties), and ‘applied oriented research’ that seeks to produce artefacts and intervention programs that bear directly on teaching and teacher education. The best examples of research that impacted practice are those that produce educational materials (such as the Freudenthal Institute) or intervention programs (such as the Cognitively Guided Instruction or the projects of Bob Wright or Julia Anghileri that Boaler describes in her paper). Boaler repeatedly presents her former research as an example of lack of impact on practice, but I find that unsurprising, since this was ‘fundamental oriented research’ and (to my knowledge) did not produce many artefacts or intervention programs susceptible of influencing teaching practices, students’ learning practices, teaching education practices or curriculum development practices. This distinction between ‘fundamental’ and ‘applied’ research is not to mean that we should only carry out applied research. By the contrary, sound applied oriented projects need to base themselves on fundamental oriented research. And because of this, some fundamental oriented research may have also an important indirect impact on practice. However, if this impact is not carefully crafted through applied projects, it may turn out to be more confusing and misleading than effective.

3. Impact on the curriculum

Discussing the issue of the gap between research and practice, Boaler considers for the most part the impact of research on teaching practice, but she also pays considerable attention to the impact of research on the curriculum. These are two natural levels to address, but we must keep in mind that teaching practice affects directly students’ learning, and that is not the case with the curriculum.

As our speaker indicates, research has a very significant impact on the curriculum in a number of countries. Let me add another example, that she did not mention, the case of Portugal. A new curriculum for basic education (grades 1-9) has just been finished by a team that I coordinated and that included teachers, mathematicians, and other mathematics educators. A quite difficult obstacle was the very narrow deadlines framed by the Ministry and this was solved by complex negotiations with the politicians. Another obstacle were the radicalized positions of different groups and organizations that intervene in mathematics education, with rather opposite views in many matters (e.g., use of calculators, rote learning of algorithms, prescription of specific teaching guides and learning objectives, etc.) and this required a careful decision of what to include. Instead of putting ‘everything research says’ in the curriculum, we weighted those ideas that were most important to stress and that were more likely to be accepted and put into practice by teachers.

Based on this strategy, we strived to design a coherent curriculum that would improve the existing curricula documents for different cycles (grades 1-4, 5-6, and 7-9) instead of having the pretension of starting everything from scratch with a brand new solution. In the same vein as the French researchers who worked on curriculum development that Boaler refers, we looked to research for ideas that were useful and sufficiently stable in the mathematics education discourse. We paid special attention to summaries of research such as Kilpatrick, Martin and Shifter (2003) and to mathematics education research handbooks and also to studies carried out in our country. Our solution was to emphasise big ideas in mathematics education such as developing number sense, spatial sense and algebraic thinking. We also brought to a similar level the mathematics themes (number and operations, geometry, data handling and algebra) and what we regarded as fundamental transversal capacities (problem solving, mathematics reasoning, and mathematics communication). So, instead of a direct translation of results of research on curriculum into a curriculum document, we experienced another kind of connection between research and practice, as we brought our knowledge and expertise as researchers to mathematics education at an institutional level.

Now that the document was approved by the Ministry of Education (in December 2007), we face a still bigger challenge. Will its main ideas find their way into the conceptions and practices of teachers and of those who are closely related to mathematics teaching and assessment such as textbook authors and writers of exam papers? We are now beginning the implementation process, in which much negotiation and careful decisions will again be required but in which drawing on the results and products of former research and the expertise of Portuguese researchers and teachers involved with research will be certainly of key importance.

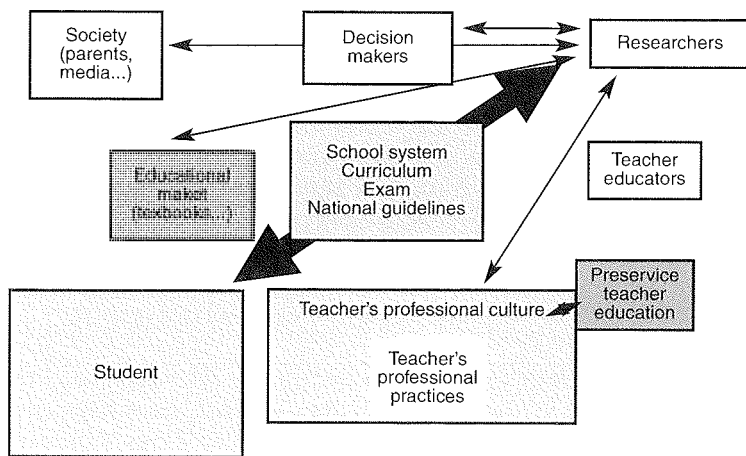
4. Looking beyond the relations between researchers and teachers

The paper by Boaler gives much attention to the relations between researchers and teachers. It also addresses, although with much lesser emphasis, the relations between researchers and policy makers. I fully support most of what is said in the paper in these two levels. However, I think that there are other relationships that need to be addressed if we want to understand how mathematics education research may influence teaching practices and learning practices and what are the obstacles in achieving these goals. In particular, we need to consider the relationships of researchers with (i) the society at large, and particularly with parent organizations and the mass media; (ii) the educational market, that has an increasing influence on education and includes the textbook industry and the educational industry that

produces all kinds of resources (especially digital resources such as interactive CD, on-line materials...); and (iii) the students themselves, since they should not be seen as passive consumers of what the system offers them, but as active participants in the learning process (Figure 1).

Another level that I find important in discussing the relationships of research and practice is preservice teacher education. Research can be brought into preservice education in several ways. For example, research results about students' learning, properly translated, may be put into the curriculum of specific courses for preservice teachers. Research results and concepts about the design and the processes of effective teacher education may be used by teacher educators to improve their teacher education models and practices. And, on a more fundamental way, teacher education may be an important laboratory allowing the fusion of expert practice of experienced teachers with the controlled innovation promoted by teacher educators and the critical energy and irreverence proper of young preservice teachers and students. Bringing all these actors working together constitutes a significant source of energy to change teaching and learning practices. Of course, similar considerations could be made regarding inservice teacher education.

Figure 1. Social players influencing mathematics teaching and learning



5. To conclude

In ending this paper, I would like to stress two ideas. The first one is that for research to bring about changes in mathematics teaching and learning *we need to act at a systemic level*. As I indicated above, mathematics education researchers need to work together with teachers and decision makers, but they also need to address the media, parents, textbook authors,

and students. And they need to work closely with school and district administrators as well. Lasting and enduring change will only come across from a change of cultures in all these settings and this requires also political and organizational changes from mathematics education researchers.

The second idea is that if we want to have a real influence on practice, *we need to see that as a problem on itself*. Influence on practice will not come just as a by product of a nice research project that was able to disseminate very well its products. It is something that applied oriented projects need to consider as part of their research problem and to carefully address on their research designs. I find that the problem of influencing practice, albeit difficult, is tractable. It all depends on the social and institutional conditions and on the position of the researchers regarding them. We can get smarter about influencing practice but we must also ask ourselves if all such impact on practice is desirable. We must not forget that other scientific fields that had a strong impact on the natural world are now plagued with unforeseen problems – just think of global warming, dilapidation of resources, and environmental disasters. Intervention towards change is barely needed, but we must be careful with the consequences of our interventions.

In sum, we may be quite positive about the power of mathematics education research as an essential way of generating knowledge about problems and processes. Taken in a broad perspective, research may be practiced not only by professional researchers but also by teachers and students, who also may experience the processes of inquiring, collecting and interpreting evidence and drawing implications from it. However, this confidence in the power of research to understand phenomena and intervene in practice must be combined with an attitude of social responsiveness, leading researchers to work closely with different social partners and being critical and reflective about what they do. That is, I suggest, an essential element of the ethics of intervention oriented research.

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The origins and early incarnations of ICMI

This lecture outlines the movement toward international cooperation and the establishment of a working network, and presents the key figures in this story. The work of IMUK/CIEM until World War II is described, as well as obstacles to cooperation, and the political pressures, which effected the dissolution of ICMI after WW I. The lecture is based on hitherto largely unexplored archival material.

1. The founding of ICMI – almost failed

As is well known, the ICMI was founded in 1908 following a proposal published in 1905 by David Eugene Smith and reiterated in the pedagogical section of the International Congress of Mathematicians (ICM) at Rome. The intense, active role Germany played in achieving the aims of ICMI is also familiar. Surprisingly, however, the founding almost failed owing to the machinations of a German participant, himself committed to mathematics education!

The imminent failure of Smith's proposal is clear from Eileen Donoghue's (1987) study. According to Smith's papers, in the fourth section of the ICM, on 9 April, while the German astronomer Archenhold was presiding, Smith presented his paper and proposed establishing the international committee. Archenhold favoured the proposal and took the vote, which passed.

Nevertheless, Smith thought it necessary to have the proposal reconfirmed at the last meeting of the fourth section, on 11 April. Then, however, the presider was the German mathematics teacher and educator Max Simon. Simon, as Donoghue reports, «was one of the few who voiced an objection to Smith's proposal» (Donoghue 1987, p. 267). She observes, «While Smith and his supporters were in the lobby discussing maneuvers, Simon adjourned the meeting».

Because it was the last meeting of the section, Smith's proposal would have been postponed until the next Congress, 4 years later! Donoghue (1987) reports how Smith prevented the premature death of his idea by trickily dealing with standing orders: «Since Italy was the official host country for the Congress, Smith urged Professors Castelnuovo and Enriques to exercise their procedural right and reconvene the meeting» (Donoghue, 1987, p. 268), which Enriques and others managed to achieve. Only then was the motion voted on and transmitted to the General Assembly for its approval. It was adopted in the closing session that same afternoon.

A second source for the near failure of Smith's proposal is a report by Walther Lietzmann, who shortly thereafter became Klein's assistant in handling ICMC matters. Lietzmann reported on an incident at the 9 April session where Smith gave his talk. August Gutzmer, one of the key German promoters, together with Klein, of the reform of mathematics teaching, reported on the movement. In the discussion following Gutzmer's talk, Max Simon criticized those reforms. Presiding at the last session, Simon used his position to defeat Smith's proposal. Lietzmann quotes Simon on the reform proposals: «The good is not new; and the new is not good!» Lietzmann characterized that opinion as a «malicious saying» (Lietzmann, 1960, p. 44; my transl., G.S.) and in his later publications never missed an opportunity to criticise Simon.

The conflict between Klein and the modernizers, on the one hand, and the defenders, like Simon, of traditional approaches, on the other hand, has been seen as a rivalry between two competing scientific schools in German mathematics education (Burscheid, 1984). Simon was a mathematics teacher at a gymnasium whose main interest was in geometry, where he advocated classical Euclidean methods. After 1903, he also lectured at Strasbourg University on the history and didactics of mathematics.

Given the positive vote after the first discussion, on 9 April, why did Smith present the proposal a second time and thus nearly fail to reach his goal? Donoghue (1987, p. 267) offers an explanation in her report on the first session: Smith's proposal was so attractive that almost everyone present expressed interest in participating in the commission. He realized «that to ensure the level of success he wanted, the commission members needed to be strategically chosen».

Smith began to reflect on detailed terms for the committee. Between 9 and 11 April, he decided to propose a three-man committee, later the *Comité Central*, to be complemented subsequently by national delegates. During the process, as Smith contacted numerous influential participants, the names of Klein, Fehr, and Greenhill emerged and were eventually accepted. Donoghue (1987, p. 268) summarizes this second stage: «With this accomplished, Smith pre-empted the haphazard nomination process

by proposing a three-man committee on organization whose members would serve as officers of the Commission».

The first proposal was a general proposition, whereas the second was a measure for concrete activity. In the first proposal, the committee was to be constituted 'permanently'; Lietzmann (1908, p. 83) speaks of a 'continuing' constitution, too. In the second proposal, the task was drastically limited: only until the next congress. Furthermore, the second version contained neither a list of countries invited to send delegates, nor the quota for the number of delegates. Those details were elaborated later. Finally, Smith had conceived of the committee's work to be restricted to secondary schools. It was upon Klein's initiative that all levels of education became included (Schubring, 2003, p. 57).

2. The "self-discovery" of the Comité Central

The next question one might ask on reading the ICM proceedings is: Why these three persons? Klein was an entirely understandable choice given his reputation as a mathematician and his active involvement in the German reform movement. The Swiss Henri Fehr was likewise obvious: As editor of *L'Enseignement Mathématique*, he was well informed about national developments in mathematics teaching and about persons active in this field. But why George Greenhill? He was an applied mathematician at the Royal Artillery Institution in Woolwich, but was retired and had not been known to be involved in questions of school teaching. He had been knighted, however, and thus had the aura of 'Sir George Greenhill'. He seems to have been nominated for purely political reasons: Smith gave as a reason that Britain would host the next ICM (Donoghue, 1987, p. 268). And the British government was to play a special role in the formation of ICMI. Lietzmann (1960, p. 45), however, was quite puzzled when Greenhill remained almost entirely silent during the first meeting of the *Comité Central* in September 1908 in Cologne.

Given that they were to collaborate to create the broader international committee, the choice of Klein, Fehr, and Greenhill posed an unprecedented challenge. How would these three succeed in establishing a common vision and arrive at working terms? None of them had previously been in cooperation or real contact with the other two—except for Klein and Greenhill, who had corresponded, although not extensively, since 1886. Although the three did not live very far apart—Göttingen, Geneva, and London—it was a person outside the group and living much farther away who got the three together, essentially serving as the midwife for ICMI: D. E. Smith again! Smith, who loved to travel and whose participation in Rome was part of a one-year sabbatical

leave, devoted much time during that summer of 1908 to the birth of the Commission.

The week after the Congress, Smith managed to see Fehr and Greenhill, who had been at Rome, meeting both—according to Donoghue (1987, p. 269)—«to explain his ideas on the aim of the commission and how it should function». She credits Smith with the next steps, but the documents show that Fehr and Smith jointly urged that the work begin. Real activity began in June. A letter of 22 June from Fehr to Smith shows Fehr's concerns about Klein's passivity. Klein had proposed that the committee meet first during the next *Naturforscherversammlung* at the end of September in Cologne. His intention had apparently been to have an extensive discussion and thus to forge a common understanding. Fehr, however, wanted immediate action—but only through correspondence. He asked Smith to visit Klein to discuss matters with him.

Klein, in contrast, fully recognized the importance of the new task but realized that it implied not merely the presidency of the international commission but also an active role in the German subcommittee, which would have to be constituted, as he told Smith in a letter of 6 October. To assume this double obligation, he needed a reliable and capable assistant. He had a candidate in mind, Walther Lietzmann, who had done an excellent comparative international analysis of current trends in geometry teaching. One day in the summer of 1908, Lietzmann received a card from Klein grudgingly asking why he had not responded. Weeks before, Klein had written a long letter inviting him to participate in the ICMI work, but it had not reached Lietzmann, because of vacations in Italy. Lietzmann accepted, and together they prepared intensely for the meeting in Cologne. Moreover, Klein seems to have been the first to think about financial matters. He was sceptical that three delegates generously allotted by Fehr to the major countries could be financed in Germany.

Smith had offered to visit Klein, and Klein invited him to come to Göttingen; he stayed there for two weeks, from 6 July on. There they discussed extensively Smith's draft of a report and Fehr's proposals for the international committee and the national subcommittees. After a U.S. student, Clifford Upton, had typed their text containing a general program for ICMI, copies were sent to Fehr and Greenhill. Fehr responded to Smith on 18 July, very happy about the outcome. By then Fehr, Klein, and Smith had reached a common understanding. In particular, Fehr appreciated the active role given to the *Comité Central* in choosing the delegates who would become members of the Commission. His and Smith's earlier draft had reserved that right to national associations or subcommittees.

Missing in the constitution of the *Comité Central* was the cooperation of Greenhill. Smith, still acting as midwife, went to England in August to bring Greenhill into the projected work. Letters from Greenhill to Smith in the sec-

ond half of August and early September show that they had met several times in London to acquaint Greenhill with the discussions among the other three. Smith had also tried to meet John Perry, the famous propagator of reforms in mathematics teaching. Yet, he never became a British delegate!

After these various bilateral meetings, the three members of the original committee, now called *Comité Central* [CC], met in Cologne on 23 and 24 September, without Smith but with Lietzmann as Klein's assistant, at the end of the *Naturforscherversammlung* and the yearly meeting of the *Deutsche Mathematiker-Vereinigung*. The meeting achieved, on the one hand, the definite consolidation, the self-discovery, of the leading group. On the other hand, it also achieved an elaboration of the plan and a definition of procedures for constituting the network (see the next section). The participants were highly satisfied. Suffice it here to quote how much Greenhill, who until then had stayed a bit outside of the communication, had become integrated into the group. After the meeting, he wrote to Smith (12.10.08):

Klein set to work at once in fine style and drew up an excellent plan of our procedure for the next years. My misgiving vanished that the work was too vast, and we should not know how and where to begin.

The plan was published (in French) in the November 1908 issue of *L'Enseignement Mathématique*. Its title was *Rapport préliminaire sur l'organisation de la Commission et le Plan général de ses travaux*.

3. Constituting the national Sub-Committees

As Lietzmann (1960, p. 45) reported, the major work of the *Comité Central* at Cologne was to find out who should be approached as a delegate to the international committee. He credits Klein with knowing whom to ask: «One has always to admire Klein's stunning capacity to select in all the countries the adapted person» (my transl.).

Actually, as the letters of the committee members show, it was a collective work, one still supported by D. E. Smith. A substantial part was, in fact, already elaborated at the first meeting. It was an unprecedentedly large and differentially structured network that the committee had undertaken to build up.

ICMI (designated at that time as CIEM, ICTM, or IMUK) should consist of the main Committee, headed by the *Comité Central*, and delegates from 'major' and 'minor' countries as members, and then non-voting representatives of associated countries, and all this backed by national subcommittees. The national subcommittees were to constitute the infra-

structure necessary for producing successful inquiries and reports from the national delegates, and there were to be 18 fully participating countries and 15 associated countries. Clearly, such an international network had never before been proposed or established. To form the network, the organizational situation of the mathematical communities and the communities for teaching mathematics in all of the countries had to be explored.

An additional complication emerged, however, with respect to finances. Klein had been aware from the beginning that the Commission would need a budget to achieve its goals. At first, he had envisaged asking the national mathematical associations for financial contributions. As those contributions would clearly be limited, a new idea arose during the first meeting in Cologne: The work of the Commission should be considered not a private initiative but a public enterprise. Each government should be asked to form a subcommittee, make nominations, and provide a budget. To make such a procedure viable, the *Comité Central* would conduct its own search for capable candidates in the respective countries and then make sure those candidates were made known to the governments and formally confirmed.

The group in Cologne had envisaged a more complicated procedure. As Klein explained in his letter of 6 October to Smith,

As soon as the situation in the main countries will be sufficiently clear, the English government shall request officially the other states to appoint delegates and to constitute national bodies. By this procedure we hope to provide the necessary stability to the endeavour, from the outset, also from the financial point of view (my transl.).

It is not clear why the British government was chosen to act as a super-power in favour of mathematics teaching, giving advice to the other governments. The idea had probably been proposed by Sir George Greenhill, but if it was, he was to be disappointed. As Lietzmann (1960, p. 45; my transl.) noted in assessing the procedures envisaged in Cologne, «The future has shown that all went well in all those countries, with just one exception, the request by the English government». The coordinated action by the government may well have failed because of the relative weakness of the disciplinary communities in Britain.

4. The ‘normal’ work, 1908 until July 1914

Lietzmann was right. Despite the enormity of the work envisaged, and although no models existed for such an international network, all went remarkably smoothly. Already in the autumn of 1908, the delegates were proposed to the respective governments, and subcommittees

were constituted for Germany and for Italy. France and other countries soon followed suit.

In Britain the process of nominating delegates proved to be the most complicated. For over 3 years, Greenhill remained the only delegate, which is all the more astonishing because of the strong and successful movement in Britain at that time for reforming mathematics teaching, including geometry teaching. One aspect of the problem was that Scotland needed to be represented by a delegate of its own. Not until 1912, the year of the ICM in Cambridge, were the three delegates appointed (all from England, none from Scotland). Moreover, Britain was the only participating country that had not succeeded in forming a proper national subcommittee. Greenhill had to address the Board of Education, hence the government, to find persons who would work on the needed national reports. Strangely, the Commission is not mentioned in any of the literature on the reform of mathematics teaching in England. Greenhill repeatedly complained about his dreadful and fruitless initiatives to obtain money from the government:

All this is easy in Germany where education is organized by the Government; and the work there may be said to be complete already. But here in England there is no centralisation and our work will be very difficult. I am warned that it is a hopeless task for any money of our Government (26.1.09).

Another point of concern was the associated countries. Although Greenhill soon succeeded in getting representatives from Canada and the Cape Colony, and although Australia, and Japan were very active, Latin America was not. Before the First World War, only Mexico and Brazil had nominated representatives. And only by the end of the war were there indications that Argentina and Cuba would join.

Despite these problems, the proper work of the *Comité Central* and of the Commission as a whole progressed remarkably well. The CC met every year, and from 1910 on, there were general meetings of the entire Commission. These meetings were, in effect, the first International Congresses on Mathematics Education. As at the ICME 3 in Karlsruhe in 1976, thematic reports were prepared, presented, and discussed in Brussels (1910), Milan (1911), Cambridge (1912), and Paris (1914), the last report having been prepared at a 1913 meeting of the CC in Heidelberg (see Schubring, 2003, for analyses of the reports and the four 'congresses'). The climax of these activities was the Paris meeting. It was prepared in the most intense manner; it had the best participation and the most vivid discussions. There was even a satellite congress on philosophy of mathematics. It is legitimate, therefore, to speak of the work of the Commission before the First World War as a success story.

In particular, it is important to see how the next ICM, at Cambridge in 1912, had dealt with IMUK/CIEM. The final vote of the General Assembly

documents the high esteem and the recognition for the work of this first international network (Proceedings 1912, p. 41).

Smith who by decision of the Congress joined the CC was by a later letter of Fehr enacted as vice-president (15.10.12). In fact, the CC felt legitimated the next year, in September 1913, to co-opt three new members and to increase thus the number of its members from four to seven – all from countries hitherto not represented in the CC: Paul Appell from France, Guido Castelnuovo from Italy and Emanuel Czuber from Austria. This increase should prove to be important in the later crisis. Since Appell did not accept, he was replaced by Hadamard.

5. *World War I and the crisis*

The Paris meeting had taken place in early April 1914. On 28 April, it was decided that the next meeting should take place in Munich from 2 to 5 August 1915. To prepare for that meeting and for the ICM in Stockholm in 1916, where the last work of the Commission was to be presented and thus the task and mandate given by the 1908 Congress would be achieved, the CC met in Göttingen from 23 to 26 July. A few days later, on 1 August, World War I began.

The former unity and cooperation within the CC ended almost immediately. Henri Fehr, citizen of a neutral state but sympathetic to French culture and scientists, acted first—on the side of the Western allies. During and after the war, he was an active promoter of aligning the Commission's policy with that of the victorious powers.

The first documented reaction to the war was a letter of 14 August 1914 from Greenhill to Smith:

This terrible catastrophe will throw Europe back a century in civilisation. It will take that time to recover. There cannot be any meeting at Munich next year, and I fear it is all [an illegible word] with the Congress at Stockholm. So it will be—*actum est de I.M.U.K.*

The next document is a letter of 3 September from Fehr to Smith, evidently written after the destruction of the Belgian town Louvain by the Germans at the end of August: «The Austro-German military caste has brought Europe to a sorry state, and I sometimes ask myself what our colleagues, and German academics in general, must in their heart of hearts think [about it]».

Fehr thought it would still be possible to hold the international conference planned for Munich by transferring it to a neutral country. Moreover, he reported the common opinion from the Göttingen meeting that the Commission's mandate should definitely be declared finished in Stockholm.

In fact, the first letter, dated 29 September, that Fehr sent to Klein after the outbreak of the war expresses optimism and a willingness to continue the work as far as possible.

In Fehr's next letter to Smith, on 30 September, he lamented that the unfortunate European war was delaying all international endeavours. He reported having proposed to Klein to continue the work of the Commission and to publish an issue of *L'Enseignement Mathématique* dedicated to the reports planned for 1915. He announced that he would write to the CC as soon as he received Klein's answer. He continued, however, with a severe critique of German efforts to convince foreigners that news of cruelties committed by German armies was mere propaganda.

On 8 November, Fehr reassured Smith that the work of the Commission would continue, but cancelling the 1915 meeting. Klein, in his response to Fehr of 7 October, had supported the proposal to continue the work as far as possible. But then he received a letter of 2 December from Fehr that Klein annotated as 'the critical letter'. Fehr repeated what he had said to Smith on 10 December: All the work had to be stopped until better times arrived. Fehr even refused to publish the questionnaire for the study on teacher training. He explained that several colleagues had expressed doubts about completing the ongoing projects. He said that numerous professors had been mobilized, not just in the countries at war but also in neutral countries, so that important collaborators were missing. But he also referred to the form that the war had taken since the destruction of Louvain.

Because Klein regarded Fehr's propositions as quite critical, he reflected intensely on his response and drafted, unusual for him, his answer of 11 December. Klein underlined the need to concentrate energy on finishing the projects and noted that what was not finished now would never be finished. He wanted that to be the CC position, too, and urged Fehr—representing a neutral country—to support that view. Klein proposed

- either to continue the work as planned, i.e. to publish all available reports in *EM*, in particular the questionnaire, while desisting to have a discussion;

- or to declare that under the given conditions the work of ICMI has to be given up, so that IMUK as an international association be formally dissolved – hoping a re-establishment under new conditions. Even in this second case, *EM* should publish the questionnaires.

Despite Klein's clear instructions, Fehr did not consult the CC but instead announced in the November issue of *L'Enseignement Mathématique* (published at the end of December) that the work of the Commission would cease during the war.

Crowning his one-sided actions, Fehr not only sent an offprint of that notice to Klein but also dealt a heavy blow in the accompanying letter of 28 December: He proposed that Klein renounce his position as president

of the Commission and assume the position of vice-president, ceding the presidency to Smith. Justifying this step, Fehr referred to Klein's signature on the scandalous *Aufruf an die Kulturwelt* of October 1914, an appeal signed by 93 German intellectuals and scientists. The appeal was a deeply deplorable document. It argued that war crimes supposedly committed by German troops were just false propaganda and even that German militarism was necessary to safeguard German culture. The appeal was one of the factors that later led to the exclusion of German scientists from international congresses and organisations. And for Klein, it led to his expulsion from the Paris Academy of Sciences. Only much later did it become known that many of the signatories, including all the non-Berliners, had never seen the text they were asked to support. They had merely reacted positively to a telegraphic inquiry to sign a patriotic appeal against war propaganda (Ungern-Sternberg, 1996, p. 23).

Fehr argued in his letter to Klein for adjournment on the one side for material reasons, the non-availability of numerous colleagues, and on the other side to national irritabilities due to the war:

As far as our committee is concerned, the issue is aggravated by the fact, that its President has signed the manifesto of German scientists. From the point of view of international relations among scientists, one has to recognize that this manifesto is deplorable, since it has dragged milieus into the fights, which had hoped to continue to work jointly on the fields of humanities, sciences and arts (my transl.).

On 9 January 1915, Klein wrote to Smith to seek his advice. He proposed that Smith, as a member from a neutral country, should take over the presidency, while he would act as one of the vice-presidents. At the same time, Klein answered Fehr briefly, saying he had contacted 'several sides' concerning the proposal and asking Fehr to wait until he had an answer. He added that if he ever renounced the presidency, it would be because of the present situation and not for reasons of health (17. 1. 1915).

It is remarkable to observe the different viewpoints of the members of the CC. Smith, responding to Klein on 5 February, did not accept the offered presidency and instead urged Klein to continue. It is a moving letter, convincingly arguing for separating politics and science:

Next, as to the Internationale Mathematische Unterrichtskommission. [...] As to your presidency, and as to the organization of the Central Committee in general, I hope that we may make no change at present. This is not a conflict of scholars, and in the academic world there is no war. I was in England until November, and I heard only the kindest words in academic circles for all German friends. It would be a *calamity* for you not to direct this great international investigation until it is completed. Of course I shall be only too glad to act as

your assistant, to send out any letters that you might suggest and to cooperate with Fehr in every possible way.

I am at your disposal. [...] Only let it be under *your direction*; let it continue to be known that you are at the head of the work. Thus shall we be sure of high international standing, and the world will be sure that the work will be worthy of the support.

On receipt of this plea, Klein felt determined to stay in his position and not to resign, at least for the time being, as he told vice-president Castelnuovo in a letter of 4 March 1915. Castelnuovo's answer of 10 March is likewise a moving document: he assured that all members of IMUK are fully aware that nobody else could replace him. Moreover, in this complicated situation, no change in the organization of international institutions should be enacted: «On the contrary, one should make all efforts to make them survive so that they might facilitate to retake normal relations among the nations» (my transl.).

Meanwhile, Fehr had informed Smith of his proposal to Klein that Smith should become the Commission president (17.2.15). Answering on 4 March 1915, Smith repeated the argument he had made to Klein, an argument quite different from that of the French and of Fehr. Smith had been in England throughout the autumn and had often communicated with the colleagues there and underlined:

that I did not believe there would be any personal feeling against a man like himself. I based this judgment upon my conversation with a number of English scholars before I returned from Europe, all of whom spoke in the kindest terms of him and of other German scholars.

Greenhill's position is likewise remarkable. In November 1914, he reacted to an apparent criticism of Klein by Smith, probably his signing the *Aufruf*:

I think you are hard on Klein. As a patriot German he must stand up for his own country. The world looks up to German science and learning as our leaders; but it is a great defeat of civilisation that they are controlled by a military despotism.

Unlike Fehr, Greenhill distinguished between scientists and the military complex. Fehr held a minority position in the CC, backed only by the French member. In that letter to Fehr, Smith said that he had just offered to assist Klein with correspondence, but then he told Fehr that he would accept the presidency for the duration of the war: «If, by taking nominal charge of the work, and acting under the direction of Professor Klein I can relieve him and assist in the progress of work, I shall be glad to do so». Fehr

had wavered several times between publishing and not publishing the questionnaire for the ongoing study on teacher training. Eventually, he followed Klein's pleas and published it in the first half of 1915. Despite his claim that many collaborators were mobilised, he said he would stay in active contact with most of the subcommittees regarding their work on the study (7.12.1915 to Smith).

Actually, Fehr did not address the CC, however, and thus Klein formally remained President. Since the USA entered the War later in 1917, too, Smith thus no longer represented a neutral country.

After 1915, correspondence became extremely rare, and the work stagnated. Even between Smith and Fehr, the correspondence stopped. Only that between Smith and Greenhill continued, but it was on literary and scientific subjects. There was no correspondence at all between Klein and Fehr between the end of 1915 and 1920!

6. *The dissolution*

Europe had experienced many wars, but never before had a war decisively interfered with cooperation between scientists of the states concerned. World War I was the first in which a ban against scientists from defeated countries was prepared and enacted. As Olli Lehto (1998) has impressively shown, it was French mathematicians who propagated the exclusion of German scientists from international cooperation and organisations. Gaston Darboux, permanent secretary of the French Academy of Sciences, inviting representatives of all Allied nations in 1916 to a meeting in Paris on international relations after the war, wrote: «Do you want, yes or no, to retake personal relations with your enemies?» (Lehto, 1998, p.16). His letter expressed the clear conviction to regard fellow scientists from the Central Powers as enemies and to exclude them hence from cooperation.

After Darboux's death, Émile Picard continued the same policy. It became official from November 1918 on. The constitutive assembly of the IRC, International Research Council, which should henceforth organise and coordinate international cooperation and associations, was held in Brussels from 18 to 28 July 1919, the ideological basis and practical implementation of the postwar international science policy were ratified. The formation of scientific unions was prepared, among them the IMU (Lehto, 1998, p. 19).

Fehr followed these developments closely and with sympathy. He was the first after the war to contact other members of the CC, but exclusively those from the victorious Allied countries. He first contacted Smith in a letter of 31 March 1919, saying that the Commission was bound by the decisions of London and Paris, that the French and Italian 'delegations' had

asked him to proceed in this manner, that the Commission could not continue to function as originally constituted, and that the representatives of the Central Powers had to be excluded. Thus, Smith should now accept the presidency, and they should continue the project on teacher education because of its key importance. Hence they should not liquidate the Commission.

In his answer of 7 May, Smith said that although he had not accepted the 1915 offer, «a change must now be made». Still, he pleaded for a president having an «international reputation as a research scholar in mathematics» and being «at the same time specially interested in the teaching of the sciences». Although nobody currently showed those qualities, he would accept «the presidency for a time». He asked that all members of the CC be consulted about this move but was not explicit as to whether he accepted the exclusions.

Fehr did not reply until 2 September, reporting that he had consulted all the members of the CC except Klein and Czuber about the question of the presidency and that he was missing only Greenhill's answer. He mentioned that he had published in his journal the new conditions determining international scientific cooperation according to the Allied decisions. Thus, he expressly accepted the reduction of internationalism (EM, 21/1920, pp. 59-60). On the other hand, he said that additional decisions had been made at a 'recent' conference of Allied countries in Brussels - actually held in July - which implied the dissolution of all organisations whose membership included the now-besieged countries. Adhering closely to all these decisions, Fehr saw no way to continue with the Commission and making Smith president.

Only in March 1920, in the wake of preparations for the Strasbourg ICM, did Fehr again contact Smith. Because the Strasbourg ICM would not include those countries from the former Central Powers, the Commission would have no relation to the Congress and therefore no report needed to be given there. Foreseeing a continuation of the exclusion policy, Fehr urged liquidation of the Commission (23.3.20).

It is not clear how strongly the so-called Italian delegation supported the French and Fehr's view, but Fehr's next letter, dated 13 April, before receiving an answer from Smith, proves that the French mathematicians were the driving force and that Fehr wholeheartedly supported them. He informed Smith that he had just met with influential French mathematicians and that he and they had agreed that the Commission should be liquidated, which he proposed should be carried out by a committee consisting of Smith, Hadamard, and himself. Only then did Fehr intend to submit the question to the other commission members. Smith answered only on 20 July, after having received word from Klein, whom he had contacted in the interim. Smith expressed clear disagree-

ment with the exclusion policy, particularly with regard to the Strasbourg Congress:

I think the general feeling among scholars in Great Britain and America, so far as I have talked with them, is that we should as soon as possible make these Congresses genuinely international, whatever our present feelings may be.

He also reported Klein's response: Klein would no longer consider himself «the President of the I.M.U.K., but [...] Sir George Greenhill and I [Smith] should act as Vice-Presidents and take what steps we think best to close up the work at Strasbourg».

Smith had written to Klein on 28 April 1920, his first letter in 4 years, telling him that he saw «no chance of general cooperation in the work of our Commission for some time to come» so that its work should be stopped «at least for the present» with regard to its international dimension. In his answer of 26 May, Klein lamented the destruction of the general association of scientific academies, which he himself had strongly promoted. But he lamented even more that the joint work would thus remain unfinished.

Fehr waited until 1 June 1920 to contact Klein—more than a year and a half after the end of the war—informing him of the inevitable liquidation of the Commission. He proposed four steps:

- The secretary-general informs the members of the forthcoming liquidation and offers to do the final work
- The complete list of reports is published
- All financial issues are resolved; and
- A closing circular is sent to the members, including in particular a financial report.

Klein answered immediately on 8 June, agreeing to the necessity of dissolving the Commission and to proceed according to the proposed four steps. Fehr affirmed on June 29th that 'our' committee had nothing to do with the Strasbourg Congress. Having received Klein's agreement of 10 July, Fehr formulated a letter—predated 5 July—to the entire Commission informing them of the liquidation process.

Greenhill's response to Klein on 26 June shows that he, too, disagreed with the French policy, not wanting to identify scientists with the policy of their states. His letter reveals, however, new dimensions and problems in international policy. Greenhill did not blame militarism or the uncivilised behaviour of the Central Powers, but rather Jews.

A number of busy bodies are at work (Jews for the most part) trying to stir up strife, instead of helping the world to settle down peacefully [...]. But considering how profitable the war has been to the Jews, and the profit they derive

from the misfortunes of their fellow creatures, it is not surprising that the Jews are striving to keep the world at war.

The object of all sensible men should be to resume harmonious relations, and carry on the comradeship of scientific work, with as little delay as possible.

Three of the four members of the CC of Cambridge were present at the Strasbourg ICM: Fehr, Greenhill, and Smith. They agreed not to speak for the Commission, and as the proceedings show, it was not mentioned. It no longer existed. Except for a letter of protest from Beke for the Hungarian Committee, there had been no resistance to the liquidation. Both Smith and Greenhill were deeply disappointed by the Strasbourg ICM. Greenhill was particularly harsh:

We were obliged to come to Strasburg, to attend the *obsèques* of IMUK. I fear our work passed unappreciated for the most part. And the new Commissions have made no sort of acknowledgement of the pioneering part we had taken (3.11.20 to Smith).

The liquidation process took a long time. First, the final list of publications had to wait until the report on mathematics teaching in Argentina, one of the new associated countries, was translated into French. That took until late 1921. Second, a number of national contributions to the budget of the International Commission were missing and had to be sought. In the last action of the Commission, on 6 September 1921, Fehr sent all members the final report as published in *L'Enseignement Mathématique* and reminded them to pay any missing contributions. This was the last action of the first IMUK/CIEM.

The only positive decision taken by the old CC, as Klein pointed out to Enriques, had been to let the national network—that is, the national sub-committees—survive.

7. Re-establishment and stagnation

After the Strasbourg ICM, Greenhill had remarked sarcastically: «I fear Fehr is saying, like Othello, his occupation is gone» (3.11. 20). In fact, although Fehr had been the active promoter of liquidation, he used CIEM letterheads from 1922 on that listed him as secretary-general and the only officer.

In 1928, the organisers of the Bologna ICM rejected the exclusion policy of the IMU (Lehto, p. 44), making it the first truly international congress after the war. In the section on teaching, Fehr presented himself as *Segretario generale* of the International Commission and gave, in its name,

a report of its activities since 1908. He did not mention the liquidation; instead, he spoke of «recent publications and report in preparation» (Atti, 1928, I, p. 111). He declared: «If the Congress wishes it, we are ready» to continue the study of teacher education. In the ensuing discussion, reference was made to the 'existing committee', and the participants voted not on the reconstitution of the Commission but on its prolongation. The officers were to be Smith as president, Castelnuovo and Hadamard as vice-presidents, and Fehr as secretary-general.

On 2 October 1928, Fehr informed Smith, who had not been able to attend, of the results and proposed that Lietzmann be co-opted as the fifth member. Smith agreed, as did Lietzmann. Smith wrote Fehr several times expressing concern that he had heard nothing concrete about the work to be done. In mid-February, Fehr wrote back, saying that things were difficult. Old members had to be replaced, and delegates from newly invited countries had to be found. But his only concrete action had been to ask Gino Loria, who had begun working on the teacher education report in 1914, to join with two other experts, Lietzmann and Archibald, and redo the work, which was finally published in 1932.

At the end of May 1929, Fehr and Smith finally met to discuss how to reconstitute the Commission. The structure they adopted followed that of the original, with some simplifications—for example, just one delegate per member country.¹ Financial contributions by each country to pay the expenses of the CC were also stipulated—without, however, charging the respective governments with those payments. It is characteristic that almost no new ideas and projects were proposed, that the only project pursued was the last one of the old agenda. Likewise, the national sub-committees functioned at a quite feeble level.

The 1932 ICM in Zürich adopted a motion proposed by its 8th section to prolong the activities of CIEM, again without an explicit limitation, and approved officers of the CC. It also asked the Commission to report on current tendencies in mathematics teaching in the different countries (Verhandlungen, 1932, pp. 59-60), a proposal that Smith had advocated. Smith was also aware of the need to have younger members and had argued for a young person as president. Despite his obvious efforts to renew programs and personnel, the decisions of the Congress were rather conservative. Reports on the state of mathematics teaching were presented at the next ICM, at Oslo in 1936. In response to a proposition by Fehr, the General Assembly agreed to continue the Commission's activities, without defining any concrete tasks or calling other persons into the CC. In *L'Enseignement*

¹ In the Smith papers there is a handwritten draft by Smith developing a much more elaborated program of work.

Mathématique after 1936, no activities of the CC or the Commission as a whole were reported, and they ceased entirely in 1939.

A new epoch confronting new challenges was necessary to revive the idea of an effective international network. Klein had died in 1925, Greenhill in 1927, and Smith in 1944. Fehr was the last of the founding activists; he was the one in 1952 who passed the torch of CIEM/ICTM/IMUK to the new ICMI.

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ICMI Renaissance: the emergence of new issues in mathematics education¹

Two facts mark the emerging of the new status of *mathematics education* as a scientific discipline: the inauguration in 1969 of the tradition of International Congresses on Mathematical Education (ICMEs), and the contemporary launch of journals related to mathematics education research. The early ICMEs acted as catalysts for new ideas that have their roots in important events that took place in the 1950s and 1960s. As a consequence new issues found their place in the international discussion on mathematics education and opened new lines of research and forms of action inside ICMI, so that we may talk about a genuine ICMI Renaissance. In our contribution we examine the background and circumstances that fostered and shaped the emergence of the new issues that have developed in mathematics education up to the present day.

1. A turning point

The history of ICMI², founded in 1908, spans the whole of the 20th century and was affected by the main events of that period. World Wars and societal and political changes led to the dissolution of many international bodies, including the ICMI. Information on the people and events of the first hundred years of ICMI is provided by the web site (Furinghetti

¹ The preparation of this paper was partly supported by the PRIN 2005019721.

² In this paper we will always use the acronym ICMI (*International Commission on Mathematical Instruction*). In fact, until the 1950s, the French CIEM (*Commission Internationale de l'Enseignement Mathématique*) and, less frequently, the German IMUK (*Internationale Mathematische Unterrichtskommission*) were used.

& Giacardi, 2008). An account of the first 75 years can be found in Howson (1984). Because ICMI was strictly connected to the IMU (International Mathematical Union), additional elements in the history of ICMI can be found in Lehto's (1988) history of IMU and Furinghetti's (2007a) analysis of the didactical sections of the ICMs (International Congresses of Mathematicians). Broadly speaking, we may identify the following main periods:

- Gestation and early period up to the first World War
- First lethargy and short-lived rebirth between the two World Wars
- The rebirth in 1952
- The Renaissance in the late 1960s and the projection to the future.

For the first two periods see Schubring (this volume). Our contribution will focus on the rebirth and on the ICMI Renaissance, which is epitomized by point 5 of the resolutions expressed at ICME-1 (The Editorial etc., 1969):

The theory of mathematical education is becoming a science in its own right, with its own problems both of mathematical and pedagogical content. The new science should be given a place in the mathematical departments of Universities or Research Institutes, with appropriate academic qualifications available (p. 284).

In the same vein, the year before, at the UNESCO international meeting (Bucharest, 23 September - 2 October, 1968), in which ICMI participated officially, Hans Georg Steiner (1968) had claimed that:

In order to get the full effect which the progress of mathematics and its epistemology can have on the teaching of mathematics a new discipline has to be established, which in the paper will be called "didactics of mathematics" and which is separate from the "methodology of mathematics teaching" (pp. 426-427).

Two main events, both promoted during the ICMI presidency of Hans Freudenthal, mark the emergence of mathematics education as a scientific discipline and the prelude to the ICMI Renaissance:

- The foundation of the journal *Educational Studies in Mathematics* in 1968 (Furinghetti, 2007b)
- The establishment of the tradition of an International Congress on Mathematical Education (ICME) in 1969.

Lehto (1988) reports that both initiatives were born thanks to the financial support of UNESCO and with only a marginal involvement of IMU. Behind these two events, there is within the community of mathematics educators the increasing perception of the inadequacy of the community of mathematicians as the place of privilege for discussing educational prob-

lems. Evidence of that is offered by the points made during the ICMI session of Utrecht, see (Commission etc., 1967)³. Freudenthal expressed his disappointment with the modality of ICMI participation at the quadrennial Congresses of Mathematicians, which was confined to the presentation of reports, and supported the idea of an ICMI congress to be held the year before the ICM, where invited talks and personal communications could be presented. He noted that, in general, the national reports were not useful. The assembly accepted the project of an ICMI Congress in 1969. The French delegate Maurice Glaymann proposed holding the congress in France. In the same session André Revuz asked for a new journal that would be closer to secondary teachers, as *L'Enseignement Mathématique* was at too high a level. As a consequence of the ICMI's struggle for autonomy of action and, more generally, for the creation of a new community for mathematics education, new academic positions were claimed⁴, as evidenced for example in Krygowska (1979) and in Wittenberg (1965).

2. New arenas for discussing mathematics education

Let's step back a moment to summarise the events that brought about these transformations concerning mathematics education as a scientific discipline.

After WWI scientific associations dissolved and there was the shocking decision to ban the researchers of the Central Powers from most international activities. This also wounded the mathematical community, which healed after ICM-1928. After WWII things went in a different way: the community of mathematicians recovered rapidly and new trends of research developed. In the proceedings of ICM-1950 there is evidence of two factors that fostered these new trends: the iron curtain and the appearance of the first computers. The place of mathematics in the world was changing.

In the first General Assembly (Rome, 9 March 1952) IMU had reconstituted the ICMI as its sub-commission. In the report of this GA, IMU president Marshall Stone (1952, p. 1974) claims that «The problem of determining the place of mathematics cannot be divorced from technical considerations concerning teaching methods». ICMI's activity, no longer

³ The colloquium was attended by 34 active and 34 passive members (*Internationale mathematische Nachrichten*, 91, 1969, 3). After the Colloquium there was an important meeting of the Executive Committee of the ICMI. The meeting was open to external participants, as shown by the intervention of Glaymann, who was not an ICMI officer.

⁴ Anna Zofia Krygowska is a rather isolated case: in 1958 she was appointed to a professorship in the Methods of Teaching Mathematics Department, within the Faculty of Mathematics and Physics in Krakow.

linked to IMU congresses, followed the tradition of the old ICMI based on international enquiries and the production of national reports published in *L'Enseignement Mathématique* and elsewhere, for example, in some ICM proceedings.

In those years other arenas were opening with the aim of finding new approaches to mathematics education suitable to the changed mathematical and social contexts. An outstanding arena was the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM). This commission was born as a study and working group, tied to the experimental mentality of its founder and animator Caleb Gattegno, who succeeded in putting together people from different backgrounds. The initial composition of CIEAEM was mainly European. Bernet & Jaquet (1998) list among the founding members the mathematician and pedagogue Gattegno (secretary), the psychologist Jean Piaget, the logician and philosopher Evert W. Beth, the epistemologist Ferdinand Gonseth, the mathematicians Gustave Choquet (president), Jean Dieudonné, Hans Freudenthal and André Lichnerowicz, and secondary school teachers Emma Castelnuovo, Lucienne Félix, and Willy Servais⁵. It should be noted that members of CIEAEM and important participants in the movements of Modern/New Math were members of ICMI Executive Committees.

In the preface to the Italian edition of (Piaget et al., 1955, It. Ed. p. VII), E. Castelnuovo and Luigi Campedelli wrote: «The need was felt for a deepening of the art of teaching and for a revision of the meaning of teaching».

This passage makes evident the growing attention to the student and to the process of teaching that would be a main concern of CIEAEM. International inquiries and comparisons of curricula, which traditionally were the core of ICMI's activity, were given less consideration. At their meetings, CIEAEM members participated as individual researchers, not as representatives of their countries. According to E. Castelnuovo (1981), the founders of CIEAEM considered it more suitable to build a new commission – one that was more flexible and free from conventions – rather than to restore the old ICMI. Felix (1985) explicitly states that CIEAEM was created to counteract the inaptness («insignificance») of ICMI.

The philosophy that underlies this new arena comes directly from Gattegno's views on mathematics education, according to which mathematics teaching involves the study of three domains: the functioning of the mind, the construction of mathematical theories, and the needs of society, sciences and technology. This study is based on practice, not on a philo-

⁵ The authors claim that the list of the founding members is controversial. They chose the largest set of members.

sophical exercise. We see that the ‘art of teaching’ is becoming ‘a specialized profession’. Thus, a new expert is needed, one who is interested in the link between students’ mental activity and mathematical knowledge. According to Gattegno, the teacher has to be such an expert and Gattegno himself creates the conditions for shaping this professional requirement. This explains the initial composition of CIEAEM, which was based on the collaboration between people of different backgrounds. This kind of collaboration will slowly foster the emergence of a new figure, the researcher in mathematics education.

The activities of CIEAEM consisted mainly in periodic meetings. Initially these meetings were restricted to about 30 participants, mostly teachers, but often students were also present. The contributions were not prepared in advance, but grew out of the discussions. No proceedings issued from the first meetings.

The warm enthusiasm and the activity of the early years of CIEAEM generated new issues that had a great impact, particularly in Europe:

- the relevance of psychology in mathematics education
- the attention to teaching methodology
- the key role of concrete materials
- the need to take all school levels (from primary to university) into consideration
- empirical research
- the relation between mental and mathematical structures.

Two important books published by some founding members focused on these new issues. In the first (Piaget et al., 1955), all authors agreed about the opportunities that Modern Mathematics offered to mathematics teaching. Dieudonné claims that the essence of mathematics is reasoning on abstracts notions. Though an intellectual effort is needed to reach the abstraction of algebraic structures, the solution to the problems of mathematics teaching is not to delaying abstraction: It is necessary to reveal to adolescents the very aspect of mathematics following the formation of their mental structures. Lichnerowicz (the future president of ICMI) asks for an experimental effort to introduce something of the spirit of modern mathematics into arithmetic, algebra, elementary geometry, by pointing out the structures of ring, group, and vector space and by adapting the language. Choquet criticizes traditional teaching and textbooks, with particular reference to geometry, and proposes the introduction of analytical geometry, Euclidean, non-Euclidean and projective spaces, and geometric transformations.

Piaget’s contribution deals with the foundations of mathematical theories and looks for links between mathematical structures, as introduced by the Bourbaki school and the structures of the mind. In his turn, Beth denies the usefulness of traditional psychology in mathematics education,

only accepting with reservation Piaget's genetic epistemology, as it deals experimentally with the mental development.⁶

Gattegno reasserts his idea that the mathematics teacher must also be a psychologist, and must foster a dynamic method that allows students to construct their mental structures. He explicitly mentions concrete teaching materials,⁷ such as Cuisenaire rods, geo-plane, and the films of Jean Luis Nicolet and Trevor Fletcher.

The use of concrete and semi-concrete materials, films and models in the activity of the pupil is the subject of the second book published by CIEAEM (Gattegno et al., 1958), which is strongly oriented to practice in the classroom. It is not coincidental that the majority of the authors are not academics. The ideas expressed in this book will have a real application in schools. For example, E. Castelnuovo designed and carried out a complete program for middle schools (ages 11-14) based on the use of concrete materials (see Castelnuovo & Barra, 1976). We must also mention the Belgian Paul Libois, initially scheduled as an author of the third part, which never appeared. For some years Libois had been the promoter of the École Decroly in Brussels and had regularly used concrete materials in the teaching of mathematics, especially geometry. One of his successful ideas was the use of the 'semi-concrete' material provided by the shadows of a figure produced by the sun or by a lamp in order to teach geometry. It is interesting to point out that the concrete materials signal a connection between psychology (Piaget's experiments) and innovation in teaching, as will emerge in some contributions to ICMEs (see below). Moreover, we may note that the ideas put forward in this book were shared by other people involved in mathematics education, as shown by a passage in the talk delivered by the ICMI vice-president Djuro Kurepa (1960) at ICM-1958:

the hands are to be active (writing, showing), the tongue, ears, brain, i.e. all organs are more or less in active interdependence and co-operation. Let us remember that for a long time, even in instruction of geometry, and still more in arithmetic, the factors action and perception were either eliminated or at least neglected (p. 570).

These two books represent the manifestos of two important innovations supported by CIEAEM: *Modern Mathematics for* (Piaget et al., 1955),

⁶ In those times the role of psychology was rather controversial, see the following passage from (Freudenthal, 1960):

All will admit that Piaget's research is highly interesting. But it is quite another thing to apply his results to teaching mathematics, firstly because Piaget's mathematical background has been rather weak, but mainly because Piaget's approach hardly reflects the teaching situation in the classroom, but the rather unusual laboratory situation of the psychologist (p. 139).

⁷ From 1980 to 1982 he created software to teach numeration, addition and subtraction.

concrete materials for (Gattegno et al., 1958). While the first innovation was mainly confined to a period of less than 20 years, at least in its most radical interpretations, the second topic is still current and comprises educational ideas and features that are still under discussion: the use of the computer, embodied cognition, gestures. The books also epitomize the two main intentions in the action of CIEAEM: the democratization of mathematics and active pedagogy.

CIEAEM's actions, containing important germs of didactic research, as it is seen nowadays, was paralleled by the New Math movement started by Max Bebermann in 1951-52 with the creation of the University of Illinois Committee on School Mathematics (UICSM) (see Raimi, 2005). The movement had roots in common with the Bourbaki school: set theory, functions, relations and logic should find their places in the new curricula, supported by the methodology of discovery. In 1958, again in the USA, Edward Begle was appointed director of the School Mathematics Study Group. The experiences of this group and other activities carried out in the USA, benefited from contributions from psychology. But, as Kilpatrick (1992, p. 18) put it, only

with the arrival of cognitive psychology in 1950s and 1960s, marked by the availability of Piaget's work in English translation and the reinterpretation of that work by Jerome Bruner, researchers in mathematics education began to have a more judicious regard for psychological theory and to collaborate more frequently with psychologists.

3. Modern Mathematics at Royaumont

Members of ICMI and CIEAEM officially met at the Conference of Royaumont (23 November - 4 December 1959). This important meeting took place in the Cultural Circle of Royaumont situated in the village of Asnières-sur Oise, near Paris. The conference, chaired by ICMI president Stone was organized by OEEC (now OECD), an international body of 18 European countries and Turkey. The USA, Canada and Yugoslavia were also invited. Its concrete aim was to produce a curricular reform that was not limited to Europe. This aim was neither utopian nor detached from the reality of the times; in fact social and political conditions were fostering reform movements all around the world. In particular, in western countries the so-called 'Sputnik shock' turned politicians' attention to educational systems.

As discussed in Menghini (2006), the contributions of Dieudonné and Servais make it evident that the conference was run under the influence of the French mathematical school, namely the Bourbaki group. In the fore-

word of the proceedings of the Conference (OEEC, 1961, pp. 11-14)⁸, the push for a reform in mathematics teaching is justified by the increasing demand for mathematics coming from society. In his introductory talk, Stone (1961) outlined a genuine program of research in the teaching of mathematics (study and experimentation) which advocated the creation of research institutes in mathematics education and the insertion of this subject into universities research projects.⁹ He pointed out that teaching has to meet the needs of applications and to devise new methodologies; he further stressed the enormous development of mathematics and the exigency of not widening the gap between school mathematics and university mathematics. In the coming years, these three main points (applications, the academic mathematics, concern for pedagogical methods) will be the key ideas of the reform movement of modern mathematics in France (Charlot, 1984).

It is remarkable that, in spite of the fact that the interests of both CIEAEM and ICMI spanned many aspects of mathematics education, at Royaumont the discussion was confined to the mathematical aspects put forward by the mathematical component of CIEAEM, and to the teacher training in relation to the new subjects. Methodology was neglected, pedagogists and psychologists played no role. Issues related to primary school were treated only in a talk by Choquet. The only contribution integrally reported in the proceedings is that by Dieudonné (1961), which, in his own words, is confined to questions of the passage from school to university. There are no considerations on methodology or on applications of mathematics; in spite of this, it became the manifesto of the conference.

In that same year, 1959, the Woods Hole Conference took place in the USA. Its aim was to improve science education, bringing together scientists, mathematicians, psychologists and others. In the book (Bruner, 1960), which summarizes the proceedings, the structural elements of all learning were emphasized as essential for understanding the whole.

One year after Royaumont, OEEC gathered in Dubrovnik (Yugoslavia, 21 August – 19 September) a group of experts appointed to produce «a modern program for mathematics teaching in secondary schools» (OEEC, 1962). A preceding meeting on the teaching of geometry had been held in Aarhus (Denmark, 30 May - 2 June 1960). In 1961 ICMI organized a meeting in Bologna (Italy, 4-7 October 1961), in which the results concerning geometry from Dubrovnik and Aarhus were discussed (contribu-

⁸ The proceedings consist in the report of the conference by Howard F. Fehr.

⁹ In the *Summary and conclusions of the seminar* it is recognized that «There is a need for large-scale, systematic, scientific research in actual classroom situations [...]. There is also a need for fundamental research into problems of intellectual development and concept formation» (p. 120).

tions are published in *L'Enseignement Mathématique*, s. 2, IX, 1963). In the programs produced by the commissions, the Cartesian plane was defined as a vector space of dimension two with a scalar product, while coordinates were introduced in some intuitive way. The discussion led to a compromise which consisted in introducing all that via axioms. Freudenthal, Libois and Tullio Viola partly disagreed with these proposals. They would have preferred maintaining continuity with the explorative and experimental methods of the lower secondary school and focusing on deductive reasoning and the proof of some theorems derived from other theorems or from axioms introduced when necessary. Freudenthal, who did not participate at the Royaumont conference, expressed his position, editing the *Report of the Subcommittee for the Netherlands of the ICMI on methods of initiation into geometry*. Freudenthal (1958, p. 7) criticizes the anti-didactic inversion introduced by the axiomatic in geometry. In the preface he draws «the reader's special attention to the work of Mr. and Mrs. Van Hiele, not only because it is the most revolutionary attempt, but because of the underlying pedagogical and didactical theory».

The new proposals were widely accepted in the spheres of both CIEAEM and ICMI, with few objections. At the end of the 19th CIEAEM meeting (in Milano Marittima, near Ravenna, Italy, 9-17 April 1965), the participants approved a motion proposed by Revuz stating that, while the introduction of notions of modern algebra in the teaching from ages 10 to 18 is being achieved without particular difficulties¹⁰, the updating of geometry is a hard task since there are still supporters of Euclid and their opposition could lead to the disappearance of geometry teaching. Instead, geometry can be perfectly inserted in a unified organization of mathematics: it is the study of a vector space of a finite dimension over the real field, with a scalar product.

Some weeks later, ICMI organized in Echternach (Luxemburg, 30 May - 4 June) a seminar on *Les répercussions de la Recherche Mathématique sur l'enseignement*, during which the CIEAEM motion was discussed. Félix (1985) notes that this was the first time the conclusions of a CIEAEM meeting had been discussed by ICMI and that the link was assured by the chair of ICMI Lichnerowicz, a member of CIEAEM since its foundation. It is probably more correct to say that at Echternach the motion was presented, but not modified by a discussion. Revuz proposed that this document, approved and signed by Choquet and Dieudonné, take the name of 'Traité d'Echternach'.

These OEEC and ICMI meetings influenced many countries, among them France, where Lichnerowicz chaired the ministerial commission for

¹⁰ The algebra proposed in Dubrovnik concerns sets, applications and functions, the introduction to real numbers, elements of number theory, combinatorics, groups and structures, linear applications and matrixes.

the teaching of mathematics (1966-1973). During that period, other countries adopted official programs based on the new contents. In the 1960s all over the world commissions and pedagogical centres were created to discuss new programs, and retraining sessions for teachers were planned.

In Europe and the USA the path of innovation went from the university to the secondary level and then to primary school, where a large scale introduction of set theory was carried out in order to present the cardinal concept of a natural number, see Pellerer (1989). Since the causes of the failure of mathematics teaching had been identified in the fact that the contents were outdated, mathematicians were supposed to play the major role in the reform. Nevertheless movements with stronger links with schools also developed. Relevant examples are the *Nuffield Project* (started in 1964) and the *School Mathematics Project* (started in 1961). The 1970s were fertile years for the creation of projects, as shown by the fact that the presentations of 15 projects are mentioned in the proceedings of ICME-3.

In the ten years following Royaumont, the paths of ICMI and CIEAEM diverged. Already in 1960 Gattegno had quit the CIEAEM, because he considered the adoption of a 'program' as the end of experimental activity. CIEAEM meetings lost their initial character and slowly became normal conferences, attended by numerous teachers.

ICMI established collaborations with other bodies (UNESCO, for one) and organised regional conferences in and outside Europe. One outstanding achievement of the collaboration with UNESCO was the launch in 1966 of the series *New trends in mathematics teaching*. Besides these politically important achievements, reports were the only very 'field-activity' carried out by ICMI.

The impetus of both ICMI and CIEAEM was running dry at the very moment in which the new social pattern required greater attention to schools. Modern Mathematics was showing its first bugs. CIEAEM was no longer the place where the cooperation among mathematicians, teachers and psychologists or pedagogists was carried out, and ICMI was not a meeting place for mathematicians and mathematics teachers. There was a need for new professional figures based on mixed areas of competence, able to develop a «frontier research» (Krygowska, 1968) and it was also necessary to change the pattern of ICMI.

So we are back to the first ICME 1969.

4. ICMEs and the new issues

The new autonomy of the community of mathematics educators, acquired through the creation of ICMEs and the emergence of mathematics education as a field of research, promoted the development of new issues. These

issues, concerning both classroom work and theoretical research, did not appear out of the blue: they have their roots in the preceding period. Analysing the evolution of these issues allows us to understand something more about current trends in research. We will start with a paradigm.

4.1. From concrete materials to technology

As we have seen, one of the main features of the activity of CIEAEM, particularly in keeping with the philosophy of Gattegno, was the use of concrete materials. This topic has a great emphasis in ICME-1 (1969) and ICME-2 (1972).

In various contributions to ICME-1 we find mention of games, worksheets, films, manipulatives, even the 'modern' overhead projector, which allowed lessons to be prepared in advance, to perform movements and overlapping. In these contributions the use of materials is put in relation to a new methodology of classroom activities that also includes working groups and classroom discussion. Among games the Minicomputer presented by Frédérique Papy at ICME-1 may be seen as somewhat similar to Seymour Papert's 'turtle geometry' presented at ICME-2. Both experiments were aimed at primary school. In ICME-1 a talk by Szoltan Dienes on mathematics in primary school was scheduled. The presence of Dienes is further evidence of the interest of the ICMI community in concrete materials, such as Dienes's logic blocks, and the beginning of a real interest in primary schools themselves. An explicit reference to the role of computers in school mathematics, especially for applied mathematics, is in the contribution by Brian Thwaites at ICME-1.

In the introduction to the proceedings of ICME-2 Geoffrey A. Howson (1973)¹¹ again discusses the role of laboratories and concrete materials, emphasizing the need for reflection on these new teaching devices. We agree with Howson's opinion that educators look at the 1960s as the period of New Math, but that actually the main long-lasting idea that gathered strength in that period was the emergence of new styles of teaching and a more systematic transfer of teaching materials and ideas in the various countries.

A rich list of products presented at ICME-2 is reported in Appendix 4 of the Proceedings and a Working Group - a new format introduced in ICME - is dedicated to the *use of television and film in the teaching of mathematics*. A tradition of presenting visual and other equipment aimed at teaching was already present at the ICMs; in the early ICMs exhibitions of mathematical models and computing machines were mounted, and from the 1950s movies were presented. This tradition was revived and updated

¹¹ The introduction is unsigned, but in a private communication Howson has confirmed to us that he is the author.

through new technologies that were soon seen as indispensable in mathematics education. Already at ICME-2 a WG explicitly addressed technology, and at ICME-3 this happened for five official activities. From that time on the programs of ICME encompassed official activities linked to computers. The tradition of the ICMI Studies started in 1985 with the theme *The influence of computers and informatics on mathematics and its teaching*. Each ICME edition marks a step forward in the evolution of the impact of computers in mathematics education. In the early contributions the emphasis was on parts of mathematics that appeared particularly affected by the presence of computers, such as algorithmic aspects or discrete mathematics. Later on, programming was considered a means for attaining rigor or abstraction and for giving meaning to symbols through computer science languages. Suitable learning environments, or microworlds, were created and studied. Certain types of software, the dynamic geometric for one, have revealed their power to revitalise parts of mathematics, for example, proofs and Euclidean geometry.

Only the aspect of Computer Assisted Learning, which was developed in the years of the early ICMEs, seems to have been rather neglected. It is likely that this is a side effect of the psychological orientation adopted in ICMEs, which was far from behaviourist theories.

4.2. *Psychology in mathematics education*

Piaget's theories are an important feature of the first two ICMEs. At ICME-2 the contribution of Piaget (not present) still outlines the analogy of Bourbaki's three mother structures with his structures of thinking; he identifies the causes of the failure of modern mathematics in the use of traditional teaching methods based on oral transmission. At ICME-2 Mary Sime explains that the value of Piaget's tests is educational rather than diagnostic, by assisting in the formation of concepts through the use of suitable materials. In his turn, Efraim Fischbein (present in the first two ICMEs) ascribes to Piaget the most important reference theory in the psychogenetic field. He compares the theories of Piaget and Bruner and proposes a compromise. He also mentions Richard Skemp.

The influence of Piaget's methods also permitted broadening the range of mathematical topics in primary school; we note that an inquiry published in the proceedings of the congress of Royaumont in 1959 concerning the training of primary school teachers speaks only of 'arithmetic' teachers. In ICME-2 both WGs on 'Pre-school and primary mathematics' and on 'Structure and activity' are linked to Piaget's experiences. In the introduction to the proceedings of ICME-2, Howson (1973) stresses the importance that Piagetian psychology had for elementary school. He also notes that the WG for «the psychology of learning mathematics» was the most attended. According to Howson, the topic dealt with «underpins the whole

of mathematics education» (p. 15). In this working group we see the premises for the creation in 1976 at ICME-3 of the Study Group PME affiliated to ICMI, which broadened the scope of the reference to psychology. The two photographs on the cover of the proceedings of ICME-2 (Howson, 1973) summarise two issues that emerged in the ICMI community: new methods of doing mathematics in the classroom (George Polya's photo) and the interlacement with psychology (Piaget's photo).

4.3. *The relationship to the world of mathematics*

Professional mathematicians have always been present in ICMI, first as founding members and supporters, and later as ICMI presidents. Moreover, for a long time ICMI inquiries were planned during the ICMs. In the first ICMEs the presence of mathematicians is rather high, while it decreases in the more recent congresses. Often the contributions of mathematicians consisted in examples of applications of mathematics to the real world and, as a consequence of this, in stressing the key role of mathematics and science in society. The presence of mathematicians offered support and encouragement for the various ICMI activities.

In the introduction to the proceedings of ICME-2 Howson (1973) recognizes the importance of the French mathematicians during in the previous 15 years and their contribution to the activities organized by OEEC/OECD. At ICME-1 and ICME-2 New/Modern Math is still present, with contributions both for and against it. The most important of them, by René Thom (1973), stresses the contradiction of a teaching that is heuristic in principle, but is based on abstract mathematics. Thom claims that Piaget is much too confident in the potentialities of mathematical formalism: Modern Mathematics has not produced new theorems and, as far as education is concerned, does not produce new knowledge. The actual problem is not rigour, but rather the development of meaning. Modern Mathematics has eliminated Euclidean geometry in favour of algebra, but it is precisely Euclidean geometry that is the link between natural language and abstraction. Because of Thom's contribution, many authors date the end of Modern Mathematics to ICME-2. In successive ICMEs we still find echoes of the debate, in particular the discussions about the movement *Back to Basics* at ICME-4 in 1980.

Polya's abstract in the proceedings does not refer to problem solving. But Polya's problem solving – which arises from the tradition of mathematicians reflecting on their own work – was the argument of one of the 38 WGs and would become a dominant theme in the successive ICMEs and in mathematics education up to the present day.

4.4. *New trends of research in mathematics education*

One of the key ideas in Gattegno's way of working, developed in the CIEAEM meetings of the 1950s, was to put the researcher in direct con-

tact with the classroom. This idea was given new life at ICME-1 by the *Mathematics Workshop*, which included a class of children at work. It was organized by the Association of Teachers of Mathematics of the UK. This signals the changes in how the study of educational problems is approached. Research in the field was maturing and at ICME-1 Begle was able to sketch an outline of the state of the art, by analysing and criticising the different typologies of mathematics education research. He noted that in the School Mathematics Study Group investigations were carried out by means of statistics, direct observation, pre- and post-tests. But statistics does not provide certain results: there is often too much data and it is too difficult to correlate. He hinted that research on large amounts of data is not necessary, and that clinical interviews, such as those used in the USSR might be used fruitfully. He complained that beliefs still existed which were not based on empirical evidence.

A call for more in-depth research in mathematics education was also present at ICME-2 in the plenary of the mathematician Hassler Whitney, who had in mind the failure of New Math. In ICME-2 a WG was devoted to *Research in the teaching of mathematics*. Successive ICMEs would show the strengthening of research in mathematics education as a scientific discipline with new results, new theoretical frameworks, new hypotheses, and new methods of gathering and recording data. In particular, technology now easily available to researchers permits new forms of data to be gathered: audiotapes, videos and devices for detecting neurological reactions provide the sources for new lines of research.

4. Conclusions

At ICME-2 Edmund Leach's contribution of some anthropological observations on number, time and common sense suggested broadening the frame of research to other domains. Moreover, one of the 38 WGs was titled *Mathematics in developing countries*. The ICMI community was ready to receive the message of Ubiratan D'Ambrosio (1986) at ICME-5, which introduced socio-cultural dimensions into mathematics education and acknowledged the dignity of various forms of mathematical culture. According to E. Castelnuovo (1998), at the beginning of the 20th century David E. Smith was inspired to conceive the ICMI by the reflections on the intellectual force brought about by the merging of cultures provoked by the children of immigrants. A century later this idea was revived by D'Ambrosio's reflections. Also, the new issues that fostered the ICMI Renaissance in the 1950s and 1960s were and are continuously revived and we hope that our historical overview contributes to put into a richer perspective the actual research in mathematics education.

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A Session at Accademia Nazionale dei Lincei

BIENVENIDO F. NEBRES, S. J.

Centers and peripheries in mathematics education

The paper looks at the impact of ICMI on East-Southeast Asian mathematics education and on mathematics education in developing countries in general. For Southeast Asia, this began in the 1970s at the invitation of Prof. Yukiyoji Kawada, then Secretary of ICMI. It led to a burst of mathematics education activity in Southeast Asia. With the opening of China after the Cultural Revolution, mathematics education activities under the sponsorship of ICMI also grew rapidly among the so-called Confucian Heritage Cultures. The International Congresses on Mathematical Education (ICME) also became a forum for mathematics educators from developing countries to share experience and research on their concerns: ethnomathematics, mathematics for all, mathematics, society and culture. The second part of the paper looks at possible lessons for the center coming from the periphery. In particular, it looks at several decades of major projects to improve mathematics education in the Philippines. It notes that the usual interventions coming from developed (OECD) countries, supported by international development agencies, have not worked. What has begun to work in the last decade have been interventions that begin with the social and cultural context (engaging the leadership, community and culture), provide interventions targeted at the actual situation and priorities of the particular group of schools, and set goals and metrics for which the leadership and community accept responsibility.

1. Introduction and context

The world of mathematics and mathematics education in which I have operated is East-Southeast Asia, including Australia. This is a corridor from Japan, Korea, China, Southeast Asia, down to Australia.

On the title of this talk: Centers and peripheries

The language of 'center' and 'periphery' is used to describe dominant voices and those whose voices are less heard. In mathematics education, the

dominant voices (centers) have been Europe and North America. The voices in mathematical education of the Confucian Heritage Cultures (China, Japan, Korea and others) can be considered in the periphery until recently. Many other developing countries, including those in Southeast Asia, remain in the periphery in the sense that, say in ICMEs, their unique voices are only heard in special sessions on mathematics education in developing countries.

ICMI and East/Southeast Asia

The context of our connecting with ICMI was the founding of the Southeast Asian Mathematical Society in 1972 and the founding of national mathematical societies in Southeast Asia, including the Mathematical Society of the Philippines, around the same time. From the beginning, these societies addressed the development of university and research mathematics as well as school mathematics and teacher training. In April of 1976, Prof. Yukiyoji Kawada, Secretary of ICMI, recommended that we organize a Regional Conference in Mathematics Education, which would be co-sponsored by ICMI. The member countries of SEAMS were also invited to apply for membership with ICMI. We did apply to ICMI and Manila accepted to host the First Southeast Asian Conference on Mathematics Education (SEACME) in 1978.

Center to periphery: Influence of ICMI on Southeast Asian mathematics education

The most immediate impact then of contact with ICMI was a burst of activity in Southeast Asia centering on organizing mathematics education conferences. Going through my notes and correspondence during those years:

Since 1978 SEAMS has organized SEACMEs regularly: 1978 First SEACME May 29 - June 3, Manila, Philippines; 1981 Second SEACME April 21-24, Kuala Lumpur, Malaysia; 1984 Third SEACME, May 22-25, Haad Yai, Thailand; 1987 Fourth SEACME, May 28 - June 1, Singapore; 1990 Fifth SEACME June 14-16, Brunei; 1993 Sixth SEACME June 7-11, in Surabaya, Indonesia; 1996 Seventh SEACME June 3-7, in Hanoi, Vietnam; 1999 Eighth SEACME May 30 - June 4, in Manila, Philippines; 2002 Ninth SEACME May 27-31, in Singapore.

Emergence of a new center for mathematics education: ICMI in East (Northeast) Asia

Since the beginning of modern international comparative studies in mathematics education, the outstanding performance of the East Asian countries (so-called Confucian Heritage Cultures) has been noted. The mathematics education comparative studies between the United States and

Japan, China, Taiwan have highlighted the role of culture, beliefs and values in mathematics education. In some ways, the East Asian countries have become a new center for studying best practices in mathematics education.

When China joined the IMU and ICMI in the 1980s, formal collaboration in mathematics education started to develop among the East Asian countries.

ICMI-China regional conferences in mathematics education

The first ICMI- East Asian mathematics education conferences were: ICMI-China Regional Conferences on Mathematics Education, Beijing, 1991; ICMI-China Regional Conferences on Mathematics Education, Shanghai, 1994.

The First East Asian Regional Conference on Mathematics Education (EARCOME) was held in Korea in 1998. Since then there have been four EARCOMES: First ICMI-EARCOME, Korea, 1998; Second EARCOME, Singapore, 2002 (this was also a SEACME); Third EARCOME, Shanghai, China, 2005; Fourth EARCOME, Penang, Malaysia, 2007; Fifth EARCOME, Tsukuba, Japan, 2009.

Southeast Asian mathematics educators always participated in these conferences. These conferences always had important speakers in mathematics education from abroad and allowed East-Southeast Asian mathematics educators to keep in contact with important regional and international trends.

Influence of the International Congresses on Mathematics Education (ICME)

We kept in contact with the ICMEs. These conferences allowed mathematicians, mathematics educators and mathematics teachers in the region to keep in touch with important developments in the field. When one goes through the talks and discussions in the various SEACMEs and EARCOMES, one sees easily the influences from international trends:

- The New Mathematics;
- Focus on Problem Solving;
- Back to Basics;
- Constructivism;
- Calculators, Computers and Technology;
- Psychology of Mathematics Education;
- Influence of the Freudenthal School.

ICME and developing countries (the periphery)

The opening of ICMI to countries from Asia, Africa and Latin America also began to bring up concerns different from the central concerns com-

ing from Europe and North America. Among the major concerns and issues that have entered the mathematics education discourse:

- Ethnomathematics, especially from Latin America and, in particular the work of Ubiratan d'Ambrosio
- Mathematics for All, coming in great part from UNESCO
- The impact of society, economics and culture on mathematics education

Voices from Latin America and from Africa have emerged in the ICMEs and other international and regional conferences.

What has been achieved through the support and encouragement of ICMI

(1) ICMI was instrumental in starting and supporting regional conferences. These conferences provided a platform for the people in the region to talk to each other. International conferences did not serve this purpose. We wanted to discuss problems that were relevant to us. The problems being discussed at the international conferences at the time were not our problems.

(2) Our participation then expanded beyond Southeast Asia (SEACME) to East/Southeast Asia (EARCOME). It introduced Southeast Asian mathematicians and mathematics educators to cultures with very successful approaches to mathematics education, which had significant differences from the approaches of the West. This was important as it invited our mathematics educators to ask whether we should not develop approaches more in conformity with our situations, rather than simply imitating trends from the developed countries. We now have larger participation in international conferences and share more of our actual experience.

(3) On the academic side, if we look at the proceedings of the different conferences, one can see easily that (a) papers have improved in number and quality; (b) there are more research papers from the local research groups, as compared to papers from overseas researchers.

(4) Other outcomes: (a) Information exchange: we found that we have much to learn from each other, not just from the advanced countries; (b) Joint work and research: the regular conferences helped to make research ventures sustainable; (c) Regional conferences are now routine, have become more international and our voices are better heard.

We can say then that the initiative of ICMI and its continuing support in East/Southeast Asia has indeed made a lot of difference in mathematics education in our region. Let me turn then to some voices from our experience.

2. Lessons from the periphery

Are there lessons from the experience of the periphery in mathematics education, namely East and Southeast Asia, that may be of importance to

the center? I would like to devote the rest of this paper to this question.

Some of these voices and lessons are already well-known, in particular the experience of the Confucian Heritage Cultures, as will be noted below. The concerns of other countries in our region, like the Philippines or Indonesia, are less well known and may be very different from the concerns of countries in the center (say the OECD countries). But I believe they may be of importance for mathematics educators in these countries. From my experience, many of the major reform initiatives in developing countries are brought in by experts from the West. This is partly because the funding agencies (World Bank, Asian Development Bank, etc.) bring in consultants from these OECD countries, who in turn bring in mathematics education experts from the same countries. Over four decades of such mathematics education reform effort in the Philippines, we have experienced a series of interventions from these experts. So far, the approaches they introduced from their own experience have not worked.

Mathematics education reform must take context and culture seriously

From the beginnings of the Southeast Asian Mathematical Society, I noted that our region in some ways lends itself as a laboratory for the impact of society and culture on mathematics and mathematics education. In my plenary address at ICME in Budapest 1988, I wrote (Nebres, 1988, p. 11):

In 1972, all our countries with the exception of Japan would have been considered developing in the economic sense. But it was clear that the development of mathematics and mathematics education was quite different in each country. We realized quite early that the categories of development or underdevelopment were inadequate to explain important differences. It could not explain the strong mathematics being done in Hanoi even during the worst years of the Vietnam war. Nor could it explain the great number of students doing pure mathematics in Singapore, when careers in pure mathematics were extremely limited in that island republic.

What emerged in the study of the differences among our countries was that there was a stronger bloc, mainly the countries within the Confucian Tradition (Japan, Korea, China, Hong Kong, Taiwan, Singapore, Vietnam), and a weaker bloc (Philippines, Indonesia, Thailand, Malaysia). The first wave of detailed studies in this area actually came from the United States. This was the cross-cultural research done by Harold Stevenson and others in the University of Michigan beginning in the late 1970s, see (Stevenson & Stigler, 1992). They provided data and analysis for our conviction that central to the differences were culture, beliefs and values. The comparison between East and West has been

studied extensively and is the subject of a very recent ICMI Study (No. 13), see (Leung, Graf, Lopez-Real, 2006).

Learning from the best in the East and in the West

In the third EARCOME held in Shanghai, the conference tackled issues coming from these different cultural approaches. The East Asian countries had shown that their approach to mathematics education led to outstanding mastery of the basics and of fundamentals. But they were less effective in encouraging creativity. EARCOME 3 thus focused on the theme of foundations and creativity: how to learn best practices in the teaching of fundamentals from the East Asian countries and at the same time to learn best practices in encouraging creativity from the West.

The usefulness of East-West comparative studies in seeking a better way forward for developing countries

What I would like to dwell on is what we have learned from the comparative studies between East and West in our own efforts to find a better way for mathematics education reform in the Philippines.

The central role of socio-economic-cultural variables in mathematics education for developing countries

After a decade of working on math education, I shared my reflections in a 1983 paper for a Regional Conference on Mathematics Education held in Japan: *Macro-Problems (Society, Economy, Culture) and Micro-Problems (Curriculum, Textbooks, Teachers, etc.)*:

We can classify problems of mathematical education into two types: the first we might call micro-problems or problems internal to mathematical education. These would relate to questions of curriculum, teacher training, textbooks, use of calculators, problem solving and the like. The second we might call macro-problems. These are problems affecting mathematics education because of pressures from other sectors of society: economy, politics, culture, language, etc.

I then went through a more detailed analysis of the challenges of mathematics education from this perspective. I concluded as follows:

The improvement of mathematics education in developing countries such as those of Southeast Asia requires continuing improvement of teacher - training, curriculum, textbooks (the internal concerns of mathematical education). However, their improvement is only possible if mathematical education has enough space and freedom (within the pressures of economics, culture, organization of education) so as to be able to plan and implement. It is the experience of developing countries that pressures from other structures of society

(economic, political, cultural) are often too strong for the system of mathematics education to work realistically on its internal concerns (p. 18).

Actual experience of attending to macro-problems and micro-problems

The opportunity to see what it might mean to attend sufficiently to macro-problems so as to make progress on improving mathematics education became possible for us in the Philippines because of major reform initiatives in the 1990s and through the early part of this decade. This was due to the confluence of two streams of reform. In 2001, then Secretary of Education, Raul Roco, invited me to discuss how we might improve mathematics and science education for the very large public school system in the country (about 14 million elementary school students and 5 million high school students). *We told him that the usual approaches of reform from the past had not worked* (Atweh, Clarkson, Nebres, 2003, pp. 214-218)¹.

The typical approach, pushed by international agencies like the World Bank and the Asian Development Bank and their consultants from the West, has been to:

- (1) Bring in a new approach, usually theory-derived and usually from the U.S. This was the method in bringing in the New Mathematics in the 1960s and in subsequent reforms in the 1970s through the early 1990s - newer trends such as back-to-basics, problem-solving, constructivism.

- (2) Develop materials based on these approaches.

- (3) Do pilot studies on small, selected scales, which always say that the new approach is better.

- (4) Then, given that necessary funding is available, implement on a national scale. Because by the time of implementation, there would be just a couple of years left in the project, the teacher training is done following what is called the *cascade model*:

National level training for regional trainers; Regional level training for division trainers; Division level training for district trainers; District level training for school trainers; School level training for teachers in the schools.

The first level training (national and regional) may be quite long, six months or so. When they get down to the level of actual teachers, it may be just two or three weeks.

As one reflects on this mode of mathematics education reform, one notes the following: The focus is on the *intended curriculum* (National Research Council, 1996). The greatest amount of time is given to the devel-

¹ One can see this in various reports included in the references, particularly on the Program for Decentralized Education (PRODED), the Secondary Education Development Program (SEDP), and, in particular, the report on national performance in TIMSS - R.

opment of the textbooks and materials and the higher-level trainers. It is also from the top, from mathematics education experts from universities and from abroad. Least attended to are the actual target implementers of the reform, the classroom teachers. The time frame is too short (Nebres, 2006, p. 279).

In talks I have given, I have compared the impact to that of a flash flood, too much in too short a time. The new curriculum and textbooks wipe out the past, but since the training gives no time for the actual classroom teachers to absorb them, they are not really learned.

In talks to various groups in the Philippines, I have been discussing a different mode of school mathematics reform. The usual way (as described above) has been to focus on the intended curriculum following major trends in the West; and to do this in detail in pilot projects and then in a hurried way for the larger scale. Not surprisingly, it succeeds on the pilot scale and fails on the larger scale (Rogan & Grayson, 2007, pp. 166-167)².

I have compared this usual process of reform to the longer 10 to 12 year cycle of school mathematics reform in Japan (Iyanaga, 1981, pp. 126-127):

- a. Immediately on implementing a new reform, a process begins of feedback on the textbooks, materials, etc. from teachers and classrooms
- b. This feedback is then processed and sifted through reports, conferences, discussions at different levels of the school system
- c. Then policies and decisions are made on the main lines of the next cycle of reform
- d. These are carried out in guidelines for new textbooks, books are written and pilot-tested
- e. Implementation begins for the new cycle.

The main feature I have pointed out is that *reform begins from the classroom, the implemented curriculum, and ideally the key players are the classroom teachers and school leaders. It is also a more evolutionary, rather than a revolutionary approach.* We begin with what we have and improve on it, rather than wipe it out and totally replace it (Nebres, 2006, p. 281).

² Rogan and Grayson (2007) state that the developing world is replete with examples of well-intentioned, well-designed curriculum reform programs that have failed to take root. One of the most important reasons seems to be a lack of clearly thought-out implementation strategies that take into account the local context and psychological factors that influence learning and change. Atweh and Ochoa (2001) have written about failure of many reform programs in mathematics education during the past 50 years. They argue that effective and long-lasting school change should start from the needs and interests of teachers in the classroom. This assures that reforms directly address the real problems that teachers face and empowers them to take control over the change process so that it becomes continuous after special projects cease.

In the meeting with Education Secretary Roco in July 2001, we suggested considering the situation in public schools, namely,

- congested classrooms (65 - 70 class size in urban areas)
- lack of textbooks
- lack of library facilities or library materials for teachers
- inadequately trained teachers
- absence of experts teachers may consult.

An initial approach would be to provide textbooks for all students, create self-contained detailed Teacher Lesson Guides per grade level, and do teacher training going through the Guide almost page by page. We also advised that the Teacher Lesson Guides should be developed by Master Teachers from the schools and not by university mathematics education experts.³

Providing enough textbooks: We were successful in having the Department of Education provide Math textbooks for all students in high school (1:1 ratio).

Teacher Lesson Guides: However, we were not successful in having the Department of Education provide copies of the Teacher Guides to all the mathematics teachers. This was due to budget constraints (a macro-problem). They were, however, given to all the teachers in an important social reform project, the Third Elementary Education Project. From an impact assessment point of view, this created an unintended control experiment.

As noted earlier, the Department of Education was not able to provide copies of these lesson guides to all the mathematics teachers.

We now turn to a reform initiative that addressed priority macro-problems and included providing the lesson guides to all the 54,000 teachers in the project.

Third Elementary Education Project: Attending to the macro-problems (creating absorptive capacity in the school and community) and micro-

³ Already in the 1970's after a few years of giving teacher-training seminars and workshops, we realized that in the Philippine context of weak teacher preparation, lack of textbooks and libraries, generic seminars/workshops were not helpful. At that time, we tried to walk teachers through the books they were using, providing problem sets they could use for class, homework, tests, etc. The proposed Teacher Lesson Guides were an effort to organize this approach. The initiative and support of Sec. Roco made this scaled-up approach possible. We decided to work with master teachers from the schools, because, in general, university mathematics and mathematics education professors have neither the experience nor the inclination to go through this very basic approach. We also have to face the fact that the reward system for university faculty, which is based on published research, does not necessarily encourage the kind of patient working with school systems needed to improve school performance on a large scale.

problems (teacher training, textbooks, lesson guides, etc.) and focusing on the implemented curriculum.

The Third Elementary Education Project (TEEP) was a nine-year PhP 12.7B public investment project (1998-2006) of the Department of Education with external financing from the World Bank and Japan Bank for International Cooperation. The project addressed improving elementary education in 23 of the poorest provinces in the Philippines, see (Bautista, 2007, pp. 7-18).

I personally directed the team that developed the plans together with these provinces from 1994 to 1998. Prior to our work on the planning, the Ateneo de Manila was commissioned to study whether it could identify the key variables that separated a high performing school from a low performing one given the same socio-economic-cultural conditions. Two key variables emerged: the leadership of the principal and the support of the community.⁴

Engaging the community and creating absorptive capacity and bringing in targeted inputs

Based on the Ateneo study (Licuanan, 1995), the Third Elementary Education Project thus focused on developing school based management (SBM), defined as the decentralization of decision-making authority from central, regional, and division levels to individual school sites, uniting school heads, teachers, students as well as parents, the local government units and the community in promoting effective schools.

In practice, the School Based Management approach means:

- starting from where the schools are rather than where they ought to be
- adopting school based management to the 'terrain' or contexts of the 23 divisions.

Impact

The results have been very good. In the review at the end of the project in 2006, the TEEP resulted in a bigger share of schools crossing the 75% mastery level and the 60% near-mastery level in the National Achievement Test (Bautista, p. 9).

Significantly Better Improvement in Mathematics

I received a phone call in August 2006 from the main evaluator of the project, Dr. Cynthia Bautista, excited about some results of their end-project eval-

⁴ The first phase of the Third Elementary Education Project was the development of the plans for about 20 of the poorest provinces in the country. This was done by a team from the Ateneo de Manila University and I personally directed the work. Implementation was delayed and actually done 2000-2006.

uation of the Third Elementary Education Project. They had noted significantly greater improvement in mathematics in the National Achievement Test (NAT) in the TEEP. When asked for possible reasons, the principals and teachers attributed the much better performance of TEEP schools in Mathematics to the Math Teachers' Lesson Guide series prepared by Department of Education and Ateneo, which TEEP printed and distributed, to all its teachers (Nuqui & Bautista, 2006, Section 2.21, pp. 2-8).

In some ways, this became an unintended control experiment, since the TEEP schools had the Lesson Guides and the others did not.

The lesson learned from this initiative on the Third Elementary Education Project and the Lesson Guides is worth noting

Our experience then of working on mathematics education reform in the Philippines (and school reform in general) validates certain approaches:

- We have to engage the culture and the community, in particular the key leaders and stakeholders. They have to want to improve their schools and they have to take responsibility for planning and achieving improvement.
- It is best to focus on the implemented curriculum, what actually happens in the classroom. Here the expert is not the university mathematics educator, but the teacher, in particular the experienced master teachers.
- It is best to have targeted, not generic, interventions, addressing the priority needs of the school.
- Clear metrics on improvement and regular evaluation.

The validity of this approach for the school system in the Philippines is shown by another major effort at school reform:

Another example: the Work of Synergeia Foundation⁵

Synergeia Foundation, Inc. focuses on improving basic education in about 200 municipalities in the country. I am the Chairman of the Foundation. Its approach is along the same lines as the TEEP. Synergeia has focused on building leadership and community support through the following:

- Focus on Local School Board (Provincial, City, Municipal). This is co-chaired by the mayor or governor and the Department of Education superintendent
- Engage whole community in assessing the situation, setting goals, deciding on priority objectives

⁵ I would like to thank Dr. Milwida M. Guevara, president of Synergeia Foundation and Theresa Anne F. Manalastas, Development Officer of Synergeia Foundation, for providing the information in this section.

- Provision of basic instructional materials (lesson plans for day to day use of teachers, workbooks for children, audio-visual materials)
- In-service training for teachers and principals.

The programs of Synergeia have already resulted in significant improvements in the reading and mathematics proficiency of elementary students, and more importantly, in local governance.

Synergeia in Bulacan. Synergeia began working in the Province of Bulacan in 2000 under the leadership of Governor Josefina de la Cruz. Over 620,000 pupils from grades one to six in 496 elementary schools in Bulacan are participating in the Synergeia program. In the 2000 National Achievement Test (NAT), pupils had an average score of 39.40% in Mathematics and 40.23% in English. Six years later, pupils achieved a NAT average score of 64.39% in Mathematics and 65.45% in English. To be noted is that this was from the start a large scale program, involving all the 620,000 elementary school pupils. We did not follow the pilot school approach to reform.

Synergeia in Lipa City, Batangas. Among the 17 communities that pioneered the reading proficiency program, the most dramatic gain was achieved in Lipa City, Batangas under the leadership of Mayor Vilma Santos. After one year, the Division Achievement test results showed that on the average, grade one pupils could already read at 54.0 percent, doubling their score in the previous year's exam.

In 2002-2003, Lipa City's overall average in the National Achievement Test was 44.85, in 2006-2007, it was 73.55. The English average in 2002-2003 was 40.15, in 2006-07, it was 73.06. The Mathematics Average in 2003 was 46.40%, in 2007, it was 76.32%. Again, this was a large scale reform involving all the elementary schools in the city.

3. *Continuing work*

Since the release of the results of the Third Elementary Education Project and the significant improvement coming from the Mathematics Lesson Guides, more and more schools and school districts have been adopting these Lesson Guides developed in 2002-2003.

Department of Education Officials and Local Governments have also noted the results of our work and we are being asked to implement similar reform efforts in their schools. The largest projects we have right now are for the whole of Quezon City (where the Ateneo de Manila University is located). This involves 98 elementary schools and 46 high schools and in 8 towns in the Province of Nueva Ecija (287 schools).

Further notes

We noted above that East-Southeast Asia seeks to learn how to strengthen creativity from the West. One way we are seeking to do this is through problem solving competitions that encourage higher order thinking skills. There are several noteworthy efforts:

The nationwide training programs and competitions run by the Mathematics Training Guild, which includes participation in regional competitions

- Regular training programs and competitions run by the Mathematics Teachers Association of the Philippines
- Our own training programs at the Ateneo de Manila University including Mathematics Olympiads with the Mathematics Training Guild
- We are beginning to introduce such mathematics competitions among the weaker schools we work with.

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Unpublished proc. of ICMI Conferences on Mathematics Education*

Unpublished proc. of SEACME Conferences on Mathematics Education*

*These proceedings were distributed only to the participants of the different conferences.

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Centres and peripheries in mathematics education: a dialogue with Professor Nebres' ideas about the theme Reaction to B. Nebres' plenary talk

I would like to thank the International Program Committee of this symposium for the invitation to establish a dialogue with the inspiring talk by Professor Nebres. The powerful ideas and insightful examples of projects developed in Philippines he described in his paper show us the challenges we are facing regarding the theme: Centres and peripheries in mathematics education.

My comments will focus precisely on these challenges. Since my research trajectory and my personal commitment have been attached to the Brazilian Landless Movement, with whom I have been working since the beginning of the 1990s, my dialogue with Prof. Nebres' ideas is based on what I have learned from this peasant movement in a peripheral country – one of the most socially unequal in the world. And here comes something which I find important to highlight: centres and peripheries – like developed and underdeveloped countries – are dichotomies which can obscure the complexity of the nowadays economic, social, political

and cultural configuration of the world. As argued by Hardt and Negri (2000, p. 15),

The sovereign power ultimately scrambled the spatial divisions between the First, Second and Third World, since we continually find the First World in the Third, the Third in the First, and the Second almost nowhere at all [...]. In the postmodernization of the global economy, the creation of wealth tends even more toward what we will call biopolitical production, the production of social life itself, in which the economic, the political and the cultural increasingly overlap and invest one another.

Thus, I would say that to different degrees, and according to their specificities, the educational challenges we are facing in countries like Brasil and Philippines have many aspects in common with what is happening in the so-called central ones. And this is why I appreciate so much what was written by Prof. Nebres, when he pointed out the importance of sharing with central countries lessons that we, from the periphery, learn from our economic, social and political disadvantages, which, of course, reach educational issues, in particular, mathematics educational ones.

I fully agree with him when he said that «the opening of ICMI to countries from Asia, Africa and Latin America also began to bring up concerns different from the central concerns coming from Europe and the North America». And he mentioned some of these concerns: Ethnomathematics, specially from Latin America and in particular the work of Ubiratan d'Ambrosio; *Mathematics for All*, coming in great part from UNESCO; the impact of society, economics and culture on mathematics education; and voices from Latin America and from Africa have emerged in the ICMEs and other international and regional conferences.

Prof. Nebres' paper – as well as the opportunity given to me to react to it – is a testimony of ICMI initiatives to have our voices heard. The ICMI study conferences organized in non hegemonic countries, like Brazil, in 2005, on the theme *The Professional Education and Development of Teachers of Mathematics* and in Hong Kong, in 2002, about *Mathematics Education in Different Cultural Traditions: A Comparative Study of East Asia and the West* definitely contributed to include other perspectives in mathematics education, which enriched our field of knowledge.

I would like to make a few remarks about Prof. Nebres' ideas regarding the repercussions to the central countries of the opening of ICMI to peripheral ones. The first is related to the field of Ethnomathematics. As he mentioned and it is well known it was the Brazilian colleague Prof. D'Ambrosio who introduced in the international context the importance of understanding mathematics and mathematics education in their relation to culture. In consonance with Paulo Freire's work on what is called, in Latin

America, *People's education*, D'Ambrosio, in his plenary talk at ICME-5, at Adelaide, presented to the international mathematics education community the ethnomathematics ideas he was developing since the mid 1970s.

To think about centres and peripheries using ethnomathematical tools will lead us to assume that different ethnomathematics, so, different mathematics educations are being produced by different cultures, by different forms of life spread on the planet. The opening of ICMI to countries from Asia, Africa and Latin America allowed the international community to be aware of the mathematics and the mathematics education developed in non hegemonic nations, giving us the opportunity of showing the specificities of their language games, but also their 'family resemblances', in Later Wittgenstein's words (2004). Such opening most probably produced new insights about a more inclusive mathematics education both in the centres and in the peripheries, as showed by the movement *Mathematics for All* mentioned by Prof. Nebres.

In countries like mine, where even nowadays we have thousands of children, young and adults excluded from the schooling processes and still a significant percentage of illiterate people, the idea of *mathematics for all* gains a special place. And Prof. Nebres is very much aware about the importance of maintaining alive this not so new idea. I share with him his arguments about the central role of socio-economic-cultural variables in mathematics education for so called developing countries. As I observe in Brazil, such variables are, ultimately, those which determine what can really be done in order to improve the access and permanency of the students in the school system. Of course, better curriculum designs and textbooks and improving the quality of in-service and pre-service teacher education are parameters that are contributing to developing education in the country. But they are subordinated to socio-economic-cultural constraints, as I learn in my work with the Landless Movement Schooling Project (Knijnik, 2006). This schooling project, according to one of the Landless Movement official documents (Kolling, Cerioli, & Caldart, 2002, p. 19), sees the need for «two articulated struggles: to extend the right to education and schooling in the rural area; and to construct a school that is *in* the rural area, but that also *belongs* to the rural area: a school that is politically and pedagogically connected to the history, culture, social and human causes of the subjects of the rural area».

The movement has dedicated itself to conceiving the schooling of its children, young and adults paying attention to these two struggles. This means that Landless educators consider peasant culture a key issue also for the teaching and the learning processes related to mathematics. But they explicitly mention that this cannot deny the relevance of acquiring mathematical tools connected to the Western European school mathematics – what they called *book mathematics* – since, as they explain, this is the math-

ematics that, different from their oral culture, has a social legitimacy that justifies its inclusion in the textbooks (Knijnik, 2007). The Landless Movement is very much aware that this hegemonic mathematics plays a key and complex role in the improving of the management of the production in rural areas and can allow the learners to go further in their schooling trajectory. But the macro social constraints they are facing are really interfering in the implementation of all these pedagogical principles.

It seems that the challenges involved in including (and maintaining) these marginalized parts of both peripheral and central countries in the educational system have some kind of connection to the idea of not restricting the mathematics curriculum to the recontextualized practices and contents of the Western European Mathematics. This means to consider the possibility of opening the mathematics curriculum to different ethnomathematics, which means to include also in our classes the mathematical knowledges produced by non hegemonic cultural groups, to which many of our students – in peripheral but also in central countries – belong. Unfortunately struggles against racism, sexism and other people's discriminations are still necessary. Mathematics is not a neutral curriculum subject. It is implicated in such discriminations and can play a role in the struggles against them. Reading Prof. Nebres' narrative about the *Third Elementary Education Project* and the work done by *Synergeia Foundation* I asked myself in what extension these issues were or were not considered in such projects.

There are other points about these projects that I would like to comment. I fully agree with Prof. Nebres position that «reform begins from the classroom, the implemented curriculum, and ideally the key players are the classroom teachers and school leaders. It is also a more evolutionary, rather than a revolutionary approach». His perspective is convergent with the in-service teacher education that the Landless Movement is developing in their eighteen hundred schools.

When referring to the Philippine educational changes, it called my attention Prof. Nebres position about the previous spiral approach to the discipline, which brought them, I quote him, «literally tearing apart the existing books and putting together the algebra parts, the geometry parts, and so on». I was wondering about how this very highly framed curriculum can help students to be more creative in mathematics, can contribute to their capacity of working with mathematical situations of the economic and social life which, if considered in a deep way, are in general extremely complex and may demand the use of sophisticated mathematical tools of different mathematics subfields. Moreover, for those students who want to and will be able to enter in a mathematics career or connected ones, how can this kind of deeply structured curriculum allow them to do better in their future professional life?

In his description of the important and so well designed and implemented project called *The Third Elementary Education Project*, Prof. Nebres mentioned that it was financially supported by the World Bank and the Japan Bank for International Cooperation. Based on our Latin American experiences, this kind of financial support may bring with it concrete political, cultural and social constraints. I was asking myself if such constraints operated or not in the Philippine project, since we are aware that the new world configuration is characterized by the 'absence of boundaries', in which «the rule of the Empire operates on all registers [...], extending down to the depths of the social world» (Hard & Negri, 2002, p. 15). And these depths of the social world include our everyday school and university practices, our classrooms, in summary, from the top to the bottom of the educational system. Then, I wonder about the repercussions of the links with the international bank system, for the work developed by the Third Elementary Education Project.

I found that it is very important – to centres and peripheries – to go deeper in our analysis about the impact of the global neoliberal economy in our everyday practices of teaching and researching, if we are interested in not accepting today's rules as the only possibility of the world configuration, in particular, of our educational systems; if we believe that it is possible to have more equal and inclusive relations among the South and the North, and inside our own continents and countries. It is clear how cultural, political and economic differences position countries in different places in our globalized planet. We are aware about how each of us, as an individual, assumes different subject-positions, based on one's cultural differences. But it is interesting to observe how the neoliberal configuration of the world subjects people from such different cultural contexts in almost the same way. Example of this is the assessment processes that are organized in order to 'measure' students, teachers and researchers locally, nationally and internationally. In the new economic, social and political designs that shape these times of Empire it is assuming new formats connected to a vast web of interests, which is enlarging its 'effects of truth', in Foucault's words. The strengths and limitations of international comparative studies in mathematics, like Pisa and TIMSS, have been analyzed in depth by the scholars from different places. Their reflections guided me to think about the evaluation process of the *Synergeia Foundation* projects at *Bulacan* and at *Lipa City* described by Prof. Nebres. What could mean, for the students and the teachers, as well as for the researches who participated in those projects, to be *measured* by the Philippines National Achievement Test?

Today it seems that the aims of the Enlightenment have failed. As pointed out by authors like Sarup (1996, p. 94), the ideas of linear progress, absolute truths, the rational planning of ideal social orders and the standardizations of knowledge and production embraced by Modernity have

been cruelly shattered. We find people of peripheral and central countries living in subhuman conditions, wars are being spread everywhere and nature is being destroyed all over the planet. Even so, or precisely because of this, our hopes of a more just and egalitarian world still remain. The 'old' project of Modernity is gone but new ones are being built. Most probably closer and less unequal relationships between centres and peripheries and also between different cultural groups in each country may exist. And Mathematics Education can contribute to them. Educational work which is being developed by social movements, such as the Brazilian Landless Movement, and the Philippine projects described by Prof. Nebres can be seen in this perspective. His reflections about centres and peripheries in mathematics education inspired me to think further about this theme.

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ICMI's challenges and future

I

MORTEN BLOMHØJ

Introduction

First of all, I wish to thank the programme committee for the invitation to this magnificent conference celebrating the first 100 years of the existence of ICMI, and for the opportunity to present what I consider to be ten major challenges for mathematics education research in the future – say, in the next ten years. For each challenge, I will discuss briefly why the challenge is important and what ICMI can do about it.

As we all know ICMI is not a strong organisation in terms of economy and ICMI cannot conduct or finance research through its own means. However, ICMI has important means of action: The ICMEs (International Congress on Mathematical Education) – with its continuing TSGs (Topic Study Groups) and DGs (Discussion groups), surveys; The ICMI-studies, ICMI affiliated studies groups, regional conference series, special international and regional conferences and the ICMI Bulletin to mention the most prominent. Through the synergy of these activities and the direct involvement of a large proportion of mathematics education researchers around the world, in my judgement, ICMI has a strong and still growing influence on our field of research. This is definitely a development worth celebrating.

The inspiration for selecting exactly the ten challenges I am going to present in this talk is hard to account for in detail. However two sources are clearly identifiable. Discussions with my good friend and colleague Mogens Niss has been a main source of inspiration. Actually all of the challenges I present here are mentioned – together with many more questions related to our field – in his closing chapter *Reflection on the state and trends in research on mathematics teaching and learning: From here to utopia* in the Second NCTM Handbook edited by Frank Lester (Niss, 2007). Another source of inspiration of a bit older date is Bent Christiansen, who left this world ten years ago and who was my Ph.D.-supervisor in the late eighties. Like Mogens Niss, Bent Christiansen was very active in the ICMI organisation – acting as a vice-president for more than ten years – and like Mogens

he was very engaged in the discussions about the nature and status of our field. I know that many of you have known him personally and worked with him in ICMEs, in the SCTP group (Systematic Co-operation between Theory and Practice) or in the Bacomet (Basic Component of Mathematics Education for Teachers) group (Christiansen, 1984, 1990).

I present the challenges in the order of decreasing generality. The first four challenges are all related to the nature and status in general of our mathematic education research as a discipline. The challenges 5-8 are related to the function of mathematics teaching and mathematics education research in societies. I end off with a more particular challenge related to some of my own research. The space allotted to this paper make it necessary to dealing only with a selection of these ten challenges, namely number 1, 2, 3, 4, 5, 6 and 10.

(1) The challenge of keeping the meta-reflections on mathematics education research alive

(2) The challenge of defining and strengthening the relations to the supporting sciences

(3) The challenge of avoiding isolation among sub-paradigms

(4) The challenge of supporting the interplay between research and development of practices

(5) The challenge of integrating mathematics in general liberal education for democracy

(6) The challenge of defining evidence based practices of mathematics teaching

(7) The challenge of mathematics education for all

(8) The challenge of improving teacher education and teachers' professional development

(9) The challenge of integrating ICT in mathematics education

(10) The challenge of integrating mathematical modelling in mathematics education.

Ten challenges

1. The challenge of keeping the meta-reflections on mathematics education research alive

This challenge is related to the nature of mathematics education as a research discipline. It is a fairly young discipline with complex relations to mathematics teaching practices and to other research disciplines. Therefore it is important for further development of the discipline to keep on posing and discussing meta-questions related to our discipline, such as: What is the object of study? What are the aims of research? What are the main

research questions? Which types of research results are produced? What criteria are or should be used to assess research?

These five meta-questions were in focus at the 12th ICMI-study entitled *Mathematics Education as a Research Domain: A search for identity* (Sierpiska & Kilpatrick, 1998, pp. 5-8). The study was framed by the general question *What is research in mathematics education and what are its results?* As a sort of general conclusion on the study the editors stated in the foreword (p. x) that:

No single agreed-upon definite answer to this question, however, is to be found in these pages As it turned out, the participants at the Study Conference were more interested in looking at the evolution of ideas and the problems yet to be solved than in taking stock of the field's achievements and results.

Ten years have passed but in my judgement this general meta-question is still relevant. The challenge is to discuss meta-issues, not necessary in order to reach agreement, but in order to foster reflections and critique in our research communities. Meta-reflections on the nature and the status of a research discipline are, as indicted by the quote, not produced naturally as part of research process in it-selves. Therefore special attention and initiatives are needed to keep the meta-discussion alive. I see at least four good reasons to do so:

Meta-discussion

- strengthens the identity and fosters reflections and critique among researchers
- is an important element in the education and enculturation of new researchers
- plays a role in legitimizing institutionalisation of mathematics education research
- is important in relation to the interplay with supporting research disciplines.

ICMI has a natural and important role to play in relation to this challenge. ICMI should maintain taking special initiatives such as the 12th ICMI Study, the continuing Discussion Groups involving meta-issues at ICMEs and conferences like this one with focus on the historical developments within our field of research. However, the effort could be reinforced by promoting integrated reflections on meta-issues in relation to ICMI-studies and other research conferences.

Of course, there are different possible answers to the meta-questions and this is exactly why it is important to keep the meta-discussion alive. For the framing of my considerations concerning the following challenges I quote Mogens Niss definition of the object of study for research in mathematics education:

The didactics of mathematics, alias the science of mathematics education, is the scientific and scholarly field of research and development which aims at identifying, characterising, and understanding phenomena and processes actually or potentially involved in the teaching and learning of mathematics at any educational level (Niss, 1999, p. 5).

According to Niss this object should be studied in its totality under the use of whatever approaches, methods, and results from other fields and disciplines that serve the purpose of uncovering and clarifying casual relationships and mechanisms in phenomena and processes of teaching and learning mathematics. I depict the object of study of mathematic education research, as represented in Figure 1, in order to emphasize that the core of mathematics education research is the students' interactions with some learning milieu planned and guided by a teacher under the intention of supporting the students' learning of mathematics in the broadest meaning. Of course, the teacher is not always present and the guiding can take many different forms. However, the intention of supporting the students' learning of some mathematics is for me a crucial element in the object of study of our discipline. This core is institutionalised in schools and the interactions are realised under the influence of many 'factors' which are themselves part of the object of study. The most important are included in the illustration in Figure 1, but the list is open and new 'factors' may be added during the development of the discipline.

In my interpretation mathematics education research is an interdisciplinary domain of research studying this object using considerations, approaches, and methods from a range of other supporting research disciplines of very different nature, such as mathematics, pedagogy, psychology, linguistics, sociology, anthropology, history, philosophy, physics, ...

This creates very complex relationships with the supporting sciences, which constitutes the second challenge.

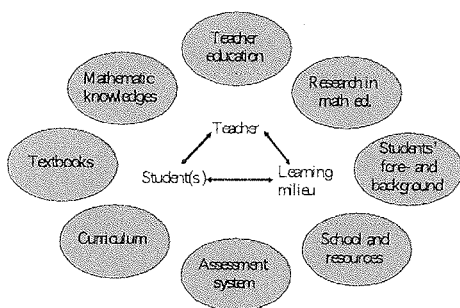


Figure 1. The object of study for mathematics education research. The 'factor' *Students fore- and background* refer to (Skovsmose, 1994, p.179).

2. The challenge of defining and strengthening the relations to the supporting sciences

This is important for several very different reasons. Due to the interdisciplinary nature of our field, scientific progress is to a large extent depending on interactions with supporting sciences. Mathematics education research needs to benefit from new developments in the supporting disciplines. However, theories or methods from supporting disciplines are seldom ready-made for application in a mathematics education context. Often, elements from 'supporting sciences' are integrated in investigative frameworks developed as part of and used in mathematics education research. Therefore, in order to enable such interplay researchers in our field need to have a quite profound knowledge about research in the supporting disciplines.

ICMI could play an important role in relation to this challenge, for example by organizing state of the art conferences or even ICMI Studies on the interplay with particular supporting sciences. Such initiatives could contribute to the development of mutual respect between researchers in mathematics education and researchers in supporting disciplines, which in many situations seem to be the most needed. A non-trivial prerequisite for lifting this part of the challenge is to find researchers from the supporting sciences that are interested in investigating and discussing the actual and potential contributions from his or her field to the mathematics education research. In order to develop truly interdisciplinary co-operation with researchers from supporting discipline we also need to focus on the possible benefits for the supporting disciplines – what can they learn from working in the field of mathematics education. Acting on an organisational level ICMI might be able to create fruitful contacts to some of the supporting sciences in order to facilitate such forms of co-operation.

On a more political level the relationships to the supporting disciplines are very important for the integration of mathematics education research in academia and thereby for the institutionalisation of our research field. We should strive for establishing mathematics education research as an interdisciplinary research discipline with very close relations to mathematics with strong connections to a number of basic research discipline form the humanities and social science, which therefore, by nature should be highly recognized by mathematics education researchers.

The relation with mathematics as a research discipline is a special case because mathematics plays an important role in relation to the object of study of our discipline and because mathematics education research is sociologically and institutionally married to mathematics. As clearly illustrated in the history of ICMI this marriage has not always been a happy one. However, divorce is impossible and also not desirable in my view. There is a strong mutual interdependence between mathematics and mathematics education research. The societal importance of mathematics is totally

depending on general mathematics education and vice versa; mathematics is depending on the continuous development of the quality of teaching and learning of mathematics on all levels; and mathematics education research draw heavily on mathematics but is also contributing to the total body of mathematical knowledge.

Fortunately, on the top organisational level, the relationship between IMU and ICMI has improved and stabilised recently through important organisational changes. However, I think it is still important and necessary for ICMI to work for improving of the relations to mathematics on all levels.

3. The challenge of avoiding isolation among sub-paradigms

In my view mathematics education research has developed to a stage close to fulfilling the criteria of being a scientific discipline but definitely not a mono-paradigmatic one. Different theoretical and methodological frameworks with the characteristics of sub-paradigms have been developed through the latest decades. APOS-studies, Theory of Didactical Situations (TDS), Mathematical Praxeologies, Realistic Mathematics Education, and socio-mathematical paradigms are some examples. This tendency is an expression of specialisation and refinement of theoretical frameworks which is a general characteristic for sound scientific development.

Many forces support such a development: the research process in itself, the sociology of a research group or community, institutional processes and funding systems, systems of publication, and conferences. Moreover, this form of specialisation is often reinforced by language and cultural preferences.

However, this development can also lead to isolation and parallelism where similar, but still different, theoretical ideas and constructs are developed within different paradigms and in different milieus around the world without being compared and discussed in a common scientific debate.

I think that we are currently experiencing this phenomenon within our field of research. Some examples are: Mathematical competencies and mathematical proficiency as two different conceptualisations of what it mean to master mathematics; APOS-theory and the reification model of concept formation; didactical engineering and RME as basis for design; and different views on mathematical modelling: emergent modelling, eliciting modelling activities, modelling competency, and critical perspectives on modelling.

In order to avoid isolation and unfruitful competition for resources and attention among different sub-paradigms these forces need to be counter-balanced by forces of coherence and identity in our international research community, and ICMI has a very important role to play in such an endeavour.

Just to make sure, I am not advocating for giving ICMI the role of a judging panel in competition between different paradigms or theories. The ICMI principle of letting the thousand flowers blossom is a fine and sound one, reflecting the state and nature of our discipline. However, in order for our field to advance it is important that the theoretical frameworks developed are analysed, compared, and discussed in order to clarify similarities and differences and in order to facilitate critique.

What can ICMI do in relation to this challenge?

- Try to create a common understanding for what is happening in our field by putting more emphasis on survey studies
- Illuminate the differences and similarities between different sub-paradigms – a task for ICME surveys?
- Create conditions for interplay between sub-paradigms and between different methodologies through special conferences.

4. The challenge of supporting the interplay between research and development of practices

Obviously, if research in mathematics education should contribute to the development of mathematics teaching practices it needs to take its points of departures in problems that appear in these practices; but not necessarily always and not in a very direct manner. The history of our discipline shows many examples of developments of theoretical ideas and concepts that have proven very useful in analysing and designing practice even though their origin might be primarily of a theoretical nature.

At least at the present stage the main influence from research on the practices of teaching lies in the processes of research and development rather than in the application of research results. Therefore the interplay between research and development of practices of mathematics teaching should be given appropriate priority in our research. I think this relates very well with Boaler's (this volume) findings concerning the influence of research on the practice mathematic teaching.

The interplay between research and practice is a classic balance problem in our field. But as expressed by Bent Christiansen (1990, p. 1) I think as researchers in mathematics education we should take on the obligation always to reflect upon the relevance of our research in relation to the development of practices of mathematics teaching.

In other words the didactical theory should be constructed and evaluated in relation to the needs and difficulties that appear in the instruction, i.e., in the practice of mathematics instruction. Thus, didactical theory, seen as systems of viewpoints, must include suggestions for strategies that can aid consideration of, as well as opposition to, the instructional needs and difficulties, which the theoretical 'core' in question relates to.

Applying the very broad notion of theory as ‘systems of connected viewpoints’ given in the quote research and development work can be seen as two processes going in different directions, from the practice (as the core of the object of study) to theory and from theory to practice, respectively. I am advocating for including both processes and the relation between them as part of our research discipline. This actually creates a new challenge, namely to develop research methods that integrate the two process. Skovsmose & Borba (2004) present a methodology with this quality within a framework of critical mathematics education.

Of course, it is possible to disagree. Freudenthal (1981) and Wittmann (2004) have advocated for going further in the direction of restricting the purpose of mathematics education research to providing a direct basis for design of mathematics teaching units accompanied by related reflections. On the other hand we find Michael Otte (1974) in his contribution to the *Sinn und Bedeutung der Mathematikdidaktik* in ZDM advocating for a much broader and more theoretical orientated delimitation of our discipline.

It is important to support the interplay between mathematics education research and development of mathematics teaching practices in order to

- ensure the relevance of research for the development of mathematics teaching practices
- support the recontextualisation of research processes and findings in the development of teaching practices
- create conditions for collaboration between researchers and teachers

ICMI can contribute by

- arranging conferences with explicit focus on connections between research and developmental processes
- including teachers involved in developmental projects in ICMI activities to give voice to the teachers in research conferences
- legitimising developmental oriented research.

5. The challenge of integrating mathematics in general education for democracy, social justice, and equality

Due to globalisation both developed and developing countries experience a growing demand on education systems for providing the society with human resources for the continuing economic and technological development. Mathematics is – for good reasons I believe – pinpointed by political systems as having a key function in this endeavour. That is also reflected in the great political interest for international assessment surveys like PISA. Mathematics education is considered to be an important parameter in the economic competition among countries.

These are strong forces acting against other important goals of mathematics education such as contributing to the maintenance and develop-

ment of societies with democracy, social justice and equality, (coming from Denmark urge me to add that I am against the use of any undemocratic means to impose democracy on other nations), and the goal of providing each individual human being with relevant mathematical competencies for taking part in, contributing to and being critical toward developments in his or her society.

In order to counter-balance the societal demand on mathematics teaching to produce an adequate qualified work force for economic development we need (more) research that:

- investigates and illuminates the possible functioning of mathematics in education for democracy, social justice and equality
- investigates the actual functioning of mathematics teaching in different educational systems as a tool for disciplining and stratifying pupils
- develops and investigates practices of mathematics teaching with the explicit aim of empowering the pupils in relation to their society locally and at large.

Of course, these issues are already addressed within ethno-mathematics and in critical mathematics education research. As expressed by Jeremy Kilpatrick in his plenary critical mathematics education is now recognized as an important new development in our field (Kilpatrick, 2008). However, ICMI can contribute to the further development by

- launching an ICMI-study on critical mathematics education
- initiating studies on the functioning of mathematics education in society and culture
- working for the inclusion of mathematics education in general education research and in pedagogy.

6. *The challenge of defining evidence based practices of mathematics teaching*

This challenge is closely related to the previous. As a consequence of the growing political interest for the rationalisation of education to serve the economic and technological development of society mathematics educators and researchers will be faced with increasing demands for providing evidence for practices of mathematics teaching.

The health-care system is in the middle of such a process in large parts of the world. And it could easily be more scaring than a night mare to see the levels of evidence for medical clinical practice being applied to mathematics education. (see *Levels of evidence from US Preventive Services Task Force*, www.ahrq.gov/clinic/uspstfix.htm)

Bureaucratic implementation of such types of systems of evidence-based practices in mathematics education will undoubtedly lead to an immense reduction of complexity with devastating consequences for mathematics teaching. Therefore it is urgent that the mathematics education research

community develops a strategic research based answer to the demand for evidence based practices of mathematics teaching.

Alan Schoenfeld (2007) in his chapter on Method in the NCTM handbook has already provided us with a very good starting point for such an endeavour. He is using, exactly, methodology from pharmacology as inspiration and as an analogy in developing a framework for evidence based research and development in mathematics education.

It is important for ICMI to engage in this challenge in order to

- develop a research-based opposition against bureaucratic attempts to rationalise mathematics teaching based on unjustified reduction of complexity
- support the transition from focus on local-based evidence for the quality of practices of mathematical teaching to focus more on evidence and conditions for qualities in large scale implementations of practices of teaching.
- discuss in depth the meaning of evidence-based mathematics teaching practices - possibly following Schoenfeld (2007)
- produce examples of different forms of good practices.

7. The challenge of integrating mathematical modelling in mathematics teaching

Mathematical modelling and applications of models have become part of the curriculum in many countries especially at secondary level. However, the educational goals, which are to be pursued through this part of the curriculum, are often not explicit. In many countries serious problems with the implementation of modelling have been reported.

Research has produced many deep analyses of the possible goals and related justifications for the teaching of mathematical modelling at the various educational levels. In my interpretation, the three main arguments for teaching mathematical modelling are (Blomhøj, 2004):

(1) Mathematical modelling bridges the gap between students' real life experiences and mathematics. It motivates the students learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means to describe and understand real life situations.

(2) In the development of highly technological societies, competences for setting up, analysing, and criticising mathematical models are of crucial importance. This is the case both from an individual perspective in relation to opportunities and challenges in education and work-life, and from the societal perspective in relation to the need for an adequately educated workforce.

(3) Mathematical models of different kinds and complexity are playing important roles in the functioning and forming of societies based on

high technologies. Therefore the development of an expert as well as layman competence to critique mathematical models and the way they are used in decision making are becoming imperatives for the maintaining and further development of democracy.

In the research literature (for instance in the proceedings from the ICTMAs) you also find an overwhelming number of developmental case-studies documenting 'successful' local implementation of a modelling course, but these are seldom evaluated against explicit goals for the students' learning. Research has also uncovered a number of specific problems connecting to the teaching and learning of different aspects of mathematical modelling (Blum et al., 2007).

Rounding off

What I have presented here is my personal view of what I consider to be ten major challenges in mathematics education research for the near future. Other researchers may of course have other priorities. Some of the challenges addressed probably could have been claimed as major and pressing even ten or twenty years ago. This is, I think, the case with challenges no.1 and with the last three challenges. However, in general I think that the choice and formulation of the ten challenges reflect the current stage of development in the field.

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II

MAMOKGETHI SETATI

The purpose of this symposium is to celebrate the 100th anniversary of the International Commission on Mathematical Instruction (ICMI) and so we have heard much about its past, its milestones, what led to its creation and what its challenges were. As we continue to experience ICMI in its present with all its complexities, we are also creating history and in a hundred years from now the mathematics education community will gather again to celebrate two hundred years of ICMI and they will most prob-

ably talk about this meeting. We are therefore creating history. It is fitting therefore that as we think about ICMI's challenges and future, we reflect on the following questions: what is the future for ICMI? What should the future be? What future are we creating? What future can we create, given the limited resources that we have? What future do we want to see as ICMI? In this paper I use the pronoun 'we' to refer to the entire international mathematics education community that ICMI represents, and I see myself, as an African, as part of this community.

I was asked to present my personal vision for ICMI, the challenges that I think ICMI is facing now and what could be expected from it as regards these in the future. Asking questions is important and reflecting on our past as an organisation gives us an opportunity to ask more questions and to think about the future. Who are we as ICMI? What is it that we are doing? What do we want to achieve? When? How can/will we know whether we are making progress or not? In my view the answers to questions that relate to our goals are not clear in our current ICMI and I want to argue that in moving forward we need to begin answering these questions. There are many mathematics educators and policy makers particularly in developing countries who have never heard of ICMI. Even those who might have heard about ICMI do not really know what it does besides the congress which happens every four years.

If we look at the ICMI website we will notice that the purpose of ICMI is clear. ICMI is a commission of the International Mathematics Union (IMU) charged with the conduct of the activities of the IMU bearing on mathematical or scientific education. According to the IMU website ICMI «shall take the initiative in inaugurating appropriate programmes designed to further the sound development of mathematical education at all levels». It is also ICMI's role to secure public appreciation of the importance of mathematics. The above description makes clear why ICMI exists. What is not clear, however, are the goals of ICMI. Goals and purposes do not necessarily mean the same thing. Purposes refer to the rationale for ICMI's existence and goals refer to ICMI's targets or ambitions. Goals, therefore, would clarify what ICMI wants to achieve by when, while purposes answer the question why do we need ICMI. In moving forward ICMI should answer questions that relate to its goals. In clarifying ICMI's goals it is important to take into consideration the fact that while ICMI focuses solely on mathematical instruction, it exists in the context of the global community, which is rife with poverty, inequity and injustice. This reality calls for ICMI to exercise its mandate while taking into consideration the challenges that exist in the world. The poor will always be among us even as we do our mathematics education research or development.

During its 2000 Millennium Summit, the United Nations signed the Millennium Declaration, which states eight goals, which range from halv-

ing extreme poverty to halting the spread of HIV/AIDS and providing universal primary education by the target date of 2015. This very aggressive plan was agreed to by 189 Heads of States and governments from all over the world. The question here is how can we as ICMI contribute to this programme? There are three specific Millennium Development Goals that I believe are relevant for ICMI

- (1) Eradicate extreme poverty and hunger
 - Reduce by half the proportion of people living on less than a dollar a day
 - Achieve full and productive employment and decent work for all, including women and young people
 - Reduce by half the proportion of people who suffer from hunger
- (2) Achieve universal primary education
 - Ensure that all boys and girls complete a full course of primary schooling
- (3) Promote gender equality and empower women
 - Eliminate gender disparity in primary and secondary education.

I am aware that others may wonder why we should be involved at all. My view is that ICMI is better positioned to contribute to this programme in the area of mathematics education because it is the only organisation in which mathematicians and mathematics educators from all over the world working in different areas of research and using different perspectives can be united against the ills of the world. We as ICMI cannot afford to stand on the sidelines and not get involved. We know very well that education is important for the eradication of poverty, empowerment of women and gender equality. Furthermore, mathematics is used as a gateway subject for many careers and jobs. Therefore the costs of not doing anything as an organisation are too high and I argue that ignoring our global context is not an option.

ICMI activities

ICMI's main activities include the quadrennial International Congress on Mathematical Education (ICME) as well as the ICMI studies. Regarding the Congresses it is important to ask, where these have been held. So far none of the congresses have been held in the developing world. This in my view is a limitation for two main reasons.

(1) Mathematics educators in developing countries do not get an opportunity to participate and therefore cannot benefit from the interactions that happen at these congresses.

(2) If the ICMEs are held only in the developed world, the experiences and voices we will hear as an organization will be those of the devel-

oped world. If we don't go to the developing world, we will never get to know or even understand the complexities of mathematical instruction or doing research in these contexts. It will also become difficult for ICMI to know how it can contribute to mathematics education development in developing countries.

So far there have been 17 ICMI studies. The questions here are: What have these studies focused on? What has not been studied? Whose issues have been dealt with through an ICMI study, and whose issues are not dealt with? What does this mean? While all the studies have focused on mathematics education, none has focused on issues that are specific to the developing world. While I accept that this is not a deliberate exclusion of issues affecting developing countries I argue that the time is now right for us to shift our focus to mathematics education matters that affect the developing world. Mathematics educators in developing countries are part of our community and so we cannot continue doing our business as usual. I therefore recommend that in instituting future ICMI studies we should consider issues that are relevant to the developing world and can also help us contribute to the attainment of the Millennium Development Goals. I therefore recommend consideration of studies on the following issues:

- Mathematics education in the context of poverty.
- Mathematics education in multilingual and multicultural contexts
- Mathematical literacy
- Teaching and learning mathematics at primary school level.

We also need to consider the impact of the ICMI studies on mathematics education all over the world. While the ICMI studies are important, in my view they do not reach other important stakeholders outside ICMI such as policy makers. We are too quiet as ICMI! Or we talk only to ourselves! It is important that we start to take leadership and represent the voice of international mathematics education. We can start doing this by formulating policy statements on matters regarding mathematics education and promoting such perspectives among member countries. These policy statements should be informed by the ICMI studies. In my view, ICMI should at the end of each ICMI study formulate a statement that can be distributed to member countries. This will not be to dictate but it will be to assume our position as the voice of the discipline internationally. In so doing policy makers can have access to the work and then critique or engage with it. This, in my view, is taking responsibility as an organisation. The work that we do is not just for us but for the growth of mathematics education internationally. We have to be bold as an organization. At the moment we are too modest, too shy. ICMI should become the important link between member countries and the international mathematics education community, it should expand the member countries' knowledge of mathematics education issues and also provide them with up to date analysis of critical issues.

Where to from here?

I have made a few suggestions in the discussion above regarding what I believe ICMI should do. We should constantly evaluate our relationship with the IMU and check whether it is enabling or constraining. We also need to revisit the manner in which we run our general assembly. Our general assembly cannot continue to function in the manner that it is functioning currently. We have a short general assembly, and many representatives do not even know about ICMI's activities and what the general assembly is about. Currently the general assembly functions as a place at which the ICMI executive presents its report and then gets members to approve. This in my view is a limitation. The general assembly needs to be an interaction between member countries and the ICMI executive. There should be reports coming from both sides and a sharing of ideas between countries. The general assembly should also interrogate policy statements formulated for distribution to member countries. Member countries pay fees to the IMU, but many would not be able to pin point the benefits they receive for their membership. We have to make it worthwhile for countries to belong to ICMI.

In conclusion, it is clear that we cannot do everything that we want to do with the level of funding we currently have. The current primary source of ICMI's funding is an annual allowance from IMU, drawn from the dues of IMU member countries (who are also de facto members of ICMI). My view is that if we can align the work that we do with the Millennium Development Goals, we will be better positioned to attract funding to do the work that we want to do. Of course we need to think carefully about: how we can get better at what we do; how we can work smarter; how we can make the most of the limited resources we have. But more than anything we need to start doing our business *unusual*!

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ICMI: A century at the interface between mathematics and mathematics education

In this text, I first consider the situation of ICMI at the interface between mathematics and mathematics education along the first century of its history, evoking successively four periods: the first years of ICMI before the first World War, the renaissance of ICMI after the second World War, the eighties and nineties marked by an increasing distance between the two communities of mathematicians and of mathematics educators, and the last decade marked by the reconstruction of relationships and increased collaboration. I then discuss some crucial challenges that mathematics education and ICMI are facing now.

1. Introduction

We have celebrated the centennial of the International Commission on Mathematical Instruction (ICMI), investigating how ICMI has both reflected and shaped the evolution of mathematics education since its birth in 1908. We have entered into the details of the history of the Commission, trying to understand how this history has made it the commission that we know today; we have pointed out its strengths and its weaknesses, we have said what we expect from it and expressed views about its future. I think that for those who have been engaged in ICMI activities for years if not decades, these days have been emotional and rewarding, and first of all I would like to express my thanks to our Italian colleagues who have invested so much in the preparation of this Symposium, and to you all for your contribution to this memorable event.

Giving a closing lecture is not the easiest thing. All you had planned to say has generally been said and often expressed much more convincingly than you could do. Your mind is full of new ideas but you

had not enough time for elaborating them. I apologize in anticipation for the repetitions and for the lack of elaboration.

The perspective I had initially chosen for this lecture is reflected by its title: looking at ICMI as an institution at the interface between mathematics and mathematics education. I will devote the first part of this lecture to this aspect while, in the second part, I will come to some challenges today crucial for mathematics education. I will consider what can be expected from ICMI regarding these challenges, keeping an interface and connection perspective but enlarging it in order to reflect insights from this Symposium.

2. *At the interface between mathematics and mathematics education*

I was not so much aware of this particular status of ICMI and of its implications at the time of my first contacts with the Commission. But in so far as I was moving from its periphery to its centre, according to the language of communities of practices, I became more and more sensitive to this characteristic and to its implications. This interface position makes ICMI something unique and thus important: we all know today the importance played by interfaces in any kind of development, be it scientific, cultural, social or economic. But such a status also generates tensions or reveals their existence. ICMI has not been, is not, exempt from these, all the more as the world of mathematics and obviously even more the world of mathematics education are themselves multifaceted worlds, expressed through a multiplicity of voices, and at times, if not often, through dissonant voices.

2.1. *The first years of ICMI: a non problematic interface*

The origin of ICMI itself illustrates its interface status. It is well known that the first President of ICMI (officially known at that time under its French name *Commission Internationale de l'Enseignement mathématique (CIEM)* or the German version *Internationale Mathematische Unterrichtskommission (IMUK)*) was the outstanding mathematician Felix Klein whose interest for education cannot be denied. What is perhaps less known, but has been clearly set out during this Symposium, is that the idea of creating an international commission to enquire into mathematics teaching was first suggested by David Eugene Smith, often considered as the father of mathematics education, in an article published by *L'Enseignement Mathématique* in 1905. First a mathematics teacher in Normal Schools, Smith was professor at Teachers College at Columbia University at the time when ICMI was created. He had developed graduate programs for secondary and postsecondary teachers, including in these courses work in the history of mathematics, and two of Smith's students had been awarded the

first American doctorates in mathematics education in 1906. Smith would become ICMI Vice-President from 1912 to 1920 and then President, after the first World War, from 1928 to 1932.

ICMI was created at the fourth International Congress of Mathematicians with the enthusiastic support of the mathematicians; its task was the study of the progress made in mathematics teaching in the various countries, (see Klein, Greenhill, & Fehr, 1908, p. 446). The importance of the ICMI production in the first years of its existence (more than 300 reports), the rapidity with which key questions which are still today with us were raised and discussed, are impressive. It gives us some evidence that the collaboration between mathematicians and mathematics educators in the Commission was immediately effective. Tensions between communities are not visible in the traces of the first ICMI years. Gert Schubring reported in his historical lecture the initial opposition by Max Simon to Smith's project and how this opposition was eventually turned thanks to the intervention of the Italian mathematician Guido Castelnuovo, but as he made clear this was not a tension between a mathematics and a mathematics education perspective: both Smith and Simon were educators. Tensions unfortunately came very soon, and ICMI nearly collapsed, but these were political tensions due to the first World War that would durably affect the academic world.

2.2. The Freudenthal era: a new regime

As very well described by our Italian colleagues in their analysis of the creation of the CIEAEM (Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques) and of the relationships between CIEAEM and ICMI, the situation changed at the time of the ICMI renaissance after the second World War. The kind of objective report on curricula and teaching practices that ICMI was used to producing no longer fulfilled the expectations of mathematics educators who claimed that a new discipline had to be established, separate from the methodology of teaching mathematics, a new discipline with ambitions to become «a science in its own right, with its own problems both of mathematical and pedagogical content» (ESM Editorial Board, 1969, p. 284). Hans Freudenthal, himself a mathematician, a mathematics educator and also a writer as presented in the portrait gallery of the ICMI historical website (www.icmihistory.unito.it), was the artisan of the ICMI evolution. This evolution was not exempt of tensions, certainly reinforced by the personal way Freudenthal managed ICMI affairs, as evidenced for instance in the interview with Bryan Thwaites. In his book on the history of the International Mathematics Union (IMU) (Letho, 1998), Letho reports (see p. 259) how Henri Cartan, President of IMU at that time, complained of the lack of information about the activities of ICMI. He was not indeed informed,

before they were taken, of two major decisions that would shape the future of ICMI: the organization of the first International Congress on Mathematical Education (ICME) in 1969 and the creation the same year of the journal *Educational Studies in Mathematics*, the official organ of ICMI, *L'Enseignement Mathématique*, being considered by Freudenthal and other scholars such as André Revuz as no longer accessible to mathematics teachers. With Freudenthal as President, ICMI affirmed its need for autonomy: the clothes designed for it by IMU had become too narrow. It is interesting to notice that this need was soon acknowledged by IMU and the associated resolution of the IMU Executive Committee in 1969 quoted in Letho's book is quite interesting. It is indeed written that:

The IMU should continue its policy of paying special attention to educational questions through ICMI, in order to ensure that the creative mathematician and the educator do not work isolated from each other (Letho, p. 259).

In spite of the problems mentioned above, the collaboration between mathematicians and educators was effective at the time of the first ICMEs. The attendance to these congresses, and their proceedings, provide evidence of this (ESM Editorial Board, 1969), (Howson, 1973). There were a lot of discussions and debates about the so-called New Maths, but the dividing line did not separate the two communities. It divided each community. It is well known for instance that Freudenthal opposed the formal and structural views of the most radical tenants of the New Maths, and that this opposition was the source of what is today known as Realistic Mathematics Education (Treffers, 1993). And the same can be said about strong Italian figures such as Emma Castelnuovo (Castelnuovo, 1969). Her fascinating text in the Proceedings of ICME-1 describing what Chevallard would label today as a three months PER (Parcours d'Étude et de Recherche) (Chevallard, 2005) on barycentric calculus, starting from physical devices, is from this point of view especially insightful.

2.3. *The eighties and nineties: an increasing distance*

As anyone knows, the disillusion of the New Maths period were as high as had been the expectations associated with it. In countries strongly engaged in the reform, as for instance mine, these disillusion resulted in a reduced engagement of the most prominent mathematicians in the reflection on mathematics education and curricular issues, and conversely in the reinforcement of mathematics education as an emergent field of research. The disillusion made evident the limitations of didactic knowledge in supporting the reform and the negative consequences of it, thus the need for a deeper understanding of the teaching and learning processes and of their interactions. But, as far as research in

mathematics education developed, building its own conceptual and methodological tools, producing specific knowledge and developing specific language for expressing it, the distance between the two communities increased, and the communication became more costly. In many places, the collaboration between mathematicians and researchers in mathematics education decreased and even collapsed. Mathematicians did not desert the mathematics education world but invested more on competitions such as the Olympiads initiated in Roumania in 1959 and the nurturing of mathematical talents, and also on various interesting innovative actions for promoting the popularization of mathematics.

ICMI was still a structure at the interface, an interface between mathematicians and an increasing number of communities that tended to be institutionalized inside the mathematics education world. The creation of the first ICMI Affiliated Study Groups in the seventies and eighties evidences this phenomenon: PME, HPM, IOWME¹. ICMI's Executive Committees (EC), whose election were controlled by IMU and its General Assembly, are insightful from this point of view: the President was always a first rank mathematician and mathematicians with an interest in education were well represented in the EC, but mathematics educators with a diversity of fields of expertise were also well represented and had officer responsibilities, being Vice-President or Secretary-General. Under the Presidency of Miguel de Guzmán, the balance between the two communities inside the ICMI executive committee progressively evolved. New tensions also arose all the more as at that time the supposed influence of mathematics educators was considered by some mathematicians as an important, if not the major, source of the observed difficulties in mathematics education, leading to such extremes as the so-called Math War in the USA. In 1998, when Hyman Bass was elected as President, Bernard Hodgson as Secretary-General, and when I entered the EC together with Nestor Aguilera as Vice-Presidents, the tension was at its maximum. At the 1998 International Congress of Mathematicians in Berlin, the project proposed by ICMI for the section on Teaching and Popularization of mathematics had been partially rejected, and the Math War in some sense had entered the section. Voices asking ICMI to take its independence from a mother institution that expressed such mistrust were becoming stronger and stronger.

¹ PME: The International Group for the Psychology of Mathematics Education (1976). HPM: The International Study Group on the Relations between the History and Pedagogy of Mathematics (1976). IOWME: The International Organization of Women and Mathematics Education (1987).

2.4. *Entering a new phase*

Retrospectively this crisis was beneficial. It obliged the ICMI EC to deeply reflect about the nature of ICMI and what we wanted ICMI to be. This led us to reaffirm the strength of the epistemological links between mathematics and mathematics education, as pointed out by Jean-Luc Dorier in his reaction to Jeremy Kilpatrick's plenary lecture at this Symposium. At the same time, we were convinced that making these links productive needed combined efforts from IMU and ICMI; the relationships could not stay as they were. Since 1998, the situation has evolved very positively thanks to the joint efforts of the successive EC and especially of their presidents: Jacob Palis and John Ball for IMU, Hyman Bass for ICMI. Hyman Bass had the credibility of a outstanding research mathematician, being known as the father of K-theory, but he was also someone having much more than the 'peripheric interest' in mathematics education shown by his predecessors, to use the same expression as he did in his opening lecture at this Symposium, Hyman Bass claimed that he is not a mathematics educator and he was certainly right saying this, but he knows research in mathematics education from the inside not just as an empathic observer. This makes a great difference. For me, there is no doubt that without him for pushing and guiding the evolution, ICMI would not be the structure it is today, and I would not be serving it as ICMI President.

ICMI is thus entering its second century of existence still at the interface between mathematics and mathematics education but certainly stronger for playing such a role and for coping with its ambitions. For the first time next July, the ICMI General Assembly will elect the new ICMI EC on the basis of a list of candidates established in full consensus by a nominating committee representing the two communities. ICMI and IMU are officially collaborating on specific projects such as the *Pipeline Study* whose aim is to analyze the flow of mathematics students from entrance to tertiary education up to graduation and entrance in the professional world, and to understand the dynamics observed in different countries. The two institutions are more and more coordinating their actions in the developing world thanks to the Developing Country Strategic Group (DCSG) and Commission for Development and Exchanges (CDE) structures where ICMI is represented. As rightly pointed out in the report of the commission recommending the creation of the DCSG in 2003, less than one third of the existing countries in the world belong to IMU, and only some fifteen more are members of ICMI without belonging to IMU. One impediment to wider membership to IMU is the requirement of independent scientific activity, which is interpreted as being some kind of sustained presence in research mathematics, but all countries are engaged in mathematics education and are thus concerned with ICMI activities. An increased collaboration between IMU and ICMI was thus considered desirable for extending their outreach and making these institutions better serve the cause of both mathematics and mathematics education worldwide.

ICMI and IMU will soon collaborate on a new project: the *Klein project*, and I am very happy to have here the opportunity of giving a few words about it. What do we mean by the Klein project? Thinking of Felix Klein we all have in mind the book *Elementary mathematics from an advanced standpoint* that Felix Klein wrote having in mind the mathematical needs of teachers. What are today the mathematical needs of secondary teachers and how to fulfill them? How to make them aware of the deepest ideas underlying the mathematics taught or susceptible to be taught in secondary schools? Even if Felix Klein's book can be still considered as a reference, our answers cannot exactly be those given by him one century ago. New domains have emerged as for instance those linked to the development of computer sciences; other domains have substantially developed and play an increasing role in mathematical sciences such as probability and statistics; even for many classical topics, perspectives have been partially renewed. In addition to that, teachers' culture has changed and we have to take this change into consideration too. Moreover, our vision of the specificities of the mathematical knowledge needed by teachers has also evolved thanks to the research developed about teachers' knowledge, practice and professional development (Adler & al., 2006; Ball & Even, 2008; Hill & al., 2007). Due to these characteristics, such a project cannot make sense today without the collaboration of different fields of expertise, without the collaboration of mathematicians and mathematics educators.

This being said, a lot remains to be done if we want to come to a satisfactory situation, and I am fully aware that the gap between the communities will not be easily reduced. As has been mentioned several times during this Symposium, mathematicians have in some sense deserted many ICMI activities, they no longer find their place and role in these, and this Symposium itself does not escape this global tendency.

The challenges that ICMI has to face are nevertheless far from being limited to issues of interface between mathematics and mathematics education communities, as evidenced in this Symposium. This leads me to the second part of this lecture where I would like to focus on some crucial challenges, and to think about what can be expected from ICMI regarding these challenges in the light of our common reflection. For giving this lecture some unity, I will nevertheless keep in some way the interface filter, extending its meaning beyond that used up to now. ICMI indeed, as has been repeated these days, is a linking structure.

3. Looking towards the future: some crucial challenges

3.1. Improving and extending ICMI outreach

ICMI as a structure has evident weaknesses. Its members are countries; each country has an ICMI representative and a few countries have created

a sub-commission of ICMI. ICMI outreach depends thus on the number of affiliated countries and on the quality of the country representatives. These receive regular information from the Secretary-General, are asked to disseminate this information, stimulate participation and contribution to ICMI activities, but also inform ICMI of activities of interest in mathematics education in their country, and make suggestions. They need to be well networked in their own country and reactive. We have to confess that this is generally not the case, and that the situation has to be seriously improved if we want ICMI to play the role we would like it to play. Some years ago, the ICMI EC decided to appoint a regional area to each of its members for improving the communication with the ICMI countries, and also for exploring the possibility of extending ICMI outreach by involving new members. Thanks to this decision, several difficult cases were solved but the efforts of the EC also made clear that increasing the quality of the ICMI structure and enrolling new members requires local action and negotiations beyond the ambit of the mere EC. It is our collective responsibility.

3.2. Increasing accessibility and visibility of ICMI activities

Increasing ICMI outreach is also increasing the accessibility and visibility of ICMI activities and productions. Along this line also, substantial improvement is needed as ICMI activities remain too confidential. Today, internet technology offers new possibilities for disseminating information and making documents widely accessible, and we must use these new possibilities. I will just mention briefly three actions recently undertaken in that direction.

At its first meeting, in June 2007, the current EC decided to launch an ICMI Newsletter on the same model as the IMU Newsletter. Jaime Carvalho e Silva accepted to take this in charge and the first issue came out last December. We have now more than 800 subscriptions and to these have to be added the close to 200 subscriptions to the Portuguese version. Moreover we encourage the production of versions in other languages for better dissemination.

We are well aware that ICMI Study books are expensive even if substantial reduction is offered to individuals belonging to the ICMI community. We have asked Springer for free on-line access to ICMI Study books three years after their publication, and this request has just been accepted.

IMU is organizing the digitalization of all IMU archives and has offered ICMI to include its documents in the process. ICMI Bulletins, ICME Proceedings, the first ICMI Study volumes are now being digitalized at the University of Bielefeld as part of the IMU project.

3.3. Accessing a wider audience: the case of ICMI Studies

Extending ICMI outreach is also making ICMI activities and work of interest for a wider audience, and I would like to evoke briefly the case of

ICMI Studies. Some years ago, the EC decided to launch a reflection on ICMI Studies and asked Steve Lerman to make an independent analysis of ICMI Study books. The discussion which followed Lerman's report helped us clarify what we wanted to achieve through an ICMI Study: not the production of a research monograph on the topic for a researcher audience, but in some sense much more than that. To make clear this position, I will just quote from a document written recently to present to the International Programme Committee of ICMI Study 19 (on proof and proving in mathematics education) the views of the ICMI EC:

An ICMI Study does not only aim at serving one category of persons: that of educational researchers, but has to make the reflection, experiences and research developed around a given theme accessible and useful to a wider audience. This implies a reflective and critical position, and a specific attention to the language used. We would also expect that an ICMI Study be a source of inspiration for educational action through the presentation and careful analysis of insightful examples, paying attention to the diversity of educational contexts and cultures, to the diversity of social and economic conditions and means for mathematics education. Said in other words, an ICMI Study does not ambition to produce a handbook more of research in mathematics education, neither a collection of brilliant but personal papers. It is a collective work where the expertise of individuals is put at the service of a community.

3.4. Fostering the development of new relationships between centres and peripheries

Even if from its beginning ICMI has included developing countries, at least as associate members, the history and interests of ICMI have been for a long time shaped by the history and interest of some few developed countries. Of course, we must not forget that ICMI has been active in the developing world since the early sixties at least, its relationships with UNESCO reinforcing its sensitivity to the specific questions that mathematics education raises in such constrained contexts. We must neither underestimate the role played by personalities such as Ubiratan d'Ambrosio in Latin-America and Africa, or Miguel de Guzmán when he created the ICMI Solidarity Fund, and many others I cannot mention here. But we cannot say that issues linked to the developing world have been at the centre of ICMI concerns during the first century of its existence.

This is no longer the case today. If I consider the actions that have mobilized a great deal of the EC energy in the recent years and the discussions we have had during the EC meetings, issues relating to the title of the Bienvenido Nebres' lecture have been a permanent focus of attention. This attention reflects for instance in:

- the choice of Mexico for the next ICME and the work attached to the preparation of this congress;

- the selection of sites for the conferences associated to the ICMI Studies (Brazil for Study 15, Vietnam for Study 17, Mexico for Study 18) and the simultaneous organization of activities for local teachers;

- the ICMI contribution to the UNESCO exhibition: Experiencing mathematics! (see www.MathEx.org) and to its travel in Southern Africa, South East Asia, and Latin-America;

the creation of new regional structures connected to the ICMI regional conferences series: EMF (Espace Mathématique Francophone) and AFRICME (Africa Regional Congress of ICMI on Mathematical Education) in Anglophone Africa.

All these actions not only attest to the increasing attention paid by ICMI to countries from the periphery, but also the new balance we try to achieve between two complementary dimensions:

- solidarity between the North and the South on the one hand,
- development of regional networking and support to regional collaboration on the other hand.

We are also more and more sensitive to linguistic issues. English has progressively established itself as the international language for communication. The CIEM/IMUK has become ICMI, the official language of the ICMEs, and of this Symposium, is English. A majority of participants to this Symposium are not native-English speakers but they are used to hearing and speaking this language due to their professional activities and status. This is not the case for many teachers and educators who have a lot to offer to ICMI activities and who can benefit from them. If ICMI wants to increase its outreach, linguistic issues cannot be under-estimated, all the more as the discussion of educational issues within a given language requires much more fluency than mathematical discussion. This was the argument at the origin of the EMF network for instance.

Beyond the development and support of such structures fostering collaboration and exchanges at the regional level, and allowing new parts of the world to benefit from and contribute to the ICMI community, beyond the organization of specific activities during the ICMEs allowing wider connections, ICMI has also to foster the critical development, capitalization and dissemination of knowledge regarding the specific challenges posed by the development of quality mathematics education in countries from the periphery. As pointed out in the discussion following the lecture by Bienvenido Nebres at this Symposium and the reaction by Gelsa Knijnik, actions such as those that they have reported raise a lot of questions. Up to what point, for instance, is there a tension between the attention to be paid to the students' culture and the attention to be paid to the mathematics knowledge and competencies that the social and economic integration in our modern societies require? I am not at all a specialist of these issues, but listening to the exchanges I had the feel-

ing that different positions were co-existing in the Assembly and I was asking myself what kind of construction, argumentation or evidence backed up these positions, what were their respective implications in terms of educational action, and how could ICMI contribute to make this clear beyond this particular community of specialists.

Another role that ICMI has to play regarding centers and peripheries is to contribute to more balanced views and relationships, making the mathematics education community at large more aware of the advances reached in developing and emerging countries and more able to benefit from these. It is not by chance that one of the two ICMI Studies to be launched this year is devoted to the teaching and learning of mathematics in constrained conditions with a specific focus on multi-linguistic contexts. We all perfectly know that such an issue is of interest now for most countries due to migration phenomena, but within the EC, this project has been especially pushed by Vice-President Jill Adler from South Africa. She knows the importance of this issue, she knows that a lot of research has been devoted to it and that countries from the periphery have a lot to offer. She easily convinced us that the time had come to capitalize on that research both in its theoretical and empirical dimensions.

3.5. *Understanding the cultural diversity and benefiting from it*

Through the theme of centers and peripheries, I have expressed some essential values of ICMI that have permeated all the discussions and contributions in this Symposium: solidarity, sensitivity to the cultural and contextual dimensions of mathematics education. This sensitivity is not only expressed in the relationships between the developed and the developing world. One of the recent achievements of ICMI has certainly been, through the development of its East Asian network EARCOME², and the ICMI Study 13: *Mathematics Education in Different Cultural Traditions - A Comparative Study of East Asia and the West* (Leung, Graf, & Lopez-Real, 2006), a substantial contribution to the understanding of the specificities of the East Asian culture in terms of education at large and of the implications of these specificities for mathematics education. As pointed out by Hyman Bass in his presidential address at the opening ceremony of ICME-9, in contrast with mathematics as a discipline, mathematics education both as a field of research and a field of practice «has a variable and culturally based character» (Bass, 2004, p. XV). Understanding better the systems of values and beliefs related to education, mathematics and mathematics education, underlying mathematics education in different countries and regional areas, how these different cultural characteristics influence research and practice in the field, how they affect educational outcomes, is a pre-condition for pro-

² EARCOME: East Asia Regional Conference on Mathematics Education.

ductive exchanges and collaborations respectful of cultural diversity inside the ICMI community. This is also a pre-condition for making ICMI able to play the role of scientific and moral authority it needs to play in an educational world where international evaluations and comparisons (whose methodology and results require careful discussion and interpretation) increasingly influence political and curricular decisions. ICMI Study 13, through the understanding it gives us of the Chinese/Confucian culture comparing it to the Greek/Latin/Christian culture, was especially timely, considering the interest that international evaluations such as TIMSS and PISA³ have generated for Asian countries and the superficial way these international comparisons are often presented and used.

3.6. Consolidating the field of mathematics education as a field of research and practice

As pointed out above, the field of mathematics education is highly dependent on contexts and cultures. On the research side, this dependence has led to a multiplicity of constructs and theories developed here and there, some internal to the field such as the theory of didactical situations initiated by Guy Brousseau (Brousseau, 1997), who received the first Felix Klein ICMI Medal in 2003, others imported and adapted from other fields such as those directly inspired by Piaget's or Vygotsky's theoretical constructions (Sierpinska & Lerman, 1996). Seen from the outside, mathematics education as a field of research gives the image of a fragmented field and the status and quality of knowledge that research in this field produces or is able to produce remains unclear. Moreover, reading the literature, you can at times get the impression that depending on the author referred to, efforts linking fundamental research and practice can lead to rather opposite strategies and didactic decisions. Due to its position at the interface between communities involved in mathematics education, ICMI has a particular role to play here. It has to foster and support the development of coherent and integrative views, to help clarify commonalities and differences. It has to foster and support reflection on the nature of the knowledge produced by didactic research, its potential and limits. It has to foster the development of productive relationships between mathematics education as a field of research and mathematics education as a field of practice, which needs to take more seriously issues of scaling-up than has been the case so far, as was pointed out by João Pedro da Ponte in his reaction to the insightful lecture by Jo Boaler at this Symposium.

3.7. Acting as a moral authority

As was evident in this Symposium, mathematics education is a domain submitted to economical, social, cultural and political pressure. We can expect ICMI

³ TIMSS : Trends in International Mathematics and Science Study. PISA: Program for International Student Assessment.

to play the role of moral authority, supporting the mathematics education community when needed, drawing attention to the possible dangers of particular decisions and practices. I have already mentioned the case of abusive use of international comparisons and evaluations. The same could be said of abusive use of high stakes tests and assessment, of reductive views about research evaluation carried out by the idea of evidenced based research, or by the use of citation indexes and impact factors.

I will briefly evoke a first attempt in that direction. Some years ago, the ICMI EC has been approached by editors of journals in mathematics education complaining about the dangers of the increasing importance given to impact factors and citation indexes for research in mathematics education. Only one journal in the field, the *Journal for Research in Mathematics Education*, is recognized by the Social Sciences Citation Index (SSCI) of ISI Thompson. This led ICMI to write a policy document on the evaluation of quality of research in mathematics education that is now accessible on the ICMI website.

4. Concluding remarks

ICMI, as an institution, is certainly highly improvable. In the second part of this talk, I have pointed out some of the main challenges it now has to face, and I have briefly presented some of the actions we have already undertaken for facing these challenges. What is important to understand, and has been made clear by this Symposium where we have extensively reflected on the first century of ICMI, is that for an institution such as ICMI the scale of time is not the scale of our quotidian life. Neither is it the usual timescale of politicians. I have been working in the ICMI EC for ten years. I have worked on projects which were launched before that, and have contributed to their success. I know that those who will come after me will do the same. We certainly progress modestly and slowly, but we progress with coherence. We know where we are and we know where we want to go. We know that those who work with us share the same desire of improving mathematics education worldwide and consider, as I said in the opening ceremony three days ago, that a high quality mathematics education is a human right. We have clearly seen at this Symposium that the new generation is there, sharp, deep, and strong. We can be confident. ICMI is moving, still at the interface, but today at a much wider interface, fostering exchanges and collaboration between the diverse communities which, all over the world, can contribute to the improvement of mathematics education. ICMI will go on moving and improving.

For an institution such as ICMI, one century is just the age of maturity!

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*Some views on ICMI at the dawn
of its second century*

In my capacity as Secretary-General of the International Commission on Mathematical Instruction, it is for me a privilege and an honour, as the tradition has it for ICMI events, to say a few words at the closing of this symposium, thus bringing our gathering to its conclusion.

The idea of organising a special activity on the occasion of the centennial of the Commission has been in the air for a long time and was in fact among the very first projects discussed by the Executive Committee of ICMI that took charge as of January 1999. A first stage on the way to the ICMI centennial was the symposium held in Geneva in October 2000 and organised jointly by the University of Geneva and ICMI to mark the one hundredth anniversary of *L'Enseignement Mathématique*. This journal, launched in 1899, was immediately adopted as the official organ of ICMI when the Commission was established at the Rome International Congress of Mathematicians, in 1908. The fact that the first Secretary-General of ICMI, Henri Fehr, was one of the two founding editors of *L'EM* is surely not without connection to this selection. But possibly even more important is the coincidence of the spirit that animated the people behind the journal and those involved in the newly born Commission, namely a spirit of internationalism and scientific solidarity (see Coray and Hodgson, 2003). The programme of the 2000 Geneva symposium was built around three main themes — geometry, analysis and applications of mathematics — and the idea was to use, among other sources, the documents that had appeared in *L'EM* to reflect on the evolution of mathematics education over the twentieth century as illustrated at three different periods: 1900, 1950 and 2000.

As far back as 2001 the ICMI Executive Committee established preliminary contacts with Italian colleagues to explore the feasibility of celebrating the centennial of the Commission in Italy, the country that hosted

the event having witnessed its official birth. We were very pleased by the enthusiastic reaction to this suggestion among both the Italian communities of mathematicians and of mathematics educators. The centennial project gradually took shape over the following years, the International Programme Committee, under the chairship of Ferdinando Arzarello, being appointed by the end of 2004. At the IPC meeting held in February 2006, the scientific programme of this symposium was defined. A wider framework than for the Geneva symposium was adopted, one of the aims then identified being to achieve an adequate balance between the consideration of historical events that marked the life of ICMI and an inspiring reflection on the past and future contribution of ICMI to mathematical education. The Local Organising Committee, lead by Marta Menghini, came with the wonderful scenario of hosting the centennial celebration at the Accademia dei Lincei, the very birthplace of ICMI, and at the Istituto della Enciclopedia Italiana — not to forget a replication of the 1908 Congress excursion to Tivoli.

Organising a symposium such as this one is impossible without the dedication of a large number of people who agree to devote much of their time and energy, sometimes over a long period, to the success of the event. I wish to stress here the hard work of all the members of both the International Programme and Local Organising Committees. And in particular I would like to mention *Ispettrice* Lucia Ciarrapico, for the role she played in the contacts between the local organisers and the Italian Department of Education; Daniela Spagnulo, for her efficient handling of the heavy secretarial duties; and Mario Barra, an essential component of the Local Committee. I also wish to acknowledge the pre-eminent contribution of two Italian historian colleagues, Fulvia Furinghetti and Livia Giacardi, who are the leading figures behind the beautiful website about the history of ICMI now being developed. But above all I want to pay a very special tribute to two wonderful colleagues already mentioned, Ferdinando Arzarello and Marta Menghini, without whose wisdom, skills and dedication the ICMI centennial celebration could hardly have materialised.

In the closing lecture she just delivered, ICMI President Michèle Artigue highlighted aspects of the sometimes fluctuating relationship over recent decades between ICMI and its mother organisation, the International Mathematical Union (IMU), and identified crucial challenges the Commission is now facing in pursuing its mission. I would like to add to these some comments of my own, especially from the perspective of my experience as ICMI Secretary-General.

I heartily testify to the fact, already mentioned by presidents Bass and Artigue, that in recent years the links between ICMI and IMU have evolved very positively, after certain periods of dormancy and at times profound distance. This improvement can be seen for instance not only in the spe-

cific financial support provided by IMU for this centennial celebration, but also in the presence at this symposium of two IMU officers, President László Lovász and Vice-President Claudio Procesi. Still much of the relationship remains somewhat fragile, and it is crucial that these stronger links between IMU and ICMI become fully institutionalised, so that they do not depend just simply on the good will of some individuals who have official responsibilities at a given time, but really become part of the life and institutional memory of both bodies. It should be noted that the recent improvement of the links with IMU first started with a series of direct informal contacts, going back to 1999, between the then newly incoming presidents and secretaries [-general] of both IMU and ICMI, Jacob Palis, Hyman Bass, Phillip Griffiths and myself. This eventually led over the following years to the regular participation of the ICMI President and Secretary-General to some of the meetings of the IMU Executive Committee, as well as to the IMU General Assembly as observers; also, as a symmetrical counterpart, the IMU EC delegated one of its members as a liaison with ICMI, and the IMU President attended some of the ICMI EC meetings. It is crucial that both IMU and ICMI remain sensitive, as institutions, to the importance of maintaining such links and direct contacts on a regular basis. While these can also materialise in the context of specific joint initiatives such as the Pipeline or the Klein projects mentioned in Artigue's lecture, it is essential to maintain regular face-to-face meeting fora like the IMU and ICMI ECs meetings.

The improvement of the links between IMU and ICMI can help resolve an important challenge ICMI is now facing as an institution, namely that of the administrative support necessary to allow ICMI to fulfil its mission. In many ways the 'life of ICMI' has become much more complex over recent decades. Not only is it the case that ICMI has substantially increased its span of activities, but the environment in which all this is happening has evolved dramatically. Everyone has experienced personally, for instance, that the technologies of information and communication, while providing extraordinary working tools supporting productivity, can also become terrible 'masters'. It was very interesting, in the interviews I made recently with two former ICMI EC members, Bryan Thwaites and Secretary Geoffrey Howson, to listen to their comments about the ICMI working environment they experienced and the kind of links the EC then had within itself and with the community: many things seemed to then happen in a more leisurely way and with much fewer interactions between people than may be the case now — if only because of the absence of email. The expectations today are totally different and it is thus hard to conceive how ICMI could continue to play its role without substantial administrative support. Still ICMI obviously does not have the financial capacity to set up such a framework of its own and continues to rely on the voluntary work of many, in particular of

the four ICMI EC officers, including of course the Secretary-General. In this regard, a very positive development occurred recently, when the IMU EC offered ICMI to be supported by a permanent secretariat that the Union is setting in place. Call for bids to host such a secretariat was recently launched to all IMU members, and ICMI was invited to define the kind of clerical support it would wish to obtain in such a context. It will still take a few years before this project materialises, but it clearly points to a better working environment for the coming ICMI ECs.

Not entirely independent of the preceding comments is the issue of the long-term financial stability of ICMI. Over the last decade, ICMI has experienced a non-negligible yearly average deficit. While the assets of ICMI allow us to face this situation for some time yet, it is clear that in the long term this leads to an impossible situation, unless the revenues of ICMI are increased substantially. It is probably no longer possible to rely essentially on IMU to provide ICMI with the financial infrastructure it needs to support its programme of activities and publications. On the other hand, in spite of the financial help brought by UNESCO for this symposium, expecting bodies such as UNESCO again to become able to provide financial support of the level they could afford, say, in the 1960s or 70s, is clearly not realistic. The ICMI EC has started reflecting on possible lines of action to meet this vital financial challenge.

I will not go back over details of the 'crucial challenges' identified by President Artigue in her talk, when looking towards the future. Still I wish to stress how fundamental is the evolution in the infrastructure and governance of ICMI, with the election of the ICMI EC now passing under the responsibility of the ICMI General Assembly. The 2003-2006 ICMI EC, which discussed this new scheme with the IMU, believed strongly in the importance of this election happening in a context of close collaboration between the IMU and ICMI communities, but with ICMI making the final decision. We see this as an essential ingredient of the 'good health' of ICMI, within itself and as regards its links with the IMU. Another crucial challenge concerns the contacts of ICMI as an organisation with the community it serves, so to face the needs and expectations of the 21st century. This clearly points to the improvement of communications through the Internet — or whatever new channel may appear in the future (one should not forget that ICME-8, in 1996, was the first ICME congress with a website). It is also connected to the 'ICMI Digital Library' project, supported technically by IMU, through which 'all' the literature produced in the context of ICMI activities (including for instance proceedings of ICMEs and ICMI Study volumes) will eventually become available freely on the Internet. A recent agreement with the publishing house of the ICMI Study volumes points to very positive developments in that connection.

In spite of the substantial challenges ICMI is now facing, I strongly believe that the history of its first century, and in particular the recent evolution of its infrastructure and programmes, point to what I perceive as a very promising and rich future.

On this note I am coming to the final part of my closing remarks. Returning to the cultural and social context that witnessed the birth of ICMI, it is possibly appropriate for me to say, thinking of the incarnation of ICMI as *IMUK* — *die Internationale Mathematische Unterrichtskommission*:

Meine Damen und Herren, liebe Kollegen, in den letzten Tagen haben wir an einem ungewöhnlichen Ereignis teilgenommen — in einer wundervollen historischen Umgebung. Ich bin sicher, dass es für Sie alle eine schöne und höchst lohnende Erfahrung war.

And then to add, having in mind the French version *CIEM* — *la Commission Internationale de l'Enseignement Mathématique*:

Je vous remercie de votre participation active à cette rencontre et d'avoir ainsi contribué à son succès. J'espère avoir le plaisir de vous revoir bientôt et vous souhaite à tous un agréable voyage de retour à la maison.

But finally coming back to today's *lingua franca*, to conclude:

I now declare the symposium celebrating the first centennial of ICMI closed.

Arrivederci!

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The participants to the Symposium. Roma, Accademia Nazionale dei Lincei.

Short Talks

COORDINATORS
ALAN BISHOP and LEE PENG YEE

EILEEN F. DONOGHUE

*The inception of ICMI:
David Eugene Smith and the founding of ICTM*

The broad role of David Eugene Smith in the founding of the International Commission on Mathematical Instruction (ICMI) one hundred years ago is not widely known. This paper introduces Smith and outlines an account of that role. The first section offers a brief profile of Smith and his professional career. The second section is presented as a question and answer ‘interview’ of Smith regarding his contributions to the founding of ICMI (also known as CIEM and IMUK). Smith’s answers have been gleaned from his published papers and his non-published memoranda, notes, and correspondence. They provide a behind-the-scenes glimpse of early efforts to organize the Commission for its work.

1. *Profile of Smith*

David Eugene Smith (1860-1944) was born in Cortland, New York. His father was a surrogate judge; his mother was an educated woman who taught her son Latin and Greek. Smith studied at nearby Syracuse University, receiving B.Ph., M.Ph., and Ph.D. degrees. As a young man in 1883, Smith embarked on an adventurous life of foreign travel, first to Central and South America and, two years later, to Europe. Eventually, he traveled to over 70 different countries. His fondness for travel abroad and his ability to garner resources to undertake these journeys were to become important factors in his chosen career, and particularly important to his role in founding and organizing the International Commission on the Teaching of Mathematics (ICTM), predecessor of ICMI.

In the early 1880s, Smith apprenticed as an attorney in his father’s law office, but he was not content with the work. A chance invitation to teach mathematics at Cortland Normal School, a local teacher training institution, led Smith toward a vocation that he could embrace. Smith’s frequent

travels provided an opportunity to collect mathematical manuscripts, textbooks, and instruments, which he used in his classes. The history of mathematics became a primary area of study for Smith and for his students (see Donoghue, 1998).

In 1891, Smith went to Michigan State Normal School in Ypsilanti (now Eastern Michigan University) to head a mathematics department in an institution on the rise. For a brief period beginning in 1898, he headed the normal school in Brockport, New York, but the offer of the chair in mathematics at Teachers College, Columbia University, drew Smith to New York City in 1901. Within five years, he had developed the first doctoral program in mathematics education in the United States and had celebrated the award of the Ph.D. to two graduate students. Three characteristics distinguished Smith's program for mathematics education: the prominent role given to a historical perspective on mathematics and teaching; the emphasis placed on acquainting oneself with international viewpoints on education; and an expectation that the teacher would take an active part in shaping mathematics as a school subject (see Donoghue, 2001, 2003).

During the period 1908 to 1912, Smith devoted much attention to assisting Felix Klein in organizing ICTM and to leading the American Commission in its work. Other professional contributions included long-term service (1902-1920) as librarian of the American Mathematical Society and associate editor of the *Bulletin*; president (1920-1921) of the Mathematical Association of America and associate editor of *The American Mathematical Monthly*; and founding president (1903) of the Association of Teachers of Mathematics in the Middle States and Maryland (a regional forerunner of the National Council of Teachers of Mathematics). Beginning in 1904, Smith joined Felix Klein, Sir George Greenhill, and others as a member of the Comité de Patronage for *L'Enseignement Mathématique*, a journal co-founded and edited by Henri Fehr. In 1928, Smith was elected to a four-year term as president of a reconstituted ICTM.

2. Q & A with Smith

Question: Professor Smith, you attended the International Congress of Mathematicians at Heidelberg in 1904. What was your reaction to that Congress?

SMITH: «I admit that I was disappointed by the nature of the work presented in the section on teaching at the Congress at Heidelberg. It did not seem to me that the subjects were considered in their true pedagogical sense. It was more often a question of certain mathematical details, and not a careful study of the general problems of mathematical education» (Smith, 1905, p. 471, E. F. Donoghue, trans.).

Q: Did you have a suggestion at that time for remedying this neglect?

SMITH: «I believe[d] that the best way to strengthen the organization for teaching pure mathematics would be to create a commission which would be named by an international Congress and which would study the problem as a whole....It seems to me that if the question were considered from an extensive and international point of view, it would result in some very useful suggestions, without bringing about a uniformity in the organization of studies» (*L'Enseignement Mathématique*, 1905).

Q: You spoke at the Rome Congress in 1908. Did you mention this idea there?

SMITH: «[I stated that] in closing, I should like to express the hope that these International Congresses may add to their already great value as clearing houses of thought, by sometime investigating, through committees, a few questions relating to secondary education. Countries cannot be uniform in their curricula, their school systems, nor their methods of teaching, but the influence of a Congress like this might greatly help many who are earnestly seeking to improve the teaching of mathematics....[S]ome of us would be glad if this Congress might deem it advisable to appoint international committees...to consider matters of this kind. An agreement is not essential, but the interchange of views and suggestions could not fail to b[e] very helpful» (*Atti del IV Congresso internazionale dei matematici, Roma, 1909*, pp. 476-477).

Q: What was the reaction to your suggestion?

SMITH: «At the conclusion of the paper Dr. Archenhold, of the Berlin Observatory, whom I had met in New York a couple of years earlier, called attention to the proposal made for an International Commission. His remarks were very favorably received, and the matter was a good deal discussed for the next two days» (ICTM: Memoranda, notes, etc., n.d.).

Q: Was there any opposition to it?

SMITH: «No one had any objection to it, except Max Simon of Strasburg, and the only trouble was that everyone wanted to go on the committee» (ICTM: Memoranda, notes, etc., n.d.).

Q: So what happened after two days of discussion? Was it all just talk?

SMITH: «It was finally decided by a few of us that the matter would be brought up at the close of the last meeting of Section IV. Unfortunately Simon [who had objected to the proposal] was then in the chair, and while we were talking it over in the lobby he adjourned the meeting» (ICTM: Memoranda, notes, etc., n.d.).

Q: Well, did that end it?

SMITH: «[No,] I then asked [Federigo] Enriques and [Guido] Castelnuovo to call the meeting again to order, as they had the official right to do. This was done and nominations were made for the Commission. These I opposed» (ICTM: Memoranda, notes, etc., n.d.).

Q: Why did you oppose open nominations from the floor?

SMITH: «[I said] that we were not prepared to consider the matter with sufficient care at that time. I therefore proposed that we elect a committee on organization, with the understanding that this committee constitute in itself the officers of the Commission, and that it make its selection of other members in such manner as it wished, consulting such learned societies as seemed to it best. I nominated Professor Klein of Göttingen, Sir George Greenhill of London, and Professor H[enri] Fehr of Geneva» (ICTM: Memoranda, notes, etc., n.d.).

Q: Why these particular individuals?

SMITH: «The reason for nominating Professor Klein was not only that he occupies a peculiar position of prominence in the mathematical world, but that he and Professor Gutzmer had recently undertaken some similar investigation in Germany, and Professor Gutzmer (whom I consulted) agreed that Klein was the best possible man for president. Greenhill was named because of his eminence in applied mathematics and because the next congress is to be held in England. Fehr was nominated because he is the working editor of *L'Enseignement Mathématique*, which will naturally be the organ of the Commission» (ICTM: Memoranda, notes, etc., n.d.).

Q: Those were strategic choices. How was your motion received?

SMITH: «Since I spoke in English, it was necessary that my motion should be put into Italian or French, and this translation was made by the secretary, Professor Castelnuovo, so that the motion appeared in his name although first made by me—a matter of no moment. Professor Castelnuovo then, after its adoption in Section IV, carried the motion to the general meeting where it was unanimously adopted» (ICTM: Memoranda, notes, etc., n.d.).

Q: So you were not a member of the original organizing commission?

SMITH: «At the time of the nominations several suggestions were made in the meeting, to the end that I be a member. These I declined to consider, for the obvious reason that the success of the movement depended on the subjection of all personal feeling—that the best men should be named independent of the chance fact that one man suggested the matter, that another spoke upon the question at the meeting, and so on» (ICTM: Memoranda, notes, etc., n.d.).

Q: Did you have any further role in determining the direction of the Commission's work?

SMITH: «[The day after the motion was adopted,] I had an interview with Professor Fehr and outlined my ideas as to the work of the Commission, and on the Sunday following I talked the matter over with Sir George Greenhill at Tivoli. The subject was then left in Professor Fehr's hands.... After the Rome Congress I went to Florence for six weeks, and then to Munich for about a month. [M]y colleague, Professor [Thomas] Fiske... had gone to Göttingen. [Fiske and Fehr sent letters urging me to go to Göttingen to consult with Klein.] To each I said that I would be glad to change my plans and go to Göttingen for a couple of weeks» (ICTM: Memoranda, notes, etc., n.d.).

Q: What happened when you got to Göttingen?

SMITH: «At Göttingen I spent about three hours a day with Professor Klein, going over the plans. I finally submitted [a draft] as a basis for discussion. The...draft was gone over by Professor Klein and myself, and I then rewrote it. It was agreed that the Commission should receive copies of my second draft and that Professors Klein, Greenhill and Fehr should settle upon the matter at a meeting to be held in Cologne on September 28, 1908» (ICTM: Memoranda, notes, etc., n.d.).

Q: What about the membership of the country delegations? How was that determined?

SMITH: «At Göttingen I went over with Professor Klein the question of the membership in the various countries. It was agreed that Perry would probably have to go on in England. He [Klein] wished me to go to Ghent and talk with Mansion as to the Belgian delegation, and this I did later, and to go to Paris and see Tannery, which I did.... [Tannery] thought that Borel and Appell and Hadamard would be good men, but said he couldn't go on himself. I communicated all this to Klein.... In London Sir George Greenhill was very kind to me personally and developed a good deal of interest in the work» (ICTM: Memoranda, notes, etc., n.d.).

Q: That entailed a substantial amount of travel. Did you attend the September 28 meeting of the Commission in Cologne?

SMITH: «[No.] I sailed from London September 5, 1908, and reached New York on September 13, after an absence of 15 1/4 months» (ICTM: Memoranda, notes, etc., n.d.).

Q: A final question, Professor Smith: You worked closely with Felix Klein to organize ICTM. How would you assess his leadership?

SMITH: «Commanding the respect of the entire mathematical world,

having given particular attention to the great problems of teaching in the past few years, and being posses[ed] of an indomitable energy, Professor Klein was the one man for the position of leader....[His degree of success] surprised even the most optimistic supporters of the Commission» (*Educational Review*, 1913, p. 1).

3. Conclusion

Indeed, the Commission was a success. At the 1912 Congress in Cambridge, over 150 reports containing thousands of pages on the teaching of mathematics from kindergarten through advanced university level were submitted by numerous countries. Smith expected that able teachers would welcome the opportunity to examine these reports and act on the findings. In doing so, they would be fulfilling the three criteria that Smith believed essential for the teacher who wished to progress in the profession: to gain a historical perspective on mathematics and its teaching, to consider international viewpoints, and to take an active role in shaping mathematics as a school subject. His work on behalf of ICTM contributed not only to his students' opportunities for advancement, but also to the establishment of mathematics education as a field of study.

One hundred years ago, leading mathematicians and mathematics educators from around the world joined in an investigation not of mathematics itself but of how and why it was taught. Efforts in a similar vein continue today, as ICMI celebrates its centenary.

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ALAN J. BISHOP

ICMI's past four decades: consolidation and diversification

1. *Introduction – the sixties*

The sixties was a time of new ideas, new horizons, and new challenges. The Russians had surprised everyone in 1957 by sending a man into space, and suddenly the 'space-race' took off. The Americans were aghast – «We must improve our science and mathematics teaching» they said. So 'New math' appeared along with 'new science', together with a plethora of new initials – SMSG (School Mathematics Study Group), UICSM (University of Illinois Committee on School Mathematics), PSSC etc. It was curriculum development at high speed, based on the traditional research-development-dissemination model used by successful industries in the USA.

In the 60s communication technologies had been growing, CCTV was becoming much more accessible, computers were growing in number, and B.F. Skinner was developing programmed instruction and teaching machines.

Behaviourism and technological developments were all the rage, and mathematics educators were responding and adapting their practices accordingly, albeit reluctantly in some cases. But change was in the air, and ICMI was part of the change.

Mathematics education was starting to be a discipline in its own right, and not just an appendage to the world of mathematics. Hans Freudenthal led the charge against the assumptions of certain mathematicians in the IMU who believed that mathematics instruction was also their area of expertise. By the end of the 60s, not only had *L'Enseignement Mathématique* been published for 70 years, but Freudenthal had also begun a new jour-

nal, *Educational Studies in Mathematics*, in 1968, and it published articles on mathematics education in English, French and German.

2. The International Congresses on Mathematical Education

International organisations such as CIEAEM which started in 1950 had demonstrated the usefulness of international conferences and collaborations in mathematics education. CIEAEM was a European-based grouping with English and French as its principal languages. The first International Congress on Mathematics Education (ICME) took place in Lyon, France in 1969 with some 650 participants, and since then the ICMEs have become regular 4-yearly conferences. To many mathematics educators around the world, the ICMEs are the public face of ICMI.

Two words are significant to me in the ICME title – the word ‘Congress’ signifies more than just a conferences, and recognises that if one is to facilitate international exchange of ideas in mathematics education, one needs to organise a meeting where people don’t just deliver academic papers ‘at’ each other. Also it recognises that mathematics education is a professional practice, not just an academic pursuit, and if the meeting is to facilitate the sharing of practical ideas, other communication vehicles should be found. The other significant word is ‘Education’. This was hugely important in the early days of the ICMEs as it signalled the wider canvas which ICMI was drawing on rather than the narrower ‘instruction’ which IMU considered relevant when it created the International Commission on Mathematics Instruction.

Despite the importance of these two words in the title, the program of the first ICME, held in France, very much followed the traditions of those of the IMU. In the planning for ICME2, this problem was noted and the format changed. Plenary lectures remained but this time there were only 6, compared with 20 at Lyons. The 15 minute contributions were scrapped completely, and in their places were national presentations.

In the 60s and 70s the core of ICMI’s activities was principally European, and this core was reflected in the fact that following the 1969 ICME in Lyons, France, the 1972 one was held in Exeter, UK, in 1972, and the 1976 one in Karlsruhe, W. Germany. This European focus built on the previous connections of international groups like CIEAEM, on the various European national meetings, and on the general cooperative frameworks being created following the disaster of World War 2. These included meetings supported by UNESCO, and the famous 1959 seminar in Royaumont under the aegis of the Organization for European Economic Cooperation (OEEC).

The 80s were a time of expansion of the international ideas and activities of mathematics education. In 1980 the ICME was held in Berkeley,

USA, and this increased the number of attendees from Canada, South America, and the 'Far East', meaning mainly at this time Japan, Hong Kong, and Taiwan. For many participants it was the first time they had heard about some of the outstanding mathematics achievements of students and their teachers from those countries.

The 1984 ICME was held in Adelaide, Australia, and this was the ICME where for the first time, the concept of 'ethnomathematics' was described internationally by Ubi D'Ambrosio. This concept highlighted not just the increasing internationalisation of ICMI's activities but also the increasing multiculturalism of mathematics education, by challenging the accepted views of the nature of mathematics itself.

This international development was given an extra boost in 1988 when the ICME in Budapest, Hungary, contained a whole day devoted to the theme of *Mathematics, Education, and Society*. During that Fifth Day Special not only were there topics related to ethnomathematics, but also there were topics focusing on historical aspects, social justice issues, policy issues and political discussions, as well as multilingual and multicultural educational situations.

The 1990s were a time to consolidate the achievements of the ICMEs in the 70s and 80s. In 1992 the ICME was held in Quebec City, Canada, and it achieved the highest number of participants of any ICME so far – 3407, from 94 countries. In 1996 the ICME was held in Seville, Spain, and in 2000 it was held in Tokyo, Japan. In this latter ICME the program consisted of the usual elements, with some additions, and the program reflected not only the increased number of participants, but also their diversity, and the desire to increase the possibilities for sharing ideas. Over 2000 people from 70 different countries attended.

3. The international consolidation of ICMI's structure

The international structure of ICMI is built on the fact that its members are not individuals but countries, and currently ICMI has some 72 member states, who generally have appointed national commissions in their own countries. This structure has facilitated the development of regional groupings in mathematics education, as well as regional congresses.

In general, as can be imagined, as the size and complexity of the ICMEs has increased, so has the cost of attendance. Given also that these congresses are organised in first-world countries (using predominantly English), participants from developing countries have to pay a high price to attend. Also it is often the case that the proceedings from these ICMEs are expensive, and also that they inadequately represent the ICME activities and outcomes. This has meant not only that the number of participants from developing

countries is low, but also they cannot easily get access to the content of the congresses through the proceedings.

In part this situation has helped to stress the importance, and increase the number and range, of regional international groupings and congresses. Two excellent examples are the Inter-American Committee on Mathematics Education (IACME), and the Southeast Asian Mathematical Society (SEAMS). IACME was created in 1960 and was the first sub-group established by ICMI. It has held a number of congresses throughout North and South America and has published its proceedings widely, and in Spanish. It is an active organisation and, as it has links with the Ibero-American Congresses of Mathematics Education (CIBEM), its influence is wider than just with the Americas.

SEAMS was formalised in Singapore in 1972, and has established contacts and links throughout the ASEAN region, and beyond. Originally a grouping of mathematicians, it held its first Southeast Asian Conference on Mathematics Education (SEACME) in Manila in 1978 with a focus on the training of mathematics teachers. It has overlaps with the East Asia Regional Conference On Mathematics Education (EARCOME) which has just celebrated its 4th conference this year in Penang, Malaysia.

A second general way that ICMI has widened its international structure has been through the development of International Study Groups. ICMI has consistently emphasised the role of Working Groups within its ICMEs, and these have sometimes grown into semi-autonomous groups, or have allied themselves with other existing discipline or academic groups.

Two of the earliest groups were the International Study Group on the Relations between the History and Pedagogy of Mathematics (HPM) group, and the International Group for the Psychology of Mathematics Education (IGPME or PME as it is better known) both of which were created in 1976 following their strong and active involvement in the first two ICMEs.

Later examples of international groupings have been the International Organisation of Women in Mathematics Education which started in 1987, the World Federation of National Mathematics Competitions founded in 1994, and the International Conference on Technology in Mathematics Education founded in 2003. These organisations not only hold their own international and regional conferences, but they also run sessions and report at the ICMEs.

Thus through these two kinds of sub-groupings, ICMI has facilitated the gradual internationalisation of mathematics education. However we should recognise that these subgroups do not necessarily solve the problems of international isolation for all countries. The regional groups do not cover the globe, the costs of attendance can still be prohibitive, and the languages used can also discriminate. The same can be said for the Study Groups.

4. ICMI's contribution to international communication in mathematics education

Over the years since its inception, ICMI has contributed to internationalising mathematics education through publications which it has fostered or with which it has been associated. Its initial contributions were reports on aspects of mathematics education, for example, an ICMI Study on the preparation of mathematics teachers was presented by Gino Loria in 1932. This tradition continues to this day with the process of writing such Study reports having been formalised in an attempt to achieve a greater degree of internationalisation in them. Nowadays one begins with appointing one or two editors, who will share the organisational and editorial responsibility for the final book. They will write the first Discussion Document, arrange the details of an invitational meeting, at which the topic is discussed in detail, and organise the writing of the final report.

The fact that now these reports are published by the international publishers Springer, has made them more accessible to the international public, and has also added to their status. The range of topics is impressive, numbering 14 now, and often builds on previous discussions at ICMEs or regional conferences. Despite the fact that the language of these reports is English, the spread of writers and editors is internationally broad, and the ICMI Studies provide an important resource for those colleagues seeking to influence policy-writers in their countries, to ensure that mathematics education gets a significant hearing when educational decisions are being made.

Another way that ICMI contributes to international communication in mathematics education is through producing proceedings of the ICMEs. As has been pointed out above, trying to summarise the activities of a huge congress in a normal sized book is all but impossible, but nevertheless it has been done and the proceedings are significant documents with important policy and historic potential. As a record of the ICMEs they are undoubtedly valuable but it was clear early on in their life that more frequent and informative material; was needed. The ICMI Bulletin has fulfilled that function, and serves both to influence externally and internationally, with reports and comments from the President and Secretary, as well as to give an account internally so to speak of its activities.

It plays a most helpful role by publicising future conferences, both regional and international, announcing relevant publications, giving information on up-coming Study Groups and other significant events. The fact that it is now available on the Web makes it a much more international document, plus this ensures that colleagues in developing countries can get easy access to it also.

5. Increasing ICMI's international contribution

It is clear that through its publications, through its member states, through its 4 yearly ICMEs and through its ever increasing range of international and regional affiliates and networks, ICMI has come a long way internationally since its second coming in 1954. But is there still a way to go? Can we be satisfied that ICMI has done all it can?

There is still language discrimination for example. It is still the case that in promoting mathematics education through its publications, it is restricting its public language to English. Others would argue that this is the way of all academic areas, to which the reply could be that mathematics education is both an academic discipline, but also a professional practice, which takes places in specific cultural and social contexts. ICMI would reply that (a) it does try to reach other language groups through its regional affiliates, who publish their reports in other languages, and (b) it does not have the human or financial resources to do anything more in that way. The problem exists in reverse also, in that it is difficult for someone not speaking English (or one of the major languages) to inform others about developments in mathematics education in their countries. Thus we are all impoverished by this problem.

I have already alluded to the financial problems for someone wanting to attend ICMEs or Regional conferences. There have been attempts to help to fund colleagues from developing countries to attend ICMEs but it is difficult to achieve a satisfactory outcome for many colleagues from those countries. The establishment of a Solidarity Program in Mathematics Education in 1992 by the then President Miguel de Guzmán was based on a 10% levy on all participants at that ICME. It was a splendid initiative, but did little to solve the under-representation from the poorest of developing countries. There have however been some successes with specific support being given for specific conferences, but without regular funds it has been difficult to achieve consistent results. The Program is still an important one to support and develop.

Finally there are still areas of the world that international mathematics education fails to reach. The prime example of a rather neglected region is Africa, which despite the activities of the African Mathematics Union (AMU) and the Southern African Mathematical Sciences Association (SAMSA), is still not well served by international or regional groups and conferences concerned with mathematics education. There has never been an ICME in Africa, and regional conferences are few and far between. I believe that improving this situation is one of the most important challenges facing ICMI.

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DEREK HOLTON

The process of an ICMI Study

1. *The start of all studies*

The idea of ICMI Studies developed from discussions held in the early 1980s at the time when Jean-Pierre Kahane became President of ICMI. Influential in these discussions were Geoffrey Howson, the Secretary of ICMI, and Bent Christiansen, ICMI Vice-President.

From its inception, ICMI had been involved in determining themes for discussion at the congresses organised by the International Mathematical Union (Howson, 1982) as well as other meetings that were devoted to specific issues of mathematics education. While all these conferences were acknowledged to be good things for ICMI to be involved in, there was a feeling that improvements could be made. For instance, it was believed that

- the meetings should have greater publicity
- there should be better preparation for the meetings
- participation should be open to anyone who was interested in the topic under consideration; and
- reports of the meetings should be properly prepared and edited and be available commercially as widely and as cheaply as possible.

The ICMI Executive Committee, EC, of the early 1980s began what resulted in the ICMI Study series, by considering the position of computers in mathematics education. An International Programme Committee, IPC, was set up for this Study and with the help of its members a discussion document was written that was sent to all national delegates and published in a selection of journals.

In order to ensure that conference participants would be genuinely involved in the meeting, potential participants were asked to submit a paper on some aspect of the Study. The members of the IPC read the papers and invited to the Study conference those authors whom they felt had most to offer. Their papers were not necessarily read at the meeting but formed the basis of general or working group sessions. In addition various experts were invited as plenary speakers.

The first Study volume was produced very quickly after the Study conference both in an extremely cheap paperback form and in hardback. Only a small selection of the submitted papers were included so the Discussion Document plus 42 other papers, which were not published in the Study volume, appeared in a collection that was published by Strasbourg University (Pluvinage, 1986). This first Study volume was reprinted and also appeared

in translation, as did Studies 2, 3, and 4. Indeed Study 2 was translated into five languages.

More on the early history of ICMI studies and an outline of the first five studies can be found in (Hodgson, 1991).

2. The start of a Study

The pattern set down by the first ICMI Study is essentially the same as that followed by subsequent studies. First a given Study topic is decided upon by the EC of the day. Possible topics are discussed by the EC and those that seem more important at the time, of general interest, and, in some way, ready for a Study, are chosen to be taken further. This choice is made with an eye for balance on mathematical area, on level of education, and what may be relevant to a worldwide audience. At the end of the day the studies are expected, through their Study volumes, to produce state-of-the-art discussions of the Study topic. More specifically, as Hodgson, not then Secretary-General, said (Hodgson, 1991):

The aim of these Studies is not to give any “ICMI-labelled” solution, to use the ICMI President’s own words, ..., but to provide an up-to-date presentation and analysis of a given theme so as to promote reflection and foster new initiatives at the national, regional or institutional level.

As Galbraith reminds us (Galbraith, to appear), the aims of ICMI, in particular through the Study programme, are to offer

researchers, teachers, curriculum designers, policy makers and others interested in mathematical education a forum promoting reflection, collaboration, exchange and dissemination of ideas and information on all aspects of the theory and practice of contemporary mathematical education as seen from an international perspective.

Having decided on a topic, the EC next chooses a Study chair, or recently two co-chairs, for their expertise in the theme of the Study. It is the chairs’ job to make sure that the Study proceeds to a satisfactory conclusion. Often, though not always, the chairs edit the final Study volume.

The EC also draws up a list of possible members for the IPC, whose job it is to determine the overall direction of the Study. This list is finalised with the involvement of the Study chairs. The IPC typically consists of people with knowledge in one or more areas of the Study and is chosen with an eye for gender and regional balance. Usually the IPC is around a dozen strong and includes the ICME Secretary-General ex-officio. A wide range

of countries and cultures have been represented on IPCs to date. However, the African continent seems to have been under-represented and Australasia has been over-represented. Further, a closer examination of the authors of articles in Study volumes reveals that 65% are from six Western countries (Galbraith, to appear).

The IPC rarely meets more than once before the Study conference so most of its work is conducted by email. Its major roles are outlined in section 3.

Nineteen studies have now been completed, are under way, or are planned¹.

3. The main steps of a Study

There are three main steps in any Study and these are the main responsibilities of the IPC. These steps are, in order, the discussion document, the Study conference, and the Study volume (see also Hodgson, 1991). Other optional extras will also be considered.

3.1. Discussion Document²

The first job of the IPC is to prepare and disseminate the Study discussion document, DD. The DD broadly sets out the aims of the Study and calls for submissions from those interested in the area.

There are no rules about the structure of a DD but they usually start with a section setting the scene of the Study within the mathematical or statistical community and within the ICMI Study framework.

This is followed by a justification for the Study. What has taken place in the area in recent times that suggests that now is a good time to review the area? For instance, Study 11 followed on the heels of some significant changes. In the mid 90s there had been a considerable increase in the number of students attending tertiary institutions in many parts of the world; major pedagogical and curriculum changes had taken place at secondary level; there was an increasing difference between secondary and tertiary mathematics education; there was an increase in the use of technology; and there was a general demand for universities to be publicly accountable. Consequently it was felt that there was a need to examine the current and future state of the teaching and learning of mathematics at university level.

¹ Further details can be found in http://www.mathunion.org/ICMI/ICMI_studies_org.html.

² The Discussion Document for Study 11, for example, can be found on-line in <http://www.mathunion/ICMI/bulletin/43/>.

Actually the DD for Study 11 then listed specific points that that Study would cover. This was then followed by an in-depth look at the themes, issues and focus of the Study. In the case of Study 11, this section considered research, practice and policy in some detail, ending with a number of specific questions to be considered.

At some point too the DD indicates the kind of groups that might contribute either with a paper or by attendance at the Study conference. For instance in the Study 11 DD, the groups suggested were «those concerned with the teaching of mathematics at the university level, ... mathematics educators undertaking research in related areas, and ... many other people with an interest in university level mathematics».

Then Study conference submissions from interested people are invited. It is expected that theoretical and practical experts in the field will offer papers on the topic of the Study.

3.2. The Study conference

One more decision that is made by the EC, in cooperation with the co-chairs, is the location of the Study conference. Often a proposal to host a conference is made by members of the IPC. Selection of a site for the conference is made on the basis of costs and local support as well as a desire by the EC to increase the involvement of developing countries. Once the venue of the conference is decided the Local Organising Committee then takes full responsibility for the running of the Study conference though the IPC decides on the programme and the plenary speakers.

It should be noted that Study conferences are very definitely working conferences whose immediate goal is to produce the skeleton from which the Study volume will ultimately be produced.

The papers that are prompted by the DD will be considered by the IPC and a limited number of authors will be invited to the Study conference. It should be underlined that Study conference attendance is by invitation only, typically to some 70 or so people. The restriction is because of the need for an optimum number to be engaged in a working conference.

3.3. The Study volume

The aim of the Study volume is to provide a state-of-the-art compendium of the Study area. It is not simply the proceedings of the Study conference. A recent principle enunciated by the EC, says that «Study volumes should contain a balance of papers that specifically critique, or develop, or apply theories or frameworks identified as particularly relevant to the topic area; noting that the balance may vary with the nature of the topic». (Galbraith, to appear) In that same paper, the EC

is reported as saying that «ICMI studies are closer to reviews of research in certain focus areas, then to reports of particular projects».

The actual format of a Study volume varies from a collection of selected and edited papers (see Study 11) to a comprehensive monographic exposition (see Study 12).

It should be emphasised that the Study volume doesn't magically appear at a Study conference. After the conference is over there is still a great deal of work that has to be done by IPC chairs. This effort is required to both gather and edit the material for the final published book.

3.4. Optional extra

After most studies the only publication is the Study volume but there are variations. For example, after Study 3 and Study 11, selected papers were published. And after Study 6, two Study volumes appeared.

In more recent times, invitations have been given from various conferences for panels to be held on certain ICMI Study themes. Often Topic Study Groups or Working Groups on a Study theme or one closely akin to a Study theme have been held at various ICMEs.

4. Reflections

This final section consists of personal reflections on my experience of being involved with two Studies, Study 11 and Study 16. I have written them in no particular order. It does seem that the Study idea is a very good one and one that should be continued. It brings experts in an area together to produce a state-of-the-art publication that can serve for many years both as a stimulus for work in the area and as a valuable reference.

4.1. Non-native English speakers

I have real concerns that the use of English as the language for both the Study conference and the Study volume is disadvantaging non-native English speakers and reducing the potential richness of Study volumes.

4.2. Study volume

Both of the studies I have been involved in seemed to require hard work in getting the ideas from the Study conference into print in the Study volume. It is difficult for the chair(s) of the IPC to get work from individuals who are generally busy people. Indeed it is very hard for the chair(s) to find the time for all the post-conference work that is necessary. Is there a way to facilitate the publication process?

Some of the laborious post-Study work of chairs and the time delay

of Study volumes (recent Study volumes seem to be appearing from 2.5 to 3 years after the conference) might be reduced if considerably more work on the Study was undertaken in advance of the Study conference. Is it possible too that the people who are responsible for writing chapters could remain at the conference venue for a further week of solid writing? Could much of their work be done in this short time frame when they had no lectures to give or meetings to attend?

In view of the comments in (Galbraith, to appear), (see the end of section 3), it would be good to widen the writing base of ICMI studies. It is very hard for many active practitioners and researchers to participate fully in both the conference discussions and the study volume writing when the language involved is English (see Galbraith, to appear, a note after Principle 6). Perhaps some effort could be put into allowing participants to submit articles in their own language and have them translated into English later. Another argument for advocating translations is that this might help developing countries to promote expertise in areas that are important both to mathematical education in general and to their countries of origin in particular.

4.3. *Web Site*

It has always seemed to me to be a pity that the Study seems to end with the production of the Study volume (except for Study 1, where UNESCO published a follow up in 1992 (Cornu & Ralston, 1992)). Would it be possible to keep some continuity by using the web and updating the material regularly? Web publication might also reduce the cost of publication and translations.

4.4. *Publicity*

One final thing, perhaps more popular events could be encouraged alongside conferences. Mathematics needs to take every chance to advertise itself to a general audience. Future studies might hold exhibitions and open lectures for this purpose. It would also be valuable, where appropriate, for local teachers to be involved in some way.

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The significance of the ICMI Study on mathematics education in East Asia and the West

1. *The ICMI Study on mathematics education in East Asia and the West*

The ICMI Study on *Mathematics Education in East Asia and the West* (hereafter referred to as the East/West Study) is the 13th of the ICMI Study series. The Study was motivated by the increasing interest in comparative studies across nations in recent years, particularly in students' mathematics achievement, as witnessed by large-scale projects such as TIMSS and PISA. Such studies provide valuable and rich sources of data, but they also generate a lot of controversy. It was felt that there is a need to interpret the findings of these comparative studies within the subtler context of the underlying cultural traditions (Leung, Graf, & Lopez-Real, 2006).

The East/West Study intended to use the lens of different cultural traditions to make analyses of mathematics education in an international context. The scope of study was confined to studying mathematics education in East Asia and the 'West'. Here East Asia and the West are cultural demarcation rather than geographic divisions. They roughly refer to the Chinese or Confucian tradition and Greek/Latin/Christian tradition.

The study encompassed the following four aspects:

- Context of mathematics education
- Curriculum (including textbooks and assessment)
- Teaching and learning
- Values and beliefs

In concluding the study, it was pointed out that the aim of such a comparative study should be for understanding and collaboration rather than for competition. The East/West study itself is an example of collaboration among mathematics education professionals from different cultural traditions. Countries with different cultural traditions should learn the best practices from each other, yet they should at the same time preserve what is valued in mathematics education in their own cultures. It is only through a better understanding of each other's cultures that we will attain a richer mathematics education worldwide (Leung, Graf, & Lopez-Real, 2006).

2. The East Asian mathematics education paradox

In trying to appreciate the significance of the East/West study, it will be instructive to consider what is referred to here as the East Asian Mathematics Education Paradox. The paradox involves four rather well known facts:

2.1. East Asian students' mathematics achievement

It is evident from results of recent international studies such as TIMSS and PISA that East Asians outperformed their counterparts in mathematics achievement, and such results are rather robust across studies and over time (Mullis et al, 2004; OECD, 2004).

2.2. Mathematics teaching in East Asian classrooms

Early studies of East Asian classrooms identified classroom practices which were rather traditional and teacher-centred, with an emphasis on drilling and rote learning. However, more recent literature based on a fine-grained analysis of East Asian classroom has reported on the high quality of teaching in the East Asian mathematics classroom (Huang &

Leung, 2005; Huang et al, 2006; Leung & Park, 2002; Ma, 1999; Mok, 2006). Among these studies, the most comprehensive has been the TIMSS 1999 Video Study.

The TIMSS 1999 Video Study was based on national probability samples of 8th grade mathematics lessons in seven countries. One lesson per teacher was videotaped, and altogether 638 eighth grade lessons were analyzed. Quantitative analysis was performed by an international team of researchers who developed and applied 45 codes to the video data. In addition, a specialist group of mathematicians and mathematics educators reviewed qualitative descriptions of a subset of 120 lessons and rated the quality of the lessons 'country-blind'.

The results obtained from the quantitative analysis of the data show that there was much dominance of teacher talk in the East Asian classroom, and the problems that students solved were procedural in nature and unrelated to real-life. However, the problems were also more complex, and there was more use of proofs in the lessons. Students also had more opportunities to learn new content (Leung, 2005). The results of the qualitative analysis of the same dataset were even more striking. East Asian students were found to be learning relatively advanced content. The lessons were more coherent and the presentation was more fully developed. Students were more likely to be engaged in the lessons, and the overall quality of the lessons was judged to be high.

2.3. Participation of East Asians in international mathematics education conferences

Notwithstanding the superior achievement of East Asian students and the relatively high quality lessons delivered by their teachers, the participation of East Asian educators in international conferences was low. In this Symposium for example, less than 6% of the participants come from East and Southeast Asia. This is significantly less than those coming from West Europe (43%) and North America (19%). This phenomenon is also observed in ICME attendances, as Table 1 shows (note a mild trend of increased participation of East Asians in ICMEs though).

2.4. East Asian scholars' contribution to the mathematics education literature.

Consistent with East Asian educators' low participation in international conferences, East Asian scholars' contribution to the international literature has been minimal. The distributions of different regions in the world from which authors of two major journals in mathematics education come are shown in Table 2.

Table 1. Attendance from different countries at ICMEs

	ICME-8, Spain 1996		ICME-9, Japan 2000		ICME-10, Denmark 2004	
	808+653=	[20.56%] ¹			523+444=	[20.58%] ³
West Europe	1461	(53.36%)	352	(16.99%)	967	(40.83%)
North America	327	(11.94%)	239	(11.53%)	411	(19.05%)
Australia/New Zealand etc	149	(5.44%)	80	(3.86%)	110	(5.10%)
Central and South America	403	(14.72%)	63	(3.04%)	80	(3.71%)
East and Southeast Asia 275(12.75%)	173	(6.32%)	945+240= 1185		[11.58] ¹	(57.19%)
East Europe	89	(3.25%)	66	(3.19%)	201	(9.32%)
Middle and South Asia	77	(2.81%)	62	(2.99%)	67	(3.10%)
Africa	59	(2.15%)	25	(1.21%)	46	(2.13%)
Total	2738	(100.00%)	2072	(100.00%)	2157	(100.00%)

Table 2. Distribution of countries of origin for authors in *Journal of Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM)

1998-2007	JRME	ESM
West Europe	39(8.99%)	317(43.07%)
North America	324(74.65%)	227(30.84%)
Australia/New Zealand	28(6.45%)	62(8.42%)
Central/South America	10(2.30%)	15(2.04%)
East and Southeast Asia	9(2.07%)	12(1.63%)
East Europe	0(0%)	8(1.09%)
Israel; Middle/South Asia	18(4.15%)	76(10.33%)
Africa	6(1.38%)	19(2.58%)
Total	434(100.00%)	736(100.00%)

3. Discussion

How can we account for this paradox between East Asian students' superior achievement in mathematics and the high quality of teaching pro-

³ Here the percentage in square brackets was calculated based on numbers with the host country excluded. For example, in ICME-8, there were 1461 participants from West Europe, and among them there were 653 from Spain.

vided by their teachers on one hand, and the low attendance of East Asian educators in international conferences and the minimal contribution they made to the literature on the other? East Asians' 'lack of contribution' to mathematics education can be explained in different ways. If a deficit model is adopted, East Asians are simply considered inferior in terms of their ability in contributing to theories of mathematics education. Some scholars indeed have attributed so-called 'deficiencies' to ethnic differences (Chamberlain, 1974; Jones, 1980).

Another model may be termed a developmental model, where East Asia is seen not to be inherently inferior, but to be at a more primitive stage of development compared to the more advanced Western countries. The modest contribution to theories of mathematics education happened to Western European countries half a century ago, and in time, the East Asian countries might be able to catch up with their more advanced counterparts (but of course the more advanced countries might then have progressed further on!) (Chilcote, 1984; Tedla, 1995).

But in trying to explain the differences in mathematics education between East Asian and Western countries, the East/West Study has taken a third line of interpretation: that of a cultural difference model. Differences, whether they are the lack of contribution to mathematics education or the superior performance of students, were explained in terms of the different characteristics of the 'Confucian-heritage culture' shared by East Asian countries (e.g., the 'East Asian pragmatism' may explain East Asians' lack of interest in theorizing) on one hand, and of the Greek/Latin/Christian traditions on the other.

There are of course historical reasons for East Asia's relatively late development in theories of mathematics education. Mathematics education as a field of study started in Europe in the late 19th/early 20th Century (Kilpatrick, this volume; Leung 2001). This was exactly the time when the East Asian countries were either closing up themselves (as in China), or were colonies of Western countries (as in Singapore). This historical and political situation deprived these countries of the opportunity of participating in the international discourse in mathematics education, and hence the chance of developing and contributing to the building of theories in mathematics education.

As for East Asians' lack of participation in the international mathematics education community, the obvious reason is clearly the language barrier. Again for historical reasons, there is clear dominance of the European languages (especially English) as international academic languages, with English becoming the *lingua franca* in most 'international' conferences (Barwell, 2003). More importantly, nearly all important journals in the field of mathematics education are in English or other European languages (and there are no citation indices or impact factors for publications written in

the East Asian languages!). Given these conditions, the lack of contribution of East Asians in the international mathematics education community is to be expected.

4. Significance of the East/West Study

The significance of the East/West Study should be understood against this background. The premises on which the Study was based is that *culture matters*, and the Study adopted a cultural difference (instead of a deficit) model in explaining of differences in educational practices, highlighting the importance of interpreting educational phenomena in terms of the underlying cultural values and traditions. There was a deliberate effort to hear a balanced voice from the East Asian and the Western traditions. For example, the were equal number of IPC members (seven each) from East Asia and the West, and of the contributors to the Study Volume, 39 of them come from East Asia and 38 of them come from the West.

More importantly, the Study explicitly acknowledged the East Asian culture as an important tradition in the international scene of mathematics education. East Asian scholars were prompted to re-evaluate and to treasure their own traditional cultural values. Practices in East Asia were seen to enrich as well as being enriched by practices in the West. Also, as a by-product, the Study resulted in more communication and cohesion among East Asian scholars (this is evident from the greater enthusiasm in participating at the EARCOME conferences), and the more active participation of these scholars in international conferences, hence enhancing the interflow of ideas between scholars from East Asia and the West as well.

5. Conclusion

The discussions above have alluded to East Asia being 'disadvantaged' for historical and language reasons. As East Asian countries are slowly 'catching up' following their economic growth, it is important to remember that there are other even more disadvantaged cultures in the world, and the international community has the obligation to help these disadvantaged countries to overcome their limitations. If these cultures are given a chance to express themselves, the international mathematics education community will surely benefit from the richness of these different traditions.

For over a century, Europe and North America have dominated the world economy, but in the last decade or so, the world has seen the emergence of East Asia as a strong competing economic power. Some people even predict that this century will be the Century of East Asia. Speaking

from the position of an East Asian, is the wish of the author that East Asia will take its turn to dominate the rest of the world? The answer is an emphatic «No»! History has taught us that *the dominance of any 'one' culture is not desirable*. East Asia should contribute to human kind by offering to the world its cultural treasures to complement those of the Western culture and not to replace them. So it is all the more important for educators from different cultures to stress collaboration and understanding. And this is the very conclusion of the East/West Study!

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ICMI and its influence in Latin America

1. *Some reflections on the ICMI, the IACME and the ICMEs.*

I will discuss the impact of ICMI on Mathematics Education in Latin America, and how the ideas exposed in the ICMEs had an influence in the region. In particular, I will discuss the creation of the affiliated committee, the IACME/Interamerican Commission of Mathematics Education, and its relations with other regions of the world.

From the first International Congress of Mathematicians, in 1897, until the ICM in Oslo, Latin American participation has been minor. Even though, in the ICM 1908, in Rome, when CIEM/IMUK was created, Argentina joined it, and in 1928, in Bologna, when CIEM/IMUK was reformulated, again Argentina was a member of it, I have noticed no relevant influences of the CIEM/IMUK in Latin America.

In the ICM 1950, in Cambridge, USA, there was a good number of participants from Latin America. In the 1954 General Assembly of IMU, it was formally decided to establish a commission concerned with the teaching of Mathematics and the official name of this commission would be *International Commission on Mathematics Instruction/ICMI*. An Executive Committee was appointed, with Henri Fehr as Honorary President, Albert Châtelet President and Heinrich Behnke as Secretary, who became its President in 1955. It is reported that Alberto E. Sagastume Berra and José Babini were delegates from Argentina. It is important to observe that no Latin American country participated in the Comparative Study of Mathematics Education, commissioned to ICMI, in 1958.

Since the early days, international mathematical organizations were concerned exclusively with Academic Mathematics, and with Mathematics Education as a mission to convey this Mathematics, as a part of conservative school traditions.

The idea of considering the cultural context in Mathematics Education was not contemplated by CIEM/IMUK. It seems that the first time an explicit reference to cultural issues as a factor to be considered in Mathematics Education is due to the Japanese algebraist Yasuo Akizuki (1960, p. 289)

[...] oriental philosophies and religions are of a very different kind from those of the West. I can therefore imagine that there might also exist different modes of thinking even in mathematics. Thus I think we should not limit ourselves to applying directly the methods which are currently considered in Europe and America to be the best, but should study mathematical instruction in Asia properly. Such a study might prove to be of interest and value for the West as well as for the East.

I see this as an important moment in the overture of the committee, then officially named ICMI, to recognize ideas coming from other cultural contexts. Indeed, I recognize this as the first opening of a space for Ethnomathematics in academic mathematical reflections. Regrettably, the remark of Akizuki did not attract the attention of his contemporaries, and it took about 20 years for the importance of such remarks to enter the academic mathematics scenario.

2. The 'Stone Age' of influence of ICMI in Latin America.⁴

In 1959 Marshal H. Stone became the President of ICMI. He announced the realization of a meeting, in Bogotá, Colombia, in 1961, under the aegis of ICMI and with financial support of UNESCO, the OAS/Organization of American States and other organizations, to discuss problems in mathematics education which are common to all countries in the region. The Organizing Committee was formed by Marcelo Alonso (Cuba), José Babini (Argentina), Howard Fehr (United States) and Leopoldo Nachbin (Brazil). In this meeting, which became known as the First Interamerican Conference on Mathematics Education, it was decided to create the *Interamerican Committee on Mathematics Education/IACME* or the *Comité Interamericano de Educación Matemática/CIAEM*.⁵

⁴ I owe this metaphor to Olli Lehto (1998).

This was decisive for the insertion of Latin America into the international movement of mathematics education. Since then, eleven Interamerican Conferences on Mathematics Education have been held. It is important to notice that the First Interamerican Conference on Mathematics Education/IACME 1, in 1961, precedes by 8 years the First International Congress of Mathematics Education/ICME 1. I will comment on the Interamerican Conferences later in this paper.

When Hans Freudenthal was President of ICMI, from 1967-1970, he championed the realization of an International Congress on Mathematics Education, with the support of the French Government and the UNESCO, in Lyons, France, 1969. This was done entirely without any connection with the IMU. Although the President of the EC of IMU is an ex-officio member of ICMI, the EC had not been told of the creation by ICMI of the new journal *Educational Studies in Mathematics*, which seemed to compete with *L'Enseignement Mathématique*. A financial contract had been signed between ICMI and UNESCO without the IMU having been informed, see Lehto (1998, p. 259). Hans Freudenthal was the major speaker of IACME 3, in Bahía Blanca, Argentina, in 1972. Since then, I met Freudenthal on several occasions, particularly in his capacity as the IMU representative (1970-1978) in the CTS/*Committee on the Teaching of Science of ICSU/International Council of Scientific Unions*. I was active in this Committee, as a proponent of the integration of Mathematics and Science Education, a trend in the seventies which was promoted in Latin America, with the support of UNESCO and the OAS. The conversations with Freudenthal allowed me to identify the serious conflicts then present in Mathematical Education. It is important to register a project of IOWO to establish a very interesting and unique project for the Formation of Mathematics Teachers in Bolivia. This was a very interesting attempt to transfer a novel and somewhat controversial approach to Mathematics Education to the, then, most backward Mathematics Education program in Latin America. But the issue of knowledge transfer is not the purpose of this paper, although these conflicts had many consequences for Mathematics Education in Latin America.

From 1971 to 1974, Sir James Lighthill was the President of ICMI and ICME 2 took place in Exeter, England, in 1972. From 1975 to 1979, the President was S. Iyanaga and the Secretary was Y. Kawada. In the information for the period 1975-1978, presented by the Secretary Kawada, we learn of a presence of Latin America in the ICMI, with Luis A. Santaló, from

⁵ Visit the site of the IACME/CIAEM, in Portuguese, Spanish and English: <http://www.furb.br/ciaem/>. The link Publications directs to the History of the IACME, written by Hugo Barrantes and Ángel Ruiz, both from Costa Rica, <http://www.furb.br/ciaem/ing/index.htm>.

Argentina, and Leopoldo Nachbin, from Brazil, listed as National Representatives. It is also reported that in the meeting of the Executive Committee of ICMI, in Vancouver, August 1974, a resolution for affiliation of IACME to ICMI was adopted.

3. *The Interamerican Conferences on Mathematics Education.*

The First Interamerican Conference on Mathematics Education, held in Bogotá, Colombia, in 1961, and the creation of the Interamerican Committee on Mathematics Education/IACME, were decisive for the insertion of Latin America into the international movement of mathematics education. The main ideas advanced in the first conference were:

- The need to change the way of teaching Geometry in Secondary Schools: to teach Geometry from the perspective of Linear Algebra, forsaking Euclidean Geometry.
- The need to teach Mathematics, in general, through the study of its fundamental structures, with the purpose of underscoring their unity. In this area, the teaching of Modern Algebra became of paramount importance.
- The above goals could only be achieved if, at the same time, a well-organized plan was carried out that was oriented to the in-service training of teachers, thus preparing new mathematics teaching professionals with the ideas of the reform, as well as improving research in mathematics.
- The above goals could not be achieved without a parallel plan, very well organized and aimed at the training of professors who were currently teaching. The aim was to instill these ideas in new teachers of Mathematics, and also to upgrade Mathematics research.

It is important to notice the European influence.

The tone was Modern Mathematics and the influence of N. Bourbaki. Mathematical content was a dominant issue. This characteristic prevailed in the next conferences: 2nd IACME, in Lima, Peru, in 1966; 3rd IACME in Bahia Blanca, Argentina, in 1972; 4th IACME, in Caracas, Venezuela, in 1975.

There was good support from the agency of Coopération Française. It is also clear, from the choice of participants, that there was an influence of the CIEAEM/*Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* in Latin American Mathematics Education at that time, more than the influence of ICMI.

The Caracas recommendations show this:

- Create research centers in each Latin American country.
- Create specific programs for teaching mathematics in the last years of secondary school.
- A specific program was not proposed, but each country should deter-

mine its own program based on its own possibilities. Some topics were recommended: real functions, linear algebra, computing, elements of infinitesimal calculus, probability and statistics.

- Organize science fairs and Olympiads.
- Improve mathematics teaching by using new technologies, collaboration with higher level mathematicians, etc.
- Improve the preparation and professional development of teachers.

4. *The Fifth Interamerican Conference on Mathematics Education, 1979*

I can see a change of influences beginning with the Fifth Interamerican Conference on Mathematics Education, held in Campinas, Brazil, in 1979. This conference represented an innovation, both in structure and in the choice of themes. It, somewhat, moved away from the influence of Marshall Stone. The role of Luis Santaló, who since the 3rd IACME in Bahia Blanca, had established an important relation with UNESCO, mainly through the *Oficina Regional de Ciencia y Tecnología para América Latina y el Caribe*/OREALC, in Montevideo, Uruguay, was crucial in the changes of IACME.

The Program was organized around only three plenary talks and four plenary panels and the invitees reflect what was regarded as important research directions for Mathematics Education.

They were given by Hassler Whitney (President of ICMI); Leopoldo Nachbin (Brazil); Emilio Lluis (Mexico); José Velázquez (Mexico); Luis Dante (Brazil); José Pascual Ibarra (Spain); Luis Santaló (Argentina); Oscar Valdivia (Peru); José von Lucken (Paraguay); Francisco Figeac (El Salvador); Jaime Michelow (Chile); José A. Valente (Brazil); Bernardo Morales (Guatemala); Enrique Góngora (Costa Rica); Saulo Rada (Venezuela); Oswaldo Sangiorgi (Brazil); Bryan Wilson (UK); Eduardo Luna (Dominican Republic); Guy Brousseau (France); Ricardo Losada (Colombia); Geraldina Porto (Brazil); Friederich Zech (Germany).

The presentations of this panel were in agreement in considering mathematics teaching as a distinct discipline with its own subject matter. There was a critique of the way the reform of mathematics teaching was carried out in the 60s, and the panelists expressed more ideas of a methodology nature. They also stressed the importance of research in the teaching (didactics) of mathematics.

I see the Fifth Interamerican Conference on Mathematics Education as a turning point in Mathematics Education in Latin America, not only by giving a predominant role to educators from Latin America, but also by proposing innovations, and discussing the modern trends in education

which begin to be considered as central in Mathematics Education.

Subsequent IACMES have exposed important accounts of innovative actions in Mathematics Education in Latin America and the establishment of new areas of interest in newly created research centers. The Interamerican Conferences on Mathematics Education were the following: VI IACME, in Guadalajara (México), 1985; VII IACME, in Santo Domingo (Dominican Republic), 1987; VIII IACME, in Miami (United States), 1991; IX IACME, in Santiago (Chile), 1995; X IACME, in Maldonado (Uruguay), 1999; XI IACME, in Blumenau (Brazil), 2003; XII IACME, in Querétaro (México), 2007.

5. The situation now

It is beyond the objectives of this talk to give an overall picture of the state of research in Mathematics Education in Latin America.

The increasing presence of Mathematics Educators from Latin America in ICMI activities becomes very clear if we look at the list of participants of the International Congresses of Mathematics Education. We see an increasing number of delegates, particularly after ICME 8, in Seville, in 1996. The list of invitees to make presentations and to organize sessions reveals the areas being studied in Latin America. The participation of Mathematics Educators from Latin America in the ICMI Studies is also noticeable.

I must also mention the recognition of Iberic ties, which resulted in the creation of a series of conferences, the CIBEM/*Conferencias Iberoamericanas de Educación Matemática*, and also the creation of the FISEM/*Federación Iberoamericana de Sociedades de Educación Matemática*.⁶ Regrettably, internal rivalries, which have consequences for rivalries in the regional organizations, still prevail.

Mathematics Educators from Latin America have been very active in the *International Study Group on the Relations between the History and Pedagogy of Mathematics/HPM* and the *International Study Group on the Psychology of Mathematics Education/PME* since their creation, in 1976. Also, *The International Organization of Women in Mathematics Education/IOWME*, *The World Federation of National Mathematics Competitions/WFNCM* and *The International Study Group for Mathematical Modelling and Applications/ICTMA*, have a relevant presence of mathematics educators from Latin America. Two groups, not formally affiliated with ICMI, but which originated from ICMI, are the *Mathematics Education and Society International Study Group/MES* and

⁶ Visit the site <http://www.fisem.org>

the *International Study Group on Ethnomathematics* / ISGEM.

There is still much to be done. I hope that the realization of ICME 11 in Monterrey, Mexico, July 2008, will have a great impact on the development of Mathematics Education in Latin America. It will bring together delegates from all the countries of the region and I hope this will be an opportunity to eliminate rivalries and reinforce our efforts towards international cooperation.

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ICMI in Africa and Africa in ICMI: The development of AFRICME

In June 2005, the first Africa regional ICMI congress (AFRICME1) was held in Johannesburg, and followed by AFRICME2 in Kenya, May 2007. Plans are underway for AFRICME3 in Botswana in 2010. The congresses have drawn participation from a large range of countries, reflecting diverse mathematics education activity. In this short paper I describe and reflect on the development of AFRICME, and the benefits of an interaction space for mathematics education in Africa.

1. Introduction

The past few years have seen increased ICMI activity in Africa, particularly through the development of regional conferences. In this short paper, I will begin with a brief discussion of the kinds of activity we are aware of in and across this vast and diverse continent. I reflect on the *role of ICMI*

in mathematics education activity in Africa, and conversely, on *the role of mathematics education in Africa* in ICMI, with focus on the AFRICME congresses.

2. Mathematics and mathematics education in Africa

There is considerable organised mathematics and mathematics education activity in Africa, some of which has been in effect for many years. The African Mathematical Union (AMU) has a long history of activity on the continent, particularly through PAMO the Pan African Mathematics Olympiad, and AMU-CHMA, the commission on the history of mathematics in Africa. More regionally, SAMSA, the Southern Africa Mathematical Sciences Association, and SAARMSTE, the Southern African Association for Research in Mathematics, Science and Technology Education, have regular conferences. No doubt there are other such initiatives across the continent.

More recently, two different kinds of regional ICMI, with conference-related activities have been launched. First to emerge is the CFEM (*Commission française pour l'enseignement des mathématiques*, the French Sub-Commission of ICMI) – an intervention driven by shared language. The CFEM has held conferences every three years since 2000. The next conference will be in Dakar in 2009. CFEM has opened space for communication on mathematics education across French-speaking countries. CFEM has strong links into French-speaking African countries, with alternate Conferences taking place on the continent. In the context of ICMI in Africa, this is an important initiative. Currently, Portuguese-speaking colleagues are in conversation about stimulating similar mathematics education activity. It is interesting how such initiatives emerge. Late last year, the ICMI executive communicated with colleagues in Mozambique around ICMI contribution to the International Commission of Scientific Unions - ICSU - pre-General assembly workshops due to take place in Mozambique in October 2008. After considerable interaction, what has emerged instead is an initiative to bring together ICMI associates across Portuguese-speaking countries. It is in this way that we see the role of ICMI in enabling collaborations across countries and continents, built on shared languages. These provide space for important collaborations across contexts and conditions.

It is fair to say that it is the initiatives of colleagues in office in ICMI, and the IMU-CDE (Commission for Development and Exchange) that have been the driving forces in developments related to Africa and other 'developing' parts of the world. My own experience in the ICMI Executive since 2003 is that the organisation has engaged actively in extending

ICMI's role in Africa. 2005 saw the first Africa regional ICMI congress in Johannesburg. Named AFRICME, AFRICME1 marked the beginning of greater interaction, collaboration and exchanges focused on mathematics education among practitioners and researchers across the continent. In May 2007, AFRICME2 was held in Kenya, and plans are underway for AFRICME3 in Botswana in 2010. Each of the two AFRICME congresses to date have drawn participants from a range of countries, and have demonstrated the worth of an interaction space dedicated to mathematics education activity on the continent.

3. *The launch and development of AFRICME*

- *AFRICME1 - Johannesburg, June 2005*

The 1st Africa ICMI (International Commission of Mathematical Instruction) Regional Congress was held at the University of the Witwatersrand from 22 – 25 June 2005. One hundred and eighty delegates from sub-Saharan Africa, and particularly Southern, Central and Eastern Africa, attended the Congress. One of the considerable challenges in organising and hosting such an Africa-focused Congress is finance. Across the continent, most practitioners in the field do not have direct access to travel funds for such activity. Fortunately, ICSU-South Africa was in a position to contribute to congress costs. While the majority of participants came from South Africa, Lesotho, Swaziland, Namibia; Botswana, Malawi, Mozambique, Zambia, Zimbabwe, Uganda and Kenya, additional funding from the IMU-CDE enabled participation from African countries further afield, particularly Tunisia and Burkina Faso.

AFRICME1 set an agenda for its regional congress, with aims to stimulate regional collaboration and activity; promote regional and global contributions and interactions; highlight issues pertinent to mathematics education in developing countries and share and showcase activities in and across countries in our region. These aims are aligned with overall ICMI goals and inform ongoing AFRICME activity.

The Congress theme of *Mathematics Teaching and Teacher Education in Changing Times*, was supported by national presentations on this theme. Participation was further invited through inputs into four focus strands, the first three of which were stimulated by an invited plenary panel presentation: Teacher Education (primary and secondary; initial and in-service); Information and Communication Technology (ICT) and Resources for teaching and learning; Indigenous Knowledge Systems (IKS), Ethnomathematics and the Curriculum; Teaching and Learning Mathematics (primary, secondary and tertiary). Members of the plenary panels were drawn from across the region, as well as from relevant international experts.

In the opening of the Congress, its significance as the first African-centred congress in Mathematics Education was fore-grounded. There was public acknowledgement of the crucial importance of mathematics as the 'underbelly' of Science and Technology development. All acknowledged the considerable challenge world wide, and particularly in Africa, of enabling all school learners to become mathematically literate, and to produce future generations of scientists. The preparation and development of quality mathematics teaching and teachers is critical to these challenges. The theme and focus of the Congress was thus fully endorsed and supported.

The Congress program included a dedicated two hour slot to national presentations. Two problems surfaced as common across countries: the recruitment, retention and support of mathematics teachers across levels; and the mathematics content preparation of teachers. There are critical shortages of well prepared mathematics teachers as well as recognition that the mathematics courses teachers are currently studying in their preparation years are not dealing with the kind of mathematical know-how needed for quality teaching. Many participants talked of the contradictory influence of the donor community in teacher education, particularly in professional development programmes. An outcome of deliberation during the Regional Meeting on the third day of the Congress was commitment to the development of a monograph, based on national presentations. The monograph was published and ready for distribution and sale at AFRICME2, see Adler et al. (2007). The contributors are themselves mathematics teacher educators in their own countries and active in the field of mathematics education community across the continent and internationally. Their discussion of teacher education in their country is grounded in knowledge of practice, as well as knowledge of debates that frame the field more widely. None are nor claim to be systematically researched surveys. Each chapter offers the authors' perspective on mathematics teacher education in their respective country. The monograph is an important collective resource highlighting common challenges facing mathematics teacher education across twelve African countries.

Reports on the congress identified key components of its resounding success. Firstly, there was extensive interaction and sharing of problems and ideas between *mathematicians, mathematics educators and teachers* across countries. This sharing across domains of expertise is a strength of ICMI. In many countries and contexts, mathematicians, tertiary level mathematics educators and government departments, typically work in separation and all too often in conflict. Yet, all, though in different ways, are involved with the preparation of mathematics teachers, and all are concerned with the quality of mathematical learners leaving school.

Secondly, the Congress was Africa-focused i.e. the research, progress, problems and challenges shared during Congress sessions assumed a shared

concern with conditions and challenges in and across African countries. This provided opportunity to grapple with real and large problems. For many of us who work extensively in Mathematics Education internationally, it is a common experience that the magnitude of the challenges we face in Africa is not easily appreciated. The scale and challenges of Africa are too often romanticised, exoticised or pathologised. We experienced a productive climate where we could share successes and challenges.

Third, this interaction across domains and across African countries enabled us to identify *key* shared problems, a central one of which is the recruitment and preparation, and then retention and professional support for teachers. This is in turn exacerbated by the language diversity in and across various countries, and their particular colonial legacy. All are further affected by prevalence and tragedy of HIV-AIDS.

- *AFRICME2 Nairobi Kenya, May 2007*

Energy released in 2005 carried into the organization of AFRICME2 in Kenya. At the same time, the relative economic strength of South Africa became visible as both costs for AFRICME2 and difficulties in raising funds increased. As is well known, travel across African countries is neither easy nor cheap, and while electronic communication improves daily, there were many more constraints in setting up AFRICME2. Added to resource constraints, the Congress was also close to national elections in Kenya, and some of the tension that have since emerged impacted on the organization and have delayed the publication of the proceedings. Constraints aside, this second Congress was also a success. Continuing with the aims and purposes that guide AFRICME activity, the Congress attracted 70 participants. Relatively large contingents were from Kenya, South Africa, Nigeria and Botswana, a function of the host country and relative ease of access to funding in those countries. Again, ICMI and CDE funding enabled subsidization of participants from Uganda, Burkina Faso, Malawi, Namibia, Zimbabwe, Mozambique and Rwanda. The theme of AFRICME2 was *Embracing Innovative Responses to Challenges in Mathematics Instruction*. National presentations took this focus, and ranged from developments in Rwanda, to embracing technology in Botswana, and large national teacher development programmes across East Africa.

That AFRICME has a value was evidenced in the bids for hosting AFRICME3 from both Botswana and Nigeria. AFRICME3 will be in Botswana in 2010. Nigerian colleagues have been encouraged to bid for AFRICME4. However, challenges are significant. In addition to cost and communication difficulties already mentioned, location and language constrain interaction across English, Portuguese and French-speaking participants. One of the more contentious sessions at AFRICME2 was

a presentation about the significance of professional associations - that professionals need to be encouraged to support these out of their own resources, at least in part. This view was not shared by all participants. Issues of dependency and entitlement vs independence and commitment were vigorously discussed, reflecting different orientations to 'aid' or 'support'.

4. Looking ahead

It is evident from the description of the developments above, that increased activity notwithstanding, there is much that needs to be done. It is easy to be positioned in a *reactive* mode, responding only as difficulties or crises emerge. One of the challenges for the community, and AFRICME congresses is an instance, is that we need to work to *stimulate* the activity needed. A critical component in this is the development of appropriate infrastructure and resources that enable activity to be initiated, embedded in its context, sustainable and with opportunities for growth. Unfortunately, much of donor driven initiatives that populate the continent have not managed to embed nor sustain their activity. One reason for this is the reluctance to invest in infrastructure. Indeed there have been many projects where, it is fair to say, benefits have accrued to the participating mathematics educationists in the donor countries, rather than to local colleagues. It is too easy to point fingers. Projects are never one-dimensional nor unidirectional. It is up to us in the region to take responsibility for the nature and direction of initiatives. Much benefit can accrue from resources made available by donor agencies.

While African countries deal with the growth of democracy under constrained socio-economic conditions, in educational terms we also bring to the wider educational community, decades of experience in challenges many developed countries now face. Rapid urbanisation and mass migration to cities has resulted in densely populated cities, and increasingly diverse urban communities. Schools in inner cities across many countries deal with large immigrant and transient populations, multiple languages, and diverse cultural home-school practices. These impact on teaching and learning. There is considerable expertise across different African countries of working successfully in what might be called constrained conditions. Eighty percent of the world's children are in developing countries. Yet, much of the research in mathematics education backgrounds this reality. It is our role in ICMI to embrace such, and take seriously the millennium goals of *Education for All* by 2015. ICMI's intention to launch a new study focused on *Re(sourcing) teaching and learning mathematics in constrained conditions: The case of multilingual classrooms*, is a welcome step in this direction.

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CLAUDI ALSINA

What is the impact of hosting an ICME for the
organizing country? The case of ICME 8 in Spain

*To Gonzalo Sánchez-Vázquez and to
Miguel de Guzmán, in memoriam*

1. *Why did we decide to host an ICME?*

Let me review, briefly, what our situation was in the 80's concerning mathematics education and why we decided to host an ICME.

For many years the Real Sociedad Matemática Española (Royal Society of Mathematics, Spain) had almost no activities in mathematics education, and mathematical teacher associations did not exist. There were in the 70's some active innovative groups of mathematics teachers (Grupo Cero, Grup Zero, Periodica Pura,...) and later three societies were founded in the Canarias (Isaac Newton), Seville (Thales) and Granada. In 1985 I became (for 12 years) the Spanish delegate in IMU and ICMI, nominated by the CSIC (Consejo Superior Investigaciones Científicas), which at that time made all the international representations in ICSU (International Council for Science). The Thales Society represented by Gonzalo Sánchez-Vázquez and Antonio Perez had the first idea of hosting an ICME in Seville, so they approached me and I decided to be involved in the project and to help them as much as possible. It was clear for us that *promoting an ICME in Spain was a great opportunity to improve mathematics education in the country*

and, in particular, to articulate a national development of mathematics teachers' associations.

2. What did we have to do to host ICME 8?

We decided to work simultaneously on two different levels, at the national level promoting activities, joining efforts and convincing authorities and, at the international level, establishing contacts with ICMI Executive Committee and promoting our candidature. The first action was to involve a big society in Andalucia (SAEM Thales) and then to look for a Federation that could articulate different regional initiatives. The development was spectacular so authorities became interested.

On the other level J. Margalef and myself approached the ICMI Executive Committee in Berkeley at an ICM, the Thales Society applied to host ICME7 in 1992, Geoffrey Howson visited Seville... but ICME 7 was awarded to Quebec.

We decided then to apply for ICME 8. To this end we organized a large group of attendants for ICME 6 in Budapest where I made a National Presentation. We also organized the ICMI Study on assessment in Calonge, we attended the ICMI Study on Popularization. But a key fact was that Miguel de Guzmán was elected President of ICMI. We finally got the responsibility of hosting ICME 8 in Seville.

To sum up these historical notes we may say that *the full process of working at the same time at the national and the international level motivated a lot of people to become involved in mathematical education activities.*

3. ICME 8

ICME 8 was for us a successful congress (3600 participants plus 400 accompanying persons) with a very rich scientific and social program. But what was really interesting was the 1992-1996 period when the Federation developed and many new educators became involved in the preparation. Miguel de Guzmán as President of ICMI, played a prominent role. Gonzalo Sánchez-Vázquez as President of the Federation and the SAEM Thales was the key person to bring together Spanish teachers and societies. Antonio Pérez, Antonio Aranda and the big group of the SAEM Thales were dedicated and efficient local organizers. I myself, as the Chair of the IPC dedicated a lot of effort to have a rich scientific program. We only held one meeting of the IPC in Seville but e-mail was already there and by exchanging thousands of e-mails we could do the work. The ICMI Executive Committee and the full IPC made essential contributions. On this occasion, as in many other ICMI initiatives, it is necessary to mention the great contributions of Mogens Niss and Bernard Hodgson.

4. What were the impacts from ICME 8?

Perhaps without hosting ICME 8, all the positive initiatives that we generated with an ICME as a goal could already have been sufficient to imply a positive change in mathematics education in Spain. But we did host ICME 8 and since then the last 12 years have been very fruitful for mathematics education development. Let me point out some outcomes that became reinforced by ICME 8.

FESPM: The Spanish Federation of Societies of Mathematical Teachers has been growing since 1996 with more regional societies founded and incorporated to the FESPM and generating as we will see new activities at all levels (<http://fespm.es/sociedadesdatos.html>).

Table 1. Spanish societies

SOCIETIES	REGIONS
Federació d'Entitats per l'Ensenyament de les Matemàtiques a Catalunya	Catalonia
Organización Española para Coeducación Matemática "Ada Byron"	Nacional
Sociedad Andaluza de Educación Matemática "Thales"	Andalucia
Sociedad Aragonesa Pedro Sánchez Ciruelo	Aragón
Sociedad Asturiana de Educación Matemática "Agustin de Pedrayes"	Asturias
Sociedad Canaria "Isaac Newton"	Canarias
Sociedad Castellano-Leonesa de Profesores de Matemáticas	Castilla-León
Sociedad de Ensinantes de de Galicia (ENCIGA)	Galicia
Sociedad Extremeña de Educación Matemática "Ventura Reyes Prósper"	Extremadura
Sociedad Madrileña "Emma Castelnuovo"	Madrid
Sociedad Matemática de Profesores Cantabria	Cantabria
Sociedad Navarra "Tornamira" Matemática Iraskasiceen Nafar Elkarte Tornamira	Navarra
Sociedad "Puig Adam" de Profesores de Matemáticas	Madrid
Societat d'Educació Matemàtica de la Comunitat Valenciana "Al-Khwarizmi"	Valencia
Sociedad Castellano Manchega de Profesores de Matemáticas	Castilla – La Mancha
Sociedad de Educación Matemática de la región de Murcia. SEMRM	Murcia
Sociedad Riojana Profesores de matemáticas. A Prima	La Rioja
Asociacion Galega Profesores de Educación Matemática AGAPEMA	Galicia
Sociedad Melillense de Educación Matemática SBM – XEIX	Melilla
Societat Balear de Matemàtiques	Balears

JAEM: Every two years the FESPM holds the national meeting of mathematical education with 800 participants. The last one was the XIII JAEM in Granada (July 2007) and the next one will take place in Girona in 2009.

Regional Activities: All societies organize their regional congresses, distribute their publications and news, i.e. there is a significant combination of national and regional activities, seminars, courses, etc.

SUMA: In a meeting held in Granada we decided to publish a journal to be distributed among all members of the FESMP and devoted to mathematics education questions that could be of interest to all associates. The first issue was published in 1988 and its directors have been Rafael Pérez-Gómez; Sixto Romero; Emilio Palacios and Julio Sancho, Francisco Martín and Inmaculada Fuentes. So far 53 issues have been published and 6000 copies have been distributed, the journal receives many articles and it is a full color publication. It is an extremely useful vehicle for the exchange of ideas and information (<http://revistasuma.es>).

The FESPM also has a Publication Service coordinated by Ricardo Lueng which publishes monographs, books of special educational interest, etc.

Mathematical competitions: At regional and national levels, with an exponential growth of participation there are many competitions: mathematical Olympiads, Cangur competition, Fem matemàtiques, Ginkamas, etc.

A Mathematics Day in the School: Every 12th of May (anniversary of Pere Puig-Adam's birthday) in many schools there is a mathematics day devoted to a topic fixed by the FESPM which distributes a working document.

ESTALMAT: Initiated and designed by Miguel de Guzmán in Madrid, the program Estalmat, devoted to develop the mathematical talents of 11-12 year-old students, has been a great initiative that many societies have also been developing in the last years.

Comisión de Educación: The Spanish Committee of Mathematics (CEMAT) coordinates today the international activities related to IMU, and promotes many different educational and research initiatives. The organization of the ICM 2006 in Madrid was for the CEMAT the equivalent to that which the organization of ICME8 was for the FESPM. Among other commissions the CEMAT has an Educational Commission (<http://www.cemat.org/educ/educ.htm>) whose President (at this moment Professor Tomas Recio) is the delegate of Spain in ICMI. Several societies of teachers and mathematics are represented and concrete national initiatives are promoted.

International projection: The FESPM has been growing by its involvement in the Iberoamerican congresses CIBEM, in the FISEM since '03 (Federación Iberoamericana de Sociedades de Educación Matemática)

which includes societies from Argentina, Brazil, Chile, Colombia, Peru, Portugal and Uruguay), and in the European federation FEAPM.

In the last decade there has been a wider participation of Spanish mathematics educators in all kinds of international activities CIAEM's, NCTM's, ICME's, ICMI studies, etc.

Mathematics Education Research: Universities in Granada, Valencia and Barcelona (UB and UAB) have been offering Masters and PhD studies on mathematics education. There is also a society devoted to research in mathematics education and an expanding number of full and assistant professors in the area.

5. Conclusion

Other countries may be interested in hosting a future ICME. The Spanish case of ICME 8 shows that such a "one-week congress" may be an excellent motivation for organizing and promoting new challenges for the national community of mathematics education. It's a lot of work but it is a good bet for the future.

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ICMI activities in East and Southeast Asia: Thirty years of academic discourse and deliberations

1. *Introduction*

Some thirty years ago, the countries of Southeast Asia and East Asia began to have cross national exchanges on mathematical education issues through the various ICMI activities. This paper will describe some of the ICMI regional activities and the benefits of such activities for the nations in the region.

Southeast Asia's connections with ICMI began with the founding of the Southeast Asian Mathematical Society (SEAMS) and the national mathematical societies in 1972. The ensuing activities took place against the backdrop of the socio-political developments in the region. Many of these Southeast Asian nations (Indonesia, Malaysia, the Philippines, Singapore, and Vietnam) were under colonial rule until after World War II. With post-war independence, some initial turbulence and tensions, greater cooperation and promotion of economic growth and peace were formally endorsed with the establishment of the Association of Southeast Asian Nations (ASEAN) forty years ago in 1967. While Thailand was held stable through governmental changes by its strong monarchy, countries like Vietnam and Cambodia had also to deal with further wars and internal struggles before nation building could proceed.

In the northern East Asian region, the nations of China, Japan, North and South Korea were all engaged in working out their international relationships within the context of post-war tensions, communism versus capitalism, and in building or re-building their economies in this context. In these past decades, the economies of Japan and in more current years, China, are important players on the world stage. These nations, together with Taiwan, Hong Kong and also Singapore, share similar oriental heritages known today as the Confucian Heritage Culture and this common heritage appears to have had much influence on the development of their mathematics education practices, a point which will be discussed subsequently.

Within this context, the expansion of ICMI activities from Europe and North American towards 'the east' since the 1970s has made important contributions to the development of mathematics education in the region.

2. *The SEACME Series*

When SEAMS was formed in 1972, the focus of the national mathematical societies was on mathematical activities rather than on Mathematics education. The national societies were housed in mathematics departments of universities and members were the mathematical elite of the countries concerned. Mathematics education, as a significant and different branch in the mathematics world together with its own form of research and theories, was yet in its infancy in the Southeast Asian region. This is not surprising considering that the education systems of the countries were grappling with literacy problems and the education of masses to meet the workforce needs of newly developing economies.

The Southeast Asia Conference on Mathematical Education (SEACME) series began in 1978 with the inaugural conference in Manila. This series of conferences was initiated by Professor Yuki Yoshi Kawada (ICMI secretary, 1975 – 1978) with the impetus and pioneering spirit provided by Professor Lee Peng Yee of Singapore and Professor Father Bienvenido Nebres of the Philippines. This first regional conference was a resounding success judging by the learning that took place through a series of activities leading up to the conference and the follow-up actions after the conference. The SEACME series “toured” the Southeast Asian countries, taking place triennially in Kuala Lumpur, Malaysia (1981), Haadyai, Thailand (1984), Singapore (1987), Brunei (1990), Surabaya, Indonesia (1993), Hanoi, Vietnam (1996) and Manila again in 1999.

The basic philosophy of the SEACME series which contributed to its success hinged on the principle that each conference was a national conference with regional and perhaps some international participation. The objective was to primarily benefit the host country and this engendered a sense of ownership and shared interest in the issues discussed and therefore better focus and relevance. Each conference was organized by the hosting nation which chose its own theme, invited speakers and encouraged attendance by mathematics teachers. The host country benefited, not only through mutual learning on relevant issues within the local mathematics education community, but also through providing their teachers and other participants the opportunity to learn from the regional and international speakers and participants. If the conference had remained totally national, the academic discourse may have suffered from parochialism. Thus, the principle of having focused national issues but with regional and even international perspectives provided for stronger discussions and the generation of new ideas. As reported in Lee (1992), «we found we could learn a lot from each other perhaps even more so than from the developed countries ... this was something that a regional conference could accomplish that no international conference could do».

3. *The EARCOME Series*

While the Southeast Asian countries are rather diverse and had come under the influence of colonial masters, the northeast Asian countries of

China, Japan and the Koreas had longer histories and greater homogeneity within each country. Formal collaboration among these north East Asian countries began in the later part of the 1980s after China joined the IMU and ICMI.

Professor Lee Peng Yee of Singapore, with his SEACME experience, worked closely with the Beijing Normal University to organize and host the first ICMI-China Regional Conference on Mathematics Education in Beijing in 1991. A large delegation of Japanese attended this conference and in fact, the presentations were all conducted in three languages: English, Chinese and Japanese with the help of very capable translators. A second ICMI-China conference was held in Shanghai three years later in 1994 with active participation from Japan and South Korea. The Southeast Asian countries also participated in these two conferences, especially Singapore, which experienced few cultural and language barriers due to her majority ethnic Chinese population.

After these two conferences, it was felt that the time had come for the series to move out of China to be more inclusive. South Korea agreed to host the first of these conferences in 1998 and the series morphed to become the ICMI-East Asian Regional Conference on Mathematics Education (ICMI-EARCOME). Although there were some language difficulties, the official language of the conference was English and the South Koreans provided their teachers with translated full conference papers prepared prior to the conference. This first EARCOME had eleven plenary keynote speakers, five of whom were from 'western' countries (USA, UK and Australia) and six from the East and Southeast Asia.

By the turn of the century, the SEACME series completed a full circle of countries with SEACME-8 back in Manila in 1999. Since Southeast Asians had participated in the ICMI-China conferences and first EARCOME, it seemed sensible to merge the two series. As a country with similar Confucian Heritage Culture as the North East Asian cultures and yet situated centrally within Southeast Asia, Singapore offered to host EARCOME-2 cum SEACME-9 in 2002. Into the twenty-first century and in a cosmopolitan Singapore, EARCOME-2 had more than 500 local participants and more than 130 foreign participants from 18 countries, including Australia, New Zealand, USA, Canada and European countries.

With this merger, the SEACME series was subsumed into EARCOME series and the subsequent two EARCOME series conferences were held in Shanghai, China in 2005 and Penang, Malaysia in 2007.

4. Changes through the years

During the first two SEACMEs, mathematics education in the region was still largely traditional in the sense of learning basic concepts and com-

putational skills. Concerns were more with raising the level of the basics for all the population. At times when economic survival was of the foremost concern, aspects such as enjoyment of mathematics or mathematics enrichment activities were of low priority. During this period, the few mathematics educators who received their graduate education in Western or Australian universities began to introduce constructivist theories into teacher education. Developing a local brand of mathematics education was a slow process since these theories had to be adapted and applied in very different cultures and contexts. Moreover, the number of mathematics education PhD holders with a strong understanding of the theories and the capability to carry out mathematics education research in local contexts was small, so their expertise went largely unrecognized and their influence was also limited.

However, the development of mathematics education research and greater understanding of mathematics learning grew steadily and, as the world entered the knowledge-based digital age with the advent of the internet, quick global communication accelerated the evolution of education systems and within them, the learning of mathematics. Such developments in education were made possible also because the nations became more developed economies and moved away from survival status. Some of the changes are:

(a) Mathematics became more widely applied and learning mathematics became more for utilitarian purposes, even at tertiary levels and hence was no longer restricted to the mathematically talented.

(b) Better education and information available to parents and students meant that teachers had to earn respect through stronger professionalism, especially in terms of their knowledge and skills. This was very different from the traditional unquestioning respect accorded to teachers in Eastern cultures.

(c) As nations became more developed, more teacher educators were also better qualified in their field and mathematics educators were no longer mathematicians with interest in education but doctorate holders with research capabilities and a good grounding of educational theories.

All these changes resulted in the learning of mathematics becoming, within each nation, hybrids between western learning theories and eastern cultures. It needs to be stressed that while there are similarities, especially among the Confucian Heritage Cultures, each hybrid is different. For example, Japanese lessons on mathematical problem solving are very 'constructivist' in stimulating mathematical thinking processes through questioning. In China, students are also engaged in problem-solving tasks of high mathematical level, with computations or algebraic manipulations performed almost effortlessly. Thus, although some mathematics lessons may appear drill-and-practice based, the motivation for procedural mastery is for mov-

ing on to higher order problem solving. In Singapore, mathematics learning tends to be pragmatic and the 'basics' and applications are well covered and understood by the pupils.

The East Asian education systems are eager to learn from others while concurrently retaining those culturally rooted aspects which work to their advantage. An example of this is parents' high expectations of their children's educational performance. Stemming from two characteristics of Confucian Heritage Cultures, firstly, strong family bonds and secondly, the respect for education and its (albeit sometimes extrinsic) usefulness, East Asian students do not have to enjoy the subject to be motivated to learn or excel in it, since family pride is sufficient motivation. The intertwining of constructivist teaching approaches coupled with high expectations seems to be a formula which has worked well for these cultures, as evidenced by TIMSS results.

By the time of the first EARCOME, the mathematics education conferences had moved from SEACME's initial 'national conference' format to one rather similar to western conferences, with a substantial number of foreign and local keynote speakers and paper presentations organized along various strands. This is a natural outcome of globalization and not unwelcome since it enables the academic discourse to be enriched with wider perspectives and diversity, to stay globally relevant and up-to-date. Nevertheless, local participation is still strong, with many paper presentations being from local participants, discussing local issues and research. For example, in EARCOME-4 (2007), the Malaysian concern about student difficulties in learning mathematics in English was a recurring theme. Also, the principle of benefiting local teachers is still applicable especially through having concurrent workshops running parallel with the conference. Practitioners who may benefit less from research presentations can learn new skills through these workshops.

A less welcome change in recent years is the decreasing participation of mathematicians in SEACME and subsequently EARCOME. The earlier strong participation of mathematicians was useful in raising the mathematicians' level of understanding of mathematics education issues at lower educational levels while the general mathematics teaching community benefited from the perspectives which mathematicians brought to the discussions. This decrease in participation is perhaps an inevitable consequence of the establishment and recognition of mathematics education as a separate field from mathematics, as the body of active mathematics education research in the region grew and developed. It would be interesting to check on mathematicians' participation in ICME congresses over the years to see if there was a similar trend. As the discourse between mathematicians and mathematics educators is valuable, ICMI may consider giving some attention to this phenomenon.

5. Conclusion

The largest and most prominent ICMI event in the region was ICME-9 (Tokyo, 2000). The holding of ICME-9 in Tokyo was a very welcome signal of Asia's importance in the more Euro-North-American-centric ICMI. Two other ICMI activities need to be mentioned: (a) ICMI Study Conference on the Teaching and Learning of Mathematics at University Level (Singapore, 1998), (b) ICMI Study Conference on Mathematics Education in Different Cultural Traditions: A Comparative Study of East Asia and the West (Hong Kong, 2002). While the first study was merely hosted by Singapore with mainly western participation, with global interest generated by TIMSS performances, Asian perspectives were central to the second study conference.

East Asia and Southeast Asia have come a long way in Mathematics Education and this is in no small way attributable to the ICMI activities begun thirty years ago. Compared to the century of ICMI activities, the East Asian region is a latecomer on the scene but with the support of the mathematics community and the fast pace of development, the region is becoming an important contributor and player in ICMI activities. Research collaboration between Asian mathematics educators and western colleagues is flourishing and Asian mathematics educators also participate in global mathematics education activities such as ICME and PME in a big way. Looking towards the future, ICMI's activities should spread to the South Asia and Middle Eastern regions to be truly global.

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Report of Working Group 1

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Disciplinary mathematics and school mathematics

The Working Group began with a Discussion Document, which prompted twenty papers from invited participants. Prior to the conference an Overview paper was prepared summarising the issues that had emerged. During the conference there were two sessions that included both brief presentations and open discussion amongst the twenty-five participants. All documents, and the names of participants, are available on the conference website (www.unige.ch/math/EnsMath/Rome2008/).

This paper summarises all the activities of the group. We wish to thank the conference organisers for producing an excellent forum for discussion of this important topic, and the participants for their outstanding participation and the insight they brought to our deliberations.

1. The Discussion Document

The document was presented as a tentative description of what the topic might encompass. A starting point was the question: What sort of mathematics do we learn at school? An answer that described school mathematics (hereafter abbreviated by SM) as a subset of topics from disciplinary mathematics (DM) was rejected as unsatisfactory, since mathematics does not reduce itself to a list of topics, but also comprises processes such as abstraction, symbolisation and axiomatisation. We raised the more fundamental question of how SM might differ in nature from DM.

An important consideration is that the nature of mathematical knowledge and mathematical certainty are not static. As we look at mathematics since 1908, the year ICMI was founded, we can identify major events which have impacted deeply on our perception of mathematics. One can think of the crisis in the foundations of mathematics, of Gödel and Russell, of constructivism and Brouwer, of Bourbaki, of the advent of computer-assisted proofs, of probabilistic proofs, and of the development of modelling in applied mathematics.

The constant evolution of DM means that one cannot simply contrast mathematics taught in school with a fixed identity. Nevertheless, there are some issues that we can use to reflect on SM and contrast it with mathematics as the subject that mathematicians have taken as their discipline. We discuss the issues of proofs and proving, the development of mathematical ideas, social aspects, applied mathematics, and high-level mathematics.

What is the role of proofs in DM and in SM? How do changes in the nature of proof in mathematics change the nature of proofs in SM? Are the standards for accepting a proof different? In what way is proving as done in SM a preparation for proving in research mathematics?

One can also think of truth or the warrant for mathematical knowledge. In mathematics, truth is not only a logical and autonomous construct: it is also a social construct and the validity of statements can be accepted by a mathematical community even if not all of its members have read (or understood) the proofs of those statements. Recent examples are Wiles' proof of Fermat's Last Theorem or the recent solution to Poincaré's conjecture by Grigori Perelman, but this is also true of much simpler results. Is it the same in SM? To what extent can the classroom mathematical community provide a warrant for mathematical statements?

A second issue is how fundamental mathematical ideas are presented to learners of mathematics. For example, is the idea of function presented in the same way in school and in undergraduate classes? Must school presentations be shadow versions of the concept as held by mathematicians? If so, how do these shadows develop more substance as the learning becomes more sophisticated? Is there a best developmental sequence for any particular mathematical idea? What are the consequences if a student stops learning mathematics before an idea is fully developed?

Social issues include the way teachers and mathematicians view their subject. Their understanding is deeply rooted in the way they do mathematics and learn mathematics themselves. Therefore there are likely to be differences between DM as experienced by undergraduates taught by mathematicians, and SM taught by mathematical teachers. How is the way we learn linked to the way our teachers see mathematics?

Although there is not a clear distinction between pure and applied mathematics, nevertheless some features embedded in this distinction cause us to reflect on the teaching of mathematics in schools. How can the extremes of mathematical investigation for its own sake and modelling real world processes both be represented in SM? How should the different philosophies inherent in each of these be presented? What are the consequences of presenting applications of mathematics as motivational material?

A final issue offered for consideration was the ways in which high-level mathematics has been made accessible to advanced school students. Is it a good idea to present mathematics differently to students of different abili-

ties or inclination? How can this be done without negative consequences?

We acknowledged the many other aspects of SM and DM that could be addressed, for example, language, assessment, formalism, the role of technology, creativity, and discovery. These initial thoughts were taken up and extended by the papers offered by members of the Working Group. We therefore invite readers of this summary to read the papers in order to appreciate their diversity and depth.

2. An Overview of the Responses

(In what follows individual papers are referred to by the surname of the author only, and readers are referred to these for relevant references). As remarked by Siu in his paper, we are first reminded of the double discontinuity described by Felix Klein. University students find themselves studying mathematics largely unrelated to what they learned in school, and young teachers are asked to teach mathematics that has little connection with the subject experienced at university. One hundred years later, we find ourselves concerned that the discontinuity has yet to be patched.

The most apparent question implicit in many of the papers is whether SM should reflect DM, or, at least, to what extent should such a connection be evident.

On the one hand, Watson argues that SM is a very different sort of thing, giving examples (see later discussion), and therefore needs to be different in type. On the other hand, in apparent contradiction to this view, Terry Wood, Maria Lucia Moro and Maria Tereza Soares express the belief that it is important that learning practices should reflect the disciplinary practices in mathematics, and thus «school mathematics would not essentially be different in nature from mathematics as a science» (Moro/Soares). While acknowledging different purposes and acknowledging the distinction between school learning practices (those that help understand existing mathematics) and disciplinary practices (that are concerned with developing mathematics), both papers argue that students learn through reasoning that resembles mathematical thought.

Many of the papers discuss other issues (the nature of university mathematics or teacher preparation) but imply by default that DM should be reflected in schools. Viviane Durand-Guerrier suggests a third option: that DM could benefit from reflecting aspects of SM.

The following questions, therefore, were proposed as the focus of early discussion within the group.

Questions 1. To what extent do we consider that SM should reflect DM? In what ways is this possible and in what ways should it be different? What are the factors that might affect the way that SM reflects DM?

The second issue that emerged from the contributions is the difficulty of describing 'disciplinary mathematics' sufficiently to know what it would look like if it was to be mirrored in schools. Johann Engelbrecht asks this question directly, David Henderson asks the same question by discussing what skills undergraduates need to have to deal with DM. Romulo Lins, by imagining Euler in a modern university context, thinks rather of a mathematical environment in which we can manage education through mathematics. Davis and Derek Holton each suggest that we should look at the whole question of mathematics knowledge generation from a wider perspective, and wondering whether DM (or SM) can be encapsulated at all. Again the issue of process versus content emerges as important, although nearly all contributions leaned towards process and mathematical habits of working rather than content.

The following set of questions was therefore proposed as a focus of our second session.

Questions 2. What aspects of DM are important to mathematicians? What descriptions and what models of these aspects are available? What historical features of mathematics have been enduring, and what features may emerge in the future? How can mathematics be seen as a coherent whole, how does it link together, and how might this be represented at school level?

Both the above sets of questions are approached in papers by Galbraith, Siu and Novotna through thinking about teacher preparation.

Another group of papers that contribute to the main questions but have a common theme are those of Zuccheri, Arlete Brito, and Klaus Volkert, who all reflect on historical exemplars of attempts to bridge the school and disciplinary divide. Zuccheri's paper describes a specific teaching method, based on cognitive theories attributed to Ernst Mach, the 'Jacob Method'. Two central points of this method are that the kernels of mathematical concepts are real situations and that concepts are the instruments made by humans to cope with experience. Brito's paper reminds us that mathematical issues are not the only ones that affect the relationship between schools and the discipline. As an example, she describes in detail the American influence on Brazilian mathematics education through USAID, both at university level through Bourbaki materials and at school level through texts from the Modern Mathematics Movement. The history of solid geometry is reviewed in Volkert's paper, showing that our pedagogical intuition does not always follow DM. For example, he describes an approach from the early 20th century based on the idea that the Euclidean simple to complex build-up does not seem a sensible approach when students come to class with wide experiences of 3-dimensional objects and their relations. However he also shows how a systematic approach to solid geometry needs more than

intuition. This contradiction is a specific example of the tensions at work if SM is to reflect the discipline.

Finally, some papers describe, or argue for, particular positions. Novotna, Kwon, and Laura Martignon each challenge us, through a particular example, to actualise our answers to the bigger questions by considering particular cases. Is the early introduction of probabilistic ideas using a heuristic approach a good example of the link between school statistics and the discipline of stochastics? Is an Inquiry-Oriented approach possible in undergraduate mathematics, and is this an example of school mathematics influencing the discipline at university level? Is exploring a non-standard algebraic structure the kind of experience that enables teachers to reflect the creative and surprising nature of mathematics in their teaching?

3. First Session: Presentations

During the conference, the Working Group had two sessions of two and a half hours each. Frédéric Gourdeau opened the first by welcoming and thanking everyone for their contributions. He noted that the complexity of the subject of the Working Group was reflected in the varied responses which were received. He introduced the first presenters (Watson and Zazkis) and respondents (Moro, Hersant, Alekseev, and Zuccheri).

Anne Watson argued strongly that a significant feature of SM is the need to be explicit about what is going on mathematically, and about what sort of thinking is taking place, in order that students learn these habits. Watson noted that SM has different purposes, different forms of reasoning, different forms of authority, and different activities, making the point that not only do these differences occur because the aims of the undertaking are different, but also because schools as institutions do not allow practices to proceed as they might for a mathematician: for example, time definition of classroom periods and curriculum imperatives.

She paints a picture of DM as a social activity and a mode of intellectual enquiry including empirical and deductive activity, reworking problems and conjecturing, proving and refuting. SM, on the other hand, has some more specific and psychological aims. Watson described a suite of developmental transitions that are particular to SM: additive to multiplicative thinking, probabilistic argumentation; following procedures to making choices; constructing multi-stage arguments; problem-solving in Polya's sense; and mathematical modelling. SM needs to facilitate these epistemological transitions, not merely reproduce the end product.

A further significant difference is the place of unifying concepts. Watson described these as vital sources of questions and answers in the discipline

of mathematics. SM needs to be the place to acquire the multiple experiences that, later, become unified under a greater concept.

Rina Zazkis proposed a picture of SM neither disjoint from, nor a subset of, DM, but overlapping with it. She argued that activity can be seen as a joining point between the two, giving examples from her own work with prospective elementary school teachers. These prospective teachers were involved in activities designed to promote ways of working which approach those of mathematics research, in form if not necessarily subject matter. Key elements of these activities were conjecturing supported by a technology that enabled effective experimentation and the use of novel visual representation, exploration, observation, and wondering.

These comments reflect Wood's paper in which she claims that this is how students come to understand the subject in a more conceptual way. Furthermore, to the extent that classrooms are an induction into a set of social norms that make up a discipline, those environments must reflect that particular habitus. Boaler's research is cited as evidence that how we learn directly impacts on how we come to see and do mathematics, hence the essence of DM must be present in the culture of learning.

Implicit in the analysis of Zazkis (and others) is the desire that SM should mirror the type of work of DM (as opposed to its content). Papers by Hersant, Alekseev, and Gila Hanna give particular examples of such processes. Descriptions of SM as a mirror give us a glimpse into the activities of mathematicians that are perceived to be worthy of pursuit. They include such aspects as conjecturing, experimenting, generalizing, abstracting, and asking questions. Other features emerged in discussion (see below).

The first of the commentators, Moro, argued that SM had to be seen as part of mathematics more broadly conceived; it was not anything else, after all. She also noted that this form of mathematics was experienced by mathematicians in their development, and therefore it is part of their mathematical knowledge. More crucially, she noted that mathematical knowledge is a cultural product, and retains this characteristic in schools. In addition to the mathematical processes that are reflected in schools, and the different goals of SM and DM, Moro mentioned five factors that would affect the way that SM would reflect the wider discipline: epistemological features of mathematics; institutional goals; student developmental characteristics; teacher expertise; and learning contexts and resources.

Magali Hersant approached the issue through her work with 'Problèmes pour chercher' (PPC) in French elementary schools. This curricular approach uses mathematical experience, generic examples, counterexamples, conjecturing and arguing, deduction and doubt as mathematical experiences, and results in the excitement and frustrations that mirror those of mathematicians. Hersant asked us to consider in what ways the students' answers approach a mathematician's point of view, and what aspects of mathe-

matician's thinking are students apparently unable to approach. While having ongoing lived experiences of this kind is critical to school mathematics, she notes that it is not possible to systematically adopt such practices alone in SM since it is not suitable for all the learning that needs to take place, and, at least in elementary schools, students' abilities to conjecture and argue are somewhat limited.

Hersant also noted that working mathematically in a classroom involves a didactical contract in the sense of Brousseau. Her research shows that the mathematical background and ability of the teacher has an important bearing on the nature of this contract, that is, on the extent to which the classroom does, in fact, mirror DM.

The next commentator, Valeri Alekseev, emphasised the need to be careful about what we mean by the terms we are using, arguing that arithmetic is just as much DM as, say, calculus. He noted, however, that SM cannot be DM because of a discontinuity in the destinations of the students. Some will have little to do with mathematics after they leave school; therefore, school mathematics must be part of culture, not an academic discipline. Thus SM cannot be a subset of DM. However there is a need for common notions, for example, that of algorithm.

Finally Luciana Zuccheri approached the same issue through considering the reasons for offering mathematics as a school subject. Apart from providing tools for life, and being a beautiful and exciting subject, doing mathematics develops many abilities, for example the skills of criticism, judgement, justification, attending to others. Therefore, she argued, as Watson had done, that students must learn to think mathematically in the sense of Polya: especially important are conjecturing, arguing, and proving or falsifying. Zuccheri elaborated by emphasising the importance of discussion and cooperative work at all levels of schooling, and hence the need for teachers to have good mathematical preparation.

In addition Zuccheri suggested that although modern research topics are beyond school students, even gifted ones, such students can approach the work of mathematicians through the history of mathematics. Here they can see how mathematicians worked on difficult problems and developed effective methods to solve them.

4. First Session: Discussion

The discussion that followed these presentations made several new points. Discussion about DM noted that it changes more quickly than SM. However it may not be so different, for example, it also happens in a restricted environment. Research is never 'pure': there are publication or conference deadlines, and disputes over who has the authority to provide warrants. Similarly,

DM also includes learning about what other mathematicians have done, and both forms involve 'knowing the rules' of how to play the game of mathematics. Other features of DM mentioned were the practice of repeatedly returning to a concept to converge on full understanding, the balance between mathematical discipline and mathematical creativity, and the practice of linking skills together and creating new ones.

A historical point of view seems to diffuse differences between the two types of mathematics. For example, the notion of rigour has changed, and the rigour of today's schools might be more comparable to that of yesterday's mathematicians.

Comments on SM made the point that, in schools, it is often necessary to teach things wrongly (or, perhaps, not fully truthfully) as a practical strategy at the stage of development of the concept being considered. Teachers look at problems in a variety of ways, including from a pedagogical point of view. These ways are not the same as a researcher bringing multiple approaches to a problem.

Another example of such difference is the use of proof. In the classroom, proof is not just a means of getting from A to B, they have a pedagogical role, as well as modelling mathematical practice. Proof, and this is also the case in DM, can also introduce new techniques, insights, and concepts, can make links across the field, can provide explanations, and can elucidate new methods. Indeed, it was argued that such use of proofs is underutilized in schools. However, in addition, the point was made that just because conjecture, argument and proof are present does not necessarily mean that mathematics is present.

The view was put forward that, if we see mathematics made up of content, activity, and competency, then it is only the latter that is essentially different in the discipline. The content, especially hypercontent such as 'pattern' might be the same for a researcher and a primary student.

The main distinguishing characteristic of SM is the presence of a teacher—how does the teacher's knowledge mediate SM? How does a student perceive their work? Possibly as working like a mathematician, but there is no pretence that they are mathematicians.

It was also noted that technology has changed the nature of both SM and DM. What we did as students is not what today's students do. Similarly young mathematicians work differently from older ones.

An important idea that was emphasised in the discussion is the need, in a school classroom, to be explicit about what is going on. For example, it is not enough to conjecture and argue, it is important to notice that we are conjecturing and arguing, and to become aware of what those practices are like.

Brent Davis rounded off the first session with a reflection on our discussion. He brought to our attention the images that we were using to frame

our comments. He noted that images like Venn diagrams already positioned SM and DM as existing places; images like reflection make SM 2-dimensional compared with DM's 3-dimensions. He asked us to consider other images: Yin/Yang to emphasise complementarity or symbiosis; pyramid structures, possibility spaces or time dependent images to frame DM as based on or emerging from SM.

5. Second Session

Bill Barton chaired the second session, introducing four presentations by Novotna, Galbraith, Siu, and Kwon. The first three related directly to teacher education, which, although not the primary focus of the working group, was felt to be a meeting place where the conceptions of DM meet the anticipation of SM.

In the first of these, Jarmila Novotna presented the use of unfamiliar mathematics in teacher education. Drawing from the work of Bromme, Nieto, and Jaworski and Gellert (who all include in the knowledge base needed for teaching a component which can be described as disciplinary mathematics even if those exact words are not used), she explained why it was important that unfamiliar mathematics situations, concerning subject matter which would not be directly taught in school, should be part of teacher education. She presented three examples from the courses for future mathematics teachers training at Charles University: functional definition of a polynomial, restricted arithmetic, and criteria for divisibility in non-decimal bases. An important reason for doing such work is the need for teachers to experience and reflect upon the reactions of a learner facing unfamiliar but not entirely new notions, the subsequent need for a refinement or reorganisation of previous knowledge, and the modification of no longer adequate understanding. Along the way, Novotna presented mathematics for teaching as including the whole of SM, part of DM, and much else besides!

Peter Galbraith argued strongly for providing teachers with authentic mathematical experiences, giving them problems that «force confrontation with understanding of mathematical specifics, using approaches that provide practice in the very attributes of argument and communication that are the lifeblood of vigorous didactics». Deliberately trying to «rattle decisively the conceptual cages of intending teachers with respect to fundamental mathematical ideas», he frames his interventions around concrete tasks which address issues at the heart of mathematics. Referring to his paper, where these are articulated around nine key aspects (for example, resolution, conflict, explanation, definition), he also indicated the importance of being able to identify potential mathematical tasks in the environment,

and being able to do creative mathematical modelling. He suggested that this is the type of DM which ought to comprise part of the education of prospective teachers.

Inspired by Freudenthal, Polya, and Wittmann, Man Keung Siu focused on the need for teachers to be involved in research while noting that the research so-intended may look different from that of a research mathematician. Explaining the differences, he indicated that part of the reason for teacher research was the necessity for teachers to develop the self-confidence needed to realise their own shortcomings while being able to think through the problem faced. He concurred with Davis and Galbraith about the need for a teacher to know mathematics differently. Two nice examples of such research are in his paper.

Oh Nam Kwon spoke from what would seem to be a standard DM setting: teaching a course on differential equations. She presented the research-based Inquiry-Oriented approach to Differential Equations project, the two main sources being instructional design of Realistic Mathematics Education from the Freudenthal Institute and literature on the social production of meaning. This is a clear example of the difficulty of describing DM: courses like the one presented profoundly challenge usual descriptions of the way mathematics is learned at the tertiary level.

Some of the following discussion centred again on the authority for deciding truth. Does it lie with the individual, with the class, with the textbook, or with the teacher? This will differ depending on our vision of SM (see Davis). We also ventured a little into the purpose of SM: is it primarily educating through mathematics, seeing the world through mathematics (which would be closer to DM), or something else?

Much of what was discussed is captured by the activity that concluded our deliberations. In small groups, participants discussed the issue of the 'mathematical awarenesses' that need to be made explicit in teaching. The task was interpreted widely, but provides a flexible framework with which to sum up our deliberations.

6. Mathematical Awarenesses needed in SM

What follows is a list of 'mathematical awarenesses' that were regarded by the group as important. They are organised so as to illustrate the group's answers to the questions about SM and DM that were identified in the preliminaries.

6.1 To what extent do we consider that SM should reflect DM?

Noting that SM is different in aims and characteristics, the reflection of DM processes is regarded as important in SM. Teachers need to be explicit to their students about these processes and the nature of mathematics.

For example:

- Mathematics grows dynamically (this is clear to mathematicians but absent in SM). Linked with this growth is the idea that the meaning of mathematical concepts emerges through the manner in which those concepts are used.
- Mathematics is not about the physical world, in the sense that the definitions and constructions of which mathematics is formed do not live in the physical world. Nevertheless, mathematics provides a good way to analyse or model reality, and modelling is an important mathematical process.
- Mathematics is a cultural construction, which could have been different and may evolve in ways which are very different to what we may anticipate.
- There is a growing belief that technology is part of mathematics. In DM, mathematics is developed through technology (Maple or Mathematica for instance) and the implications for SM can be important.
- Mathematics contains implicit associations (metaphors, images, exemplars, gestures, etc.) that give shape to a concept.

6.2 What aspects of DM are important to mathematicians?

Processes, rather than topics, are the most important characteristics of DM. For example:

- Important mathematical processes include conjecturing, experimenting, generalizing, abstracting, arguing, doubting, and asking questions.
- Much of mathematics involves looking for invariance against the background of variance. Symmetry, a key concept, can be seen in this light.

6.3 What of the role of proof in SM and DM?

Proof is not just a record of a logical argument, nor the basis for belief. It has many other roles, pedagogical in SM and creative in DM. Examples of important features are:

- Logic is an important and key aspect of DM. However, it is often not formal logic which is employed, but contextual logic. For example, the distinction between implicit versus universal quantification may need to be drawn out in SM.
- Making personal sense of mathematical statements and having personal proofs or justifications is part of doing mathematics. One does not simply accept statement after statement without building a personal mathematical architecture of some sort.
- Mathematicians make mistakes! While this is clear to all mathematicians, this does not mean that one cannot trust statements in mathematics.

Three important questions that were left unanswered by the group were:

- Is there a best developmental sequence for any particular mathematical idea?
- What are the consequences if a student stops learning mathematics before an idea is fully developed?
- Is it a good idea to present mathematics differently to students of different abilities or inclination?

7. Conclusions

Working Group 1 of the Rome Symposium on the Occasion of the 100th Anniversary of ICMI was asked to address the issue of school and disciplinary mathematics. Rather than taking a historical view of this relationship, the group used the dichotomy produced by constructing these two forms of mathematics as a means to investigate the nature of the educational task in mathematics. We did not take these constructs as 'real', but rather used them as a means to better understanding.

Where did it lead us? At the same time as describing much detail that enables us to see learning mathematics in the light of the discipline of mathematics (and vice versa), we felt that an important idea was that of 'awarenesses'. When undertaking mathematics as a research endeavour, a mathematician deals with content, and performs mathematical processes with particular goals in mind. The mathematician is aware, at a meta-level, of different options and self-regulates his or her activity within the confines of the field.

In schools, the content and processes bear a resemblance to those of the mathematician. This occurs at all levels of mathematics education, from primary to tertiary. It was felt to be possible (if desired) to communicate some sense of every aspect of DM in any mathematics classroom. The mathematical authenticity of the experience depends on many factors; critical amongst these is the mathematical knowledge of the teacher.

However, the goals of SM are different from those of DM, and, in particular, the need for awareness of what is happening mathematically is important. Some of these awarenesses are listed above. In addition, students need to develop an awareness of the epistemology of mathematics, how mathematical knowledge has been generated historically, and how it develops as one learns mathematics. We feel that communicating mathematical awareness is at the essence of the mathematics teachers' task vis a vis that of the research mathematician (or, perhaps, of the task of the mathematician when they are teaching, compared with when they are researching).

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Report of Working Group 2

BARBRO GREVHOLM and DEBORAH LOEWENBERG BALL

The professional formation of mathematics teachers

1. *Introduction*

Working Group 2 focused on the knowledge and skills needed for teaching practice, drawing on theoretical, mathematical, and practical perspectives, and on how these ideas have evolved over time.

The aims of the group were to examine the following key issues:

- ways in which such knowledge and skills can be developed (opportunities for learning);
- teaching as professionally skilled work, and how this influences teacher education; and
- the role and the work of mathematics teacher educators.

The group attracted 32 paper contributors and three additional participants in addition to the two co-chairs of the group. Although contributors to WG2 were originally asked to focus on a broad range of specific aspects of teachers' education, some issues and phenomena aroused more interest than others and these were the focus of many papers. In this synthesis paper we provide a brief overview of issues and questions that emerged across the papers and that were discussed during the five hours of session time in Rome.

As co-chairs of Working Group 2, we were invited to write a paper that synthesized in broad strokes the work and deliberations of our group. The papers provided a sampling of current research in the area of mathematics teacher formation. The papers altogether comprise about 170 pages of single-spaced text; it is therefore of course not possible for us to do justice to the details. Due to space limitations, we do not include many references beyond the 32 contributed papers. The papers themselves give an overview of relevant references in the area of mathematics teacher formation. It is only possible to offer glimpses of the content of the papers and the rich discussions that took place in the group sessions. We hope that this survey will stimulate interest in reading the complete set of papers and to thus get a more detailed perspective on the available research studies and their results.

2. *The historical development of mathematics teacher education*

In a separate paper (Grevholm, 2008) we have given an account of how we see the role of ICMI and the ICMEs in the development over time of mathematics teacher education. It is evident that teachers' education has, from the very beginning, been a matter of direct interest to the organization. One of the first ICMI documents is an overview of structure of mathematics teacher education in different countries (Loria, 1933). Moreover, it is obvious that there has been much change in teacher education over the last 100 years. We can summarize this change saying that mathematics teacher education has shifted from:

- training to education, a professionalization of the mathematics teacher
- teachers' colleges or pedagogical seminars to universities, representing an academization of and/or inclusion in the university system of the education
- from education for an elite to education for all, a massification of education with obvious consequences for teacher education
- from experience-based to research-based, from valuing skilled practice to theoretical professional knowledge of teachers.

This history of mathematics teacher education also became visible in the discussions of the working group in Rome. It is obvious that the current situation is influenced by the history of teacher education and the roots are important for understanding future development.

3. *Structure and focus of the Group Work at the Symposium*

The working group focused on the historical evolution of conceptions of the various forms of knowledge and skills for mathematics teaching, and the role of ICMI in this domain. The group also discussed how teachers might be helped to acquire such knowledge and skills. The working group examined, in particular, the traditional dichotomy between content knowledge and pedagogical knowledge and considered how the standing of the teacher as a professional has evolved over time. Because the theme of the working group was so broad, we attempted to focus the work with nine specific questions, listed on the webpage for our group. From these nine the following topics were distilled, and these were discussed in smaller groups:

- *Theoretical frameworks used in studies of mathematics teacher education*

Among the group's papers, there is a rich diversity of frameworks used. In addition, many studies do not rely on a specific framework but center

instead on some crucial concepts. Moreover, a few foundational papers strongly dominate the references on teacher education and teacher knowledge (e.g., Shulman, 1986). What are the reasons for such a phenomenon and how does it influence the development of research and practice in mathematics teacher education?

- *Different conceptions of the mathematical knowledge needed for teaching and how it can be acquired, and its relation to pedagogical knowledge*

Is it fruitful to talk about mathematical knowledge needed for teaching mathematics or would it be more useful to strive for knowing how to act in a teaching situation in order to create better learning opportunities? Both views were discussed in the working group papers. We discussed what 'pedagogical content knowledge' is and how it differs from the subject knowledge needed for teaching mathematics. Danielle Huillet discusses this issue in her thesis, and she questions the concept of pedagogical content knowledge (PCK). What are the appropriate warrants for a theory of the mathematical knowledge needed for teaching? How universal or culture- or country-specific is the mathematical knowledge needed for teaching? The closer we get to practice in considering the knowledge needed for teaching, the more questions there are about whether cultural differences in teaching practices may affect the knowledge demands of the work.

- *The role of teacher educators in mathematics teacher education – their practice and development*

We considered the mathematics teacher educator and we wondered whether teacher educators need a more formal education. Currently there is little or no preparation for the work of mathematics teacher education. Does such preparation exist in some countries? What are the practices of teacher educators and how are they developed? How are they documented and shared with other teacher educators?

- *The tensions between the academic foundations of mathematics teacher education and well grounded experience in what helps to develop a good teacher*

What are the desirable academic foundations of mathematics teacher education? What constitute educative experiences in practice? What are, or might be, the tensions between the academic foundations of mathematics, teacher education, and educative experience?

Do we have criteria for judging what we mean by a good teacher? How-and under what circumstances-does classroom experience lead to improvements in teaching? In some countries what is valued in teacher edu-

cation has changed from clinical experience to more theoretically based knowledge about teaching and learning. Often this process has been linked to the merging of teacher education into universities. What might be the long-term effects of such a process?

- *The changes in structures of mathematics teacher education, division between practice and theory, the demand for research-based teacher education*

There are many different meanings underlying the notion of 'research-based teacher education'. What is it that we are actually striving for? Teacher training as a concept seems to have changed into teacher education based on more theoretical perspectives. What are the consequences of such a change in structural views? What are the possibilities and implications for structures of mathematics teacher education? What are the tensions between practice and theory and how might—or ought—they be addressed in these possible structures?

- *ICMI contributions to the dialogue between the mathematics teacher education community and other disciplines*

What are the practices of mathematics teacher education and how are they developed? In what ways are they different from or similar to the practices of mathematics teaching? What kinds of knowledge/knowing are reflected through the practices of (mathematics) teacher educators? Can formal education for teacher educators support the development of such knowing?

Based on the discussions of the above issues and questions, and on the contributed papers, we turn now to explore some aspects and areas of interest from Working Group 2.

4. *Theoretical frameworks, central concepts and constructs*

Research on teachers and teacher education has been based on a variety of different theoretical frameworks. No single theory or framework dominates scholarship in this area (da Ponte). In the papers we see examples of studies on teachers' mathematical knowledge (Neubrand, Potari, Proulx), teachers' knowing, pedagogical knowledge and pedagogical content knowledge (Adler), teachers' practices and teachers' professional identity, professional development (Even, Koellner, Naik, da Ponte), teachers' beliefs, assumptions and awareness (Kotsopoulos, Liljedahl, Malara, Mason, Morselli). Not always are these central concepts explained or defined generally in studies where they are used. Among overarching theories used we find appren-

ticeship models, legitimate participation, socio-cultural theory, design research approaches, interventions and many other frameworks.

Mason, in his contribution, writes about the borrowed approaches taken: «Examples of ‘approaches’ include behaviourism, pragmatism, genetic epistemology (often translated into simple constructivism), psychological, radical, and social constructivisms, sociocultural-historical, activity theory (in three generations), discourse, enactivism, distributed cognition, and complexity. All of these originate in other disciplines. What seems to be difficult is to weave a fabric out of all of these so as to maintain the complexity of learning and teaching mathematics, while at the same time informing practice and structuring systematic enquiry, but without losing the force and insight of particular approaches».

Mason contends that, «rather than ploughing established furrows and probing more deeply in order to find something fresh which is unencumbered with a burgeoning literature, researchers turn to new approaches, new disciplines, new methods, new discourses which might shed different light, or at least offer a rich seam of new papers to contribute to the literature (and so achieve institutionally imposed publication targets)». Could this be a sign of the need to reconsider what has been done so far and to try to cultivate what we have without taking stock of progress thus far?

Mason argues that, «the key question is how can teachers and pre-service teachers be stimulated and prompted to probe beneath the surface of their current awareness and sensitivities in order to notice, regarding mathematics; mathematical pedagogy; mathematical didactics; the structure of a mathematical topic; the structure and role of attention, will, desire and habit; possible criteria for selecting and presenting tasks, and for choosing ways of fostering and sustaining learner work on those tasks to pedagogic advantage; and the psychology and sociology of learners in institutions, including the potential influences of assumptions and beliefs about, and attitudes to learning mathematics held and manifested by learners, institutions, and others in their ambit». To achieve this would require a major shift in modal approaches to professional education, both preservice and inservice.

The outcomes of studies on teacher education and teachers’ work are also not easy to combine when the assumptions for the conclusions are different and the central concepts used are interpreted in different ways. Our knowledge about teacher education and teachers’ practices tends to be fragmented and difficult to interpret. In spite of the many studies with convincing results, it is challenging for designers of teacher education programmes to choose from among all these results pointing in different directions. In her plenary lecture, Jo Boaler emphasized that the mathematics education research that seems to have most success in influencing classroom practice consists of studies of teacher learning. These research programmes often included periods where the researchers worked with teach-

ers in trying to include the results of the research in teachers' practices and in classroom activities.

5. Methodological issues in studies of mathematics teacher education

Studies of teacher education are often carried out by teacher educators (Adler, Ball, Krainer, Lin, & Novotná, 2005). Most of these are case studies or small-scale studies; longitudinal studies are less common. Most studies seem to be qualitative studies but exceptions exist — such as the ongoing international comparative TEDS-M study (2008), for example, which is mainly quantitative and built on questionnaires for mathematics teachers.

With respect to the methods used in the studies reported in our working group, we see examples of all the common methods of qualitative research in mathematics education, including observations (classrooms or teacher education), interviews, video recordings, audio recordings, questionnaires, story writing, field notes, self observations, analysis of classroom transcripts and similar techniques. Adler uses in-depth case studies, others use classroom observations in combination with interviews (Potari). We find examples of intervention projects (Čeretková, Even, Millman et al, Morselli), design-based approach (Berg), combinations of action research and design-based approach (Amato), studies in the tradition of research for innovation (Malara) and longitudinal studies (Anthony). Some of the papers are more theoretical, including analyses or discussions of parts of the field, concepts or special questions (da Ponte, Liljedahl, Mason, Skott) and a few papers are historical and grow from the analysis of historical texts and documents (Bjarnardottir, Malloy). A number of the papers are linked to larger studies like the Coactive project that relates to PISA (Blum, Neubrand) or the Learning Communities of Mathematics project, a large project in Norway (Berg). In those larger projects often many instruments are used in the varied methods applied (Anthony, Even, Blum, Neubrand, Berg). Some researchers report from projects where mathematical tasks, problems or examples are used as part of the method (Pehkonen, Kotsopoulos et al, Mewborn, Zaslavsky, Borko, & Koellner). That there is such a rich variety of methods used across the studies represented in our working group indicates the large range of theoretical perspectives and methods underlying current work in the field.

6. Content knowledge, pedagogical content knowledge, mathematics for teaching, and teacher competencies

The meanings of these central concepts in the rubric were repeatedly discussed in the group and they are also present in many of the contribu-

tions. Adler, for example, provides both theoretical and empirical reflections on what it means to research mathematics for teaching and relates it to a study in South Africa. Proulx claims that one source of the conflict of mathematics content knowledge and pedagogical knowledge appears to be situated in the nature of the mathematics content offered to future teachers. He shares experiences from a teacher education project focused on the exploration of school mathematics. It is through alternative experiences that teachers will have the opportunity to continue growing in mathematics and enhance their ways of knowing and doing the mathematics, and consequently of developing alternative orientations toward mathematics and its teaching.

Neubrand asked: What does it mean to say that a mathematics teacher has 'professional' knowledge? From where do teachers get this knowledge? In what ways is the professional knowledge of mathematics teachers influencing classroom instruction? Does professional knowledge of the teacher have an effect on students' achievement? The results from the Coactive project support fostering a strongly subject-bounded, but nevertheless pedagogically oriented education of teachers, in the universities as well as in further education.

Törner et al., in the context of mathematical literacy, relates an intensive discussion in Germany about the concept of learners' competency, which then opens the question of what competency is for teachers. The theorizing around this concept, by not paying sufficient attention to teachers' professional action schemes, has pushed certain aspects of their scope of action into the background. Long stretches of teaching are driven by well-executed professional routines while performance in class is a more elusive but equally decisive factor. They claim that the most complete command of the relevant knowledge, paired with a perfect mastery of the required know-how, might still result in a poor performance. The teacher fails to hold the audience's attention. An actor, say Törner et al., must engage the spectator's sympathy with the character portrayed. Likewise, the teacher must elicit the learner's sympathy for the subject of the lesson. This can be done only if she - like the actor - *lives* it as though it were for the first time. In preparation for such a lesson, she would not only review the appropriate sequence of professional routines, but revisit the subject itself like a novice, unarmed and willing to be surprised. In other words, she has to practice what she preaches - live, real-time understanding.

Mewborn discusses learning to teach as assisted performance. She concludes that, «freeing ourselves from the idea that teacher *preparation* is just that—preparation for the future in the form of practice—and opening ourselves to the idea that teacher *education* is about structuring learning opportunities for future teachers, using the same principles we use to design educational opportunities for children, provides a wide variety of interesting

avenues for future scholarly work. The unarticulated and untested assumption within the idea of assisted performance, however, is that somehow the teachers will be able to 'pull it all together' when they leave us in order to do the things that teachers do». Here she touches upon the question of whether we should use the same theory of learning for teachers' learning that we do for students' learning?

7. *Research-based teacher education*

Berg reports a design-based study, where student teachers were engaged as research assistants in a larger research study. She claims that through becoming legitimate peripheral participants in the research project, the student teachers are engaged in critical discussions and negotiation of meaning concerning theoretical ideas from the research literature, in interviews where they search to follow the pupils' thinking, and in writing an essay aiming to link theory and practice. These activities act synergistically and promote critical thinking.

Frant presents an investigation on how reflecting and discussing about one's practice can contribute towards a professional development of calculus professors. Focus was on investigating how theories can be used to help studying about practice; to explore and raise ideas that may help the process of professional development of the involved professors. She concludes that the meetings were a unique opportunity to reflect and share about each other's practices. The theories helped to develop new ways of looking at old practices because they provoke the participants to pursue a direction that was not thought about before.

Skott sees problems in considering the teacher as an implementer of reform initiatives developed elsewhere. He makes the point that we should reconsider the ease with which we expect the results of small-scale studies to transform teaching-learning practices in ordinary classrooms. We should take it seriously that contexts matter, not only as starting points for research, but also when we consider the potential impact of research results.

8. *Mathematics teacher education, its practices and activities*

Čeretková works with a project whose main aim addresses the problem of the shortage of young people, who are attracted to study and enter teacher training in the mathematical and scientific subjects of the school curriculum. She relates to Comenius' book *Didactica Magna* as a background in the Slovak school system. In this project the role of didactics of mathematics, as the important subject in university programmes in teachers edu-

cation, is to explain or show to students as much about methods as it is possible, to prepare didactical situations so they can take part in exercising methods and evaluate themselves and colleagues in using the method from at least two position: as student and as teacher.

Da Ponte indicates that there is growing consensus that the mathematics teacher plays a fundamental role in the process of education and that, to be successful, any change or curriculum innovation needs to be accepted and put into practice by teachers. This requires a serious transformation of the professional culture of teacher educators and of teachers in different school levels. Such transformation emphasizes the concept of teachers' professional knowledge, values his/her practice as the center of the formative process, and stimulates processes of collaboration, reflection and projects, often framed as research on teachers' professional practice.

Chapman argues that self-study can lead to the necessity for preservice teachers to be aware of their initial and alternative ways of thinking and patterns of behaviour. In general, it can play an important role in mathematics teacher education in order to help to transform preservice teachers in ways more desirable for the teaching of mathematics.

Millman et al. explored the use of assessment in mathematics content courses for future elementary school teachers. These courses are often taught by mathematicians whose emphasis is on learning or relearning the mathematics rather than pedagogical implications of mathematical knowledge. The courses are found to be difficult by a large percentage of students for whom mathematics is a major constraint. In collaboration between the departments of mathematics and education the idea to infuse assessment into mathematics content courses was explored in two ways, self-assessment and peer assessment. Students were involved in self-assessment of their procedural knowledge, conceptual knowledge, and problem solving. Through the addition of a required oral mathematics presentation, students were also engaged in peer evaluation of other students. Results of the study do suggest that the infusion of assessment into mathematics content (subject matter) courses holds great potential to strengthen mathematical understanding among pre-service elementary teachers. The researchers were particularly encouraged by the finding that the level of the oral mathematical presentation was associated with aspects of conceptual understanding or knowledge.

9. Professional development of mathematics teachers

Anthony reports from a longitudinal project for beginning teachers. She concludes that beginning teachers have legitimate learning needs that cannot be assessed in advance or outside the contexts of their teaching. Schools need to adapt their advice and guidance programme to suit the relevant

context in the situation, and to match the individual teacher's levels of experience and preparedness. Equally, beginning teachers need to be aware of both non-formal and formal learning needs and be equipped and prepared to take more responsibility for own professional growth.

Guided by principles derived from a situative perspective, Borko and Koellner designed and studied the Problem-Solving Cycle model (PSC) of mathematics professional development (PD). Three major design principles are described, that provided the conceptual framework for the PD program and research: establishing a professional learning community, using artifacts of practice, and establishing community around video. The participation in the PSC positively impacted teachers' professional knowledge and classroom instructional practices, according to the authors' findings.

10. What is the present role of ICMI with respect to the development of teacher formation, and what might be its role in the future?

ICMI offers a unique venue for cross-national exchange with respect to teacher formation. ICMEs have regularly included topic study groups and working groups that focused on teachers' mathematical knowledge, teacher development, and initial teacher preparation. In 2005, a study conference was held in Brazil on the professional education of teachers of mathematics. This five-day study conference focused exclusively on the following issues related to the practices and disciplined study of initial and ongoing teacher training.

Reflecting on the opportunities for cross-fertilization and collaboration that the organization can provide, the group considered how to mobilize collective exchange and work on the professional formation of teachers. It would be wise to create some opportunity to work on teacher education at ICMEs and in Studies. Possible future study foci include:

- Opportunities to consider cultural dimensions of teaching and how these bear on the work of teacher education
- Study focused on teacher educators—who they are, what they do, what they know, how they learn their work

Looking ahead, what knowledge of teacher education, teachers' professional development, and teachers' practices is it that we need for the future? Where are the blank areas that we need to investigate? Knowledge about students' learning is growing and knowledge about teachers and their practices is becoming substantial but do we actually know much about how teachers do influence students' learning? What is the relation between teachers' teaching and students' learning? It is easier to

find studies showing what students do not learn from teachers' teaching than the opposite. Are there consequences of the fact that there is such a spread of theoretical frameworks? Or that most frameworks are imported from other domains? Would it be possible to try to unify the meanings of central concepts related to teacher education and teachers and to develop frameworks that are indigenous to teaching practice? Or is the complexity of the teaching situations such that a variety of frameworks are necessary in order to meet different viewpoints and perspectives?

The working group exemplified the value of cross-national discussion of teacher formation. Participants shared an interest in professional formation over time, and in determining the optimal balance of academic and formal training, on one hand, and experience or work in practice, on the other. Teachers' involvement in research was also valued, and common interest emerged around policy and instructional improvement.

Although the products of the work in Rome were more questions than answers, we clarified common interests - such as the content knowledge needed for teaching - and questions (What is meant by 'research-based'? or 'practice-based'?). We noticed that we use different words for the same idea; same terms for different things; it was illuminating and also useful to realize this. The word 'practice' is used in several different and distinct ways—for example, teaching practice, mathematical practices, learners' practices, practicing—but the word 'method' (as in teaching methods) was rarely used.

Language, concepts, and evidence—concerns for each of these shaped the work in Rome. Teachers' professional formation, central to the quality of teachers' practice and the improvement of students' learning, was at the core of the working group's attention.

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Report of Working Group 3

HILARY POVEY and ROBYN ZEVENBERGEN

Mathematics education and society

1. *Introduction*

ICMI has played an important role in giving impetus and space for the emergence and growth of the study of social, cultural, political and economic issues in mathematics education. These have spawned diverse areas of scholarship - such as the impact of gender and class, ethno-mathematics, critical mathematics education, equity and social justice - which have found expression in the scientific activities and programmes of ICMI. This paper is intended to provide an overview of the field and to introduce the other papers submitted to the working group. It is organised around three themes: conducting research, sites of structural disadvantage and the pedagogic implications of critical mathematics education.

2. *Overview and contribution papers*

Following the initial invitation, an overview paper setting out key themes was circulated and papers were invited. Fourteen papers were submitted and were published on the ICMI Centennial website. A list of these appears at the end of this report. The overview paper was then re-written to include reference to all the contributed papers and to suggest a possible structure for our discussions. We include the text here.

There has been an increasing recognition over the last two decades that the context in which learning occurs profoundly affects what is learnt and by whom. In much early mathematics education research, knowledge was conceived as being simply a property of the individual consciousness. The realisation that knowledge is produced in situations (Lave, 1988; Wenger, 1998; Lerman, 2000) requires us to move beyond an analysis of learning which is dependent on a psychological representation of the mind alone and to consider instead the setting – its social relationships, its cultural locality, the discursive frameworks available in the locale, the social and politi-

cal environment which frames it – and how that setting functions generatively in the construction of knowledge. In other words, mathematics education research has taken ‘a social turn’ (Lerman, 2000, p. 19). Indeed, as Heather Mendick et al. state in their seminar paper, there is a recognition that mathematics education is always already social.

However, those researchers who are mostly likely to be found together in a forum focused on Mathematics education and society usually mean something more than this. Such researchers understand mathematics education as being a profoundly political activity – political in the sense of being intimately bound up with issues of power, authority and the legitimisation of knowledge: who is able to decide what happens, who is recognised as having the authority to set the agenda, whose interests are served by currently dominant conceptions of learning, whose voice ‘counts’, whose knowledge is deemed legitimate and authoritative and so on (Hardy and Cotton, 2000; Klein, 2002). Education is seen as being deeply value-laden, a moral activity. It follows that enquiry in mathematics education must engage with issues of social justice and that a fundamental role of mathematics education research is critique and transformation.

The papers being presented to the Working Group cover three interconnected themes, many of the papers picking up more than one of them.

- A number of the authors are concerned with specifically methodological questions. What are the implications of an equity agenda for research methods and methodologies? Are there specific methodological implications for critical mathematics education researchers?
- Some authors take as particular focus one of the systematic sites of disadvantage in mathematics education. How do class, gender, ‘race’, ethnicity and global position - what has been called the North/South divide - impact on mathematics education? How are the sites of structural disadvantage conceptualised? How do they interact?
- A third focus, and the central concern of some of the papers, is the practices associated with teaching, learning and accrediting mathematics. What educational practices promote or inhibit a social justice agenda? What might a critical mathematics pedagogy look like?

This overview briefly considers each theme in turn and draws on the working group papers to illustrate and extend them.

3. Conducting research

As well as mathematics education itself, conducting mathematics educational research is also a political and moral activity involving issues of values, power, authority and legitimacy. Researchers interested in Mathematics education and society are likely to suggest that the aim of inquiry for social

justice is «the critique and transformation of the social, political, cultural, economic, ethnic, and gender structures that constrain and exploit humankind» (Guba and Lincoln, 1998, p. 211). Such a stance may have implications at a methodological level and a number of the contributors to this seminar make such methodological considerations a focus of their papers.

Research methods and methodologies are not neutral with respect to social justice; what are needed are approaches that explicitly acknowledge the politics of methodology and its impact on research (Vithal, 2003). Power differentials abound in the relationships between the researcher and the researched, with consequent questions about the status - an acknowledgement of the 'limits of knowing' (Walshaw, 2002 p. 346) - and ownership of the tales that are told (Povey & Angier, 2006). In her contribution to this seminar, Tansy Hardy suggests an understanding of research as discursively and culturally constituted and explores the implications for how 'data' are read and how and which stories are told as a result, focusing, in part, on the discursive practices themselves. Dave Wagner is also concerned with 'storylines' but, in contrast, his paper draws on positioning theory and favours an approach where interpretation «considers only the interactions of the immediate participants in a conversation» since «discourse systems are solely experienced locally through the medium of individuals in interactions». He raises concerns about authority structures and about how communities are represented, illustrating these ideas by reporting on work with Canadian Aboriginal communities. He identifies colonial storylines and their connection with transmission metaphors for knowledge acquisition; and he contrasts these with 'more respectful' ones which show distributed agency.

Others who have become frustrated with mathematics education research which does not deal with social justice issues in a practical way have argued that there is a strong need to bring teachers into the research process and focus on issues of equity as they relate to classroom practice (Rousseau & Tate, 2003). The concern here is to avoid conducting research on, rather than with, teachers (Setati, 2000). In her contribution to this seminar, Marilyn Goos examines how teachers and researchers can work together in ways which allow genuine collaboration to be brought into being rather than in ways which simply serve the academic researchers' agenda. She draws on three, methodologically distinct, research projects to 'generate questions about the role of mathematics education research with respect to critique and transformation of the researcher and the researched'. The paper contributed by Tony Cotton is also concerned to critique and transform this accepted set of relationships:

Research in mathematics education is surely conducted for the benefit of teachers and the children they work with. Yet so often the voices of these key bene-

ficiaries are marginalised within research to play the roles of clipped commentators allowed in only so long as they offer sound bites that sit neatly in the researcher's preferred story.

He suggests an alternative and more democratic purpose for mathematics education research, that of opening up spaces for marginalised or 'silenced' groups to be heard. He describes using new, more collaborative methods to enable researchers, teachers and pupils to 'speak to' both the academy and to those who form policy, methods which privilege experience over theory as a basis for understanding, allowing us to think how «our worlds may be changed».

A warning is given in the contribution from Bill Atweh about the complexity of achieving change in education. He argues that, «in spite of the long history of research and practice, social justice remains under theorised in mathematics education». He notes that, because mathematics acts as a gate-keeper for entry to many aspects of society, programmes have been developed to meet the needs of students who are deemed not to be engaging or not to be achieving as a result of factors of social background. He demonstrates some tensions between different possible understandings of social justice and associated practices; basing his argument on the writings of Nancy Fraser, he points up that such understandings and practices may achieve the opposite of what was intended.

The gap between desired change and the results of attempts to achieve it is also taken as a starting point in the paper from Candia Morgan, this time in the context of teacher development and pedagogical change. She argues that, currently, evaluation explanations of the gap tend to focus on the characteristics of the individual teachers involved; and there is an associated tendency to frame such explanations «in terms that suggest teacher deficit, often related to their pedagogic subject knowledge or mismatch between their beliefs and reform principles».

The paper argues for the value of a Critical Discourse Analytic approach which takes into account 'the social and cultural contexts in which innovation takes place', contexts which structure the sense-making of both teachers and students. The approach involves detailed attention to and analysis of texts produced by the full variety of participants in the educational system, texts which are likely to produce multiple competing discourses, with a view to understanding better the positions available to teachers and students and the impact of these on their identities and on how they engage with pedagogic innovation, transformation and change.

4. Sites of structural disadvantage

It has long been recognised that neither education systems in general nor mathematics education in particular is neutral in terms of learners' posi-

tionings with respect to class, gender, 'race', ethnicity and global position. With respect to each of these (and other) positionings, some learners are systemically, structurally disadvantaged. For some of these sites, there is a considerable body of related mathematics education research; others have been more neglected. One key area of structural disadvantage which has been a focus of considerable research attention over the last few decades is that of gender (Barnes, 2000; Becker, 1996; Burton, 1999; Fennema, 1996; Forgasz et al, 2000; Hanna, 1995; Leder, 2001; Mendick, 2006). Much of this will be well known to ICMI participants since this concern led to the creation of the ICMI Affiliated Study Group, the International Organisation of Women and Mathematics Education (IOWME), which has been active for more than twenty years. During this time, the attainment profile for girls in mathematics has changed significantly in a number of countries but issues remain: young women opting out of mathematics; who identifies with mathematics and how; the ways that mathematics classrooms permit and perpetuate unhelpful stereotypes; and many more.

Other research has focused on 'race' and ethnicity (Atweh et al, 2001; Ladson-Billings, 1997; Powell, 2002); or on class (Frankenstein, 1990; Povey & Boylan, 1988; Zevenbergen, 1999; Lubienski, 2002). Others have specifically identified the issues of indigenous people coming to learn mathematics (Zevenbergen, Mousley and Sullivan 2004). Tate (1997) also draws on all these areas to identify the difficulties of multiple areas of disadvantage on the learning of mathematics. And, through much of this research, these sites of structural disadvantage have been conceptualised as fundamentally interconnected (Keitel, 1998).

Structural sites of disadvantage form the focus for a number of the papers being presented at this seminar. Dave Wagner's paper points up both the difficulty and the possibilities of research in the contexts of indigenous peoples. Of his own work, he writes,

Colonialist storylines seem inevitable in this community that experiences the fallout of colonialism daily. Though respectful research and dissemination is challenging in such an environment, it can improve intercultural understandings.

Two other contributors also seek to research respectfully with the mathematics education experiences of indigenous peoples. Robyn Zevenbergen's seminar contribution is concerned with Indigenous students who live in remote communities in Australia. Alerted by PISA to the continuing profound inequality between different social, cultural and geographic Australian communities, she uses ethno-mathematical ideas to explore the tensions between school mathematics practices and the mathematical knowledge the students bring to school. The focus for Gelsa Knijnik's seminar paper is the effect of 'regimes of truth' on adult Brazilian peasant mathematics educa-

tion. She discusses how statements come to be made about what it is to learn mathematics in this social, political and cultural context and how such statements circulate in peasant pedagogical culture. She further reflects on the interconnections between such truths and school mathematics processes.

Two of the contributors to the seminar take as their context issues related to taking a global perspective. Bill Atweh illustrates his theoretical argument mentioned earlier with material from a study in the Asia Pacific region concerned with the internationalisation and globalisation of mathematics education: he draws on the views of mathematics educators from around the world to report on and develop understanding of collaboration projects between industrialised and less industrialised countries. Paola Valero's contribution draws attention to the fact that

mathematics education research, in general, has been operating in a reality that is far from being the reality of most classrooms in the world.

She argues that mathematics education researchers, moving between utopia and reality and seeking to contribute to creating better utopias, need to give serious consideration to the reality of the vast majority of the world's mathematics classrooms. A research agenda is required which generates knowledge about what it is to teach and to learn mathematics in classrooms in situations of poverty and conflict.

The effects of class and home upon the lived experiences of pupils is the focus of the contribution from Vanessa Roper and Peter Gates. They use Bourdieu's concept of cultural capital to frame their analysis of two case studies of pupils near the beginning of their secondary schooling, exploring the factors which influence success at school. They examine the influence of home and family, including socio-economic status and linguistic background, on mathematics attainment; they also consider the effects of schooling itself. The pupils' resulting mathematical learning trajectories are examined and the claim is made that what the pupils achieve in the future will, «in all likelihood, be shaped by their social background».

In their contribution, Heather Mendick et al. argue that issues of gender, class and ethnicity are all deeply implicated in the construction of what it takes to be a mathematician. Drawing on a study conducted in England and Wales and based on group interviews with 15-16 year old school students and with humanities and mathematics undergraduates, they build up a picture of what it means to be a 'real' mathematician and of what it means to be mathematically 'able'. They write,

Participants' strong default image of a mathematician is of a white, middle-class man, and is associated with markings onto and into the body including

states of clothing, posture and mental health ... It is apparent that particular mathematical stories stick to some bodies and slide off others.

They note that a process of Othering is at play and that the close connection between mathematical identities and inscription on the body makes current perceptions 'natural', thus cutting short social understandings.

Issues of identity and of how identity is performed are also concerns addressed in the paper from Tansy Hardy. She takes up the notion of subjectivity as a theoretical frame: «this framing presents identification not as state but a process through which learners are constituted and through which, in often implicit ways, they constitute themselves». Juxtaposing material from a wide range of sources - textual fragments, images from interviews, questionnaires, research journals, published research - and drawing on the lived experiences of pre-service teachers, she seeks to explore the ways in which ambiguities are caught up in the process of identity formation, the ways in which identity is constantly rearranged.

5. The pedagogic implications of critical mathematics education

If young people are to learn to think mathematically, to manifest mathemacy (Alro & Skovsmose, 2002), to develop as persons and to acquire those democratic competencies needed to live as citizens – critical consciousness, sustained and sustainable action and co-operation (Moreira, 2002) – there are implications for mathematics classrooms. They will need to be places where learners set up productive relationships with the process of coming to know. For many learners in mathematics classrooms these disciplinary relationships are fraught with difficulty. Mathematics is experienced as being only a body of already established abstract knowledge, always known and belonging to experts, a discipline which is «without fuzziness or debateable results ... no experiment, no interpretation of evidence, no comparison of criticisms» (Rodd, 2002, p. 2). Learning mathematics becomes only a process of acquiring received knowledge of already existing rules and procedure and doing mathematics becomes performance. Rather, tasks are needed which can be approached in a variety of ways, and for which a wide range of tools can be offered as appropriate; which provide useful opportunities for learners to see themselves as active, as choosing, deciding, producing arguments for and against, assessing validity and generating questions and ideas. Such practices profoundly affect the nature of the resulting knowledge. How we know, and how we come to know, are inseparable from what we know.

The contribution to the seminar from Ole Skovsmose deals with this directly. He is interested in the notions of critique and of critical mathematics education and, specifically, in their relationship to both uncertainty and possi-

bility. He argues that mathematics «operates as part of a world-wide distributed technical rationality», in everyday practices, in professional contexts and in technological enterprises in a complex web of social and political framings. It operates in a globalised economy with the accompanying locales of disadvantage and is implicated in the processes of inclusion and exclusion. Drawing on examples from developing world contexts, the paper attempts to formulate what a critical mathematics education for the future would look like.

In their contribution to the seminar, João Filipe Matos and Madalena Santos point out that research in mathematics education can contribute to help in understanding how different people organize their mathematical knowledge. They point to the ways in which both cultural studies and the ethno-mathematics tradition have provided accounts of «the particular ways people organize, adapt and build up mathematical structures and forms of thought in order to make sense of everyday activities»; these will typically be very different from the ways validated by schooling. Currently in Europe there is a movement to recognise and accredit such experiential mathematical competences for the very many people who have not followed regular compulsory schooling; but the frameworks being devised for such accreditation seem only to be able to take schooling and academic learning as their reference point.

As well as setting up productive relationships with coming to know, and as part also of that agenda, social justice demands that teachers and learners set up productive relationships between and amongst themselves. Relational equity is the theme of Jo Boaler's contribution to the seminar. She uses this term to «describe equitable relations in classrooms; relations that include students treating each other with respect and responsibility». Her report is based on a four year study in Californian high schools. In one of the study schools, a low-income, ethnically diverse, urban high school, the pupils learned to respect others in the class who were differently positioned with respect to cultural background, social class, gender and attainment level. In addition, their behaviour overall improved and they achieved better results, with differences in attainment between groups either reduced or eliminated altogether. The classroom approach was multi-dimensional, nurturing «social and intellectual commitment, communication, responsibility, and respect».

6. *Conclusion*

In the interests of equity, there is a continuing and pressing need for research which advances our understanding of the issues related to mathematics education and society. The papers presented at the Working Group deepen our knowledge of the field and provide an enriched perspective on the possibilities for effective action for transformative change.

7. The seminar at the centennial

All working group participants were asked to read all the contributed papers before the meeting in Rome in order to maximise the opportunities for discussion when we met face to face. Each paper was then very briefly introduced and the group used the overview structure to focus the discussion in the first session. The second session, was focused in response to the first.

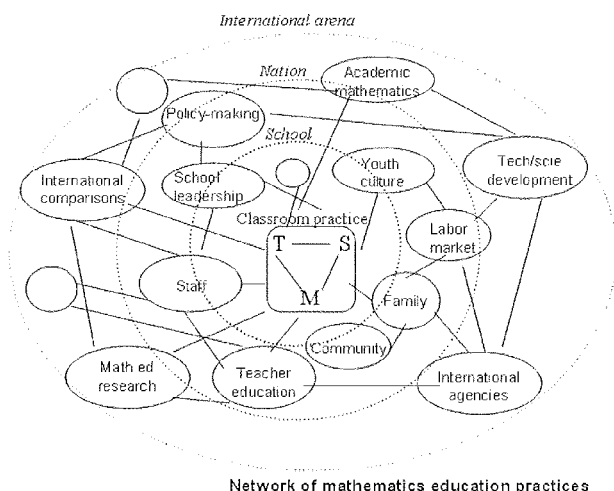
The Working Group reported very favourably on the inclusion of this group in the Centenary celebrations of ICMI. As recently as 30 years ago, such a group would not have been seen to be an aspect of a mathematics education conference. We acknowledge that there has been considerable recognition of the importance of the contexts within which mathematics is learned and shaped during the past three decades. The history of engagement with the social aspects of learning mathematics was covered from D'Ambrosios's keynote in ICME5, through PDME, IOWME and MES to the present day was discussed.

The consensus is that much of the past 100 years of mathematics education has been shaped by three very general trends. In the beginning there was mathematics which was the key focus of ICMI's work. The second wave of ICMI was the recognition that mathematics could only be learned through some form of education so the second epoch was that of *mathematics education*. As educational theories became more complex, it was realised that the focus simply on cognition failed to account for the complex milieu in which learning occurred. This has seen the birth of the third epoch, that of *mathematics education and society* where the complexity of learning mathematics is more comprehensive viewed through the recognition of the contexts within which learning occurs. It is this final epoch that was the focus of this working group. Some of this complexity is encapsulated in this diagram.

Looking forward to challenges for the next century of ICMI, the group drew considerably on the figures cited by Skovsmose in this forum, in which most of the cited research in mathematics education focuses on only 10% of the world population; the majority still do not have the luxury of attending school or any form of quality schooling. Thus, there is a considerable skewing of published research. One of the challenges for researchers in mathematics education is how can more children/learners access formal schooling and how can the quality of educational provision in many countries be enhanced so as to redress the educational inequalities faced by too many learners. The provision of quality education for all became a key point for discussions

One of the key concerns for this group was the inclusion of non-Western perspectives in coming to learn mathematics. It was notable in this group

that non-European peers were absent from the discussions, in some cases despite having indicated a desire to attend. As noted by some of our Asian colleagues' addresses in other forums, the perspectives of minority groups in the dominant discourses of the field are difficult to find. In mainstream journals, Asian perspectives (for example) are notably missing. Some journals, such as the *International Journal for Mathematics and Science Education* (Springer) actively encourage inclusions of papers from non-Western researchers. However, the barriers of language and cultural nuances and perspectives remain challenges. For us, one of the challenges for ICMI in the coming century is one of inclusion of all social and cultural groups – whether as research subjects or participants in forums. This seems to be one of the major challenges in this coming epoch and we actively encourage our peers to consider seriously ways of including those who are traditionally excluded from participation.



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The papers for WG 3 are available on the web-page of the Symposium at <http://www.unige.ch/math/EnsMath/Rome2008/>

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Report of Working Group 4

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Resources and technology throughout the history of ICMI

1. Introduction

A variety of papers were presented in WG4 that differed with respect to the tools and objects they addressed and the theoretical perspectives they adopted. The papers were made available before the meeting and are the main references that we use to discuss artifacts and technology over the 100 years of ICMI. Here we will try to map some of the issues that characterize this wide variety of papers.

We will cluster the papers around some different issues: the history of different technologies throughout ICMI; resources and technologies used by participants; and the different theoretical issues raised. In the last section of the paper, we report on a small fraction of the discussions that took place during the face-to-face sessions of the working group. A list of the WG members who were able to attend the meeting in Rome appears at the end of the paper.

2. The history of ICMI

Ruthven reconstructs a partial history of the early decades of ICMI from the perspective of the role given to methods of intuition and experiments (hence of physical ‘tools’) as they appear in the official documents. This ‘laboratory approach’ to mathematics included geometrical drawing, graphical methods, practical measuring, and numerical computation. A tool that is now commonplace in the classroom, squared/graph paper, was nearly unknown until the end of the nineteenth century, because of the cost, and was gradually disseminated in schools together with the entrance of new educational philosophies, which are mirrored in ICMI documents. Hence the use of squared paper can be seen as a sign (metonymy) indicative of the laboratory approach.

Whilst Ruthven’s references are mainly from the Anglo-Saxon literature, Trouche and Maschietto offer a complementary view from France,

recalling the early documents published in *L'Enseignement Mathématique* (which became the official journal of ICMI following its creation in 1908). In the same period, Borel (1904) claimed: «It is necessary to do more and to create a genuine laboratory for mathematics. I believe that this issue is very important and has to be studied in a very serious way» (our translation). Borel emphasized manipulative aspects and methods involving work with small groups under the supervision of the teacher.

Another window into the history of tangible school materials (paper, notebooks, blackboards, and so on) is provided by Villarreal, who reconstructs the development of attitudes towards media and manipulative materials in Latin America, drawing on a joint project with Borba: every new medium gave rise to new educational practices that strongly shaped the classroom environment, both for classical and for information technologies.

In his contribution, Isoda reconstructs the history of the use of technologies in Japan in relation to their contemporaneous development in Europe. The main contact was between Japan and Germany, hence curriculum development in Japan was influenced by Felix Klein, the German mathematician who was the first president of ICMI. The plenary speech given by Shimada at ICME2 (1972, Exeter, UK) is testimony to the continuation of this collaboration. The Japanese case is, in a sense, exceptional, as it shows that a region far from Europe and America maintained a systematic relationship with ICMI from the very beginning. This was not the case for other parts of the world. For instance, Gerdes (from Mozambique) describes the scanty representation of African countries at ICME (with the exception of South Africa). He recommends a larger role for the African Mathematical Union for sharing and debating the problems of re-rooting African education, in light of the neglect of cultural issues in the colonial and post-colonial transplanted school system, especially with respect to the potential of technologies. Similarly, Ekol (from Uganda) recommends increasing the role of ICMI and widening the scope and coverage of ICMI activities in African countries to strengthen regional portals or contacts.

Needless to say, the historical reconstructions that go back to the very early years of ICMI show examples of ancient technologies, as ICT was not yet available at that time. Felix Klein himself paid attention to ancient instruments such as curve drawing devices and transformation pantographs. Villarreal points out Klein's contribution in defending the function of calculating machines «for teaching arithmetic to children».

Klein can be credited with being a pioneer in proposing the use of technology in schools. To evoke Klein's very important contribution, Bartolini Bussi presented, at the meeting of WG4 in Rome, some working copies of the instruments cited by him (1924, 1939):

- a commercial copy of a modern mechanical calculating machine for children;

– a copy of the ‘Nürnberg shears’ (see *figure 1*), a mechanism designed by one of Klein’s students composed of a chain of jointed rods that forms a series of similar parallelograms. According to Klein’s explanation (1939, p. 75): «the corners common to two successive parallelograms A, B, C, ... under all the deformation of the jointed system, form similar point rows on the line g which joins them, the common diagonal of the parallelograms». The same property applies to C, D, E and E, F, A. As an affine transformation is determined by the correspondence between two triples of points, if one knows the images of A, C and E, he may construct the image of B, D and F, and by using additional shears of the same sort, he may «set up plane models of affine transformations of the greatest variety».

– a copy of the Peaucellier invensor (Klein, 1939, p. 100), which executes a circular inversion, and can therefore draw a straight line in a rigorous way (see *Figure 2*). Both copies were produced and are still used for educational purposes by the Laboratory of Mathematical Machines, at the Department of Mathematics of Modena (www.mmlab.unimore.it).

Trouche and Maschietto address the question «Do new technologies transform old ones or erase them?», claiming that an *old* technology is not a *dead* technology. They highlight the relevance of history for the teaching and learning of mathematics, not only for general debates and educational research projects, but also to examine in greater depth the question of technology integration. The ICMI Study 10 (Fauvel & van Maanen, 2000), and other activities promoted by the Study Group HPM (History and Pedagogy of Mathematics), attest to this interest.

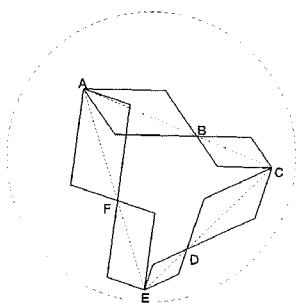


Figure 1

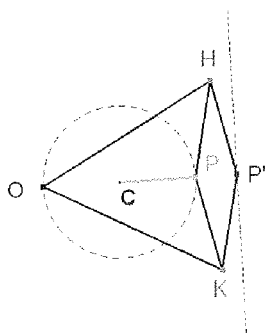


Figure 2

Some authors reconstruct the history of ICMI in the post-ICT era. As Lingefjärd reminds us, the introduction of calculators into ICMI history dates back to ICME1 (1969, Lyon, see www.icmihistory.unito.it). This introduction, in a sense, represented a continuation of the previous emphasis on experimental mathematics and the laboratory approach. Robutti

reports on the elaboration of the idea of *mathematics laboratory* within the Italian community, with deep roots in the historical tradition, on the one hand, and with space for ICT, on the other. According to her:

A mathematics laboratory is a methodology[...], aimed at the construction of meanings of mathematical objects [...]. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together. It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning cannot be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself.

Laborde considers the proceedings and programs of the last ICMEs over the past decade. She notes the decreasing number of Working, Topic and Discussion Groups on technology since ICME 8, which could be related to the fact that the integration of technology into mathematics teaching had become less extraordinary. Technology was considered a catalyst for change in the curriculum or in the teaching practice; a tool changing mathematical activity (modelling or processing data in statistics, experimenting in algebra or geometry, visualizing in geometry); an aide to improve students' understanding (as an intermediate level between the physical reality and the formal mathematical model). Only recently have studies begun to address the integration of technology by ordinary teachers in ordinary classes. These studies have revealed that integrating technology is not an easy task for teachers, who may constrain the potential of technology in order to retain control of the classroom. They also pointed to future avenues of research related to the knowledge needed by teachers to use technology.

Strässer continues this historical analysis, comparing the structures of the two ICMI Studies devoted to technology: Study 1 (Churchhouse et al., 1986) and Study 17 (Hoyle & Lagrange, to appear). This analysis shows that development within the discipline of mathematics itself has become less important for the educational use of technologies. There is, according to him, a relative autonomy of the educational system, of research in didactics of mathematics, and of classroom practice that leads to the use as well as the rejection of new technology. This process is not fully controlled by developments within the discipline of mathematics. The 'royal road' to the educational use of computers, software and communication technology within mathematics teaching and learning is yet to be discovered - if such a thing can be said to exist. Strässer also focuses on the emergence of competing theoretical frameworks for research in the field, pointing out that a

mere analysis of the artifacts proves to be insufficient to guarantee their use in the teaching and learning of mathematics.

A report on the 17th ICMI study is provided by the two chairs of the study, Hoyles and Lagrange, who write that, in a varied and evolving context, ICMI 17 sought a balance between two potentially contradictory aims: to reflect on *current* uses of technology in mathematics education, avoiding mere speculation on hypothetical prospects, on the one hand, and to address the range of hardware and software with *potential* to impact or contribute to mathematics teaching and learning, on the other hand. These aims are reflected in the ICMI Study book (in progress), which address themes such as:

Implementation of technology-rich curricula; issues of access and equity; theoretical perspectives and classroom implementation; learning and assessing mathematics through digital technologies; mathematical knowledge and practices resulting from access to digital technologies; collaborative work and communication; communities of inquiry, learning, and practice; design of learning environments and curricula for different geometries and for modeling.

3. Resources and technology: which objects?

We have already highlighted the visible shift from strictly classical technologies to ICT in the history of ICMI. Some of the papers address only one kind of technology, while others addressed both. Hence, one easy way to classify papers is in terms of the kinds of tools they refer to.

3.1 Classical technologies

Gerdes' paper is concerned with traditional artifacts developed in African countries and the way that computer technology can transform the research on such artifacts, to counter the impression that mathematics is something alien to or not rooted in African cultures. He cites several examples: squares of cardboard paper imitating a basket weaver's technique to produce a pyramidal funnel used for salt production; hexagonal bands and patterns to weave traditional hats, baskets, and fishing traps; hexagonal tiling; a non-ahedral musical instrument; drawings in the sand, and so on.

Activities with mathematical machines are reported by Trouche and Maschietto, who refer to the chapter on media in Fauvel & van Maanen (2000). They focus on the contribution of two different authors: Bartolini Bussi, who addressed the exploration of instruments already built, and Isoda, who involved students in the construction of instruments.

Taimina reports on the Cornell Reuleaux Kinematic Model Collection, which has been designated by the American Society of Mechanical Engineers

(ASME) as a (United States) National Historic Landmark. The precious collection, which evokes the years when the ICMI culture of experimental approach to mathematics was established (at the end of the nineteenth century), is today at the disposal of area schools. Digital technologies have also been used to allow virtual visitors to the KMODDL website to enjoy the exploration of instruments. As Taimina writes:

This Digital library of Kinematic Models is one of the examples of how the Internet can enrich mathematical education. It is allowing this originally constructed classroom set of models to become widely available – anywhere where there is an access to the Internet one can play with these models interactively. Besides the direct use in the classroom this digital library allows further exploration on your own. It is an interesting example of historic models transformed into digital tools and this way opening new possibilities in geometric design of motion. It is also a rich resource to show our students interdisciplinary connections and practical applications of pure geometric ideas.

The interaction between old technologies and ICT is also addressed by Olive, who reconstructs his personal ‘odyssey’ in the use of artifacts, materials and tools, from Dienes’ blocks to Java bars. In this paper, new ‘hybrid’ objects are introduced, i.e., computer simulation of manipulatives that are ‘handled’ through the computer mouse. Classical teaching aids (e.g. Dienes’ logic blocks, logic tracks and multibase arithmetic blocks) are compared with their ‘copies’ in the *National Library of Virtual Manipulatives (NLVM)* and with the new *Computer-Based Interactive Learning Environments*. The author concludes that structured materials have been assimilated as well as transformed by the new technologies developed over the past 40 years. «Dienes’ theory of multiple embodiments together with Kaput’s notion of multiple-linked representations that the new technologies provide, are key aspects of today’s dynamic tools for teaching and learning mathematics». Olive’s paper is connected with one of the questions posed in the discussion document: «The present technologies (ICT) are, in some respects, the heirs of this long tradition [of different tools and artifacts in mathematics education]. A widespread misunderstanding is that, due to their effectiveness, they may replace the old instruments: why use abaci when pocket calculators are available? Why refer to ruler and compass construction when powerful software is available?»

This point is addressed by Noss, who, following Artigue, distinguishes between the *pragmatic* and *epistemic* roles of the use of instruments, the former focusing on the productive potential (in terms of efficiency, cost, etc.), and the latter focusing on the contribution to understanding mathematical objects. He claims, however, that as technologies evolve, the balance between pragmatic and epistemic roles change over time. When ‘obso-

lete' technologies are supported for explicitly didactical reasons, sometimes a substantial price is paid, as students are being asked to work with obsolete technologies considered important for them in the longer term, but which the students may not see as important themselves. The author concludes: «we have a responsibility – not simply to preserve the obsolete in the name of epistemic function – but to design new objects and representations, and connect them as richly as we can to what we aim for students to learn».

3.2 Information and Communication Technology

As we have seen above, some classic artifacts have become virtual objects as well, in the sense that they are presented with the 'participation' of computers. At first, such technologies were labelled information technology, and more recently with the availability of the Internet, and in particular of the 'www' interface, they have come to be referred to as information and communication technology (ICT) in many ICMI-related events.

Healy and Kynigos both address the use of microworlds, the notion introduced by Papert, to express the idea that we can create an environment in which a context is 'created'. Healy stresses that Papert meant to 'reconstruct mathematics' in an appropriate way that is tailored to the learner. She shows how the notion of microworld has remained relatively stable in spite of being applied within different theoretical frameworks, such as the Piagetian, the Vygotskian and the semiotic. Kynigos also discusses how different theoretical frameworks have shaped research on technology, particularly the notion of a microworld. He proposes that microworlds should be 'half-baked': they should be incomplete environments that would invite learners to complete them or to take pieces and develop them.

Robutti illustrates how graphing calculators, embedded with information technology, can be combined with other media, seen as means of communication, to reorganize students' thinking in the classroom. The key idea is that calculators alone can be considered old technology, compared to calculators with this added software, because the dynamics of the activity in the classroom, the role of the teacher and the interaction among students undergo deep changes. The added value of this new form of mathematics laboratory is based on the possibility of sharing, in real time, the work done by each student on her/his calculator screen, or the work done by the students together on a common screen.

Lavicza, addressing the difference between the way students and mathematicians produce knowledge, argues that mathematicians have actually used technology more extensively than teachers have. She claims that researchers should be paying closer attention to the way mathematicians use software in their research and teaching. Kaino illustrates, in his paper, how software such as *Mathematica* or calculators can transform the nature

of mathematical learning. He illustrates how problems related to systems of linear equations acquire a different nature for students depending on the technology used – software, calculator, or paper and pencil. Trouche also deals with the reverse side of this issue, addressing the way different software is ‘shaped’ by students’ actions. Trouche and Robutti address an important issue for this group: the interaction of handheld devices with IT.

In ICMEs held during the past decade or so, the combination of what has been referred to in this paper as classical technology with IT seems to be a new trend. What seems to be a definite trend at ICME is the use of the Internet. Gadanidis shows how multimodal Wiki pages allow for online collaboration. In such environments, collaboration is possible between researchers, teachers, and students in different geographical regions of the world. This author also discusses how different media – video, regular text, and other modalities of language, or ICT – shape the mathematics that is produced.

Using a similar conceptualization of technology, Villarreal emphasizes the way the Internet has come to play an important role, not only in online courses, but also in the transformation of the notion of problem itself. As pointed out by these last two authors, Kaino also emphasizes the use of the Internet as a means for creation of groups that are interested in a given subject, despite being in different geographical areas (also from non-affluent countries, like Uganda).

The sample of technologies used and/or referred to by the authors of this WG cannot be considered to be statistically representative of those being used throughout the world. However, the studies discussed in this section seem to point out that new technologies, rather than erasing old technologies, tend to reshape, reallocate and replace classical technologies. Hybrid technologies – which can be understood as the combination of classical artifacts with IT – are illustrative of such a phenomenon. Examples of this include the Lego-Logo experience and handheld devices. Similarly, the Internet seems to be transforming activities in face-to-face laboratories as well as enabling the development of virtual laboratories and databases of films used in mathematics education. On the other hand, there are many indications that some artifacts considered classical still have a life of their own with little influence from ICT.

4. Theoretical issues

In the early decades of ICMI, for the most part, professional mathematicians debated the issue of mathematics teaching. This seems to have been a general trend in Germany (as Klein’s involvement clearly shows), the UK and USA (see Ruthven’s paper), France and Italy (see Trouche and Maschietto’s

paper), Japan (see Isoda's paper), and Latin America (see Villarreal's paper). The professional mathematicians who founded ICMI had a deep knowledge of the epistemology of mathematics. In the decades that followed, however, it became clear that the problems of mathematics teaching could not be tackled without considering the complexity of the educational system. This implied that outcomes produced by other fields of scientific research, as well as pressures coming from school teachers, needed to be taken into consideration. A systemic approach to mathematics education came to the foreground (see also Furinghetti & Giacardi, 2008). When the problem of mathematics education started to be considered in a systemic way, and the community of 'professional mathematics educators' was created, the need to frame the processes that were designed, implemented and studied came to the foreground. Hence, mathematics education researchers started to wonder about the models of learning, teaching, using artifacts, communicating, and producing representations, and so on.

It is not the aim of this chapter to review all the theoretical frameworks used in the didactical approach to technologies in mathematics education (a thorough review, at least of ICT, is contained in the forthcoming ICMI Study 17 volume). As already mentioned, mathematics educators have developed different theoretical frameworks to study the interaction between tools (including all kinds of technologies) teachers and learners in educational settings, as is evident in some of the contributions to this WG that address experimental research in the mathematics classroom. These different frameworks, based on different authors, have differences and commonalities.

One difference among authors throughout ICMI history, and within our group as well, has to do with the status of mathematics as it interacts with ICT. The question «Does mathematics remain untouched as different technologies are used?» may help to frame the issue. Some authors seem to view technology as juxtaposed to mathematics, while at the other end of the spectrum, mathematics is believed to be deeply changed, or the technology or media are viewed as co-authors of the mathematics produced. This issue seems to be important at an epistemological level, regarding the nature of mathematics, but also has implications for the classroom, as the view regarding the role of technology also helps to shape curriculum development.

There are also issues that are approached in a similar fashion by different theoretical frameworks. For example, even though they base their research on different authors, many members of this Working Group emphasized that learners do make different use of artifacts (whether classic or ICT). They emphasize that tools not only shape the knower, but the knower also shapes the tool.

Also, some authors blended theoretical constructs from semiotics, philosophy, anthropology, sociology, cognitive and social psychology, ergonomics and computer science. Questions remain, however, regarding the sta-

tus of gestures and other body-rooted concepts in the classroom, and how an embodied cognition approach influences curricula. *How do the different authors situate themselves in this landscape?*

Similarly, the discussion about visualization and its role in mathematics education seems to have evolved and, at the same time, raised new questions. The authors who approached the issue seem to agree on the importance of visualization in mathematics education and the use of graphical representations. On the other hand, it is not clear what should be accepted as a legitimate mathematical argument, and at what level. *Can a visual proof be accepted at any level? How has software that emphasizes multiple representations of concepts like functions, or dynamic geometry software, changed the status of visualization in mathematics education?*

Although we have a variety of theoretical frameworks, we seem to have generated many questions that are still inadequately addressed by the different theories we have developed. Besides the questions already raised in this section, several others come to mind:

Are there differences between the use of the graphing calculator and use of function software in the classroom? What kinds of changes does the Internet bring to mathematics education? Do new technologies transform old ones or erase them? What is the relation between human thinking and the tools that have been developed throughout history? How is the very notion of human body affected by artifacts in general and by computer technology in particular? Are representations of a given mathematical concept being transformed by the different technologies we have experienced in these 100 years of ICMI? Is knowledge produced by a human, by humans, or by collectives of humans-with-media? Has the notion of a problem in the mathematical classroom been affected by different technologies? How has the classroom been affected by the Internet? Are teachers in a new 'risk zone' as they cope with new uncertainties resulting from the availability of information and communication technology?

No contribution offers a full-fledged presentation of a theoretical framework connected to empirical work, and this may be related to the limited length of the papers. However, it seems that the issue of theoretical frameworks suitable for interpreting, designing, and analyzing classroom processes, where all kinds of technologies are intentionally introduced, is still open and requires further debate. Geiger, following an historical reconstruction of research on the role of collaborative interaction in technology-rich classrooms, examines papers from ICMI Study 17 concerning the use of technology to promote social activity. He concludes that research is still in an early stage, and further research is required before any mature understanding is achieved regarding the roles played by digital tools in fostering the type of social interaction that supports the learning of mathematics.

5. Concluding remarks

In this paper, we attempted to summarize some points regarding the role of artifacts and computer technology throughout the 100 years of ICMI's history. We based this on a set of papers specially prepared for the conference that celebrated the 100 years of ICMI. This set of papers, still available on the website of the Symposium, helped us to see how the educational use of artifacts seems to coincide with the beginning of ICMI and with the very notion of mathematics education, or similar terms used during these 100 years to attempt to describe the concern with teaching mathematics in an inclusive way that makes it accessible to all. In pointing out this coincidence, it is not our intention to claim that artifacts were not used before the birth of ICMI, but only that artifacts were being used as ICMI initiated its activities. In the last thirty years, the use of information and communication technology has become increasingly intense without erasing the use of classic artifacts, and in many cases, it has transformed them. It is important to keep in mind, however, as both Lavicza and Ekol suggest, that the use of computer technology – and one can easily imply use of artifacts in general – is very unequal, at least in England and Uganda. Even when technology is used, Gadanidis reminds us that its meaningful use still appears to be a 'rare event'. This should remind all of us to be very humble about the conclusions we may draw from the theories we have been building. The expression 'tangible objects' was created to denote objects that are impregnated with sensors, computer chips, etc. Some of them, such as robots and cars, have already been used in education. The Internet seems to add another layer to this matter, and it seems to be rearranging the places of both classic artifacts and 'classic' information technology.

The papers show that the use of technology varies considerably from country to country, within the same country, and among different professional communities (e.g. mathematicians or math teachers). Although different sub-groups tackled some of the questions raised above, no definitive answers resulted from the discussion carried out by the group. All groups seem to experience the tension between trying to understand the current uses of technology and artifacts, and at the same time, envisioning new ways of using them in mathematics teaching and learning. Moreover, new questions and issues seem to have arisen. The relationship between textbooks and technologies was discussed, showing that there are different ways of situating both in the classroom, regardless of whether they are digital or traditional textbooks. The way that different artifacts control or constrain mathematical activity in educational settings was debated, showing that it is a pertinent concern, whether classical machines are considered or the virtual Internet world is taken into account. There seems to be a consensus that the Internet will transform education, but it was not clear how, nor if the theoretical frameworks we have today will be sufficient to deal with phenomena that are emerging with the increasing popularization of

Internet. Finally, the group discussed how assessment should be transformed in the digital technological world, as well as the nature of digital technology: *Is digital technology a tool?*

The attempt to understand the role of artifacts and technology in 100 years of ICMI history was fruitful, and it is fair to say that it may become an increasingly relevant theme for research in years to come. But it is also fair to say that the preparation of WG, based on interaction supported by digital technology, and the face-to-face meeting, did not enable us to envision the place of technology in mathematics education 100 years from now!

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Report of Working Group 5

GILAH LEDER and LUIS RADFORD

Mathematics education: an ICMI perspective

1. Preamble

The *Symposium on the Occasion of the 100th Anniversary of ICMI* attracted considerable attention within the mathematics education research community. The working groups, an important component of the program, were appropriately seen as a productive opportunity for participants to discuss and showcase aspects of their own research. The parameters for participation in Working Group 5 were broad and were summarised as follows:

It can be argued that ICMI's impact on the field of mathematics education is both defined and reflected through the affiliated groups that it has recognized (and in some cases incubated) and through the ICMI studies it has spawned. What has been gained and lost by the mathematics education communities through the choices made? For example, what has been the influence of psychology, philosophy, history, ethnography, anthropology, [...] on mathematics education. Have their approaches been adopted? Have some been privileged over others?

We refined the overall focus further: The broad theme of this working group can clearly be addressed in diverse ways. We considered the following topics particularly relevant:

- The evolution of theoretical frameworks in mathematics education;
- The influence of other disciplines in this evolution;
- ICMI contributions to the dialogue between the mathematics education community and other disciplines – for example anthropology, ethnography, and philosophy;
- Outcomes of interdisciplinary dialogue;
- Further possible ICMI roles in promoting the dialogue (for example, should new disciplines be considered?).

In the remainder of this report we have captured the overall thrust of the material presented, the discussions these generated, and our more general deliberations as best we can within the space allocated.

2. Report

In response to the call for contributions, a number of papers were submitted. These are summarised below.

The fuller papers, prescribed to be limited to five pages, can be downloaded from www.unige.ch/math/EnsMath/Rome2008. A number of participants, not listed in the table, also contributed to the discussions in the WG.

As planned, participants in WG5 were drawn from a range of different countries, with Africa the only continent not represented among the presenters. Collectively, presenters touched – in varying degrees – on the sub themes listed above.

Summary of the paper presentations

Authors: Marcelo Salles Batarce (London South Bank University & Universidade Estadual do Mato grosso do Sul – Brazil) & Adriana Cesar de Mattos (London South Bank University & Methodist University, University of Piracicaba, Brazil, SP)

Title: The ICMI's Grammar.

Brief summary based on the written paper: Issue raised: Is ICMI a chapter in the history of mathematics education or has ICMI set the scene for (shaped) mathematics education? Reform of the school mathematics curriculum needs a language to express its aims and new directions. ICMI and its affiliates often mirror (or anticipate?) these needs. What is needed to achieve the aims of mathematics to be both a universal and international (i.e., non political) science? Are new concepts and terminology needed to achieve this?

Key points raised during the presentation & discussion: The *Commission Internationale de l'Enseignement Mathématique* was established in 1908; the *International Commission in Mathematics Instruction* in 1952. How closely are these two linked? The concepts 'internationalisation of the educational system' and 'mathematics universality' are not synonymous. The former is broader than the latter.

Authors: Christer Bergsten (Linköping University, Sweden).

Title: On home grown and borrowed theories in mathematics education research – the example of embodied cognition.

Brief summary based on the written paper: Using embodied cognition [EC] as an exemplar, three broad issues are explored in this paper: the dimensions of relevance; the influence on scientific discourse, the furthering of scientific knowledge, and on educational practice; and issues of compatibility with other theoretical perspectives already applied in the field.

Key points raised during the presentation & discussion: What happens

when a 'new' theory from outside feeds into an existing field of study? And in particular, what has been the impact on mathematics education of embodied cognition? Two separate waves can be identified, with the first in particular leading to research presentations at PME, one of ICMI's affiliated groups. The influence of embodied cognition on the field can usefully be considered under three headings: dimensions of relevance; implications; and compatibility. Those who have drawn on the theory include those interested in the link between the acquisition of mathematical knowledge, gestures, and more general semiotic perspectives, and those exploring the epistemological foundations of mathematics and its relevance for education.

Authors: Paolo Boero (Università di Genova, Italy).

Title: Processes and products, structures and meaning in mathematics classroom: some snapshots from the last century.

Brief summary based on the written paper: Various 'movements' in mathematics education are discussed. Evidence is presented to illustrate how positions in other disciplines have influenced changes and priorities in the teaching of school mathematics. It is argued that mathematics education should not merely rely on tools and theories from other disciplines but should also develop as a relatively autonomous scientific discipline.

Key points raised during the presentation & discussion: To show the importance of developing mathematics education as an autonomous scientific discipline, using tools from other disciplines when helpful, was a core theme. Changing priorities in the teaching and learning of mathematics in school were driven by dominating positions in epistemology and psychology, and in particular by: Formalism and logicism (from Hilbert, Frege, Russell); by Modern (or New) Mathematics (Piaget + Bourbaki); and by Constructivism(s). However, the inertia of the school system (teachers, parents, etc.) proved a force against change.

Authors: Rolf Biehler (University of Kassel (Germany) & Andrea Peter-Koop (University of Oldenburg, Germany). The authors were unable to attend the Symposium.

Title: The development of mathematics education as a scientific discipline – some reflections from a German perspective.

Brief summary based on the written paper: The authors provide, and build on, three historical snapshots – involving Felix Klein, ICME3 in Karlsruhe, and the work of the *Institute for Didactics of Mathematics*, highlighting the link between ICMI and developments in mathematics education in Germany.

Authors: Gerd Brandell (Lund University, Sweden).

Title: Using multiple theoretical perspectives to connect, clarify and convey research results.

Brief summary based on the written paper: Do different theoretical perspectives facilitate or impede insights into complex teaching situations? Similarities/overlap between apparently different theoretical approaches to understanding how learning takes place can be highlighted by using the overlay of the characteristics of a 'deep' and 'surface' approach to learning. It is argued that diversity can be handled constructively by exploring unifying, integrating, competing, and networking notions and that the characteristics of deep and surface approaches to learning are particularly useful for considering similarities and overlap between different theories.

Key points raised during the presentation & discussion: Several examples were used to illustrate that the same empirical data can be analyzed using different theories or approaches. A more global theory can help to communicate research results to an audience outside the mathematics education community (e.g. mathematicians). There are various ways to build on different theoretical perspectives: unifying; integrating; competing and comparing; and networking the different and possibly competing interpretations.

Authors: Tony Brown (Manchester Metropolitan University, UK).

Title: Subjectivity: An alternative to the psychology of mathematics education.

Brief summary based on the written paper: How should mathematics education research position itself? «Mathematics education research needs to move away from earlier instrumentalist tendencies concerned with understanding and improving mathematical performance against unproblematised social registers».

Key points raised during the presentation & discussion: Mathematics education research is widely premised on psychologically oriented research perspectives centred on control and perspectives which often rely on questionable assumptions. But using a 'Lacanian conception of subjectivity' may be more profitable. Using this lens, the student is understood through participation in the linguistic structure of learning; teachers are understood through the filters of curriculum structure; and researchers are considered according to how they conceptualise psychology/ subjectivity.

Authors: Bruno D'Amore & Martha Isabel Fandiño Pinilla (University of Bologna, Italy).

Title: Change of the meaning of mathematical objects due to the passage between their different representations.

Brief summary based on the written paper: Transforming 'everyday language' into algebraic expressions involves «a constant change of meaning ... within various semiotic systems». Examples are given to illustrate how disciplines outside mathematics education are already

influencing our explanations of mathematical learning and activity.

Key points raised during the presentation & discussion: What are the causes of the changes of meaning, what origin do they have? Some theories 'external' to mathematics education – and in particular philosophy, sociology, anthropology, and psychology – have a strong influence on the ways in which different interpretations are analyzed.

Authors: Nadia Douek (IUFM de Nice, France).

Title: The determination of mathematical objects of didactical activities.

Brief summary based on the written paper: Theoretical constraints influence (limit or expand) the theoretical and practical dimensions of mathematics education. Particular emphasis is placed on the work of Vergnaud and Vygotsky.

Key points raised during the presentation & discussion: The idea of a mathematical object is problematic, the author proposed, because the boundaries of a mathematical object (its determination) seem to depend on the philosophical choices made, for instance on the epistemology and/or the cognitive psychology perspective(s) chosen to guide our analysis. Thus, it is argued, it is preferable to consider mathematical objects as emergent from activity which is dynamic and rich with its socio-cultural components.

Authors: Helen Forgasz (Monash University, Australia).

Title: Positioning gender and mathematics education research.

Brief summary based on the written paper: Evidence is presented of diverse ways in which ICMI has broadened the research horizons of gender and mathematics.

Key points raised during the presentation & discussion: ICMI's decision to host a Study on 'mathematics and gender' stimulated international academic debate on this topic, strengthened the area as a credible research domain, highlighted different theoretical perspectives, and also drew attention to gender differences within ICMI's organizational structures. Further work on gender has been promoted and supported by two affiliated groups, IOWME and PME. Has ICMI's influence on the field waned in recent years? And what can we realistically anticipate as the future contributions of ICMI?

Authors: Michael N. Fried (Ben Gurion University of the Negev, Israel).

Title: History of mathematics and the future of mathematics education.

Brief summary based on the written paper: The founders of ICMI (including F. Klein and D.E. Smith) considered that history of mathematics was important to mathematics education. Yet can the history of mathe-

matics be readily incorporated into the school curriculum?

Key points raised during the presentation & discussion: The personalities and activities connected with ICMI have, in the past, shown a deeply rooted interest in history of mathematics. But what practical problems are posed when the history of mathematics is incorporated into mathematics education, and are these practical problems, in fact, entwined with more deeply seated theoretical matters? How can curriculum requirements be accommodated? To what extent does history become a tool rather than a subject that is studied? How can mathematics education be reconceptualized to allow the history of mathematics to become an integral part of mathematics studies? And what is lost if this is not done?

Authors: Fulvia Furinghetti (Università di Genova, Italy).

Title: Mathematics education in the ICMI perspective.

Brief summary based on the written paper: The development of mathematics education as a discipline is traced. Lively descriptions are given of cooperation, hostilities, and frictions between the communities of mathematicians and mathematics educators.

Key points raised during the presentation & discussion: The aims of 'communication', 'solidarity', and 'internationalization' underpinned the establishment in 1899 of the Franco-Swiss journal *L'Enseignement Mathématique* which in its turn contributed to the eventual creation of ICMI. The current more equal relationship between ICM and ICMI is in sharp contrast with past traditions. Though part of the International Congresses of Mathematicians, ICMI was for many years 'the poor relative': the 'pedagogy of mathematics' was not considered worthy of a plenary session, ICMI presentations were invariably relegated to one of the final sessions of the congress, and – for the first 100 years – the ICMI president was a male mathematician and not a mathematics educator. Only through a partial rupture with the community of mathematicians could the mathematics education community construct its own identity as an academic discipline.

Authors: Solomon Garfunkel (COMAP, Inc., U.S.A.).

Title: The Faffufni-Chaim Yankel effect: A cautionary tale.

Brief summary based on the written paper: Reflections on «the patterns of how projects and programs are evaluated». Implicitly: how can we move beyond 'accepted' recipes for the conduct of research? Or why good ideas are lost ...

Key points raised during the presentation & discussion: What have we (in the USA) gained and lost by moving from earlier funding approaches to the current mechanisms for funding projects? How can we ensure that an appropriate balance is retained between quality based and popularity based approaches to project filtering? Are we now so weighed down by

bureaucratic processes that innovative, formula-challenging projects are doomed not to be supported? Should ICMI become involved in championing certain projects?

Authors: Gabriele Kaiser (University of Hamburg, Germany).

Title: Meaning in mathematics education. Reflections from various perspectives.

Brief summary based on the written paper: Meaning in mathematics education is influenced by diverse factors. Experimental evidence is provided that modelling examples can make mathematical learning more meaningful and that comparisons with students from different cultural backgrounds might enhance our understanding of the meaning students develop in mathematics education.

Key points raised during the presentation & discussion: What is meant by 'meaning' in mathematics education? Will modelling examples increase the likelihood that students will find mathematics learning more meaningful? Should we distinguish between personal and objective meaning? Two specific teaching 'experiments' set in two different cultural contexts are used to explore these and related issues.

Authors: Stephen Lerman (London South Bank University, UK).

Title: Theories as intellectual resources in mathematics education research.

Brief summary based on the written paper: A rich overview is provided of different theoretical perspectives relevant to mathematics education research, their evolution, potential, and limitations.

Key points raised during the presentation & discussion: Mathematics educators often draw on a range of disciplines – frequently psychology, anthropology, and sociology – as they grapple to understand better issues of teaching and learning mathematics. How best to make sense of multiple theories was explored and the danger of mindless application of theory to educational research stressed. The concluding section of the written paper captures some of the thrust of the presentation: «I am not surprised by the multiplicity of theories in our field and the debates about their relative merits, nor do I see it as a hindrance. ...I am particularly troubled by the attacks on educational research as an inadequate shadow of a fetishised image of scientific, psychological or medical research».

Authors: Po-Hung Liu (National Chin-Yi University of Technology, Taiwan).

Title: A 4-dimensional analysis of the practice in mathematics education.

Brief summary based on the written paper: Practice in mathematics education can conveniently be considered in terms of four components: global vision, local focus, mathematical knowledge, and time span. Such

an analysis highlights not only tensions and contradictions but also opportunities and strategies for further developments.

Key points raised during the presentation & discussion: Mathematics education issues, it was forcefully argued, are not only pedagogical, but also epistemological, social, and even political. ICMI should respond to diverse calls and needs among all countries. Rather than merely look for solutions to problems, ICMI should provide a structure for facilitating interactions and exchange of views. A four-dimensional view, described in some detail in the written paper, for achieving this was discussed. Meanwhile: whose voices are given prominence in research journals and at conferences and whose voices are ignored? Whose problems are considered worthy of research and whose situations are seen as unproblematic? Might research in mathematics education *damage* rather than facilitate mathematics teaching/learning under certain circumstances and in certain settings?

Authors: Immaculate Namukasa (The University of Western Ontario, Canada). The author was unable to attend the Symposium.

Title: The contribution of multiple disciplines of influence to mathematics education: A complexity science interpretation.

Brief summary based on the written paper: Different paradigms for conceptualizing mathematical thinking are described: Individual and content psychology, cognitive and information processing, co-emergent and context paradigm; coherence and post-structuralism, and ecological and systems paradigm. Drawing from complexity theory the author demonstrates how considering varied perspectives in light of each other might bring forth novel perspectives on mathematical thinking.

Authors: Marie-Jeanne Perrin-Glorian (DIDIREM, Université Paris Diderot et IUFM Nord-Pas-de-Calais, France).

Title: From producing optimal teaching to analysing usual classroom situations... The notion of milieu.

Brief summary based on the written paper: The theory of didactic situations, its evolution, and the integral component of milieu are described in some detail. Strong reference is made to Brousseau's work.

Key points raised during the presentation & discussion: Various aspects of the theory of didactic situations were discussed: the theory's historical development, the notion of *milieu* and its constituent elements; the contribution of the theory to analysing regular lessons. The theory's assumptions, viability, and broad applicability generated considerable audience debate.

Authors: Norma Presmeg (Illinois State University, USA).

Title: Semiotic theoretical frameworks: creativity and imaginative rationality in mathematics education.

Brief summary based on the written paper: Theories from many different disciplines are germane to mathematics education research. Considerable emphasis was placed on the work of Charles Sanders Peirce. Various rich examples are provided.

Key points raised during the presentation & discussion: The strong focus on logical thinking in mathematics education should not exclude recognition of the creative elements that can enrich mathematics learning. Similarities and differences between deduction, induction, and abduction, and their relevance to problem solving in mathematics, were considered at some length. The different representations of mathematical objects and processes embodied in the Peircean triad of iconic, indexical, and symbolic signs were also highlighted.

Authors: Wolfgang Schläglmann (Johannes Kepler Universität, Austria).

Title: Is cognitive neuroscience relevant to mathematics education research?

Brief summary based on the written paper: Learning is now regarded as an increasingly complex process, involving cognitive and social processes, affect, emotions and motivation. Concepts from many fields are invoked in our attempts to explore and understand how it occurs. An argument is made to focus as well on neuroscience and areas that may benefit from neuroscience based research are listed.

Key points raised during the presentation & discussion: Recent developments and findings in cognitive neuroscience appear to be opening interesting new research pathways for increasing our understanding of factors facilitating or inhibiting mathematics learning. Various examples, discussed in the written paper, were highlighted.

Authors: Nathalie Sinclair (Simon Fraser University, Canada).

Title: Notes on the aesthetic dimension of mathematics education.

Brief summary based on the written paper: Aesthetics, it is argued, contributes to 'understanding the rationality of mathematics itself, and to enriching existing theories in mathematics education'. Various examples are given and issues worthy of further investigation are put forward.

Key points raised during the presentation & discussion: Different disciplines raise different dimensions and questions with respect to aesthetic values in mathematics and mathematics education. For example, should aesthetic considerations in mathematics and school mathematics be the same or different? What can mathematics educators learn from mathematicians? Is aesthetic sensibility linked to mathematical motivation and curiosity? How do theories of embodied cognition relate to aesthetic perception?

Authors: Heinz Steinbring (Universität Duisburg-Essen, Germany).

Title: Mathematical knowledge as a social construct of teaching / learning processes – the epistemology oriented mathematical interaction research.

Brief summary based on the written paper: Three broad areas are covered in this paper with its focus on constructing meaning for mathematical signs: mathematics teaching as an autonomous culture; epistemological constraints of mathematical signs in the culture of teaching; and the interactive constructions of mathematical knowledge – social and epistemological conditions.

Key points raised during the presentation & discussion: The theoretical bases and practical implications of Epistemology Oriented Mathematical Interaction Research were outlined in more detail and illustrative examples were provided. The specific rules of the social behavior of the participants is investigated through an interaction analysis of the classroom culture. How the mathematical signs of concepts and of operations are used and interpreted is examined via an epistemological analysis of the classroom culture.

3. *Synthesis and discussion*

The contributions, it can be seen, were diverse and often provocative. A number of broad themes emerged.

3.1 *The evolution of theoretical frameworks in mathematics education*

Historical identification of changes:

Formalism and logicism; Modern mathematics; Constructivism(s); Sociocultural approaches.

Two contemporary trends were highlighted:

Mathematics Education modeled after the natural sciences;

Mathematics Education modeled after the social sciences.

3.2 *Interdisciplinary perspectives*

Semiotics, neuroscience, critical theory in art and philosophy, embodied cognition, epistemology, history, psychology were all areas on which mathematics education research has drawn.

3.3 *ICMI Future contributions*

ICMI, it was also argued, could profitably shape the research landscape of the future in a number of ways, and in particular:

By fostering a dialogue between the mathematics education commu-

nity and other groups (policy makers, government...) to ensure that sensible and practical criteria be applied to prioritizing and funding research;

To maintain, and if necessary reinforce, a challenge to the role of dominant (male) Western paradigms as drivers for our research and interpretation of international results.

3.4 Some issues

A number of issues attracted particularly lively debates. These included:

The role of creativity and aesthetics in mathematics and the different theoretical perspectives within which this might be explored;

Whether educational research might harm students, e.g., by frequent advocacy of new, and not necessarily well tried, teaching approaches rather than offering stability of learning environments.

Perhaps the most vigorous discussions involved trying to reconcile different theoretical perspectives. Attempts to do so were not always successful!.

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ST:Short Talk; PL: Plenary; WG: Working Group;

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